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A categorial approach to relativistic locality☆

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Abstract

Relativistic locality is interpreted in this paper as a web of conditions expressing the compatibility of a physical theory with the underlying causal structure of spacetime. Four components of this web are distinguished: spatiotemporal locality, along with three distinct notions of causal locality, dubbed CL-Independence, CL-Dependence, and CL-Dynamic. These four conditions can be regimented using concepts from the categorical approach to quantum field theory initiated by Brunetti, Fredenhagen, and Verch [1]. A covariant functor representing a general quantum field theory is defined to be causally local if it satisfies the three CL conditions. Any such theory is viewed as fully compliant with relativistic locality. We survey current results indicating the extent to which an algebraic quantum field theory satisfying the Haag-Kastler axioms is causally local.

Keywords: Quantum field theory, category theory, operator algebra theory, causality

1. The main claims

The question of whether quantum theory is compatible with relativity theory is a central issue in philosophy and foundations of physics. The compatibility in question would manifest in quantum theory satisfying "relativistic locality" conditions that express harmony of quantum theory with the conceptual picture of the physical world according to theory of relativity. Whether such a harmony

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is possible depends on both what quantum theory is taken to be and on how the relativistic locality conditions are specified.

The debate on whether standard, non-relativistic quantum mechanics of finite degrees of freedom satisfies locality conditions motivated by the theory of relativity goes back to the early days of quantum mechanics: referring to "locality" played a crucial role in the EPR argument, and "locality" was also central in Bell's analysis of the problem of "local" hidden variables. These early debates resulted in "no-go" theorems, typical interpretation of which was that "... some sort of action-at-a-distance or (conceptually distinct) nonseparability seems built into any reasonable attempt to understand the quantum view of reality." [2][p. 169] (Redhead's book [2] is a classic reference providing a detailed analysis of the different "locality" conditions formulated in connection with standard quantum mechanics and the related "no-go" results, including the EPR argument and Bell's work.)

With the emergence of (relativistic) quantum field theory, where the physical systems to describe have infinite degrees of freedom, the question of the relation of quantum theory to "relativistic locality" got sharpened for two reasons: First, because developing relativistic quantum field theory was in part motivated by the intention to create a quantum theory that is "relativistically local" by its very construction. Second, as Howard argued by reconstructing the gradual changes in Einstein's views about quantum mechanics and "relativistic locality" in the years 1935-1949 [3], [4], Einstein's worry about ordinary quantum mechanics of finite degrees of freedom was not so much about completeness of the theory; rather, the worry was about the theory not being field theoretical: Einstein thought that quantum mechanics did not fit into a field theoretical paradigm (see [5] and [6] for further details of the analysis of Einstein's views from this perspective). Thus it is natural to ask if (relativistic) quantum *field* theory itself is "relativistically local"? Of course, the answer to this question depends sensitively on how relativistic locality is specified.

One can take the position (see for instance [7]) that absence/presence of entanglement across spacelike distances according to a theory is the single most crucial feature that is decisive from the point of view compatibility of a theory with relativistic locality. Adopting this position leads one to the conclusion that, ironically, relativistic quantum field theory is even *less* compatible with relativistic locality than is non-relativistic quantum mechanics of finite degrees of freedom, because entanglement is even *more* prevalent in quantum field theory than in non-relativistic quantum theory (see the papers [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18] for the issue of entanglement in quantum field theory). Another position can be that "... all the special theory of relativity (STR) can be taken to demand of a theory set in Minkowski sapcetime is that it exhibit Lorentz covariance." [19][p. 109] – this, she claims, already establishes peaceful coexistence of quantum field theory with STR [19][p. 109]. Earman and Valente [20][p. 2] regard the local primitive causality condition (time slice axiom) the one on which relativistic locality of quantum field theory crucially rests.

The present paper takes a somewhat different position, one that is probably closer to the intuition of a theoretical physicist: The first claim in this paper is that relativistic locality is *not a single property* a physical theory can in principle have but an intricately interconnected web of features. Each of those individual features express some important aspect of relativistic locality, and a physical theory can in principle have some of these features but not others. A physical theory is in full compliance with relativistic locality if it possesses *all* the features in this web however – in this case the physical theory is fully compatible with the causal structure of the underlying spacetime. Section 2 describes informally the web of relativistic locality concepts, pointing out some of its general features. These features are more or less straightforward but it is important to be aware of them when it comes to the technically explicit specification of the relativistic locality concepts and to the question of whether one can have a relativistically local quantum theory.

The second claim of this paper is that category theory provides a general, useful and flexible framework in which the web of relativistic locality concepts can be formulated in a technically clean manner. The advantage of using category theory to discuss relativistic locality is at least two-fold: First, category theory provides a unified language to help organize the multitude of locality concepts that occur in the vast literature on "locality". The locality concepts can differ robustly, expressing conceptually very varied contents; and they also can differ subtly, expressing nuanced mathematical variations of a particular type of locality concept. Category theory helps to bring order in the occasionally confusing maze of locality concepts. Second, category theory helps to formulate quantum field theory on curved spacetimes. This is a non-trivial task because some of the crucial components of quantum field theory on a flat spacetime loose their meaning in a quantum field theory over curved spacetime (Pincaré covariance is one example). Section 3 describes the main elements of the categorial approach to relativistic quantum field theory initiated by Brunetti, Fredenhagen and Verch [1]. In sections 4 and 5 it will be seen that one can naturally formulate additional, physically motivated locality conditions in this categorial framework, and one can raise then the problem of whether those additional locality conditions are satisfied by relativistic quantum field theory (section 6).

The third claim of this paper is that, although there remain some open problems about the status in quantum field theory of some of the categorial relativistic locality conditions, the available evidence (presented in section 7) suggests that relativistic quantum field theory behaves well from the perspective of relativistic locality. This confirms von Neumann's view about the relation of quantum theory and the theory of relativity:

> "And of course quantum electrodynamics proves that quantum mechanics and the special theory of relativity are compatible "philosophically" – quantum electrodynamic fails only because of the concrete form of Maxwell's equations in the vicinity of a charge."
> (von Neumann to Schrödinger, April 11, 1936), [21][p. 213]

von Neumann suggests here that, once one has been able to handle mathematically the singularity arising from the (physically unrealistic) assumption of *point-like* localizability of electrodynamic fields and charges, there should not be any other conceptual obstacle in the way of creating a (non-pointlike) localized and causally well-behaving quantum field theory.

2. The relativistic locality conditions informally

Informally put, relativistic locality conditions express harmony of a physical theory with the conceptual picture of the physical world according to the theory of relativity. The harmony, or rather: compatibility, has two major components: *Spatio-Temporal Locality* and *Causal Locality*. The Causal Locality condition consists of three elements: Causal Locality – *Independence*, Causal Locality – *Dependence* and Causal Locality – *Dynamic*. These conditions are described in this section informally.

- **Spatio-Temporal Locality**: This condition requires that physical systems are regarded as localized *explicitly* in spacetime regions.
- **Causal Locality**: This condition requires that the observational-operational properties of the physical systems localized in spacetime regions are in harmony with the causal relations between the spacetime regions:
  - A spacetime has a causal structure that specifies causally *independent* and causally *dependent* spacetime regions.
  - **Causal Locality – Independence**: This condition requires that physical systems localized in *causally independent* spacetime regions are *independent*.
  - **Causal Locality – Dependence**: Any correlation between physical systems localized in *causally independent* spacetime regions is explainable in terms of matters of fact localized in the *common causal past* of the causally *independent* regions to which the correlated systems belong.
  - **Causal Locality – Dynamic**: The dynamical evolution of a system localized in a region determines the system in the region's causal closure.

