Disclosures and Asset Returns

By

Hyun Song Shin

DISCUSSION PAPER 371

March 2001

FINANCIAL MARKETS GROUP
AN ESRC RESEARCH CENTRE

LONDON SCHOOL OF ECONOMICS

ISSN 0956-8549-371

Disclosures and Asset Returns*

Hyun Song Shin
London School of Economics
Houghton Street
London, WC2A 2AE
h.s.shin@lse.ac.uk

February 2001

Abstract

Public information to financial markets often arrives through the disclosures of interested parties who have a material interest in the reactions of the market to the new information. When the strategic interaction between the sender and the receiver is formalized as a disclosure game with verifiable reports, market prices observed in equilibrium can be given a simple characterization that relies only on the face value of the announcement. Also, this characterization predicts that the return variance following a bad outcome is higher than it would have been if the outcome were good. When investors are risk averse, this leads to negative serial correlation of asset returns.

*I am grateful to Sudipto Bhattacharya, Neil Shephard and David Webb for encouragement and advice during the preparation of this paper.

1. Introduction

The arrival of public information influences asset prices, but frequently such information is provided by interested parties who have a material interest in the reactions of the market to such news. Managers of a firm will consider carefully claim that one knows a particular feature of the world, and contrast this with the difficulty of proving the negative - that one is ignorant of some feature of the world. In a recent paper, Bull and Waton (2000) have sought to clarify the foundational issues concerning verifiability by endogenizing the generation and disclosure of evidence.

There main contribution of the framework developed here is a simple pricing rule for equilibrium prices that can be used to investigate some of the empirical predictions concerning the time series properties of asset returns. In particular, the theory generates the following prediction. Following a 'bad' outcome, the uncertainty over subsequent outcomes increases. Thus, for example, if the stock price of a firm falls following an unfavourable disclosure, the variance of subsequent returns increases.

Such an increase in stock return volatility following a bad outcome is familiar from the lop-sided "smile" or "smirk" in implied volatility in option prices, but this feature has also been the topic of an active literature in the time series properties of stock returns. Black (1976) documented how low stock return was associated with an increase in the subsequent return volatility, and suggested the hypothesis that the reduced proportion of equity within the total assets of the firm may be one explanation for this empirical regularity. This "leverage hypothesis" has received much attention since, and the terminology has stuck even though the increased leverage has been found to be too small to account for the size of the effect on volatility (Christie (1982), Schwert (1989), Figlewski and Wang (2000)). Econometric techniques have evolved to address this regularity, such as the EGARCH approach of Nelson (1991), and more recent techniques such as Barndorff-Nielsen and Shephard (1998). Engle and Ng (1993) survey some of the earlier techniques. Campbell and Kyle (1993) assume this regularity as one of the building blocks of their study of asset returns. However, in spite of the voluminous empirical literature, much remains to be learned of the underlying microeconomics of this phenomenon. One motivation for developing the theory below is to close this gap.

As well as the "leverage hypothesis", the theory may also be useful in addressing another active area in finance - that of the pricing of defaultable securities. Since the payoff to a creditor is akin to having a short position in a put option on the assets of the firm, with a strike price equal to the face value of the debt, option pricing techniques can be used to determine the price of debt. This was the contribution of Merton's (1974) classic paper. Nevertheless, the empirical success of this approach has been mixed, with the usual discrepancy appearing in the form of the overpricing (by the theory) of the debt, and especially of the lower quality, riskier debt. Anderson and Sundaresan (1996) address this problem

2. Model

The firm undertakes N independent and identical projects, where each project succeeds with probability r, and fails with probability $1-r$. Each successful project raises the return by Δ, where Δ is a positive constant. The liquidation value of the firm given s successes and $N-s$ failures is given by

$$(1+\Delta)^s .$$

These payoffs can be seen as a special case of the binonomial tree model of Cox, Ross and Rubinstein (1979) where an "up" move is of size $(1+\Delta)$, while a "down" move leaves the value unchanged.

The ex ante value of the firm, denoted by V_0, is the expected liquidation value obtained from the binomial density with success probability r. Thus,

$$\begin{aligned} V_0 &= \sum_{s=0}^{N} \binom{N}{s} \left[r\left(1+\Delta\right)\right]^s \left(1-r\right)^{N-s} \\ &= \left(1+r\Delta\right)^N \underbrace{\sum_{s=0}^{N} \binom{N}{s} \left[\frac{r\left(1+\Delta\right)}{1+r\Delta}\right]^s \left[\frac{1-r}{1+r\Delta}\right]^{N-s}}_{=1} \\ &= \left(1+r\Delta\right)^N , \end{aligned} \tag{2.1}$$

which coincides with the price of a risk-free bond with return $r\Delta$ for each project.

There are three dates - *initial*, *interim* and *final*. We index these dates by $0, 1$ and 2 respectively. At the initial date, nothing is known about the value of the firm other than the description above. As time progresses, the projects begin to yield their outcomes. At the interim date, not enough time has elapsed for the manager to know the outcomes of all the projects. However, the outcomes of *some* of the projects will have been realized. In particular, there is a probability

$$\theta$$

that the outcome of a project is revealed to the manager by the interim date. This probability is identical across all projects, and whether the outcome is revealed is independent across projects. By the final date however, all uncertainty is resolved. The outcomes of all the projects become common knowledge. The firm is liquidated, and consumption takes place.

There is differential information at the interim date between the *manager* of the firm and the rest of the market. The manager is able to observe the success and failure of each project as it occurs, and hence knows the numbers of successes and failures at the interim date, but the rest of the market does not. Instead, the only information available to the market at the interim date is a *disclosure* by the manager. The manager is free to disclose some or all of what he knows, by actually exhibiting the outcome of those projects whose outcomes have already been determined. However, he cannot concoct false evidence. If he knows that project j has failed, he cannot claim that it has succeeded. In this sense, although the manager has to tell the truth, he cannot be forced to tell the whole truth.

The implicit understanding is that the manager's disclosures are verifiable at a later date by a third party, such as an auditor, who is able to impose a very large penalty if the earlier disclosure is exposed to be untrue, i.e. inconsistent with evidence made available by the manager. But how much private information the manager has at the time of disclosure is not verifiable even at a later date. So the manager is free to withhold information if such information is deemed to be unfavourable. Although managers may dress up the results in ways that place a firm's prospects in the best light possible, there are well-established accounting principles which impose broad limits on what is possible. The ultimate sanction is the one against fraud.

