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Identification of Global and Local Shocks in International Financial Markets via General Dynamic Factor Models

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Abstract

We employ a two-stage general dynamic factor model to analyze co-movements between returns and between volatilities of stocks from the US, European, and Japanese financial markets. We find two common shocks driving the dynamics of volatilities – one global shock and one US-European shock – and four local shocks driving returns, but no global one. Co-movements in returns and volatilities increased considerably in the period 2007-2012 associated with the Great Financial Crisis and the European Sovereign Debt Crisis. We interpret this finding as the sign of a surge, during crises, of interdependencies across markets, as opposed to contagion. Finally, we introduce a new method for structural analysis in general dynamic factor models which is applied to the identification of volatility shocks via natural timing assumptions. The global shock has homogeneous dynamic effects within each individual market but more heterogeneous effects across them, and is useful for predicting aggregate realized volatilities.

JEL subject classification: C32, C55, C38, G01, G15.

Key words: Dynamic factor models, volatility, financial crises, contagion, interdependence.

1 Introduction

In the last few decades characterized by increasing financial integration, growing attention has been devoted to financial market dynamics as an international phenomenon. The strong international co-movements observed across different stock markets during periods of turmoil affirmed the idea of financial contagion, and an extensive literature has focused on correlations and international spillovers in returns and volatilities. Examples, among many others, are King and Wadhwani (1990), Lin et al. (1994), King et al. (1994), Karolyi (1995), Wongswan (2006), Diebold and Yılmaz (2009), and Corradi et al. (2012), for volatility spillovers, and Forbes and Rigobon (2002), Karolyi (2003), Corsetti et al. (2005), and the surveys by Claessens et al. (2001) and Karolyi and Stulz (2003), with references therein, for the analysis of contagion and return correlations.

Our goal here is to better understand the co-movements within and between different stock markets. We do so by disentangling, from a large dataset containing US, European, and Japanese stocks, the various sources, national and international, of return and volatility dynamics. Our approach is based on dynamic factor models in presence of a block structure, which allows us to assess the importance of these sources. We also contribute to the international finance literature by investigating whether the global financial crisis and the European sovereign debt crisis were characterized by financial contagion or interdependence. Furthermore, we develop a new methodology for structural analysis in dynamic factor models, allowing us to identify volatility shocks and their predictive power for aggregate realized volatilities.

Our methodology is based on a two-step *general dynamic factor model*,[1] which extends the original Forni et al. (2000) model by tacking into account (i) the possibly distinct dynamics in returns and volatilities, and (ii) the block structure induced by the various markets (US, Europe, Japan). The two steps consist of a factor decomposition of the panel of returns, followed by a further one for the resulting volatilities. Our analysis combines for the first time the methods originally proposed by Hallin and Liška (2011) in presence of blocks, by Forni et al. (2015) on one-sided estimation, and by Barigozzi and Hallin (2015) for the joint analysis of returns and volatilities. Note that in this setting the common factors are in fact mutually orthogonal white noise processes loaded by the data along with their lags, and for this reason are referred to as *common shocks*.

When compared with alternative methodologies for the analysis of high-dimensional time series, our dynamic factor approach enjoys a number of advantages. First, being based on non-parametric estimation, it is not plagued by the well-known *curse of dimensionality* affecting previous parametric approaches to multivariate volatilities (see the surveys by Asai et al., 2006; Bauwens et al., 2006; Silvennoinen and Teräsvirta, 2009, and the literature cited therein).

Second, our approach compares quite favorably to other existing factor models, mainly of the *static* or the *exact* type. Unlike the static factor model of Chamberlain and Rothschild (1983), Stock and Watson (2002), and Bai and Ng (2002), the dynamic approach does not impose any particular restrictions on factor dynamics, and takes into account the whole second-order dynamic structure of the data rather than restricting to contemporaneous covariances.[2] Moreover, we do not require the assumption of cross-sectional or serial orthogonality

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[1]"Generalized factor models" and "general factor models" are used interchangeably in the literature; for simplicity, we are sticking to "general factor models".

