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Information acquisition, price informativeness, and welfare ☆

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Abstract

We consider the market for a risky asset with heterogeneous valuations. Private information that agents have about their own valuation is reflected in the equilibrium price. We study the learning externalities that arise in this setting, and in particular their implications for price informativeness and welfare. When private signals are noisy, so that agents rely more on the information conveyed by prices, discouraging information gathering may be Pareto improving. Complementarities in information acquisition can lead to multiple equilibria.

JEL classification: D82; G14

Keywords: Heterogeneous valuations; Information acquisition; Learning externalities; Welfare

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1. Introduction

We study the market for a risky asset for which agents have interdependent private valuations. Heterogeneous valuations may arise for various reasons. For example, agents may differ with respect to the uses they have for the asset, their liquidity needs, their investment opportunities, or the regulatory constraints they face. Diversity in valuations can be thought of as an indirect way to capture idiosyncratic preference or endowment shocks.[1] It can also be interpreted in purely behavioral terms – for example, agents could "agree to disagree" about the distribution of the asset payoff, or a subset of traders could be subject to psychological biases or misperceptions. Each trader is uncertain about his own valuation, and has the opportunity to acquire private information about it prior to trade. Equilibrium prices reflect some of this information.

We use a standard competitive rational expectations setup, with Gaussian shocks and constant absolute risk aversion, that nests the classical models of Grossman and Stiglitz (1980) and Hellwig (1980). Essentially the only difference with respect to the classical framework is that we allow agents' valuations to be imperfectly correlated. This gives us a tractable model of partial revelation without resorting to exogenous noise trade, with a unique linear equilibrium price function for any allocation of private information. The model highlights the role played by learning externalities in determining the information content of prices and the welfare of market participants.

To exposit our main results, it suffices to consider a symmetric version of our model. There are several types of agents distinguished by their valuations. Agents of type i have valuation θ_i and a proportion λ_i of these agents acquires private information about θ_i. The equilibrium price takes a very simple form: it is proportional to $\sum_i \lambda_i \theta_i$. Since agents differ in their valuations, they also differ in the information that they extract from prices. In particular, type i agents make inferences about θ_i, inferences that are necessarily imperfect due to the dependence of the equilibrium price on the valuations of other types.

Complementarities in information acquisition, that give agents a greater incentive to gather information when others do so, arise naturally in this setting. To understand how, suppose that there are only two types, and the proportions of informed agents, λ_1 and λ_2, are exogenously given. Let ρ be the correlation coefficient between θ_1 and θ_2, $|\rho| < 1$. The equilibrium price is proportional to $\lambda_1\theta_1 + \lambda_2\theta_2$. Price informativeness for type 1 is decreasing in λ_2, as long as $\lambda_2 \leq \lambda_1$. This is true regardless of the value of ρ, though it is easiest to see when $\rho = 0$ as in that case the valuation of type 2 appears as "pure noise" in the price function from the perspective of type 1. This is an *across-type complementarity* wherein agents of a given type learn less from the price if more agents of another type acquire information.

Now observe that, as long as $\rho \neq 0$ and $\lambda_2 > 0$, the price conveys some information to type 1 agents about their valuation θ_1 even if none of them acquires information about it ($\lambda_1 = 0$). If $\rho < 0$, price informativeness for type 1 is in fact decreasing in λ_1, for $\lambda_1 < |\rho|\lambda_2$. In this interval, as more agents of type 1 acquire information, the price becomes an increasingly mixed signal about θ_1; for example, a high price can result from a high θ_1 ("good news" for type 1) or from a high θ_2 ("bad news" for type 1). Thus, if $\rho < 0$, a *within-type complementarity* can arise, wherein price informativeness for a given type is lower when more agents of that type acquire information.

[1] These shocks may depend, for example, on group affiliations or on the geographic location of traders. See Rostek and Weretka (2012) for further discussion and interpretation.

We endogenize the information acquisition decisions of agents for an arbitrary number of types. Agents of type i can choose to pay a cost c_i to acquire a private signal about their valuation θ_i. We characterize the equilibrium allocation of private information, i.e. the equilibrium value of λ_i for each type i. Naturally, the λ_i's depend on the c_i's as well as on the correlation of valuations across types. Given the learning externalities discussed above, we do not expect information gathering to be efficient, however.

The welfare analysis is complicated by the fact that price informativeness is a multidimensional object in an economy with heterogenous valuations and, moreover, there is no unambiguous link between price informativeness for a given type and the welfare of that type. Agents can make better portfolio decisions if prices are more informative about their valuation. But more informative prices are also closer to their true valuation, reducing profitable trading opportunities. We find that increasing the cost of information acquisition for agents of the highest cost type leads to a reduction in the proportion of these agents who acquire information, lowering price informativeness for them and improving their welfare. Price informativeness for other types is higher, on the other hand, while the effect on their welfare depends on how precise their private information is. When their private signals are noisy, so that they have more to gain from learning from prices, they are better off. This is the case in which discouraging information acquisition by the highest cost type makes all types better off. Notice that it is precisely when prices have an important role to play in aggregating and transmitting private information that curtailing the collection of private information (by a subset of agents) is Pareto improving. A more general takeaway is that private information collection can impact different groups of agents differently, both in terms of the information conveyed by prices and welfare.

Across-type complementarities, wherein information gathering by one type interferes with learning from prices by other types, are an important ingredient of our welfare result. Within-type complementarities play no role here, but are crucial when we consider equilibrium multiplicity. It can turn out that there is an equilibrium in which no agent of type i (for some i) acquires information and another equilibrium in which all of these agents do. In fact, in the equilibrium in which no type i agent is informed, prices are more informative for all types, including type i. Both across-type and within-type complementarities are at play here.

1.1. Related literature

Vives (2014) studies a competitive rational expectations equilibrium model with private valuations. As in our paper, there is no equilibrium with a high correlation of types, and when an equilibrium does exist the price function is partially revealing. However, price informativeness does not depend on the mass of informed agents (as long as this mass is positive) – the price reveals the average type of all agents regardless of how many are informed. This in turn implies that the information acquisition decisions of agents are independent.

Our stochastic environment shares some features with that of Rostek and Weretka (2012, 2015), insofar as they allow heterogeneity in the correlations between the private valuations of traders. They impose an "equicommonality" assumption, namely that the average correlation between the valuation of a trader and those of the remaining traders is the same for all traders. We do not impose any restriction on the correlation structure for our results on the characterization of equilibrium and price informativeness with exogenous private information (though we do impose symmetry in our analysis of information acquisition). The aims of the Rostek–Weretka papers are different from ours – they study the effect of an exogenous increase in the number of traders on price informativeness (which, in contrast to our setting, is the same for all traders) and on market power.

Our framework generalizes the models of Grossman and Stiglitz (1980) and Hellwig (1980), as well as several later extensions of these models. We go beyond this literature in looking at information acquisition by different groups of traders, and analyzing the learning externalities that arise both within and across groups.

While the social value of a public signal in a symmetric information economy has been the subject of a voluminous literature going back to Hirshleifer (1971) (see Gottardi and Rahi, 2014 and the references cited therein), not much research has been done on the welfare properties of private information production when prices reflect some of this information. In particular, the literature gives little guidance on the circumstances in which policies that affect private information collection can improve market outcomes in a rational expectations economy. In Vives (2014), information acquisition is socially efficient provided the marginal cost of information is sufficiently low. This efficiency result is not surprising given that there are no learning externalities in this model. Allen (1984) shows that imposing a tax on information gathering in a variant of the Grossman–Stiglitz model can make all agents better off. But the welfare analysis is compromised by the presence of noise traders.[2] In fact, most of the rational expectations literature relies on exogenous noise trade and hence does not provide a suitable framework for welfare analysis. Usually a proxy for welfare is employed, such as price informativeness, price volatility or some measure of liquidity. There are a few papers that feature fully optimizing traders but, with the exception of Vives (2014) cited above, they do not address the question of the optimality of the equilibrium allocation of private information.

There is a large literature on complementarities in information gathering. The closest to the present paper are competitive models in which these complementarities arise because prices become less informative as more agents acquire information.[3] Stein (1987) provides an early example of the entry of informed speculators reducing price informativeness for existing traders, in a setting where agents seek to forecast shocks to the supply of the underlying in a futures market. In an environment closer to that of Grossman and Stiglitz (1980), but with different assumptions on preferences and distributions, Barlevy and Veronesi (2008) find that a complementarity can arise because the asset payoff and noise trader demand are negatively correlated. Their mechanism has a similar flavor to our within-type complementarity which is due to a negative correlation between the valuations of traders from different groups. Price informativeness can be decreasing in the incidence of informed trading in Ganguli and Yang (2009) and Manzano and Vives (2011) because agents have access to two sources of information (about the asset payoff and the asset supply), in Goldstein et al. (2014) because agents with different trading opportunities in segmented markets may trade on the same information in opposite directions,

2 While the liquidity traders in Allen (1984) do have a utility function, it is contrived to generate exogenous noise trade exactly as in Grossman and Stiglitz (1980).

3 Other mechanisms have also been explored in the literature. Complementarities in information acquisition arise in Goldstein and Yang (2015) because agents collect different pieces of information about the asset value (as more agents of one group acquire information, the uncertainty about the asset payoff is reduced for the other group, increasing the return from gathering information for them), in Mele and Sangiorgi (2015) because of Knightian uncertainty (as prices become more informative, uninformed agents have a greater incentive to acquire information to resolve the ambiguity and thus "decode" the information contained in prices), in García and Strobl (2011) due to relative wealth concerns (as the proportion of informed agents rises, so does the average wealth of all agents, giving the uninformed an additional incentive to gather information), and in Veldkamp (2006a,b) because of increasing returns to scale in the supply of information (information gets cheaper as more agents acquire information).

and in Breon-Drish (2012) due to non-normality of shocks. Relative to this literature, our model admits a more pronounced multiplicity of equilibria, including equilibria in which agents who collect information have a higher cost of information acquisition than those who do not, in an otherwise symmetric economy.

The paper is organized as follows. We describe the economy in Section 2. In Sections 3–5 we take the information acquisition decisions of agents as given. We characterize the unique linear equilibrium price function in Section 3. Then, in Section 4, we provide several examples in which this characterization can be employed. In Section 5 we analyze the information content of the price for each type. We endogenize information acquisition in Section 6, and discuss learning externalities within and across types. Section 7 is devoted to welfare and Section 8 to equilibrium multiplicity. Section 9 concludes. Most of the proofs are in the appendices.

2. The economy

There is a single risky asset in zero net supply, and a riskfree asset with the interest rate normalized to zero. There are N types of agents, $N \geq 2$, and a continuum of agents of each type. Formally, we index agents of any given type by the unit interval, endowed with Lebesgue measure. The private valuation for the risky asset of an agent of type i is given by $v_i = \theta_i + \eta_i$. Prior to trade, type i agents can acquire a private signal about θ_i by incurring a cost c_i; for agent n of type i (agent in for short) this signal takes the form $s_{in} = \theta_i + \epsilon_{in}$. In other words, type i agents are distinguished by their valuation v_i, and θ_i is the part of v_i about which they can gather information at some cost; their signals can have some idiosyncratic variation, however.

The random variables $\{\theta_i, \eta_i, \{\epsilon_{in}\}_{n\in[0,1]}\}_{i=1,\dots,N}$ are joint normal with mean zero. Let $\boldsymbol{\theta} := (\theta_i)_{i=1}^N$ and $\boldsymbol{\eta} := (\eta_i)_{i=1}^N$. For each type i, the valuation shock η_i is independent of $\boldsymbol{\theta}$ (but may be correlated with η_j, $j \neq i$), the signal shock ϵ_{in} is independent of $(\boldsymbol{\theta}, \boldsymbol{\eta})$, and the signal shocks across agents, $\{\epsilon_{in}\}_{n\in[0,1]}$, are i.i.d. We adopt the convention that the average of a continuum of i.i.d. random variables with mean zero is zero. Then, the average signal of agents of type i, $\int_n s_{in} dn$, is equal to θ_i.[4] To ensure that the problem is nontrivial, we assume that the covariance matrix of $\boldsymbol{\theta}$ is positive definite.[5]

If agent in buys q_{in} units of the risky asset at price p, his "wealth" is $W_{in} = (v_i - p)q_{in}$. Given his information set $\mathcal{I}_{in}$, which consists of all the random variables that he observes prior to trade, he solves $\max_{q_{in}} E[-\exp(-r_i W_{in})|\mathcal{I}_{in}]$. Agents have rational expectations – they know the price function, a function of the private signals of all agents in the economy, also denoted by p, and condition on the price when making their portfolio decisions. Thus $\mathcal{I}_{in} = \{s_{in}, p\}$ if agent in is informed, and $\mathcal{I}_{in} = \{p\}$ if he is uninformed.

We denote the proportion of agents of type i who choose to become informed by $\lambda_i \in [0, 1]$. An equilibrium consists of a vector $\boldsymbol{\lambda} := (\lambda_i)_{i=1}^N$, and a price function p, such that agents optimize and markets clear. Agent optimization requires that each agent is happy with his information acquisition decision (to acquire private information or not) given the price function p, and subsequently, for any realization of p, he chooses an optimal portfolio given his information. Letting $q_i := \int_n q_{in} dn$, the aggregate trade of type i, the market-clearing condition is $\sum_i q_i = 0$.

[4] See the technical appendix of Vives (2008) for a discussion of the use of the strong law of large numbers in this context. For ease of exposition, we drop the qualifier "almost surely".

[5] Note that agent valuations cannot in general be written as the sum of a common value component and an idiosyncratic private value component. Indeed, if $v_i = v + u_i$, where $\mathrm{Cov}(v, u_i) = 0$ and $\mathrm{Cov}(u_i, u_j) = 0$ for all i, j, then $\mathrm{Cov}(v_i, v_j) = \mathrm{Var}(v)$, which is positive and the same for all i, j.

For random variables x and y, we denote the covariance of x and y by σ_{xy}, the variance of x by σ_x^2, and the conditional variance of x given y by $\sigma_{x|y}^2$.

3. The equilibrium price function

In this section we solve for a rational expectations equilibrium price function for given $\boldsymbol{\lambda}$. We conjecture a linear price function of the form

$$p = \sum_{i=1}^{N} a_i \theta_i, \tag{1}$$

for some coefficients $(a_1, \ldots, a_N)$, not all zero. Thus the private signals of type i agents are reflected in the price only through their average signal, which is equal to θ_i. Given the linear-normal structure of the model, agents have a mean-variance objective function. The optimal portfolio of agent in is:

$$q_{in} = \frac{E(v_i|\mathcal{I}_{in}) - p}{r_i \operatorname{Var}(v_i|\mathcal{I}_{in})}. \tag{2}$$

To calculate agents' portfolios, we use the standard projection theorem for normals.[6] Let $\beta_i := \sigma_{\theta_i p}/\sigma_p^2$. Then

$$\sigma_{\theta_i|p}^2 = \sigma_{\theta_i}^2 - \beta_i \sigma_{\theta_i p}. \tag{3}$$

We proceed under the provisional assumption that p does not (fully) reveal θ_i for any i, i.e. $\sigma_{\theta_i|p}^2 > 0$. We will show later, in the proof of Proposition 3.2, that this assumption is in fact satisfied at any equilibrium. We use the superscripts I and U to distinguish between the portfolios of informed and uninformed agents.