A semi-formal specification of the above conditions is the following. Let I be a set of regions of a spacetime M, which is assumed to be equipped with two relations

$\times_M$	and	$\prec_M$
symmetric		transitive

where

- $V_1 \times_M V_2$ expresses the causal *independence* of regions $V_1, V_2 \subset M$

- $V_1 \prec_M V_2$ expressing: V_1 is in the *causal past* of V_2

Using this notation, the relativistic locality conditions can be formulated more explicitly as follows.

- **Spatio-temporal Locality**:
  - Each physical system $\mathcal{S}$ is labeled by V from a set $(I, \times_M, \prec_M)$ of spacetime regions indicating where the system S is located: $\mathcal{S}(V)$.
  - The labeling is consistent in the sense that $\mathcal{S}(V_1)$ is a subsystem of $\mathcal{S}(V_2)$ if $V_1 \subseteq V_2$.
- **Causal Locality**
  - **Independence**:
    $\mathcal{S}(V_1),\ \mathcal{S}(V_2)$ are independent whenever $V_1 \times_M V_2$.
  - **Dependence**:
    If $\mathcal{S}(V_1)$ and $\mathcal{S}(V_2)$ are correlated and $V_1 \times_M V_2$ holds then the correlation between $\mathcal{S}(V_1)$ and $\mathcal{S}(V_2)$ is explainable in terms of matters of fact in local system $\mathcal{S}(V)$ with

$$V \prec_M V_1 \qquad \text{and} \qquad V \prec_M V_2 \tag{1}$$

  - **Dynamic**:
    Observables of system $S(\overline{V})$ in the causal closure $\overline{V}$ of region V are determined by observables of system $S(V)$.

In what follows, **Causal Locality – Independence**, **Causal Locality – Dependence** and **Causal Locality – Dynamic** will be referred to as CL-Independence, CL-Dependence and CL-Dynamic, respectively.

The following features of the above semi-formal definition of relativistic locality are worth pointing out:

1. The specific features of the background spacetime are left open. This is useful on this level of generality because locality should make sense in any spacetime, locality conditions should not be spacetime-specific.
2. Relativistic covariance of the theory is *not* assumed; under the present interpretation of relativistic locality, relativistic covariance is therefore not part of the notion of relativistic locality. This is advantageous because a general spacetime might *not* have any non-trivial symmetry with resepct to which covariance of the theory could be required, but locality should be meaningful with respect to any spacetime.
3. It is left open in what sense the systems localized in causally independent regions are supposed to be *independent*. This ambiguity is deliberate at this point because, as will be seen later, independence is not a uniquely fixed notion.

4. It also is left open in the description of relativistic locality what kind of *correlation* there could exist between systems localized in causally independent spacetime regions. This is again deliberate because, as we will see, there exist different types of correlations between distant physical systems.
5. It is not specified what it means to give an explanation of correlations between physical systems. This lack of specificity is explained by the fact that the notion of explanation also is well known to be not unique – one can explain the same phenomenon in many different ways.
6. The Spatio-Temporal Locality and the Causal Locality conditions are *not* independent: Spatio-Temporal Locality is a conceptual presupposition for Causal Locality: It should be clear that without Spatio-Temporal Locality the Causal Locality conditions cannot be formulated at all. Note that Spatio-Temporal Locality is first and foremost (non-pointlike) locality of *observables*; as a consequence, *states* are also local to the extent they are defined on the local observables.
7. The CL-*Independence* and CL-*Dependence* conditions are logically *independent*: CL-Independence does not entail CL-Dependence, nor conversely.
8. CL-Dependence presupposes that independence is not the same as absence of correlation. This is indeed the case: We will see that independence understood as co-possibility is co-possible with correlation.
9. The CL-Independence and CL-Dependence conditions are conceptually *independent* of CL-Dynamics.

3. Categorial quantum field theory

There are several approaches to quantum field theory. The approaches can be grouped into two broad classes: heuristic and axiomatic. In heuristic approaches mathematical precision is compromised *in a disciplined manner* in favor of descriptive and predictive usefulness of the theory; in axiomatic approaches mathematical rigor is maintained at the expense of descriptive breadth and predictive strength. Thus heuristic and axiomatic quantum field theories complement each other in a natural way, they should be seen not as competitors but as closely related attempts to understand nature. These two approaches have been developing in harmony, mutually influencing each other in a constructive manner. Summers' paper [22] gives a review of the status of axiomatic quantum field theories (also called "constructive field theories" because in axiomatic approaches the emphasis is not so much on the axioms themselves but on constructing physically relevant models of the axioms). For some features of axiomatic quantum field theory in a historical perspective, see [23] and the references therein.

A particular approach to quantum field theory in the tradition of mathematical physics is categorial quantum field theory. This approach goes back to the works of Segal [24] an Atiyah [25] in the eighties[1]. There are two discernible trends in categorial approaches: Topological quantum field theory and the categorial generalization of the algebraic quantum field theory; the (simplifying) slogan[2] is that the former is the categorial (re)formulation of field theory in the Schrödinger picture, whereas the latter is a categorial formulation of the Heisenberg picture. Topological quantum field theory is motivated by the difficulties of the path integral formalism, and it circumvents path integrals by specifying functors embodying crucial features of time evolutions. The work by Bartlett [26] gives an overview of the main ideas of topological quantum field theory, including some of its history; Baez [27] provides a compact introduction. The categorial approach generalizing the algebraic axiomatization [28], [29], [30] was initiated by Brunetti, Fredenhagen and Verch [1]. The main motivation for the Brunetti-Fredenhagen-Verch approach stems from the fact that we want to be able to develop quantum field theory in a general (curved) spacetime. We therefore need a formalism that is flexible enough to accommodate any (physically reasonable) background spacetime. In addition, "standard" relativistic quantum field theory relies on certain axioms (e.g. covariance, spectrum condition, existence of vacuum state) which are framed in terms of a preferred representation of the Poincaré group. In a typical curved spacetime, however, there are no non-trivial global symmetries; hence none of the standard axioms that rely on the existence of a global symmetry make sense in a general curved spacetime. Thus we need a way to reformulate these axioms in a generally covariant fashion. Brunetti, Fredenhagen and Verch formulate these motivations thus:

> Quantum field theory incorporates two main principles into quantum physics, locality and covariance. Locality expresses the idea that quantum processes can be localized in space and time (and, at the level observable quantities, that causally separated processes are exempt from any uncertainty relations restricting their commensurability). The principle of covariance within *special* relativity states that there are no preferred Lorentzian coordinates for the description of physical processees, and thereby the concept of an absolute space as an arena for physical phenomena is abandoned. Yet it is meaningful to speak of events in terms of spacetime points as entities of a given, fixed spacetime background, in the setting of special relativistic physics.
>
> In general relativity, however, spacetime points loose this a priori meaning. The principle of general covariance forces one to regard spacetime points simultaneously as members of several, locally diffeomorphic spacetimes. It is rather the relations between distinguished events that have physical interpretation.

