Robot Arithmetic: New Technology And Wages

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Abstract:

Existing economic models show how new technology can cause large changes in relative wages and inequality. But there are also claims, based largely on verbal expositions, that new technology can harm workers on average or even all workers. This paper shows – under plausible assumptions - that new technology is unlikely to cause wages for all workers to fall and will cause average wages to rise if the prices of investment goods fall relative to consumer goods (a condition supported by the data). We outline how results may change with different assumptions.

Introduction

There are widespread concerns among commentators from many different backgrounds (including science, philosophy, business as well as economics: see, for example, Brynjolfsson and McAfee, 2014, Ford, 2015, Frey and Osborne, 2017, Susskind and Susskind, 2015, Bostrom, 2014, White House, 2016, Peter Stone et al., 2016)) about the current and likely future impact of new technology (mostly robots and artificial intelligence) on the demand for labor. There is also a growing empirical literature on the impact of new technology and robots on the labor market (Autor and Dorn, 2013, Goos et al., 2014, Graetz and Michaels, 2015, Acemoglu and Restrepo, 2017).

Fears about the impact of new technology on workers are not new, although the technology feared has varied over time (Bowen, 1966, Autor, 2015,). Past fears proved unfounded, but it is argued (not for the first time) that 'this time is different', and that the impact of past technologies can be no guide to the impact of future technologies. But many of the current analyses of the likely impact of new technology on workers rely on verbal or partial equilibrium analysis without a formal model of the economy as a whole. The risk is that the conclusions are not based on underlying consistency of reasoning. This paper is about the conditions in which new technology can or cannot harm workers, and is motivated by the belief that the existing literature largely consists of a set of special models while this paper aims for results that are as general as possible.

The existing economics literature provides examples of models in which new technology causes large changes in relative wages and increases in inequality (e.g. Acemoglu and Autor, 2011). And if capital is regarded as fixed then it is simple to write down a constant returns to scale production function in which wages for all workers fall. If the production function is $F\left(L,K,\theta\right)$ where θ is new technology, and $\frac{\partial F}{\partial \theta}>0$ so new technology raises output but $\frac{\partial^2 F}{\partial L \partial \theta}<0$ then the marginal product of labour and the wage falls with new technology.

But it has proved much harder to write down models with endogenous capital in which wages fall. For example Acemoglu and Restrepo (2017a) identify what they call the 'productivity effect' that causes wages to rise with new technology in their model once capital is endogenous. But their model has one type of labour and one good (which can be a capital or consumption good) and a very particular form of new technology, leaving open the question of whether there are alternative models in which wages can fall. The aim of this paper is to present simple but general models to address this issue.

This paper considers models in which there are an arbitrary number of types of labor, and arbitrary number of goods that may be used for consumption or capital or both, and arbitrary forms of new technology, seeking necessary and sufficient conditions for when wages may rise or fall.

In our benchmark model we assume that labor is the only fixed factor, that the interest rate is unaffected by new technology, that there are constant returns to scale and perfect competition, and that we are comparing economies with different levels of technology in steady-state (an approach also taken in Acemoglu and Autor, 2011; Acemoglu and Restrepo, 2018).

In the benchmark model our first result is that at least one type of worker must be made better-off by new technology. A corollary is that models with only one type of labor must have the feature that real wages rise with new technology. This leaves open the possibility that workers as a group lose.

Our second main result is that new technology must cause the average wage of workers to rise if the price of capital goods falls relative to consumption goods, a condition that seems satisfied in the data. This leaves open the possibility that there might be very large rises in inequality but we also show that if the supply of labor of different types is perfectly elastic, then all workers must gain.

Taken together these results are more optimistic about the impact of new technology on workers than many current discussions. But they are results in models and are based on assumptions that may not be satisfied – the paper concludes with a discussion of how alternative models might lead to different conclusions.

Benchmark Model and Main Results

We denote the supply of the many types of labour in the economy by a (row) vector L. For the moment we assume the supply of each type of labor is fixed but we return to this below. We denote wages by a (column) vector w and assume all workers supply labour inelastically and will work for any non-zero wage.

We assume there are many types of consumption, intermediate and capital goods. Denote the price vector of consumer and/or intermediate goods as p and the rental price of capital goods as p^K.

Assumption CRS: The production function has constant returns to scale in every sector.

