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On a Dilemma of Redistribution

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Abstract

McKenzie Alexander (2013) presents a dilemma for a social planner who wants to correct the unfair distribution of an indivisible good between two equally worthy individuals or groups: *either* she guarantees a fair outcome, *or* she follows a fair procedure (but not both). In this paper I show that this dilemma only holds if the social planner can redistribute the good in question at most once. To wit, the bias of the initial distribution always washes out when we allow for sufficiently many redistributions.

1 Introduction

McKenzie Alexander (2013) presents a dilemma for a social planner who wants to correct the unfair distribution of an indivisible good between two equally worthy individuals or groups (call them a and b):

Dilemma *Either* she guarantees a fair outcome, *or* she follows a fair procedure (but not both).

The argument is disconcertingly simple. Suppose the initial distribution is biased against b. If b nevertheless receives the good against all odds, as it were, it would seem unfair to take it away from her. However, if a receives the good then the social planner would want to intervene and redistribute. There are two strategies the social planner could follow when redistributing: the redistribution could be fair, offering equal chances to a and b of winning the good redistributed, or it could be unfair. McKenzie Alexander proves that if the social planner follows the former strategy, then *ex ante*, a and b have unequal chances of receiving the good. The procedure for redistributing is fair, but the outcome is that b is favoured (overall). On the other hand, if the social planner follows the latter strategy, then equal chances can be guaranteed *ex ante*, assuming the social planner chooses the appropriate biased lottery, but the redistribution would be biased against b. To wit, the social planner can either employ a fair redistribution procedure or guarantee a fair distributive mechanism *ex ante*. But not both!

McKenzie Alexander doesn't explicitly mention Broome's (1990) theory of fairness, but his *Dilemma* poses an interesting challenge to it. Broome famously construed fairness as the proportional satisfaction of claims. In the case in which a social planner is deciding on the distribution of an indivisible good between two equally worthy candidates, Broome's theory requires that both candidates be given an equal chance of getting the good (Broome, 1990, 96). However, assume that a miscalculation took place in the distribution of an indivisible good. Can someone trying to act

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fairly, given Broome's understanding of the term, redress the unfairness of the initial allocation? If the answer to this question is 'no', then Broome's theory of fairness would be in trouble as one would expect the demands of fairness in distribution to cohere with the demands of fairness in redistribution. McKenzie Alexander's *Dilemma* seems to suggest Broome's theory fails this test.

In this paper I show that *Dilemma* only holds if the social planner can redistribute the good in question at most once. More precisely, irrespective of the bias of the initial unfair distribution, a social planner can secure a fair overall outcome through fair redistributions. Consequently, *Dilemma* doesn't pose a challenge to Broome's theory of fairness as long as we allow for iterated rounds of redistribution.

2 McKenzie Alexander's argument

Consider the following formal representation of the initial (unfair) distribution McKenzie Alexander's social planner is trying to correct:

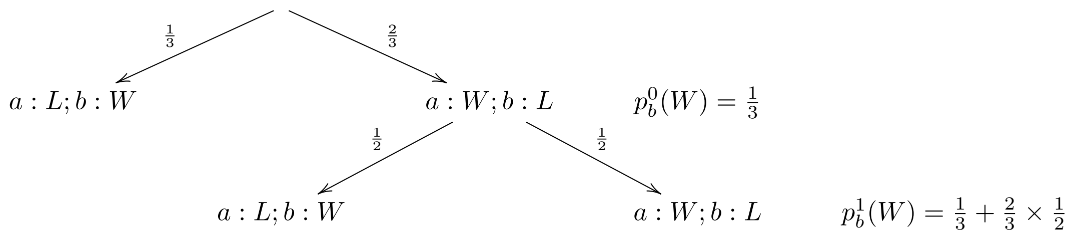
$$\begin{tikzcd} & {} \arrow[dl, "\frac{1}{3}"'] \arrow[dr, "\frac{2}{3}"] & & \\ a : L; b : W & & a : W; b : L & p_b^0(W) = \frac{1}{3} \end{tikzcd}$$