[2]Throughout, we call "static" a factor model in which factors are loaded in a static way, namely through

of idiosyncratic components as in the exact factor models of Geweke (1977) and Sargent and Sims (1977). Reflecting those features, the model is referred to as *generalized* or *general* because it generalizes alternative factor models and arises naturally without particular assumptions on the data-generating process (see Forni and Lippi, 2001 and Hallin and Lippi, 2013 for discussions of this point). In this sense, we extend previous studies which considered static and/or exact factor models for analyzing large panels of financial data.[3]

Third, the approach consisting in a joint factor model analysis of returns *and* volatilities, originally proposed by Barigozzi and Hallin (2015, 2017), has the advantage of allowing us to compare the common shocks in returns with those in volatilities. Specifically, it is shown in Barigozzi and Hallin (2015) that shocks which are common to volatilities do not necessarily originate in the volatilities of the common components of returns, and that the volatilities of the idiosyncratic components of returns also are affected, in a highly non-negligible way, by those volatility shocks.

Fourth, the block-wise analysis originally exploited in general dynamic factor models by Hallin and Liška (2011), is based on the canonical decomposition into orthogonal subspaces of the Hilbert space spanned by the random variables considered. As such, the decomposition always exists and is unique, thus allowing us to identify the origin of all common shocks driving returns and volatilities. Specifically, we can classify those shocks into "global" and "local" shocks, where the "global" shocks are driving the intersection between the common spaces of the three markets under study, while the "local" ones are common to one single market, or to two of them. As shown in this paper our approach nests the hierarchical factor model of Moench et al. (2013), where by assumption factors can only be either global or local to just one single market.

We consider a large panel of daily stock returns including the constituents of three major market indices: Standard & Poor 500 for the US, Standard & Poor Europe 350 for Europe, and Nikkei 225 for Japan. This yields daily observations for a total of $N = 830$ stocks, over a period of about 15 years, which corresponds to a sample size of $T \simeq 4000$ days.

As a first step of our methodology, we consider a general dynamic factor model for the panel of stock returns, for which four common shocks are identified. Estimation of the model for returns provides also the estimated innovations of the common and idiosyncratic returns, from which we construct a panel of volatility proxies. In the second step of our methodology a general dynamic factor model is then estimated for those volatility proxies, and two common shocks are found. Once those common sources of variation are estimated we carry out four empirical exercises:

(i) a classification of the common shocks of returns and volatilities into "global" and "local" shocks;

(ii) the assessment of interdependencies and contagion in returns and volatilities panels during the turmoil period (2007-2012) of the past decade, characterized by the Great Financial Crisis and the European Sovereign Debt Crisis;

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loading matrices, whereas in our dynamic approach we consider more general loadings given by matrix polynomials in the lag operator, i.e. filters.

[3]See for example: Connor and Korajczyk (1986, 1988), Diebold and Nerlove (1989), Ng et al. (1992), Harvey et al. (1992), Jones (2001), Sentana and Fiorentini (2001), Van der Weide (2002), Fiorentini et al. (2004), Connor et al. (2006), Sentana et al. (2008), and Aramonte et al. (2013), for conditionally heteroskedastic factor structures in returns, and Engle and Marcucci (2006), Rangel and Engle (2012), Ghysels (2014), Pakel et al. (2014), Barigozzi et al. (2014), Fan et al. (2015), and Luciani and Veredas (2015), for factor structures in volatilities.

(iii) the identification of the volatility shocks and their impulse response functions by observing that US-European shocks cannot affect contemporaneously the Japanese market;

(iv) the evaluation of the explanatory power of the identified volatility shocks when predicting aggregate realized variances.

The following is a summary of our main findings.

(i.a) The market for returns has no global shock, and the contribution of foreign shocks, in general, is relatively modest. These findings are consistent with the idea that the domestic assets are largely overweighted in portfolios; in other words, we find evidence of the so-called *home bias* phenomenon, as previously pointed out, for example, by French and Poterba, 1991, Tesar and Werner, 1994, Coeurdacier and Rey, 2013, or Petzev et al., 2016.