Denote the unit cost function for consumption/intermediate goods by the vector $c\left(w, p, p^K, \theta\right)$ and for investment goods by $c^i\left(w, p, p^K, \theta\right)$ where θ is the level of technology. This set-up allows for the possibility that some goods may be impossible to produce (have an infinite cost) at some technologies

Improvements in technology are captured by assuming that the cost function must be non-increasing in θ for all goods (both consumption and investment) at constant wages and prices:

$$c_\theta \equiv \frac{\partial c\left(w, p, p^K, \theta\right)}{\partial \theta} \leq 0, \qquad c^i_\theta \equiv \frac{\partial c^i\left(w, p, p^K, \theta\right)}{\partial \theta} \leq 0 \tag{1}$$

with a strict inequality for at least one good. For the analysis to be interesting we also need to assume that, for given wages, improvements in technology reduce the price of at least one good that is demanded by consumers. This does not require technology to affect the cost function for these goods directly, it might be an indirect effect through an impact on the costs of producing intermediate or investment goods that are used in that sector. This rules out the possibility that new technology only affects the production of a set of goods that have no link, direct or indirect, to consumption goods. Such a set of goods will have zero production in equilibrium as they serve no purpose.

We need to make some assumptions about how prices are determined.

Assumption RK: There are financial assets paying an interest rate r, unaffected by changes in technology.

The constant interest rate assumption implies that the supply of capital is perfectly elastic in the long-run: this could be derived from an underlying model of consumer choice when, in a static steady-state, the interest rate would be the rate of time preference, but we do not go into the details here. We also assume that capital goods depreciate at a constant rate δ. Then, a conventional no-arbitrage argument implies that the relationship between the cost of capital and the price of investment goods is $p^K = \left(r+\delta\right) p^i$, where p^i is the price of investment goods.

About the nature of markets we assume:

Assumption PC: Output and input markets are perfectly competitive.

PC implies that prices equal unit costs so that we have:

$$p = c\left(w, p, \left(r+\delta\right) p^{i}, \theta\right) \tag{2}$$

$$p^{i} = c^{i}\left(w, p, \left(r+\delta\right) p^{i}, \theta\right) \tag{3}$$

Finally, we make the following assumption, largely for tractability

Assumption HOM: Consumers' preferences are homothetic so there is a unique consumer price index, denoted by $e\left(p\right)$.

Results

In what follows we simply compare steady-states with constant levels of technology, asking whether wages are higher or lower in economies with more advanced technology. This approach allows us to be as general as possible about the way in which technology affects production opportunities. But there is a cost – we do not model the transition from one steady-state to another, nor do we model an economy in which technology is changing over time. Economic models of growth typically make quite restrictive assumptions about how new technology affects productive opportunities, often to have a model that is analytically tractable and displays balanced growth (Uzawa, 1961, Acemoglu, 2008, Grossman et al., 2017).

The first result of this paper is the following (proved in the Appendix).

Result 1: Improvements in technology must raise the real wage of at least one type of worker.

The intuition for this is that, conditional on prices and wages, costs are weakly decreasing in technology. This means that, conditional on wages, all prices must be weakly decreasing in technology. This means that no price increase can be larger than the largest wage increase. For this group of workers, real wages must therefore be rising.

A corollary of Result 1 is that in models with only one type of labor, new technology of any form must raise real wages for all workers.

Result 1 says that at least one group of workers must gain from new technology but leaves open the possibility that almost all workers lose. However the next result provides a sufficient condition for the average wage to rise.

Result 2: Improvements in technology raise the average real wage of workers if the price index of investment goods does not increase relative to the price index of consumption goods.

This result is the main result of the paper so the proof is in the main text. Given assumption HOM the expenditure function for workers can be written as $e(p)u^w$ where $e(p)$ is the price index and u^w is the (column) vector of utilities for each type of worker. In equilibrium total expenditure must equal total income for each type of worker which gives us:

$$w = e(p)u^w \tag{4}$$

The total utility of workers will be Lu^w, which can be interpreted as (total) real wages. Using (4) and taking logs we have that:

$$\log Lu^w = \log Lw - \log e(p) \tag{5}$$

Now consider a change $d\theta$ in technology. From (5) we have that:

$$\begin{aligned}\frac{Ldu^w}{Lu^w} &= \frac{Ldw}{Lw} - \frac{e_p(p)dp}{e(p)} = \frac{Ldw}{Lw} - \frac{X^w dp}{Lu^w e(p)} \\ &= \frac{1}{Lw}\left[Ldw - X^w dp\right]\end{aligned} \tag{6}$$

Where X^w is the vector of consumption demands by workers, and we have used Shephard's Lemma to substitute out for e_p.