[1] Segal's paper [24] was originally written in 1989 but remained in manuscript form until it got published in 2004, see Segal's note in [24].

[2] See the formulation at *nLab*: http://ncatlab.org/nlab/show/FQFT.

> This principle should also be observed when quantum field theory in presence of gravitational fields is discussed.
>
> Quantum field theory ... is a *covariant functor* ... in the ... fundamental and physical sense of implementing the principles of locality and general covariance... [1][p. 61-78]

The covariant functor Brunetti, Fredenhagen and Verch refer to in the above quotation is between two categories:

- $(\mathfrak{Man}, hom_{\mathfrak{Man}})$
  The category of spacetimes with isometric embeddings of spacetimes as morphisms.

- $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$
  The category of C^*-algebras with injective C^*-algebra homomorphisms as morphisms.

Before listing the properties of these two categories and defining precisely the functor representing quantum field theory (Definition 1), it is useful to describe informally its main features. The functor $\mathcal{F}$ assigns to any spacetime manifold M an operator algebra $\mathcal{F}(M)$, selfadjoint elements of which are interpreted as representing the set of observables measurable in M. This explicit association of the observables with a specific spacetime embodies a basic aspect of locality: the idea that any measurement, observation, and interaction can only take place at a particular location in spacetime – this was called in section 2 Spatio-Temporal Locality. This interpretation of the assignment $M \to \mathcal{F}(M)$ makes it very natural to stipulate a number of properties for the functor $\mathcal{F}$; the features express general covariance and the causal locality conditions discussed in section 2. General covariance is expressed by the requirement that the functor be covariant: If a spacetime M is embedded into spacetime M' via a map g, and M' also is embedded into spacetime M'' via embedding g', with g and g' both preserving the spacetime structures, then M is embedded into M'' via the composition $g' \circ g$. The functor $\mathcal{F}$ should then yield algebra embeddings $\mathcal{F}(g)$ and $\mathcal{F}(g')$ that embed the corresponding operator algebras: $\mathcal{F}(M)$ into $\mathcal{F}(M')$ and $\mathcal{F}(M')$ into $\mathcal{F}(M'')$ via embeddings $\mathcal{F}(g)$ and $\mathcal{F}(g')$ that preserve the structure of the algebra of observables; furthermore, these assignments of algebra embeddings to spacetime embeddings must be such that $\mathcal{F}(g') \circ \mathcal{F}(g)$, which embeds $\mathcal{F}(M)$ into $\mathcal{F}(M'')$ is equal to $\mathcal{F}(g' \circ g)$. Note that this requirement of general covariance does not assume any non-trivial symmetry of the embedded spacetimes and is meaningful for any spacetime having some features that once can minimally expect a spacetime to possess on physical grounds. The covariance ensures that isometric, physically equivalent spacetimes have the same set of observables associated with them; individual points in particular spacetimes loose their physical meaning, only the relation of such points matters. A further, crucial feature of the functor $\mathcal{F}$ is a causal locality condition: Operator algebras $\mathcal{F}(M_1)$ and $\mathcal{F}(M_2)$ are demanded to commute within the algebra $\mathcal{F}(M)$ if the spacetimes M_1 and M_2 are spacelike related when considered as embedded into

M. This feature, called Einstein Locality, is an expression of independence and is the categorial formulation of the well-known local commutativity (also called microcausality) requirement. Further causal locality conditions can be (and will be) formulated for the functor in sections 4 and 5.

It should be clear now how the notion of quantum field theory as a covariant functor captures the crucial components of what could be called a "field theoretical paradigm", which was informally articulated by Einstein in his critique of standard, non-relativistic quantum mechanics of finite degrees of freedom (see [5] and [6] for a more detailed discussion of this historical aspect from the perspective of the less, general, non-categorially formulated algebraic approach to quantum field theory): Physical systems represented by the observables that one can measure on them are always considered as "located somewhere" in spacetime, and their association with particular spacetime regions is in harmony with the causal structure of spacetimes in the spirit of the theory of (general) relativity. In short: Categorial quantum field theory is a mathematically precise general specification of the field theoretical paradigm, no matter whether the spacetimes are flat or not.

We turn now to the technically more explicit specification of the covariant functor representing quantum field theory.

The most important features of the category $(\mathfrak{Man}, hom_{\mathfrak{Man}})$ are the following (see [1] for more details):

- The objects in $Obj(\mathfrak{Man})$ are 4 dimensional C^∞ spacetimes (M, g) with a Lorentzian metric g and such that (M, g) is Hausdorff, connected, time oriented and globally hyperbolic.

- The morphisms in $hom_{\mathfrak{Man}}$:

$$\psi \colon (M_1, g_1) \to (M_2, g_2)$$

  are isometric smooth embeddings such that

  - ψ preserves the time orientation;
  - if the endpoints $\gamma(a), \gamma(b)$ of a timelike curve $\gamma \colon [a, b] \to M_2$ are in the image $\psi(M_1)$, then the whole curve is in the image: $\gamma(t) \in \psi(M_1)$ for all $t \in [a, b]$.
  - The composition of morphisms is the usual composition of maps.

The category $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$:

- The objects in $Obj(\mathfrak{Alg})$ are unital C^*-algebras.

- The morphisms are injective, unit preserving C^*-algebra homomorphisms

$$\alpha \colon \mathcal{A}_1 \to \mathcal{A}_2$$

  The composition of morphisms is the usual composition of C^*-algebra homomorphisms.

Definition 1. A locally covariant quantum field theory is a covariant functor $\mathcal{F}$ between the categories $(\mathfrak{Man}, hom_{\mathfrak{Man}})$ and $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ (pictured in the diagram below) which satisfies the Einstein Causality and Time Slice features specified after the diagram.

$$\begin{array}{ccc} (M,g) & \xrightarrow{\quad\psi\quad} & (M',g') \\ \mathcal{F}\Big\downarrow & & \Big\downarrow\mathcal{F} \\ \mathcal{F}(M,g) & \xrightarrow{\ \mathcal{F}(\psi)\ } & \mathcal{F}(M',g') \end{array}$$

$$\begin{aligned} \mathcal{F}(\psi_1 \circ \psi_2) &= \mathcal{F}(\psi_1) \circ \mathcal{F}(\psi_2) \\ \mathcal{F}(id_{\mathfrak{Man}}) &= id_{\mathfrak{Alg}} \end{aligned}$$

- Einstein Causality:
  The functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ is called *Einstein Causal* if
  $$\Big[\mathcal{F}(\psi_1)\Big(\mathcal{F}(M_1,g_1)\Big), \mathcal{F}(\psi_2)\Big(\mathcal{F}(M_2,g_2)\Big)\Big]_-^{\mathcal{F}(M,g)} = \{0\} \qquad (2)$$
  whenever
  $$\begin{aligned} \psi_1 &: (M_1,g_1) \to (M,g) \\ \psi_2 &: (M_2,g_2) \to (M,g) \end{aligned}$$
  and $\psi_1(M_1)$ and $\psi_2(M_2)$ are spacelike in M, where $[\ ,\ ]_-^{\mathcal{F}(M,g)}$ in (2) denotes the commutator in the C^*-algebra $\mathcal{F}(M,g)$.