(i.b) In contrast with this, the panel of volatilities is clearly driven by a global shock explaining about 17%, 16%, and 13% of the total variance of volatilities in the US, Europe, and Japan, respectively. As a consequence, volatilities are strongly interconnected, thus confirming the view that risk premia have an international flavor (see e.g. Karolyi and Stulz, 2003, and references therein).

(ii) By investigating how explained variances in each subspace change during turbulent times, we confirm the widespread belief that co-movements are largely increased during crises but we do not observe amplified effects of foreign shocks on a given market. Therefore, our results provide some support for the hypothesis of increased interdependencies as opposed to very limited evidence in favor of financial contagion (see Forbes and Rigobon, 2002; Karolyi, 2003, for similar results).

(iii) Analysis of impulse responses to the global volatility shocks reveals within-market homogeneity but between-markets heterogeneity.

(iv) Although the estimated volatility shocks are extracted from a panel of closing prices that does not incorporate information on intraday volatility, we find that they explain about 30% of the aggregated realized variances of the markets under study and in one-step-ahead predictive regressions. Encompassing regressions in the tradition of Mincer and Zarnowitz (1969) allow us to infer that in most markets and samples considered our shocks add useful information to predictions based on the heterogenous autoregressive (HAR) models proposed by Corsi (2009).

The outline of the paper is as follows. In Section 2 the dynamic factor model for returns and volatilities is presented. Section 3 deals with the estimation of the model, and the separation between global and local shocks. In Sections 4 and 5 we apply our model to the analysis of a large panel of stocks in the US, Japanese and European stock markets, with focus on the aspects of interdependence, contagion and spillovers (Section 4), and on dynamic effects and volatility prediction (Section 5). Section 6 concludes.

2 General dynamic factor models of returns and volatilities

We describe in detail our two-stage model for returns and volatilities when considering all three markets jointly, while in the next section we show how we can exploit the block structure

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of the data in order to gain more insight into the origin of the dynamic sources of variation. The dataset we consider consists in an $N \times T$ panel $\mathbf{Y} := \{Y_{it}; \ i = 1, \ldots, N; \ t = 1, \ldots, T\}$ of stock returns. A generic element of $\mathbf{Y}$ is denoted as $Y_i := \{Y_{it}; \ t = 1, \ldots, T\}$. Throughout, we assume each Y_i to be zero-mean strongly stationary and we also require some very general regularity assumptions ensuring the existence of the (multivariate) spectrum of $\mathbf{Y}$ (see Forni et al., 2015 for a more formal description and details).

2.1 Stage 1: A dynamic factor model for returns

The most general factor model representation of $\mathbf{Y}$, known as the *general* (or *generalized*) *dynamic factor model* (GDFM), is

$$Y_{it} = X_{it} + Z_{it} = \sum_{k=1}^{Q} b_{ik}(L) u_{kt} + Z_{it}, \qquad i = 1, \ldots, N, \tag{1}$$

where the following conditions hold

(C1) the process $\mathbf{u} = (u_1, \cdots, u_Q)'$ is orthonormal white noise (typically, $Q \ll N$);

(C2) the polynomials $b_{ik}(L)$ are one-sided and have square-summable coefficients for any $i = 1, \ldots, N$ and any $k = 1, \ldots, Q$;

(C3) the *common* component $\mathbf{X} := \{X_{it}; \ i = 1, \cdots, N; \ t = 1, \cdots, T\}$ is driven by pervasive factors, that is, the Qth eigenvalue of its spectral density matrix diverges as $N \to \infty$ for almost all[4] frequencies in the range $[-\pi, \pi]$;

(C4) the *idiosyncratic* component $\mathbf{Z} := \{Z_{it}; \ i = 1, \cdots, N; \ t = 1, \cdots, T\}$ is stationary and possibly autocorrelated, but only mildly cross-correlated, that is, the eigenvalues of its spectral density matrix are uniformly bounded as $N \to \infty$;

(C5) the common component and the idiosyncratic component are mutually orthogonal, that is, uncorrelated, at all leads and lags;

(C6) Q is the smallest integer for which (C1)-(C5) hold.