More formally, the information available to the manager at the interim date can be summarized by the pair

$$(s, f)$$

where s is the number of successes observed, while f is the number of failures observed. The manager's disclosure strategy $m(\cdot)$ maps his information (s, f) to the pair (s', f'), giving the number of *disclosed* successes and failures, where the requirement of verifiability imposes the constraint that

$$s' \leq s \quad \text{and} \quad f' \leq f. \tag{2.2}$$

This constraint reflects the requirement that the disclosure takes the form of actually exhibiting a subset of the realized outcomes to the market.

We assume that the disclosure policy of the manager is motivated by the objective of maximizing the price of the firm. Since the initial and final price of the firm is based on symmetric information, the focus of the analysis will be on the interim price V_1. The market, however, anticipates the manager's disclosure policy, and prices the firm by discounting the manager's disclosures appropriately. This gives rise to a game of incomplete information. We will model the "market" as a player in the game who sets the price of the firm to its actuarially fair value based on all the available evidence, taking into consideration the reporting strategy of the manager.

More formally, the market's strategy is the pricing function

$$(s', f') \mapsto V_1 \tag{2.3}$$

We ensure that the market aims to set the price of the firm to its actuarially fair value by assuming that its objective in the game is to minimize the squared loss function:

$$(V_1 - V_2)^2 \tag{2.4}$$

where V_2 is the (commonly known) liquidation value of the firm at the final date. The market then sets V_1 equal to the expected value of V_2 conditional on the disclosure of the manager, as generated by his disclosure strategy. The manager, on the other hand, anticipates the optimal response of the market, and chooses the disclosure that maxmizes V_1.

3. Properties of Equilibrium

3.1. Full Revelation Cannot Occur in Equilibrium

One immediate conclusion we can draw is that a policy of full disclosure by the manager can never be part of any equilibrium. To see this, suppose for the sake of argument that the manager always discloses fully, so that the disclosure strategy is the identity function:

$$m(s, f) = (s, f).$$

The best reply by the market is to set V_1 to be

$$V_1(s, f) = (1 + \Delta)^s (1 + r\Delta)^{N-s-f},$$

since there are $N - s - f$ unresolved projects, and the expected value of the firm is

$$\begin{aligned} & (1+\Delta)^s \sum_{i=0}^{N-s-f} \binom{N-s-f}{i} [r(1+\Delta)]^i (1-r)^{N-s-f-i} \\ = \; & (1+\Delta)^s (1+r\Delta)^{N-s-f}. \end{aligned} \tag{3.1}$$

But then, the manager's disclosure policy is sub-optimal, since the feasible disclosure $(s, 0)$ that suppresses all failures elicits the price:

$$(1+\Delta)^s (1+r\Delta)^{N-s}, \tag{3.2}$$

which is strictly higher than (3.1) for positive f. Hence, we are led to a contradiction if we suppose that full disclosure can figure in an equilibrium of the disclosure game.

3.2. Some Failures May Be Disclosed in Equilibrium

Although full revelation cannot figure in an equilibrium, it would be wrong to conclude that failures are never disclosed in equilibrium. Consider the following example with $N = 2$. Figure 3.1 illustrates the argument.

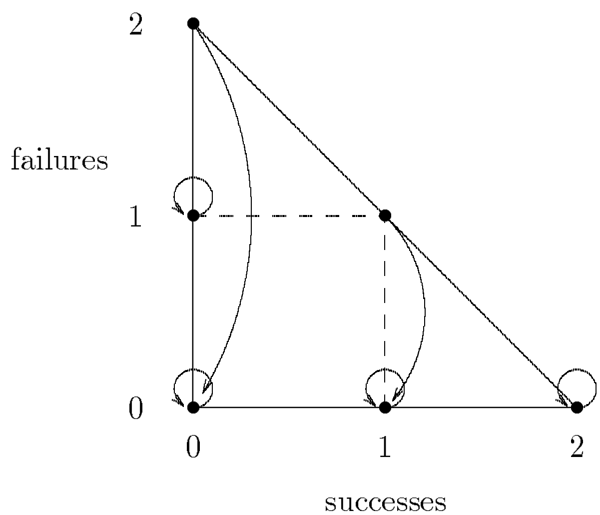

Figure 3.1: Equilibrium Strategy with Reported Failure

The solid dots in figure 3.1 represent the possible types of the manager in the Bayesian game, and the arrows represent the disclosures of the respective types. In the disclosure strategy depicted in figure 3.1, every type except type $(0, 1)$ follows the rule of suppressing any failures. Type $(0, 1)$, however, reveals his type truthfully. We will show that such a strategy can form part of an equilibrium for some parameter values. It is useful to have some notation for the full-revelation value of the firm, as derived in equation (3.1) above. Define:

$$\rho(s,f) \equiv (1+\Delta)^s (1+r\Delta)^{N-s-f}$$

$\rho(s,f)$ is the full-revelation value of the firm in the sense that, if the type (s,f) fully disclosed everything that he knew, then the market value of the firm would be $\rho(s,f)$.

Consider the following pricing rule chosen by the market

failures \ successes	0	1	2
2	$\rho(0,2)$		
1	$\rho(0,1)$	$\rho(1,1)$	
0	$V_1(0,0)$	$V_1(1,0)$	$\rho(2,0)$

Table 3.2

where $V_1(0,0)$ and $V_1(1,0)$ are the best replies by the market against the disclosure strategy in figure 3.1. Thus, $V_1(0,0)$ is the convex combination of $\rho(0,0)$ and $\rho(0,2)$ weighted by the posterior probability of types $(0,0)$ and $(0,2)$ respectively. $V_1(1,0)$ is similarly a convex combination of $\rho(1,0)$ and $\rho(1,1)$. In particular,

$$\begin{aligned} V_1(0,0) &= \frac{(1-\theta)^2 \rho(0,0) + \theta^2 (1-r)^2 \rho(0,2)}{(1-\theta)^2 + \theta^2 (1-r)^2} \\ V_1(1,0) &= \frac{(1-\theta) r \rho(1,0) + \theta^2 r (1-r) \rho(1,1)}{(1-\theta) r + \theta^2 r (1-r)} \end{aligned}$$

Now, consider a set of parameter values such that

$$\rho(0,1) = V_1(0,0) \tag{3.3}$$

For example, when $\Delta = 1$ and $r = 0.5$, this equality holds when θ solves

$$4(1-\theta)^2 (\rho(0,0) - \rho(0,1)) + \theta^2 (\rho(0,2) - \rho(0,1)) = 0$$

for which there is a root at $\theta \approx 0.71$. Given (3.3), we claim that the reporting strategy in figure 3.1 and the valuation rule in table 3.2 constitute an equilibrium. We show this for the notion of *sequential equilibrium*, due to Kreps and Wilson (1982), but the example would fit any standard version of Bayesian Nash equilibrium.