Given the assumption that existing capital all depreciates at a rate δ, to maintain capital stocks of K in a steady-state requires investment of $I = \delta K$. Capital-owners have total income per period of $(r+\delta)p^i K$ but have to spend $\delta p^i K$ on maintaining their capital holdings so have total consumption expenditure of $rp^i K$.

Since the prices of consumption goods equal their unit costs, the change in prices from a change $d\theta$ in technology can be written as:

$$dp = c_w dw + c_p dp + c_k dp^k + c_\theta d\theta$$
$$= c_w dw + c_p dp + (r+\delta) c_k dp^i + c_\theta d\theta \tag{7}$$

And the change in the price of investment goods can be written as:

$$dp^i = c_w^i dw + c_p^i dp + c_k^i dp^k + c_\theta^i d\theta$$
$$= c_w^i dw + c_p^i dp + (r+\delta) c_k^i dp^i + c_\theta^i d\theta \tag{8}$$

From Shephard's Lemma, total demands for intermediate goods, X^d can be written as:

$$X^d = Xc_p + Ic_p^i = Xc_p + \delta K c_p^i \tag{9}$$

There is also an equivalent equation for the demand for capital goods:

$$K^d = Xc_k + Ic_k^i = Xc_k + \delta K c_k^i \tag{10}$$

And total demands for labor can be written as:

$$L^d = Xc_w + Ic_w^i = Xc_w + \delta K c_w^i \tag{11}$$

In equilibrium, we must have the complementary slackness condition $\left(L - L^d\right) w = 0$. This implies that if wages for a particular type of labor is positive then demand for that type of labor must equal supply. But if the wages for a particular type of labor are zero then it is possible that demand is less than supply and there is some unemployment of that type of labor. Here we assume that $L = L^d$ for all types of labor as this makes the algebra simpler. But we discuss the other case in the Appendix – it does not alter the result.

Now pre-multiply (7) by X, the total vector of consumption goods (some of which are used as intermediate goods) and (8) by I and sum them to have:

$$Xdp + Idp^i$$
$$= Xc_w dw + Ic_w^i dw + Xc_p dp + Ic_p^i dp + (r+\delta) Xc_k dp^i + I(r+\delta) c_k^i dp^i + \left(Xc_\theta + Ic_\theta^i\right) d\theta \tag{12}$$

Using (9)-(11) this can be written as:

$$\left[X - X^d\right] dp + Idp^i = Ldw + (r+\delta) Kdp^i + \left[Xc_\theta + Ic_\theta^i\right] d\theta \tag{13}$$

Now $X - X^d = X^w + X^k$ where X^w is consumption of workers and X^k is consumption of capitalists, and $I = \delta K$ in steady-state in which case we have:

$$\left[Ldw - X^w dp\right] + \left[rKdp^i - X^k dp\right] = -\left[Xc_\theta + Ic_\theta^i\right] d\theta > 0 \tag{14}$$

The first term in square brackets is, from (6), the change in the total utility of workers. The second term in square brackets is related to the change in the total utility of capitalists. The sum of these terms must be positive saying that the gains from new technology must flow either to workers or capitalists. But this does not say that workers must get some share of the gains. But (14) can be written as:

$$\begin{aligned} Ldu^w &= \left[Ldw - X^w dp \right] = \left[X^k dp - rKdp^i \right] - \left[Xc_\theta + Ic_\theta^i \right] d\theta \\ &= \left[\left(p \circ X^k \right) \frac{dp}{p} - r \left(p^i \circ K \right) \frac{dp^i}{p^i} \right] - \left[Xc_\theta + Ic_\theta^i \right] d\theta \end{aligned} \tag{15}$$