Hereafter, we call $\mathbf{X}$ the *level-common* component and $\mathbf{Z}$ the *level-idiosyncratic* component to emphasize the fact they are the common and idiosyncratic components of the panel of returns. Moreover, since the components of $\mathbf{u}$ are white noises driving the *level-common* component they are called the *level-common shocks*.

The number Q of level-common shocks is identified as the number of eigenvalues of the spectral density matrix of $\mathbf{Y}$ that are diverging, as $N \to \infty$, almost everywhere in the frequency range $[-\pi, \pi]$.[5] The method proposed by Hallin and Liška (2007), which is based on this asymptotic behavior of eigenvalues, is used to determine Q.

As it stands, the GDFM in (1) arises as a representation result that essentially does not place any restriction, besides stationarity and the existence of a spectrum, on the dynamics

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[4]Spectral densities are only defined up to a set of frequency values contained in a Borel set with Lebesgue measure zero, and this "almost all" or "almost everywhere" restriction should be part of most statements involving spectral densities; it has no practical implications, though, and, for the sake of simplicity, we often omit it.

[5]This is a simple consequence of Weyl's inequality, see also Proposition 1 in Forni et al. (2000).

of the panel; this is why it is referred to as "general" (see the discussion in Hallin and Lippi, 2013). On the other hand, in the more popular *static approach* to factor models (see Stock and Watson, 2002; Bai and Ng, 2002, among others), dynamics are inferred from $\mathbf{Y}$'s contemporaneous covariances only rather than from its full second order structure, which, if consistency is to be achieved, imposes stronger restrictions on the data-generating process.

Recent results by Anderson and Deistler (2011) on vector autoregressions with singular spectra have been exploited by Forni et al. (2015) to derive, under the mild additional assumption of a rational spectrum, a block-diagonal autoregressive representation for the GDFM equivalent to (1). Specifically, using obvious notation $\mathbf{Y}_t = (Y_{1t}, \cdots, Y_{Nt})'$, $\mathbf{Z}_t = (Z_{1t}, \cdots, Z_{Nt})'$ and $\mathbf{u}_t = (u_{1t}, \cdots, u_{Qt})'$, and assuming, without loss of generality, that N factorizes into $N = K(Q+1)$ for some $K \in \mathbb{N}$, they show that we always can write

$$(\mathbf{I} - \mathbf{A}\left(L\right)) \mathbf{Y}_t = \mathbf{H}\mathbf{u}_t + (\mathbf{I} - \mathbf{A}\left(L\right)) \mathbf{Z}_t, \tag{2}$$

where (a) $\mathbf{H}$ is an $N \times Q$ matrix of loadings, (b) $\widetilde{\mathbf{Z}}_t := (\mathbf{I} - \mathbf{A}(L))\mathbf{Z}_t$ is idiosyncratic, thus satisfying condition C4 above, and (c)

$$\mathbf{A}\left(L\right) = \begin{pmatrix} \mathbf{A}^{(1)}\left(L\right) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{(2)}\left(L\right) & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}^{(K)}\left(L\right) \end{pmatrix},$$

