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Not a Behaviorist

Samuelson's Contributions to Utility Theory

in the Harvard Years, 1936–1940

Ivan Moscati*

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* Department of Economics, University of Insubria, Varese, Italy, and Centre for Philosophy of Natural and Social Science, LSE; email: ivan.moscati@uninsubria.it. I am grateful to Wade Hands and Philippe Mongin for several discussions on Samuelson's theory of choice; these discussions were the main motivation for writing this paper. Some portions of this chapter draw in part on work previously published elsewhere, namely Moscati 2013 and Moscati 2018. Any errors are mine.

Abstract

In this chapter I review the contributions to utility theory that Samuelson made when he was a Ph.D. student at Harvard, from the first scientific papers he began writing in 1936 to the Ph.D. dissertation he submitted in November 1940. Based on this review, I make three points: (1) after exploring contrasting research paths during the years 1936–1937, Samuelson's stance on utility analysis quickly stabilized and, from around mid-1938, he became an advocate of an ordinal-utility approach to choice theory; (2) accordingly, the widespread image of the young Samuelson as a committed behaviorist who wanted to free economic analysis from the utility concept is misleading; (3) the so-called Das Paul Samuelson Problem, that is, the question of whether Samuelson changed his mind on utility analysis between 1938 and 1948–1950, has either a negative answer or is ill-posed.

Keywords: Paul A. Samuelson; Utility theory; Ordinal utility; Revealed preference theory; Cardinal utility.

JEL codes: B21; B31; B40; D11; D15.

Introduction

When economists, historians of economics, and economic methodologists refer to the work of Paul Samuelson in choice and demand theory, they typically cite the "Note on the Pure Theory of Consumer's Behaviour" that he published when he was a twenty-two-year-old graduate student at Harvard University (Samuelson 1938a, 1938b). In that article, Samuelson put forward an approach to demand analysis, later called revealed preference theory, whose postulates concern choice behavior, rather than, as with other economists of the period, the properties of ordinal utility functions or the shape of indifference curves. The typical reference to Samuelson's 1938 "Note" includes the following quotation: "I propose, therefore, that we start anew in direct attack upon the problem, dropping off the last vestiges of the utility analysis" (1938a, 62).

The steady association of Samuelson with the 1938 "Note" in general and this quotation in particular has had two interrelated effects. First, it has contributed to an image of Samuelson, and specifically of the young Samuelson, as a committed behaviorist who wanted to free economic analysis from the psychological concepts of utility and preference and give demand theory a purely behaviorist configuration centered on those choice postulates later called the Axioms of Revealed Preference (see e.g. Wong [1978] 2006). Second, when this behavioristic image of the young Samuelson was juxtaposed with his later presentation of revealed preference theory as equivalent to ordinal utility theory (Samuelson 1948; 1950), the question arose of whether he had changed his mind between 1938 and 1948–1950. Wade Hands (2014, 86–87) gives a name to this question: "Did he [Samuelson] change his mind about revealed preference, particularly between the time he was writing the original 1938 paper [the "Note"] and his second round of contributions (Samuelson 1948, 1950)? I will call this . . . question Das Paul Samuelson Problem."[1]

In the present paper, I do three things. First, I review the several contributions to utility theory that Samuelson made when he was a Ph.D. student at Harvard, from the first scientific papers he began writing in 1936 to the

[1] The reference is to *Das Adam Smith Problem*, that is, the extensive late nineteenth-century German debate over the apparent inconsistency between the "sympathetic" conception of human nature put forward by Smith in his *Theory of Moral Sentiments* ([1759] 1976) and the "selfish" conception advanced in the *Wealth of Nations* ([1776] 1976). More on Das Adam Smith Problem in Tribe 2015.

doctoral dissertation he submitted in November 1940. In his first article, composed in 1936 and published in February 1937, Samuelson put forward a seminal model for intertemporal choices that relies on what today we call cardinal utility (Samuelson 1937). In the "Note" (1938a; 1938b), I will argue, Samuelson did not want to eliminate utility from consumer choice theory but presented his revealed preference approach as complementary to the utility-based approach. In a paper published in October 1938, he explored the empirical implications of ordinal utility theory (Samuelson 1938c). In another paper, also published in October 1938, he stated conditions that make utility cardinal rather than just ordinal – thereby popularizing the very expression "cardinal utility" – but judged these conditions implausible (Samuelson 1938d). In a brief comment published in February 1939, he opposed Harro Bernardelli's attempt to reintroduce the non-ordinal notion of diminishing marginal utility (Samuelson 1939). In a paper completed in the fall of 1939, but published only three years later, Samuelson (1942) criticized the non-ordinal assumption that the marginal utility of monetary income is constant. Finally, in his Ph.D. dissertation, which was entitled "Foundations of Analytical Economics" (Samuelson 1940), and later became *Foundations of Economic Analysis* (Samuelson 1947), he downplayed the revealed preference approach and presented the theory of consumer behavior adopting an ordinal utility approach. A comprehensive review of Samuelson's early work in utility analysis has hitherto been missing in the rich literature about him, so the present chapter fills a lacuna.[2]

[2] In the first volume of his impressive intellectual biography of Samuelson, covering the period 1915–1948, Roger Backhouse (2017) discusses most of the works in utility analysis that Samuelson carried on between 1936 and 1940. However, Samuelson's utility theory plays only a marginal role in Backhouse's narrative and, I find, gets somehow lost in his encyclopedic study. Moreover, I do not always share Backhouse's interpretation of Samuelson's stance on utility. Wade Hands (2001; 2013; 2014; 2017) has thoroughly discussed Samuelson's revealed preference theory between 1938 and 1950, but has never explicitly targeted Samuelson's early work in utility analysis. John Chipman's essay on Samuelson's consumption theory (1982) is very helpful in many respects, but examines Samuelson's work from an economic-theoretic rather than a historical perspective. Even Philippe Mongin's illuminating article on Samuelson's revealed preference theory (2000) says little about Samuelson's early work in utility analysis. In previous work, I have discussed individual items of Samuelson's early research in utility theory (Moscati 2013; 2018), but this is my first attempt to connect the different pieces in a systematic picture.

Second, based on this review of Samuelson's early contributions to utility theory, I contend that the image of the young Samuelson as a committed behaviorist who consistently attempted to eliminate notions of utility and preference from economic analysis is misleading. Rather, I will argue, in 1936–1937 he was "exploring different research paths," that is, more prosaically, he was a young man who wrote different things in different papers. However, Samuelson's stance on utility analysis quickly stabilized, and from around mid-1938 he became a consistent advocate of an ordinal-utility approach to choice theory.

Samuelson's intellectual trajectory – *from exploration to ordinalism* – is in fact similar to that of another eminent demand theorist of the period, namely the English economist John Hicks. In a famous paper coauthored with Roy Allen, Hicks put forward a utility-free approach to demand analysis based on the notion of indifference curves (Hicks and Allen 1934). In subsequent works, however, Hicks (1937; 1939) set forth his analysis in terms of ordinal utility indices, and in *Value and Capital* (1939) he fine-tuned the ordinal approach to utility theory.

Third, I argue that the answer to Das Paul Samuelson Problem is either negative or the question itself is ill-posed. It is negative in the sense that Samuelson did not change his mind between 1938 and 1948–1950, since he had already changed it in 1938. Alternatively, the problem itself is ill-posed in the sense that the claim of a change of mind in Samuelson's exploratory path between 1936–1937 and mid-1938 is preposterous.