First consider the market's valuation rule in table 3.2. We have already noted that $V_1(0,0)$ and $V_1(1,0)$ constitute the best reply for the market against the disclosure strategy of the manager. The disclosures $(2,0)$ and $(0,1)$ are truthful and fully revealing, so that the best replies are $\rho(2,0)$ and $\rho(0,1)$ respectively. The remaining cells in the table are for disclosures that receive zero probability in the manager's reporting strategy. The values $\rho(0,2)$ and $\rho(1,1)$ are supported by the off-equilibrium belief that the types $(0,2)$ and $(1,1)$ have "trembled" and have disclosed truthfully by mistake.

Now consider the reporting strategy of the manager as depicted in figure 3.1. Since $V_1(1,0) > \rho(1,1)$ the suppression of the one failure by type $(1,1)$ is optimal given the valuation rule. For type $(0,2)$, since $V_1(0,0) = \rho(0,1) > \rho(0,2)$, he cannot do better than to suppress both failures. Types $(0,0)$, $(1,0)$ and $(2,0)$ must report truthfully, since their feasible set of disclosures are singletons. This just leaves type $(0,1)$. Since $\rho(0,1) = V_1(0,0)$, type $(0,1)$ cannot do better than to reveal himself truthfully. Hence, the reporting strategy in figure 3.1 is a best reply against the valuation rule in table 3.2. Finally, the beliefs that underlie the values in table 3.2 can be obtained as the limit of a sequence as ε tends to zero of full support beliefs in which types $(0,2)$ and $(1,1)$ reveal themselves by mistake with probability ε. Hence, the strategies form a sequential equilibrium.

3.3. Generic Absence of Reported Failures

The example above shows that one can construct equilibria in which some failures are reported, but the construction relies on the manager being indifferent between suppressing a failure and revealing it. If type $(0,1)$ strictly preferred to reveal the single failure, implying that $\rho(0,1) > V_1(0,0)$, then type $(0,2)$ would strictly prefer to deviate, and suppress just one failure rather than suppressing both of them. In this way, the example above rests on a knife-edge property of the payoffs, and any perturbation of the parameters would upset the equilibrium. We can show that the parameter values for which we can construct such examples are of measure zero in the space of parameter values.

In conducting our argument, let us introduce the following terminology. Say that a disclosure is *sanitized* if it reports all the successes but suppresses all the failures. The str

Let $m\left(\tilde{t}\right)$ be the report of type $\tilde{t}$ in equilibrium and consider the set of types

$$A \equiv \left\{ (s,f) | \, m\left(\tilde{t}\right) \leq (s,f) \text{ and } s \leq \tilde{s} \right\}$$

The set A is depicted in figure 3.2 as the shaded region. The set of types for whom their equilibrium disclosure is $m\left(\tilde{t}\right)$ must be a subset of A. That is, the inverse image of the equilibrium disclosure strategy m evaluated at $m\left(\tilde{t}\right)$ must be a subset of A. This is so for two reasons. First, verifiability requires that if type (s,f) discloses $m\left(\tilde{t}\right)$, then $m\left(\tilde{t}\right) \leq (s,f)$. Second, by construction, any type which has more than $\tilde{s}$ successes will sanitize its report, and hence cannot be one of those types reporting $m\left(\tilde{t}\right)$.

The equilibrium V_1 evaluated at $m\left(\tilde{t}\right)$ is a convex combination of the full relevation values $\rho(t)$ of types $t \in A$. In other words, there is some probability distribution μ over the types in A such that

$$V_1\left(m\left(\tilde{t}\right)\right) = \sum_{t \in A} \mu(t)\,\rho(t)\,. \tag{3.4}$$

Now, consider the set of types whose report is $\hat{t} \equiv (\tilde{s}, 0)$. This is the set $m^{-1}\left[\hat{t}\right]$, the inverse image of the equilibrium strategy evaluated at $\hat{t}$. This set is non-empty, since $\hat{t}$ itself belongs to that set. By hypothesis, type $\tilde{t}$ strictly prefers the report $m\left(\tilde{t}\right)$, even though the sanitized report $\hat{t}$ is available to him. This implies that $m^{-1}\left[\hat{t}\right] \cap A$ is empty. Together with the fact that every type to the right of $\tilde{t}$ chooses a sanitized report, we conclude that $m^{-1}\left[\hat{t}\right]$ consists of types that have as many successes as $\tilde{t}$, but with strictly fewer failures than $m\left(\tilde{t}\right)$. Therefore the full revelation value of any type in $m^{-1}\left[\hat{t}\right]$ must be strictly greater than that of any type in A. In other words,

$$\rho(t) > \rho(t') \text{ for any } t \in m^{-1}\left[\hat{t}\right] \text{ and } t' \in A \tag{3.5}$$

From (3.4) and (3.5), we have

$$V_1\left(\hat{t}\right) > V_1\left(m\left(\tilde{t}\right)\right)$$

which contradicts the hypothesis that type $\tilde{t}$ prefers the disclosure $m\left(\tilde{t}\right)$ over the sanitized disclosure $\hat{t}$. This proves theorem 3.1.

Theorem 3.1 confirms the suspicion that any non-sanitization equilibrium relies on the knife-edge property of indifference. Since the full revelation value $\rho(t)$ of any type t is strictly increasing in r and Δ, the set of parameter configurations in the space of triples (r, Δ, θ) that give rise to an equilibrium that uses non-sanitized reports is of measure zero. We can thus claim that, generically in the space of fundamentals (r, Δ, θ), only the sanitized disclosures occur with positive probability in equilibrium.

To be sure, we cannot claim uniqueness of *equilibrium* - not even generically. An equilibrium strategy must specify a value of the firm for every conceivable report, even those that receive zero probability in equilibrium. There are few compelling arguments that tie down the value of the firm given these off-equilibrium reports. However, we have a result that is almost as good. We have shown that, generically in the space of parameters, there are no reported failures in equilibrium. To the extent that the motivation for studying this game lies in the empirical task of explaining asset returns, our theorem lends support to examining only those equilibria in which bad news is suppressed by the manager.

4. Equilibrium Prices

Having established the importance of sanitized disclosures for equilibrium, we now turn to the main business of this paper - that of characterizing the empirical properties of asset returns. Our focus is on the posterior distribution over successes conditional on the manager's disclosure. This is the key question for us, since the equilibrium prices at the interim date are determined by this posterior distribution. We can pose our question by first noting the following pair of observations.