- Time Slice axiom:
  If (M,g) and (M',g') and the embedding
  $$\psi\colon (M,g) \to (M',g')$$
  are such that $\psi(M,g)$ contains a Cauchy surface for (M',g') then
  $$\mathcal{F}(\psi)\mathcal{F}(M,g) = \mathcal{F}(M',g')$$

The definitions of Einstein Causality and Time Slice axiom above are the categorial versions of the familiar definitions of Einstein causality and the local version of the time slice axioms in algebraic quantum field theory ([29][p. 57-58; 110-111]). The categorial formulations of these postulates differ from their standard versions only in that in the categorial approach no background spacetime is fixed; hence these two concepts have to be made relative to the objects in $Obj(\mathfrak{Man})$ and their embeddings via morphisms in $hom_{\mathfrak{Man}}$. But the physical motivation for them is the same as for their more familiar formulations in algebraic quantum field theory: The Einstein Causality postulate ensures that

"Two observables with spacelike separated regions are compatible. The measurement of one does not disturb the measurement of the other." [29][p. 107] Time Slice axiom "... stipulates that there is a dynamical law respecting the causal structure. It corresponds to the hyperbolic propagation character of the fields..." [29][p. 111] One can indeed show that (under certain assumptions) the categorial Time Slice axiom entails the existence of a well-behaving dynamics [1][section 4.]. This makes it possible to interpret the Time Slice axiom as ensuring that the condition we called CL-Dynamic holds in categorial quantum field theory.

We are now in the position to formulate, in terms of categorial quantum field theory, the relativistic locality conditions described informally in section 2. The first concept in that list is Spatio-temporal locality. It is clear that this Spatio-temporal locality condition holds in categorial quantum field theory since it is expressed by the stipulation that quantum field theory is a functor from the category of manifolds to the category of C^*-algebras: By setting up a connection between spacetime manifolds and C^*-algebras, the functor $\mathcal{F}$ specifies explicitly which manifold (in particular which spacetime region (M,g)) a particular set of observables represented by the C^*-algebra $\mathcal{F}(M,g)$ belongs to. Next, the Relativistic Locality condition CL-Independence has to be specified. This is done in the next section. To simplify notation, in what follows, g is dropped from $\mathcal{F}(M,g)$ and $\mathcal{F}(M)$ denotes the C^*-algebra associated with spacetime (M,g) by the functor $\mathcal{F}$.

4. Causal Locality – Independence in terms of categorial concepts

The notion of independence is a very rich one: there exists a great variety of (logically non-equivalent) concepts of independence. Typically, in the context of quantum physics, one encounters the problem of specifying independence of subsystems S_1, S_2 of a larger physical system S. The core idea of subsystem independence is that anything that is possible for subsystems S_1 and S_2 considered in their own right, are co-possible from the perspective of the large system S. For instance, if ϕ_1 and ϕ_2 are possible states of systems S_1 and S_2, respectively, then the two states are jointly realizable as a single state of system S. This kind of independence is known as C^*-independence, or W^*-independence, depending on whether the states are required to be normal or not (cf. [14]). Another type of independence expresses the mutual compatibility of operations carried out on systems S_1 and S_2; this kind of independence is called operational C^*-or W^*-independence (see [31], [32] and section 7 for more details). Category theory helps to give a unified formulation of all these types of independence: realizing that one can regard both states and more general operations as morphisms between algebras, one can re-state the standard independence concepts in the form of categorial independence as morphism co-possibility. In this section this notion is defined precisely.

Let $Mor_{\mathfrak{Alg}}$ be some class of morphisms in the class of C^*-algebras (possibly different from $Hom_{\mathfrak{Alg}}$). Informally, the idea of $Mor_{\mathfrak{Alg}}$-independence as co-possibility is that, given *any* two morphisms $T_1, T_2 \in Mor_{\mathfrak{Alg}}$ on objects (C^*-

algebras) $\mathcal{F}(M_1)$ and $\mathcal{F}(M_2)$ that are embedded into object (C^*-algebra) $\mathcal{F}(M)$, there is a morphism $T \in Mor_{\mathfrak{Alg}}$ on $\mathcal{F}(M)$ that extends *both* T_1 *and* T_2. To make the notion of $Mor_{\mathfrak{Alg}}$-independence technically explicit, we need to define the extension of morphisms:

Definition 2 (ψ-extension of $Mor_{\mathfrak{Alg}}$ morphisms). Given

$$\begin{array}{rcl} \psi & : & (M,g) \to (M',g') \\ T & \in & Mor_{\mathfrak{Alg}}(\mathcal{F}(M),\mathcal{F}(M)) \\ T' & \in & Mor_{\mathfrak{Alg}}(\mathcal{F}(M'),\mathcal{F}(M')) \end{array}$$

The morphism T' is called a ψ-extension of T if the following diagram is commutative:

$$\begin{array}{ccc} \mathcal{F}(M) & \xrightarrow{\mathcal{F}(\psi)} & \mathcal{F}(M') \\ T \big\downarrow & & \big\downarrow T' \\ \mathcal{F}(M) & \xrightarrow{\mathcal{F}(\psi)} & \mathcal{F}(M') \end{array}$$

Remark 1. Note that morphisms need not be extendable in the following sense: Given

$$\begin{array}{rcl} \psi & : & (M,g) \to (M',g') \\ T & \in & Mor_{\mathfrak{Alg}}(\mathcal{F}(M),\mathcal{F}(M)) \end{array}$$

we have

$$\begin{array}{ccc} \mathcal{F}(M) & \xrightarrow{\mathcal{F}(\psi)} & \mathcal{F}(\psi)\mathcal{F}(M) \\ T \big\downarrow & & \big\downarrow T'_0 \\ \mathcal{F}(M) & \xrightarrow{\mathcal{F}(\psi)} & \mathcal{F}(\psi)\mathcal{F}(M) \end{array}$$

with

$$\mathcal{F}(\psi)\mathcal{F}(M) \ni \mathcal{F}(\psi)X \mapsto T'_0(\mathcal{F}(\psi)(X)) \doteq \mathcal{F}(\psi)T(X)$$

but a ψ-extension T' of T, i.e. an extension of T'_0 from $\mathcal{F}(\psi)\mathcal{F}(M)$ to a morphism on $\mathcal{F}(M')$ may *not* exist. This is the case, for instance, if one takes the operations (completely positive, unit preserving linear maps, see section 7) as morphisms: Operations defined on sub-C^*-algebras of C^*-algebras need not be extendable from the subalgebra to the superalgebra [33], and this complicates assessment of the status of operational independence in quantum field theory (see [34], [6], [32], [35] for further discussion of this point.)

In view of the above Remark, the following definition is not redundant:

Definition 3. The class of morphisms $Mor_{\mathfrak{Alg}}$ is said to have the unrestricted extendability feature if for any C^*-algebras $\mathcal{A}_0, \mathcal{A} \in \mathfrak{Alg}$, where $\mathcal{A}_0$ is a C^*-subalgebra of $\mathcal{A}$, if T_0 is a morphism on $\mathcal{A}_0$, then there exists a morphism $T \in Mor_{\mathfrak{Alg}}$ on $\mathcal{A}$ that extends T_0.

