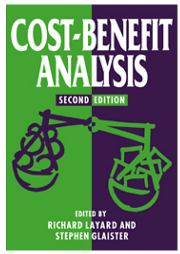
Richard Layard and Stephen Glaister

Introduction



Cost-Benefit Analysis

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INTRODUCTION

Should Chicago build a new airport, or India a new steel mill? Should higher education expand, or water supplies be improved? How fast should we consume non-renewable resources and what are the costs and benefits of protecting the environment? These are typical of the questions on which cost–benefit analysis has something to say. However, there is no problem, public or personal, to which its broad ideas could not be applied.

The chapters in this book concentrate on those issues that are common to all cost–benefit appraisals. We have cast this introduction in a form that is theoretical and also shows how one might tackle a particular problem.[1] We take an imaginary project and show how the chapters throw light on the analysis of this project. But first a few remarks are needed on the general ideas involved.

The basic notion is very simple. If we have to decide whether to do *A* or not, the rule is: Do *A* if the benefits exceed those of the next best alternative course of action, and not otherwise. If we apply this rule to all possible choices, we shall generate the largest possible benefits, given the constraints within which we live. And no one could complain at that.

Going on a step, it seems quite natural to refer to the 'benefits of the next best alternative to *A*' as the 'costs of *A*'. For if *A* is done those alternative benefits are lost. So the rule becomes: do *A* if its benefits exceed its costs, and not otherwise.

So far so good. The problems arise over the measurement of benefits and costs. This is where many recoil in horror at the idea of measuring everything on one scale. If the objection is on practical grounds that is fair enough; and many economists have swallowed hard before making a particular quantification, or have refused to make one. But if the objection is theoretical it is fair to ask the objector what he means by saying that *A* is better than *B*, unless he has some means of comparing the various dimensions along which *A* and *B* differ. There are some people who believe that one particular attribute of life, such as the silence of the countryside, is of absolute importance. For them cost–benefit analysis is easy: the value of all other benefits and costs is negligible. More problematical are those people who believe in the absolute importance of two or more items, for they are doomed to intellectual and spiritual frustration. Whenever *A* is superior to its alternative on one count and inferior on another, they will feel obliged to do both. Choices between such alternatives have to be made only too often. However, rational choice may be possible even if not every item has a unique price. Often it will be sufficient to know that the prices lie within some finite range; the answer will be unaffected by exact values. The only basic principle is that we should be willing to assign numerical values to costs and benefits, and arrive at decisions by adding them up and accepting those projects whose benefits exceed their costs.

But how are such values to be arrived at? If we assume that only people matter, the analysis naturally involves two steps. First, we must find out how the decision would affect the welfare of each individual concerned. To judge this effect we must ultimately rely on the individual's own evaluation of his mental state. So the broad principle is that 'we measure a person's change in welfare as he or she would value it'. That is, we ask what he or she would be willing to pay to acquire the benefits or to avoid the costs. There is absolutely no need for money to be the numeraire (i.e., the unit of account) in such valuations. It could equally well be bushels of corn but money is convenient.

The problems of inferring people's values from their responses to hypothetical questions or from their behaviour are clearly acute and present a central problem in cost–benefit analysis.

The second step is to deduce the change in social welfare implied by all the changes in individual welfare. Unless there are no losers, this means somehow valuing each person's £1. If income were optimally distributed, £1 would be equally valuable regardless of whose it was; that is what 'optimally distributed' means. Each person's £1 has equal weight. And if income is not optimally distributed most economists would argue that it should be redistributed by cash transfers rather than through the choice of projects. But what if we think that cash will not be distributed, even if it should be? Then we may need to value the poor person's extra £1 more highly than the rich person's.

However, this raises the kind of question that underlies almost all disputes about cost–benefit analysis, or indeed about moral questions generally: the question of which constraints are to be taken as given. If a government agency knows for certain that cash will not be redistributed to produce a desirable pattern of income, even if it should be, then the agency should allow for distributional factors when evaluating the project. Equally, it should not allow for these factors if it can ensure that redistribution is achieved by some more appropriate method. In practice it may not know whether this can be ensured, or it may decide that the issue is outside its competence. Until this is settled a rational project appraisal is impossible.

Likewise, in the personal sphere it is reasonable to take unalterable features of one's character into account in deciding what is right. But which features are unalterable? In each case the issue is which constraints are exogenous to the decision maker.

This brings us to the relation between cost–benefit analysis and the rest of public policy. The government's overall aim is presumably to ensure that social welfare is maximized subject to those constraints over which it has no control such as tastes, technology and resource endowments. In any economy this objective requires some government activity owing to the failure of free markets to deal with the problems of externality, economies of scale, imperfect information and inadequate markets for risky outcomes, and also because of the problem of the maldistribution of wealth.

Three main methods of intervention are open: regulation, taxes and subsidies, and public direction of what is to be produced, be it *via* public enterprise or purchase from private firms. Each of these types of government activity can be subjected to cost–benefit analysis, as the case studies in this volume illustrate. In the case of public production the great strength of cost–benefit analysis is that it permits decentralized decision making. This is needed because, even if the public sector is small, no one office can hope to handle the vast mass of technical information needed to decide on specific projects. Decentralization deals with that problem, just as the price mechanism does, but it raises the obverse problem that the right decisions will only result if the prices used by the decision makers correctly reflect the social values of inputs and outputs at the social optimum: what are usually called their 'shadow prices'.

In a mixed economy market prices often do not do this. So the main problem in cost–benefit analysis is to arrive at adequate and consistent valuations where market prices fail in some way. If production is to take the form of public enterprise, government will generally lay down some of the prices (like the discount rate) to be used by public-sector enterprises, as well as their decision rules. A similar adjustment is to be made in the price quoted when purchase is from private firms. In a more planned economy the government will lay down more of the prices and might, in principle, by iterative search find a complete set of prices, which, if presented to producers, would lead to their production decisions being consistent with each other and with consumers' preferences.[2] However, in practice no government has tried this and most rely in part on quantitative targets or quotas, as well as taxes and subsidies, to secure consistency in at any rate some areas of production.

If any of the activities of government agencies are non-optimal, the cost–benefit analysis is faced with a second source of difficulty in finding relevant prices: whether and how to allow for those divergences between market prices and social values that arise from the action or inaction of government itself.

Broadly the valuations to be made in any cost–benefit analysis fall under four main headings:

1 The relative valuation of costs and benefits at the time when they occur.
2 The relative valuation of costs and benefits occurring at various points in time: the problem of time preferences and the opportunity cost of capital.
3 The valuation of risky outcomes.
4 The valuation of costs and benefits accruing to people with different incomes.

Part I of this book deals with the various issues of principle involved in cost–benefit analysis. Part II shows how to evaluate time savings, safety, the environment and exhaustible resources. Part III contains case studies which illustrate some of the problems to which the techniques of cost–benefit analysis have been applied.

1 THE OVERALL APPROACH

Suppose there is a river which at present can only be crossed by ferry. The government considers building a bridge, which, being rather upstream, would take the traveller the same time to complete a crossing. The ferry is a privately owned monopoly and charges £0.20 per crossing, while its total costs per crossing are £0.15. It is used for 5,000 crossings per year. The bridge would cost £30,000 to build but would be open free of charge. It is expected that there will be 25,000 crossings a year with the bridge and that the ferry would go out of business. The government send for the cost–benefit analyst to advise them on whether to go ahead with the bridge.

In any cost–benefit exercise it is usually convenient to proceed in two stages:

(a) Value the costs and benefits in each year of the project;
(b) Obtain an aggregate 'present value' of the project by 'discounting' costs and benefits in future years to make them commensurate with present costs and benefits, and then adding them up.

At each stage the appraisal differs from commercial project appraisal because (i) costs and benefits to all members of society are included and not only the monetary expenditures and receipts of the responsible agency, and (ii) the social discount rate may differ from the private discount rate. The main work goes into step (a) and we shall concentrate on this for the present.

Consumers' surplus and willingness to pay

We need to avoid logical errors in deciding which items are to be included as costs and benefits and to value those that are included correctly. The guiding principle is that we list all parties affected by the project and then value the effect of the project on their welfare as it would be valued in money terms by them. In this case there are four parties: taxpayers, ferry owners, existing travellers and new travellers (who previously did not cross but would do so at the lower price).

1 The taxpayers lose £30,000, assuming the bridge is financed by extra taxes.
2 The ferry owners lose their excess profits of £250 [i.e., $0.05 \times 5{,}000$] in each future year for ever (area A on figure 1).

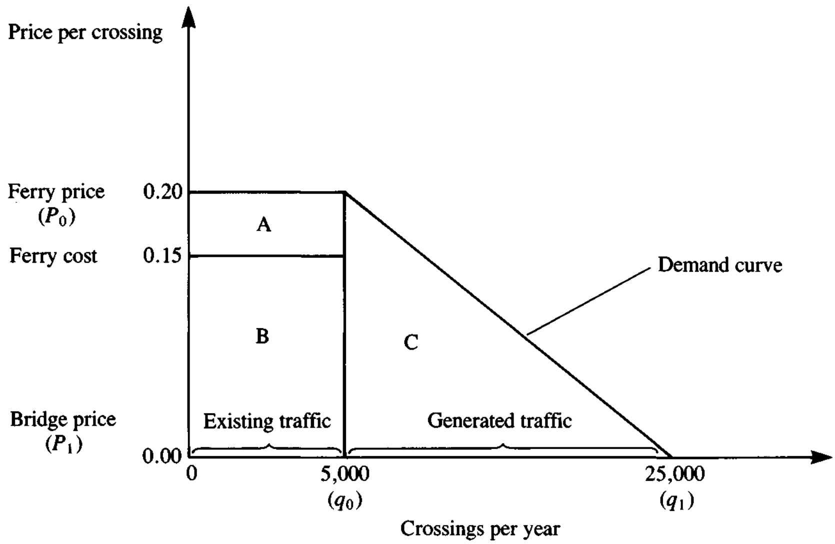

Figure 1

3 The existing travellers gain £1,000 [0.20 × 5,000] in each future year for ever, due to the fall in price (areas A + B).

4 The new travellers: to evaluate their gains is more difficult. We know that the new journey which is most highly valued was nearly made by ferry, and is therefore worth very nearly £0.20; while the journey which is least highly valued is valued only marginally more than its price of £0.00. For the intermediate journeys we have to make some arbitrary assumption and will assume that the value per journey falls at a constant rate from £0.20 to zero. In other words we are assuming that the demand curve is a straight line. So the average gain to new travellers is £0.10 per crossing and the total gain £2,000 [0.10 × 20,000] per year for ever. This figure corresponds to the gain in consumers' surplus on the part of the new travellers, since it represents the sum of the differences between the maximum they would be willing to pay for their journeys and the amount they actually pay, which in this case is zero.[3] Geometrically it is represented by area C, the area under the demand curve and above a horizontal line at the final price (which here is zero). In this special case where the demand curve is a straight line the value of generated sales will always equal $\frac{1}{2}(p_0 - p_1)(q_1 - q_0)$, i.e., half the price fall times the quantity increase. This is a formula which is used over and over again in cost–benefit analysis, especially for small changes in prices so the linearity assumption is a reasonable approximation to *any* actual demand curve.

Criteria for a welfare gain

We can now tabulate (table 1) the overall picture of net benefits (benefits minus costs), discounting all the future permanent flows of net benefits by an arbitrary 10 per cent per annum to obtain their present values.[4]

Can these now be added up? It depends entirely on our approach to the problem of income distribution. If we wish to use the very restrictive Pareto criterion of a welfare improvement, we shall support a project only if some people gain and nobody loses. But if some people gain while others lose, the Pareto criterion provides no guidance. If we follow it we must add up the net receipts of all the parties concerned. Then we only support the project if the sum is positive and compensation is paid to the losers. However, there is in practice almost no case where it is feasible to compensate everybody and if the Pareto rule were applied no projects would ever get done. Therefore many cost–benefit analysts have fallen back on the Hicks–Kaldor criterion, which says that a project can be supported provided the gainers could, in principle, compensate the losers even if they do not.[5]

In this case net receipts can always be added. However, there is no ethical justification for the Hicks–Kaldor criterion; where compensation will not be paid there seems no alternative to interpersonal comparisons of the value of each person's gains and losses.

This brings us to a second case for unweighted adding up – where interpersonal comparisons are made but it is judged that in the prevailing income distribution £1 is equally valuable to all the parties concerned. This may in some cases be quite a reasonable procedure. If not, there are only two alternatives: to use some system of distributional weights or simply to show the net benefits to each party and let the policy maker apply his own evaluation. If distributional weights are used they need not be unique: it may be that the weights can take a wide range of alternative values and yet provide an unambiguous verdict on a project. Viewed from this angle the Pareto criterion is just an extreme case where the distributional weights are allowed to vary infinitely.

For the time being we shall assume, as many cost–benefit analysts do, that unweighted adding up is permissible. We now learn an important lesson: that the area A disappears from the calculation. The reason is of course that it was a transfer payment (monopoly rent) rather than a payment for real goods and services; and if everybody's £1 is equally valuable transfers cannot change social welfare. Consumers used to pay this rent and now they do not, but there is no saving in resource cost as a result of the non-payment after the bridge is built. The economic cost–saving from the demise of the ferry comes from the liberation of resources worth B for production elsewhere in the economy. The only other economic change is the value of the additional consumption (C) – the real value of the first 5,000 journeys is neither more nor less than it was before. Thus, if one had wanted to take a short cut to estimating future net benefits (granted all

Table 1

	Future net benefits per year for ever		
	£	Area in figure 1	Present value at 10% discount rate
Ferryowners	− 250	− A	− 2,500
Existing consumers	+ 1000	A + B	+ 10,000
New consumers	+ 2000	C	+ 20,000
Taxpayers	—	—	− 30,000
Society			?

pounds equally valuable) one could have straightaway identified only the changes of real economic significance, i.e., the cost–saving on the ferry (B) and the consumers' surplus on the generated traffic (C).

Suppose the shares of the ferry company, which were previously worth £2,500, fall to zero. Should this also be included as a cost? Clearly not, since when we calculated the capitalized value of the ferry's profits we were in fact approximating the change in welfare of the shareholders, which is measured by changes in the value of their equity. We can use one measure or the other but not both, and it may generally be easier to measure the yield and to ignore changes in asset values.

So let us bravely add up the table. The present value turns out to be negative (− £2,500) and the project should be turned down. At this point someone might suggest that the project could be made 'viable' by charging a toll on the bridge. This is nonsense – all that it does is to reduce the number of journeys and hence reduce the gain in the real value of additional consumption, without any corresponding reduction in cost. The reader might like to confirm his understanding of the argument so far by recalculating the table for the case where the government levies a toll of £0.05 per crossing. The moral of this is that the prices charged for the output of a project may profoundly affect its economic desirability.[6] The correct price is the marginal social cost per unit of output, which is zero in the case of journeys across an uncongested bridge.

2 MEASURING COSTS AND BENEFITS WHEN THEY OCCUR

The concept of a shadow price

The concept of the shadow price associated with a constraint is of fundamental importance.