1. Setting the stage: utility and demand theory in the 1930s

To understand Samuelson's early work in utility and demand theory it is essential to bear in mind the state of the theory in the 1930s. That period was one of intense debates associated with the conclusive phase of the so-called ordinal revolution.

1.1. From Pareto to Hicks and Allen, 1900–1934

The ordinal revolution, initiated by Vilfredo Pareto around 1900, consisted in the gradual construction of a theory of demand and equilibrium independent of the assumption that utility is measurable. Pareto and subsequent "revolutionaries" pursued the goal of superseding measurable utility along two distinct lines, which

can be called the preference-based approach and the choice-based approach. Pareto explored both research lines, and in this he was followed by both Samuelson and Hicks.

In the *Manual of Political Economy* ([1906/1909] 2014), Pareto adopted the preference-based approach. In this approach, the primitive concept is that of preference: individuals have well-behaved preferences between commodity bundles, and are able to rank bundles according to their preferences. Utility is just an ordinal numerical index that represents the individual's preference ranking between bundles by assigning higher numbers to more preferred bundles. In mathematical terms, the ordinal nature of utility is expressed by the fact that, if the utility function U(x) represents the individual's preference ranking, any other utility function U*(x)=F[U(x)], where F is any increasing function, also represents the individual's preference ranking.

In other writings, Pareto ([1900] 2008; [1911] 1955) advocated a choice-based approach in which the primitive elements are the individual's indifference curves. Pareto conceived of an indifference curve as something that can be elicited experimentally by observing the individual's choices, that is, without any reference to psychological introspection or the utility notion. More precisely, Pareto imagined an experiment in which the individual is asked to choose between two commodity bundles x and y. If the individual chooses bundle x, the composition of y is changed up to the point where the individual becomes indifferent between x and y, thus determining two points on the indifference curve. This procedure can be repeated until a sufficient number of points on the same indifference curve are identified. Pareto's experiment, however, was hypothetical; there was no actual experimental subject and no commodity bundles.

Pareto's analysis was highly innovative but, as observed by many authors from the 1930s on, defective on both the preference-based and the choice-based lines of attack. In the 1910s, 1920s, and early 1930s, Pareto's idea of restating demand and equilibrium analysis independently of measurable utility found many supporters, especially in its preference-based version (see e.g. Johnson 1913; Slutsky [1915] 1952; Amoroso 1921; Bowley 1924; Schultz 1933). However, these supporters did not address the problems that Pareto had left open. Things changed quite abruptly in the mid-1930s, when a new generation of economists carried out Pareto's ideas by solving most of the problems he had left open. The

conclusive phase of the ordinal revolution was initiated in 1934 by an article jointly co-authored by John Hicks and Roy Allen, two members of the circle of brilliant young economists who formed around Lionel Robbins at the LSE in the early 1930s.

Hicks and Allen (1934) followed the choice-based approach, and attempted to construct demand theory without introducing utility indices into the picture. As for Pareto, the cornerstone of Hicks and Allen's analysis was the indifference curve, and more precisely the marginal rate of substitution (MRS), which corresponds to the slope of the indifference curve. Hicks and Allen defined the MRS between commodities x and y as the quantity of commodity y that just compensates the individual for the loss of a marginal unit of x. This is a definition in terms of commodity quantities and is independent of utility. Based on the MRS so defined, and the assumption that the MRS is diminishing or, equivalently, that the indifference curves are convex, Hicks and Allen were able to determine the relationships between the demand for goods, their price and consumer income in elasticity terms, and to decompose the effect of a price change on demand into what in current microeconomics are called the substitution effect and the income effect. Hicks and Allen's 1934 article quickly became the new reference point for utility and demand theorists.

1.2. The debate on the determinateness of the utility function, 1934–1936

For our narrative, it is important to call attention to a less well-known part of the conclusive phase of the ordinal revolution, namely the debate on the determinateness of the utility function. In one digression in his *Manual*, Pareto ([1906/1909] 2014, 132–133) cursorily suggested that individuals are not only able to rank combinations of goods (an assumption that in the 1930s was labeled "Pareto's postulate 1") but might even be capable of ranking transitions from one combination to another (as an assumption this suggestion became "Pareto's postulate 2"). Moreover, Pareto argued, if an individual prefers passing from combination x to combination y over passing from combination y to combination z, then for him the utility difference between x and y is larger than the utility difference between y and z, that is, $U(y)-U(x)>U(z)-U(y)$.

From the mid-1910s to the early 1930s, a number of eminent economists from different quarters picked up on Pareto's discussion about the ranking of transitions and endorsed his postulate 2 (Osório 1913; Edgeworth 1915;

Amoroso 1921; Bowley 1924; Rosenstein-Rodan [1927] 1960; Morgenstern 1931). In an article published in 1934, however, the Polish-American economist Oskar Lange of the University of Chicago argued that, when added to postulate 1, postulate 2 implies a return to measurable utility or, as he called it, "determinate" utility. More precisely, Lange (1934) claimed that postulate 2 restricts the admissible transformations of the utility function representing the individual's preferences to a subset of the increasing transformations, namely the positive linear transformations. This means that if the utility function $U(x)$ represents the individual's preferences, only utility functions $U^*(x)$ obtained by multiplying $U(x)$ by a positive number α and then adding any number β, that is, transformations $U^*(x)=\alpha U(x)+\beta$, with $\alpha>0$, also represent the individual's preferences. Today we call a utility function with this feature a cardinal utility function, but Lange did not employ that expression.

Lange's article initiated a significant debate between 1934 and 1936. In particular, Henry Phelps Brown (1934) of Oxford University pointed out that ranking transitions from one combination to another is different from ranking utility differences. That is, even if an individual prefers passing from combination x to combination y over passing from combination y to combination z, this does not imply that for him $U(y)-U(x)>U(z)-U(y)$. Therefore, Phelps Brown continued, Lange's claim that postulates 1 and 2 are sufficient to restrict the admissible transformations of the utility function to the positive linear ones is unwarranted. Phelps Brown did not, however, investigate what assumptions, if any, should be added to postulates 1 and 2 to obtain measurable or "determinate" utility. This issue was addressed by Viennese mathematician Franz Alt. In an article in German, published in the Austrian journal *Zeitschrift für Nationalökonomie* and escaping the attention of many Anglo-American economists, Alt ([1936] 1971) showed that in order to obtain a utility representation of preferences that is unique up to positive linear transformations, Pareto's postulates 1 and 2 must be integrated by five additional assumptions on the individual's preferences.

1.3. Supporters of measurable utility

One final element of the situation of utility analysis and demand theory in the 1930s should be recalled here, namely that not everybody was in favor of abandoning measurable utility and embracing an ordinal utility approach. At the University of Cambridge, for instance, economists such as Arthur Cecil Pigou and

Dennis Robertson ignored the ordinal approach and continued teaching Alfred Marshall's ([1920] 1961) theory of utility and demand, which largely relies on measurable utility.