Definition 4. The functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ is said to satisfy the $Mor_{\mathfrak{Alg}}$-Causal Independence condition, if whenever

$$\begin{aligned} \psi_1 &: (M_1, g_1) \to (M, g) \\ \psi_2 &: (M_2, g_2) \to (M, g) \end{aligned}$$

and $\psi_1(M_1)$ and $\psi_2(M_2)$ are spacelike in M, then

$$\text{for any } T_1 \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M_1), \mathcal{F}(M_1)) \text{ and any } T_2 \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M_2), \mathcal{F}(M_2)) \tag{3}$$

there is a

$$T \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M), \mathcal{F}(M)) \tag{4}$$

which is a ψ_1-extension of T_1 and a ψ_2-extension of T_2.

Another categorial independence condition closely related to $Mor_{\mathfrak{Alg}}$-Causal Independence is $Mor_{\mathfrak{Alg}}$-Causal *Separability*:

Definition 5. The functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ is said to satisfy the $Mor_{\mathfrak{Alg}}$-Causal Separability, if whenever

$$\begin{aligned} \psi_1 &: (M_1, g_1) \to (M, g) \\ \psi_2 &: (M_2, g_2) \to (M, g) \end{aligned}$$

and $\psi_1(M_1)$ and $\psi_2(M_2)$ are spacelike in M, then for *any* $T_1 \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M_1), \mathcal{F}(M_1))$ there is a $T \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M), \mathcal{F}(M))$ which is a ψ_1-extension of T_1, and the restriction of T to $\mathcal{F}(\psi_2)\mathcal{F}(M_2)$ is the *identity* morphism on $\mathcal{F}(\psi_2)\mathcal{F}(M_2)$.

$Mor_{\mathfrak{Alg}}$-Causal Separability is the categorial version of what is called the *no-signaling* prohibition: If a morphism T_1 represents an operation performed on a physical system localized in spacetime (region) M_1 with observabes described by C^*-algebra $\mathcal{F}(M_1)$ then this operation can be carried out as an operation on a larger system localized in $M \supset \psi_1(M_1)$ in such a way that the operation leaves intact the physical system localized in M_2, with $\psi_2(M_2)$ being spacelike in M from $\psi_1(M_1)$ (see [35] for further discussion).

It is obvious that

$$\Big[\ Mor_{\mathfrak{Alg}}\text{-Causal Independence}\ \Big] \Rightarrow \Big[\ Mor_{\mathfrak{Alg}}\text{-Causal Separability}\ \Big]$$

but the converse is not obvious; in fact we conjecture that it does not hold in general:

Conjecture:

$$\Big[\; Mor_{\mathfrak{Alg}}\text{-Causal Independence} \; \Big] \not\Leftarrow \Big[\; Mor_{\mathfrak{Alg}}\text{-Causal Separability} \; \Big]$$

Remark 2. If

$$\begin{aligned} \psi_1 \quad &: \quad (M_1, g_1) \to (M, g) \\ \psi_2 \quad &: \quad (M_2, g_2) \to (M, g) \end{aligned}$$

and $\psi_1(M_1)$ and $\psi_2(M_2)$ are spacelike in M, then Einstein causality of $\mathcal{F}$ requires

$$\Big[\mathcal{F}(\psi_1)\mathcal{F}(M_1), \mathcal{F}(\psi_2)\mathcal{F}(M_2) \Big]_-^{\mathcal{F}(M)} = \{0\}$$

which does *not* entail that for all A in the intersection

$$\mathcal{F}(\psi_1)\mathcal{F}(M_1) \cap \mathcal{F}(\psi_2)\mathcal{F}(M_2) \tag{5}$$

we have

$$T_1(A) = T_2(A) \tag{6}$$

for every $T_1 \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M_1), \mathcal{F}(M_1))$ and for every $T_2 \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M_2), \mathcal{F}(M_2))$, which is obviously a necessary condition for the existence of a T on $\mathcal{F}(M)$ that is both a ψ_1-extension of T_1 on $\mathcal{F}(M_1)$ and a ψ_2 extension of T_2 on $\mathcal{F}(M_2)$. From this it follows that

$$\Big[\; \text{Einstein causality} \Big] \not\Rightarrow \Big[\; Mor_{\mathfrak{Alg}}\text{-Causal Independence} \; \Big]$$

In other words, $Mor_{\mathfrak{Alg}}$-Causal Independence is an independence condition that does not obviously hold as a consequence of Einstein Causality (local commutativity); the status of $Mor_{\mathfrak{Alg}}$-Causal Independence in categorial quantum field theory is therefore not a straightforward matter. If, however, $\mathcal{F}$ is a tensor functor, then this entails $Mor_{\mathfrak{Alg}}$-Causal Independence under some natural further assumptions on the morphisms $Mor_{\mathfrak{Alg}}$ and their extendability; this will be discussed further in section 8.

It is clear that the physical content of $Mor_{\mathfrak{Alg}}$-independence depends on the nature, i.e. on the physical interpretation, of the morphisms in $Mor_{\mathfrak{Alg}}$. One can view $Mor_{\mathfrak{Alg}}$ as a *variable* in the problem of relativistic locality: Taking different types of morphisms one obtains different independence concepts. Thus the notion of independence is not unique, and the different independence concepts might be logically independent. A possible specification of $Mor_{\mathfrak{Alg}}$ is to take it to be the class of *operations*: completely positive unit preserving linear maps (non-selective operations) between C^*-algebras. We will consider the resulting operational independence concepts in section 7.

Next, we formulate the relativistic locality condition we called CL-Dependence.

5. Causal Locality – Dependence in terms of categorial concepts

As it was formulated in section 2, the CL-Dependence condition is a generalization of what became called the Principle of the Common Cause. This principle, which goes back to Reichenbach's work [36], states that correlations need to be explained causally: either by displaying a causal connection between the correlated entities, or by displaying a common cause – if a direct causal link between the correlated entities is excluded. Typically, the correlation the Common Cause Principle refers to is taken to be the usual correlation of random events with respect to a probability measure. (For a detailed analysis of the Common Cause Principle in non-categorial terms see [37].) The CL-Dependence condition is a substantial generalization of the "standard" Common Cause Principle in that CL-Dependence requires a causal explanation of *any* type of correlation, i.e. of correlations between any type of morphisms. This generalization emerges naturally by taking a categorial viewpoint of the standard situation: classical probability measures are states on commutative operator algebras, and states are special morphisms in the category of operator algebras. As long as there are correlated morphisms, the correlations they represent cry out for explanation just as much as correlations of random events do. Another feature of the CL-Dependence that was not part of the original idea of Reichenbach is the explicit stipulation requiring the spatio-temporal location of the common cause. This additional demand makes assessing the status of the CL-Dependence condition in categorial quantum field theory very difficult, as we shall see. To formulate the CL-Dependence condition explicitly, we need to define the notion of correlated morphism first:

Definition 6. Given

$$\begin{aligned}\psi_1 &: (M_1, g_1) \to (M, g)\\ \psi_2 &: (M_2, g_2) \to (M, g)\end{aligned}$$

with $\psi_1(M_1)$ and $\psi_2(M_2)$ spacelike in M, the morphism $T \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M), \mathcal{F}(M))$ is said to be (ψ_1, ψ_2)-correlated if for some $X \in \mathcal{F}(M_1)$ and $Y \in \mathcal{F}(M_2)$ one has

$$T(\mathcal{F}(\psi_1)(X)\mathcal{F}(\psi_2)(Y)) \neq T(\mathcal{F}(\psi_1)(X))T(\mathcal{F}(\psi_2)(Y)) \tag{7}$$

The above notion of correlated morphism is a natural generalization of the standard notion of correlation: Taking as morphism T a state ϕ on the C^*-algebra $\mathcal{F}(M)$ via the identification $T(A) \doteq \phi(A)I$, condition (7) states that observables X and Y are correlated in the state ϕ.