Consider first a simple example: you are to find two numbers, each greater than or equal to zero, which maximizes the sum of their squares. The *objective* is clear enough, but, as stated, this problem is ill defined. There is no solution because one can make the sum of the squares as big as one wishes by simply increasing the numbers themselves without bound. The problem can be made well defined by adding a *constraint*: for example, the sum of the first number and twice the second number must be less than or equal to ten. The solution is easy to see. One wants to make both numbers as large as possible, but for each unit subtracted from the second number, the first number can be increased by two units whilst remaining within the constraint, with a net gain to the sum of the squares. Therefore the solution is that the first number should be 10, the second number should be zero and the sum of the squares will be 100. So 100 is the value of the objective at the solution of the problem.

Now, suppose the constraint is relaxed by one unit. The first number plus twice the second number must be less than or equal to 11. Clearly, the solution will be to put the first number at 11 and leave the second number at zero. The objective will increase to 121 at the new solution.

> The shadow price on the constraint is the increase in the maximized objective when the constraint is relaxed by one unit.

In this case the shadow price is $121 - 100 = 21$.

The shadow price therefore measures how much better we could do, measured by the stated objective, if the constraint were relaxed by one unit: it is the penalty of having to observe the constraint. Put another way, it is the maximum we should be willing to pay to secure a relaxation of the constraint.

In practical work we have to face a series of constraints. If there were no constraints then there would be few interesting problems! Each constraint will have its own shadow price. It will be apparent how very useful that information is. It tells us which are the most important (or damaging) constraints and therefore where it would be most productive to direct effort towards relaxing constraints.

Consider now a simplified economic example which illustrates the way in which these concepts arise in planning.

A planner in a developing country has to set the outputs of two commodities: manufactured goods and food. He values a unit of manufactured goods at US $4 and a unit of food at US $7. To produce a unit of manufactured goods requires two units of trained labour and one unit of capital equipment. To produce a unit of food requires three units of trained labour and one unit of capital equipment. Twelve units of labour and five units of capital are available. What output levels should be set in order to maximize the value of the plan?

This is a simple Linear Programming problem. It is linear because the objective and the technical constraints all have coefficients which are independent of the volumes produced: doubling all outputs doubles the value of the plan and also doubles the input requirements.

Denote the quantity of manufactures by M and the quantity of food by F. Then the objective is to

Maximize $4M + 7F$

The constraints say that the consumption of each of the two factors cannot exceed the total amounts available

Capital constraint	$M + F \leq 5$
Labour constraint	$2M + 3F \leq 12$
and	$M \geq 0$ and $F \geq 0$

This problem can be solved using the diagram in figure 2. The diagram shows the line representing points such that

$$M + F = 5 \text{ or equivalently } F = 5 - M$$

The capital constraint dictates that the planner must choose a point on that line or below it. The line representing the labour constraint is similarly shown. Since the constraints must be observed simultaneously the *feasible region* is the area bounded by all the constraints.

The diagram also shows the lines representing

$$4M + 7F = 42 \quad \text{and} \quad 4M + 7F = 49$$

These are sometimes referred to as 'iso-revenue lines'. If they were feasible then all points on these lines would yield a value to the plan of 42 and 49 respectively. The problem is solved by finding a point on a line parallel to these two lines, as far away to the north east of the origin as possible whilst being within the feasible region.

This must occur at one of the three vertices and these are the only points we need to consider. $M = 5$ and $F = 0$ yields a value of \$20; $M = 3$ and $F = 2$ yields a value of \$26; $M = 0$ and $F = 4$ yields a value of \$28. As is obvious from the relative slopes of the lines in the diagrams, this last point is the solution to the problem: produce no units of manufactures, four units of food, giving a value of \$28.

Now, what of the shadow prices of the constraints? Note that the solution point lies inside the capital constraint. The capital constraint turns out to be 'non-binding', that is, irrelevant! Therefore the value to the plan of an extra unit of capital is zero – this is its shadow price. This is a general result. *Any resource which is in excess supply at an optimum allocation of resources will have a social value of zero.* In the context of our example note that this shadow price may be quite different from the international market price of capital (denominated in US \$). So we see that there may be a distinction between the market price of a resource and its 'true' value measured relative to the objective. Note also that the shadow price is a marginal concept – we say that the value of an extra unit of capital is zero, *on the margin*. This is not to deny the value of intra-marginal units.

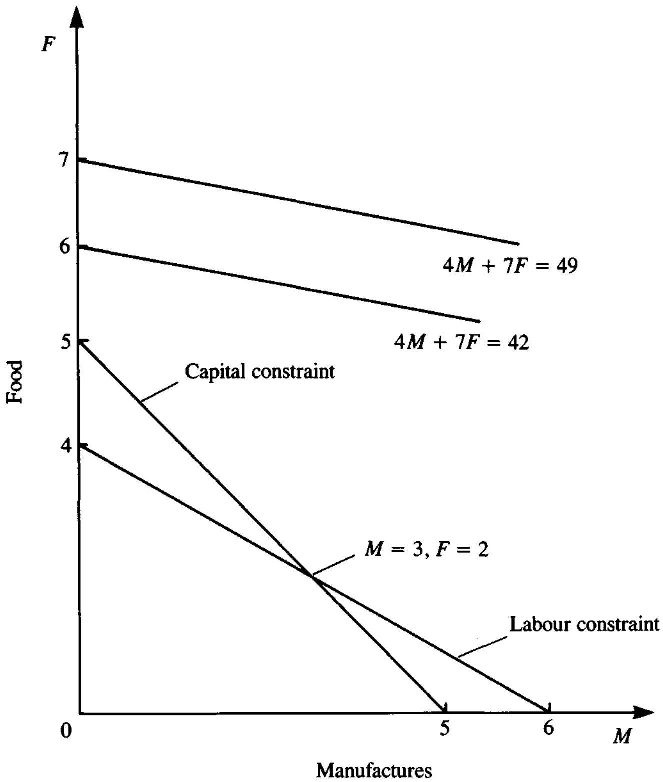

Figure 2

The technology dictates that we cannot produce anything unless we have some capital. It is just that too much is available, so it is not worth paying anything to get some more. Indeed, it would be sensible to sell the one unit of surplus capital on the international market for whatever positive price it would fetch, providing that it were feasible to do so at a sufficiently low selling and transport cost.

Finally, note that the shadow price is only meaningful in relation to the particular objective. For instance, if the relative valuations of the two products changed from the present ratio of 4:7 then the slope of the iso-revenue line would change. It might become sufficiently steep for the optimum point to shift to $M = 3$ and $F = 2$, or even to $M = 5$ and $F = 0$. In either case the capital constraint binds and the shadow price is definitely positive. So shadow prices depend upon the nature of the technical constraints, the quantities of resources available *and* the particular objectives.

What is the shadow price of skilled labour? If the quantity of skilled labour that is available increases from 12 to 13 then the intercept of the labour constraint on the vertical axis of figure 2 will increase from $12/3 = 4$ to $13/3$. Therefore the output of food can increase by $1/3$. The value of this incremental output is \$7/3 and so that is the shadow price of the skilled labour constraint.

In order to facilitate the reading of Drèze and Stern's survey in this volume it is worth expressing these results more formally. Suppose the problem is to

$$\text{Maximize} \quad f(x_1, x_2, \ldots, x_n)$$

subject to a constraint

$$g(x_1, x_2, \ldots, x_n) \leq b$$

Imagine that this problem has been solved. If the value of b is increased then the solution values for each of the x_i will vary and so will the maximized value of the objective. Let $V(b)$ describe the relationship between the maximized value of the objective and b. Then the shadow price of the constraint is given by

$$\partial V(b)/\partial b$$

In words, the shadow price on the constraint is the rate of change of the maximized objective with respect to a unit relaxation of the constraint.

Many readers will be aware that a standard approach to solving problems of this kind is to introduce a new variable, λ, known as a Lagrangean multiplier, and to maximize the Lagrangean function

$$L = f(x_1, x_2, \ldots, x_n) + \lambda\{b - g(x_1, x_2, \ldots, x_n)\}$$

with respect to the x_i. This procedure will provide the optimal values for the x_i and also a value for $\lambda \geq 0$. It can be shown that this value for λ at the optimum is precisely the shadow price on the constraint.

An elementary result is now apparent which essentially justifies the existence of cost–benefit analysis as a subject. Suppose that, in some economy, an objective function has been specified. A 'project' is proposed which will consume some goods and services and produce others. An age-old question is whether the 'market test' is a good test of whether the project is worthwhile: is it the case that the project is worthwhile if and only if it is predicted that the money values of the revenues will exceed the money values of the costs? It is almost a tautology that the answer is 'yes', but if and only if the prices used in calculating values are the correct shadow prices. By definition the shadow prices measure the 'worth' of a unit of each of the commodities as measured by the objective function. This is demonstrated formally by Drèze and Stern. Therefore a correct measure of the value of a nation's net national product must use shadow prices, see Dasgupta and Maler in this volume. As a corollary, if market prices differ significantly from market prices then a simple commercial, financial appraisal of the project will not give a correct answer. Therefore, if one believes that market prices always reflect shadow prices there is no reason to carry out cost–benefit analysis: straight financial appraisal will suffice. Cost–benefit analysts earn their living through the proposition that this is not the case.

Market prices and shadow prices

As a practical matter it is necessary to start with market prices as our first estimate of shadow prices where they exist. Very often they will be immediately acceptable as shadow prices. But it is not just a matter of convenience to use market prices as a starting point. Under certain circumstances market prices have desirable properties. In fact relative market prices are often excellent approximations to relative shadow prices. This result is one of the oldest and most famous results in economics. It has come to be known as 'the fundamental theorem of welfare economics'. Roughly speaking this states that, under the right conditions, a competitive market equilibrium will be an efficient allocation of resources and, conversely, any particular allocation of resources can be achieved as the outcome of competitive equilibrium if the initial distribution of assets to consumers is appropriate.

The bones of the first part of the fundamental theorem can be seen from the familiar, partial equilibrium, demand and supply model. Consider figure 3 which represents the market for a particular commodity. By definition the demand curve represents the number of units consumers *would like* to purchase at each price. The supply curve is the number of units the producing industry *would like* to sell at each price. We are particularly interested in the point of intersection of the two curves. This is an equilibrium because at this price the wishes of the two sets of agents are consistent with one another. The elementary analysis of markets predicts that there will be a tendency for this price to become established in a freely competitive market.

There is another way to look at this analysis. The demand curve can be thought of as the price that consumers are willing to pay for each quantity of the product. It therefore represents their marginal valuation of the product as we have already noted in the context of our bridge example (figure 1). In a competitive industry the supply curve is derived as the 'horizontal sum' of individual firms' marginal cost curves. It therefore represents the marginal cost of producing one more unit. Therefore, the point of intersection is special in that the value to consumers of an extra unit coincides with the cost of producing it. On the face of it the market price is the shadow price, providing that the objective is to maximize the value to consumers net of costs of supply.

To illustrate how this argument may break down, consider the case of environmental pollution by producers. (The following argument applies equally to other negative externalities like congestion, and, with an obvious modification, to positive externalities such as education.) If a firm creates pollution then we have to recognize a distinction between the cost to firms of their production (the *private cost*) and the total cost to society (the *social cost*). Consider figure 3.[7]

The additional feature here is the new curve representing the marginal social cost of supply. At any level of output the vertical distance between this and the supply curve is the money value of the additional social pollution cost of the

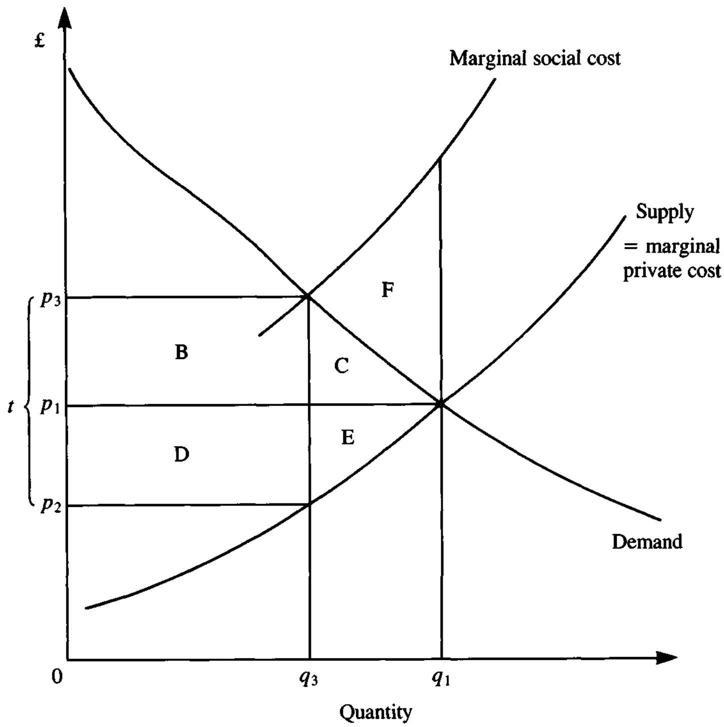

Figure 3

marginal unit produced. Therefore, the area C + E + F is the total social cost of pollution attributable to increasing production from q_3 to q_1.

The free market will set demand equal to supply; i.e., marginal private cost equal to the marginal willingness to pay, p_1. But this price does not equal the shadow price of the commodity. Consider a proposal to impose a tax t equal to $p_3 - p_2$. Consumers pay p_3 but producers now receive p_2.

Let us appraise this from the point of view of the consumers. They face a unit price increase of $p_3 - p_1$. The area B represents extra cash paid for the same units. The area C represents the loss to consumers because of the deterred consumption. These areas correspond to the areas (A + B) and C respectively of figure 1.

Now consider the point of view of producers. Since the supply curve is the marginal cost curve, the area below it is the total variable cost. Therefore the area D + E is the loss of profit due to the tax.

The area B + D is the product of t and the quantity traded, q_3, so it is the tax revenue. This represents benefits for others in the economy perhaps because it facilitates reduction in taxation elsewhere.

Taking all these items we find that there is a net loss from the tax of (B + C) + (D + E) − (B + D) = (C + E). If there were no pollution then that would be the end of it. We would have demonstrated that the imposition of a tax creates a net economic loss represented by the two triangles. This is known as the *deadweight loss*. It comes about because of the 'wedge' which is driven between marginal producer costs and producer benefits. For all units between q_3 and q_1 the benefits exceed the (private) costs.

However, in this case the situation is complicated by the presence of pollution. By reducing output from q_1 to q_3 social benefits of C + E + F are created. These exceed the deadweight loss to give a net gain represented by area F.

Notice that at the solution the tax has the property that the price paid by consumers is just equal to the total marginal social cost of their consumption. This is the correct shadow price for this commodity. Notice also that a proposal to impose this tax is silent on the issue of the distribution of the costs and benefits.