Harro Bernardelli, a Viennese of Italian extraction who studied economics in Germany before joining the Robbins circle at LSE in 1933, criticized Pareto as well as Hicks and Allen because their theories entailed "the relinquishing of many propositions which until now have been considered as undoubtedly belonging to the body of Economic Theory" (1934, 71), such as the principle of diminishing marginal utility.[3] For Bernardelli, the theories of Pareto and Hicks–Allen were "axiomatic experiments" that showed how much our economic knowledge depends only on the assumption that individuals are able to rank combinations of goods. As axiomatic experiments, these theories resembled "the behaviour of a man who cuts off one of his legs, in order to see how he gets on as a cripple" (71). For Bernardelli, such amputation was unnecessary.

The Norwegian Ragnar Frisch also continued to believe that utility is measurable and, in a pioneering econometric study, he attempted to actually measure the marginal utility of monetary income (Frisch 1932). In the early 1930s, Frisch's work was widely discussed, but most commentators stressed that his utility measurements relied on doubtful assumptions, most notably the assumption that the marginal utility of each commodity does not depend on the quantities of other commodities, that is, that the utility function is additively separable (see e.g. Bowley 1932; Schultz 1933; Allen 1933).

With this picture of the state of utility theory in the 1930s before us, we can now turn to Samuelson and his formative years at Chicago and Harvard.

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[3] As noted by Hicks and Allen (1934, 55–57), an ordinal approach to utility implies the dismissal of notions that are not invariant to increasing transformations of the utility function, such as the principle of diminishing marginal utility. To see this, let $U(x_1, \ldots, x_n)$ be the utility function, and denote U_i the first-order partial derivative of U with respect to x_i, and U_{ii} the second-order partial derivative of U with respect to x_i. The principle of diminishing marginal utility implies that $U_{ii}<0$. Consider now an increasing transformation F(U) of U with $F'>0$. The second-order partial derivative of F(U) is $F'\times U_{ii} + F''\times(U_i)^2$. Now, even if $U_{ii}<0$, if $F''\times(U_i)^2$ is large enough, $F'\times U_{ii} + F''\times(U_i)^2$ can be positive.

2. Samuelson's training in utility and demand theory

Samuelson (1915–2009) entered the University of University of Chicago in January 1932, and then in fall 1935 moved on to graduate school at Harvard. Roger Backhouse's (2017) intellectual biography of Samuelson makes clear that when he began writing research papers on utility theory around mid-1936, he was familiar with the state of utility theory sketched in the previous section.

In the winter quarter of 1935, his senior year at Chicago, Samuelson took the graduate course in Price and Distribution Theory taught by Jacob Viner. As Samuelson (1972, 7–8) later wrote, Viner covered the subject by using indifference-curve analysis, which was then exceptional at Chicago: "Although I had the best undergraduate education in economics that opportunity could provide at that date, only once, and then in Viner's graduate course, was I exposed to the mysteries of indifference curves and the production possibility frontier."

A more systematic introduction to the recent theory of utility demand came from Wassily Leontief's seminar in Price Theory, which Samuelson attended in fall 1935 during his first term at Harvard. Leontief, who had recently published an article in which he used indifference curves to analyze international trade (Leontief 1933), presented demand theory according to an indifference-curve approach that was analogous to Hicks and Allen's. Moreover, like the two English economists, Leontief avoided introducing utility indices into the picture. As Samuelson recalled in an unpublished note on the origin of revealed preference theory, "Leontief went deliberately slowly, eschewing use of 'utility' functions, and steadfastly adhering to 'marginal rate of substitution' concepts" (Samuelson 1996). Leontief's seminar had a great impact on Samuelson, who declared that "no other course I ever took so profoundly set me on the way of my life career" (Samuelson 2004, 6).

Samuelson's assimilation of utility and demand theory continued in the spring of 1936, when he took the course Topics in Statistical Theory taught by polymath Edwin Bidwell Wilson. Wilson also covered some utility theory, using as a textbook *The Mathematical Groundwork of Economics* (1924) by the English economist and statistician Arthur Bowley. This was a slender treatise in mathematical economics that attempted to harmonize traditional utility analysis *à la* Marshall with Pareto's novel ordinal approach. In an article published in

1935, Wilson had criticized Bowley for making an erroneous point about the implications of a change of the marginal utility of money (Wilson 1935). He repeated this criticism in the course attended by Samuelson (see Backhouse 2017, 151–152; Carvajalino 2018).

For the development of Samuelson's ideas on demand and utility theory, the course on International Trade he took with Gottfried Haberler in the fall of 1936 was also important. Unlike Leontief, Haberler did not use indifference curves to analyze international trade because, among other reasons, he did not see any compelling reason to assume that the curves have the curvature required to identify a unique equilibrium point in international trade (see Backhouse 2017, 180–181). As we will see, Haberler's skepticism about the curvature of indifference curves was at the origin of Samuelson's 1938 "Note" on the theory of consumer's behavior.

To summarize, through Viner's course at Chicago and the courses by Leontief, Wilson, and Haberler at Harvard, between 1935 and 1936 Samuelson became familiar with up-to-date utility analysis and demand theory, and was thereby prepared to contribute to the subject.

3. Discounted utility, February 1937

Probably, Samuelson began writing what came to be his first published article during the summer of 1936 – completing it the following fall (Backhouse 2017, 168). The article appeared in the February 1937 issue of the *Review of Economic Studies* under the title "A Note on Measurement of Utility" (Samuelson 1937). The *Review* was the junior LSE economics journal, founded in 1933 by Ursula Webb, who belonged to the Robbins circle and in 1935 married Hicks, Abba Lerner, another member of the Robbins group, and Paul Sweezy, a Harvard graduate student who visited the LSE in the academic year 1932–1933.

In his 1937 article, Samuelson adopted a traditional, almost pre-Paretian approach. The work is not based on preference rankings represented by ordinal utility functions, nor on indifference curves and marginal rates of substitution. The declared goal of the paper is to delineate a method to measure the marginal utility of money that provides an alternative to, and is arguably more effective than, the method proposed by Frisch (1932; see section 1.3 above). In order to measure the marginal utility of money, Samuelson (1937, 156) put forward a

model of choice over time according to which the individual behaves so as to maximize the discounted sum of all future utilities. Basically, this means that, if (x_0, x_1, x_2, . . . , x_T) is a stream of monetary payments from the present time t=0 until a future time t=T, the individual behaves so as to maximize the discounted sum of future utilities $\sum_{t=0}^{T} U(x_t)e^{-\pi t}$, whereby π is a parameter capturing how the individual discounts the future. Samuelson claimed that, if this model correctly describes the intertemporal choice behavior of an individual, and if we have sufficient observations about the individual's actual choices, we can inductively measure his marginal utility of money.[4]

The details of the method Samuelson suggested to measure utility are not relevant for our concerns. For us it is more important to note that Samuelson was well aware that his discounted utility model relied on a number of questionable assumptions, such as: "at every instant of time the individual's satisfaction depends only upon the consumption at that time" (159). Samuelson also remarked that the maximization of the discounted sum of future utilities implies that the individual is able to rank utility differences, that is, Pareto's assumption 2: "Reflection as to the meaning of our Assumption Two [that the individual maximizes the sum of future utilities] will reveal that . . . we must invoke Pareto's postulate Two, which relates to the possibility of ordering *differences* in utility by the individual" (160–161). Following Lange (1934), for Samuelson postulate 2 implies that the utility function U(x) featured in the discounted utility model is not ordinal but "determinate," that is, unique only up to positive linear transformations. However, in the 1937 article, Samuelson did not profess himself scandalized by this implication. In the three articles on utility theory and demand analysis that Samuelson published in 1938, he took a different approach.