- On the one hand, the ex ante probability distribution of the true number of successes is binomial with success probability r.
- Meanwhile, when the manager uses sanitized disclosures, the distribution of *disclosed successes* at the interim date is also a binomial distribution, with success probability θr. This is because the manager observes a success at the interim date with probability θr, and observations of successes are independent across projects.

What is the posterior distribution over realized successes conditional on a particular disclosure? Consider the joint probability density over *disclosed* successes at date 1 and the *realized* successes at date 2. We can depict this density in tabular form as below.

		realized successes				
		0	$\cdots$	k	$\cdots$	N
	0					
	$\vdots$					
disclosed	s			$h(s,k)$		
successes	$\vdots$					
	N					

Since the number of disclosed successes cannot exceed realized succeses, all entries below the leading diagonal are zero. $h(s,k)$ is the probability that the manager discloses s successes when in fact, the realized successes turns out to be k. Denoting by

$$h(k|s)$$

the probability of k realized successes conditional on disclosure of s successes, we have the following result, whose proof is given in the appendix.

Lemma 4.1. *Let $q = (r - \theta r) / (1 - \theta r)$. Then,*

$$h(k|s) = \begin{cases} \binom{N-s}{k-s} q^{k-s} (1-q)^{N-k} & \text{if} \quad s \leq k \\ \\ 0 & \text{otherwise} \end{cases} \tag{4.1}$$

In other words, the posterior distribution over realized successes conditional on disclosed successes is also binomial, where the success probability q is given by $(r - \theta r) / (1 - \theta r)$. We can understand this result better by appealing to a closure property of binomial distributions which states that a mixture of binomial distributions with binomial weights is also a binomial distribution. To state the

Lemma 4.2. $B(p)\,B(q) = B(p+q-pq)$

Proof. The (i,s)th entry of $B(p)\,B(q)$ is given by

$$\begin{aligned}
&\sum_{j=i}^{s} \tbinom{N-i}{j-i} p^{j-i} (1-p)^{N-j} \tbinom{N-j}{s-j} q^{s-j} (1-q)^{N-s} \\
=\ & [(1-p)(1-q)]^{N-s} \sum_{j=i}^{s} \underbrace{\tbinom{N-i}{j-i}\tbinom{N-j}{s-j}}_{=\binom{N-i}{s-i}\binom{s-i}{j-i}} p^{j-i} [(1-p)\,q]^{s-j} \\
=\ & \tbinom{N-i}{s-i} [(1-p)(1-q)]^{N-s} (p+(1-p)\,q)^{s-i} \underbrace{\sum_{j=i}^{s} \tbinom{s-i}{j-i} \left[\tfrac{p}{p+(1-p)q}\right]^{j-i} \left[\tfrac{(1-p)q}{p+(1-p)q}\right]^{s-j}}_{=1} \\
=\ & \tbinom{N-i}{s-i} (p+q-pq)^{s-i} [(1-p)(1-q)]^{N-s},
\end{aligned}$$

which is the (i,s)th entry of $B(p+q-pq)$. This proves the lemma.

Since $0 \leq p+q-pq \leq 1$ for any probabilities p and q, binomial matrices are closed under multiplication. In particular, for $p = r\theta$ and $q = (r - r\theta)/(1 - r\theta)$, we have

$$B(p)\,B(q) = B(r). \tag{4.3}$$

Since $p = r\theta$, the distribution over *disclosed successes* is given by the top row of $B(p)$. The top row of $B(r)$ gives the ex ante distribution over successes. Since the ex ante distribution over successes must be equal to the average of the posterior distributions weighted by the probability of each disclosure, equation (4.3) confirms the result stated in lemma 4.1 that the ith row of $B(q)$ gives the distribution over realized successes conditional on disclosed successes of i.

Note that $B(q)$ tends to the identity matrix as $\theta \to 1$. This has a natural interpretation. When θ is large, the manager is well informed about the true number of successes, and the disclosure is informative. In the limit, the manager is fully informed, so that there is full revelation of the ex post number of successes. The market's response is to put all the weight on the worst possible outcome consistent with the manager's disclosure. This is the so-called "unravelling" argument discussed in Milgrom (1981), Grossman (1981) and Milgrom and Roberts (1986), and much of the literature has focused on the conditions under which full revelation takes place (such as Lipman and Seppi (1995), Seidmann and Winter (1997)). However, as long as $\theta < 1$, the manager is not always fully informed, and the market must make allowance for some pooling between genuinely uninformed types of the manager and those types that are fully informed but are withholding information[2].

At the opposite extreme to the case of full revelation, we have the case in which $\theta = 0$. Then, we have $B(q) = B(r)$, so that the conditional density is obtained from the prior probability of success given by r. The market thus takes the manager's disclosures at face value. For intermediate values of θ, the market discounts the manager's disclosures reducing the probability q in the pricing formula. When $\theta = 0$, only one disclosure receives positive weight, namely the one in which no successes are reported. Any favourable shift in the disclosure of successes (an increase in p) is matched by a corresponding opposite shift in the posterior probability of success (a decrease in q), and vice versa. The market discounts the opportunistic disclosures of the manager by adjusting downwards the price of the firm at the interim stage. For any given r (ex ante probability of success), the trade-off between p and q is given by

$$\frac{dq}{dp} = -\frac{1-q}{1-p}. \tag{4.4}$$

Since failures are never disclosed under the sani

$$
\begin{aligned}
&= (1+\Delta)^s \sum_{i=0}^{N-s} \binom{N-s}{i} [q(1+\Delta)]^i (1-q)^{N-s-i} \\
&= (1+\Delta)^s (1+q\Delta)^{N-s} . \qquad (4.6)
\end{aligned}
$$

Figure 4.1 depicts the equilibrium prices over time. The lowest price observed at date 1 is when the manager reports no successes at all, in which case price is $V_1(0) = (1+q\Delta)^N$. This compares with the ex ante price of $V_0 = (1+r\Delta)^N$. The highest possible price at date 1 is when the manager reports 100% success (i.e. when $s = N$). For intermediate disclosure k, the interim price is given by $(1+\Delta)^k (1+q\Delta)^{N-k}$. Since k successes have been disclosed, the final liquidation value of the firm lies between $(1+\Delta)^k$ and $(1+\Delta)^N$. The theorem tells us that the factor $(1+q\Delta)^{N-k}$ is the appropriate scaling factor for this residual uncertainty.