Incidentally, we can immediately see the solution to the classical problem of what charge to make for crossing an uncongested bridge – one of the first problems to be subject to this kind of analysis (Dupuit, 1844). If there is no congestion and if there are no other significant externalities, then the optimum price is the marginal cost. If the marginal cost of an extra crossing is zero, as in the situation illustrated in figure 1, then the optimum price is zero, as previously noted.

The matter, however, is not usually as easy as it has been made to appear so far. There are two main problems that arise in valuing net benefits when they occur.

1 For market items, market prices may not represent the true social value of the items. They may be either distorted (e.g., by taxes or monopoly) or there may be a market disequilibrium (e.g., unemployment or balance of payments troubles).

2 We need to devise methods of valuation (e.g., for time, recreational amenities, life and so on) for non-market items including public goods and the external effects of market items.

Both problems are handled by the use of shadow prices. The fundamental importance of shadow prices is shown by Drèze and Stern in this volume. In all economies the price structure is so distorted that it is impossible to estimate with precision the price for each item which would lead to the 'second-best optimum' choice and scale of projects, given the constraints. All that is possible is a partial approach in which the adjustments made to market prices make it more likely that the right decision is made than would be the case using unadjusted prices.

Some examples of market failures

Monopoly

Suppose first that our bridge project uses cement produced by a monopolist and sold at well above its marginal cost. Should we use its market price or its marginal production cost? The answer, as to all questions in shadow pricing, is that it depends on how one expects the rest of the economy to adjust when the project is undertaken. If the national production of cement is expected to increase by the full amount used by the project, then clearly it should be charged at marginal cost. If, on the other hand, no extra cement will be produced, then the project should value its cement at its alternative use value, which is given by its (market) demand price. If we expect a mixture of the two responses we use a weighted average of price and marginal cost.[8]

Indirect taxes

Similar principles apply when an input is subject to indirect tax. We use the producer's supply price if we expect production to increase by the full amount of the project's demand, and the demand price if we expect no growth in output. For a mixture of the two responses the relevant weighting depends on the elasticities of supply and demand as explained in Harberger (1969). Thus if the project consumes $a + b$ units of the input, where b is the number of extra units produced as a result of the project, the relevant price becomes approximately $a/(a + b)$ times the demand price (p^D) plus $b/(a + b)$ times the supply price (p^S), see figure 4.

This whole argument assumes of course that there is no economic rationale for the indirect tax other than the need to raise revenue. Where there is a rationale, such as the correction of external diseconomies (e.g., via fuel tax), the discouragement of 'merit bads' (like smoking) or the redistribution of income, no adjustment may be called for.

Unemployment

Suppose that the bridge is built by workers who would otherwise be unemployed. If there are no macroeconomic costs of employing them (such as more inflation, or reductions in private investment) the cost is not their wage but the value of their lost leisure. This is less than their wage, if they were involuntarily unemployed. Employing people on the project may mean reducing employment and output elsewhere, and the cost of a worker is then measured by his wage. However, in most countries there is a degree of structural unemployment, and the inflationary effects of employing an extra worker depend on who he is and where he lives. So different costing procedures are needed, relevant to the particular circumstances. In countries worried by sub-optimal investment, the effects on

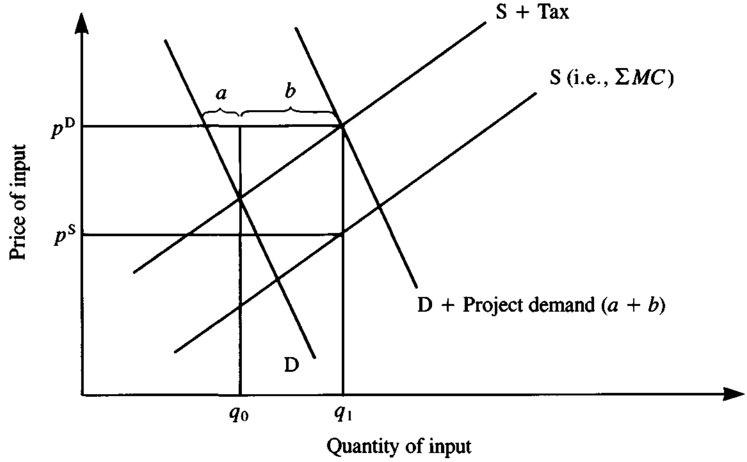

Figure 4

consumption of employing more people must also be allowed for (see Sen in this volume).

Pearce in this volume on investment in forestry illustrates how a shadow wage can be estimated and how the value chosen can affect the outcome of an evaluation.

Property rights

A market can only function if it is clear who owns what. Dasgupta and Mäler in this volume show how inadequate specification of property rights and lack of trust between potential traders can lead to important failures in environmental economics. For example, if a timber merchant purchases the right to deforest an upland then he will take no account of the fact that in doing so he may reduce the ability of the forest to restrain heavy rain, thereby increasing the risk of damaging floods to farmers in the lowlands. In principle the lowland farmers could get together and compensate the timber merchant for not damaging their interests. However, as the authors say, 'when the cause of the damages is hundreds of miles away and when the victims are thousands of impoverished farmers, the issue of a bargained outcome usually does not arise'. The result may be overharvesting of the forest which would be socially inefficient. They show how, because of this kind of market failure, different institutional arrangements for assigning property rights can lead to differing outcomes, some of which are less socially efficient than others.

Foreign exchange

Another thorny problem arises over the foreign exchange component of cost and benefits. This is often considerable, especially when indirect effects are allowed

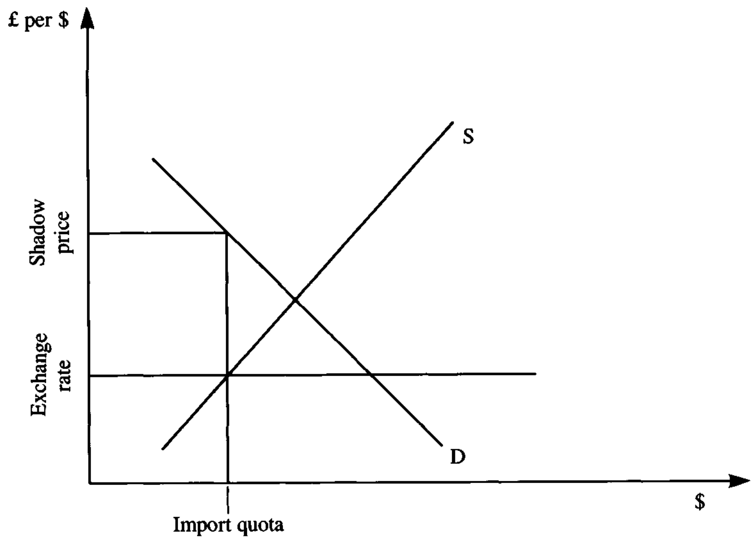

Figure 5

for. On our bridge, for example, the steel used may be imported; or, if produced at home, it may be made by imported equipment or it could be exported. Similarly, the extra journeys over the bridge may carry goods to the ports.

Now suppose that the country has an overvalued currency, causing a balance of payments problem which is handled by exchange controls. This means that the demand price for foreign exchange is greater than the official price which the recipients of import licences pay (see figure 5). How then should we price imports and exports? The standard approach is to use a shadow price of foreign exchange corresponding to the demand price rather than the official price. In this way imports and exports receive prices higher than their nominal prices, so as to measure their social value in terms of the prices of domestic (non-traded) goods. It is easy to see why imports have to be raised in value – their scarcity value is indicated by the demand price for foreign exchange. Export prices have to be raised *pari passu*, assuming that £1 worth of extra exports would be used to pay for £1 worth of extra imports.

However, Little and Mirrlees (1969) argue that it is far preferable to price exports and imports (and indeed all goods which in *optimal conditions* would be traded internationally) at their unadjusted prices, and to adjust downwards the value of 'non-traded' goods. (Strictly, imports are to be valued at their marginal import cost and exports at their marginal export revenue.) Their proposal really has two elements. The first is to take as numeraire the bundle of goods which a unit of domestic currency will buy on the world market rather than on the domestic market. This is a matter of convenience rather than of principle which Little and Mirrlees justify on the grounds that there are more traded than non-traded goods, which is probably the case for industrial projects with which their *Manual of Industrial Project Analysis* is concerned.

The second element of their proposal is that all goods which are 'tradable', that is that would be traded in optimal conditions, ought generally to be valued at world prices, even if they are not currently being traded freely because of non-optimal government policies. The purpose here is moralistic: to ensure that countries have the least possible excuse for overlooking the possible gains from trade. As is well known, a price-taking country producing two traded outputs maximizes its welfare by maximizing the value of its output valued at world prices and thus expanding its consumption possibly set to the maximum (OAB in figure 6). If the world price of guns is one ton of butter and a country has an import quota of guns raising their domestic price to 1.1 tons of butter, it should still not produce guns at any cost above one ton of butter unless it is determined to maintain the quota. Doing cost–benefit at world prices is intended to focus planners' minds on the cost of such distorted trading policies. Drèze and Stern demonstrate this point clearly.

Not surprisingly, the Little–Mirrlees proposals aroused a minor hornets' nest. Some critics consider them to have the wrong ideals. For example, Stewart and Streeten (1971) accuse them of an unduly piecemeal approach to development and of ignoring the linkages in the economy which necessitate a strategy of growth in which free trade may not be optimal. Sen, in this volume, though more in favour of the open economy approach to development, points out that, if this aim is not shared by other government agencies, that constraint should be allowed for in project appraisal. For example, if it would be rational to import a certain input but this is banned by the government, the input should in his view be valued not at its world price but rather at its domestic cost of production.[9] In the final chapter in part I of this volume Little and Mirrlees review the debate and discuss the impact that it has had on actual project appraisal.

Pricing of non-market items

With non-market goods and services, we do not even have a market price as a starting point. The marginal value which people place on a good (A), for which they do not directly pay money, has therefore to be inferred from the money they would have to pay for a good (B) which they show themselves to value equally. In this book we illustrate this approach to a few examples of the many items to which it can be applied.

Time

Suppose that our bridge will in fact save travellers time (say ten minutes) as well as money (the ferry charges). We have a good deal of evidence on how to value

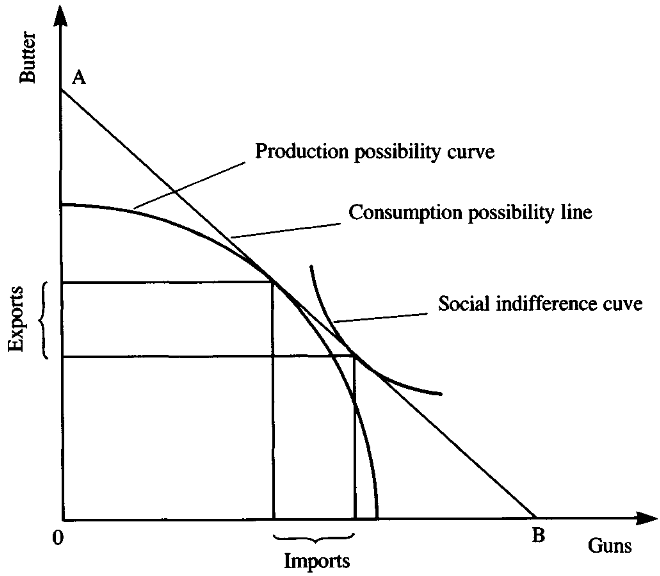

Figure 6

savings of time. Take leisure time first. A crude approach would be based on the theory of labour supply. This asserts that if a worker has freedom to vary his hours of work, the marginal value of one hour's leisure can be inferred from his gains from one hour's work, since at the margin he must be indifferent between the two. So one hour's leisure is worth the post-tax wage rate minus the marginal psychic disvalue of one hour's work.

One problem with this approach is that the disvalue of work has to be guessed. Equally problematical, an hour's travelling during one's leisure hours does not deprive one of an hour's leisure; it merely involves using leisure one way rather than another. We want to know the difference in value between an hour's leisure travelling and an hour's leisure in its alternative use.

We have therefore to find cases where individuals can choose to save travelling time at the cost of spending money. Such choices frequently arise in the choice between two methods of transport (say bus versus rail, on the journey to work). Suppose that everybody equally dislikes spending time travelling (T), in the sense that they agree on the money equivalent (say b) of a marginal minute spent at home rather than travelling. Then we can describe the costs (C) of the bus and underground journeys by the following expressions

$$C_B = a_B + bT_B + M_B$$

$$C_R = a_R + bT_R + M_R$$

where a_B and a_R are parameters reflecting the intrinsic pleasures of bus and rail respectively and T and M represent the time and money costs involved. The difference in these costs (ΔC) depends on where a person lives, and turns out to be a good predictor of the probability (P_B) of his travelling by bus rather than rail. One can for example fit a function such as the following

$$\log\left(\frac{P_B}{1 - P_b}\right) = B\Delta C = B(a_U - a_B + b\Delta T + \Delta M)$$

From it we can estimate b and hence the cost of commuting time in terms of its money equivalent.

In fact, if data can be obtained on the options facing *individuals* and the choices they actually make then it is more efficient in the statistical sense to develop models of individual behaviour. 'Behavioural (binary, disaggregate) choice modelling' has developed considerably over the 1970s and 1980s both in the theory and in the techniques available for collecting suitable data.

Of particular note has been the development of 'Stated Preference' techniques, where individuals respond to hypothetical questions, as distinct from 'Revealed Preference' techniques, where actual behaviour is observed. Stated preference methods have considerable cost advantages and allow the exploration of future options where no present-day analogue can be found.

Other advances have been in the fall in the cost of computing, the development of statistical models and the associated software. These have made easy things which would have been prohibitively expensive a few years ago.

These matters are explored in detail in this volume by MVA *et al.* It is shown how they have been applied in the context of value of time studies to derive estimate values for savings in leisure time in many different circumstances.

For time saved in the course of work the problem is simpler because the main cost is the loss of the worker's output, measured roughly by his wage. However, we should still take into account the worker's excess psychic (dis)satisfaction from travelling rather than working.

Recreational facilities

For recreational facilities too, attempts have been made to infer valuations from behaviour. Following the method developed by Clawson (1959), Mansfield (1971) estimated the value to the community of the English Lake District National Park countryside viewed as a recreational facility. For this we need to know the excess of the total value which people put upon their visits over the costs they incur in making them. Thus we need to know the demand curve for visits. Since people come to the Lake District from differing distances they incur differential costs in getting there. It seems reasonable to suppose that the number of visits from an area would be proportional to the population of the area, but that this proportion would depend on the costs (and also on car ownership per head of population in the area). Mansfield fits a demand curve using these variables for each area from which visitors come. For visitors from each area the surplus can be readily computed as the area under the implied demand curve. The total surplus is simply the sum of these surpluses.

There are many problems with the method, in particular its failure to include the price of substitute recreational facilities in the regression analysis. (The reader may like to work out for himself the direction of bias which would result in the estimated surplus, supposing, as seems likely, that the price of substitutes rises as the cost of reaching the Lake District rises.) In addition, as Mansfield points out, it is very difficult to use evidence on existing facilities to evaluate new and potentially competitive ones, such as a lake trapped by a barrage across nearby Morecambe Bay.