Where $\circ$ denotes a Hadamard product and dp / p is vector of proportional changes in prices. (15) can then be written as:

$$\begin{aligned} Ldu^w &= pX^k \left[\frac{\left(p \circ X^k \right) \frac{dp}{p}}{pX^k} - \frac{\left(p^i \circ K \right) \frac{dp^i}{p^i}}{p^i K} \right] - \left[Xc_\theta + Ic_\theta^i \right] d\theta \\ &= pX^k \left[\frac{d\tilde{p}}{\tilde{p}} - \frac{d\tilde{p}^i}{\tilde{p}^i} \right] - \left[Xc_\theta + Ic_\theta^i \right] d\theta \end{aligned} \tag{16}$$

Where the first line uses the fact that from the capitalists' budget constraint $pX^k = rp^i K$ and $\tilde{p}$ is the consumer price index and $\tilde{p}^i$ the investment goods price index. The term in the difference in inflation rates is positive if investment goods prices fall faster than consumer goods prices (e.g. because consumer goods involve more labour-intensive services), proving the result.

The importance of the price of investment goods relative to consumption goods can be understood through a simple model. In clarifying the role of the assumptions it is useful to reduce the number of goods and labor to one but to assume the relative price of investment goods is affected by technology, $p^i(\theta)$ (what Greenwood et al, 1997, term investment-specific technical change). Represent the gross output of the economy through the use of a production function:

$$Y = F\left(L, X, K, \theta\right) \tag{17}$$

Where labor is, L, intermediate inputs, X, and capital, K. and the state of technology that we denote by θ. Normalize the price of the consumption good to 1 and denote the wage paid to labor by w. With a constant interest rate the cost of capital will be $p^i(\theta)(r+\delta)$.

With constant returns to scale the total payment to inputs exhausts total output. So total payments to labor can be written as:

$$wL = F(L, X, K, \theta) - X - p^i(\theta)(r+\delta)K \tag{18}$$

i.e. gross output net of the intermediate goods used and the payments to the owners of capital. Differentiating (18) with respect to new technology leads to:

$$\begin{aligned} L\frac{\partial w}{\partial \theta} &= \frac{\partial F}{\partial \theta} + \left[\frac{\partial F}{\partial X} - 1\right]\frac{\partial X}{\partial \theta} + \left[\frac{\partial F}{\partial K} - p^i(\theta)(r+\delta)\right]\frac{\partial K}{\partial \theta} - (r+\delta)K\frac{\partial p^i}{\partial \theta} \\ &= \frac{\partial F}{\partial \theta} - (r+\delta)K\frac{\partial p^i}{\partial \theta} \end{aligned} \tag{19}$$

Where the second equality follows because the terms involving X and K cancel under the assumption that these inputs are paid their marginal product. The first term in the second line is positive as is the second term if the relative price of investment goods is falling. Result 2 simply shows the same is true with many goods and types of labor[1].

The intuition for Result 2 is that new technology allows more output to be produced than before. This extra output might go to labor or the owners of capital. But if the impact of the new technology is to reduce the price of investment goods relative to consumption goods, then the return to existing capital must fall, causing a rise in the overall return to labor. And any additional capital must be paid its marginal product so its return cannot be at the expense of labor. Result 2 does not imply that the labor share of national income rises (Karabarbounis and Neiman, 2013) because the stock of capital might increase enough to more than off-set the fall in relative investment goods prices.

It is obviously important to consider whether the condition that investment good prices fall relative to consumption goods is likely to be satisfied in practice. Most data suggests that it is (Krusell et al., 2000, Jones, 2016, IMF, 2017). But one implication is that it is possible to

[1] Casual inspection of (19) might lead one to think that if the relative price of investment goods rises enough, wages could fall even with one type of labor which would contradict Result 1. But if there is no technical regress in producing capital goods there is an upper bound on the increase in the price of investment goods which is that $\frac{\partial \ln p^i}{\partial \theta} \leq \frac{\partial \ln w}{\partial \theta}$, in which case (19) implies real wages must rise.

come up with an example in which new technology raises the relative price of capital goods and the average wage of workers falls - this is done in the Appendix.

Results 1 and 2 do not say anything about whether all or most workers benefit – it is possible that the majority of workers lose or that there will be no demand for some types of workers even if their wages fell to zero. So Results 1 and 2 do not say that new technology will not have serious consequences for inequality in labor income. But there are policies that could ensure all workers gain. With the assumption that the number of different types of workers is fixed, one can simply tax the winners and distribute to the losers without affecting any production decisions. Note that one can achieve this by taxing only labor – one does not need to tax 'robots' as has been suggested by, among others, Bill Gates (though seen Guerreiro et al., 2017, for a different argument supportive of taxing robots). This process of redistribution may be politically difficult - especially if the winners and losers are in different countries – and one should not be complacent about the ability of political processes to restrain rises in inequality. But it is important to understand that there is a simple policy to ensure that all gain.