There are two ways the scenario can play out. Either a wins (W, and b loses, L) or b wins. The chance of a winning is $\frac{2}{3}$ and the chance of b winning is the complement, $\frac{1}{3}$. McKenzie Alexander assumes that because a and b are equally worthy they should get an equal claim to the good (which is what Broome's theory would require, as well). However, since b's chance of receiving the good is less than $\frac{1}{2}$ as a result of this distribution, it means she is aggrieved. If the b nevertheless wins, Mckenzie Alexander contends the social planner should refrain from interfering. Taking the good away from b would be like punishing her for making it despite the odds which were stacked against her. So the social planner should only interfere when a wins this distribution. In other words, the protocol McKenzie Alexander believes a social planner motivated by consideration of fairness should be following is the following:

1. In an unfair decision procedure, the aggrieved has the right to demand an appeal, using a fair decision procedure, if she loses;
2. In an unfair decision procedure, the loser does not have the right to demand an appeal, using a fair decision procedure, if he was favoured. (McKenzie Alexander, 2013, 228)

These two principles are minimal under Broome's theory of fairness. I take the first principle to be a self-evident consequence of Broome's theory of fairness. In order to establish the second principle, assume first that each individual (or group) has an equal chance of receiving the good (i.e. their chances would be proportional to their claims, which are equal in McKenzie Alexander's scenario). Then, according to Broome, the allocation of the good would be fair and hence the party that does not receive the good cannot claim that it has been treated unfairly and ask for any kind of redress - they were given "a sort of surrogate satisfaction" (Broome, 1990, 98). Now imagine one of the parties has a chance of receiving the good that far exceeds their claim but nevertheless doesn't receive the good. It seems that if they didn't have a claim when their chances were proportional with their claim, they shouldn't have a claim when their chances exceed their claim as their surrogate satisfaction now exceeds what they were owed.

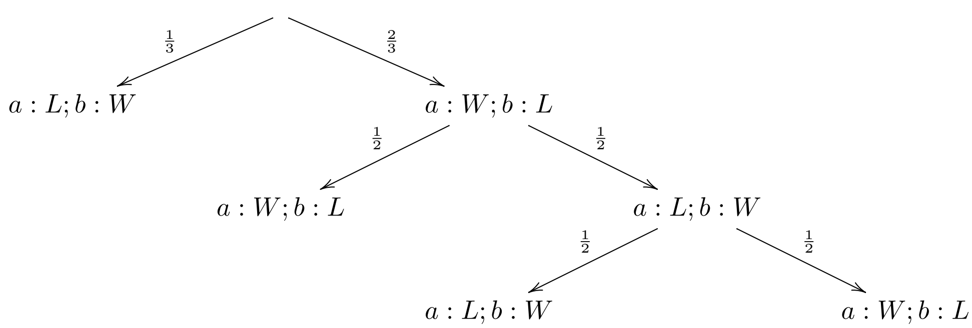
Returning to McKenzie Alexander's scenario, assume the social planner decides to redistribute the good through a fair procedure (in which the probability with which the two individuals receive the good is commensurate to their claim).



At the level of the redistribution both a and b are given an equal chance of winning the good by the social planner. This is in line with their (equal) claim. However, if this is how the social planner interferes, the redistributive mechanism he thus creates awards b an *ex ante* higher chance of winning the good than her claim, $p_b^1(W) = \frac{2}{3}$. If we assume that a had no doing in the initial bias in his favour, we have a strong intuition this set-up is unfair.