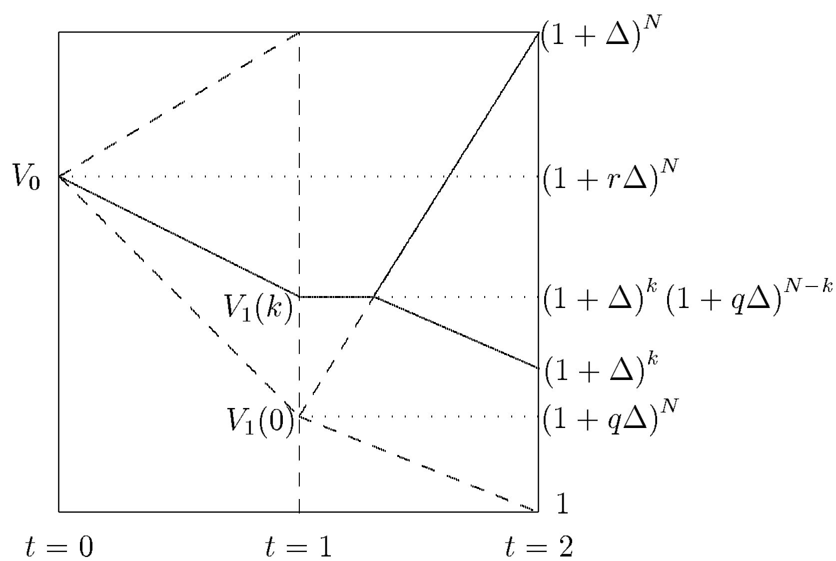

Figure 4.1: Equilibrium Prices

5. Variance of Returns

One virtue of the simple pricing rule outlined above is that empirical predictions on the return process can be obtained with minimal effort. In particular, we can show that the return variance following a bad first period return is higher than the return variance following a good first period return. More precisely, define the *first period return* as

$$R_1 = \frac{V_1}{V_0}$$

and define *second period return* as $R_2 = V_2/V_1$. Then consider the variance of second period return conditional on first period return, denoted by $\mathrm{Var}(R_2|R_1)$. We can prove:

Theorem 5.1. *$Var(R_2|R_1)$ is a decreasing function of R_1.*

The intuition for this result is easy to grasp from the fact that the posterior densities over successes are given by the q-binomial matrix. First period return is high when a large number of successes are disclosed at date 1. This implies that the residual uncertainty is small thereafter. In particular, the residual uncertainty is given by the binomial density with success probability q where the number of trials is equal to the number of undisclosed projects. Thus, the higher is the first period return, the lower is the uncertainty governing second period return.

Let us prove theorem 5.1 by deriving explicitly the conditional variance of second period return. Price given disclosure s is $V_1(s) = (1 + \Delta)$

for small Δ. So when θ is high (so that the manager is well informed), the disclosure s must be high to match the heightened expectation of the market. Any disclosure that falls short of this will result in a fall in price.

The second period return conditional on disclosure s by the manager is the random variable $V_2/V_1(s)$, which takes the value $(1+\Delta)^j \big/ [1+q\Delta]^{N-s}$ with probability $\binom{N-s}{j} q^j (1-q)^{N-s-j}$. Since

$$\mathrm{E}\left(R_2^2|s\right) = \frac{\mathrm{E}\left(\left.(1+\Delta)^{2j}\right| s\right)}{[1+q\Delta]^{2(N-s)}} = \left[\frac{q(1+\Delta)^2+1-q}{(1+q\Delta)^2}\right]^{N-s},$$

the variance of second period return conditional on disclosure s is

$$\mathrm{Var}\left(R_2|s\right) = \left[\tfrac{1+q\Delta(2+\Delta)}{(1+q\Delta)^2}\right]^{N-s} - 1. \tag{5.4}$$

Conditional variance is decreasing in s. Using the relation between disclosure s and the first period return R_1 from (5.2), we can derive the expression for the variance of second period return conditional on R_1. Let $R_1^{\max} \equiv [(1+\Delta)/(1+r\Delta)]^N$ be the highest possible return at date 1 (when the manager announces a 100% success rate on the projects). Then,

$$\mathrm{Var}\left(R_2|\, R_1\right) = Q^{\log\left(R_1^{\max}/R_1\right)} - 1 \tag{5.5}$$

where

$$Q \equiv \left[\frac{1+q\Delta(2+\Delta)}{(1+q\Delta)^2}\right]^{1/\log\left[\frac{1+\Delta}{1+q\Delta}\right]}. \tag{5.6}$$

This proves the theorem. A more useful expression for the conditional variance of R_2 can be obtained when the increment Δ is small. Since $\log(1+x) \approx x$ for small x, we have the following approximation for $\log Q$ when Δ is small.

$$\log Q = \tfrac{\log\left(1+q\left(2\Delta+\Delta^2\right)\right)-2\log(1+q\Delta)}{\log(1+\Delta)-\log(1+q\Delta)} \approx \frac{q\Delta}{1-q}. \tag{5.7}$$

Then (5.5) has the approximation:

$$\mathrm{Var}\left(R_2|\, R_1\right) \approx \left(\frac{R_1^{\max}}{R_1}\right)^{\frac{q\Delta}{1-q}} - 1. \tag{5.8}$$

Further simplifications are possible through more formal asymptotic analysis. Before we do so, we examine the pricing consequences of risk aversion.

5.1. Negative Serial Correlation of Returns

So far we have examined pricing in a risk-neutral setting. This entails that both the expected first period return and the expected second period return are 1. We now examine the consequences of introducing risk aversion into the pricing formula.

Since conditional return variance is decreasing in first period return, a low first period return will be associated with large variance in subsequent returns. If prices reflect risk aversion, the first period price must be correspondingly lower in order to induce risk averse traders to hold the asset.

Explicit pricing formulae are made easier if we assume that the representative investor has the constant relative risk aversion utility function. Thus, for this sub-section only, we assume that

$$u(c) = \frac{c^{1-\alpha}}{1-\alpha}. \tag{5.9}$$

Then, the price of the firm at the interim date is obtained from the state prices across the ex post numbers of successes. Each state price is the normalized value of the product of the probability of that state and the marginal utility of consumption at that state. Since marginal utility is given by $u'(c) = c^{-\alpha}$, the price of the firm at the interim date as a function of the disclosed number of successes s is given by:

$$\begin{aligned} \widehat{V}_1(s) &= \frac{\sum_{i=0}^{N-s} \binom{N-s}{i} q^i \left[(1+\Delta)^{s+i}\right]^{1-\alpha} (1-q)^{N-s-i}}{\sum_{i=0}^{N-s} \binom{N-s}{i} q^i \left[(1+\Delta)^{s+i}\right]^{-\alpha} (1-q)^{N-s-i}} \\ &= (1+\Delta)^s \left[\frac{q(1+\Delta)^{1-\alpha} + 1 - q}{q(1+\Delta)^{-\alpha} + 1 - q}\right]^{N-s} \end{aligned}$$