Choice of Occupation

The models used so far have assumed that the supply of different types of workers is fixed. In the long-run that is not a plausible assumption - think of types of labor as occupations and that workers can choose their occupation at the start of their careers. It is plausible that the numbers of workers choosing different careers depends on the wages on offer, the costs of training for different occupations, and how pleasant or unpleasant is the nature of the work. One prominent case is that the labor supply to different occupations is perfectly elastic, which means that relative wages are fixed - occupations which require longer periods in education or are more unpleasant have to be compensated by higher wages.

The perfect elasticity model may seem extreme but is not a bad approximation to the data – over time there have been huge changes in the level of employment in different occupations but relatively modest changes in relative wages.

Result 3: If labor of different types is in perfectly elastic supply, then workers of all types must gain from technological progress.

The intuition for this result is that perfectly elastic labor supply between occupations means that wage differentials are fixed, so that all wages must go up or down together, reducing the model effectively to one with only one type of labor. And a corollary of Result 1 is that if there is only one type of labor, then new technology of any form must raise real wages for all workers. The benchmark model does not model the costs of the acquisition of human capital but, with perfectly elastic supply of labor to occupations, utility must be equalized across them and if there is an 'entry-level' occupation without training costs then utility net of training costs must also rise as the wage in the entry-level occupation will be rising as wages rise.

The Role of the Assumptions of the Benchmark Model

As indicated in the introduction, these models are only as good as their assumptions. Here we indicate how the results can change if the assumptions are violated. It is easiest to consider relaxing assumptions in the context of the one type of labor, one good model with the price of investment relative to consumption goods fixed at 1. Results 1 and 2 then imply that workers must gain from new technology if the assumptions of constant returns to scale, a constant interest rate and perfect competition are satisfied. Now consider how one might try to over-turn this result by varying these assumptions.

Decreasing Returns to Scale

A very simple example of a decreasing returns to scale production function where the wage can fall is $\left(L+\theta X\right)^{\alpha}$ for $\alpha<1$. But decreasing returns is often thought implausible because one can always replicate existing activities so decreasing returns to scale is often thought to result from an 'omitted' fixed factor. So this result could be interpreted to say that while new technology increases the returns to fixed factors as a whole, it is just that labor is not the only fixed factor. Although it is a common and plausible assumption that labor is currently the main fixed factor, it is possible that some other fixed factor comes to be important, e.g. if robots required some rare earth in their manufacture. In that case it is possible that the benefits from new technology go to the owners of that scarce factor and not to labor. But this is a different argument from most accounts of the impact of new technology. The models of Hanson (2001) and Susskind (2017) in which workers are harmed by new technology rely on assuming decreasing returns or that labor is not the only fixed factor.

Imperfect Competition

If there is imperfect competition then prices will be a mark-up on marginal costs. Mark-ups do not necessarily cause the results outlined above to fail. For example, Result 1 will still apply if one inserts mark-ups (possibly different for different goods), as long as mark-ups are constant. But it is conceivable that technical change causes mark-ups to rise for some goods in which case it is possible for wages to fall. The simplest way to see this is to note that $\frac{\partial p^i(\theta)}{\partial \theta}$ in (19) could be very large and positive if technology causes investment goods industries to become less competitive and the mark-up to rise. Some concern about rising mark-ups has been expressed (Autor et al., 2017, De Loecker and Eeckhout, 2017), but, even if relevant, it is less about the direct impact of technology and more about the way technology affects market competition. One could also get falling real wages if there was imperfect competition in the labor market and technology increased the market power of employers. Imperfect competition also allows for increasing returns to scale in production. The Appendix presents a simple model where each individual firm has increasing returns to scale and some market power, but there is free entry of firms into industries (i.e. a model of monopolistic competition). It shows this is isomorphic to the models already considered if the fixed and variable costs of firms use inputs in the same proportions. So our main results would apply in this case but one could conceivably get different results if fixed and variable costs use different types of inputs.