3 From one-shot to iterated redistributions

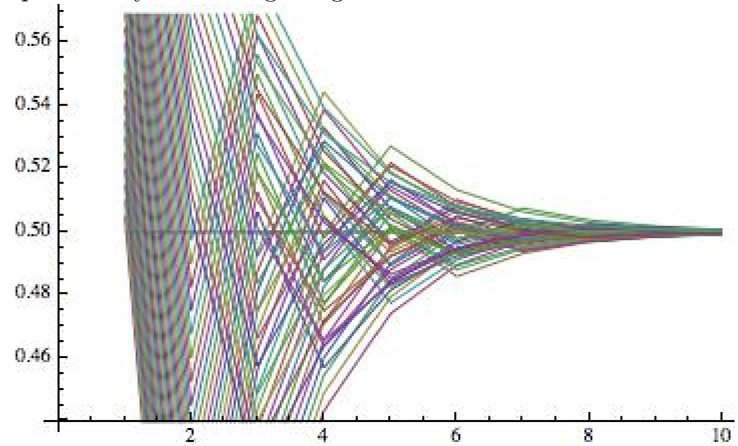
By redistributing fairly, i.e. according to the claims of the two individuals involved, the social planner generated an *ex ante* unfair mechanism. Can the social planner do anything to correct this *ex ante* unfairness? The answer is YES. He can redistribute once again if the individual aggrieved by the last redistribution performed does not win the good. In this case, after the first redistribution, a is left with a chance of winning less than his claim, and hence now becomes the aggrieved party. So whenever a loses the good, the social planner seems entitled to offer him another, fair chance (as per McKenzie's first principle).



$$p_b^0(W) = \frac{1}{3} < \frac{1}{2}$$
$$p_b^1(W) = \frac{1}{3} + \frac{2}{3} \times \frac{1}{2} = \frac{2}{3} > \frac{1}{2}$$
$$p_b^2(W) = \frac{1}{3} + \frac{2}{3} \times \frac{1}{2^2} = \frac{1}{2} = \frac{1}{2}$$

Evaluate the situation after the first redistribution: a is now aggrieved since a's *ex ante* chance of winning the good, $p_a^1(W) = \frac{1}{3} < \frac{1}{2}$. Therefore the social planner can redistribute again whenever a loses the redistribution. After the second redistribution, both individuals a and b now have equal *ex ante* chances of winning the good which is being distributed and hence there is no need for the social planner to correct when one of them loses. This is good news, but notice that the analysis was dependent on the initial bias. It worked for $p = \frac{1}{3}$. Does the solution work for all initial biases (for all unfair distributions)? The answer is again YES. Figure 1 tracks how the *ex ante* chances of winning the good evolve over 10 redistributions for values of the initial bias between 0 and $\frac{1}{2}$ in 0.01 increments. A formal proof is provided in the Appendix.

Figure 1: *Ex ante* probability of winning the good over 10 redistributions for different initial biases



McKenzie Alexander writes that

> [s]ometimes the correct response to an injustice generated by an unfair decision procedure is to use another unfair decision procedure, which appears to disadvantage (in some sense) the same person again. In these cases, *two wrongs do make a right.* (McKenzie Alexander, 2013, 230, my emphasis)

The result of this paper then is that the bias against an aggrieved individual (or group) always washes out when we allow for sufficiently many redistributions. In other words, we do not have to make a second wrong in order to make right by the aggrieved: at most infinitely many rights will do. This is encouraging. But even if it is always the case that a social planner can correct an initial unfair distribution by behaving fairly towards both the aggrieved and the party favoured in the initial distribution, no social planner has infinite time and resources. Can anything better be done for real social planners? The answer is one last time YES.

I contend it is unproblematic to assume people are not sensitive to minute differences in probabilities. Then let the sensitivity of the most sensitive member of the two person/group society we are concerned about in this paper be δ. I investigated two possible values for δ: $\delta_1 = 0.001$ and $\delta_2 = 0.01$. Under δ_1 the individuals in the society cannot tell a .500 chance of winning the good apart from a .5001 chance. Under δ_2 they cannot tell apart a .50 from a .51 chance of winning the good. It turns out that for δ_1 it takes *at most* nine redistributions for the probability of winning for b to reach the interval $[.499, .501]$ and hence become identical to $\frac{1}{2}$. For δ_2, the probability of winning for b reaches a value in $[.49, .51]$ in *at most* six redistributions.This resulted by testing all values of the initial bias, p, between 0 and $\frac{1}{2}$ in 0.01 increments in Mathematica 9. That is, the question I asked was "in how many redistributions does the probability of winning for the aggrieved reach a value in the interval $[.499, .501]$ (for δ_1)/$[.49, .51]$ (for δ_2)"? And I investigated the following values for the initial bias against the aggrieved $p \in \{.01, .02, .03, \ldots, .48, .49\}$.[1]