4. The "Note," February-August 1938

In September 1937 Samuelson was appointed a Junior Fellow in Harvard's prestigious Society of Fellows (Samuelson 1998; Backhouse 2017, 193–198). The Society had been created by Lawrence Henderson and other Harvard professors to give a selected group of promising Harvard students the possibility

[4] More on the fortunes and limitations of the discounted-utility model in Frederick, Loewenstein, and O'Donoghue 2002.

of pursuing their own research without the formal obligations associated with a standard Ph.D. program. The fellowship lasted three years and fellows were forbidden to work toward a Ph.D. degree. Samuelson took full advantage of the opportunity represented by the fellowship, and during his three years in the Society he published thirteen articles in major economic journals on topics ranging from consumer theory to international trade. The first article in this impressive series was the celebrated "Note on the Pure Theory of Consumer's Behaviour," which appeared in the February 1938 issue of *Economica*, the senior LSE economics journal, when he still was twenty-two years old (1938a). In the August 1938 issue of the journal, Samuelson published a brief Addendum that simplified his theory, and provided an alternative definition of its key postulate (1938b).

On various occasions, Samuelson (1950; 1996; 1998; 2004) recounted that the "Note" grew out of a remark made to him by Haberler in his 1936 course on International Trade. In international trade theory indifference curves were used to represent the preferences of a country but, as mentioned in section 2, Haberler did not use indifference curves because he was skeptical about assumptions concerning their curvature. So he asked his student, Samuelson, who was then enthusiastic about Leontief indifference-curve analysis: "How do you know indifference curves are concave?" (Samuelson 1950, 369).

Samuelson gave a quick response, but then came to see that Haberler's question had in fact wide-ranging implications, not only for international trade theory but also for consumer theory: how do we know that the indifference curves of individuals are convex? The convexity assumption made by Hicks and Allen now became suspect in Samuelson's eyes, relying on dubious introspective considerations. In the "Note" he wrote: "Just as we do not claim to know by introspection the behaviour of utility, many will argue we cannot know the behaviour of . . . indifference directions. Why should one believe in the [*diminishing*] *rate of marginal substitution*?" (Samuelson 1938a, 61).

Haberler's remark was made at some point in the fall of 1936, but the "Note" was written later, probably in the second half of 1937. Originally, it bore a title more explicit that the final one: "New Foundations for the Pure Theory of

Consumer's Behavior."[5] According to Samuelson (1996), the final version was completed in early 1938: "I wrote all this up for publication early in 1938. Marion Crawford, soon to become Marion Samuelson, wrote up my dictation. It was all done in great haste at the insistence of the Editors of *Economica*, who wished to include it in an early issue." Samuelson never saw the proofs of the article and had to accept that the editors had deleted some sections of the manuscript.[6]

As an alternative to Hicks and Allen's convex-indifference-curve approach, in the "Note" Samuelson proposed working out consumer theory on the basis of three postulates that directly concerned the individual's demand behavior. In the Addendum of August 1938, these postulates were reduced to one, which was later called the Weak Axiom of Revealed Preference. In particular, Samuelson proved that almost all the restrictions on the demand functions that derive from the constrained maximization of an ordinal utility function can also be obtained starting from the Weak Axiom.[7] The only restriction on demand functions that derives from utility maximization but not from the Weak Axiom is the so-called "integrability condition" for demand function, that is, the symmetry of the compensated variation of the demand for a good when the price of another good varies.[8]

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[5] An undated draft of the "Note" bearing this title is contained in box 152 of the Samuelson Papers. The published article is shorter than the draft, but there are no significant conceptual differences between the two.

[6] Given what Samuelson (1996) remarks about the alphabetical symbols used in the paper ("x's which Marion had written down . . . appeared in print as Greek ψ's"), it appears likely that the typescript sent to *Economica* in early 1938 is the draft of the paper contained in box 152 of the Samuelson Papers (see footnote 5). One of the parts that were significantly shortened in passing from the draft to the published article is the reconstruction of the history of utility theory and demand analysis that opens the paper.

[7] These restrictions are expressed as features of certain mathematical matrices, but their economic meaning can be summarized as follows: (i) the substitution effect is negative, that is, when the price of a good increases, the compensated demand for that good decreases; (ii) the income effect, that is, the effect of a price increase on demand due to the decrease of purchasing power, can be either positive or negative; (iii) when the price of a good increases, the uncompensated demand for the good can either decrease or increase, where the latter case is that of so-called Giffen goods; (iv) if the prices of all commodities and the consumer's income change in the same proportion, the quantities of goods demanded by the consumer do not change.

[8] More on the importance of the integrability condition in the history of demand analysis in Mongin 2000; Hands 2006; Hands 2014.

This is not the place for discussing the descriptive validity or the normative plausibility of the Weak Axiom, nor, more generally, to appraise the pros and cons of the revealed preference approach to choice theory.[9] Here I would like to make only three relevant points.

First, even in the "Note", that is, even in the most behavioristic paper – in which he proposed to drop "the last vestiges of the utility analysis" (1938a, 62) – Samuelson did not completely preclude the introduction of the utility notion; nor did he intend to contradict the results obtained by using utility-related constructs. Rather, he took a pluralist stance, and argued that his novel approach allowed a more direct analysis of consumer behavior than did the utility-based analysis. This pluralist stance is evident in the two sentences of the "Note" that follow the sentence about the removal of the last vestiges of the utility analysis. Here is the complete paragraph:

> I propose, therefore, that we start anew in direct attack upon the problem, dropping off the last vestiges of the utility analysis. This does not preclude the introduction of utility by any who may care to do so, nor will it contradict the results attained by use of related constructs. It is merely that the analysis can be carried on more directly, and from a different set of postulates. (62)

As remarked in the introduction to this chapter, scholars who claim that the young Samuelson was a committed behaviorist typically quote the first sentence of this paragraph. However, they typically omit the rest of the paragraph, and thus provide an incomplete picture of the stance on utility analysis of the young Samuelson.

Second, in the Addendum of August 1938, the Weak Axiom is presented in terms of preference, rather than choice. In the "Note," Samuelson explained the postulate in terms of selection, i.e., choice, of one batch of goods over another: "In words this [the postulate] means that if an individual select batch one over batch two, he does not at the same time select two over one" (1938a, 65). However, in the Addendum Samuelson switched to a preference terminology,

[9] On these issues, see e.g. Sen 1973; Wong [1978] 2006; Pollack 1990; Sen 1997; Spiegler 2008; Hausman 2012; Hands 2013.

and explained the Weak Axiom as imposing a consistency condition over preferences: "The individual always behaves consistently in the sense that he should never 'prefer' a first batch of goods to a second at the same time that he 'prefers' the second to the first" (1938b, 353). This explanation of the Weak Axiom in terms of the consistency condition over preferences supports the idea that, even in the "Note," Samuelson did not want to eliminate utility and preference from consumer choice theory. Rather, he conceived of his revealed preference theory as an approach compatible with, and to some extent complementary to, utility-based analysis.