Rising Interest Rate

The Appendix shows that if new technology causes the interest rate to rise then this causes a rise in the return to capital and possible falls in real wages. In most standard economic models the interest rate is a function of the underlying growth rate (zero in our steady-state) and the rate of time preference. There is no particular reason why new technology would affect the rate of time preference so the mechanism for why interest rates might rise are not clear to us but we outline it as a hypothetical possibility.

New technology might increase the growth rate of the economy which would be expected to increase the interest rate. This would tend to reduce the real wage but the higher growth rate may cause wages to rise at a faster rate ultimately leading to higher real wages. But our model with its comparison of steady states is silent about what might happen in a dynamic economy. But the current problem facing many advanced economies does not seem to be one of fast productivity growth and high real interest rates.

Non-Steady States

Our comparison of steady states allows us to be relatively general about the way that new technology affects production, but does come at the cost that we do not analyse the transition from one steady-state to another, and do not analyse an economy in which technology changes over time. This leaves open the possibility that new technology causes real wages to fall along the growth path. However, it is well-known that these analyses are hard – one has to impose more restrictions than we have done to have attractable model of economic growth. One way in which our comparison of steady-states may be limited is in its analysis of a singularity if it becomes possible to produce robots that are identical to (or better than) people. If this is the case then labor is no longer effectively a fixed factor. In a steady-state this is a situation in which wages would fall to zero and prices of all goods would also fall to zero if there is perfect competition. This would be an economy of total abundance because there is no longer a natural limit to the level of production caused by the existence of labor as a fixed factor. But one could not get to the point of total abundance instantaneously so a model of transition would be needed. Aghion et al (2017) provide a useful discussion of this case emphasizing the restrictive conditions under which singularities might be relevant.

Conclusion

The possible impact of new technology on workers has attracted a lot of attention. Although there are many specific models which seek to investigate this, the existing literature does not lay out conditions under which new technology can be expected to harm no workers, all workers or reduce the average wage. This paper has tried to do this using very simple underlying models. One underlying theme is that it is harder than one might think to write down economic models in which workers as a group are harmed by new technology: the reason is that if labor is the only fixed factor and the terms of trade shift in favour of workers as the relative price of investment goods decline, then workers as a whole are likely to gain from new technology. And if the supply of labor to different occupations is perfectly elastic, then all will gain. The threats to wages from new technology may come more from impacts on the competitiveness of markets.

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Appendix: Proof of Results and Specific Examples

Proof of Result 1

Stack the prices of consumption and investment goods into a single vector ρ. Combine the cost functions into a single vector as well – continue to denote this by c. Write the stacked prices as:

$$\rho = c\left(w, \rho, \theta\right) \tag{1}$$

Taking logs and differentiating leads to:

$$\frac{\partial \log \rho}{\partial \theta} = \Lambda^{\rho} \frac{\partial \log \rho}{\partial \theta} + \Lambda^{w} \frac{\partial \log w}{\partial \theta} + \frac{\partial \log c}{\partial \theta} \tag{2}$$

Where Λ^{ρ} is a non-negative matrix whose ijth element, γ_{ij}^{ρ}, is given by:

$$\gamma_{ij}^{\rho} = \frac{\rho_j}{c_i} \frac{\partial c_i}{\partial \rho_j} \tag{3}$$

From Shephard's Lemma we know that the derivative of the cost function with respect to a price is the per output demand for that input. Hence γ_{ij}^{ρ} is the share of the cost of input j in the production of good i. Similarly, Λ^{w} is a non-negative matrix whose ijth element, γ_{ij}^{w}, is given by:

$$\gamma_{ij}^{w} = \frac{w_j}{c_i} \frac{\partial c_i}{\partial w_j} \tag{4}$$

γ_{ij}^{w} is the share of the cost of type of labor j in the production of good i. The ith row of Λ^{ρ} must sum to one minus the share of labor costs in the production of good i, and the ith row of Λ^{w} must sum to the share of labor in the production of good i. Denote the vector of shares of labor costs by s.