with $(Q+1) \times (Q+1)$-dimensional diagonal blocks $\mathbf{A}^{(1)}\left(L\right), \cdots, \mathbf{A}^{(K)}\left(L\right)$. Moreover, for any $i = 1, \ldots, K$, $\det \mathbf{A}^{(i)}(z) \neq 0$ for $z \in \mathbb{C}$ such that $|z| \leq 1$, the filters are one-sided, and each polynomial $\mathbf{A}^{(i)}$ has finite degree. Existence of such filters is due to the fact that, under model (1), the moving average representation of any $(Q+1)$-dimensional subset of common components $(X_{(i-1)(Q+1)+1}, \cdots, X_{i(Q+1)})'$, for $i = 1, \ldots, K$, is *tall* and *generically* zeroless in the sense of Anderson and Deistler (2011). That is, apart from a measure-zero subset in the parameter space, the K autoregressive operators $\mathbf{A}^{(i)}\left(L\right)$ all invert into *fundamental* moving average filters which are unique for $(X_{(i-1)(Q+1)+1}, \cdots, X_{i(Q+1)})'$, i.e. have no roots inside the complex unit disk (the *zeroless* property).[6]

Then, the level-common component $\mathbf{X}_t$ admits the singular moving average representation

$$\mathbf{X}_t = (\mathbf{I} - \mathbf{A}\left(L\right))^{-1} \mathbf{H}\mathbf{u}_t, \tag{3}$$

and the common shocks u_{kt} in (2) and (3) are the same as in (1).

The main advantage of representation (2) over (1) is that, with K simple $(Q+1)$-dimensional autoregressions, it allows us to estimate the GDFM using one-sided filters only. Furthermore, estimation based on (2) is found to perform better than earlier approaches based on (dynamic) principal components (see for example the empirical results in Forni et al., 2016, for macroeconomic data, and in Barigozzi and Hallin, 2017, for financial data).

Estimation of (2) is studied in detail by Forni et al. (2017) under the additional assumptions that all Y_i's have finite fourth moments and geometrically declining physical dependence in the sense of Wu (2005), a condition which controls the amount of serial dependence. Such condition on serial dependence is automatically satisfied by all X_i's, because of (C1)-(C2), and

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[6] On the other hand, non-zeroless moving average representations are non-unique in the sense that they allow for multiple non-fundamental representations and invert into non-causal – i.e. two-sided – autoregressive representations (see e.g. Soccorsi, 2016).

it can be shown to be fulfilled by any idiosyncratic component, i.e. satisfying (C4), admitting a Wold representation. The following is a summary of the key estimation steps:

1. *Filtering.* A non-parametric estimate of the spectral density of $\mathbf{Y}$ is decomposed via dynamic principal components analysis into the sum of a common and an idiosyncratic spectrum (Forni et al., 2000). All autocovariance matrices of the common component $\mathbf{X}$ are then obtained as inverse Fourier transforms of the common spectrum and used to estimate the autoregressive filter $\mathbf{A}(L)$ by means of (low-dimensional) Yule-Walker equations.[7]

2. *Principal components.* Letting $\widetilde{\mathbf{Y}}_t := (\mathbf{I} - \mathbf{A}(L))\mathbf{Y}_t$, (2) yields a static factor model for $\widetilde{\mathbf{Y}}$, with Q common factors $\mathbf{u}$ loaded only contemporaneously by means of the loadings $\mathbf{H}$. Therefore, linear invertible transformations of $\mathbf{u}$ and $\mathbf{H}$ are estimated by means of principal component analysis on $\widetilde{\mathbf{Y}}$ (Bai and Ng, 2002; Stock and Watson, 2002). The filtered idiosyncratic component is obtained from $\tilde{\mathbf{Z}}_t = \widetilde{\mathbf{Y}}_t - \mathbf{H}\mathbf{u}_t$.

We conclude the analysis of returns with three final comments on this procedure. First, in order to prevent the estimation results in step 1 to depend on the (arbitrary) ordering of the cross-sectional items, we estimate the model for 100 random permutations of the cross-section and compute averages. In this respect, it has to be noticed that stabilization of the results takes place very quickly and in particular the larger the panel sizes, the smaller the number of permutations required for stabilization (see also the numerical results in Forni et al., 2016 and empirics in Forni et al., 2017, for similar evidence). Second, although the loadings $\mathbf{H}$ and shocks $\mathbf{u}$ are in general not identified. Full identification of $\mathbf{u}$ is beyond the scope of this paper and hereafter we identify them as the Q largest standardized principal components $\widetilde{\mathbf{Y}}$, which are orthonormal by construction. Moreover, this indeterminacy does not affect the next stage of the model, where only the well-identified product $\mathbf{H}\mathbf{u}$ is needed. Last, note that, under assumptions (C1)-(C6), model (2) also could be estimated by means of a purely spectral approach (see Forni et al. (2000)). The latter, however, relies on the empirical dynamic principal components of $\mathbf{X}$, yielding two-sided filters and estimates of the common component as a weighted averages of present, past, but also future values of the data. That two-sidedness issue was addressed in Forni et al. (2015, 2017): see the next section.