Lemma 3.1 *(Optimal portfolios). Suppose $\sigma_{\epsilon_i}^2$ and $\sigma_{\eta_i}^2$ are not both zero, and $\sigma_{\theta_i|p}^2 > 0$. Then the optimal portfolios of type i agents are given by*

$$q_{in}^I = \frac{1}{r_i} \cdot \frac{\sigma_{\theta_i|p}^2 s_{in} - \left[\sigma_{\theta_i|p}^2 + (1-\beta_i)\sigma_{\epsilon_i}^2\right] p}{(\sigma_{\theta_i|p}^2 + \sigma_{\epsilon_i}^2)\sigma_{\eta_i}^2 + \sigma_{\theta_i|p}^2 \sigma_{\epsilon_i}^2},$$

$$q_{in}^U = -\frac{1}{r_i} \cdot \frac{1-\beta_i}{\sigma_{\theta_i|p}^2 + \sigma_{\eta_i}^2} p.$$

From Lemma 3.1, the aggregate trade of type i agents is

$$q_i = \frac{\lambda_i}{r_i} \cdot \frac{\sigma_{\theta_i|p}^2 \theta_i - \left[\sigma_{\theta_i|p}^2 + (1-\beta_i)\sigma_{\epsilon_i}^2\right] p}{(\sigma_{\theta_i|p}^2 + \sigma_{\epsilon_i}^2)\sigma_{\eta_i}^2 + \sigma_{\theta_i|p}^2 \sigma_{\epsilon_i}^2} - \frac{1-\lambda_i}{r_i} \cdot \frac{1-\beta_i}{\sigma_{\theta_i|p}^2 + \sigma_{\eta_i}^2} p, \tag{4}$$

[6] Consider random vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, $(\mathbf{x}_1, \mathbf{x}_2) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, and partition $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ as follows:

$$\boldsymbol{\mu} := \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}, \qquad \boldsymbol{\Sigma} := \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix},$$

where $\boldsymbol{\mu}_i := E(\mathbf{x}_i)$ and $\boldsymbol{\Sigma}_{ij} := \operatorname{Cov}(\mathbf{x}_i, \mathbf{x}_j)$, $i, j = 1, 2$. If $\boldsymbol{\Sigma}_{22}$ is nonsingular, we have

$$(\mathbf{x}_1|\mathbf{x}_2) \sim N\big(\boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\mathbf{x}_2 - \boldsymbol{\mu}_2), \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21}\big).$$

which is linear in θ_i and p. We can now solve for the price function using the market-clearing condition, $\sum_i q_i = 0$. Before proceeding with this task, we impose some further assumptions which will stay in force for the remainder of the paper:

A1. $\lambda_k > 0$ and $\lambda_\ell > 0$ for at least two types k and ℓ.
A2. The equilibrium trade of agent in is measurable with respect to his information $\mathcal{I}_{in}$.
A3. For any type i, one of the following information structures applies:
 (a) *Asymmetric information:* $\sigma^2_{\epsilon_i} = 0$ and $\sigma^2_{\eta_i} > 0$; or
 (b) *Differential information:* $\sigma^2_{\epsilon_i} > 0$ and $\sigma^2_{\eta_i} = 0$.

Assumption A1 is only provisional – we endogenize $\boldsymbol{\lambda}$ in Section 6 where we show that, in any equilibrium, λ_i is indeed positive for at least two types (see Lemma 6.3). Assumption A2 rules out some trivial equilibria. Assumption A3 is for tractability, and gives us two canonical information structures that have been employed in the literature. Under information structure (a), type i is asymmetrically informed in the sense that the informed agents of type i know θ_i while the uninformed of that type do not. Under information structure (b), type i is differentially informed in the sense that the informed agents of type i have conditionally i.i.d. signals about θ_i; moreover, the restriction $\sigma^2_{\eta_i} = 0$ implies that $v_i = \theta_i$, so that their pooled information reveals their type.[7] Note that Assumption A3 allows some types to be asymmetrically informed and others to be differentially informed.

Proposition 3.2 *(Equilibrium price function). There is a unique linear equilibrium price function given by*

$$p = k \sum_i \gamma_i \theta_i, \qquad k \neq 0, \tag{5}$$

where

$$\gamma_i = \begin{cases} \lambda_i (r_i \sigma^2_{\eta_i})^{-1} & \textit{if type } i \textit{ is asymmetrically informed,} \\ \lambda_i (r_i \sigma^2_{\epsilon_i})^{-1} & \textit{if type } i \textit{ is differentially informed.} \end{cases}$$

The price function does not (fully) reveal θ_i for any i.

From Lemma 3.1, we see that the coefficient on the private signal in the optimal trade of an informed agent of type i is $(r_i \sigma^2_{\eta_i})^{-1}$ in the asymmetric information case, and $(r_i \sigma^2_{\epsilon_i})^{-1}$ in the differential information case. We can think of this as the "trading intensity" of an informed agent. Thus the coefficient of θ_i in the price function is proportional to the trading intensity of the informed agents of type i, times their mass λ_i. The proposition does not require any condition on the correlations between the θ_i's (other than positive definiteness of the covariance matrix). These correlations do affect the value of the constant k in the price function.

It is instructive to compare the revelation properties of our price function with that of Vives (2014). Vives assumes that there is a continuum of types, with the result that the price reveals

[7] While the terms "asymmetric" and "differential" are a useful way to distinguish between one-sided and multifaceted private information, we should point out that our usage is somewhat loose – we say that type i is asymmetrically informed even if $\lambda_i = 1$ so that all type i agents know θ_i, and we refer to a type as differentially informed even though, strictly speaking, this label only applies to the informed agents of that type.

the average type, which for any trader is a sufficient statistic for the information of all *other* traders. Thus every trader effectively has access to the pooled information of all traders in the economy (Vives calls this a "privately revealing" equilibrium). In our model, on the other hand, there are finitely many types, with a continuum of each type. While the price does not reflect any idiosyncratic variation *within* types, it is affected by idiosyncratic variation *across* types. An agent of type i, who seeks to learn θ_i, knows θ_i in equilibrium only if his private signal already tells him what θ_i is. If he does not observe θ_i directly, how much he learns from the price depends on the mass of informed agents of every type. Price informativeness in Vives' model is the same regardless of the mass of informed agents (as long as this mass is positive).

4. Examples

In this section we provide a number of examples of our general framework. They include the economy in Grossman and Stiglitz (1980) with noisy aggregate supply, and the competitive limit of the economy in Hellwig (1980) with noise traders. In both cases, the noise is easily endogenized as optimizing trade arising from liquidity or hedging considerations. It can also be interpreted in purely behavioral terms. In these examples, agents have CARA utility and all random variables are joint normally distributed with mean zero. In Examples 4.1–4.3, optimal portfolios are given by Lemma 3.1 and the equilibrium price function by Proposition 3.2. In Example 4.4, Lemma 3.1 does not hold but an agent's portfolio is linear in the price and his private signal (if he is informed), so that the equilibrium price is still a linear combination of the θ_i's as in Proposition 3.2.

Example 4.1 *(Grossman–Stiglitz).* The asset payoff is $v = \theta + \eta$. There is a unit mass of investors, of whom a proportion $\lambda \in (0, 1)$ privately observes θ. In addition, there is a unit mass of "noise traders" whose private valuation is $u + \eta$, of which they privately observe the component u. The random variables θ, η, and u are mutually independent. This fits into our model with two types, both of which are asymmetrically informed: $\lambda_1 = \lambda, \lambda_2 = 1, \theta_1 = \theta, \theta_2 = u$, $\eta_1 = \eta_2 = \eta$, and $\sigma^2_{\epsilon_1} = \sigma^2_{\epsilon_2} = 0$. The optimal portfolios are

$$q^I_{1n} = \frac{\theta - p}{r_1 \sigma^2_\eta}, \qquad q^U_{1n} = \frac{E(v|p) - p}{r_1 \mathrm{Var}(v|p)}, \qquad q^I_{2n} = \frac{u - p}{r_2 \sigma^2_\eta}.$$

The price function is

$$p = k \left[\frac{\lambda}{r_1 \sigma^2_\eta} \theta + \frac{1}{r_2 \sigma^2_\eta} u \right],$$

which takes the same form as in Grossman and Stiglitz (1980), with u playing the role of the random aggregate supply or noise trade.

Notice that our noise traders do not trade an exogenous amount, as they are typically assumed to do in the noisy rational expectations literature. They can be thought of as "sentiment traders", with u being the sentiment shock, as in Mendel and Shleifer (2012) (whose model is a variant of the above example), or as investors who trade on noise as though it were information, as in Banerjee and Green (2015) and Peress (2014). ‖

Example 4.2 *(Grossman–Stiglitz with optimizing liquidity traders).* Rather than mimic the standard assumption of independent noise trade, suppose we replace the type 2 traders in Example 4.1

with optimizing "liquidity traders". These traders perceive the asset value to be $v = \theta + \eta$, just like the type 1 traders, but also have an endowment, which is the product of two normal random variables, y and e.[8] The random variable y is independent of e and η, and is not perfectly correlated with θ; it can be interpreted as the size of the liquidity shock. The covariance $\sigma_{\eta e}$ is nonzero. Each liquidity trader privately observes θ and y prior to trade. His optimal portfolio is given by

$$q_{2n}^{I} = \frac{\theta - r_2\sigma_{\eta e} y - p}{r_2\sigma_\eta^2}. \tag{6}$$

This fits into our model as in Example 4.1, except that here $\theta_2 = \theta - r_2\sigma_{\eta e} y$.[9] The price function is

$$\begin{aligned} p &= k\left[\frac{\lambda}{r_1\sigma_\eta^2}\theta + \frac{1}{r_2\sigma_\eta^2}(\theta - r_2\sigma_{\eta e} y)\right] \\ &= \frac{k}{\sigma_\eta^2}\left[(\lambda r_1^{-1} + r_2^{-1})\theta - \sigma_{\eta e} y\right]. \end{aligned}$$

If y is independent of θ, the price function is of the same form as in Grossman and Stiglitz (1980). ‖

Example 4.3 *(Hellwig)*. The asset payoff is θ. There is a unit mass of differentially informed agents who receive conditionally i.i.d. signals about θ. In addition, there is a unit mass of "noise traders" who are differentially informed about their private valuation u, which is independent of θ. There are no uninformed agents. This is a special case of our general setup with two types, both of which are differentially informed: $\lambda_1 = \lambda_2 = 1, \theta_1 = \theta, \theta_2 = u$, and $\sigma_{\eta 1}^2 = \sigma_{\eta 2}^2 = 0$. The optimal portfolios are:

$$q_{1n}^{I} = \frac{E(\theta|s_{1n}, p) - p}{r_1 \mathrm{Var}(\theta|s_{1n}, p)}, \qquad q_{2n}^{I} = \frac{E(u|s_{2n}, p) - p}{r_2 \mathrm{Var}(u|s_{2n}, p)}.$$

The price function is

$$p = k\left[\frac{1}{r_1\sigma_{\epsilon_1}^2}\theta + \frac{1}{r_2\sigma_{\epsilon_2}^2}u\right].$$

This is essentially the limiting equilibrium in Hellwig (1980), as the number of informed traders goes to infinity, with u playing the role of the exogenous noise trade as in Example 4.1.

Just as we replaced the noise traders in Example 4.1 with optimizing liquidity traders in Example 4.2, we can do that here as well. The optimal portfolio of type 2 agents is then given by (6). We assume that ϵ_{1n} is independent of y. This fits into our general model as follows: $\lambda_1 = \lambda_2 = 1, \eta_1 = 0, \eta_2 = \eta, \sigma_{\epsilon_1}^2 = \sigma_\epsilon^2, \sigma_{\epsilon_2}^2 = 0, v_1 = \theta_1 = \theta, v_2 = \theta_2 + \eta$, and $\theta_2 = \theta - r_2\sigma_{\eta e} y$.[10] The price function is

[8] A number of papers in the CARA-normal REE literature have used such a specification of the endowment to generate a hedging motive for trade. See, for example, Rahi (1996).

[9] The models of Ganguli and Yang (2009) and Manzano and Vives (2011) cannot be reduced to such a specification since in their setting the coefficient of the hedging term is not exogenous (it depends on the price function).

[10] For example, we can think of the asset payoff being $\theta + \eta$, with type 1 agents having the ability to hedge the risk η.

$$p = k\left[\frac{\theta}{r_1\sigma_\epsilon^2} + \frac{\theta - r_2\sigma_{\eta e}y}{r_2\sigma_\eta^2}\right]$$
$$= k\left[\left(\frac{1}{r_1\sigma_\epsilon^2} + \frac{1}{r_2\sigma_\eta^2}\right)\theta - \frac{\sigma_{\eta e}}{\sigma_\eta^2}y\right].$$

Notice that, unlike the noise traders in the first part of this example, who are differentially informed, all the liquidity traders have the same information. ‖

Thus our model provides a parsimonious framework that nests the classical models of Grossman and Stiglitz (1980) and Hellwig (1980), as well as some extensions of these models that have been studied in the literature. The above examples also suggest more general settings that have not been considered in the literature and where our analysis is applicable. Example 4.1 can easily be extended to multiple categories of "sentiment traders" who may or may not have private information about their own sentiment factor u_i. Multiple types of fully rational traders whose valuations are heterogeneous because of differing liquidity or hedging needs can likewise be considered along the lines of Example 4.2. A difficulty arises when we allow agents with a hedging motive to choose whether or not to acquire information: hedgers, as modeled in Example 4.2, do not have a mean-variance objective function if they are uninformed. Our analysis still applies, however, as we see in the following example:

Example 4.4 *(Multiple hedgers).* This example is fully worked out in Appendix B; we provide the salient details here.

The asset payoff is v. There are N types of traders with stochastic hedging needs. The wealth of agent n of type i is given by

$$W_{in} = (v - p)q_{in} + y_i e_i,$$

where $y_i e_i$ is the initial endowment. We assume that $(y_i)_{i=1}^N$ is independent of $(e_i)_{i=1}^N$ and also of v, $\sigma_{ve_i} \neq 0$ for all i, and the covariance matrix of $(y_i)_{i=1}^N$ is positive definite. If agent in chooses to acquire information, he observes y_i. The proportion of informed agents of type i is λ_i.

Lemma 3.1 does not hold in this example, so we calculate portfolios from scratch. We conjecture that p is a linear combination of the y_i's. If agent in is informed, his optimal portfolio is analogous to (6):

$$q_{in}^I = \frac{-r_i\sigma_{ve_i}y_i - p}{r_i\sigma_v^2}.$$

Note that $E(v|y_i, p) = E(v) = 0$. Defining the "valuation" of type i agents as $\theta_i := \sigma_{ve_i}y_i$, we can write q_{in}^I as a linear combination of θ_i and p. If agent in is uninformed, his wealth is not normally distributed conditional on his information at the time of trade; in particular his endowment $y_i e_i$ is not normal. Hence he does not have a mean-variance objective function. In Appendix B we show that q_{in}^U is a scalar multiple of p (see equation (53)). Thus optimal portfolios take the same linear form as in Lemma 3.1, but with different coefficients. Using the market-clearing condition $\sum_i[\lambda_i q_{in}^I + (1-\lambda_i)q_{in}^U] = 0$, we can calculate the price function. It is linear in agent valuations just as in Proposition 3.2, and can be written as

$$p = k\sum_i \lambda_i\theta_i,$$

for some nonzero scalar k. ‖

5. Price informativeness

In this section we study the informativeness of the equilibrium price function for given proportions of informed agents $\boldsymbol{\lambda}$ (we endogenize $\boldsymbol{\lambda}$ in the next section). Letting $\boldsymbol{\gamma} := (\gamma_i)_{i=1}^N$, we can write the price function (5) as $p = k\boldsymbol{\gamma}^\top\boldsymbol{\theta}$.[11] We denote the covariance matrix of $\boldsymbol{\theta}$ by $\mathbf{V}$, assumed to be positive definite, and the i'th column of $\mathbf{V}$ by $\mathbf{V}_i$. Due to the symmetry of $\mathbf{V}$, the i'th row of $\mathbf{V}$ is $\mathbf{V}_i^\top$. Then we have

$$\sigma_p^2 = k^2\boldsymbol{\gamma}^\top\mathbf{V}\boldsymbol{\gamma}, \qquad \text{and} \qquad \sigma_{\theta_i p} = k\mathbf{V}_i^\top\boldsymbol{\gamma}.$$

For the uninformed agents of type i, we use the following measure of price informativeness:

$$\mathcal{V}_i := \frac{\sigma_{\theta_i}^2 - \sigma_{\theta_i|p}^2}{\sigma_{\theta_i}^2}. \tag{7}$$

Clearly, $\mathcal{V}_i \in [0, 1)$. Substituting from (3), we get

$$\mathcal{V}_i = \frac{\sigma_{\theta_i p}^2}{\sigma_{\theta_i}^2\sigma_p^2} = \frac{1}{\sigma_{\theta_i}^2} \cdot \frac{(\mathbf{V}_i^\top\boldsymbol{\gamma})^2}{\boldsymbol{\gamma}^\top\mathbf{V}\boldsymbol{\gamma}}. \tag{8}$$

If $\sigma_{\epsilon_i}^2 > 0$, the informed agents of type i also learn from the price. For these agents, the corresponding measure of price informativeness is:

$$\mathcal{V}_i^I := \frac{\mathrm{Var}(\theta_i|s_{in}) - \mathrm{Var}(\theta_i|s_{in}, p)}{\mathrm{Var}(\theta_i|s_{in})}.$$

Lemma 5.1. *Suppose $\sigma_{\epsilon_i}^2 > 0$. Then $\mathcal{V}_i^I \in [0, 1)$ and is a strictly increasing function of $\mathcal{V}_i$.*

In view of this result, we will use $\mathcal{V}_i$ as our measure of price informativeness for agents of type i, whether or not they observe a noisy private signal in addition to the price. If type i is differentially informed, all type i agents make inferences from the price. If type i is asymmetrically informed, on the other hand, only the uninformed agents learn from the price; the informed already know θ_i.