Denote the maximum goods price change as $\frac{\partial \log \rho^{\max}}{\partial \theta}$ and the maximum wage change as $\frac{\partial \log w^{\max}}{\partial \theta}$. Then (2) implies that, for all goods, we must have:

$$\begin{aligned} \frac{d \log \rho}{d\theta} &\leq \Lambda^{\rho} \frac{d \log \rho^{\max}}{d\theta} + \Lambda^{w} \frac{d \log w^{\max}}{d\theta} + \frac{\partial \log c}{\partial \theta} \\ &= \left(1 - s\right) \frac{d \log \rho^{\max}}{d\theta} + s \frac{d \log w^{\max}}{d\theta} + \frac{\partial \log c}{\partial \theta} \end{aligned} \tag{5}$$

With equality only for goods which are produced only use goods and labor with the maximum price and wage changes. (5) applies for all goods, including goods with prices increasing at the fastest rate. For these goods we can re-arrange (5) to yield:

$$\frac{d\log \rho^{\max}}{d\theta} \leq \frac{d\log w^{\max}}{d\theta} + \frac{1}{s}\frac{\partial \log c}{\partial \theta} \leq \frac{d\log w^{\max}}{d\theta} \tag{6}$$

Where s is the labor share for that good. This proves the result but is only valid if $s > 0$. What happens if the good with the highest price increase is produced using no labor? If this good is produced using some goods with price increases below the maximum then it cannot be the good with the highest price index as (5) will be a strict inequality leading to a contradiction if $s = 0$. If it is only produced using goods with the highest price increase, this is a contradiction if there is any technical change in that sector. If there is not, there is a set of goods with no technical change produced with no labor and only each other as intermediate or capital goods. Because these goods are produced without fixed factors, there is no limit to the supply of them so the price of them will always be zero, contradicting the fact that they have the highest inflation rate.

Ultimately workers are only interested in the price of consumption goods and this result seems to leave open the possibility that prices only fall for investment goods. But if these investment goods are used, directly or indirectly (meaning it might only be used to produce investment goods but those investment goods are ultimately used in the production of consumption goods through some chain), this must be transmitted to the price of some consumption good. There will be no production, in equilibrium, of a set of investment goods only used to produce themselves in which case the result says that technological change in goods that are not produced will have no benefit.

Result 2 when the wage of some types of workers are zero

It is possible that the wages of some types of labor fall to zero and there is possibly some unemployment for those types. This case can be allowed for in the following way. Remove these types of workers from the cost functions as the zero wage allows us to do this. The result above then goes through for the set of workers with non-zero wages. But the average wage result applies to all workers if we include the unemployed as having zero earnings. The set of types of workers with zero wages may change with the technology but the formulae above remain valid even for this case.

If all types of workers have zero wages then we are in a situation where all prices (as well as wages) will be zero, i.e. this is a situation of total abundance. Our static analysis is not well-suited to this case – it is discussed at the end of the paper.

An example where the average wage of workers falls

The example outlined here shows how the real wage of workers can fall if new technology causes the price of investment goods to rise relative to consumption goods i.e. the condition of Result 2 is not satisfied. Such an example must have at least two types of goods (to allow relative prices to change) and two types of labor (otherwise Result 1 would imply that real wages would rise).

Assume that there are two sectors, a consumption good sector and an investment good sector. The consumption good is assumed to be produced by one type of labor – call it c-labor – and capital goods, according to the production function:

$$X = L_c f\left(\frac{K_c}{L_c}, \theta\right) = L_c f\left(k_c, \theta\right) \tag{7}$$

The investment good is assumed to be produced by a different type of labor – call it i-labor – and capital goods according to the production function:

$$I = L_i g\left(\frac{K_i}{L_i}, \theta\right) = L_i g\left(k_i, \theta\right) \tag{8}$$

Assume the price of consumption good is numeraire – set it equal to 1.

The wage of c-labour will be given by:

$$w_c = f - k_c f_k \tag{9}$$

And the demand for capital in consumption good sector will be given by:

$$\left(r + \delta\right) p_i = f_k \tag{10}$$

The wage of i-labour will be given by:

$$w_i = p_i\left[g - k_i g_k\right] \tag{11}$$

And the demand for capital in the i-sector will be given by:

$$(r+\delta)\, p_i = p_i g_k \tag{12}$$

Note that (12) implies that the capital-labour ratio in the i-sector solves the equation:

$$g_k\left(k_i,\theta\right) = (r+\delta) \tag{13}$$

Which, conveniently, is independent of prices. Given the inelastic supply of i-labour this also fixes the amount of i-capital. Now the total supply of c-capital must satisfy the equation

$$\delta\left(K_c + L_i k_i\right) = L_i g \tag{14}$$

Which can be re-arranged to give:

$$K_c = L_i \frac{g-\delta k_i}{\delta} \tag{15}$$

Which implies that the amount of c-capital can also be solved for independent of prices. This then implies that the total capital stock is given by:

$$K = L_i \frac{g}{\delta} \tag{16}$$

Note that the production function for the consumer good plays no role in determining the total level of capital or its allocation across sectors.