Cost-effectiveness analysis

There is a wide range of non-market items which can be valued in one way or another. For example, the cost of noise is discussed in Mishan (1970). Nevertheless, there remain other goods for which no meaningful valuation can be made especially pure public goods, which can jointly benefit many people and where it is difficult to exclude people from the benefits. Owing to the difficulty of exclusion, there is no way of getting people to reveal their valuations of such goods. Whenever cost–benefit analysis becomes impossible, since the benefits cannot be valued, it is still useful to compare the costs of providing the same beneficial outcome in different ways. This is called cost-effectiveness analysis and is regularly used in defence, public health and other fields.

Avoiding the problem of the value of saving life

Perhaps the most difficult item of all to value is human life. Yet it is quite clear that countless policy decisions affect the incidence of death. So each decision implies some valuation of human life.

Since valuation of life is a difficult and controversial subject it is worthwhile asking how far one can go in analysing safety without having to make this judgement. The answer is: quite a long way!

Consider the following simplified problem. A manager in a health service is given a fixed budget. There are several quite different ways in which that budget might be spent: renal dialysis units, heart surgery, hip replacements, preventative screening, health education, etc. Assume that accurate estimates are available of the number of lives that would be saved by each kind of expenditure, related to each level of expenditure. The following objective is set – 'allocate the budget so as to maximize the total number of lives saved'. (Wagstaff in this volume discusses some more sophisticated objectives that have been applied in practice.)

The reader may wish to reflect on the characterstics of the solution to this problem. At first sight it might seem attractive to spend a large proportion of the budget on areas of treatment of conditions which cause a large number of deaths.

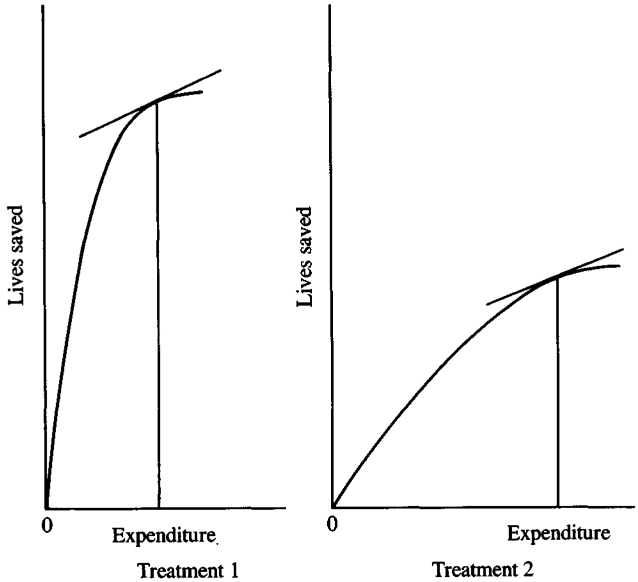

Figure 7

This is not necessarily correct. It may be that increasing expenditure on such a treatment would have little effect on the death rate. Or to put it another way, it may be that the expenditure per life saved is very high. There may be other ways of spending the money which will save more lives per £1 saved.

If the problem has been solved it must be the case that the number of extra lives saved per £1 of extra expenditure is the same for every kind of treatment. Otherwise it would be possible to spend less on one treatment, transfer the funds to another treatment and end up with a net increase in lives saved with the same total expenditure budget.

The solution is illustrated in figure 7 in a case where there are two treatments. The slopes of the curves must be equated to each other and to a value such that the implied expenditures exhaust the total available.

Several points emerge from this simple analysis. If we observe areas of public policy where these margins are not equated then it must be the case that there are more lives being lost than necessary, given the overall level of public expenditure. Yet it is a commonplace that safety regulations and other public expenditure decisions imply very widely varying valuations per life saved. Moody (1977) cites values ranging from £50 per life, implied by the decision not to spend on a particular medical screening programme to £20,000,000 per life saved by a change in building regulations following the collapse of a block of flats caused by a gas explosion.

Second, in many fields it is not even possible to investigate the subject simply because the technical relationships – like those drawn out in figure 7 – have not been established. For instance, design engineers in the transport industries will often say that they have worked in accordance with professional guidelines and standards which assure safety. Yet, it is not possible to evaluate the safety of the designs because no one is able to state the change in accident rate and the change in cost as the various characteristics of the designs are changed.

If our problem has been solved then we know how many lives would be saved if an extra £1 were spent. This is the shadow price on the budget constraint for this problem. It gives us a way of judging whether a proposed *new* way of spending money is going to be worthwhile – it must offer at least as many lives saved per £1 of expenditure as the established shadow price.

A great deal can be achieved by cost-effectiveness analysis. Note that all of this discussion has avoided the need to specify an *ex ante* value of life. But *ex post* a value is implied – the shadow price. It is this which should be considered when decisions are made about whether the overall budget should be increased. However, sooner or later we have to face up to the fact that we cannot work out the overall worth of projects which affect safety (and most of them do) without deciding what value to give to saving lives and avoiding injuries.

The value of saving life

Suppose that each worker on our bridge has a 1 per cent probability of being killed on the job, and there are twenty-five workers. One approach would be to cost the risk of loss of life at 25 per cent times the (average) human capital value per worker. But this immediately raises the question of 'value to whom?' Weisbrod (1961) distinguishes two measures, without committing himself to saying which is relevant to public decision. The first consists of the present (discounted) value of a person's future lifetime production, measured by his earnings. This measure implies that production is equally valuable, regardless of how many people there are to consume the fruits of it. Alternatively he assumes that all that matters is consumption per living person, in which case the value of someone's life is measured by the effects of his death on the welfare of those who would survive him. This equals the excess of his expected future production over his future consumption, again discounted to the present.

However, neither of these approaches is very satisfactory. The former would, in most countries, support population policies which maximized the birthrate, while the latter would support policies which minimized it. An alternative that has been proposed is the use of life insurance values, but these could only reflect, if anything, a view of what dependants would need in the event of the death of the insured. It does not tell us anything about the benefits of a change in the probability of death occurring – and that is what a project does change.

Mishan (1971b), after reviewing the methods that have been used, concluded that there are three cost consequences of a policy decision which affects the probability of a person's (Jack's) death: First, there is the way in which Jack himself values the extra risk. If he voluntarily assumes the risk, this value is already included in the price he requires for accepting it. This is why bridge builders get relatively high wages, and there is no need to make any additional allowance for the risk cost to them of their work. If, on the other hand, the labour was conscript we should have to find a way of valuing the risk. Likewise, we should allow for any risk imposed on people living beneath the bridge who might get killed. The second item is the value which others put on the financial consequences for them of Jack's increased risk of death. This may bear some relation to the measure of his future production less consumption. Where Jack has allowed for this in the terms on which he accepted the risk, it should not of course be double counted. Finally, there is the psychic value to others of the increased risk of Jack's death, for which again any double counting must be avoided.

The key feature of this approach is that it does not value death (or life) as such, but only change in the probability of death. All the valuations are thus *ex ante* and no attempt is made to value the suffering which will actually (*ex post*) be caused if a typical worker dies.

Is this right? Suppose, for example, that the suffering which a worker's partner will experience if he or she dies is equivalent to $-£x$. The expected *ex post* suffering in our example is then $-£0.25x$. What is the *ex ante* valuation of it? If the partner values risky outcomes in terms of expected utility and knows that the probability of the partner's death is 0.01 and that its utility would be $-£x$, then his or her *ex ante* valuation of the risk is $-£0.01x$ and the overall *ex ante* valuation (of all twenty-five partners) is $-£0.25x$. The *ex ante* valuation is thus the same as the *ex post* one quoted above. But individuals may instead be ill-informed about the probability of death or may not make a realistic estimate of the suffering they would experience in the event of their partner's death. In this case what does the planner put into his calculation: his own expected *ex post* evaluation or the *ex ante* one expressed by the individuals? The answer to this quite general question depends on the degree of paternalism the planner is willing to exercise. If he supports the prohibition of taxation of 'merit bads', like cigarette smoking, on the grounds that the government can foresee the suffering these will cause better than the individual can, then it seems quite logical to adopt the same approach in cost–benefit analysis. The present valuations of the parties affected are then disregarded in favour of their future valuations. How often one would be willing to do this is debatable, but it seems absurd to argue that one should always regard people's present preferences as decisive.

Reverting to the valuation of life, it remains clear that *ex ante* states of mind are relevant in the sense that, if a husband's life is blighted by the fear of his wife's death, this needs to be taken into account, quite independently of how he would value the actual event of her death. As far as the individual who may die is concerned this fear of death is in fact the only element that enters the calculation, since she will feel nothing once she is dead.

So where does this leave us? How could we calculate the various magnitudes? The answer is that it is very difficult, but, as Mishan points out, it is better to know what it is one should know, even if one cannot know it, than to know something that is irrelevant.

Rosen, in this volume, shows how to interpret labour market data as a reflection of differences in risk. Jones-Lee gives a comprehensive survey and critique of the various estimates that have been made. He reports some of his pioneering work in developing stated preference techniques in this area in the book from which the extract in this volume is taken. In an area where there is so much doubt and room for debate it is not surprising that there is a range of estimates to choose from. What *is* surprising is that the evidence, taken as a whole, strongly suggests an order of magnitude for the value of saving a life: a 'reasonable' value (at 1992 prices) might be £500,000 per life or £2 million per life, but it would not be consistent with the evidence to use a value of £20 million per life saved. In safety appraisal getting the correct orders of magnitude is often all one can aspire to. The literature can now offer a solid contribution to that.

3 THE SOCIAL TIME PREFERENCE RATE AND THE SOCIAL OPPORTUNITY COST OF CAPITAL[10]

Suppose we have satisfactorily computed the net benefits of a project in each future year. So we know $B_0 - C_0$, $B_1 - C_1$, . . ., $B_n - C_n$, where B indicates benefits, C costs and n is the number of periods during which the project produces its effects. We still have the problem of aggregating these, to see if they are positive in total. If we have done the job properly, these net benefits represent the changes in consumption brought about by the project, for it is only consumption (rightly defined) that ultimately affects human welfare. So we need to know the value of next year's consumption relative to this year's, and so on for all future years. This relative value is expressed in terms of a time preference rate, or (the same thing) a discount rate. If this rate is 5 per cent per annum, this means that £1 of next year's consumption is worth only about £0.95 of this year's. More generally, if r is the rate, we value each £1 of next year's consumption the same as £$1/(1 + r)$ of this year's, r being from this point of view the rate at which we discount next year's benefits. Or, to put the matter the other way round, £1 of this year's consumption is worth £$(1 + r)$ of next year's, r being the rate by which present consumption is preferred to future consumption. As Feldstein (1964) points out, there is no *a priori* reason why the rate at which we discount 1994s pounds to make them equivalent in value of 1993s should equal the rate at which we discount 2004s pounds to make them equivalent to 2003s. But his assumption is almost invariably made. If we make it, the value of a project (valued in terms of today's consumption) becomes

$$PV = B_0 - C_0 + \frac{B_1 - C_1}{1 + r} + \frac{B_2 - C_2}{(1 + r)^2} + \ldots + \frac{B_n - C_n}{(1 + r)^n}$$

and the project should be undertaken if it has a positive value. The project could, of course, be valued in terms of pounds in year n (by multiplying the present value by $(1 + r)^n$) or in any other year, but this makes no difference to the decision, since if the value is positive in one year it is positive in all other years and vice versa.

The problem is: How do we choose the discount rate? This is an issue of considerable practical importance. In poor countries in particular, the choice between present and future consumption is truly agonizing. How far is it legitimate to depress present levels of living for the sake of the future? The horizon for a road scheme might well extend to forty years into the future. A benefit valued at £100 in forty years' time will have a present value of £14.2 at a discount rate of 5 per cent, £4.6 at 8 per cent and only £2.2 at 10 per cent! A high rate of discount will count against projects which take a long time to gestate. For instance, the case of forestry is discussed by Pearce in this volume. As Pearce notes, it is not legitimate to use an artificially low discount rate just because of this consideration.

To think about this question it is most useful to begin with a single-person economy (Robinson Crusoe) and to apply to his situation the tools of capital theory developed by Irving Fisher.[11]

Suppose that Crusoe's consumption consists only of fish and that he knows he will live for only two years. At present he spends all his time fishing with his existing equipment and catches 100 fish a year. He will do the same next year unless he takes time off to improve his equipment. If he does some investment of this type, he opens up the production possibility curve shown in figure 8. This schedule consists of a series of projects arrayed from right to left in order of their rate of return – the cost in year 0 is the sacrifice of consumption involved and the return is the gain of consumption in year 1. Only one of these projects is shown separately in the diagram – the most profitable project, costing two fish now and yielding six in the following year.

Crusoe chooses from the various production possibilities so as to reach his highest inter-temporal indifference curve. In other words he calculates the amount by which he can increase future consumption by sacrificing a further unit of present consumption and invests until this is equal to the minimum amount of extra future consumption required to induce him to sacrifice a further unit. The first of these amounts is the (absolute) slope of the production possibility curve. If B_1 is the benefit of the marginal investment project and C_0 is its cost then its value corresponds to B_1/C_0. This can be written as $[(B_1 - C_0)/C_0] + 1$, or $\rho + 1$, where ρ is the (marginal) rate of return on investment. The second is the slope of the indifference curve. Its value is MU_0/MU_1, which can be written $(MU_0 - MU_1)/MU_1 + 1$, or $r + 1$, where r is the (marginal) rate of time preference. So the

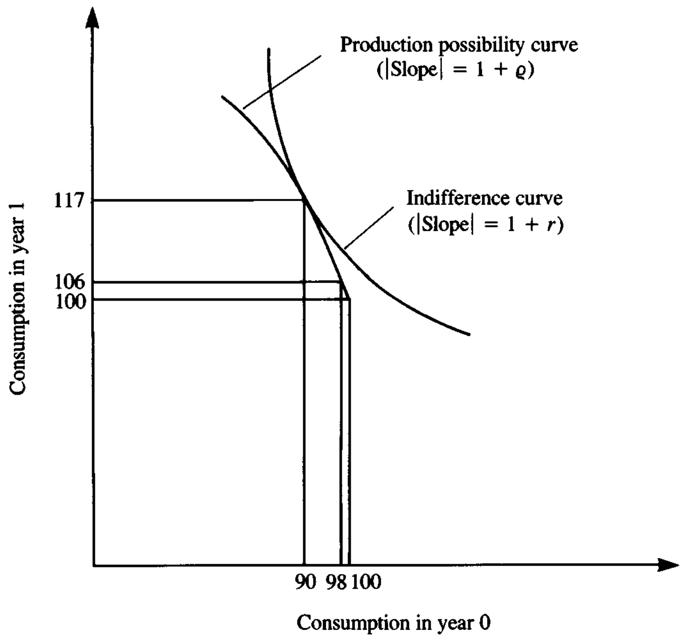

Figure 8

process of choice involves equating the rate of return on capital to the rate of time preference.