We say that the economy is *symmetric* if the risk aversion coefficients r_i and the shock variances, $\sigma_{\theta_i}^2, \sigma_{\eta_i}^2$ and $\sigma_{\epsilon_i}^2$, are the same for all i (in which case, we drop the subscript i on these parameters), and all types are either asymmetrically informed or differentially informed. For a symmetric economy, $\boldsymbol{\gamma}$ is proportional to $\boldsymbol{\lambda}$ by Proposition 3.2. Moreover, the restriction that $\sigma_{\theta_i}^2 = \sigma_\theta^2$ for all i allows us to characterize $\mathcal{V}_i$ in terms of the correlation matrix $\mathbf{R} := (\sigma_\theta^2)^{-1}\mathbf{V}$, with ij'th element $\rho_{ij} := \mathrm{corr}(\theta_i, \theta_j)$. Let $\mathbf{R}_i$ be the i'th column of $\mathbf{R}$. We write $x \propto y$ to indicate that x and y have the same sign.

Proposition 5.2 *(Price informativeness). For a symmetric economy, price informativeness for type i is given by*

$$\mathcal{V}_i = \frac{(\mathbf{R}_i^\top\boldsymbol{\lambda})^2}{\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda}}. \tag{9}$$

Furthermore,

[11] All vectors are taken to be column vectors unless transposed.

$$\frac{\partial \mathcal{V}_i}{\partial \lambda_i} \propto \mathbf{R}_i^\top \boldsymbol{\lambda}. \tag{10}$$

Notice that $\mathcal{V}_i$ is homogeneous of degree zero in $\boldsymbol{\lambda}$: price informativeness depends only on the relative proportions of informed agents across types. For type i, we say that there is a *within-type complementarity* if $\partial\mathcal{V}_i/\partial\lambda_i < 0$, and an *across-type complementarity* if $\partial\mathcal{V}_i/\partial\lambda_j < 0$ for some $j \neq i$. We see from (10) that a necessary condition for within-type complementarity for type i is that $\rho_{ij} < 0$ for some j.

6. Endogenous information acquisition

So far we have taken the allocation of private information to be exogenous. We now endogenize information acquisition. In order to become informed, a type i agent must pay a positive cost c_i. He takes as given the vector $\boldsymbol{\lambda}$ and the corresponding price function $p = k\boldsymbol{\gamma}^\top\boldsymbol{\theta}$. We wish to find $\boldsymbol{\lambda}$ such that, for any type, both the informed and uninformed find their decision with regard to information acquisition optimal.

It is convenient to use the following monotonic transformation of ex ante expected utility:

$$\mathcal{U}_{in} := \left(E[\exp(-r_i \hat{W}_{in})]\right)^{-2}, \tag{11}$$

where $\hat{W}_{in} = W_{in} - c_i$ if agent in acquires information, and $\hat{W}_{in} = W_{in}$ if he does not. Using superscripts I and U for informed and uninformed agents respectively, we have

Lemma 6.1 *(Utilities). For given $\boldsymbol{\lambda}$,*

$$\mathcal{U}_{in}^I = e^{-2r_ic_i} \cdot \frac{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{v_i-p}}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}}, \qquad \textit{and} \qquad \mathcal{U}_{in}^U = \frac{\sigma^2_{v_i-p}}{\sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}}.$$

Since the utility of an agent depends only on his type, and on whether he is informed or uninformed, we shall henceforth drop the subscript n. Thus $\mathcal{U}_i^I$ will denote the utility of all informed agents of type i, and $\mathcal{U}_i^U$ the utility of all uninformed agents of this type. An equilibrium $\boldsymbol{\lambda}$ is characterized by

$$\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} \quad \text{is} \quad \begin{cases} \geq & 1 \quad \text{for} \quad \lambda_i = 1 \\ = & 1 \quad \text{for} \quad \lambda_i \in (0,1) \\ \leq & 1 \quad \text{for} \quad \lambda_i = 0. \end{cases} \tag{12}$$

Notice that, if $\lambda_i \in (0,1)$, the ex ante expected utility of an informed agent of type i (after paying the cost c_i) must be equal to the ex ante expected utility of an uninformed agent of that type. We now compute the utility ratio:

Lemma 6.2 *(Utilities of informed vs uninformed). For given $\boldsymbol{\lambda}$,*

$$\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} = e^{-2r_ic_i}\left[1 + \frac{\sigma^2_{\theta_i}}{\sigma^2_{\eta_i}}(1-\mathcal{V}_i)\right], \tag{13}$$

if type i is asymmetrically informed. If type i is differentially informed, we get the same expression with $\sigma^2_{\eta_i}$ replaced by $\sigma^2_{\epsilon_i}$.

For both information structures, the utility ratio is decreasing in $\mathcal{V}_i$. As one would expect, the incentive to become informed is lower if prices are more informative.

From Proposition 3.2 and Lemma 6.2, it is apparent that the two cases of type i being asymmetrically or differentially informed are formally identical as far as equilibrium is concerned. We will present our equilibrium results for asymmetric information. If type i is differentially informed, the corresponding results are obtained simply by replacing $\sigma^2_{\eta_i}$ by $\sigma^2_{\epsilon_i}$.

We assume that agents have an incentive to acquire information if they cannot learn anything from the price. Letting

$$\bar{c}_i := \frac{1}{2r_i} \log\left[1 + \frac{\sigma^2_{\theta_i}}{\sigma^2_{\eta_i}}\right],$$

this assumption is equivalent to the following condition (from (13)):

A4. For each type i, $c_i < \bar{c}_i$.

This will be a standing assumption (along with assumptions A1–A3 imposed in Section 3) for the rest of the paper. It says that the cost c_i is low relative to the signal-to-noise ratio $\sigma^2_{\theta_i}/\sigma^2_{\eta_i}$. This leads us to the following result, which ensures that equilibrium does not (fully) reveal θ_i for any type i (see Proposition 3.2):

Lemma 6.3 *(Partial revelation). An equilibrium vector* $\boldsymbol{\lambda}$ *has at least two elements that are strictly positive.*

Proof. If $\lambda_i = 0$ for all i, the price does not reveal any information to any type. By Assumption A4, all types have an incentive to acquire information, a contradiction. If $\lambda_j > 0$, and $\lambda_i = 0$ for all $i \neq j$, the price fully reveals θ_j, so that there is no incentive for type j agents to engage in costly information acquisition in the first place. $\square$

We now specialize the discussion to symmetric economies, i.e. those for which r_i, $\sigma^2_{\theta_i}$, $\sigma^2_{\eta_i}$ and $\sigma^2_{\epsilon_i}$ are the same for all i. Then the upper bound on the information acquisition cost $\bar{c}_i$ is also the same for all i, and as with the other parameters we drop the subscript i. Let

$$\alpha_i := 1 - (e^{2rc_i} - 1)\frac{\sigma^2_\eta}{\sigma^2_\theta}. \tag{14}$$

This expression is obtained by setting the utility ratio in (13) to one and solving for $\mathcal{V}_i$. Then, from (9) and (12), we have:

Lemma 6.4 *(Price informativeness vs cost). For a symmetric economy, an equilibrium* $\boldsymbol{\lambda}$ *is characterized by*

$$\mathcal{V}_i = \frac{(\mathbf{R}_i^\top \boldsymbol{\lambda})^2}{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}} \quad is \quad \begin{cases} \leq & \alpha_i \quad for \quad \lambda_i = 1 \\ = & \alpha_i \quad for \quad \lambda_i \in (0,1) \\ \geq & \alpha_i \quad for \quad \lambda_i = 0. \end{cases}$$

The parameter α_i lies in the interval (0,1), due to Assumption A4, and is a strictly decreasing function of the cost c_i. Whether agents of type i acquire information or not depends on the magnitude of c_i, or equivalently of α_i, relative to the informativeness of the price $\mathcal{V}_i$. The indifference condition is $\mathcal{V}_i = \alpha_i$. Without loss of generality, we order the types so that the c_i's are in ascending order ($c_1 \leq c_2 \leq \ldots \leq c_N$), or equivalently the α_i's are in descending order ($\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_N$). It will often be easier to think in terms of the α_i's rather than the c_i's.

We say that an equilibrium $\boldsymbol{\lambda}$ is *stable* if $\partial\mathcal{V}_i/\partial\lambda_i \geq 0$ for all i satisfying $\mathcal{V}_i = \alpha_i$. If this condition is violated, i.e. if $\mathcal{V}_j = \alpha_j$ and $\partial\mathcal{V}_j/\partial\lambda_j < 0$ for some j, a small increase in λ_j will lead to prices being less informative about θ_j, making the increase in λ_j self-fulfilling; if $\lambda_j = 1$, we can apply the same logic for a small decrease in λ_j. No condition is needed if $\mathcal{V}_i \neq \alpha_i$ since this inequality continues to hold for a small change in λ_i. We will only be concerned with stable equilibria in this paper. We shall therefore drop the adjective "stable" in what follows, without any risk of confusion – henceforth, whenever we refer to an equilibrium, it is implied that it is stable.

For the remainder of the paper, we shall restrict ourselves to symmetric economies satisfying the additional assumption that $\rho_{ij} = \rho$, $i \neq j$. This simplifies the analysis and provides us with the clearest intuitions. We then need to impose a lower bound on ρ due to

Lemma 6.5 *(Lower bound on ρ). Suppose $\rho_{ij} = \rho$, for all $i \neq j$. Then the correlation matrix* $\mathbf{R}$ *is positive definite if and only if*

$$\rho > \rho_{min} := -\frac{1}{N-1}.$$

We denote by $\mathcal{E}$ the set of symmetric economies with $\rho_{ij} = \rho > \rho_{\min}$, $i \neq j$.

Our first equilibrium characterization result says that types with a positive mass of informed agents can be ranked by price informativeness: for a lower cost type, the proportion of informed agents is higher, and price informativeness is higher as well.

Proposition 6.6 *(Ranking by price informativeness). Consider an economy in $\mathcal{E}$. Suppose λ_i and λ_j are nonzero and not both equal to 1. Then the following statements are equivalent: (a) $c_i < c_j$, (b) $\lambda_i > \lambda_j$, and (c) $\mathcal{V}_i > \mathcal{V}_j$. The following statements are also equivalent: (a) $c_i = c_j$, (b) $\lambda_i = \lambda_j$, and (c) $\mathcal{V}_i = \mathcal{V}_j$.*

Notably missing from the proposition is a ranking of types for which no agent acquires information relative to types for which some agents do. Later, in Proposition 8.2, we show that it is possible to have $\lambda_i = 0$ and $\lambda_j = 1$ even though $c_i < c_j$. This counterintuitive situation arises because of complementarities in information gathering.

The incentive to acquire information depends on the value of the common correlation coefficient ρ. Our next result characterizes the values of ρ for which either all agents acquire information or none do. More precisely, in the latter case, all agents have an incentive to free ride on the information gathering of others, and hence there is no equilibrium.

Proposition 6.7 *(Information acquisition: polar cases). Consider an economy in $\mathcal{E}$. There is no equilibrium if $\rho \geq \sqrt{\alpha_2}$.*[12] *There is an equilibrium with $\lambda_i = 1$ for all i if*

[12] It can be shown that this bound is tight, in the sense that if $\rho \in (\rho_{\min}, \sqrt{\alpha_2})$, there are parameter values such that an equilibrium exists.

$$\rho \leq \frac{\alpha_N N - 1}{N - 1}.$$

If $\rho \geq \sqrt{\alpha_2}$, then in fact $\rho \geq \sqrt{\alpha_i}$ for all $i \geq 2$ (since $\alpha_2 \geq \ldots \geq \alpha_N$). Due to the high correlation of types, agents of type i, $i \geq 2$, have an incentive to free ride on the information revealed by the price. But we know from Lemma 6.3 that in equilibrium there must be at least two types with a positive mass of informed agents. Hence there is no equilibrium. Notice that the cutoff value $\sqrt{\alpha_2}$, beyond which the correlation of types induces too much free riding, is decreasing in the cost of information c_2 and the noise-to-signal ratio $\sigma_\eta^2/\sigma_\theta^2$. Evidently, agents are more prone to free ride on others' information if the relative gain from acquiring their own information is small. The second part of Proposition 6.7 says that if ρ is sufficiently small, price informativeness is low enough to sustain an equilibrium in which all agents of all types acquire information.

The nonexistence result for $\rho \geq \sqrt{\alpha_2}$ shows how the Grossman–Stiglitz paradox can arise in an economy with correlated types, in both the asymmetric information and the differential information cases (the Grossman and Stiglitz, 1980 setting with no noise in the aggregate supply can be seen as a limiting case of our model with asymmetrically informed types as $\rho \to 1$).

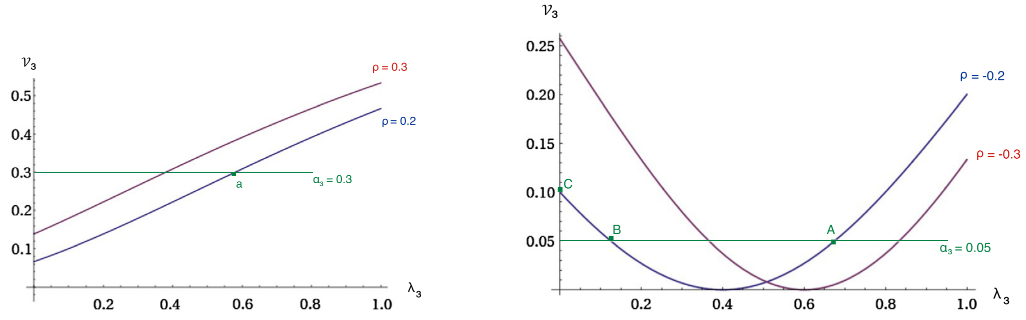
A general characterization of parameters for which an equilibrium exists, beyond the polar cases discussed in Proposition 6.7, is an interesting open question that we leave for future work. Instead, we focus on a class of equilibria for which we can not only provide readily interpretable conditions for existence, but which also lend themselves to a tractable analysis of welfare and multiplicity. An equilibrium in this class has the property that $\mathcal{V}_i < \alpha_i$, for $i \neq N$, regardless of the value of λ_N. It allows us to study the impact of information acquisition by agents of the highest cost type, type N, on price informativeness and welfare of all types without having to worry about the effect of a change in λ_N on $\lambda_i, i \neq N$, since the latter values remain fixed at one by Lemma 6.4. Such an equilibrium is parametrized by λ_N; accordingly we refer to it as a λ_N-equilibrium. Before providing conditions for existence of a λ_N-equilibrium, we exogenously set $\lambda_i = 1$ for $i \neq N$, and investigate the learning spillovers that arise when we perturb λ_N. Let $\mathcal{V}_j(\lambda_N)$ denote the dependence of $\mathcal{V}_j$ on λ_N, fixing $\lambda_i = 1$ for $i \neq N$, and let $\lambda^* := -\rho(N-1)$; note that $\lambda^* \in (0, 1)$ if and only if $\rho \in (\rho_{\min}, 0)$.