Defining the constant

$$\pi \equiv \frac{q(1+\Delta)^{-\alpha}}{q(1+\Delta)^{-\alpha} + 1 - q} \tag{5.10}$$

the price can be written as

$$\widehat{V}_1(s) = (1+\Delta)^s (1+\pi\Delta)^{N-s} \tag{5.11}$$

Comparing this expression with the corresponding pricing formula for the risk-netural case (4.6), we see that the effect is to replace q by π. Since $\pi < q$, the effect of risk aversion is to reduce the price of the firm at the interim date. The price of the firm at the interim date with risk aversion coincides with the price in the risk-neutral case where the posterior probability of success has been reduced from q to π. Second period return conditional on disclosure s is a random variable that takes the value $\frac{(1+\Delta)^j}{[1+\pi\Delta]^{N-s}}$ with probability $\binom{N-s}{j} q^j (1-q)^{N-s-j}$. Hence expected second period return conditional on disclosure s is given by

$$\mathrm{E}\left(R_2|s\right) = \left[\frac{1+q\Delta}{1+\pi\Delta}\right]^{N-s}. \tag{5.12}$$

Expected second period return is decreasing in the disclosure s. Following good news, subsequent expected return is low, but following bad news, subsequent expected return is high.

The price of the firm at the initial date, denoted by $\widehat{V}_0$, can be derived in a similar way. It is given by

$$\begin{aligned} \widehat{V}_0 &= \frac{\sum_{i=0}^N \binom{N}{i} r^i \left[(1+\Delta)^i\right]^{1-\alpha} (1-r)^{N-i}}{\sum_{i=0}^N \binom{N}{i} r^i \left[(1+\Delta)^i\right]^{-\alpha} (1-r)^{N-i}} \\ &= \left[\frac{r(1+\Delta)^{1-\alpha}+1-r}{r(1+\Delta)^{-\alpha}+1-r}\right]^N \\ &= (1+\chi\Delta)^N \end{aligned} \tag{5.13}$$

where

$$\chi = \frac{r(1+\Delta)^{-\alpha}}{r(1+\Delta)^{-\alpha}+1-r} \tag{5.14}$$

Figure 5.1 depicts the consequences of risk aversion for asset prices at dates 0 and 1. Risk aversion lowers prices both at the initial and interim dates, thereby raising the expected return above the actuarially fair rate of 1.

First period return when s successes are disclosed is given by

$$R_1(s) = \frac{\widehat{V}_1(s)}{\widehat{V}_0} = \left[\frac{1+\Delta}{1+\pi\Delta}\right]^s \left[\frac{1+\pi\Delta}{1+\chi\Delta}\right]^N.$$

so that from (5.12), we have an expression for the expected second period return conditional on first period return. It is

$$\mathrm{E}\left(R_2\,|R_1\right) = \left[\frac{1+q\Delta}{1+\pi\Delta}\right]^{\frac{1}{\log\frac{1+\Delta}{1+\pi\Delta}}\left\{\log\left(R_1^{\max}/R_1\right)\right\}} \tag{5.15}$$

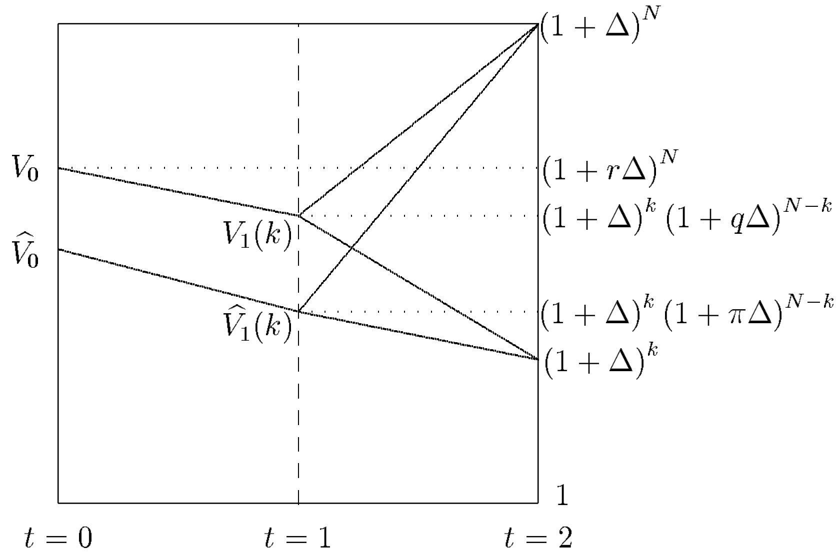

Figure 5.1: Prices with risk aversion

where $R_1^{\max}$ is the highest possible first period return, given by $[(1+\Delta)/(1+\chi\Delta)]^N$. We therefore have the following result.

Theorem 5.2. *$E(R_2\,|R_1)$ is a decreasing function of R_1.*

This result is a natural consequence of the fact that conditional variance of second period return is decreasing in first period return. For risk averse investors to be induced to hold the asset following a bad outcome at date 1, the expected second period return must be higher. We can obtain simpler expressions for conditional return when Δ is small. Using the approximation $x \approx \log(1+x)$, we can write (5.15) as

$$\mathrm{E}\left(R_2\,|R_1\right) \approx \left(\frac{R_1^{\max}}{R_1}\right)^{\frac{q-\pi}{1-\pi}}. \tag{5.16}$$

5.2. Asymptotic Analysis

We would now like to take limits where $\Delta \to 0$ and $N \to \infty$ in such a way that the return distributions are non-degenerate. We do this for the risk-neutral case only. Similar arguments can be deployed for the risk averse case. In particular, we would like to impose the constraint that

$$\mathrm{Var}\left(R_1\right) = \mathrm{Var}\left(R_2|1\right) = \sigma^2 \tag{5.17}$$

for constant $\sigma > 0$, so that the first period return variance is equal to the second period return variance at the mean. In order to find the right limiting procedure, it is useful to note the following approximations for small Δ in the risk neutral case.

$$\begin{cases} \log R_1^{\max} \approx N\Delta\left(1-r\right) \\ \\ \mathrm{Var}\left(R_1\right) \approx \exp\left\{N\Delta^2\left(p+q^2-pq^2\right)\right\} - 1 \\ \\ \mathrm{Var}\left(R_2|\, R_1\right) \approx \exp\left\{\dfrac{\Delta q}{1-q}\left(N\Delta\left(1-r\right) - \log R_1\right)\right\} - 1 \end{cases} \tag{5.18}$$