Definition 7. The functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ is said to satisfy the $Mor_{\mathfrak{Alg}}$-Causal *Dependence* condition, if whenever some morphism $T \in Mor_{\mathfrak{Alg}}(\mathcal{F}(M), \mathcal{F}(M))$ is (ψ_1, ψ_2)-correlated (on operators $X \in \mathcal{F}(M_1), Y \in \mathcal{F}(M_2)$), then there exist a spacetime (M_0, g_0) and an embedding $\psi_0\colon (M_0, g_0) \to (M, g)$ with

$$\psi_0(M_0) \prec_M \psi_1(M_1), \psi_2(M_2) \tag{8}$$

and there exists a morphism

$$T_0 \in Mor_{\mathfrak{Alg}}\big(\mathcal{F}(M_0), \mathcal{F}_0(M_0)\big)$$

which *screens off* the correlation displayed by morphism T between X and Y in the following sense: T_0 has a ψ_0-extension $T_0^{\psi_0}$ from $\mathcal{F}(M_0)$ to $\mathcal{F}(M)$ for which we have

$$(T\circ T_0^{\psi_0})(\mathcal{F}(\psi_1)(X)\mathcal{F}(\psi_2)(Y))=(T\circ T_0^{\psi_0})(\mathcal{F}(\psi_1)(X))(T\circ T_0^{\psi_0})(\mathcal{F}(\psi_2)(Y)) \quad (9)$$

The $Mor_{\mathfrak{Alg}}$-*Causal* Dependence condition (Definition 7) requires that a (possibly operator valued) correlation predicted by a morphism between operators lying in algebras pertaining to spaceike separated spacetime regions is “explainable” by a morphism on a local algebra associated with a region lying in the common causal past of the regions containing the correlated operators; where “explainable” means: manipulating (i.e. conditionalizing) the correlated morphism with (the extension of) a morphism in the causal past of the correlated operators makes the correlation disappear. Definition 7 can be viewed as a formulation in categorial quantum field theory of the concept of explaining correlations between causally independent quantities in terms of common causes ([37]). Note that $T_0^{\psi_0}$ depends on the elements X, Y and it is *not* required that the conditioned operation $T \circ T_0^{\psi_0}$ is *totally* uncorrelated, i.e. that eq. (9) holds for elements X', Y' different from X, Y.

6. Relativistic locality as a causally local covariant functor

We are now in the position of formulating the concept of relativistic locality in a technically explicit manner using the introduced causal locality concepts for a covariant functor:

Definition 8. A covariant functor $\mathcal{F}\colon (\mathfrak{Man}, hom_{\mathfrak{Man}}) \to (\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ which satisfies

- Einstein Causality
- Time-Slice axiom
- $Mor_{\mathfrak{Alg}}$-Causal Independence
- $Mor_{\mathfrak{Alg}}$-Causal Dependence

is called a *causally* $Mor_{\mathfrak{Alg}}$*-local functor*.

The main claim of this paper is then that a causally $Mor_{\mathfrak{Alg}}$-local functor captures, in terms of category theory, our intuition about what it means for a quantum theory to be relativistically local: A particular quantum field theory is in compliance with relativistic locality if it can be formulated as a a covariant, causally $Mor_{\mathfrak{Alg}}$-local functor for some physically interpretable class of morphisms $Mor_{\mathfrak{Alg}}$.

Analysis of relativistic locality in this framework proceeds therefore by doing the following:

1. One has to re-state a quantum field theory in terms of a covariant functor $\mathcal{F}$.
2. One has to specify a class $Mor_{\mathfrak{Alg}}$ of morphisms that has a clear physical interpretation.
3. One has to check whether the functor $\mathcal{F}$ is causally $Mor_{\mathfrak{Alg}}$-local.
4. One can try to vary the class $Mor_{\mathfrak{Alg}}$ of morphisms to explicate different independence concepts and their status from the perspective of relativistic locality.

Where do we stand in this analysis?

1. Some quantum field theories have been re-stated on the basis of concepts of category theory; in particular, the Haag-Kastler local algebraic quantum field theory ([29], [30], [28], [38], [39], [40]) can be recovered in categorial terms (see [1] and Proposition 1 below).
2. There is a good candidate for $Mor_{\mathfrak{Alg}}$ with a clear physical interpretation: the operations, $Op_{\mathfrak{Alg}}$: completely positive, unit preserving maps between C^*-algebras. The operations are generalizations of measurements, in particular of the projection postulate (see section 7).
3. The available evidence indicates that the functor $\mathcal{F}$ recovering the Haag-Kastler local algebraic quantum field theory is causally $Op_{\mathfrak{Alg}}$-local; although no full proof has been found yet (see section 7).
4. Morphisms other than $Op_{\mathfrak{Alg}}$ have been considered: the subclass $Op^*_{\mathfrak{Alg}}$ of $Op_{\mathfrak{Alg}}$ composed of *normal* operations on von Neumann algebras; i.e. operations that are continuous in the ultraweak operator topology. One has a number of open problems in this direction.

In the next section some results are recalled that can be interpreted as positive evidence that the covariant functor describing the Haag-Kastler algebraic quantum field theory is causally $Op_{\mathfrak{Alg}}$-local in the sense of Definition 8.

7. Relativistic locality and Haag-Kastler quantum field theory

Proposition 1 (Brunetti-Fredenhagen-Verch 2003, Proposition 2.3)**.** *The Haag-Kastler algebraic quantum field theory can be recovered as a particular case of categorial quantum field theory as follows: Given a covariant functor $\mathcal{F}$ in the sense of Definition 1, take*

- *the flat Minkowski spacetime (M, g) as an object in $Obj(\mathfrak{Man})$;*
- *open bounded regions $O \subset M$ with restriction of g to O as spacetimes in their own right (element of $Obj_{\mathfrak{Man}}$);*
- *$\psi\colon (O, g) \to (O', g')$ as the* identity map *on O' restricted to O;*
- *$\mathcal{A}(O) \doteq \mathcal{F}(O)$ $\qquad C^*$-algebra of local observables.*

Then

- *The group of isometric diffeomorphisms of M is represented on the quasilocal algebra $\mathcal{A} = \overline{\cup_{O\subset M}\mathcal{A}(O)}$ by C^*-algebra automorphisms acting covariantly on $\mathcal{A}$.*
- *The Time Slice axiom holds and becomes what is known as Local Primitive Causality.*

A net of local von Neumann *algebras satisfying the Haag-Kastler axioms also can be recovered in the above manner as a particular case of categorial quantum field theory.*

Let $Mor_{\mathfrak{Alg}}$ be the class of non-selective operations (unit preserving completely positive, linear maps) $Op_{\mathfrak{Alg}}$ (see [33], and [41] for the definition and elementary facts about operations). Recall some elements of $Op_{\mathfrak{Alg}}$:

- States:

$$\phi \quad \Leftrightarrow \quad \mathcal{A} \ni A \mapsto \phi(A)I \in \mathcal{A}$$