Lemma 6.8. *Consider an economy in $\mathcal{E}$. Suppose $\lambda_i = 1$ for all $i \neq N$. Then we have:*

i. $\partial \mathcal{V}_i / \partial \lambda_N < 0$, *for $i \neq N$;*
ii. $\partial \mathcal{V}_N / \partial \lambda_N \propto \lambda_N - \lambda^*$. *If $\rho < 0$, $\mathcal{V}_N(\lambda^*) = 0$.*

Thus there is an across-type complementarity: information acquisition by type N agents reduces price informativeness for all other types. A higher weight on θ_N in the price function has the effect of increasing the "noise" in the price signal for agents of types other than N. Of course, this complementarity translates into a learning externality only in the differential information case; in the asymmetric information case, at a λ_N-equilibrium, agents of type $i, i \neq N$, do not need to extract any information from the price as they already know θ_i.

Within-type learning externalities depend on the sign of ρ. Fig. 1 depicts the 3-type case. If $\rho \geq 0$ (and hence $\lambda^* \leq 0$), information acquisition by type N makes prices more informative for type N itself. However, if ρ is negative, $\mathcal{V}_N$ is not monotonic in λ_N. It is decreasing until it reaches its minimum value of zero at $\lambda_N = \lambda^*$, after which it is increasing. The within-type complementarity for low values of λ_N arises because an increase in λ_N confounds the price signal for type N: a higher p could be the result of a higher θ_N ("good news" for type N) or because of a higher $\theta_i, i \neq N$ ("bad news" for type N, since $\rho < 0$).



(a) $\rho \geq 0$. (b) $\rho < 0$.

Fig. 1. Price informativeness $\mathcal{V}_3$ as a function of λ_3, given $\lambda_1 = \lambda_2 = 1$.

We now turn to the question of existence. A λ_N-equilibrium exists if c_i is lower than some cut-off level c^*, for $i \neq N$, in order to induce all agents of these types to acquire information. An interior λ_N-equilibrium, i.e. one in which $\lambda_N \in (0, 1)$, exists under the additional condition that c_N lies in an interval to the right of c^*, so that some but not all agents of type N become informed.

Proposition 6.9 *(λ_N-equilibrium). Consider an economy in $\mathcal{E}$. Then there are scalars c^*, c^{**} and c^{***}, satisfying $0 < c^* < c^{**} < c^{***} \leq \bar{c}$, such that*

i. A λ_N-equilibrium exists if $N \geq 3$, and $c_i \in (0, c^*)$ for $i \neq N$. It is unique if $\rho \geq 0$.
ii. An interior λ_N-equilibrium exists if $c_i \in (0, c^*)$ for $i \neq N$, and $c_N \in (c^{**}, c^{***})$. It is unique regardless of the value of ρ, with $\lambda_N > \lambda^*$.

The condition $N \geq 3$ in Proposition 6.9 (part (i)) is needed to ensure that an equilibrium exists even if λ_N happens to be zero (if $N = 2$, there would be no such equilibrium due to Lemma 6.3).

Consider the 3-type case shown in Fig. 1. If $\rho \geq 0$, there is a unique λ_3-equilibrium. For $\rho = 0.2$ and $\alpha_3 = 0.3$, this equilibrium is at point a in Fig. 1a. The possibility of multiple λ_3-equilibria arises if ρ is negative. We will discuss multiplicity in Section 8. For now we note that even if ρ is negative, there is a unique interior λ_3-equilibrium. For the case where $\rho = -0.2$ and $\alpha_3 = 0.05$, this equilibrium corresponds to point A in Fig. 1b. Point B is not an equilibrium because it is not stable.

Information spillovers at an interior λ_N-equilibrium will feature prominently in our welfare analysis. At such an equilibrium $\lambda_N > \lambda^*$ by Proposition 6.9 (part (ii)). It then follows from Lemma 6.8 (part (ii)) that there is no within-type complementarity for type N. In Fig. 1b, we see that such a complementarity is excluded by the stability criterion (which is satisfied at A but not at B). On the other hand, across-type complementarities for types other than N apply at any λ_N-equilibrium by Lemma 6.8 (part (i)). To summarize:

Corollary 6.10. *Consider an economy in $\mathcal{E}$. At an interior λ_N-equilibrium, we have $\partial\mathcal{V}_N/\partial\lambda_N > 0$, and $\partial\mathcal{V}_i/\partial\lambda_N < 0$ for $i \neq N$.*

We conclude this section by reconsidering Example 4.4. In this example, ex ante utilities are not given by Lemma 6.1, but the utility ratio $\mathcal{U}_i^I/\mathcal{U}_i^U$ is similar to that given by (13). We show in Appendix B (see equation (57)) that

$$\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} = e^{-2r_i c_i}\left[1 + \frac{1}{\sigma_v^2\left[r_i^2\sigma_{\theta_i}^2(1-\mathcal{V}_i)\right]^{-1} - \left(\rho_{ve_i}^2\right)^{-1}}\right],$$

where $\rho_{ve_i} := \mathrm{corr}(v, e_i)$. As in (13), $\mathcal{U}_i^I/\mathcal{U}_i^U$ depends on $\boldsymbol{\lambda}$ only through price informativeness $\mathcal{V}_i$, and is decreasing in $\mathcal{V}_i$. This leads to a different cutoff $\bar{c}_i$, a slightly different definition of a symmetric economy (in this case it is one in which r_i, $\sigma_{\theta_i}^2$ and $\rho_{ve_i}^2$ are the same for all i), and a different (decreasing and invertible) mapping $\alpha_i : (0, \bar{c}) \to (0, 1)$, analogous to (14). With these modifications, all the results in this section from Lemma 6.3 onwards apply to Example 4.4 (and so do our multiplicity results in Section 8; however, the welfare analysis in the next section does not, since it relies on Lemma 6.1).

7. Welfare

In this section we consider the effect of information acquisition by agents of a given type on their own welfare as well as on the welfare of agents of other types.[13] A key role is played by the information conveyed by prices. The effect of price informativeness on welfare is not unambiguous, however. On the one hand, higher price informativeness for agents of type i (a higher $\mathcal{V}_i$) leads to better portfolio decisions for these agents. On the other hand, it is associated with prices being closer to their valuation v_i, so that the gains from trade that they can exploit are smaller.[14] As we shall see, the welfare of type i agents is increasing in $\sigma_{v_i-p}^2$. While $\mathcal{V}_i$ and $\sigma_{v_i-p}^2$ are not bound by a tight functional relationship, these two variables tend to be inversely related. If agents have sufficiently precise private information, so that learning from prices is relatively unimportant for them, they prefer prices to be less revealing. Conversely, if there is a lot of noise in their signals, they are better off if prices are more informative.

Imagine a hypothetical planner who can perturb the cost of gathering information and thereby the information acquisition decisions of agents. In general, a change in the cost of information for any one type will affect the proportions of informed agents of every type. We sidestep this difficulty by restricting ourselves to a λ_N-equilibrium. At such an equilibrium, a local change in c_N affects λ_N, and hence $\mathcal{V}_i$ for all i, but for $i \neq N$ leaves λ_i unchanged at 1. Moreover, at an interior λ_N-equilibrium, it is straightforward to characterize the effect of a local change in c_N:

Lemma 7.1. *Consider an economy in $\mathcal{E}$. At an interior λ_N-equilibrium, we have $\partial\lambda_N/\partial c_N < 0$, $\partial\mathcal{V}_N/\partial c_N < 0$ and $\partial\mathcal{V}_i/\partial c_N > 0$ for $i \neq N$.*

Thus, in a neighborhood of an interior λ_N-equilibrium, a higher cost of information acquisition for type N agents results in fewer of them acquiring information. By Corollary 6.10, this reduces price informativeness for type N while increasing it for all other types.

[13] We carry out a conventional welfare analysis under the assumption that agents' objective functions are a faithful representation of their welfare. This may not be the case if the heterogeneity in valuations arises from behavioral considerations.

[14] We can see from the optimal portfolio of type i, given by (2), that these agents tend to go long when $v_i > p$ and short when $v_i < p$; there are gains from trade for type i only to the extent that the price does not perfectly reflect their valuation.

We now investigate the welfare effects that arise. For an economy in $\mathcal{E}$, all types are either asymmetrically informed ($\sigma^2_{\epsilon_i} = 0$, for all i) or differentially informed ($\sigma^2_{\eta_i} = 0$, for all i). As far as information acquisition is concerned, these two cases are formally identical, involving only a change of notation (due to Lemma 6.2). However, as is apparent from Lemma 6.1, they do require a separate welfare analysis. We provide results for the more interesting case of differential information (as we noted earlier, across-type learning externalities do not arise at a λ_N-equilibrium in the asymmetric information case, since agents of type $i, i \neq N$, already know θ_i). From Lemma 6.1, ex ante utilities in the differential information case are:

$$\mathcal{U}_i^I = e^{-2rc_i}\left[\frac{\sigma^2_\theta}{\sigma^2_\epsilon} + (1-\mathcal{V}_i)^{-1}\right]\frac{\sigma^2_{v_i-p}}{\sigma^2_\theta}, \qquad i \neq N, \tag{15}$$

$$\mathcal{U}_N^U = \mathcal{U}_N^I = (1-\mathcal{V}_N)^{-1}\cdot\frac{\sigma^2_{v_N-p}}{\sigma^2_\theta}. \tag{16}$$

Notice that, for any type j, for given price informativeness $\mathcal{V}_j$, utility is increasing in $\sigma^2_{v_j-p}$; likewise, for given $\sigma^2_{v_j-p}$, it is increasing in $\mathcal{V}_j$.

Proposition 7.2 *(Welfare). Consider an economy in $\mathcal{E}$. Suppose $N \geq 3$ and all types are differentially informed. Then, in a neighborhood of an interior λ_N-equilibrium:*

i. The utility of type N agents is strictly increasing in c_N;
ii. The utility of type i agents, $i \neq N$, is strictly increasing in c_N if $\sigma^2_\theta/\sigma^2_\epsilon$ is sufficiently low; and
iii. The utility of type i agents, $i \neq N$, is strictly decreasing in c_N if $\sigma^2_\theta/\sigma^2_\epsilon$ is sufficiently high.

The welfare effects of an increase in c_N are all mediated by the induced reduction in λ_N. A lower λ_N makes type N agents better off. The effect on the welfare of other types depends on the signal-to-noise ratio $\sigma^2_\theta/\sigma^2_\epsilon$. If private signals are sufficiently noisy (the signal-to-noise ratio is sufficiently low), they are better off. On the other hand, if private information is sufficiently precise, their welfare is lower. In particular, in the case of noisy private signals, there is excessive information acquisition in equilibrium: reducing the proportion of informed agents of the highest cost type leads to a Pareto improvement.

For type N agents, incentives to gather information are misaligned with their own objectives: (at the margin) they choose to collect information even though they are worse off in the ensuing equilibrium. Restricting information acquisition by these agents reduces the amount that they learn from prices. This adverse information effect, and the higher cost of acquiring private information, are outweighed by greater trading gains (as measured by a higher $\sigma^2_{v_N-p}$).[15]

The learning externality for the other types goes in the opposite direction. Information acquisition by type N agents interferes with learning from prices by agents of types other than N (and this is true regardless of the value of ρ). This across-type complementarity is responsible for the somewhat counterintuitive result that discouraging information acquisition is Pareto improving precisely when prices have an important role to play in information aggregation.

[15] This is reminiscent of the result in Kurlat and Veldkamp (2015) that investors may be better off with no information, though in our case $E(v_i - p)$ is always zero, so the utility gain cannot be attributed to investors being able to trade a "high-risk, high-return asset".

8. Complementarity and multiplicity

In this section we show how the presence of a within-type complementarity can lead to multiple equilibria. We also compare price informativeness across equilibria. Given two equilibria E and E', we say that E *informationally dominates* E' if price informativeness is strictly higher at E for every type. Recall that $\lambda^* := -\rho(N-1)$.

Proposition 8.1 *(Multiple equilibria). Consider an economy in $\mathcal{E}$. Suppose $N \geq 3$, $\rho < 0$, and $c_i \in (0, c^*)$ for $i \neq N$. Then there is a $c_N^* \in (0, \bar{c})$ such that if $c_N \in (c_N^*, \bar{c})$, there are two λ_N-equilibria: with $\lambda_N = 0$ and with $\lambda_N \in (\lambda^*, 1]$. The first equilibrium informationally dominates the second one.*

The condition that $c_i \in (0, c^*), i \neq N$, ensures that a λ_N-equilibrium exists. This is taken from Proposition 6.9, which also tells us that ρ must be negative for there to be multiple λ_N-equilibria. The lower bound on c_N is needed to sustain the equilibrium in which $\lambda_N = 0$.[16]

When ρ is negative, $\mathcal{V}_N$ is not monotonic in λ_N – there is a within-type complementarity for $\lambda_N < \lambda^*$ (Lemma 6.8 (part (ii)). In order to understand how this complementarity drives multiplicity, it is instructive to take a detailed look at the 3-type case, depicted in Fig. 1b. Consider first the plot of $\mathcal{V}_3(\lambda_3)$ for $\rho = -0.2$. There is a unique equilibrium for $\alpha_3 \geq 0.1$: λ_3 lies in the interval $[0.8, 1]$ and is increasing in α_3 (or, equivalently, decreasing in c_3). If $\alpha_3 = 0.1$, we have $\mathcal{V}_3 = \alpha_3$ at $\lambda_3 = 0$. However, this does not qualify as an equilibrium by our definition since it is not stable. The cutoff value c_3^* corresponds to $\alpha_3 = 0.1$. The case of $\alpha_3 = 0.05$ is shown in the figure. There are two equilibria,[17] indicated by points A and C (as we noted earlier, B is unstable).[18] Perversely, type 3 agents learn more from the price when none of them acquire information (point C). Agents of types 1 and 2 also learn more from the price at C than at A; this is a consequence of the across-type complementarity identified in Lemma 6.8 (part (i)).

Suppose we are initially at point C, with α_3 just below 0.1. Consider an increase in α_3. As α_3 crosses 0.1, there is a discontinuous jump in λ_3 from 0 to 0.8. A small decrease in the cost of information sets off a "frenzy" of information gathering for type 3 agents, with the proportion of informed agents jumping from 0 to 80%. As soon as α_3 exceeds 0.1, the cost of information is low enough to justify acquiring it. But as more agents acquire the information, prices become less informative, inducing even more agents to acquire information. The same discontinuous jump in information acquisition arises if α_3 is just below 0.1, and there is a small increase in the uncertainty facing uninformed agents, as measured by σ_θ^2. This has the effect of reducing $|\rho|$, shifting the curve downwards.[19]

Now consider the plot for $\rho = -0.3$. Again start with $\lambda_3 = 0$ with $\alpha_3 < 0.257$. As α_3 increases beyond this cutoff value, the equilibrium jumps to $\lambda_3 = 1$. A small decrease in the cost

16 For sufficiently low values of ρ it turns out that $c_N^* < c^*$, though of course $c_N \geq c_i$ for all i.

17 Equilibrium multiplicity is generated by the non-monotonicity of price informativeness as a function of the proportion of informed traders. This is in contrast to Ganguli and Yang (2009) where multidimensional information leads to two equilibrium price functions for any given allocation of private information. For one price function price informativeness is monotonically increasing in the proportion of informed traders, while for the other price function it is monotonically decreasing.

18 Note that for $\alpha_3 < 0.1$, $\lambda_3 = 0$ is a stable equilibrium. If λ_3 increases by a small amount from 0, price informativeness $\mathcal{V}_3$, which is continuous in λ_3, still remains above α_3.