This choice simultaneously determines not only the discount rate but also the rate of saving of the economy (here 10 per cent, 10/100) and its rate of growth (here 17 per cent, 17/100).[12] Any reader who thinks it is right to maximize the rate of growth of an economy should think again. That is done by nearly starving. The real problem is to decide on the right rate of growth.

If the level of investment and its consumption is determined simultaneously with the discount rate, one might well ask what is the use of the discount rate in individual project appraisal? The answer, as mentioned earlier, is that in the real world there are many potential decision makers who, though they cannot see the overall scene, have, taken together, a much clearer view of the detailed possibilities than could possibly be comprehended in a central planning agency. There is therefore a strong case for decentralized decision making, provided the central government sets prices (including the discount rate) and decision rules to the individual agencies which ensure consistency in their decisions.

There remains, however, the acute problem of finding a figure for the discount rate. Three main approaches have been suggested.

1 Use post-tax interest rates on long-term risk-free bonds.
2 Make assumptions about the desired growth rate and the inter-temporal indifference map.

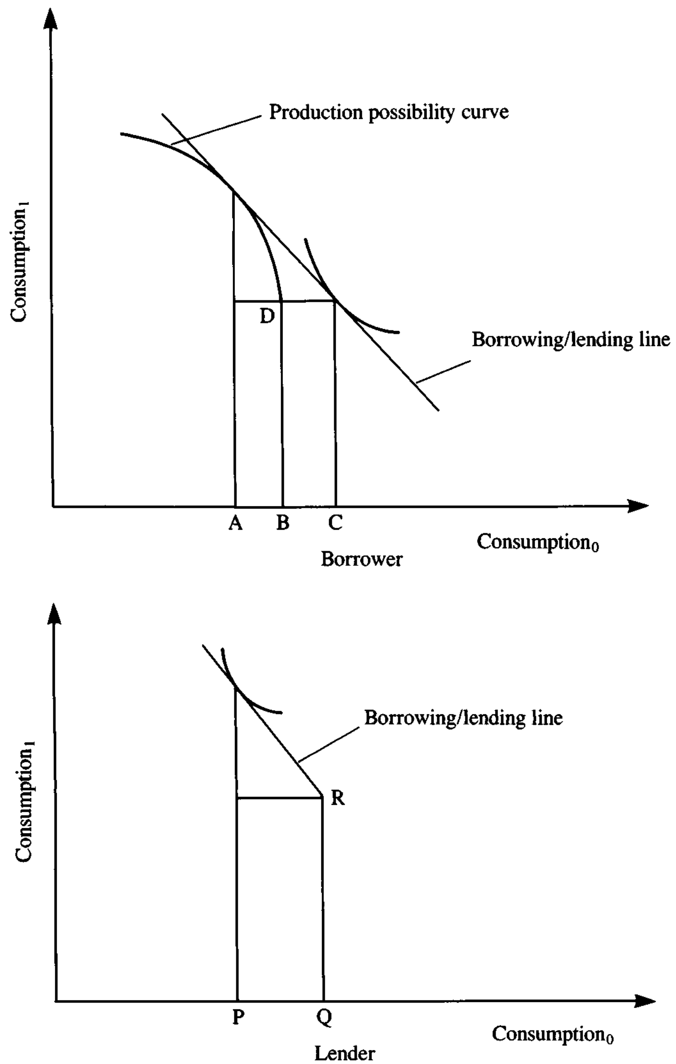

Figure 9

3 Make assumptions about the desired growth rate and the production possibility curve.

The market rate of interest

In a mixed economy the most obvious indicator of time preference is the market rate of interest. To see how this is determined in a perfectly competitive Walrasian system of general equilibrium we modify the economy of Robinson Crusoe in two ways only: we multiply the human species, and we allow for the possibility of borrowing and lending. The interest rate then adjusts until it equals simultaneously the rate of time preference of all individuals in the society and the rate of return on productive investment (see Stigler, 1966, or Stiglitz in this volume). This is illustrated in figure 9. The representative borrower, with initial endowment indicated by point D, invests AB and borrows AC and thereby increases the consumption available in year 0 whilst leaving consumption in period 1 unchanged. The representative lender, initially endowed at point R, lends PQ. If there are as many borrowers as lenders, QP must equal AC – the function of the interest rate is to equate the two, and it varies until they are equal (compare Dasgupta in this volume).[13] At their equilibrium levels of consumption the borrowers' and lenders' rates of time preference are equated to the interest rate. So does this mean that the interest rate indicates the true rate of social time preference?

This is a subject of controversy, but before we embark on the controversy there is one point on which everyone is agreed. If there is inflation the money rate of interest will be adjusted to some extent to allow for the expected rate of change of prices. But we are interested in the subjective rate of substitution between real consumption in one period and another, and so we need to find a 'real' rate of interest which reflects this.[14]

If the money interest rate indicates that people are indifferent between £1 now and £1 + i next year, it follows that, if I expect prices to rise by a proportion p during the year, I must be indifferent between £1 worth of goods now (at today's prices) and the goods which £$(1 + i)/(1 + p)$ would buy tomorrow if today's prices still prevailed. This last figure equals approximately £1 + $i - p$, so that the $i - p$ is the real rate of time preference and the real rate of interest. How price expectations are formed is something we only partially understand. If expectations equalled actual rates of price change, one would have to conclude that the real rate of interest in Britain in 1971 was negative! What can be said quite dogmatically is that money interest rates must be substantially reduced in inflation to obtain a measure of the real rate of interest.

This much is agreed, but there remain many problems with using even an adjusted interest rate as a measure of society's time preference.

Uncertainty and imperfect capital markets

The future is uncertain and this leads to a multiplicity of interest rates depending on the credit risks involved. Which rate should be used in cost–benefit analysis? It would of course be possible to vary the discount rate according to the riskiness of the net returns of the project, but it is preferable to express all net returns in terms of their certainty equivalent. We then need a standard discount rate relevant to certainly known levels of consumption. This is why the most commonly used estimate of private (and social) time preference is the rate on long-term government bonds (reduced to allow for expected inflation and income tax payments).

These bonds are generally considered free of risk. However, since the future price level is uncertain, there is in fact no truly risk-free rate of interest, nor are there any long-term bonds whose nominal yield is independent of unknown future interest rates, unless they are held to maturity (Arrow, 1969). Nevertheless it remains true that, for the ordinary lender, the adjusted long-term bond rate probably gives as good evidence as we are likely to get on his risk-free time preference.[15]

But whether this is what his time preference would be, given an optimal allocation of resources, is another matter. The fact of uncertainty means that, in the absence of a complete system of futures markets, market prices are of limited welfare significance. For the savings and investment decisions of each individual depend upon his forecast of future incomes and prices, yet these will themselves depend on the decisions made by others. This some believe to be a potent source of underinvestment.[16]

Moreover, the imperfections of the capital market mean that the time preferences of borrowers will almost always exceed those of lenders. In principle, the discount rate for each project should be a weighted average of the time preferences of all those affected by the project (Musgrave, 1969).[17]

The influence of money on interest rates

Interest rates have also been criticized on the grounds that they reflect monetary as well as real forces, and therefore should not be used for decisions of real resource allocation. There are, of course, some who maintain that money cannot alter real rates of interest except where there are unanticipated changes in the rate of change of money creation. But most economists would accept that money creation can change real interest rates, through its effect on prices and thus on the public's perception of its real wealth (Metzler, 1951; Mundell, 1963). Given these influences, do interest rates retain any normative significance?

This, as usual, is a question of which constraints are to be taken as given. There must, in principle, be an optimum quantity of money, which could be solved for simultaneously with all the other magnitudes in the economic system, including the implied rate of interest. However, in all probability we do not have the optimum quantity, and project planners, unless they are trying to secure changes in monetary policy, must accept whatever preferences the monetary authorities have helped to create.

Myopic (or 'pure time preference')

A third problem with interest rates is that they reflect each individual's *ex ante* anticipation of the relative value of his future consumption. But individuals may underestimate the pleasure which future consumption will in fact give them, if, as Pigou (1920) alleged, they are victims of 'defective telescopic faculty'. According to Pigou, an individual with the same commitments in both years must, in fact, be as well off if he consumes £100 in year 0 and £150 in year 1 or the reverse. That is to say, his indifference map in figure 8 ought to look the same whichever year's consumption is measured along the horizontal axis. So the indifference curves should be symmetrical about the 45° line from the origin, with the time preference rate being zero when consumption in the two periods is equal. But in fact Pigou surmises that people have 'pure time preference': they value present consumption more highly than future consumption even on the 45° line.

The problem with myopia lies partly in telling whether people have it, though there are certainly stagnant economies with higher interest rates than can be readily explained by risk and capital market imperfections. But even if people do have it and their *ex ante* evaluations fail to equal what the planner knows they will feel *ex post*, there is still the question of how far the planner should impose his own long sight upon the myopic multitude. Marglin (1963a) adopts the view that it is undemocratic for him to do so. But cost–benefit analysis is not in itself a 'democratic' process. It is a thought process which attempts to throw light on what is right, given our knowledge of the consequences of certain actions. If the planners really knew there was myopia they should surely use this knowledge. The main argument against overriding current preferences is that planners may be unlikely to know better than the individuals actually affected, and this argument is a strong one.

External effects in consumption

The problems discussed so far relate to the ability of interest rates to produce an optimum, assuming that all that matters is the welfare of individuals now living and that each individual's welfare is affected only by his own family's consumption stream. But, the reader may say, what about the welfare of future generations? After all, public investments may augment or reduce the consumption of people yet unborn. Surely this should be taken into account?

There are two approaches to this problem. In the first we continue to value only the welfare of those now living, but recognize that this is affected by the consumption available to future generations. Now if my welfare depended only on the consumption available to me and my own heirs, there would be no problems. But most of us do in fact derive pleasure from contemplating the welfare of others and their heirs, and this poses a classic case of externality. For in a free market these external effects are ignored and individuals maximize their own utility, taking the savings decisions of others are exogenous to their own decisions. But if they could bargain or take a collective decision *via* the political process a better outcome could be obtained.

The originial argument about this 'isolation paradox' was propounded by Sen (1961), developed by Marglin (1963a), criticized by Lind (1964) and reformulated in a more general form by Sen (1967). His argument is this.

Suppose each individual in society makes the following valuation of one unit of consumption according to who consumes it:

consumed by him now 1
consumed by his heir γ
consumed by others now β
consumed by others' heirs a

Suppose also that one unit of consumption foregone by the present generation leads to k units extra of consumption by the next.

(a) Consider the equilibrium of a competitive market in which each individual makes his own independent saving decision. Assume that, if an individual saves one unit, his heir will benefit by λk and others' heirs by $(1-\lambda)k$, λ depending on the level of taxes, such as death duties. The individual will save till his future return from saving equals his present sacrifice, so that in equilibrium

$$\lambda k\gamma + (1-\lambda)ka = 1$$

The marginal rate of return on capital $(k-1)$ is thus given by

$$k = \frac{1}{\lambda\gamma + (1-\lambda)a}$$

(b) Now suppose that whenever one individual saves one unit, every other individual does the same. This could be the case for example if all investment was financed by taxes and all individuals were equally rich. And suppose the decision on whether everyone shall save one more unit is taken by referendum. Then each individual will vote for more saving until the future return (in terms of his own utility) from such saving equals his present sacrifice. If there are N members of the community, each with one heir, each individual's heir can expect to get a $(1/N)$th share of the Nk units of extra future consumption; so in equilibrium

$$k\gamma + (N-1)ka = 1 + (N-1)\beta$$

In this case the marginal rate of return on capital $(k-1)$ is given by

$$k = \frac{1+(N-1)\beta}{\gamma + (N-1)a}$$

The vital question is whether this equilibrium rate is the same as under competition (i.e., as in (a)). Sen's argument is that it would be pure chance if it was and in fact he thinks it is lower. It certainly would be if both $\lambda < 1$, as it would if inheritance taxes spread the proceeds of saving, and $1/\beta > \gamma/a$, as it would be if people were more egoistic when comparing their own consumption with their own contemporaries' than when comparing their heir's consumption with their heirs' contemporaries'. And in this case a free market will produce underinvestment and an excessive interest rate. Whatever one thinks of the values of the various parameters, the approach at least draws attention to the issues raised by external effects in consumption. However, none of those who have argued on these lines has suggested how one would actually get a social discount rate which did allow for these external effects.

The welfare of future generations per se

Nor have we yet considered what allowance should be made for the welfare of future generations *per se*. Is it really right that projects should be judged exclusively in terms of their effects on the welfare of those now living? Most economists would say 'Yes' on the grounds that cost–benefit analysis should be democratic.[18] However, if one takes the alternative view that cost–benefit analysis aims to throw light on what is right, it is difficult to think of any ethical justification for ignoring future generations.[19] Suppose, for example, that one were to subscribe to the widely held ethical position that the best course of action is that which has consequences one would prefer if one had an equal probability of being each of the people affected by the action.[20] Then clearly future generations must be taken into account in so far as they are affected by the action. But how would one do this by means of a discount rate, when it is essentially a problem of income distribution? The answer is that conceptually the discount rate is an untidy way of proceeding. We should really work out the value of the project to each of the people affected (including the unborn), choosing as the unit for each person £1 of his consumption at some specified point in his life. We should then aggregate across persons using as weights for each individual the relative social value of his consumption at that point in his life. However, suppose that we regard 'me today' as a different person from 'me tomorrow'. Then all inter-temporal problems become distributional. If we can find a satisfactory general solution to the problem of distribution we shall also have solved the inter-temporal problem. This, in a sense, is the approach of 'optimal growth theory' to the discount rate problem.

Making assumptions about the social inter-temporal utility function

The theory of optimal growth, stemming from the seminal work of Ramsey (1928),[21] assumes that there is a cardinal utility-of-consumption function $U(C)$ common to all persons, and that total inter-temporal welfare consists of an aggregation of the utilities enjoyed by each person in each period. So, beginning again with Robinson Crusoe, his total inter-temporal welfare is[22]

$$W = U(C_0) + U(C_1)$$

The absolute slope of the indifference curve is

$$\frac{\partial W/\partial C_0}{\partial W/\partial C_1} \quad \text{or} \quad \frac{U'(C_0)}{U'(C_1)}$$

Now, suppose we assume that the marginal utility of consumption falls steadily, in such a way that for each 1 per cent increase in consumption the marginal utility falls by ϵ per cent. Then $U'(C) = C^{-\epsilon}$.

The slope of the indifference curve is now

$$\frac{U'(C_0)}{U'(C_1)} = \frac{C_0^{-\epsilon}}{C_1^{-\epsilon}} = [C_1/C_0]^{\epsilon} = (1+g)^{\epsilon} \simeq 1 + g\epsilon$$

where g is the proportional rate of growth of consumption.[23] So the discount rate equals $g\epsilon$, which is the rate of growth of consumption times the elasticity of the marginal utility of consumption with respect to consumption. If our economy were growing at 2 per cent a year and the elasticity of marginal utility with respect to consumption were, say, 1.5, the discount rate would be 3 per cent. The faster the rate of growth of the economy the higher the appropriate discount rate, since future income is that much higher than present income and therefore that much less valuable at the margin.