Third, the "Note" is a purely theoretical contribution that does not deal with actually observed human behavior. The "Note" contains neither statistical data about demand, like those used by Frisch in his econometric study of the marginal utility of money (1932), nor experimental data, like those obtained by the American psychologist Louis Leon Thurstone (1931) in a pioneering laboratory experiment to elicit the indifference curves of an individual.[10] Moreover, in the "Note" Samuelson did not attempt to test his choice postulate, i.e., the Weak Axiom, against actual data on choice behavior but took its validity as self-evident. Therefore, I contend, if any behaviorism is present in the "Note" it is little more than a rhetorical behaviorism.

The two papers on consumer theory that Samuelson published in October 1938 support the view that, even in 1937–1938, he was not opposed to utility analysis and, more specifically, ordinal utility analysis, but considered it as a scientifically legitimate approach to the study of consumption choices. Indeed, both papers feature the word "utility" in their titles.

5. Empirical implications of ordinal utility, October 1938

One of the two papers was titled "The Empirical Implications of Utility Analysis." Samuelson probably wrote it in late 1937, and presented it on December 27 of that year at a meeting of the Econometric Society held in Atlantic City (Leavens 1938). Around the same time, Samuelson also sent the paper to Wilson, his teacher at Harvard, who sent back comments. In early 1938, that is, almost at

[10] More on Thurstone's experiment and its reception among utility theorists in the 1930s in Moscati 2007.

the same time as he sent the final version of the "Note" to *Economica* (see section 4), Samuelson submitted the paper to *Econometrica*, the Econometric Society's journal. The paper was in fact reviewed by Wilson, who recommended publication (see Backhouse 2017, 201–202). The paper was published in the October 1938 issue of *Econometrica* (Samuelson 1938c).

Although the paper on the empirical implications of utility analysis is almost contemporary with the "Note," its stated goals are quite different. Here Samuelson did not argue that the last vestiges of the utility analysis should be dropped. Rather, he claimed that ordinal utility theory is scientifically sound because it does have refutable implications in terms of demand behavior: "It is the purpose here to demonstrate that the utility analysis in its ordinary form does contain empirically meaningful implications by which it could be refuted" (345). Notably, by "utility analysis in its ordinary form," Samuelson meant ordinal utility theory plus the assumption that, in the "Note," he criticized for relying on dubious introspective considerations, namely the assumption that indifference curves are convex:

> Only the most general assumptions are made: that there exists an ordinal preference field satisfying everywhere curvature conditions [i.e. convexity conditions] sufficient to insure a proper relative maximum under the constraint of a fixed total budget. (345)

In the *Econometrica* paper, Samuelson derived the empirical implications on demand behavior of ordinal utility theory, such as the negativity of the substitution effect, and then stressed that the same implications can be derived more easily and directly from the postulates on choices he put forward in the "Note": "Recently I proposed a new postulational base upon which to construct a theory of consumer's behavior. It was there shown that from this starting point could be erected a theory which included all the elements of the previous analysis [ordinal utility analysis]" (346).

Samuelson saw no inconsistency between his *Econometrica* article and his "Note." In the "Note," he criticized the convex-indifference-curve assumption and explored the implications on demand behavior of a set of postulates that do not concern utility, preference, or the shape of indifference curves. In the *Econometrica* article, he adopted the ordinal utility framework, assumed that

indifference curves are convex, and explored the implications on demand behavior of this approach. The conclusions of the two articles are similar: the ordinal-utility or indifference-curve approach and the approach based on the choice postulates have almost the same empirical implications on demand behavior.

My point here is not to criticize Samuelson's "explorative" method as bad scientific practice. From the viewpoint of the philosophy of science, in fact, investigating the empirical implications of different sets of assumptions can readily be defended as a convenient scientific method. Even from the viewpoint of the psychology of science, the circumstance that a twenty-two-year-old scientist explores different research path appears hardy censurable. The only point I want to make is that the fact that Samuelson adopted both the ordinal utility approach and the choice-based approach, in two separate papers that were submitted almost at the same time (both in early 1938), is incompatible with the picture of the young Samuelson as a committed and consistent behaviorist who wanted to free economic analysis from the utility concept.

This image is also incompatible with the content of the other article Samuelson published in October 1938. This article was titled "The Numerical Representation of Ordered Classifications and the Concept of Utility," and appeared in the October 1938 issue of the *Review of Economic Studies*.

6. Conditions for cardinal utility, October 1938

His *Review of Economic Studies* article was Samuelson's contribution to the debate on the determinateness of the utility function initiated by Lange in 1934. As discussed in section 1.2, Lange (1934) had claimed that Pareto's postulate 1, according to which individuals can rank combinations of goods, and postulate 2, according to which individuals can even rank transitions from one combination to another, restrict the admissible transformations of the utility function to the positive linear ones. Phelps Brown (1934) had pointed out that Lange's claim was based on an incorrect implicit assumption, and that postulates 1 and 2 are not sufficient to obtain "determinate" utility. Then, Alt ([1936] 1971) had showed that, in order to restrict the admissible transformations of the utility function to the positive linear transformations, postulates 1 and 2 must be integrated with five additional postulates. Without mentioning Alt's paper, Samuelson (1938d)

took up the problem where Phelps Brown had left it, first providing alternative conditions that make utility unique up to positive linear transformations, and then criticizing them from an ordinal-utility viewpoint.

6.1. Equation 15

Following Phelps Brown, Samuelson noted that postulates 1 and 2 concern only preference order. Postulate 1 refers to preference order over combinations of goods and allows for the introduction of a numerical index U that assigns larger numbers to more preferred combinations. Thus, if an individual prefers combination z to combination y, and combination y to combination x, then postulate 1 implies that $U(z)>U(y)>U(x)$. Postulate 2 refers to the preference order over transitions from one combination to another and allows for the introduction of another index that Samuelson called G. The index G assigns larger numbers to more preferred transitions: if the individual prefers transition from x to y to transition from y to z, then postulate 2 implies that $G(x, y)>G(y, z)$. Phelps Brown had stressed two main issues with the indices U and G. First, the numbers associated by G with transitions need not be equal to the differences between the numbers associated by U to combinations. That is, postulate 2 does not imply that $G(x, y)=U(y)-U(x)$ or that $G(y, z)=U(z)-U(y)$.[11] More generally, since postulate 2 refers only to the ranking of transitions and G numbers, it has no implications for the differences between the U numbers. Second, since the G numbers have only an ordinal meaning, it does not make sense to sum them. Thus, for instance, if the individual considers transition from x to y equally preferable to transition from y to z, then the G number associated with both transitions is the same, say, 7. But postulate 2 does not warrant that the G number associated with the transition from x to z is $7+7=14$.

To solve these two issues, Samuelson assumed that the preference order over combinations and the preference order over transitions are both transitive. He connected them by arguing that if an individual prefers the transition from x

[11] Consider the following numerical example. If the individual prefers z to y, and y to x, we can assign the following U numbers to the three combinations: $U(z)=10$, $U(y)=3$, and $U(x)=1$. If the individual prefers transition from x to y to transition from y to z, we can assign to the two transitions the G numbers $G(x, y)=5$ and $G(y, z)=2$. Although these U numbers and G numbers are perfectly consistent with postulates 1 and 2, it turns out that $U(y)-U(x)=2$ while $G(x, y)=5$, and $U(z)-U(y)=7$ while $G(y, z)=2$.

to y to the transitions from x to z, that is, if G(x, y)>G(x, z), then combination y must be preferred to combination z, that is, U(y)>U(z). Subsequently, Samuelson introduced the key postulate of his article as equation 15 (1938d, 68). This equation overcomes the problem that G numbers cannot be summed by simply assuming that G numbers can indeed be summed. That is, if G(x, y) is the number associated with the transition from x to y, and G(y, z) is the number associated with the transition from y to z, Samuelson's equation 15 requires that the number G(x, z) associated with the transition from x to z is equal to the sum of G(x, y) and G(y, z), that is, G(x, y)+G(y, z)=G(x, z).