19 The magnitude of ρ can also be affected by a public signal about $\boldsymbol{\theta}$, or by market size as in Rostek and Weretka (2012).

of gathering information (or a small increase in σ_θ^2), leads to *all* agents of type 3 acquiring information. In this case, there is also a discontinuous downward jump in price informativeness for type 3 agents. Of course, at $\lambda_3 = 1$, the information revealed by the price is only relevant in the differential information case (in the asymmetric information case, the agents' private signals already tell them what θ_3 is). With differential information, it is indeed possible that each type 3 agent learns less about his valuation when all type 3 agents acquire information, even when he combines his private signal with the information contained in the price.

Next, we show that, under a tighter condition on ρ, equilibrium multiplicity can be much more pronounced than is suggested by Proposition 8.1. For this result, we focus on equilibria wherein λ_i is either 0 or 1 for all i. We say that type i is uninformed if $\lambda_i = 0$ and informed if $\lambda_i = 1$.

Proposition 8.2 *(Multiple equilibria II). Consider an economy in $\mathcal{E}$. Suppose $N \geq 3$, and $\rho < -N^{-1}$. Then, for an open set of cost parameters $(c_i)_{i=1}^N$, and for all integers m satisfying $N/2 < m \leq N$, there exists an equilibrium in which m types are informed and the remaining $(N - m)$ types are uninformed. At any such equilibrium, price informativeness for the uninformed types is strictly higher than price informativeness for the informed types. Furthermore, the equilibrium in which all types are informed is informationally dominated by any equilibrium in which some types are uninformed.*

The proposition says that, in the presence of a sufficiently strong complementarity, there is a plethora of equilibria. The equilibrium in which all agents of all types acquire information is actually the worst in terms of price informativeness: prices would be more informative for everyone if one or more types switch to not acquiring any information. The allocation of types to the informed and uninformed groups is arbitrary. Thus there are equilibria in which the types that acquire information have a higher cost than the types that do not.

The results of this section require a negative correlation between agent valuations. We focus on the more tractable case where all pairwise correlations are the same, but this is not an essential assumption. Negative correlations can arise due to hedging motives, which can easily be incorporated in our model as in Example 4.4. In many markets, negative correlations are a natural consequence of hedgers being on opposite sides in another market. Suppose, for instance, that the asset in Example 4.4 is a wheat futures contract and types i and j are producers of wheat and of bread respectively. Then the covariances σ_{ve_i} and σ_{ve_j} are of opposite sign, while the scale factors y_i and y_j, which we can think of as the projected size of the wheat crop and the demand for bread respectively, comove with the economy as a whole, and hence are positively correlated. Thus the correlation between the valuations of these two types, which is given by $\mathrm{corr}(\sigma_{ve_i} y_i, \sigma_{ve_j} y_j) = -\mathrm{corr}(y_i, y_j)$, is negative.

The complementarity result in Goldstein et al. (2014) has a similar flavor to ours: it is driven by a sufficiently strong hedging motive that makes a subset of informed investors trade in the opposite direction to others who only have a speculative motive. Another plausible scenario that can generate negatively correlated valuations, described by Barlevy and Veronesi (2008), is one where some agents have access to a private technology the returns on which are higher in good times, when the asset fundamental is also high. These agents sell the asset in order to free up resources for other projects.

While it should be clear that our results on both welfare and equilibrium multiplicity are driven by information spillovers, the nature of these spillovers is different for these two sets of results. Multiplicity is a consequence of the non-monotonicity of $\mathcal{V}_i$ with respect to λ_i, i.e. within-type complementarities. Across-type complementarities account for one equilibrium informationally dominating another, but not for the multiplicity itself. Our welfare result, on the other hand, is entirely governed by across-type complementarities, in particular the fact that less information acquisition by type N makes prices more informative for all other types. Within-type complementarities play no role here. Even in the case where $\mathcal{V}_N$ is not monotonic in λ_N, it is nevertheless increasing in a neighborhood of an interior λ_N-equilibrium. Our welfare result is a local one, and locally there is no within-type complementarity, regardless of the sign of ρ.

9. Concluding remarks

We study competitive rational expectations equilibria in an economy in which agents have interdependent private valuations for the risky asset. For any given allocation of private information, there is a unique linear equilibrium price function that takes a very simple form. We characterize the endogenous distribution of private information when agents can choose whether or not to pay for it. We highlight the role of learning externalities within and across types of agents. When private signals are noisy and agents rely primarily on the information transmitted by prices, raising the cost of information collection for the highest cost type, and thereby curtailing their information gathering activities, can make all types better off. When valuations across types are negatively correlated, multiple equilibria can arise.

A number of open questions remain. Relaxing our symmetry assumptions, especially with regard to the correlations ρ_{ij}, could lead to a deeper understanding of learning spillovers and their effect on incentives to produce information. Our welfare analysis is also incomplete, since we restrict ourselves to λ_N-equilibria. An interesting extension of our framework would be to allow agents to choose what information to acquire, as they may well prefer to be informed about the valuation of a type other than their own.

Appendix A. Proofs

Proof of Lemma 3.1. The assumption that $\sigma^2_{\theta_i|p} > 0$ ensures that the covariance matrix of (s_{in}, p) is nonsingular even if $\sigma^2_{\epsilon_i} = 0$. The conditional expectations of v_i, given $\{s_{in}, p\}$ and p, respectively, are:

$$
\begin{aligned}
E(v_i|s_{in}, p) &= [\sigma^2_{\theta_i} \quad \sigma_{\theta_i p}] \begin{bmatrix} \sigma^2_{\theta_i} + \sigma^2_{\epsilon_i} & \sigma_{\theta_i p} \\ \sigma_{\theta_i p} & \sigma^2_p \end{bmatrix}^{-1} \begin{bmatrix} s_{in} \\ p \end{bmatrix} \\
&= \frac{1}{(\sigma^2_{\theta_i} + \sigma^2_{\epsilon_i})\sigma^2_p - \sigma^2_{\theta_i, p}} \cdot [\sigma^2_{\theta_i} \quad \sigma_{\theta_i p}] \begin{bmatrix} \sigma^2_p & -\sigma_{\theta_i p} \\ -\sigma_{\theta_i p} & \sigma^2_{\theta_i} + \sigma^2_{\epsilon_i} \end{bmatrix} \begin{bmatrix} s_{in} \\ p \end{bmatrix} \\
&= \frac{1}{(\sigma^2_{\theta_i} + \sigma^2_{\epsilon_i})\sigma^2_p - \sigma^2_{\theta_i, p}} \cdot \left[(\sigma^2_{\theta_i}\sigma^2_p - \sigma^2_{\theta_i, p}) s_{in} + \sigma_{\theta_i p}\sigma^2_{\epsilon_i} p \right] \\
&= \frac{1}{\sigma^2_{\theta_i} + \sigma^2_{\epsilon_i} - \beta_i \sigma_{\theta_i p}} \cdot \left[(\sigma^2_{\theta_i} - \beta_i \sigma_{\theta_i p}) s_{in} + \beta_i \sigma^2_{\epsilon_i} p \right] \\
&= \frac{1}{\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i}} \cdot \left(\sigma^2_{\theta_i|p} s_{in} + \beta_i \sigma^2_{\epsilon_i} p \right),
\end{aligned}
$$

and

$$
E(v_i|p) = \frac{\sigma_{\theta_i p}}{\sigma^2_p} p = \beta_i p.
$$

The conditional variances are:

$$\begin{aligned}
\operatorname{Var}(v_i|s_{in},p) &= \sigma^2_{v_i} - [\sigma^2_{\theta_i} \quad \sigma_{\theta_i p}]\begin{bmatrix} \sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} & \sigma_{\theta_i p} \\ \sigma_{\theta_i p} & \sigma^2_p \end{bmatrix}^{-1}\begin{bmatrix} \sigma^2_{\theta_i} \\ \sigma_{\theta_i p} \end{bmatrix} \\
&= \sigma^2_{v_i} - \frac{1}{(\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i})\sigma^2_p - \sigma^2_{\theta_i,p}} \cdot [\sigma^2_{\theta_i} \quad \sigma_{\theta_i p}]\begin{bmatrix} \sigma^2_p & -\sigma_{\theta_i p} \\ -\sigma_{\theta_i p} & \sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} \end{bmatrix}\begin{bmatrix} \sigma^2_{\theta_i} \\ \sigma_{\theta_i p} \end{bmatrix} \\
&= \sigma^2_{v_i} - \frac{1}{(\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i})\sigma^2_p - \sigma^2_{\theta_i,p}} \cdot \left[(\sigma^2_{\theta_i}\sigma^2_p - \sigma^2_{\theta_i,p})\sigma^2_{\theta_i} + \sigma^2_{\theta_i,p}\sigma^2_{\epsilon_i}\right] \\
&= \sigma^2_{v_i} - \frac{1}{\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} - \beta_i\sigma_{\theta_i p}} \cdot \left[(\sigma^2_{\theta_i} - \beta_i\sigma_{\theta_i p})\sigma^2_{\theta_i} + \beta_i\sigma_{\theta_i p}\sigma^2_{\epsilon_i}\right] \\
&= \sigma^2_{\eta_i} + \frac{(\sigma^2_{\theta_i} - \beta_i\sigma_{\theta_i p})\sigma^2_{\epsilon_i}}{\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i} - \beta_i\sigma_{\theta_i p}} \\
&= \sigma^2_{\eta_i} + \frac{\sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}}{\sigma^2_{\theta_i|p}+\sigma^2_{\epsilon_i}},
\end{aligned}$$

and

$$\operatorname{Var}(v_i|p) = \sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}.$$

Plugging these conditional moments into (2), we get the desired result. □

Proof of Proposition 3.2. We proceed under the provisional assumption that $\sigma^2_{\theta_i|p} > 0$ for all i, i.e. the price function does not (fully) reveal θ_i for any i. We will verify later that this assumption does in fact hold. From (4),

$$q_i = \gamma_i\theta_i - k_i p, \tag{17}$$

where

$$\gamma_i := \frac{\lambda_i}{r_i} \cdot \frac{\sigma^2_{\theta_i|p}}{(\sigma^2_{\theta_i|p}+\sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}}, \tag{18}$$

and

$$k_i := \frac{\lambda_i}{r_i} \cdot \frac{\sigma^2_{\theta_i|p} + (1-\beta_i)\sigma^2_{\epsilon_i}}{(\sigma^2_{\theta_i|p}+\sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}} + \frac{1-\lambda_i}{r_i} \cdot \frac{1-\beta_i}{\sigma^2_{\theta_i|p}+\sigma^2_{\eta_i}}. \tag{19}$$

Using the market-clearing condition, $\sum_i q_i = 0$, we obtain:

$$\Big(\sum_i k_i\Big)p = \sum_i \gamma_i\theta_i. \tag{20}$$

Suppose first that $\sum_i k_i = 0$. Then, $\sum_i \gamma_i\theta_i = 0$. Due to the positive definiteness of the covariance matrix of $\boldsymbol{\theta}$, we must have $\gamma_i = 0$ for all i. Since $\sigma^2_{\theta_i|p} > 0$ by assumption, it follows from (18) that $\lambda_i = 0$ for all i. But this contradicts Assumption A1, which says that $\lambda_i > 0$ for at least two types. We conclude that $\sum_i k_i \neq 0$.

We can now solve (20) for the price function p, and we see that it is indeed given by (5), with $k = (\sum_i k_i)^{-1}$. From (18), it is immediate that $\gamma_i = \lambda_i(r_i\sigma^2_{\eta_i})^{-1}$ if $\sigma^2_{\epsilon_i} = 0$ (type i is asymmetrically informed), while $\gamma_i = \lambda_i(r_i\sigma^2_{\epsilon_i})^{-1}$ if $\sigma^2_{\eta_i} = 0$ (type i is differentially informed).

Finally, we verify that a price function of the form (1) does not reveal θ_i for any i ($\sigma^2_{\theta_i|p} > 0$ for all i). Suppose not, say p reveals θ_j. Then, since the covariance matrix of $\boldsymbol{\theta}$ is positive definite, so that θ_j is not perfectly correlated with any linear combination of the remaining θ_i's, we must have $p = a_j\theta_j, a_j \neq 0$. Since p does not reveal θ_i for $i \neq j$, equations (17)–(19) still hold for $i \neq j$. For type j, Lemma 3.1 does not apply, but q_j can be calculated directly from (2). Assuming for the moment that $\sigma^2_{\eta_j} > 0$, $q_j = (\theta_j - p)(r_j\sigma^2_{\eta_j})^{-1}$. Thus q_j is given by (17), with $\gamma_j = k_j = (r_j\sigma^2_{\eta_j})^{-1}$. From (20), we see that $\sum_i k_i \neq 0$, for otherwise $\gamma_i = 0$ for all i, a contradiction. Hence the price function is given by (5). But since $p = a_j\theta_j$, $\gamma_i = 0$ for all $i \neq j$, which in turn implies that $\lambda_i = 0$ for all $i \neq j$. This contradicts Assumption A1. For the case where $\sigma^2_{\eta_j} = 0$, we must have $p = \theta_j$, and the optimal trade of a type j agent is indeterminate. However, the aggregate trade of type j is pinned down by market clearing, i.e. $q_j = -\sum_{i\neq j} q_i = \sum_{i\neq j}(k_i p - \gamma_i\theta_i) = \sum_{i\neq j}(k_i\theta_j - \gamma_i\theta_i)$. Invoking Assumption A3, q_j is measurable with respect to the information of type j agents. Hence, we must have $\gamma_i = 0$ for all $i \neq j$, leading to the same contradiction that we arrived at above. □

Proof of Lemma 5.1. We have

$$\mathcal{V}_i^I = \frac{\left(\sigma^2_{\theta_i} - \frac{\sigma^4_{\theta_i}}{\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i}}\right) - \frac{(\sigma^2_{\theta_i} - \beta_i\sigma_{\theta_i p})\sigma^2_{\epsilon_i}}{\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i}-\beta_i\sigma_{\theta_i p}}}{\sigma^2_{\theta_i} - \frac{\sigma^4_{\theta_i}}{\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i}}}$$

$$= \frac{\beta_i\sigma_{\theta_i p}\sigma^2_{\epsilon_i}}{\sigma^2_{\theta_i}(\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i}-\beta_i\sigma_{\theta_i p})}$$

$$= \frac{\mathcal{V}_i\sigma^2_{\epsilon_i}}{(1-\mathcal{V}_i)\sigma^2_{\theta_i}+\sigma^2_{\epsilon_i}}.$$

The result follows. □

Proof of Proposition 5.2. From (8), it is immediate that

$$\mathcal{V}_i = \frac{(\mathbf{R}_i^\top\boldsymbol{\lambda})^2}{\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda}}.$$

Differentiating this expression, we obtain

$$\frac{\partial\mathcal{V}_i}{\partial\lambda_i} = \frac{2\mathbf{R}_i^\top\boldsymbol{\lambda}\left[\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda} - (\mathbf{R}_i^\top\boldsymbol{\lambda})^2\right]}{(\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda})^2}$$

$$= \frac{2\mathbf{R}_i^\top\boldsymbol{\lambda}(1-\mathcal{V}_i)}{\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda}}$$

$$\propto \mathbf{R}_i^\top\boldsymbol{\lambda},$$

where we have used the fact that $\mathcal{V}_i \in [0, 1)$. □

Proof of Lemma 6.1. With the understanding that the cost c_i is paid by agent in only if he is informed, his ex ante expected utility is

$$\begin{aligned} E[-\exp(r_i c_i - r_i W_{in})] &:= -e^{r_i c_i} E\big[E(\exp(-r_i W_{in})|\mathcal{I}_{in})\big] \\ &= -e^{r_i c_i} E\big[\exp(-r_i \mathcal{E}_{in})\big], \end{aligned} \tag{21}$$

where

$$\begin{aligned} \mathcal{E}_{in} &:= E(W_{in}|\mathcal{I}_{in}) - \frac{r_i}{2}\mathrm{Var}(W_{in}|\mathcal{I}_{in}) \\ &= \big[E(v_i|\mathcal{I}_{in}) - p\big]q_{in} - \frac{r_i}{2}q_{in}^2\mathrm{Var}(v_i|\mathcal{I}_{in}). \end{aligned}$$