To many people this approach seems to provide the only valid rationale for discounting, rather than investing until the marginal rate of return is zero.[24] Geometrically, it provides an explanation for the convexity of the indifference curves in figure 8, if one remembers that the rate of growth of the economy is indicated by the slope of a ray through the origin to the point in question on the curve.

Dasgupta, in this volume, illustrates the ethical problems posed by the utilitarian approach. He shows that on certain assumptions any positive rate of impatience, no matter how small, will imply that it will be optimal to allow an economy with exhaustible resources to decay in the long run, even though it would be feasible to avoid the decay. Current generations will not accumulate sufficient capital in the early years to offset the declining resource use inevitable in later years, at the costs of later generations' welfare.

The practical importance of these arguments is illustrated by Pearce in this volume. In the context of renewable resources he records the view held by some that 'sustainability' is somehow an ideal to strive for in determining the rate of extraction. But as Dasgupta and Dasgupta and Mäler (both in this volume) note, there will, in principle, be an optimal rate of depletion and there is generally no reason to think that the historically determined stocks of renewable resources happen to be at an optimal level or that the level should be sustained at its present level: 'Whether or not policy should be directed at expanding environmental resource bases is something we should try and deduce from considerations of population change, intergenerational well-being, technological possibilities, environmental regeneration rates and the existing resource base.'

So much for Robinson Crusoe. If we turn to an economy of many persons we should presumably calculate social welfare as some aggregation of individual utilities. Supposing, however, that at each point in time income is equally distributed. Then social welfare in a particular period depends on the utility of consumption per head. But is this all or does it also depend on the number of people enjoying that consumption per head? This is a matter of controversy, which profoundly affects one's approach to the problem of optimal population. However, as far as the discount rate goes, it is enough to note that with a static population the discount rate is the same whichever we assume. With a population growing at rate n the time preference rate is $(g - n)\epsilon + n$, if welfare depends only on the utility of consumption per head; if it depends on that times the number of people, the rate is $(g - n)\epsilon$.[25] Which ever assumption we make, the discount rate is lower the faster the rate of population growth, provided ϵ exceeds unity.

How can we determine the magnitude of ϵ? It can be regarded in two separate lights. One can take the view that there is no place in positive economics for a utility function which says by what proportion a person prefers A to B. And if the same utility function is held to apply to more than one person, this is said to be even more unreasonable, involving as it does inter-personal comparisons of utility. From this point of view therefore the utility-of-consumption function becomes a purely normative device for representing one's ethical position – it is, if you like, a particular form of Bergsonian social welfare function (see Bator, 1957).

Alternatively, it can be argued that we have, from the fact of insurance, some positive evidence that individuals do compare the psychic loss from losing their insurance premium with the psychic loss from losing their houses (Friedman and Savage, 1948). The trouble is that, though the fact of insurance provides evidence of a diminishing marginal utility of income over some range, the fact of gambling suggests that for some people the marginal utility is increasing over some range. We can therefore get no estimate of ϵ from behaviour towards risk. An alternative approach is one in which the static utility function of the individual is assumed to be additive in two of the items of consumption, such as food and other goods, i.e., the marginal utility of food is independent of the quantity of other goods and vice versa. If this highly questionable assumption is made, the elasticity of the marginal utility of consumption can be calculated from the price and income elasticities of food and from its share in income. The kind of values which are found for ϵ in this way tend to fall between 1 and 2.5 (Fellner, 1967).[26]

Clearly, the use of any single value of ϵ is exceedingly arbitrary. Some people's marginal utility of income may indeed rise over some ranges making ϵ negative; and inter-personal comparisons, though they conform to some of our deepest intuitions about the human situation, are bound to be based on the most intuitive of judgements. Most of those who use the utility-of-consumption function would regard it as having some basis in positive reality, but as being so incapable of empirical investigation that the value assumed for ϵ is essentially normative: the larger ϵ the more egalitarian the approach.

And what of the rate of growth of consumption (g)? The problem here is that this is one of the variables which in principle we want to optimize at the same time as the discount rate (see figure 8). It is of course true that once we have chosen the growth rate we have settled the rate of time preference. But then, assuming our initial decision was correct, we have also settled the marginal rate of return on capital, making it equal to the rate of time preference.

Making assumptions about the production possibility curve

This has led Marglin (1963a) to put forward as an alternative approach the following strategy of decision. First decide on the growth rate you want and then see what investment that would require (see figure 8). If this is unacceptable, consider different combinations of investment and growth till you have found the one you prefer. This will imply a marginal rate of return on capital, which, since the choice is optimal, must by definition equal the rate of time preference. Use this rate as the discount rate. Critics have argued that this approach is impractical (Prest and Turvey, 1965) but it is less likely to produce inconsistent decisions than the above approach, where the rate of growth (g) and the discount rate ($g\epsilon$) are separately decided, independently of the production possibilities which must in fact link them.

So we are left with two main alternatives. The first allows adjusted market interest rates to determine the government's discount rate and hence the rate of investment and the rate of growth of the economy. The other starts by deciding on the rate of growth and infers from it the appropriate discount rate for use in decentralized decision making. The latter is widely regarded as in-feasible, and most cost–benefit analysts have followed some procedure where interest rates are taken as starting points for a measure of the social rate of time preference.

The social opportunity cost of capital

However, all the discussion so far has been based on the assumption that the government can bring the economy somehow to the optimal rate of saving and investment, either by inducing the private sector to act or by acting itself. But this may be impossible for various reasons, in which case the cry will be heard, as it is in so many countries: too little growth. We now have a problem in that capital investment, and hence investment displaced by the current project, is socially more valuable than consumption of equivalent monetary value. So if the resources used on the current project would otherwise have been producing investment goods valued at £x in the market, the cost of these resources is greater than £x, since £1 of investment is worth more than £1 of consumption and we choose to measure values in units of consumption.

Let us take a concrete example. Suppose certain products (say shirts) are only produced in the private sector and the government is not permitted to invest funds in the private sector. Suppose too that long-term lending rates (after income tax) accurately reflect private time preference. Nevertheless, owing to Income Tax and Corporation Tax, investors will require from investments in shirts a pre-tax rate of return (ρ) that far exceeds their rate of time preference. The government now considers building our bridge, financing it by borrowing on the capital market, and this borrowing will displace an equal amount of investment in the shirt industry. At what discount rate should the bridge project be evaluated?

The answer is simple. The stream of benefits (consumption generated) and cost (consumption displaced) should (by definition) be discounted at the rate which represents the social value of consumption in different periods, i.e., at the rate of social time preference (r). But what is the consumption stream foregone when £1-worth of resources are diverted from investment in the shirt industry to bridge building? The investment would have had a rate of return of ρ, so the stream of consumption from an investment of £1 can be approximated by the 'permanent' income stream, ρ, ρ, ρ and so on for ever. The present value of this stream when discounted at the time preference rate (r) is ρ/r.[27] So each £1 of cash spent on the bridge should now be costed at £ρ/r (Marglin, 1963b). But the social discount rate remains r, and the present value of the project, where C is the initial money cost and B the additional consumption generated per year for ever, is

$$V = -C\frac{\rho}{r} + \frac{B}{r}$$

The project should be undertaken if this is greater than zero.

The reader will see that this requires that

$$-C + \frac{B}{\rho} > 0$$

In other words, if the costs are counted in money terms (unadjusted), we get the correct decision if we then compute the present value, using ρ as the discount rate. It can, however, be confusing to do so and is sometimes incorrect; and the reader is recommended to hold on to the notion of discounting by the time preference rate. We shall revert to this point later.

The case we have just quoted, where any government investment displaces an investment of equal monetary cost in the private sector, occurs when there is an absolute savings constraint (i.e., total overall capital rationing). This is sometimes regarded as the standard position in underdeveloped countries, and in such a case the social time preference rate assumed will make little difference to the selection of projects.

However, the most general case is where the government is subject to some constraints – it cannot ensure that all projects which ought to be undertaken are, but equally it does have power to alter the overall rate of investment in the economy. Suppose, for example, that it finances the bridge by taxes, and each

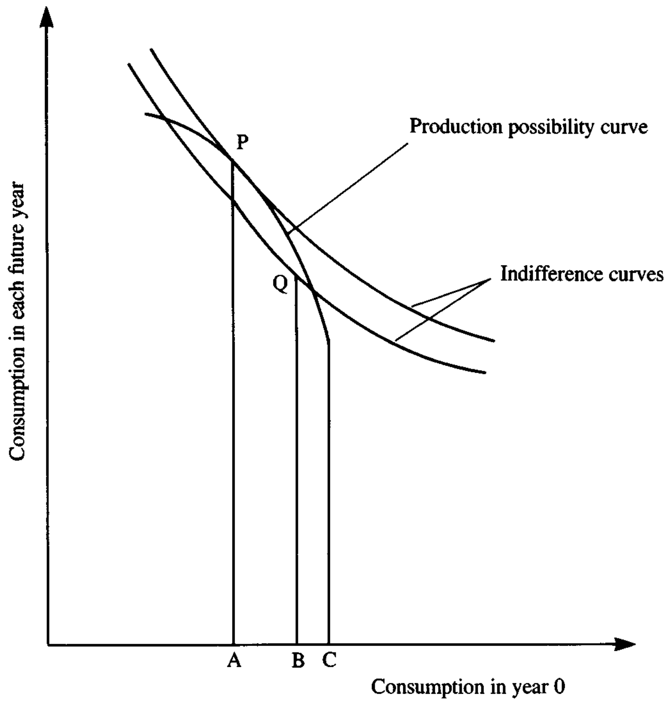

Figure 10

£1 spent reduces private saving and investment by £θ and consumption by £$(1 - \theta)$. Then the cost per pound of bridge building is $\theta\rho/r + (1 - \theta)$ pounds.

The crucial issue, as usual, concerns which constraints are considered binding. For clarity let us briefly repeat the argument, using the basic Fisher diagram in figure 10.

1 If the government can ensure that all projects which ought to be done are done (i.e., B is invested) the time preference rate is the (absolute) slope of the indifference curve at P and the opportunity cost of capital is £1 per pound spent.

2 If there is an absolute savings constraint CB, the time preference rate is the slope of the indifference curve at Q; the opportunity cost of capital is the slope of the production possibility curve at Q divided by the time preference rate.

3 If the private sector will undertake no projects less profitable than the marginal project at Q but the public sector can undertake projects falling within AC, then the time preference rate is the slope of the indifference curve at the final point chosen in consumption space (call it r) and the opportunity cost of capital is $\theta\rho/r + (1 - \theta)$, where ρ is the final marginal rate of return on private investment and θ is the rate of displacement.[28]

An even more constrained position is where the volume of private investment and of public investment are regarded as being separately constrained. In this case, the relevant opportunity cost of capital for public-sector projects is ρ'/r where ρ' is the marginal rate of return in the public sector.

If the disequilibrium at Q is caused by Corporation Tax, Income Tax, or the like, an alternative to public-sector operation of projects AC (where these produce private goods) would be a cut in the tax or the equivalent use of investment grants or allowances. This broad approach is preferred by Musgrave (1969). Monetary policy could also be used to reduce interest rates. But the final cost–benefit decision must be consistent with whatever other arrangements are finally decided on, and the calculation of opportunity cost should proceed as indicated.

The approach outlined above has been frequently criticized. Pioneered by Krutilla and Eckstein (1958), and formalized by Marglin (1963b), it was sharply attacked by Baumol (1968) in a celebrated article which begins by arguing in favour of using the social opportunity cost rate (ρ) as the social discount rate. However, as Arrow (1969, p. 58) points out, he seems here to be dealing with the case of an absolute savings constraint, where, as we have shown, it may in practice be all right to discount by ρ. When, later, he allows for a variable savings rate, he agrees that we have a second-best problem and, given this, the Marglin approach seems correct (Diamond, 1968; James, 1969; Usher, 1969). Mishan (1967) has also argued in favour of using the social opportunity cost rate, provided the government has the power to invest in the private sector. This again seems consistent with the approach outlined above.

There remain three problems. First there is the question of the 'synthetic discount rate'. We saw before that where the cost of capital is ρ/r the correct decisions could sometimes be obtained by valuing all costs and benefits at their nominal value and discounting by ρ. Similarly, if the cost of capital were $\theta/r + (1 - \theta)$ we could use a synthetic discount rate $\theta\rho + (1 - \theta)r$. This is the approach used by some economists (Diamond, 1968; Usher, 1969) and practised by many governments. However, there are many cases where the approach breaks down, and Feldstein (1974) makes out a cogent case against it.

The second problem is of course the assessment of the actual borrowing. Harberger (1969) outlines a fruitful approach to the problem in the case of borrowing, even though it is couched in terms of the search for a synthetic discount rate. Feldstein (1974) argues that it is by no means self-evident that the cost of £1 financed by taxes is less than that of £1 financed by borrowing, once one has allowed for the possible reinvestment of tax-financed interest on the part of those from whom the government borrows.

The third problem is that of reinvestment of the benefits of a project. If savings are sub-optimal, reinvested benefits of nominal value £1 are worth more than benefits which are consumed. In underdeveloped countries one of the arguments put forward for public-sector rather than private-sector production has been that the rate of reinvestment out of profits may be higher. Marglin (1963b) shows how the rate of reinvestment can be allowed for by adjusting the cost of capital, but the same result could equally well be achieved by adjusting the benefit stream. Marglin has been criticized by Arrow (1966) for treating the rate of reinvestment as exogenous, rather than as something optimized over time simultaneously with current investment. One should also, in a full treatment, allow for the possibility that ρ and r would change over time and optimally tend to converge (Little and Mirrlees, 1969).

We can now apply the notions we have been developing to our original simple problem of the bridge. Suppose the risk-free rate on government bonds is 10 per cent per annum, the rate of inflation in recent years 4 per cent per annum and the marginal tax rate 0.33. Then we might put the rate of time preference at 4 per cent [(10 − 4)0.66]. (This assumes a fairly perfect market for borrowing and lending.) But the pre-tax nominal rate of return on private investment is, say, 14 per cent, or 10 per cent in real terms. If the bridge is financed by taxation, each £1 spent may reduce private investment by approximately the marginal propensity to save (or rather more) – by say £0.2. Thus the opportunity cost of the bridge is £30,000 times $(0.2 \times 0.10/0.04 + 0.8)$ = £39,000. The present value of the perpetual stream of future benefits, assuming none is reinvested, now becomes £2,750/0.04 = £68,750. So the project should be done.