One may discuss whether the postulated equation 15 simply begs the issue posed by Phelps Brown. For our concerns here, however, this point is not relevant. More important is that Samuelson showed that his postulate, together with the other assumptions mentioned above, is necessary and sufficient to make the G numbers associated with transitions equal to the difference between the U numbers associated with combinations, that is, to have G(x, y)=U(y)–U(x). In turn, and as Lange had already showed, G(x, y)=U(y)–U(x) if and only if the utility function U is unique only up to linear increasing transformations (69–70).

In the final part of his paper, Samuelson discussed the plausibility of the condition G(x, y)+G(y, z)=G(x, z) from the viewpoint of the ordinal approach to utility and preference: "What is the meaning of this condition in terms of the individual's ordinal classification of movements? Can such a relationship in general be satisfied?" (70). His answer was negative. He saw "no *a priori* reason why the individual's preference scale . . . should obey this arbitrary restriction," and he regarded the chance that some individual actually satisfy it as "infinitely improbable" (70).

6.2. Samuelson, Lange, and Alt

As mentioned above, in his article Samuelson did not mention the 1936 article by Alt, in which the Viennese mathematician had provided a different solution to the issue posed by Phelps Brown. One question that naturally arises is whether Samuelson knew of Alt's 1936 article. We can say that he was at least aware of its existence.

Presumably in early 1938, Samuelson sent a draft of his paper to Lange, who replied in a letter dated 10 May 1938 (Samuelson Papers, Box 48). Lange declared Samuelson's manuscript "a contribution which really helps to clarify the

subject" and judged Samuelson's equation 15, that is, his postulate G(x, y)+G(y, z)=G(x, z), a satisfactory condition for restricting the admissible transformations of the utility function to the positive linear ones. Moreover, Lange was more positive than Samuelson about the validity of the condition: "I do *not* share your object[ion] to equation (15). . . . Acceptance of (15) neither runs us into contradictions or violates observations."

In his letter, Lange also explicitly invited Samuelson to look at Alt's article and pointed out the possible relationship between Samuelson's postulate 15 and Alt's postulates: "I would suggest that you look up the article of Alt, Über die Messbarkeit des Nutzens, Zeitschr. F. Nat.- Oeconomie, Bd. VII (1936). If I am not mistaken your equation (15) corresponds to his postulates IV and V." We know from a letter of one of the editors of the *Review of Economic Studies* to Samuelson that he did not see the proofs of his article.[12] Therefore, even if he did look at Alt's article between May and October 1938, he could not have added a reference to Alt. Be that as it may, in his subsequent writings of the 1930s and 1940s, Samuelson did not refer to Alt's 1936 article.

6.3. Naming Cardinal Utility

A final, terminological feature of Samuelson's (1938d) article should be mentioned. In current economic theory, a utility function representing the preferences of an individual is called "ordinal" if it is unique up to any increasing transformations, and "cardinal" if it is unique only to positive linear transformations. While the notion of ordinal utility had already stabilized in the writings of Pareto, that of cardinal utility stabilized only much later.

In effect, until Hicks and Allen (1934, 54–55) referred to "a 'cardinal' conception of utility" in a passage of their celebrated paper, the very expression "cardinal utility" was not used in economics; and even Hicks and Allen did not make clear what they meant by cardinal utility. Apparently, they used cardinal as a residual notion, in the sense that they considered cardinal everything not ordinal, that is, not invariant to increasing transformations of the utility function. Certainly, Hicks and Allen did not associate cardinal utility with positive linear

[12] Letter of Ursula Webb Hicks to Samuelson, 4 October 1938, Samuelson papers, box 37.

transformations of the utility function, not least because in their article there is no sign of them.

The first article in which utility unique up to positive linear transformations was explicitly and consistently coupled with the terms "cardinal" and "cardinal measurability" was Samuelson's *Review of Economic Studies* article of October 1938. The association occurs ten times in this paper, of which here is one example: "Dr. Lange has not proved satisfactorily that from these two assumptions [Pareto's assumptions 1 and 2] can be derived the cardinal measurability of utility (subject to a linear transformation involving scale and origin constants)" (1938d, 66). I argue, therefore, that cardinal utility acquired its current technical meaning in Samuelson's 1938d article.

6.4. Summing up

The papers on intertemporal choice and consumer behavior that the young Samuelson wrote between mid-1936 and early 1938, which were published between February 1937 and October 1938, reveal him as very far from a committed behaviorist who aimed at dropping the last vestiges of utility analysis. In contrast, the young Samuelson dealt intensively with utility theory and made important contributions to it: he put forward a model for intertemporal choices based on the maximization of cardinal utility (1937), explored the empirical implications of ordinal utility theory on demand behavior (1938c), and stated conditions that make utility cardinal rather than just ordinal, yet judged these conditions implausible (1938d). Even in the "Note" (1938a, 1938b), Samuelson took a pluralist stance, and presented his revealed preference approach as compatible with, and to some extent complementary to, ordinal utility analysis.

Arguably, in the years 1936–1938 the young Samuelson was exploring different research paths, ranging from cardinal utility analysis to the revealed preference approach, passing through ordinal utility theory. His explorations, however, display a definite trend in the direction of a refusal of cardinal utility assumptions, which are accepted in the February 1937 article but rejected as "infinitely improbable" in the October 1938d article, and the endorsement of the ordinal utility approach, which is adopted in the two articles of October 1938 (1938c, 1938d). As for the revealed-preference approach, Samuelson advocated it in the "Note" (1938a, 1938b), but played it down in the two articles of October 1938 (1938c, 1938d). As we will see in the following sections, in the papers and

the Ph.D. dissertation that he wrote in 1939–1940, Samuelson also played down the revealed-preference approach and presented the theory of consumer demand according to ordinal utility theory. Before discussing these works, however, a brief overview of the main developments in utility analysis after the publication of Hicks and Allen's 1934 article is in order.

7. Further developments in utility and demand theory, 1935–1939

7.1. Slutsky and Allen

Russian economist and statistician Eugen Slutsky was an admirer of Pareto. In 1915, he published in the *Giornale degli Economisti*, the Italian journal in which Pareto had published most of his contributions, an article which anticipated many of the results later obtained by Hicks and Allen. Unlike the two LSE economists, however, Slutsky ([1915] 1952) expressed his theory in terms of a utility function and its derivatives. Moreover, Slutsky did not make clear whether his results were ordinal in nature, that is, whether they were invariant to increasing transformations of the utility function. At any rate, for reasons about which we can only speculate, for almost twenty years Slutsky's paper was completely neglected. It was rediscovered only in the early 1930s, by Valentino Dominedò (1933) in Italy, Henry Schultz (1935) in the United States, and Allen in England.[13]

In an article published in the *Review of Economic Studies*, Allen (1936) called attention to Slutsky's paper, acknowledged his priority with respect to a number of results, and stressed the differences between Slutsky's utility-based approach and the utility-free approach he and Hicks had put forward in their 1934 article: "Slutsky's starting point is different from that of Hicks and myself. Our theory was constructed so as to be independent of the existence of an index of utility. . . . Slutsky expresses his theory in terms of one selected utility function and its partial derivatives" (127). Allen showed, however, that Slutsky's results are in fact independent of measurability assumptions on the utility function and hold also in a purely ordinal framework.