From (2), $E(v_i|\mathcal{I}_{in}) - p = r_i q_{in}\mathrm{Var}(v_i|\mathcal{I}_{in})$. Therefore,

$$\mathcal{E}_{in} = \frac{r_i}{2}q_{in}^2\mathrm{Var}(v_i|\mathcal{I}_{in}).$$

Substituting for q_{in} from Lemma 3.1,

$$\begin{aligned} -r_i\mathcal{E}_{in}^I &= -\frac{1}{2}\cdot\frac{\Big[\sigma^2_{\theta_i|p}s_{in} - \big[\sigma^2_{\theta_i|p} + (1-\beta_i)\sigma^2_{\epsilon_i}\big]p\Big]^2}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\big[(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}\big]}, \\ -r_i\mathcal{E}_{in}^U &= -\frac{1}{2}\cdot\frac{(1-\beta_i)^2}{\sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}}p^2. \end{aligned}$$

In order to evaluate (21), we invoke the fact that if $x \sim N(0, \sigma^2)$, then $E[e^{-\frac{1}{2}x^2}] = (1+\sigma^2)^{-\frac{1}{2}}$. Using the definition of $\mathcal{U}_{in}$ given by (11), we obtain

$$\mathcal{U}_{in}^I = e^{-2r_i c_i}\left[1 + \frac{(\sigma^2_{\theta_i} + \sigma^2_{\epsilon_i})\sigma^4_{\theta_i|p} + [\sigma^2_{\theta_i|p} + (1-\beta_i)\sigma^2_{\epsilon_i}]^2\sigma^2_p - 2[\sigma^2_{\theta_i|p} + (1-\beta_i)\sigma^2_{\epsilon_i}]\sigma^2_{\theta_i|p}\sigma_{\theta_i p}}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})[(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}]}\right],$$

which, after some algebraic manipulation, gives us the expression for $\mathcal{U}_{in}^I$ in the statement of the lemma. Also,

$$\mathcal{U}_{in}^U = 1 + \frac{(1-\beta_i)^2\sigma^2_p}{\sigma^2_{\theta_i|p} + \sigma^2_{\eta_i}},$$

which yields the desired expression for $\mathcal{U}_{in}^U$. □

Proof of Lemma 6.2. From Lemma 6.1:

$$\begin{aligned} \frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} &= e^{-2r_i c_i}\cdot\frac{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})(\sigma^2_{\theta_i|p} + \sigma^2_{\eta_i})}{(\sigma^2_{\theta_i|p} + \sigma^2_{\epsilon_i})\sigma^2_{\eta_i} + \sigma^2_{\theta_i|p}\sigma^2_{\epsilon_i}} \\ &= e^{-2r_i c_i}\left[1 + \frac{\sigma^4_{\theta_i|p}}{\sigma^2_{\theta_i|p}(\sigma^2_{\epsilon_i} + \sigma^2_{\eta_i}) + \sigma^2_{\epsilon_i}\sigma^2_{\eta_i}}\right]. \end{aligned}$$

If type i is asymmetrically informed ($\sigma^2_{\epsilon_i} = 0$), we get

$$\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} = e^{-2r_i c_i}\left[1 + \frac{\sigma^2_{\theta_i|p}}{\sigma^2_{\eta_i}}\right].$$

Substituting for $\sigma^2_{\theta_i|p}$, using (7), gives us the desired expression for the utility ratio. In the differential information case ($\sigma^2_{\eta_i} = 0$), we get the same expression with $\sigma^2_{\eta_i}$ replaced by $\sigma^2_{\epsilon_i}$. □

Proof of Lemma 6.5. Let $\mathbf{1} := (1, \ldots, 1)^\top$ and $\mathbf{v}_j := (-1, 0, \ldots, 0, 1, 0, \ldots, 0)^\top$, where the 1 is in the j'th place. Then $\mathbf{R1} = [1 + \rho(N-1)]\mathbf{1}$, and $\mathbf{R}\mathbf{v}_j = (1-\rho)\mathbf{v}_j$, for $j = 2, 3, \ldots, N$. Thus the eigenvalues of $\mathbf{R}$ are $[1 + \rho(N-1)]$ and $(1-\rho)$, the latter with multiplicity $N-1$. Since $\mathbf{R}$ is a symmetric matrix, it is positive definite if and only if all its eigenvalues are positive, i.e. if and only if $1 + \rho(N-1) > 0$. □

Proof of Proposition 6.6. Specializing (9) to the case where $\rho_{ij} = \rho$ for all $i \neq j$, we obtain the following expression for price informativeness for type i, for an economy in $\mathcal{E}$:

$$\mathcal{V}_i = \frac{\left[(1-\rho)\lambda_i + \rho \sum_k \lambda_k\right]^2}{(1-\rho)\sum_k \lambda_k^2 + \rho\left(\sum_k \lambda_k\right)^2}. \tag{22}$$

Now consider an equilibrium $\boldsymbol{\lambda}$. Due to the stability condition, for all ℓ satisfying $\mathcal{V}_\ell = \alpha_\ell$, we have $\partial \mathcal{V}_\ell / \partial \lambda_\ell \geq 0$, and hence $\mathbf{R}_\ell^\top \boldsymbol{\lambda} \geq 0$ (from (10)). In addition, $\mathcal{V}_\ell = \alpha_\ell$ implies that $\mathcal{V}_\ell > 0$, so that $\mathbf{R}_\ell^\top \boldsymbol{\lambda} \neq 0$ (from (9)). Thus $\mathbf{R}_\ell^\top \boldsymbol{\lambda} > 0$, i.e.

$$(1-\rho)\lambda_\ell + \rho \sum_k \lambda_k > 0, \tag{23}$$

for all ℓ satisfying $\mathcal{V}_\ell = \alpha_\ell$. In particular, this is the case if $\lambda_\ell \in (0, 1)$. Moreover, if $\lambda_\ell = 1$, the LHS of (23) is equal to $1 + \rho \sum_{k \neq \ell} \lambda_k$, which is positive since $\rho > \rho_{\min}$. Thus (23) applies as long as $\lambda_\ell \in (0, 1]$, and in particular for $\ell = i, j$ in the statement of the proposition. It follows that $\lambda_i > \lambda_j$ if and only if $\mathcal{V}_i > \mathcal{V}_j$, and $\lambda_i = \lambda_j$ if and only if $\mathcal{V}_i = \mathcal{V}_j$. It remains to show that $c_i < c_j$ if and only if $\mathcal{V}_i > \mathcal{V}_j$ (since we can reverse the indices i and j, this in turn implies that $c_i = c_j$ if and only if $\mathcal{V}_i = \mathcal{V}_j$).

Suppose first that $\mathcal{V}_i > \mathcal{V}_j$. Then $\lambda_i > \lambda_j$ and hence $\lambda_j \in (0, 1)$. Furthermore, $\alpha_i \geq \mathcal{V}_i > \mathcal{V}_j = \alpha_j$, so that $c_i < c_j$. Next suppose that $c_i < c_j$, or equivalently $\alpha_i > \alpha_j$. If $\mathcal{V}_i \leq \mathcal{V}_j$, we have $\lambda_i \leq \lambda_j$, implying that $\lambda_i \in (0, 1)$, and $\alpha_i > \alpha_j \geq \mathcal{V}_j \geq \mathcal{V}_i = \alpha_i$, a contradiction. Therefore $\mathcal{V}_i > \mathcal{V}_j$. □

Proof of Proposition 6.7. Let

$$\hat{\lambda}_i := \frac{\lambda_i}{\sum_{k \neq i} \lambda_k}, \qquad \text{and} \qquad \delta_i := \frac{\sum_{k \neq i} \lambda_k^2}{\left(\sum_{k \neq i} \lambda_k\right)^2},$$

which are well-defined for any i since $\sum_{k \neq i} \lambda_k > 0$ by Lemma 6.3. Using (22), we can write $\mathcal{V}_i$ as follows:

$$\begin{aligned}
\mathcal{V}_i &= \frac{\left[\lambda_i + \rho \sum_{k \neq i} \lambda_k\right]^2}{(1-\rho)\left[\lambda_i^2 + \sum_{k \neq i} \lambda_k^2\right] + \rho\left[\lambda_i + \sum_{k \neq i} \lambda_k\right]^2} \\
&= \frac{(\hat{\lambda}_i + \rho)^2}{(1-\rho)(\hat{\lambda}_i^2 + \delta_i) + \rho(\hat{\lambda}_i + 1)^2} \\
&= \frac{(\hat{\lambda}_i + \rho)^2}{(\hat{\lambda}_i + \rho)^2 + (1-\rho)(\delta_i + \rho)}.
\end{aligned} \tag{24}$$

Notice that $\mathcal{V}_i$ is strictly decreasing in δ_i and, if $\rho \geq 0$, strictly increasing in $\hat{\lambda}_i$. Hence, provided $\rho \geq 0$, a lower bound for $\mathcal{V}_i$ is obtained from (24) by setting $\hat{\lambda}_i$ equal to its lowest possible value, which is 0, and δ_i equal to its highest possible value, which is 1 ($\delta_i = 1$ if and only if there is only one type k, $k \neq i$, for which $\lambda_k > 0$). This gives us $\mathcal{V}_i \geq \rho^2$; if $\lambda_i > 0$, we have $\mathcal{V}_i > \rho^2$.

By Lemmas 6.3 and 6.4, there are at least two types for which $\lambda_i > 0$ and consequently $\mathcal{V}_i \leq \alpha_i$. If $\rho \geq 0$, we must therefore have $\rho^2 < \mathcal{V}_i \leq \alpha_i$ for these two types, implying that $\rho < \sqrt{\alpha_2}$ (recall that we have ranked the α_k's in descending order). Thus there is no equilibrium if $\rho \geq \sqrt{\alpha_2}$.

Now suppose $\lambda_i = 1$ for all i. Then $\mathcal{V}_i = \bar{\mathcal{V}}$, where

$$\bar{\mathcal{V}} = \frac{1 + \rho(N-1)}{N}.$$

This is an equilibrium provided all agents have a (weak) incentive to acquire information, i.e. if $\bar{\mathcal{V}} \leq \alpha_i$ for all i, or $\bar{\mathcal{V}} \leq \alpha_N$. From this we obtain the upper bound on ρ in the proposition. □

Proof of Lemma 6.8. From (22), imposing the condition that $\lambda_i = 1$ for $i \neq N$, we have:

$$\mathcal{V}_i = \frac{\left[(1-\rho) + \rho(N-1+\lambda_N)\right]^2}{(1-\rho)(N-1+\lambda_N^2) + \rho(N-1+\lambda_N)^2}, \qquad i \neq N, \tag{25}$$

$$\mathcal{V}_N = \frac{\left[(1-\rho)\lambda_N + \rho(N-1+\lambda_N)\right]^2}{(1-\rho)(N-1+\lambda_N^2) + \rho(N-1+\lambda_N)^2}. \tag{26}$$

We can rewrite $\mathcal{V}_N$ as follows:

$$\mathcal{V}_N = \frac{(\lambda_N - \lambda^*)^2}{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}}. \tag{27}$$

Using (25) and (27), and the fact that $\rho > \rho_{\min}$, we can directly verify both statements of the lemma. □

Proof of Proposition 6.9. For $i \neq N$, we fix $\lambda_i = 1$ and look for a condition on the α_i's such that $\mathcal{V}_i(\lambda_N) < \alpha_i$, irrespective of the value of $\lambda_N \in [0, 1]$. From Lemma 6.8 (part (i)), we see that $\mathcal{V}_i(\lambda_N)$ is maximized at $\lambda_N = 0$. Hence the following condition suffices for the existence of a λ_N-equilibrium (using (25)):

$$\alpha_i > \alpha^* := \mathcal{V}_i(0) = \frac{1 + \rho(N-2)}{N-1}, \qquad i \neq N. \tag{28}$$

Next, we look for further conditions that ensure that $\lambda_N \in (0, 1)$. For this we need to consider the function $\mathcal{V}_N(\lambda_N)$ given by (26), and in particular its shape as described by Lemma 6.8 (part (ii)). By Lemma 6.4, we must have $\mathcal{V}_N = \alpha_N$. We consider separately the cases of $\rho < 0$ and $\rho \geq 0$.

If $\rho < 0$, there are two candidates for an interior equilibrium. These are illustrated in Fig. 1b as points A and B. However, at B the stability condition, $\partial \mathcal{V}_N / \partial \lambda_N \geq 0$, does not hold. This leaves us with the equilibrium corresponding to point A, at which $\lambda_N > \lambda^* > 0$. To ensure that $\lambda_N < 1$, we impose the following condition:

$$\alpha_N < \alpha^{**} := \mathcal{V}_N(1) = \frac{1 + \rho(N-1)}{N}. \tag{29}$$

Note that the foregoing analysis depends only on Lemma 6.8 (part (ii)), not on the particular parameters ($N = 3$, $\rho = -0.2$) chosen for Fig. 1b.

If $\rho \geq 0$, $\mathcal{V}_N$ is increasing in λ_N on $[0, 1]$. Condition (29) guarantees that $\lambda_N < 1$, just as in the case where $\rho < 0$. In order to ensure that $\lambda_N > 0$, we require that

$$\alpha_N > \alpha^{***} := \mathcal{V}_N(0) = \frac{\rho^2(N-1)}{1+\rho(N-2)}. \tag{30}$$

It is straightforward to check that $\alpha^* > \alpha^{**}$ regardless of the sign of ρ, and $\alpha^{**} > \alpha^{***}$ if $\rho \geq 0$.

Provided the conditions for existence are satisfied, the uniqueness results follow from Lemma 6.8 (part (ii)). If $\rho \geq 0$, $\mathcal{V}_N(\lambda_N)$ is strictly increasing on $[0, 1]$. Hence there is a unique λ_N-equilibrium whether or not it is in the interior. If $\rho < 0$, at an interior λ_N-equilibrium we have $\lambda_N > \lambda^*$; it must be unique since $\mathcal{V}_N(\lambda_N)$ is strictly increasing on the interval $[\lambda^*, 1]$.