Total income to workers must be the difference between the production of consumption goods and the consumption of capitalists which is the part of their income not used to cover depreciation i.e. $rp^i K$ of capitalists. This implies:

$$\begin{aligned} Lw = L_c w_c + L_i w_i &= L_c f - rp^i K \\ &= L_c f - \frac{r}{r+\delta} f_k K = L_c f - \frac{r}{r+\delta}\frac{g}{\delta} f_k L_i \end{aligned} \tag{17}$$

Now suppose that the nature of the new technology is that it does not affect production of the investment good (this is an example so this is not meant to be plausible). In this case g is fixed and the capital-labor ratios in the two sectors are unaffected by the new technology. Differentiating (17) we have that:

$$\frac{d(Lw)}{d\theta} = L_c f_\theta - \frac{r}{r+\delta}\frac{g}{\delta} f_{k\theta} L_i \tag{18}$$

The first term is positive but the second term can outweigh it if new technology heavily raises the marginal product of capital in the consumption goods sector.

Note the link to the condition in Result 1. (10) implies that the relative price of investment goods rises (resp. falls) if $f_{k\theta} > (<)0$. If $f_{k\theta} < 0$ the relative price of investment goods falls and (18) says that average wages must rise, consistent with Result 1. But if $f_{k\theta} > 0$ then the relative price of investment goods rises and and (18) says that average wages can fall.

Proof of Result 3

If there are many types of labor but they are in perfectly elastic supply then relative wages are constant. The cost functions can be written as a function of the wage of one type of labor chosen as numeraire and the relative wages which are exogenous. The model is then reduced to one in which there is only one wage and as result 1 implies the real wage must rise for one type of worker, they must rise for all types of labor.

Increasing Returns and Imperfect Competition

Many current models of the economy assume that individual firms have increasing returns to scale. This section considers what happens if that is the case. Continue to use c to denote marginal costs but now assume that firms have to pay a fixed cost $c^f(w,\rho,\theta)$ to enter an industry – for simplicity here we use the stacked price approach of Result 1 rather than distinguish between consumption and investment goods.

Increasing returns at the individual firm level is not compatible with perfect competition so we assume that price is a mark-up, μ, possibly varying across sectors, on marginal costs i.e. we have:

$$\rho = (1+\mu)c \tag{19}$$

We treat μ as exogenously given though it is usually derived from other parameters in the model – for our purpose this is not important.

Free entry into an industry implies that total revenue of the industry must equal total costs which can be written as:

$$(\rho - c)X = Nc^f \tag{20}$$

Where X is gross output and N is the number of firms. Using (19) this can be written as:

$$N = X\frac{\mu c}{c^f} \tag{21}$$

Now consider input demands. Total demand from this sector for input j can be written as:

$$X\frac{\partial c}{\partial p_j} + N\frac{\partial c^f}{\partial p_j} \tag{22}$$

Using (21), (22) can be written as:

$$X\frac{\partial c}{\partial p_j} + X\mu c\frac{\partial \log c^f}{\partial p_j} = X\frac{\partial c}{\partial p_j}\left[1 + \mu\frac{\frac{\partial \log c^f}{\partial p_j}}{\frac{\partial \log c}{\partial p_j}}\right] \tag{23}$$

If marginal costs and fixed costs using inputs in the same proportions this implies that $\frac{\partial \log c^f}{\partial p_j} = \frac{\partial \log c}{\partial p_j}$ the total factor demands can be written as:

$$X\frac{\partial c}{\partial p_j}[1 + \mu] \tag{24}$$

Which is completely isomorphic to our standard model using $(1+\mu)c$ as the cost function. One can derive a similar expression for the demand for a type of labor replacing the price with the wage.

This leaves open the possibility that technology might harm workers if it affects fixed costs in a different way to marginal costs. And it does assume that all firms within an industry are identical – many models assume heterogeneity which gives rents to the more productive firms. It is possible that new technology might disproportionately advantage these firms. The analysis of these models is left for later research.