[13] On the rediscovery of Slutsky's 1915 article, see Chipman and Lenfant 2002.

7.2. Hicks the Ordinalist

Allen's article paved the way for the ordinal restatement of Slutsky's findings and the subsequent establishment of ordinal utility theory as the mainstream approach to demand analysis. Although for a while Allen (1938) insisted on the utility-free approach, after 1936 Hicks (1937; 1939) set forth his analysis in terms of ordinal utility indices.[14] Most notably, in *Value and Capital* (1939), Hicks fine-tuned the ordinal approach to utility theory. He now re-presented Slutsky's results in a systematic and mathematically clear way and demonstrated, more thoroughly than had Allen, that these results are ordinal in nature. Hicks also showed that the results he and Allen had obtained in 1934 using the marginal rate of substitution could be obtained through ordinal utility indices in a theoretically rigorous and much simpler way.

Hicks's intellectual trajectory – from the choice-based approach explored in the article with Allen to the full-fledged ordinalism expounded in *Value and Capital* – was thus similar to the intellectual trajectory of Samuelson as reconstructed in the present paper.

8. Samuelson's ordinalism stabilizes, 1938–1939

Between the fall of 1938 and the fall of 1939, Samuelson wrote two papers related to consumer and demand theory. Both works confirm the stabilization of his ordinal approach to utility analysis.

8.1. The dispute with Bernardelli on diminishing marginal utility

The first paper was titled "The End of Marginal Utility: A Note on Dr. Bernardelli's Article," and was published in the February 1939 issue of *Economica*. As its title indicates, the article was a comment on a paper published by Harro Bernardelli, one of the opponents to the ordinal-utility approach mentioned in section 1.3.[15]

In his paper, Bernardelli (1938) had defended the scientific legitimacy of the principle of diminishing marginal utility and argued that it can obtained by a

[14] For more on the differences between Hicks's and Allen's approaches, see Fernandez-Grela 2006.

[15] In the mid-1930s Bernardelli left the LSE for the University of Liverpool, and in 1937 he moved eastward, to universities in, first, Burma (Rangoon) and then New Zealand (Otago). More on Bernardelli in Donoghue 2007.

novel and plausible set of postulates that are invariant to increasing transformations of the utility function. This would render the principle of diminishing marginal consistent with the ordinal approach to utility analysis. In his comment, Samuelson (1939, 87) claimed that in fact Bernardelli's postulates "are *not* invariant under a monotonic renumbering of the indifference loci" and, therefore, the principle of diminishing marginal utility remains incompatible with the ordinal-utility approach.

Bernardelli (1939) replied that Samuelson had misconstrued his postulates. Without entering here into the details of the Samuelson–Bernardelli exchange, we can say that, although in effect Samuelson missed Bernardelli's main point, Bernardelli's mathematical demonstrations were nonetheless flawed.[16] More important for us is that Samuelson criticized Bernardelli's postulates from a purely ordinalist viewpoint, and without any reference to the revealed preference approach put forward in the "Note" of February 1938.

8.2. Against the constancy of the marginal utility of income

In the fall of 1939, Samuelson completed a further paper in utility analysis. It discussed the notion of the marginal utility of monetary income, that is, the marginal utility a consumer can obtain by spending an additional unit of his income, and the assumption, often made by Marshall and other demand theorists, that the marginal utility of income is constant. Samuelson sent the paper to Oskar Lange, who was editing a volume of essays in memory of Chicago economist and statistician Henry Schultz, who had died in a car accident in 1938 (see Backhouse 2017, 209–210). The volume, and therefore Samuelson's paper, were published only three years later (Samuelson 1942).

Samuelson began his essay by observing that the very notion of marginal utility of income is not invariant to increasing transformations of the utility function and therefore is cardinal, rather than ordinal, in nature (76–77). Subsequently, he showed that the assumption that the marginal utility of income is constant implies that the income elasticity of demand for each commodity is unitary, that is, that a given percentage increase in the consumer's income is reflected in an equal percentage increase in the consumer's demand for each

[16] On the limits of Bernardelli's approach, see Lancaster 1953.

commodity. However, Samuelson stressed, consumers typically do not react to income increases in this way: "As far as I know, every investigation contradicts flatly this basic assumption" (81). Finally, Samuelson investigated what happens when the assumption of constant marginal utility of income is combined with another typically Marshallian assumption, namely that the marginal utility of each commodity is independent of the quantities of other commodities, that is, that the utility function is additively separable. Samuelson proved that in this case the consumer would spend a fixed fraction k_i of his income on each commodity x_i. Again, this conclusion appeared implausible to him: "It need hardly be said that no empirical observations justify the imposition of such a definite form upon the . . . demand functions" (83).

Samuelson's endorsement of the ordinal utility approach found its most systematic expression in the Ph.D. dissertation he wrote between mid-1940 and November 1940.

9. The "Foundations of Analytical Economics," 1940

In the middle of 1940 the prohibition of work toward a Ph.D. degree prescribed by the Society of Fellows expired, and Samuelson began writing his Harvard Ph.D. dissertation. The writing proceeded at "fever pace" (Samuelson 1998, 1377), with some parts dictated to Marion, who had married Samuelson in July 1938. The dissertation was entitled "Foundations of Analytical Economics" (Samuelson 1940), and Samuelson submitted it in November 1940, after he had already left Harvard for the Massachusetts Institute of Technology, the institution where he would spend the rest of his academic life.[17]

Samuelson successfully defended his Ph.D. thesis on December 4, 1940. His examiners were Harvard economists Joseph A. Schumpeter, Edward Chamberlin, and Overton Taylor. Wilson, Samuelson's mathematical mentor, was also on the examination panel (Backhouse 2017, 453–454). After a seven-year delay, partly due to the war, the thesis was published as *Foundations of Economic Analysis* (1947). The book contained some new chapters not in the dissertation, but the dissertation material was incorporated into the book with

[17] For more on Samuelson's move to MIT, see Backhouse 2014.

few modifications. This is particularly true for the parts on consumer and demand theory.[18]

In the Ph.D. dissertation and, in identical form, in *Foundations*, Samuelson further downplayed the revealed preference approach he had proposed in his 1938 "Note" and, in chapter five (of both the dissertation and the book), he presented the theory of consumer behavior following an ordinal utility approach substantially equivalent to that used by Hicks in *Value and Capital* (Samuelson 1940, 110–146; 1947, 90–124). The starting point of his analysis was the consumer's ordinal preferences, represented by a utility function that is invariant to any increasing transformation. Samuelson then expressed the equilibrium conditions for the consumer in terms of the determinants of the matrix of the second-order derivatives of the ordinal utility function. Only at the end of chapter five did Samuelson mention the postulate of the 1938 "Note," that is, the Weak Axiom, and then only as an alternative way of characterizing one of the implications of ordinal-utility analysis, namely that the substitution effect is negative (1940, 139–146; 1947, 111–117).[19]

In chapter six of the Ph.D. dissertation (which became chapter seven of *Foundations*), Samuelson discussed "Special Aspects of the Theory of Consumer's Behavior" (1940, 147–164; 1947, 172–202). At the beginning of the chapter, he argued that "the content of utility analysis in its most general form [involves] only an ordinal preference field," and he dismissed "the cardinal measure of utility" as a "special and extra" assumption by which "nothing at all is gained" (1940, 147–150; 1947, 172–173). Samuelson then discussed other special and extra assumptions of utility theory, such as the additive separability of the utility function and the constancy of the marginal utility of income and showed that these assumptions often imply cardinal utility. However, Samuelson rejected these other special assumptions, too, judging them "not generally applicable," "arbitrary," "dubious," "highly unrealistic," "superfluous," and leading to "really fantastic conclusions" (1940, 150–189; 1947, 174–202).