By setting:

$$\Delta^2 = \frac{\log\left(1+\sigma^2\right)}{K \cdot N} \tag{5.19}$$

for constant K, and taking limits as $N \to \infty$, we have

$$\begin{aligned} \mathrm{Var}\left(R_1\right) &\to \exp\left\{\frac{p+q^2-pq^2}{K} \cdot \log\left(1+\sigma^2\right)\right\} - 1 \\ \mathrm{Var}\left(R_2|\, 1\right) &\to \exp\left\{\frac{q\left(1-r\right)}{\left(1-q\right)K} \cdot \log\left(1+\sigma^2\right)\right\} - 1 \end{aligned}$$

Then, for

$$\begin{aligned} \theta &= \frac{2-r-\sqrt{4-8r+5r^2}}{2r} \\ K &= r - \frac{1}{2}\left(2-r-\sqrt{4-8r+5r^2}\right) \end{aligned}$$

we have

$$K = p + q^2 - pq^2 = \frac{q\left(1-r\right)}{\left(1-q\right)}$$

so that $\mathrm{Var}(R_1) = \mathrm{Var}(R_2|\, 1) = \sigma^2$ in the limit, as desired.

Note, however, that this particular limiting procedure has the consequence that the volatility smile disappears as N becomes large. This can be seen from the expression for $\mathrm{Var}(R_2|\, R_1)$ in (5.18). The term involving $\log R_1$ enters with the same order of magnitude as Δ, which in turn is of the order of $N^{-\frac{1}{2}}$. The disappearance of the volatility smile in the limit suggests that we should exercise some caution in using the large N approximation for the general case with finite N.

The reason for the disappearnce of the volatility smile in the limit can be understood in terms of the different rates at which the support of the success distribution shifts as compared to the rate at which the standard deviation shifts. The maximum number of successes is increasing linearly in N, but the standard deviation increases at a much slower rate - at the rate of $\sqrt{N}$. The volatility smile is a phenomenon that is tied up with the maximum first period return $R_1^{\max}$, and hence is inherently a feature that appeals to the size of the support of the distribution over successes. As N becomes large, the support is increasing at a much quicker rate than the standard deviation of first period return.

5.3. Alternative limiting procedure

Another way to illustrate the role of the finite support in generating the volatility smile is to examine an alternative limiting procedure that results in a finite support in the limit. This can be accomplished by setting

$$\Delta = \frac{\log(1+\sigma^2)}{\sqrt{N}} \tag{5.20}$$

$$r = 1 - \frac{1}{\sqrt{N}} \tag{5.21}$$

$$\theta = 1 - \frac{1}{\log(1+\sigma^2)} \tag{5.22}$$

Then, we achieve (5.17), and have the following limiting expressions.

$$R_1^{\max} \rightarrow 1+\sigma^2 \tag{5.23}$$

$$\mathrm{Var}(R_2|\, R_1) \rightarrow \frac{1+\sigma^2}{R_1} - 1 \tag{5.24}$$

and thus obtain the volatility smile in the limit.

However, the limiting return distributions display a number of unconventional features. The first period return R_1 has mean of 1 and variance σ^2, but its support is bounded. The maximum return is given by $1+\sigma^2$, and its minimum return (given by the limit of $\left[(1+q\Delta)/(1+r\Delta)\right]^N$) can be shown to converge to

$$\left[\frac{1}{1+\sigma^2}\right]^{\log\left(1+\sigma^2\right)-1}. \tag{5.25}$$

which is less than 1 from (5.22). Thus, R_1 lies between (5.25) and $1+\sigma^2$. The second period return R_2 conditional on disclosure s has the lowest value of $\left[1/(1+q\Delta)\right]^{N-s}$ (when no further successes result from the unresolved projects) and the highest value of $\left[(1+\Delta)/(1+q\Delta)\right]^{N-s}$ (when all remaining projects turn out to be successful). In the limit, the former tends to zero, while the latter tends to $(1+\sigma^2)/R_1$, where R_1 is the first period return corresponding to disclosure s. Since the second period conditional return has mean of 1, the shape of the distribution differs markedly depending on first period return. Given low first period return, the distribution of second period return is positively skewed, while given a high first period return, it is negatively skewed. Although these features of the return distribution are unconventional, such an approach may deserve further attention given the empirical findings of Krishnan et al. (1999) which documents evidence that the skewness of disclosures and returns may have a role in explaining the degree of scepticism exercised by the market on the disclosures of managers.

6. Concluding Remarks

The theory in this paper has been developed in the context of *corporate disclosure*, and this has motivated our choice of using the framework of verifiable reports in setting up our game. There are many other contexts in which the disclosing party has an interested in the reactions of the market to its disclosures. Not all of these cases would be best dealt with by using the verifiable reports framework. For brokers' stock recommendations, for example, the regulatory constraints on the generally accepted accounting principles (GAAP) would not apply, and it would be more reasonable to employ the cheap talk framework. Morgan and Stocken (2000) examine this issue.

There are some cases for which the choice of framework is more finely balanced. Disclosures by governments is one of these cases. Sovereign risk has been notoriously difficult to capture in a formal asset pricing setting since the notion of default is even less clear than in the case of corporate default. Opportunistic behaviour on the part of the debtor cannot be ruled out, where the *willingness* to repay is more relevant than the ability to repay. The series of international financial crises in recent years has been a salutary reminder of the shortcomings of our current understanding of financial distress in international finance.

One policy response to the turbulence in international markets has been to call for increased transparency of disclosure from government and other official sources, as well as major market participants. A series of initiatives are under way from multilateral organizations towards greater transparency (see BIS (1999), IMF (1998)). The theory presented in this paper could be seen as one way to formalize the notion that *uncertainty increases during a crisis*. To the extent that governments and monetary authorities have an interest in the reactions of the market towards a particular set of outcomes, its disclosure policy will be necessarily influenced by this. The analogy between the disclosures by governments and official bodies on the one hand, and the accounting disclosures by firms on the other is only as strong as the assumption that disclosures by governments are verifiable. To the extent that the analogy can be pushed further, the absence of news is seen as bad news by sophisticated market participants, and it has the effect of increasing uncertainty (by raising the variance of subseqent returns). When expressed in this way, the debate on transparency of disclosures can then be placed more firmly within familiar theoretical categories. Further exploration of this approach to official disclosure policy would seem to be promising, and more work on the institutional foundations of verifiability, such as the work by Bull and Watson (2000), is called for.