2.2 Stage 2: A dynamic factor model for volatilities

In order to proceed with the analysis of volatilities, we need appropriate residuals from the model of returns from which we can build volatility proxies. In particular, our modelling approach for the analysis of volatilities follows closely Barigozzi and Hallin (2015) and is based on the following two key ingredients.

1. *Returns residuals.* Consider the GDFM decomposition (2) of the panel of returns $\mathbf{Y}$ and its estimation as described above. First, let $\mathbf{e}_t := \mathbf{H}\mathbf{u}_t$: the $\mathbf{e}_t$'s then are the N-dimensional white noise level-common residuals. Second, note that the filtered process $\tilde{\mathbf{Z}} = (\tilde{Z}_1 \ldots \tilde{Z}_N)'$ still is idiosyncratic, hence mildly cross-correlated; therefore, it can be modelled, without much loss of information, as an N-tuple of univariate autoregressions. Denote by $v_{it} := (1 - c_i(L))\tilde{Z}_{it}$ the innovation resulting from fitting an

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[7]The orders of the K autoregressive models $\mathbf{A}^{(k)}(L)$ are determined by means of standard (low-dimensional) identification methods such as, for example, AIC or BIC.

autoregressive model to $\tilde{Z}_i$. The N-tuple of level-idiosyncratic residuals $\mathbf{v} = (v_1 \ldots v_N)'$ then is an N-dimensional white noise process of "componentwise innovations" for $\tilde{Z}_i$. The same $\mathbf{v}$ also is the white noise residual resulting from fitting to $\mathbf{Z}$ a VAR of the form

$$\begin{pmatrix} 1 - c_1(L) & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & 1 - c_N(L) \end{pmatrix} (\mathbf{I} - \mathbf{A}(L))\mathbf{Z}_t = \mathbf{v}_t.$$

Therefore, because the two processes $\mathbf{X}$ and $\mathbf{Z}$ are mutually orthogonal at all leads and lags, $e_{it} + v_{it}$ by construction is the (white noise) residual obtained by projecting Y_{it} onto the Hilbert space spanned up to time $(t-1)$ by $\mathbf{X}$ and Z_i. Thus, because Z_i is idiosyncratic, and neglecting idiosyncratic mild cross-correlations (see (C4)), $\mathbf{e}_t + \mathbf{v}_t$ is the white noise residual obtained by projecting $\mathbf{Y}_t$ onto its past.

2. *Volatility proxies.* For each stock, we define (see Engle and Marcucci, 2006) the centered volatility proxies

$$s_{it} := \left\{ \log \left[(e_{it} + v_{it})^2 \right] - \mathsf{E} \left[\log \left((e_{it} + v_{it})^2 \right) \right] \right\} \qquad i = 1, \ldots, N : \qquad (4)$$

$s_i := \{ s_{it}; \ t = 1, \ldots, T \}, i = 1, \ldots, N$ then is an $N \times T$-dimensional panel $\mathbf{s} = (s_1 \ldots s_N)'$ of centered volatility proxies.[8]