The result is interesting as it tells us that no matter what the bias of an initial distribution is, it is always possible for a social planner to offer the two participants to the distribution equal *ex ante* chances of winning the good if at most six rounds of fair redistributions are available (assuming that the most sensitive of the aggrieved and the favoured of the original distribution has sensitivity δ_2).

4 Conclusion

To sum up, contrary to McKenzie Alexander's point, there is no tension between procedural and outcome fairness as long as the social planner is given the opportunity to redistribute sufficiently many times. It may be the case that "sometimes... two wrongs make a right" but so do *a wrong and infinitely many rights.* And in fact, a wrong and *sufficiently* many rights (depending on p and δ) are right *enough.*[2]

References

Broome, J. (1990). Fairness. *Proceedings of the Aristotelian Society*, 91:87–101.

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[1]Notebooks used are available upon request.

[2]I am grateful to Jason McKenzie Alexander, Luc Bovens, Richard Bradley and Graham Oddie for valuable feedback on early versions of this paper.

Appendix

Let x_i stand for the probability of the aggrieved of the initial distribution winning after the i^{th} redistribution,

$$\begin{aligned} x_1 &= p + (1-p)\frac{1}{2} \\ x_n &= x_{n-1} + (1-p)\frac{1}{2^n} \text{ iff } x_{n-1} < \frac{1}{2} \\ &= x_{n-1} - (1-p)\frac{1}{2^n} \text{ iff } x_{n-1} > \frac{1}{2} \\ &= \frac{1}{2} \text{ iff } x_{n-1} = \frac{1}{2} \end{aligned}$$

We can now formally state the main result of this paper: $lim_{n\to\infty}(x_n) = \frac{1}{2}$. Let the following sequences stand for the elements in (x_n) less than $\frac{1}{2}$ and greater than $\frac{1}{2}$, respectively:

$$\begin{aligned} (a_n) &= \{a \in (x_n) : a < \frac{1}{2}\} \\ (b_n) &= \{b \in (x_n) : b > \frac{1}{2}\} \end{aligned}$$

In order to establish the result in the paper it is enough to prove that the limit of both (a_n) and (b_n) is $\frac{1}{2}$. The proofs are symmetrical and we will only show the proof for (b_n). We will first show that (b_n) is decreasing and then (by the Squeezing Theorem) that its limit is indeed $\frac{1}{2}$.

Take $b_m \in (b_n)$. By construction, $b_m \in (x_n)$. Suppose it corresponds to element $x_n \in (x_n)$. Remark that m may differ from n. Since $b_m > \frac{1}{2}, x_n > \frac{1}{2}$. Therefore $x_{n+1} = x_n - (1-p)\frac{1}{2^{n+1}}$. If $x_{n+1} > \frac{1}{2}$ then $x_{n+1} = b_{m+1}$ if not, $x_{n+2} = x_n - (1-p)\frac{1}{2^{n+1}} + (1-p)\frac{1}{2^{n+2}}$ and so on. Therefore, depending on the value of p, $b_{m+1} = x_{n+k_n}$, for some natural number k_n.[3]

$$\begin{aligned} b_{m+1} &= x_n - (1-p)\frac{1}{2^{n+1}} + (1-p)\frac{1}{2^{n+2}} + \cdots + (1-p)\frac{1}{2^{n+k_n}} \\ &= x_n - (1-p)(\frac{1}{2^{n+1}} - \frac{1}{2^{n+2}} - \cdots - \frac{1}{2^{n+k_n}}) \\ &= x_n - (1-p)\frac{1}{2^{n+k_n}} \end{aligned}$$