- Conditional expectations:

$$T\colon \mathcal{A} \to \mathcal{A}_0 \qquad T(A_0) = A_0 \qquad A_0 \in \mathcal{A}_0$$

  - In particular the conditional expectation (non-selective projection postulate):

$$\begin{aligned} \mathcal{N} \ni X \mapsto T(X) &= \sum_i P_i X P_i \\ P_i \text{ projections in } \mathcal{N}, &\qquad \sum_i P_i = I \end{aligned}$$

- Kraus operations:

$$\begin{aligned} \mathcal{N} \ni X \mapsto T(X) &= \sum_i W_i X {W_i}^* \\ W_i \in \mathcal{A}, &\qquad \sum_i W_i {W_i}^* = I \end{aligned}$$

If one takes $Mor_{\mathfrak{Alg}}$ to be the class $Op_{\mathfrak{Alg}}$ of operations, then the problem of causal $Op_{\mathfrak{Alg}}$-locality of the covariant functor describing the Haag-Kastler quantum field theory emerges. The status of CL-Independence in Haag-Kastler quantum field theory is stated by the following

Proposition 2 ([31], [32])**.** *The functor $\mathcal{F}$ describing a net of local von Neumann algebras satisfying the Haag-Kastler axioms satisfies the $Op_{\mathfrak{Alg}}$-Causal Independence condition for*

$$\begin{aligned} \psi_1\colon D_1 &\to D \\ \psi_2\colon D_2 &\to D \end{aligned}$$

where D_1, D_2 are strictly *spacelike separated double cone regions in double cone D.*

Proposition 2 is just the categorial reformulation of the *operational* C^*-independece of von Neumann algebras associated with strictly spacelike separated double cones in algebraic quantum field theory satisfying the Haag-Kastler axioms. The mathematical content of the $Op_{\mathfrak{Alg}}$-Causal Independence condition is that the von Neumann algberas $\mathcal{A}(D_1), \mathcal{A}(D_2)$ in a net of von Neumann algebras satisfying the Haag-Kastler axioms are operationally C^*-independent in the sense that any two operations T_1 on $\mathcal{A}(D_1)$ and T_2 on $\mathcal{A}(D_2)$ are copossible: can be extended to an operation on $\mathcal{A}(D)$. The physical content of this independence condition is clear: Assume that two physical systems S_1 and S_2 are localized in strictly spacelike separated (hence causally disjoint) double cone regions D_1, D_2 of spacetime and that their observables are represented by von Neumann agebras $\mathcal{A}(D_1), \mathcal{A}(D_2)$. If S is a larger system localized in double cone region D, then S_1 and S_2 are physically independent as subsystems of the larger system S in the sense that any two physical interaction with systems S_1 and S_2 can be jointly realized as a single interaction with system S.

An immediate corollary of Proposition 2 is:

Proposition 3 ([35]). *The covariant functor $\mathcal{F}$ describing a net of local von Neumann algebras satisfying the Haag-Kastler axioms satisfies the $Op_{\mathfrak{Alg}}$-Causal Separability condition for*

$$\begin{aligned} \psi_1 \colon D_1 &\rightarrow D \\ \psi_2 \colon D_2 &\rightarrow D \end{aligned}$$

where D_1, D_2 are strictly *spacelike separated double cone regions in double cone D.*

The above proposition is the proper formulation of the no-signaling condition for general operations in quantum field theory (it was shown in [35] that local commutativity, i.e. Einstein Causality, is not sufficient to exclude signaling with respect to operations that are not representable by Kraus operators).

The available evidence indicates that the functor describing a net of local von Neumann algebras satisfying the Haag-Kastler axioms (including Local Primitive Causality) also *might* satisfy the $Op_{\mathfrak{Alg}}$-Causal Locality – Dependence condition; no proof known is however. The evidence is the following:

Proposition 4 ([42], [43], [37]). *If*

- *$\omega \in Op_{\mathfrak{Alg}}$ is a state: $A \mapsto \omega(A)I$ such that*
- *ω is (ψ_1, ψ_2)-correlated on X, Y ($X \in \mathcal{A}(M_1), Y \in \mathcal{A}(M_2)$), where M_1, M_2 are spacelike separated subregions of region M;*

then there exist selective *operations T_0 on $\mathcal{A}(M_0)$ such that*

- *the extension T of T_0 to $\mathcal{A}(M)$ screens of the correlation:*

$$(\omega \circ T)(XY) = (\omega \circ T)(X)(\omega \circ T)(Y)$$

with

- $M_0 \subseteq \Big[$*causal past of* $M_1 \cup$ *causal past of* $M_2\Big]$

The reason why Proposition 4 is not sufficient to conclude that the covariant functor describing a net of local von Neumann algebras satisfying the Haag-Kastler axioms (including Local Primitive Causality) also satisfies the $Op_{\mathfrak{Alg}}$-Causal Dependence condition is three-fold: First, the operations in Proposition 4 that screen-off the correlation between X and Y in state ω are *selective* (not unit preserving), as opposed to be non-selective as required by Definition 7. Second, such selective screener-off operations are known to exist only for the particular correlated operations known as states but not for other, more general types; although many other, non-state-like operations are also correlated. Third, the selective operations that screen off the correlations predicted by states are localized in the *union* of the causal pasts of M_1 and M_2 rather than in the *intersection* of the causal pasts; hence condition (8) in Definition 7 of CL-Dependence does not hold for the operation T in Proposition 4.

8. Einstein Causality and tensor property of the covariant functor

More recently, the Einstein Causality condition has been replaced by another requirement (Axiom 4 in [44]; also see [45], [46], [47]). This new axiom entails Einstein Causality (under some additional hypotheses it is equivalent to Einstein Causality); thus Brunetti and Fredenhagen interpret it as an independence condition [44][p. 134]. It is argued in this section however that this Axiom 4. also entails a $Mor_{\mathfrak{Alg}}$-type independence generally under some further assumptions (see Proposition 6).

To formulate Axiom 4. one extends the category $(\mathfrak{Man}, hom_{\mathfrak{Man}})$ to a tensor category denoted by $(\mathfrak{Man}^{\otimes}, hom^{\otimes}_{\mathfrak{Man}})$, and, taking the category $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$ of C^*-algebras as a tensor category with respect to the minimal C^*-tensor product of C^*-algebras, the covariant functor $\mathcal{F}$ can be extended naturally to a tensor functor $\mathcal{F}^{\otimes}$ between these two tensor categories. The tensorial property of $\mathcal{F}^{\otimes}$ embodies then Einstein Causality.