Finally, we write the conditions (28), (29) and (30) on the α_i's in terms of the corresponding c_i's from (14). The scalars c^* and c^{**} correspond to α^* and α^{**}, respectively. The scalar c^{***} corresponds to α^{***} if $\rho \geq 0$; if $\rho < 0$, c^{***} is just the assumed upper bound on c_N, given by $\bar{c}$. Thus we have $0 < c^* < c^{**} < c^{***} \leq \bar{c}$. □

Proof of Lemma 7.1. At an interior λ_N-equilibrium, writing λ_N as a function of c_N, we have the identity $\mathcal{V}_N(\lambda_N(c_N)) = \alpha_N(c_N)$. Hence

$$\frac{\partial \mathcal{V}_N}{\partial c_N} = \frac{\partial \mathcal{V}_N}{\partial \lambda_N} \cdot \frac{\partial \lambda_N}{\partial c_N} = \frac{\partial \alpha_N}{\partial c_N}.$$

Since $\partial\mathcal{V}_N/\partial\lambda_N > 0$ (Corollary 6.10), and $\partial\alpha_N/\partial c_N < 0$ (from (14)), it follows that $\partial\lambda_N/\partial c_N < 0$, and $\partial\mathcal{V}_N/\partial c_N < 0$. For $i \neq N$, $\partial\mathcal{V}_i/\partial c_N > 0$ since $\partial\mathcal{V}_i/\partial\lambda_N < 0$ (Corollary 6.10). □

Proof of Proposition 7.2. In order to evaluate the utility expressions (15) and (16), we begin by calculating $\sigma^2_{v_i-p}/\sigma^2_\theta$. From Proposition 3.2,

$$p = \frac{k}{r\sigma_\epsilon^2} \cdot \boldsymbol{\lambda}^\top \boldsymbol{\theta}.$$

Therefore,

$$\sigma_p^2 = \sigma_\theta^2 \left(\frac{k}{r\sigma_\epsilon^2}\right)^2 \boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda},$$

$$\sigma_{\theta_i p} = \sigma_\theta^2 \left(\frac{k}{r\sigma_\epsilon^2}\right) \mathbf{R}_i^\top \boldsymbol{\lambda},$$

so that

$$\beta_i = \frac{\sigma_{\theta_i p}}{\sigma_p^2} = r\sigma_\epsilon^2 k^{-1} \cdot \frac{\mathbf{R}_i^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}},$$

$$\sigma^2_{\theta_i|p} = \sigma_\theta^2 - \beta_i \sigma_{\theta_i p} = \sigma_\theta^2 (1 - \mathcal{V}_i).$$

From (19):

$$k_i = \frac{\lambda_i}{r\sigma_\epsilon^2} + \frac{1}{r} \cdot \frac{1-\beta_i}{\sigma^2_{\theta_i|p}}$$

$$= \frac{1}{r\sigma_\epsilon^2}\left[\lambda_i + \frac{1 - r\sigma_\epsilon^2 k^{-1} \cdot \frac{\mathbf{R}_i^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda}}}{\zeta(1-\mathcal{V}_i)}\right],$$

where $\zeta := \sigma_\theta^2/\sigma_\epsilon^2$. Summing over i, and recalling that $k = (\sum_i k_i)^{-1}$, we obtain

$$k^{-1} = \frac{1}{r\sigma_\epsilon^2}\left[\sum_i \lambda_i + \sum_i \frac{1 - r\sigma_\epsilon^2 k^{-1} \cdot \frac{\mathbf{R}_i^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda}}}{\zeta(1-\mathcal{V}_i)}\right].$$

Now we can solve for k:

$$k = r\sigma_\epsilon^2 \cdot \frac{\zeta + \sum_i (1-\mathcal{V}_i)^{-1} \frac{\mathbf{R}_i^\top \boldsymbol{\lambda}}{\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda}}}{\zeta \sum_i \lambda_i + \sum_i (1-\mathcal{V}_i)^{-1}}. \tag{31}$$

We have

$$\begin{aligned}
\sigma_{v_i-p}^2 &= \sigma_\theta^2 + \sigma_p^2 - 2\sigma_{\theta_i p} \\
&= \sigma_\theta^2 + \sigma_\theta^2 \left(\frac{k}{r\sigma_\epsilon^2}\right)^2 \boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda} - 2\sigma_\theta^2 \left(\frac{k}{r\sigma_\epsilon^2}\right) \mathbf{R}_i^\top \boldsymbol{\lambda},
\end{aligned}$$

so that

$$\frac{\sigma_{v_i-p}^2}{\sigma_\theta^2} = 1 + \frac{k}{r\sigma_\epsilon^2}\left[\frac{k}{r\sigma_\epsilon^2}\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda} - 2\mathbf{R}_i^\top \boldsymbol{\lambda}\right].$$

Let

$$\phi := \frac{k}{r\sigma_\epsilon^2}\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda}, \qquad \text{and} \qquad \phi_i := \phi - \mathbf{R}_i^\top \boldsymbol{\lambda}. \tag{32}$$

Then we can write

$$\begin{aligned}
\frac{\sigma_{v_i-p}^2}{\sigma_\theta^2} &= 1 + (\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda})^{-1}\phi(\phi - 2\mathbf{R}_i^\top \boldsymbol{\lambda}) \\
&= 1 + (\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda})^{-1}\left[\phi_i^2 - (\mathbf{R}_i^\top \boldsymbol{\lambda})^2\right] \\
&= 1 - \mathcal{V}_i + (\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda})^{-1}\phi_i^2.
\end{aligned}$$

From (15) and (16),

$$\mathcal{U}_i^I = e^{-2rc_i}\left[\zeta + (1-\mathcal{V}_i)^{-1}\right]\left[1 - \mathcal{V}_i + (\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda})^{-1}\phi_i^2\right], \qquad i \neq N, \tag{33}$$

$$\mathcal{U}_N^U = 1 + (1-\mathcal{V}_N)^{-1}(\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda})^{-1}\phi_N^2. \tag{34}$$

Our calculations so far apply for arbitrary $\boldsymbol{\lambda}$. Now we restrict ourselves to an interior λ_N-equilibrium. At such an equilibrium, $\mathcal{U}_i^I$ and $\mathcal{U}_N^U$ depend on c_N only through λ_N, and hence it suffices to sign their derivatives with respect to λ_N, and use the fact that $\partial\lambda_N/\partial c_N < 0$ (Lemma 7.1). We do not need to consider $\mathcal{U}_N^I$ separately since, at an interior λ_N-equilibrium, it is equal to $\mathcal{U}_N^U$, and these two utilities remain equal as we perturb c_N.

Setting $\lambda_i = 1$, for $i \neq N$, we get

$$\begin{aligned}
\mathbf{R}_i^\top \boldsymbol{\lambda} &= \mathbf{R}_1^\top \boldsymbol{\lambda} = 1 + \rho(N-2) + \rho\lambda_N, \qquad i \neq N, \\
\mathbf{R}_N^\top \boldsymbol{\lambda} &= \rho(N-1) + \lambda_N, \\
\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda} &= (N-1)\mathbf{R}_1^\top \boldsymbol{\lambda} + \lambda_N \mathbf{R}_N^\top \boldsymbol{\lambda} \\
&= (N-1)[1 + \rho(N-2)] + 2(N-1)\rho\lambda_N + \lambda_N^2.
\end{aligned}$$

Notice that

$$\mathbf{R}_1^\top \boldsymbol{\lambda} - \mathbf{R}_N^\top \boldsymbol{\lambda} = (1-\rho)(1-\lambda_N). \tag{35}$$

For $i \neq N$, we have $\mathcal{V}_i = \mathcal{V}_1$. From (9),

$$(1-\mathcal{V}_1)^{-1} = \frac{1}{1-\rho} \cdot \frac{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}}{D}, \tag{36}$$

where

$$D := (N-2)[1+\rho(N-2)] + 2(N-2)\rho\lambda_N + (1+\rho)\lambda_N^2,$$

and

$$(1-\mathcal{V}_N)^{-1} = \frac{1}{1-\rho} \cdot \frac{\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda}}{(N-1)[1+\rho(N-1)]}. \tag{37}$$

Note that $(1-\mathcal{V}_i)^{-1} > 0$ for all i. In particular, $D > 0$ and $1+\rho(N-1) > 0$ (the latter is just a restatement of the condition that $\rho > \rho_{\min}$). From (31) and (32),

$$\phi = \frac{\zeta \boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda} + (N-1)(1-\mathcal{V}_1)^{-1} \mathbf{R}_1^\top \boldsymbol{\lambda} + (1-\mathcal{V}_N)^{-1} \mathbf{R}_N^\top \boldsymbol{\lambda}}{\zeta(N-1+\lambda_N) + (N-1)(1-\mathcal{V}_1)^{-1} + (1-\mathcal{V}_N)^{-1}}.$$

For $i \neq N$, $\phi_i = \phi_1$. Using (32) and (35),

$$\begin{aligned}
\phi_1 &= \frac{\zeta\left[\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda} - (N-1+\lambda_N)\mathbf{R}_1^\top \boldsymbol{\lambda}\right] + (1-\mathcal{V}_N)^{-1}\left[\mathbf{R}_N^\top \boldsymbol{\lambda} - \mathbf{R}_1^\top \boldsymbol{\lambda}\right]}{\zeta(N-1+\lambda_N) + (N-1)(1-\mathcal{V}_1)^{-1} + (1-\mathcal{V}_N)^{-1}} \\
&= -\frac{(1-\rho)(1-\lambda_N)\left[\zeta\lambda_N + (1-\mathcal{V}_N)^{-1}\right]}{\zeta(N-1+\lambda_N) + (N-1)(1-\mathcal{V}_1)^{-1} + (1-\mathcal{V}_N)^{-1}} \\
&= -\frac{(1-\rho)(1-\lambda_N)}{1+(N-1)\cdot \frac{\zeta+(1-\mathcal{V}_1)^{-1}}{\zeta\lambda_N+(1-\mathcal{V}_N)^{-1}}},
\end{aligned} \tag{38}$$

and

$$\begin{aligned}
\phi_N &= \frac{\zeta\left[\boldsymbol{\lambda}^\top \mathbf{R} \boldsymbol{\lambda} - (N-1+\lambda_N)\mathbf{R}_N^\top \boldsymbol{\lambda}\right] + (N-1)(1-\mathcal{V}_1)^{-1}\left[\mathbf{R}_1^\top \boldsymbol{\lambda} - \mathbf{R}_N^\top \boldsymbol{\lambda}\right]}{\zeta(N-1+\lambda_N) + (N-1)(1-\mathcal{V}_1)^{-1} + (1-\mathcal{V}_N)^{-1}} \\
&= \frac{(N-1)(1-\rho)(1-\lambda_N)\left[\zeta + (1-\mathcal{V}_1)^{-1}\right]}{\zeta(N-1+\lambda_N) + (N-1)(1-\mathcal{V}_1)^{-1} + (1-\mathcal{V}_N)^{-1}} \\
&= \frac{(N-1)(1-\rho)(1-\lambda_N)}{N-1+\frac{\zeta\lambda_N+(1-\mathcal{V}_N)^{-1}}{\zeta+(1-\mathcal{V}_1)^{-1}}}.
\end{aligned}$$

From (34) and (37), $\partial\mathcal{U}_N^U/\partial\lambda_N \propto \phi_N \cdot \partial\phi_N/\partial\lambda_N$. Clearly ϕ_N is positive. Since $\partial\mathcal{V}_N/\partial\lambda_N > 0$ and $\partial\mathcal{V}_1/\partial\lambda_N < 0$ (Lemma 7.1), we see that $\partial\phi_N/\partial\lambda_N < 0$. Hence, $\partial\mathcal{U}_N^U/\partial\lambda_N < 0$. This establishes statement (i) of the proposition.

In order to prove statements (ii) and (iii), we show that $\lim_{\zeta\to 0} \partial\mathcal{U}_1^I/\partial\lambda_N < 0$, and $\lim_{\zeta\to\infty} \partial\mathcal{U}_1^I/\partial\lambda_N > 0$, ignoring the dependence of the equilibrium value of λ_N on ζ (thus showing that the inequalities hold for arbitrary $\lambda_N \in (0,1)$). From (33), for $i \neq N$,

$$\begin{aligned}
e^{2rc_i}\mathcal{U}_i^I &= \Big[\zeta + (1-\mathcal{V}_1)^{-1}\Big]\Big[1-\mathcal{V}_1 + (\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda})^{-1}\phi_1^2\Big] \\
&= \zeta\Big[1-\mathcal{V}_1 + (\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda})^{-1}\phi_1^2\Big] + 1 + (1-\mathcal{V}_1)^{-1}(\boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda})^{-1}\phi_1^2 \\
&= 1 + \zeta L^{-1}\Big[(1-\rho)D + \phi_1^2\Big] + (1-\rho)^{-1}D^{-1}\phi_1^2,
\end{aligned}$$

where $L := \boldsymbol{\lambda}^\top \mathbf{R}\boldsymbol{\lambda}$. Therefore (primes denote derivatives with respect to λ_N):

$$\begin{aligned}
e^{2rc_i}\cdot\frac{\partial \mathcal{U}_i^I}{\partial\lambda_N} &= \zeta L^{-2}\big[(1-\rho)(LD'-L'D) + (2L\phi_1' - L'\phi_1)\phi_1\big] \\
&\quad + (1-\rho)D^{-2}\phi_1\big[2D\phi_1' - D'\phi_1\big].
\end{aligned} \tag{39}$$

Note that L and D do not depend on ζ. From (38), $\phi_1 < 0$ and

$$\lim_{\zeta\to 0}\phi_1 = -\frac{(1-\rho)(1-\lambda_N)D}{D+(N-1)^2[1+\rho(N-1)]}, \tag{40}$$

$$\lim_{\zeta\to 0}\phi_1' = (1-\rho)\cdot\frac{D^2 + (N-1)^2[1+\rho(N-1)][D-(1-\lambda_N)D']}{\Big[D+(N-1)^2[1+\rho(N-1)]\Big]^2}, \tag{41}$$

$$\lim_{\zeta\to \infty}\phi_1 = -\frac{(1-\rho)(1-\lambda_N)\lambda_N}{\lambda_N+N-1}, \tag{42}$$

$$\lim_{\zeta\to \infty}\phi_1' = (1-\rho)\cdot\frac{\lambda_N^2+(N-1)(2\lambda_N-1)}{(\lambda_N+N-1)^2}. \tag{43}$$

From (39),

$$\lim_{\zeta\to 0}\frac{\partial \mathcal{U}_i^I}{\partial\lambda_N} \propto -\lim_{\zeta\to 0} X, \qquad \text{and} \qquad \lim_{\zeta\to \infty}\frac{\partial \mathcal{U}_i^I}{\partial\lambda_N} \propto \lim_{\zeta\to \infty} Y,$$

where

$$X = 2D\phi_1' - D'\phi_1, \tag{44}$$

$$Y = (1-\rho)(LD'-L'D) + (2L\phi_1' - L'\phi_1)\phi_1. \tag{45}$$

From (40), (41) and (44),

$$\lim_{\zeta\to 0} X = 2D\Big[D+(1-\lambda_N)D'/2\Big] + 2(N-1)^2[1+\rho(N-1)]\Big[D-(1-\lambda_N)D'/2\Big].$$

We now show that the two terms in large square brackets are positive (and hence $\lim_{\zeta\to 0} X > 0$). We have

$$D + (1-\lambda_N)D'/2 = (N-2)[1+\rho(N-1+\lambda_N)] + (1+\rho)\lambda_N.$$

This expression is increasing in ρ. It is easy to check that it is equal to zero when evaluated at $\rho = \rho_{\min} = -(1-N)^{-1}$. Hence it must be positive. Moving on to the second term, we have

$$\begin{aligned}
D - (1-\lambda_N)D'/2 &= (N-2)[1+\rho(N-3)] + 3(N-3)\rho\lambda_N \\
&\quad + 2\rho\lambda_N(1+\lambda_N) + \lambda_N(2\lambda_N-1).
\end{aligned}$$

This expression is increasing in ρ (this is the first time in the proof that we use the condition that $N \geq 3$), hence greater than its value at $\rho_{\min}$:

$$D-(1-\lambda_N)D'/2 > \frac{2(N-2)-3(N-3)\lambda_N-2\lambda_N(1+\lambda_N)}{N-1}+\lambda_N(2\lambda_N-1)$$
$$=\frac{2(N-2)}{N-1}(1-\lambda_N)^2,$$

which is positive.