The cost of capital under structural unemployment

Before leaving the question of opportunity cost we must look briefly at a special case which is important in economies suffering from structural unemployment. When we consider the cost of doing something we are normally interested in the value of the activities we shall have to give up, if the general level of economic activity is to be held constant. This corresponds to the procedure, advocated by Musgrave (1959), of making the 'allocation branch' assume that the 'stabilization branch' is doing its job properly. In this case, if resources costing £1 are used in this project, there must be a corresponding reduction of £1's worth in the output of other investment or consumption goods (valued at market prices).[29]

But, especially in underdeveloped countries, there may often be resources which are unemployed, even though there are no reasons from a stabilization policy point of view why they need be. In such cases of non-demand–deficient unemployment, the spending of £1 which displaces £θ of private investment may displace less than £$(1 - \theta)$ of consumption. It may even increase consumption by, say, £φ. In this case we subtract from the cost because it is a positive gain to have this consumption being undertaken. The cost per pound is now £$\theta\rho/r - \varphi$, which may or may not exceed £1.

The most obvious relevance of this is to the shadow wage of previously underemployed rural labourers now employed in urban industry. Suppose their marginal product in agriculture was zero. If no output of investment or consumer goods elsewhere in the economy were displaced by employing them, the cost of employing them would be zero. But the very act of employing them will probably raise their consumption, because they now receive a positive wage. This extra consumption could, of course, be provided by cutting down the consumption of other workers by the same amount (by stiffer taxes, for example); and if this were done the cost of employing them remains zero – there is no change elsewhere in the output of investment or consumption goods. But more likely there will be some increase in the output of consumption at the expense of investment. Suppose in fact that the gain in consumption equalled the full wage (W) and that the project itself produced none of these extra consumption goods. Then each £1 spent in wages would reduce investment by the same amount and the shadow wage would be $W(\rho/r - 1)$ (Marglin, 1967, pp. 56–7). This is clearly a special case and a general presentation of the problem can be found in Sen (1972).

But the basic point is that, when unemployed resources exist in the presence of sub-optimal saving, we have the dual problem that, while the resources can be employed with little loss of output elsewhere, the act of employing them is likely to raise the demand for consumption. Thus, a project may displace other investment, not only through its method of finance, but through the payments which it makes to the hitherto unemployed factors it employs.

It will be apparent from this discussion that the choice of a discount rate is always going to be a tricky matter. Spackman (1991) gives a practitioner's account of how the problem has been approached by the UK Treasury.

By way of summary we cannot do better than quote from the final paragraph of Stiglitz's survey in this volume:

> The value of the social rate of discount depends on a number of factors, and indeed I have argued it might vary from project to project depending, for instance, on the distributional consequences of the project.[30] . . . The decision on the appropriate rate of discount thus inevitably will entail judgements concerning what are the relevant constraints. I have suggested . . . that the distortionary consequences of taxation and the implications of imperfect risk markets are significant. Both lead to social rates of discount that normally exceed the consumer rate of interest. Indeed, under not unreasonable circumstances, they may exceed the producer rate of interest.

Present value versus internal rate of return rules

So far we have assumed that the most convenient way to maximize the present value of consumption is to choose all projects having positive present values or, where there are two or more mutually exclusive projects, to choose the one with the highest present value. However, there is an alternative approach which has often been advocated and even more often used: the rate of return approach. The

Table 2

	Stream	IRR	PV at $r = 0.05$
A	− 100, 110	0.10	5
B	− 10, 12	0.20	1.5
C	− 100, 6, 6, (for ever)	0.06	20

rate of return is that rate (ρ) which sets the present value of the project at 0. Thus we solve for ρ in

$$0 = B_0 - C_0 + \frac{B_1 - C_1}{1 + \rho} + \ldots + \frac{B_n - C_n}{(1 + \rho)^n}$$

The rule then is: undertake the project if ρ exceeds the discount rate r.

In very many cases the two approaches give the same answer, and often the rate of return is an interesting and suggestive statistic. However, there are three main arguments against using the rate of return rule for specific decisions. First, it is not the intrinsically correct rule: it is merely a procedure which often gives the same answer.[31] In cases where the discount rate changes from period to period (e.g., falls over time) there is no one value of r with which ρ can be compared, while the present value rule remains well defined. Ignoring this, a second problem arises in the case of mutually exclusive projects, where the internal rate of return may provide the wrong ranking. For example, the table above compares two projects of different size, A and B, in which this is the case. It also compares two projects of different length, A and C, where again the rate of return gives the wrong ranking.

As Feldstein and Flemming (1964) point out, this problem can, in principle, be overcome by calculating not the separate internal rates of return for each project but the rate of return on the difference between each project being considered (say A) and each of its alternatives. For example, taking A and B above, this is the rate of return on − 90, 98 which at nearly 9 per cent is well above the discount rate of 5 per cent. However, the more projects there are the more pairs of projects have to be considered.

Moreover, there remains a third objection. The rate of return calculations may not give a unique answer and in fact may give as many solutions which could have economic meaning as there are sign changes in the stream of net returns (this is an application of Descartes' 'rule of signs'). For example, if we take the stream − 1, 5, − 6 and plot its present value this becomes zero at both 100 per cent and 200 per cent (see figure 11). At any discount rate on either side its present value is, not surprisingly, negative. Each calculated rate of return tells us that at that discount rate the present value is moving from negative to positive or vice versa, and nothing more. However, if a project can be terminated at will at

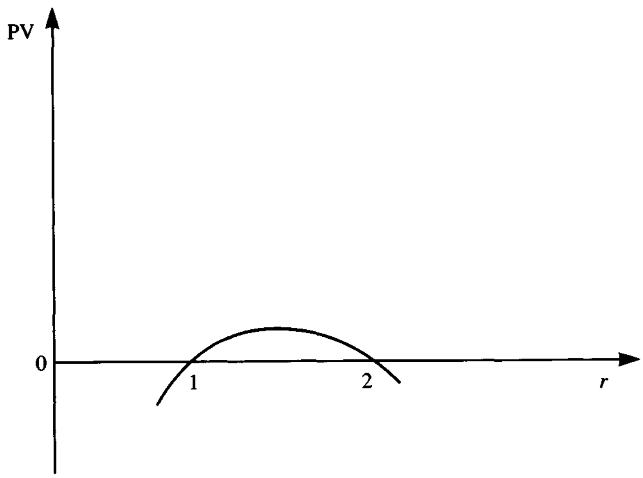

Figure 11

any point in time, it will have a unique internal rate of return, assuming that for each possible discount rate we choose the optimal life of the project (Arrow and Levhari, 1969). But it is not clear how often projects can be usefully regarded as terminable in this way. In real life there may of course be few projects for which the net returns stream changes sign more than once, except in industries where there may be heavy terminal costs (such as filling in mines, decommissioning nuclear power stations and so on). But the differences in net returns between projects may change signs frequently, and thus for mutually exclusive projects the internal rate of return rule may frequently break down.

Finally, there is the problem of capital rationing, where the correct approach is to select projects in order of their present value per unit of constrained cost until the cost constraint is exhausted.[32] Selection in order of rate of return provides a less general approach.

The first year rate of return

A project may pass the test of showing a positive net present value; but might it be a better project if it were delayed by one year? By delaying we would defer the capital expenditures but lose a year's benefit.

Suppose that the initial cost in the coming year is C_0 and the net benefits in following years are $N_1, N_2, \ldots, N_T$, where T is the horizon. Then the PV of the scheme would be

$$-C_0 + N_1/(1+r) + N_2/(1+r)^2 + \ldots + N_T/(1+r)^T$$

However, if we delay by one year then the PV of the scheme would be

$$- C_0/(1 + r) + N_2/(1 + r)^2 + \ldots + N_{T+1}/(1 + r)^{T+1}$$

If we can ignore the present value of the benefits in the final year, N_{T+1} then the gain from delay is

$$- C_0/(1 + r) + C_0 - N_1/(1 + r)$$

This will be positive if

$$N_1/C_0 < r$$

The quantity on the left of this expression is known as the first year rate of return. We conclude that if the first year rate of return is less than the rate of discount then the benefits of one year's delay exceeds the costs and the project should be delayed, and in doing so we will increase the overall worth of the project. Of course, delay may have other advantages in that more information may become available in the meantime or some adverse and unforeseen factor may emerge.

In practice it is rare for this test to be applied. The question posed is usually 'whether' rather than 'when?'.

4 THE TREATMENT OF RISK

We have so far studiously avoided the problem of risk. But suppose that we really do not know how much the bridge will cost. It could cost £25,000, but equally well £35,000. How do we proceed? If the project were being undertaken by a private firm, which bore all costs and reaped all benefits, and whose discount rate for certainly known income streams was 5 per cent, it would be likely to cost the bridge at its 'expected' (average) cost of £30,000 and then use a higher discount rate than 5 per cent. The reason is simply that the owners of the firm are averse to risk and to anyone who is risk averse a certain prospect of receiving £b is worth more than a 50–50 chance of £$0.5b$ and £$1.5b$.

Much of human behaviour towards risk can be explained by the hypothesis that people maximize their expected utility, using a cardinal utility-of-income function of the kind we have already discussed. Thus if a person is a risk averter, whose marginal utility of income falls as income rises, we can see from figure 12 that the utility of £b exceeds the expected utility of a 50 per cent chance of £$0.5b$ plus a 50 per cent chance of £$1.5b$. The latter is

$$E(U) = 0.5U(0.5b) + 0.5U(1.5b)$$

that is, a weighted sum of the possible utilities resulting, the weights being the probabilities attaching to each.[33] The cost of risk is the difference between the mean or 'expected value' of the prospect (here $0.5 \times 0.5b + 0.5 \times 1.5b = b$) and

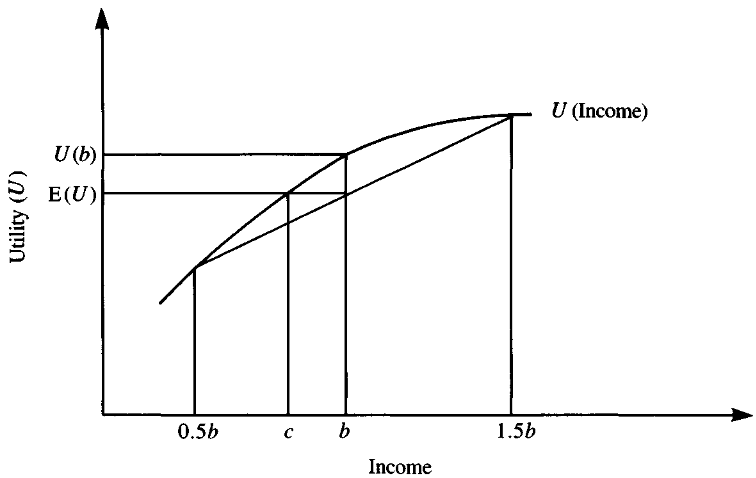

Figure 12

the value which the individual actually places on the uncertain prospect (i.e., the certain prospect which he rates as of equal value, here c).[34]

Now, if there were perfect (and costless) markets for insurance, the firm would not need to bear the cost of risk and could happily discount this project at 5 per cent. But, largely because of the problem of 'moral hazard' such markets generally exist in only a limited form. The securities market makes possible a good deal of pooling of risk among ultimate wealth owners (Pauly, 1970), but even so private firms generally use discount rates much higher than the rate applying to certainly known costs and returns, the excess depending largely on the degree of risk of the project.[35]

Should the public sector follow suit? It has been argued that, unless it does, it will be led to undertake projects identical in their net returns to those which the private sector would have rejected, and this is sub-optimal. However, Arrow and Lind in this volume argue otherwise.

They start with our standard assumption that net returns should be valued as they would be valued by the people to whom they accrue. Thus a risky project undertaken by one person ought to be valued at less than its expected value. If insurance is impossible it is absolutely right for a one-man firm with a certainty discount rate of 5 per cent not to undertake a project with only a 5 per cent rate of return and he ought not to be subsidized to encourage him to do so – the subsidy cannot reduce the social cost of risk.

However, the public sector (or indeed General Motors) is a very large firm with very many shareholders. Our difference between £35,000 and £25,000 when averaged over, say, ten million taxpayers amounts to no more than £1/1,000. The question is: Is a spread of possible project costs of this order sufficient to make us value the cost of the project to each taxpayer at something higher than its expected value of £3/1,000 (assuming each taxpayer pays the same)? Arrow and Lind prove that as the number of taxpayers tends to infinity the cost of the risk tends to zero. The verbal part of their article should be comprehensible to any reader, but the proof of this proposition is presented in formal mathematics and it may be helpful here to provide an intuitive illustration of it. Suppose that ignoring the cost of the bridge my income is y. If the bridge is built and paid for out of taxes, my income will fall to an expected value of $y - 0.003$. However, there is in fact a 50–50 chance that it will be either $y - 0.0025$ or $y - 0.0035$. Do I value this uncertain prospect significantly less than a certain prospect of $y - 0.003$? The answer is that it depends on whether my marginal utility of income falls significantly as y rises from $y - 0.0035$ to $y - 0.0025$. It seems unlikely that the fall is significant. Moreover, if my share in the cost were halved, my cost of risk would be more than halved. So the more taxpayers a given risky project is spread over, the smaller is the total cost of risk. In practice it seems reasonable, for most investment projects, to assume that the costs and benefits accruing to taxpayers have no risk cost and should therefore be discounted at the risk-free rate of time preference.[36]

However, not all the benefits or costs of public projects do accrue to taxpayers as a whole. In the case of our bridge, the cost accrues to the taxpayers but the benefits accrue to the travellers. Whether or not the benefits are sufficiently large and uncertain for any one traveller for them to be valued at less than their expected value is an empirical question.

For some projects it is unquestionably the case that the projects impose substantial risk cost. Suppose that the bridge was designed to be safe in all normal weathers but would collapse under the influence of a tidal wave of a kind that has happened once in the last 100 years. If one assumed that the probablility of disaster in each future year was 1 per cent and one knew the consequences of disaster, one could in principle make an evaluation along the lines we have been discussing if some utility-of-income function were assumed.

However, sometimes the probabilities attaching to the outcomes of a project may not even be guessable. This provides a case of what Knight called uncertainty and here the only approach is to fall back on game theory. The various possible strategies are discussed in Dorfman (1962). No one of them can be said to be correct, but they do at any rate provide a way of marshalling one's thoughts.

5 THE TREATMENT OF INCOME DISTRIBUTION

Finally, we revert to the problem of the distribution of income. Suppose that travellers over our bridge are on average twice as rich as taxpayers but half as rich as ferry owners. Clearly the project will redistribute income. Should this be taken into account in deciding whether to build the bridge? And if so, how?

As we saw earlier, there is no need to allow for distribution if lump-sum transfers will be made so as to compensate the losers. For in that case, if the present value of the project to the gainers exceeds its cost to the losers, the gainers will still be better off than before, even after they have compensated the losers.

But in practice redistribution cannot be effected by lump-sum transfers. It normally requires taxation which imposes an 'excess burden' upon those taxed, representing a loss of efficiency in the economy. Moreover, there may be political objections to cash redistribution, and it is often administratively difficult to devise a tax which falls specifically on the beneficiaries of a project and a transfer which goes specifically to the losers. If redistribution to offset the losses due to the project is not implemented, then the project cannot be justified on the grounds that it is a Pareto-improvement, since at least some people are worse off.