[18] More on the relationship between dissertation and book, and the reasons for the book's delayed publication in Samuelson 1998 and Backhouse 2015.

[19] On Samuelson's approach to consumer's choice theory in the Ph.D. dissertation and *Foundations*, see also Hands (2014), especially p. 102.

10. The alleged Das Paul Samuelson Problem, and conclusions

Samuelson's ordinalist stance did not change over the 1940s. As just discussed, *Foundations of Economic Analysis*, published in 1947, reproduced the ordinal utility approach to consumer and demand theory expounded in the Harvard thesis of 1940. *Foundations* quickly became a reference book for postwar students of economics of no less importance than Hicks's *Value and Capital*. The two books provided a systematized version of the ordinal-utility approach to consumer and demand theory that has remained canonical until today.

In November 1948, Samuelson published an article titled "Consumption Theory in Terms of Revealed Preference," which consolidated the bridge between the choice-based and the preference-based approaches to demand theory. At the theoretical level, Samuelson (1948) showed that, in the case of only two goods, the observation of a sufficient number of a consumer's choices satisfying the Weak Axiom makes it possible to elicit the consumer's indifference curves. At the terminological and conceptual level, by introducing the very expression "revealed preference," Samuelson suggested that consumer's preferences exist prior to consumer's choices, and in fact cause them.

In 1950, an article by the Dutch-American economist Hendrik Houthakker (1950), and a prompt follow-up by Samuelson (1950), transformed the bridge between the two approaches into a comfortable revolving door. Houthakker introduced a coherence assumption on consumer behavior stronger than Samuelson's Weak Axiom – the so-called Strong Axiom of Revealed Preference – and proved that if the choices of a consumer satisfy the Strong Axiom, these choices can be interpreted as the result of the constrained maximization of the consumer's well-behaved ordinal preferences. Samuelson (1950) completed Houthakker's contribution by showing that also the reverse is true, that is, that if a consumer maximizes his well-behaved ordinal preferences under the budget constraint, his choices satisfy the Strong Axiom. Houthakker's and Samuelson's articles showed that the choice-based approach and the preference-based approach to demand theory are substantially equivalent, and thus vanished the opposition between the two approaches.

Samuelson interpreted this equivalence result not as a refutation of the revealed preference approach he had advanced in the 1938 "Note," but as a full realization of it. As mentioned in section 4, the Weak Axiom does not imply all

empirical restrictions on demand functions that are implied by the constrained maximization of an ordinal utility function. In particular, the Weak Axiom does not imply the so-called "integrability condition" for demand function, that is, the symmetry of the compensated variation of the demand for a good when the price of another good varies. In his 1950 article, Samuelson claimed that in 1938 he had conjectured about the existence of a strengthened version of the Weak Axiom that had exactly the same empirical implications of the constrained maximization of ordinal utility, including the integrability conditions. However, he was unable to find such a strengthened version of the Weak Axiom, which was eventually provided by Houthakker. The relevant passage from Samuelson's 1950 article is worth quoting at some length:

> We are now in a position to complete the programme begun a dozen years ago of arriving *at the full empirical implications for demand behaviour of the most general ordinal utility analysis*. . . . I soon realised that this [the Weak Axiom] could carry us almost all the way along the path of providing new foundations for utility theory. But not quite all the way. . . . I held up publication on the conjecture that if the axiom were strengthened . . . then non-integrability could indeed be excluded. . . . But no proof was forthcoming for all these years, until Mr. Houthakker's paper arrived in the daily mail. Not only had he provided the missing proof, but in addition he had independently arrived at precisely the same strong axiom as I had hoped would save the day. (Samuelson 1950, 369–370)[20]

Based on this claim, several interpreters of Samuelson's economic thought have argued that, between 1938 and 1948–1950, he changed his mind about the utility concept and ordinal utility analysis: while in 1938 he wanted to free the theory of consumer behavior from the last vestiges of the utility concept, and saw revealed preference theory as a research program wholly alternative to ordinal utility theory, in 1948–1950 he conceived of revealed preference theory as wholly equivalent to ordinal utility theory (for this interpretation, see e.g. Houthakker 1983, and Wong [1978] 2006). Hands (2014) has labelled *Das Paul*

[20] Samuelson made similar claims also in correspondence; see in particular the letter to Houthakker of December 23, 1949, and the letter to Hicks of January 25, 1952. Relevant passages from both letters are quoted in Hands 2014, 99.

Samuelson Problem the question of whether Samuelson changed his mind between 1938 and 1948–1950. Based on the present review of the contributions to utility theory that Samuelson made when he was a Ph.D. student at Harvard, I argue that Das Paul Samuelson Problem has either a negative answer or is ill-posed.

We saw that in 1936–1938 Samuelson explored different research paths, which ranged from cardinal utility analysis to the revealed preference approach, passing through ordinal utility theory. Around mid-1938, however, his stance on utility analysis stabilized: he firmly refused cardinal utility assumptions, downplayed the revealed preference approach, and fully endorsed ordinal utility theory. The articles on utility theory he published after mid-1938, as well as his Harvard Ph.D. dissertation of November 1940, clearly express this position. Therefore, Samuelson did not change his mind between 1938 and 1948–1950. If he changed his mind, he did so much earlier on, namely between 1936 and mid-1938. In this sense, Das Paul Samuelson Problem has a negative answer.

Alternatively, Das Paul Samuelson Problem is ill-posed in the sense that it is preposterous to represent the intellectual trajectory of the young Samuelson, which brought him from the explorations of the period mid-1936–mid-1938 to the ordinalist stance maintained after mid-1938, as involving a change of mind. As noted above, Hicks's ideas also followed a similar trajectory – from the utility-free approach to demand analysis pursued in the 1934 paper co-authored with Allen to the full-fledged, ordinal-utility approach expounded in *Value and Capital*. However, and rightly, nobody has argued that there exists a *Das John Hicks Problem*.

In my opinion, Das Paul Samuelson Problem is an artifact that draws from the erroneous conviction that the young Samuelson was a committed and consistent behaviorist. This conviction, in turn, draws from two main errors of interpretation: (1) the neglect of the several, important contributions to utility analysis that Samuelson made during his years at Harvard; and (2) an incomplete reading of his "Note" of 1938, in which revealed preference theory is already presented as compatible with, and to some extent complementary to, utility-based analysis. In this chapter I have attempted to refute the image of the young Samuelson as a committed behaviorist, and thus to show that Das Paul Samuelson Problem that is drawn from that image is a false problem.

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