To be more specific, let $\mathcal{A}_1 \otimes_{min} \mathcal{A}_2$ be the minimal tensor product of C^*-algebras $\mathcal{A}_1$ and $\mathcal{A}_2$, and let $(\mathfrak{Alg}^{\otimes}, hom^{\otimes}_{\mathfrak{Alg}})$ denote the tensor category of C^*-algebras with respect to this tensor product, with the set of complex numbers as unit object and with the homomorphisms $hom^{\otimes}_{\mathfrak{Alg}}$ being identical to $hom_{\mathfrak{Alg}}$: the class of injective C^*-algebra homomorphisms. (To simplify notation, in what follows, the subscript *min* will be omitted from the tensor product $\otimes_{min}$.) The category $(\mathfrak{Man}^{\otimes}, hom^{\otimes}_{\mathfrak{Man}})$ has, by definition, as its objects *finite* disjoint unions of objects from $\mathfrak{Man}$ and the empty set as unit object. (Thus the objects in $\mathfrak{Man}^{\otimes}$ are no longer connected spacetimes.) By definition, the morphisms $\psi^{\otimes}$ in $hom^{\otimes}_{\mathfrak{Man}}$ are maps of the form

$$\psi^{\otimes}\colon M_1 \sqcup M_2 \sqcup \ldots \sqcup M_n \to M \qquad (10)$$

($\sqcup$ denoting the disjoint union) such that

(i) the restriction of $\psi^{\otimes}$ to any M_i are morphisms in the category $(\mathfrak{Man}, hom_{\mathfrak{Man}})$;

(ii) the images $\psi^{\otimes}(M_i)$ of the spacetimes M_i are spacelike in M:

$$\psi^{\otimes}(M_i) \times_M \psi^{\otimes}(M_j) \qquad i \neq j \tag{11}$$

To define the tensorial features of the functor, we need some notation first. Let $\psi_i \colon M_i \to N_i$ be embeddings of disjoint spacetimes M_i $(i = 1, 2)$ such that the images $\psi_1(M_1)$ and $\psi_2(M_2)$ are causally disjoint in $N_1 \cup N_2$. Then $\psi_1 \otimes \psi_2$ denotes the map

$$(\psi_1 \otimes \psi_2) \quad : \quad M_1 \sqcup M_2 \to N_1 \cup N_2 \tag{12}$$

$$(\psi_1 \otimes \psi_2)(x) \quad \doteq \quad \begin{cases} \psi_1(x) & \text{if } x \in M_1 \\ \psi_2(x) & \text{if } x \in M_2 \end{cases} \tag{13}$$

Clearly, the map $(\psi_1 \otimes \psi_2)$ is a morphism in the category $(\mathfrak{Man}^{\otimes}, hom^{\otimes}_{\mathfrak{Man}})$. The tensor product $\alpha_1 \otimes \alpha_2$ of two injective C^*-algebra homomorphisms α_1 and α_2 on the tensor product $\mathcal{A}_1 \otimes \mathcal{A}_2$ of C^*-algebras $\mathcal{A}_1$ and $\mathcal{A}_2$ is defined in the usual way as the extension to $\mathcal{A}_1 \otimes \mathcal{A}_2$ of the map

$$(\mathcal{A}_1 \otimes \mathcal{A}_2) \ni A_1 \otimes A_2 \mapsto \alpha_1(A_1) \otimes \alpha_2(A_2) \tag{14}$$

Let $\iota_1 \colon M_1 \to M_1 \otimes M_2$ denote the trivial embedding of spacetime M_1 into the disjoint union $M_1 \otimes M_2$. One can then require that the covariant functor $\mathcal{F}$ be a tensor functor in the sense of the following definition:

Definition 9. The covariant functor

$$\mathcal{F}^{\otimes} \colon (\mathfrak{Man}^{\otimes}, hom^{\otimes}_{\mathfrak{Man}}) \to (\mathfrak{Alg}^{\otimes}, hom^{\otimes}_{\mathfrak{Alg}}) \tag{15}$$

is called a *tensor* functor if for any two spacetimes $M_1, M_2 \in \mathfrak{Man}$ with $M_1 \cap M_2 = \emptyset$ and embeddings $\psi_1 \colon M_1 \to N$ and $\psi_2 \colon M_2 \to N$ with causally disjoint images in N we have

$$\mathcal{F}^{\otimes}(\emptyset) \quad = \quad \mathbb{C} \tag{16}$$

$$\mathcal{F}^{\otimes}(\iota_1)(A_1) \quad = \quad A_1 \otimes I \qquad A_1 \in \mathcal{F}^{\otimes}(M_1) \tag{17}$$

$$\mathcal{F}^{\otimes}(\iota_2)(A_2) \quad = \quad I \otimes A_2 \qquad A_2 \in \mathcal{F}^{\otimes}(M_2) \tag{18}$$

$$\mathcal{F}^{\otimes}(M_1 \otimes M_2) \quad = \quad \mathcal{F}^{\otimes}(M_1) \otimes \mathcal{F}^{\otimes}(M_2) \tag{19}$$

$$\mathcal{F}^{\otimes}(\psi_1 \otimes \psi_2) \quad = \quad \mathcal{F}^{\otimes}(\psi_1) \otimes \mathcal{F}^{\otimes}(\psi_2) \tag{20}$$

One has then

Proposition 5 ([44], Theorem 1)**.** *If $\mathcal{F}^{\otimes}$ is a covariant tensor functor, then it satisfies Einstein Causality. Conversely, if $\mathcal{F}$ is a covariant functor in the sense of Definition 1 (so in particular $\mathcal{F}$ satisfies Einstein Causality) then it can be uniquely extended to a tensor functor $\mathcal{F}^{\otimes}$ of the form (15).*

The essential equivalence of Einstein Causality and the tensor feature of the covariant functor $\mathcal{F}^{\otimes}$ stated in Proposition 5 makes it possible to formulate sufficient conditions that entail $Mor_{\mathfrak{Alg}}$-Causal Independence in general:

Proposition 6. *If the class of morphisms $Mor_{\mathfrak{Alg}}$ is closed with respect to the tensor product in the tensor category $(\mathfrak{Alg}^{\otimes}, hom^{\otimes}_{\mathfrak{Alg}})$ and has the unrestricted extendability feature, then a quantum field theory given by a covariant tensor functor $\mathcal{F}^{\otimes}$ satisfies the $Mor_{\mathfrak{Alg}}$-Causal Independence condition.*

A class of morphisms that does have the unrestricted extendability feature is the class of C^*-algebra states (states defined on C^*-subalgebras of C^*-algebras are extendable by the Hahn-Banach theorem, see e.g. [48]); moreover the products of states defined on components of tensor products extend naturally to the tensor product; hence the class of states is closed with respect to the tensor product in the tensor category $(\mathfrak{Alg}, hom_{\mathfrak{Alg}})$. Thus Propositions 6 and 5 contain, in a categorial formulation, the well-known C^*-independence (in fact the C^*-independence in the *product sense* [14][p. 208-209]) of local C^*-algebras in the Haag-Kastler quantum field theory.

The class $Op_{\mathfrak{Alg}}$ of morphisms containing general operations does *not* have the unrestricted extendability feature however: completely positive maps defined on C^*-subalgebras of C^*-algebras are not in general extendable to a completely positive map on the larger algebra [33]. Thus, without further assumptions, Propositions 6 and 5 do *not* entail the $Op_{\mathfrak{Alg}}$-Causal Independence in general. The assumption in Proposition 2 on the shape of the spacetime regions (double cones) is thus important: it ensures the hyperfiniteness of the algebras associated with double cones [49], [29][p. 225], which is a sufficient condition to ensure extendability of operations [50], [51][Theorem 6]. Note that hyperfiniteness of the double cone algebras does not ensure extendability of *normal* operations to *normal* operations; hence it is not clear if the W^*-version of Proposition 2 also holds – the status of the $Op^*_{\mathfrak{Alg}}$-Causal Independence in quantum field theory is an open problem.

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