Then a wider criterion has to be introduced to decide whether or not the project increases social welfare – a criterion in which the changes of income to each of the parties affected are weighted by the marginal social values attaching to the income of each group. The criterion is thus that the project should be undertaken if $\Sigma a_i \Delta Y_i$, is positive, where ΔY_i, is the present value of the project to the ith person and a_i is the marginal social value of the ith person's wealth.

However, we need to distinguish between two separate cases. The first is the regular situation where there is a possibly profitable project available, which yet has distributional implications. Musgrave (1969) argues that the excess burden of most taxes is small and therefore one should in general judge projects on efficiency grounds and leave taxation to perform the redistributional objective. But it is hard to see how in our illustrative project or in many others specific taxes and transfers could be devised which actually compensated the losers; and to say that the government undertakes many projects and everyone will therefore come out better off in the end is to put excessive trust in princes. So, for many specific projects that have been devised for other reasons, there seems no alternative to looking at their distributional implications.

It is quite another matter, however, to devise projects specifically as a means of redistributing income. There is always a strong argument for redistribution in cash rather than kind, when redistribution in kind may lead individuals to consume more of any good than they would if given the equivalent command over resources in the form of cash. However, even here there may be cases where redistribution in kind may be essential if it is to avoid humiliating its recipients or to benefit children rather than adults; any such projects to redistribute in kind need to be evaluated using some system of weights (a_i).

The obvious system of weights come from a utility-of-income function of the kind we have used in discussing discount rates and risk. If social welfare (W) is the sum of individual utilities, and the ith individual's utility depends on his income Y_i, then $W = \Sigma U(Y_i)$ and $\partial W / \partial Y_i = U'(Y_i)$.[37] So the appropriate weight (a_i) is $U'(Y_i)$ and the project should be undertaken if $\Sigma U'(Y_i) \Delta Y_i$ is positive. The simplest assumption as before would be that $U'(Y_i) = Y_i^{-\epsilon}$. If some value of ϵ is assumed we can now proceed to recompute the present value of our bridge project. For example, if (for convenience of computation) $\epsilon = 1$, then, on our hypothesis about the relative incomes of the groups, the present values to taxpayers, travellers and ferry owners are weighted in the ratios 4:2:1 and the total present value of the project (in units of ferry owners' present value) is now proportional to $4(-39{,}000) + 2(75{,}000) + 1(6{,}250) = 250$.[38] It is just worth doing.

The problem as usual is in determining ϵ. Eckstein (1961) argued that the government's estimate of ϵ should be implicit in the structure of marginal income tax rates. However, as Freeman (1967) has pointed out, this is not the case if the government seeks to maximize the sum of individual utilities. To do this a government with a given national income to distribute would need to equalize the marginal utility of income of all individuals, i.e., it would aim at complete income equality, by setting taxes equal to $y - \bar{y}$, where y is individual income and $\bar{y}$ is average income.[39]

Weisbrod (1968) has therefore suggested that the government's weights for different income groups should be inferred from its previous decisions on whether or not to adopt the various projects open to it. This involves the solution of a set of simultaneous equations. There is an obvious logical objection to this approach: either the government's decisions so far have been consistent; in which case why worry about helping it continue to be consistent, or they have been inconsistent, in which case why pretend they were consistent. There is much force in this criticism, especially when it is linked to the difficulties of estimation that arise. However, Weisbrod's article is important in stressing that from now on government decisions should embody some consistent set of distributional weights.

A third, and more limited, approach has been suggested by Marglin (1967). In this, total consumption is maximized subject to some minimum consumption being secured to a given underprivileged group or region. Alternatively, the consumption of the underprivileged may be maximized subject to some minimum total consumption. Any constrained maximization of this kind implies in its solution a relative weight attaching to consumption of the underprivileged as against consumption in general. This is measured by the shadow price on the constraint. But this value is determined *ex post*.

It is a less general approach to the income distribution problem, but, if no other is available, it is one way of allowing for an important dimension of public policy.[40] The only alternative is that pursued by the Roskill Commission on the Third London Airport (1970), which is to show the costs and benefits of different groups in society separately and let the policy makers decide their own weights.[41]

The question of the criteria for a welfare improvement is discussed in more detail by Layard and Walters in this volume. This shows how welfare changes for individuals can be estimated, but that the question of whether a social gain has occurred cannot be separated from the issue of the social valuation of benefits to the rich compared with benefits to the poor.

NOTE TO P. 35

If welfare depends on consumption per head, we can write

$$W = \frac{1}{1-\epsilon}\left[\frac{C_0}{N_0}\right]^{1-\epsilon} + \frac{1}{1-\epsilon}\left[\frac{C_1}{N_1}\right]^{1-\epsilon},$$

$$\frac{\partial W/\partial C_0}{\partial W/\partial C_1} = \frac{(C_0/N_0)^{-\epsilon}(1/N_0)}{(C_1/N_1)^{-\epsilon}(1/N_1)}$$

$$\simeq (1 + g - n)^{\epsilon}(1 + n)$$

$$\simeq \{1 + (g - n)\}\epsilon(1 + n)$$

Even if consumption per head is going to be roughly constant, additional consumption will be less useful in period 1 than in period 0 because it is spread over more people. (In this respect Eckstein's approach (1957, p. 75) seems inappropriate. The expression given above is taken from Sen (1968, p. 16, footnote 16) where, however, C should refer to total rather than per capita consumption.)

If welfare depends on population times the utility of consumption per head, we can write

$$W = \frac{N_0}{1-\epsilon}\left[\frac{C_0}{N_0}\right]^{1-\epsilon} + \frac{N_1}{1-\epsilon}\left[\frac{C_1}{N_1}\right]^{1-\epsilon},$$

$$\frac{\partial W/\partial C_0}{\partial W/\partial C_1} = \frac{(C_0/N_0)^{-\epsilon}}{(C_1/N_1)^{-\epsilon}}$$

$$\simeq (1 + g - n)^{\epsilon}$$

$$\simeq 1 + (g - n)\epsilon$$

If consumption per head is going to be roughly constant, the effect on welfare per head of a unit gain in consumption per head is equal in the two periods. Additional consumption sufficient to produce a unit gain in consumption per head in period 0 will produce a $1 - n$ gain in consumption per head in period 1 and therefore a gain in utility per head $1 - n$ as large as the gain in period 0. But this gain is spread over a population $1 + n$ as large. So if consumption per head is constant the discount rate is zero.

As to which assumption is reasonable see Meade (1955, ch. 6). The discussion in Feldstein (1964) is more general and assumes simply that

$$W = f\left(\frac{C}{N}, N\right)$$

NOTES

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1 Good textbooks are Mishan (1971a), UNIDO (1972) and Pearce and Nash (1981). Comprehensive survey articles are Prest and Turvey (1965), Henderson (1965) reprinted in R. Turvey (1968) and Eckstein (1961) reprinted in R.W. Houghton (1970).

2 This proposition assumes no increasing returns to scale.

3 This statement is true if the marginal utility of income is constant. In this case the statistic also measures the 'compensating variation' in income – i.e., the amount by which these consumers would have to be taxed after the price fall to make them no better off than before – and the 'equivalent variation' – i.e., the amount which consumers would need to be given if the price fall did not occur to make them as well off as if it did. Opinions differ on which of these measures is the most relevant indicator of welfare gain. Mishan (1971a, pp. 48–9) favours the compensating variation, assuming, as he does, that the Hicks–Kaldor criterion is used in aggregating the gains of different consumers. See Layard and Walters in this volume for a discussion of these issues.

4 The reasons for discounting and the choice of discount rate are discussed by Stiglitz in this volume. Below we explain why the present value of a stream of £*a* per year for ever at a 10 per cent discount rate equals £10*a*.

5 On welfare criteria see Mishan (1971a) or Layard and Walters in this volume. For a passionate plea in favour of the Hicks–Kaldor criterion see Harberger (1971). Mishan (1971a) argues that if this criterion is used, as he believes it should be, this is because it corresponds to the 'virtual constitution' of the society.

6 See for example Beesley and Foster (1965), and Newbery and Morrison and Winston in this volume.

7 This figure and the analysis in the text is based closely on Alasdair Smith's (1982) excellent text, p. 129.

8 This, and what follows, assumes strictly that other goods are competitively produced and untaxed. It also ignores questions of income distribution. In general if other goods are taxed an average rate t marginal factor cost (MC) should be replaced by $MC(1 + t)$.

9 For reply to criticisms of their approach see Little and Mirrlees (1972) and their chapter in this volume. On other approaches to the problem see Dasgupta (1972) and Bacha and Taylor (1971).

10 For a full survey of this topic, with a special emphasis on the point that the rate to be used must depend upon which constraints have to be accepted as binding, see Stiglitz in this volume.

11 For an excellent exposition of Fisherian capital theory see Hirshleifer (1958) reprinted in B.V. Carsberg and H.C. Edey (1969).

12 A savings rate of 10 per cent would not normally produce a growth rate of 17 per cent; this results from the use of the two-period model.

13 This diagram is extremely partial and does not bring out the simultaneous need for aggregate saving to equal aggregate investment.

14 In principle, it makes no difference whether we use a money rate of interest and nominal consumption valued at forecast rates of inflation, or a real rate of interest and real consumption at constant prices. However, it is difficult to forecast inflation, so C–B analysis is normally done in constant prices.

15 Spackman (1991) contains a review of the long-term history of these rates in the UK experience.

16 For a good discussion of the optimality or otherwise of saving/investment decisions in a market economy see Phelps (1965, ch. 4), reproduced in A.K. Sen (1970b).

17 Strictly the only correct procedure is to evaluate, separately for each individual, the present value of his net benefit stream, discounted at his own time preference rate.

18 This view is held most strongly by those who would also use the Hicks–Kaldor welfare criterion in project evaluation. It is easy to see why the two approaches are natural bedfellows. The Hicks–Kaldor criterion aims, as far as possible, to separate decisions about production and investment from those about distribution, and acquires whatever plausibility it has from the thought that we could always alter the income distribution by cash transfers. However, distribution between non-overlapping generations in a closed economy can only be altered by decisions about investment. It is therefore convenient if future generations need not be considered.

19 A practical argument is sometimes put forward for ignoring them, in that we cannot know their preferences. However, there are many items (like life) where we do not know how they are valued by present generations, and many (like bread) where we can be fairly sure what future generations will feel.

20 On possible ethical approaches to the question of fairness, see Sen (1971).

21 This is reproduced in A.K. Sen (1970a). See also Dasgupta in this volume.

22 Compare Dasgupta in this volume, equation 4.

23 The reader will find this approach presented in Eckstein (1957, pp. 75ff.) and in Eckstein (1961), which is reproduced in R.W. Houghton (1970) where, however, the printing of ϵ on p. 232 is in some places misplaced. For a more elaborate application of optimal growth theory to the choice of discount rate, see Arrow (1966 and 1969); Arrow however discounts future utilities to allow for some element of pure time preference. See also Arrow and Kurtz (1970), Stiglitz in this volume and Dasgupta in this volume.

24 For a full discussion of the rationale of discounting, along these lines, see Feldstein (1964). Feldstein does not, however, assume a constant ϵ and therefore provides no actual formula. The reader should note that in optimal growth theory it is necessary for optimality, but not sufficient, that the marginal rate of return equal the proportional rate of decline of marginal utility.

25 For proof of these propositions see the note at the end of the Introduction.

26 Harberger (1969) argues that such values are too high to be consistent with the limited amount of insurance purchased by most individuals.

27 If the rate of interest is r per annum, this means that 1 now has the same value as r per year for ever. Therefore $1/r$ now is equivalent to 1 per year for ever and ρ/r is equivalent to ρ per year for ever.

28 In an open economy there are possibilities of lending and borrowing abroad which need to be added to the production possibility curve of figure 10, before we can claim to have society's true consumption possibility frontier. Thus, in principle, we might need to allow for the effect of a project in displacing foreign lending. Equally, if the country

is a net borrower, the cost of capital is the present value of the debt service at the margin.

29 As Baumol (1968, p. 792) points out, this need not imply that for a tax-financed project the extra taxes should equal the expenditure on the project. This depends on the propensities to consume of all the parties affected. If the MPC is uniform for all, more taxes need to be raised than expenditure incurred if employment is to be held constant.

30 On the distributional consequences see later in this introduction.

31 Hirschleifer (1958) shows why the present value rule is intrinsically correct for the private investor or for a society which can borrow or lend abroad. In the case of a closed economy the optimum occurs at a direct tangency between the indifference surface and the production possibility surface. Hirschleifer also sets out the main problems with the IRR approach. For a simplified exposition of some of these points see Baumol (1965, pp. 422–7 and 437–47).

32 At the level of the economy this is the problem of the savings constraint. Here the problem can be handled in the simplified case by costing capital at ρ/r (where ρ is the rate of return on the marginal project within the constraint) and then doing all projects with positive present values. The rule given above in the text is a general rule which should be used also by separate agencies subject to budget constraints.

33 More generally $E(U) = \Sigma p_i U(Y_i)$, where p_i is the probability of obtaining income Y_i. Thus in the example we should, strictly, be concerned with the utilities of $y + 0.5b$ and $y + 1.5b$, where y is normal income. For a full analysis see Friedman and Savage (1948).

34 See Layard and Walters (1978) for a full treatment which relates the cost of risk to the degree of risk and aversion and the variability of the prospect.

35 This is partly because, unlike the stockholder, a manager cannot insure against the personal risk associated with a project which he initiates.

36 This assumes that the returns to each separate project are independent of national income, that is, that stabilization policy is successful.

37 In fact it is much more satisfactory to assume that $W = W[U_1(Y_1), \ldots, U_n(Y_n)]$, in which case $\partial W/\partial Y_i = (\partial W/\partial U_i)U_i'(Y_i)$. This satisfies the notion of fairness, that additional happiness for a miserable man is more desirable than additional happiness for someone who is already happy. However, it renders the problem of quantification even more intractable, even if we assume that all individuals have the same utility-of-income function.

38 The present values to each party are those computed on p. 40.

39 (i) Put another way, the government equalizes the marginal sacrifice (of utility) per dollar of tax. By contrast, the Eckstein approach implies that governments equalize the marginal sacrifice per dollar of income due to tax; if so, a man with a marginal tax rate of 0.25 could then be assumed to have a marginal utility of income double that of a man with a marginal tax rate of 0.50. However, there is no reason to think that governments do or should pursue this principle – nor the principles of equiproportional sacrifice or equal absolute sacrifice used by Mera (1969) – to derive implicit utility-of-income functions.
(ii) If the government maximizes the sum of individual utilities but tax rates affect production, it should equalize the marginal rates of transformation between pairs of individual incomes to the ratios of marginal utilities of income (Freeman, 1967).

40 For a strong plea to exclude income distribution from formal economic welfare analysis see Harberger (1971).

41 Roskill Commission on the Third London Airport (1970, ch. 29). Separate calculations are not shown in the final report. The Roskill Commission treated foreigners on the same footing as British nationals, whereas some cost–benefit analyses assign them a distributional weight of zero (often without even discussing the issue).

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