In consequence,

$$\begin{aligned} b_m - b_{m+1} &= x_n - x_n + (1-p)\frac{1}{2^{n+k_n}} \\ &= (1-p)\frac{1}{2^{n+k_n}} > 0 \end{aligned}$$

This concludes the proof that (b_m) is a decreasing sequence. In order to show that the limit of all elements in (x_n) greater than $\frac{1}{2}$ when $n \to \infty$ is $\frac{1}{2}$ it is enough to show that

[3] k_n has to be at least 1, in which case both x_n and x_{n+1} are greater than $\frac{1}{2}$; and $k_{n+1} \geq k_n$

$$\frac{1}{2} - \frac{1}{2^n} \leq (x_n)_{x_n \geq \frac{1}{2}} \leq \frac{1}{2} + \frac{1}{2^{n+1+k_{n+1}}}$$

If this is the case, by the Squeeze Theorem

$$lim_{n\to\infty}(x_n)_{x_n \geq \frac{1}{2}} = lim_{n\to\infty}(\frac{1}{2} - \frac{1}{2^n}) = lim_{n\to\infty}(\frac{1}{2} + \frac{1}{2^{n+1+k_n}}) = \frac{1}{2}$$

The first inequality obviously holds since all elements of the sequence $(\frac{1}{2} - \frac{1}{2^n})$ are at most $\frac{1}{2}$. And all elements of $(x_n)_{x_n \geq \frac{1}{2}} \geq \frac{1}{2}$, by construction. Then we only need to check the second inequality. We do this by induction:

$$x_1 = p + (1-p)\frac{1}{2} \leq \frac{1}{2} + \frac{1}{2^{1+1+k_1}}$$

Since k_1 is 0 (as $x_1 \geq \frac{1}{2}$ for all values of p), the right hand side of the inequality will equal $\frac{3}{4}$ which is the highest value (x_n) reaches:

$$\begin{aligned}
x_n &\leq \frac{1}{2} + \frac{1}{2^{n+1+k_n}} \\
x_n - (1-p)\frac{1}{2^{n+k_n}} &\leq \frac{1}{2} + \frac{1}{2^{n+1+k_n}} - (1-p)\frac{1}{2^{n+k_n}} \\
x_{n+k} &\leq \frac{1}{2} + \frac{1}{2^{n+1+k_n}} - (1-p)\frac{1}{2^{n+k_n}}
\end{aligned}$$

What the induction aims to establish is that $x_{n+k_n} \leq \frac{1}{2} + \frac{1}{2^{n+2+k_{n+1}}}$. So we need to show that (the following reasoning steps are all equivalent):

$$\begin{aligned}
\frac{1}{2} + \frac{1}{2^{n+1+k_n}} - (1-p)\frac{1}{2^{n+k_n}} &\leq \frac{1}{2} + \frac{1}{2^{n+2+k_{n+1}}} \\
\frac{1}{2^{n+1+k_n}} - \frac{1}{2^{n+2+k_{n+1}}} &\leq (1-p)\frac{1}{2^{n+k_n}} \\
2^{1+k_{n+1}-k_n} - 1 &\leq (1-p)2^{2+k_{n+1}-k_n} \\
2^{1+k_{n+1}-k_n} - (1-p)2^{2+k_{n+1}-k_n} &\leq 1 \\
2^{1+k_{n+1}-k_n}(1-2+p) &\leq 1 \\
2^{1+k_{n+1}-k_n}(p-1) &\leq 1 \\
&\text{But } p-1 < 0 \text{ for all values of } p
\end{aligned}$$

Therefore, for all values of p, all n: $x_n \leq \frac{1}{2} + \frac{1}{2^{n+1+k_n}}$. This concludes the proof.