Appendix

Proof of lemma 4.1. Since the disclosed number of successes at the interim date cannot exceed the realized number of successes, $h(i,s) = 0$ for $i > s$. When $h(i,s)$ is positive, it is the product of two numbers - the probability that the *realized* number of successes is s, and the probability that i of these successes is realized by the interim date. In other words, for $i \leq s$,

$$\begin{aligned} h(i,s) &= \binom{N}{s} r^s (1-r)^{N-s} \cdot \binom{s}{i} \theta^i (1-\theta)^{s-i} \\ &= \frac{N!}{(N-s)!i!(s-i)!} r^s (1-r)^{N-s} \theta^i (1-\theta)^{s-i} \end{aligned}$$

Then,

$$\begin{aligned} \frac{h(i,s)}{h(i,s-1)} &= \frac{\frac{N!}{(N-s)!\,i!\,(s-i)!} \cdot r^s (1-r)^{N-s} \theta^i (1-\theta)^{s-i}}{\frac{N!}{(N-s+1)!\,i!\,(s-i-1)!} \cdot r^{s-1} (1-r)^{N-s+1} \theta^i (1-\theta)^{s-i-1}} \\ &= \frac{N-s+1}{s-i} \cdot \frac{r(1-\theta)}{1-r} \\ &= \frac{\frac{(N-i)!}{s!\,(N-i-s)!} \cdot r^s (1-\theta)^s (1-r)^{N-i-s}}{\frac{(N-i)!}{(s-1)!\,(N-i-s+1)!} \cdot r^{s-1} (1-\theta)^{s-1} (1-r)^{N-i-s+1}} \\ &= \frac{\binom{N-i}{s} q^s (1-q)^{N-i-s}}{\binom{N-i}{s-1} q^{s-1} (1-q)^{N-i-s+1}} \end{aligned}$$

where

$$q = \frac{r(1-\theta)}{(1-r) + r(1-\theta)} = \frac{r - r\theta}{1 - r\theta}$$

This proves lemma 4.1.

References

[1] Anderson, R. and S. Sundaresan (1996) "Design and Valuation of Debt Contracts" *Review of Financial Studies*, 9, 37-68.

[2] Austen-Smith, D. (1994) "Strategic Transmission of Costly Information" *Econometrica*, 62, 955-963.

[3] Barndorff-Nielsen, O. E. and N. Shephard (1998) "Incorporation of a leverage effect in a stochastic volatility model" Nuffield College discussion paper W98-14, http://www.nuff.ox.ac.uk/economics/papers/index.html.

[4] Basel Committee on Banking Supervision (1999) *Recommendations for Public Disclosure of Trading and Derivatives Activities of Banks and Securities Firms*, Bank of International Settlements, Basel.

[5] Black, F. (1976) "Studies in Stock Price Volatility Changes" *Proceedings of the Business and Economic Statistics section of the American Statistical Association*, 177-181.

[6] Bull, J. and J. Watson (2000) "Evidence Disclosure and Verfiability" unpublished paper, UC San Diego.

[7] Campbell, J. Y. and A. S. Kyle (1993) "Smart Money, Noise Trading and Stock Price Behaviour" *Review of Economic Studies*, 60, 1 - 35.

[8] Christie, A. (1982) "The Stochastic Behavior of Common Stock Variances: Value, Leverage and Interest Rate Effects" *Journal of Financial Economics*, 10, 407-432.

[9] Cox, J. C., S. A. Ross and M. Rubinstein (1979) "Option Pricing: a Simplified Approach" *Journal of Financial Economics*, 7, 229-263.

[10] Crawford, V. and J. Sobel (1982) "Strategic Information Transmission" *Econometrica*, 50, 1431-1451.

[11] Dye, R. A. (1985a) "Disclosures of Nonproprietory Information" *Journal of Accounting Research*, 23,123-145.

[12] Dye, R. A. (1985b) "Strategic Accounting Choice and the Effect of Alternative Financial Reporting Requirements" *Journal of Accounting Research*, 23, 544-574.

[13] Engle, R. and V. Ng (1993) "Measuring and Testing the Impact of News on Volatility" *Journal of Finance*, 48, 1749-1778.

[14] Figlewski, S. and X. Wang (2000) "Is the 'Leverage Effect' a Leverage Effect?" unpublished paper, NYU.

[15] Grossman, S. J. (1981) "The Informational Role of Warranties and Private Disclosures about Product Quality" *Journal of Law and Economics*, 24, 461 - 483.

[16] International Monetary Fund (1998) *Report of the Managing Director to the Interim Committee on Strengthening the Architecture of the International Monetary System,* http://www.imf.org/external/np/omd/100198.htm

[17] Kreps, D. and R. Wilson (1982) "Sequential Equilibria" Econometrica, 50, 863-894.

[18] Krishnan, M., S. Sankaraguruswamy, and H. S. Shin (1997) "Skewness of Earnings and the Believability Hypothesis: How Does the Financial Market Discount Accounting Earnings Disclosures?" unpublished paper, http://www.nuff.ox.ac.uk/users/Shin/working.htm

[19] Lewis, T. R. and D. E. M. Sappington (1993) "Ignorance in Agency Problems" *Journal of Economic Theory*, 61, 169-183.

[20] Lipman, B. L. and D. J. Seppi (1995) "Robust Inference in Communication Games with Partial Provability" *Journal of Economic Theory*, 66, 370-405.

[21] Milgrom, P. R. (1981) "Good News and Bad News: Representation Theorems and Applications" *Bell Journal of Economics*, 12, 350-391.

[22] Milgrom, P. R. and J. Roberts (1986) "Relying on the Information of Interested Parties" *Rand Journal of Economics,* 17, 18-32.

[23] Morgan, J. and P. Stocken (2000) "An Analysis of Stock Recommendations" unpublished paper, Princeton University.

[24] Nelson, D. B. (1991) "Conditional Heteroskedasticity in Asset Returns: a New Approach" *Econometrica*, 59, 347-370.

[25] Okuno-Fujiwara, M., A. Postlewaite and K. Suzumura (1990) "Strategic Information Revelation," *Review of Economic Studies*, 57, 25-48.

[26] Schwert, G. (1989) "Why Does Stock Market Volatility Change over Time?" *Journal of Finance*, 44, 1115-1153.

[27] Seidmann, D. J. and E. Winter (1997) "Strategic Information Transmission with Verifiable Messages" *Econometrica*, 65, 163-170.

[28] Shin, H. S. (1994) "News Management and the Value of Firms" *Rand Journal of Economics*, 25, 58-71.

[29] Verrecchia, R. E. (1983) "Discretionary Disclosure" *Journal of Accounting and Economics*, 5, 179-194.