It remains to establish that $\lim_{\zeta\to\infty} Y > 0$. This involves some tedious but straightforward calculations, of which we provide only the salient details. From (42), (43) and (45), we find that $\lim_{\zeta\to\infty} Y \propto f(\rho)$, where

$$\begin{aligned} f(\rho) := {} & [1+\rho(N-1)](\lambda_N+N-1)^3\mathbf{R}_1^\top\boldsymbol{\lambda} \\ & + (1-\rho)(1-\lambda_N)^2(N-1)(\lambda_N+N-1)\mathbf{R}_1^\top\boldsymbol{\lambda} \\ & - (1-\rho)(1-\lambda_N)\lambda_N N\boldsymbol{\lambda}^\top\mathbf{R}\boldsymbol{\lambda}. \end{aligned}$$

We consider f as a function defined on $\mathbb{R}$, with exogenously specified $\lambda_N \in (0,1)$. We show that $f'' > 0$, $f'(\rho_{\min}) > 0$, and $f(\rho_{\min}) > 0$. It follows that $f > 0$ over the relevant interval $(\rho_{\min}, 1)$. Hence $\lim_{\zeta\to\infty} Y > 0$. □

Proof of Proposition 8.1. Proposition 6.9 assures us that a λ_N-equilibrium exists: we can fix $\lambda_i = 1$, for $i \neq N$, and focus solely on the determination of λ_N. From Lemma 6.8 (part (ii)), $\mathcal{V}_N(\lambda_N)$ is minimized at λ^*; $\lambda^* \in (0,1)$ since $\rho \in (\rho_{\min}, 0)$. There is an equilibrium with $\lambda_N \in (\lambda^*, 1]$; indeed, there is a unique equilibrium in this interval (see Fig. 1b for the case of $N = 3$). There is a second equilibrium, with $\lambda_N = 0$, if

$$\alpha_N < \alpha_N^* := \mathcal{V}_N(0) = \frac{\rho^2(N-1)}{1+\rho(N-2)}.$$

The cutoff value c_N^* corresponds to α_N^*, using (14).[20]

At the equilibrium with positive λ_N, we have $\mathcal{V}_N \le \alpha_N < \mathcal{V}_N(0)$, i.e. price informativeness is strictly lower for type N. From Lemma 6.8 (part (i)), this is the case for the other types as well. □

Proof of Proposition 8.2. We consider a situation in which m types are informed and the remaining $(N-m)$ types are uninformed. By symmetry, all informed types have the same price informativeness, which we denote by $\mathcal{V}_{(1)}$. All uninformed types also have the same price informativeness, $\mathcal{V}_{(0)}$. From (22):

$$\mathcal{V}_{(1)} = \frac{1+\rho(m-1)}{m}, \qquad \text{and} \qquad \mathcal{V}_{(0)} = \frac{\rho^2 m}{1+\rho(m-1)}. \tag{46}$$

This is an equilibrium provided $\mathcal{V}_{(0)} > \alpha_1 \ge \alpha_2 \ge \ldots \ge \alpha_N > \mathcal{V}_{(1)}$, by Lemma 6.4. Thus we require $\mathcal{V}_{(0)} > \mathcal{V}_{(1)}$. Using (46), and noting that $1+\rho(m-1) > 0$ due to the fact that $\rho > \rho_{\min}$, we get the following condition

$$\rho_{\min} = -\frac{1}{N-1} < \rho < -\frac{1}{2m-1}.$$

[20] This value of c_N^* is the same as that of c^{***} in Proposition 6.9, for the case where $\rho \ge 0$. In the present result, however, we have $\rho < 0$.

In particular, we require that m be an integer strictly greater than $N/2$, or $m \geq (N+1)/2$. The condition $\rho < -N^{-1}$ ensures that $\rho < -(2m-1)^{-1}$ for all such values of m. The open subset of cost parameters $(c_i)_{i=1}^N$ for which the proposition holds corresponds to the possible choices of $(\alpha_i)_{i=1}^N$, with $\alpha_i \in (\mathcal{V}_{(1)}, \mathcal{V}_{(0)})$. The same choice must apply for all $m \geq (N+1)/2$. Since $\mathcal{V}_{(1)}$ is decreasing in m and $\mathcal{V}_{(0)}$ is increasing in m, the appropriate interval is the one for $m = (N+1)/2$.

Now let us compare an equilibrium with $m = N$ to one in which $m < N$. Since $\mathcal{V}_{(1)}$ is strictly decreasing in m, price informativeness for the types who remain informed is higher for $m < N$ than at $m = N$. For any type i that switches from being informed to being uninformed, price informativeness must go up, since $\mathcal{V}_i < \alpha_i$ in the first case and $\mathcal{V}_i > \alpha_i$ in the second. □

Appendix B. Example 4.4

Here we provide a complete analysis of Example 4.4. We will need the following result, which is a special case of Theorem 3.2a.1 in Mathai and Provost (1992):

Lemma B.1. *Suppose* $\mathbf{A}$ *is a symmetric* $n \times n$ *matrix,* $\mathbf{b}$ *is an* n*-vector,* c *is a scalar, and* $\mathbf{x}$ *is an* n*-dimensional normal random variable:* $\mathbf{x} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$*,* $\boldsymbol{\Sigma}$ *positive definite. Then* $E[\exp(\mathbf{x}^\top \mathbf{A}\mathbf{x} + \mathbf{b}^\top \mathbf{x} + c)]$ *is well-defined if and only if* $|\mathbf{I} - 2\mathbf{A}\boldsymbol{\Sigma}| > 0$*, and is given by*

$$\left|\mathbf{I} - 2\mathbf{A}\boldsymbol{\Sigma}\right|^{-\frac{1}{2}} \exp\left[\frac{1}{2}(\boldsymbol{\mu} + \boldsymbol{\Sigma}\mathbf{b})^\top (\mathbf{I} - 2\mathbf{A}\boldsymbol{\Sigma})^{-1} \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} + \boldsymbol{\Sigma}\mathbf{b}) - \frac{1}{2}\boldsymbol{\mu}^\top \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu} + c\right].$$

We assume that $1 - r_i^2 \sigma_{e_i}^2 \sigma_{y_i}^2 > 0$, which ensures that $|\mathbf{I} - 2\mathbf{A}\boldsymbol{\Sigma}| > 0$ in all the cases where we apply Lemma B.1 below.

Since W_{in} is normally distributed conditional on (y_i, p), we have

$$E\left[-\exp(-r_i W_{in}) | y_i, p\right] = -\exp(-r_i \mathcal{E}_{in}), \tag{47}$$

where

$$\begin{aligned}\mathcal{E}_{in} &:= E(W_{in}|y_i, p) - \frac{r_i}{2}\mathrm{Var}(W_{in}|y_i, p) \\ &= -\frac{r_i}{2}\sigma_v^2 q_{in}^2 - (p + r_i \sigma_{ve_i} y_i) q_{in} - \frac{r_i}{2}\sigma_{e_i}^2 y_i^2.\end{aligned} \tag{48}$$

If agent in is informed he chooses q_{in} to maximize (47), or equivalently (48). We obtain:

$$q_{in}^I = \frac{-r_i \sigma_{ve_i} y_i - p}{r_i \sigma_v^2}. \tag{49}$$

If agent in is uninformed, he maximizes

$$\begin{aligned}E[-\exp(-r_i W_{in})|p] &= E\Big[E[-\exp(-r_i W_{in})|y_i, p]\Big|p\Big] \\ &= -E\left[\exp\left(-r_i \mathcal{E}_{in}\right)|p\right].\end{aligned}$$

From (48).

$$-r_i \mathcal{E}_{in} = \frac{r_i^2}{2}\sigma_{e_i}^2 y_i^2 + r_i^2 \sigma_{ve_i} q_{in} y_i + \frac{r_i^2}{2}\sigma_v^2 q_{in}^2 + r_i p q_{in},$$

which is of the form $Ay_i^2 + by_i + c$, where

$$A = \frac{r_i^2}{2}\sigma_{e_i}^2, \qquad b = r_i^2 \sigma_{ve_i} q_{in}, \qquad c = \frac{r_i^2}{2}\sigma_v^2 q_{in}^2 + r_i p q_{in}.$$

Hence we can apply Lemma B.1 to the agent's objective function, with

$$\mu = E(y_i|p) = \frac{\sigma_{y_i p}}{\sigma_p^2} p, \qquad \Sigma = \sigma_{y_i|p}^2 = \sigma_{y_i}^2 - \frac{\sigma_{y_i p}^2}{\sigma_p^2},$$

to obtain

$$\begin{aligned} E\big[-\exp(-r_i W_{in})|p\big] &= -(1-2A\Sigma)^{-\frac{1}{2}} \\ &\quad \cdot \exp\Big[(1-2A\Sigma)^{-1}\Big(A\mu^2 + b\mu + \frac{1}{2}\Sigma b^2 + (1-2A\Sigma)c\Big)\Big] \\ &= -\Big(1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2\Big)^{-\frac{1}{2}} \exp\Big[\Big(1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2\Big)^{-1} H\Big], \end{aligned} \tag{50}$$

where

$$\begin{aligned} H &:= \frac{r_i^2\sigma_{e_i}^2\sigma_{y_i p}^2}{2\sigma_p^4}p^2 + \frac{r_i^2\sigma_{ve_i}\sigma_{y_i p}}{\sigma_p^2}pq_{in} + \frac{r_i^4}{2}\sigma_{ve_i}^2\sigma_{y_i|p}^2 q_{in}^2 \\ &\quad + (1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\Big[\frac{r_i^2}{2}\sigma_v^2 q_{in}^2 + r_i p q_{in}\Big] \\ &= \frac{r_i^2}{2}\Big[r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2 + (1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2\Big]q_{in}^2 + r_i\Big[\frac{r_i\sigma_{ve_i}\sigma_{y_i p}}{\sigma_p^2} + (1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\Big]pq_{in} \\ &\quad + \frac{r_i^2\sigma_{e_i}^2\sigma_{y_i p}^2}{2\sigma_p^4}p^2. \end{aligned} \tag{51}$$

The agent's portfolio choice problem boils down to minimizing H. The first-order condition is

$$r_i^2\Big[r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2 + (1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2\Big]q_{in} + r_i\Big[\frac{r_i\sigma_{ve_i}\sigma_{y_i p}}{\sigma_p^2} + (1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\Big]p = 0, \tag{52}$$

which gives us

$$q_{in}^U = -\frac{\frac{r_i\sigma_{ve_i}\sigma_{y_i p}}{\sigma_p^2} + (1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)}{r_i\Big[r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2 + (1 - r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2\Big]}p. \tag{53}$$

Using the market-clearing condition $\sum_i[\lambda_i q_{in}^I + (1-\lambda_i)q_{in}^U] = 0$, we see that the price function takes the form $p = k\sum_i \lambda_i\theta_i$. The constant k is well-defined and nonzero, from the same argument as in the proof of Proposition 3.2.

We now calculate ex ante expected utilities. Consider first the informed agent. Using (48) and (49),

$$\begin{aligned} \mathcal{E}_{in} &= \frac{r_i}{2}\big(\sigma_v^2 q_{in}^2 - \sigma_{e_i}^2 y_i^2\big) \\ &= \frac{1}{2r_i\sigma_v^2}\big[-r_i^2(\sigma_v^2\sigma_{e_i}^2 - \sigma_{ve_i}^2)y_i^2 + p^2 + 2r_i\sigma_{ve_i}y_i p\big]. \end{aligned}$$

Hence $-r_i\mathcal{E}_{in}$ is of the form $\mathbf{x}^\top\mathbf{A}\mathbf{x}$, where $\mathbf{x}=(y_i\ \ p)$, and

$$\mathbf{A}=\frac{1}{2\sigma_v^2}\begin{bmatrix} r_i^2(\sigma_v^2\sigma_{e_i}^2-\sigma_{ve_i}^2) & -r_i\sigma_{ve_i}\\ -r_i\sigma_{ve_i} & -1\end{bmatrix}.$$

Using Lemma B.1 (and noting that $E(\mathbf{x})=0$),

$$\begin{aligned}-e^{r_ic_i}E[\exp(-r_iW_{in})]&=-e^{r_ic_i}E[\exp(-r_i\mathcal{E}_{in})]\\&=-e^{r_ic_i}|\mathbf{I}-2\mathbf{A}\boldsymbol{\Sigma}|^{-\frac{1}{2}}.\end{aligned}$$

Calculating the determinant, we get

$$|\mathbf{I}-2\mathbf{A}\boldsymbol{\Sigma}|=(\sigma_v^2)^{-1}\Big[r_i^2\sigma_{ve_i}^2\sigma_{y_i}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i}^2)\sigma_v^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_p^2+2r_i\sigma_{ve_i}\sigma_{y_ip}\Big].$$

Using the definition of $\mathcal{U}_{in}$ given by (11), we have

$$\begin{aligned}\mathcal{U}_{in}^I&=e^{-2r_ic_i}|\mathbf{I}-2\mathbf{A}\boldsymbol{\Sigma}|\\&=\frac{e^{-2r_ic_i}}{\sigma_v^2}\Big[r_i^2\sigma_{ve_i}^2\sigma_{y_i}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i}^2)\sigma_v^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_p^2+2r_i\sigma_{ve_i}\sigma_{y_ip}\Big].\end{aligned}\tag{54}$$

Next, consider the uninformed agent. From (51)–(53),

$$\begin{aligned}H&=\frac{r_i^2\sigma_{e_i}^2\sigma_{y_ip}^2}{2\sigma_p^4}p^2-\frac{r_i^2}{2}\Big[r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2\Big]q_{in}^2\\&=\frac{r_i^2\sigma_{e_i}^2\sigma_{y_ip}^2}{2\sigma_p^4}p^2-\frac{\Big[\frac{r_i\sigma_{ve_i}\sigma_{y_ip}}{\sigma_p^2}+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\Big]^2}{2\Big[r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2\Big]}p^2\\&=-\frac{1}{2\sigma_p^2}hp^2,\end{aligned}\tag{55}$$

where

$$\begin{aligned}h&:=-\frac{r_i^2\sigma_{e_i}^2\sigma_{y_ip}^2}{\sigma_p^2}+\frac{\sigma_p^2\Big[\frac{r_i\sigma_{ve_i}\sigma_{y_ip}}{\sigma_p^2}+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\Big]^2}{r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2}\\&=\frac{1}{r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2}\Bigg[-\frac{r_i^2\sigma_{e_i}^2\sigma_{y_ip}^2}{\sigma_p^2}\big[r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2\big]\\&\qquad+\frac{r_i^2\sigma_{ve_i}^2\sigma_{y_ip}^2}{\sigma_p^2}+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)^2\sigma_p^2+2r_i\sigma_{ve_i}\sigma_{y_ip}(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\Bigg]\\&=\frac{1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2}{r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2}\\&\qquad\cdot\Bigg[\frac{r_i^2\sigma_{ve_i}^2\sigma_{y_ip}^2}{\sigma_p^2}-\frac{r_i^2\sigma_{e_i}^2\sigma_{y_ip}^2}{\sigma_p^2}\sigma_v^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_p^2+2r_i\sigma_{ve_i}\sigma_{y_ip}\Bigg].\end{aligned}$$

Using Lemma B.1 to evaluate the expected value of (50), and substituting the expression for H given by (55), we get

$$
\begin{aligned}
E\big[-\exp(-r_i W_{in}\big] &= E\big(E[-\exp(-r_i W_{in})|p]\big) \\
&= -(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)^{-\frac{1}{2}} E\left(\exp\left[-(2\sigma_p^2)^{-1}(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)^{-1}hp^2\right]\right) \\
&= -(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)^{-\frac{1}{2}}\left[1+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)^{-1}h\right]^{-\frac{1}{2}} \\
&= -\left[(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)+h\right]^{-\frac{1}{2}}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\mathcal{U}_{in}^U &= (1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)+h \\
&= \frac{1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2}{r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2}\Bigg[r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2 \\
&\qquad +\frac{r_i^2\sigma_{ve_i}^2\sigma_{y_i p}^2}{\sigma_p^2}-\frac{r_i^2\sigma_{e_i}^2\sigma_{y_i p}^2}{\sigma_p^2}\sigma_v^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_p^2+2r_i\sigma_{ve_i}\sigma_{y_i p}\Bigg] \\
&= \frac{1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2}{r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_v^2} \\
&\quad \cdot\left[r_i^2\sigma_{ve_i}^2\sigma_{y_i}^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i}^2)\sigma_v^2+(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)\sigma_p^2+2r_i\sigma_{ve_i}\sigma_{y_i p}\right].
\end{aligned}
\tag{56}
$$

From (54) and (56), we can calculate the utility ratio:

$$
\begin{aligned}
\frac{\mathcal{U}_i^I}{\mathcal{U}_i^U} &= e^{-2r_ic_i}\left[1+\frac{r_i^2\sigma_{ve_i}^2\sigma_{y_i|p}^2}{\sigma_v^2(1-r_i^2\sigma_{e_i}^2\sigma_{y_i|p}^2)}\right] \\
&= e^{-2r_ic_i}\left[1+\frac{1}{\sigma_v^2(r_i^2\sigma_{\theta_i|p}^2)^{-1}-(\rho_{ve_i}^2)^{-1}}\right] \\
&= e^{-2r_ic_i}\left[1+\frac{1}{\sigma_v^2\big[r_i^2\sigma_{\theta_i}^2(1-\mathcal{V}_i)\big]^{-1}-(\rho_{ve_i}^2)^{-1}}\right],
\end{aligned}
\tag{57}
$$

where $\rho_{ve_i} := \mathrm{corr}(v, e_i)$.

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