Considering a GDFM decomposition (for which conditions (C1)-(C6) hold)

$$s_{it} = \chi_{it} + \xi_{it} = \sum_{j=1}^{Q^s} d_{ij}(L) \varepsilon_{jt} + \xi_{it}, \qquad i = 1, \ldots, N, \qquad (5)$$

of the volatilities $\mathbf{s}$ seems quite natural. The number Q^s can be determined by means of the Hallin and Liška (2007) criterion and, proceeding as in (2), we can recover the common volatility shocks $\boldsymbol{\varepsilon} = (\varepsilon_1 \ldots \varepsilon_{Q^s})'$, along with the related impulse response functions $d_{ij}(L)$. As for the case of returns, we require the volatility proxies s_i's to have finite fourth-order moments and geometrically declining physical dependence (Wu, 2005). Moreover, note that, as explained above, e_i and v_i are the white noise residual obtained from two mutually orthogonal components, then by construction they are uncorrelated, therefore justifying our definition of s_i.

Finally, we show in this paper that standard identification approaches can then be employed to characterize the shocks $\boldsymbol{\varepsilon}$. Consider the vector notation

$$\mathbf{s}_t = \mathbf{D}(L) \boldsymbol{\varepsilon}_t + \boldsymbol{\xi}_t$$

for the GDFM decomposition (5) of the volatility panel. Then, we have a class of equivalent moving average representations for the common components, of the form

$$\boldsymbol{\chi}_t = \mathbf{D}(L) \mathbf{R}^{-1} \mathbf{R} \boldsymbol{\varepsilon}_t, \qquad (6)$$

where the $Q^s \times Q^s$ matrix $\mathbf{R}$ can be determined by imposing appropriate exogenous restrictions. Among all possible choices we restrict our search to orthogonal transformations as is customary in structural VAR models and we refer to the empirical analysis in Section 5 for the choice of $\mathbf{R}$. At this point, however, the block structure of the panel (that is, the geographical origin of each observation), which so far has not been exploited, is to play a major role, as explained in the next section.

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[8]In practice, expectations in (4) are replaced with arithmetic means.

3 Disentangling global and local shocks

The panel $\mathbf{Y}$, in our dataset, is the union of three subpanels or blocks corresponding to returns of companies from the US, European, or Japanese markets. That block structure, of course, is essential if intermarket dependencies are to be studied. Using obvious notation, we denote those subpanels as $\mathbf{Y}^{\text{US}}$, $\mathbf{Y}^{\text{EU}}$, $\mathbf{Y}^{\text{JP}}$, with observations Y_{it}^{US}, Y_{it}^{EU}, and Y_{it}^{JP}. Likewise, the volatility panel built from returns splits into three subpanels, $\mathbf{s}^{\text{US}}$, $\mathbf{s}^{\text{EU}}$ and $\mathbf{s}^{\text{JP}}$, with elements s_{it}^{US}, s_{it}^{EU}, and s_{it}^{JP}, respectively. Let N^{US}, N^{EU}, and N^{JP} stand for the cross-sectional sizes of the blocks, with $N^{\text{US}} + N^{\text{EU}} + N^{\text{JP}} = N$.

The methodology described so far is focusing on the analysis of one single panel; it allows us to recover level- and volatility-common shocks, but does not take into account any information about the block structure of the observations. As a consequence, it cannot provide any insight into the sources of these shocks. Taking into account the block structure of a large panel is precisely the aim of the method proposed by Hallin and Liška (2011).

In particular, let us use the expression "space of a (sub)panel" for the Hilbert space of all quadratic mean-convergent linear combinations of the elements of the (sub)panel, and the limits of quadratic mean-convergent sequences thereof.[9] Likewise, the "common (idiosyncratic) space" of a (sub)panel is the Hilbert space spanned by common (idiosyncratic) components of that (sub)panel. The methodology we employ relies on the natural decomposition of the spaces of $\mathbf{Y}$ or $\mathbf{s}$ into subspaces that are common or idiosyncratic to each block, along with their intersections. Such decompositions lead to a refinement, based on the block structure, of the GDFM decompositions in (1) and (5).

The intuition is that some common shocks might be block-specific. For example, a shock that is common in the Japanese subpanel, but does not belong to the US or European common spaces can be considered as a Japanese "local" shock: pervasive (in the sense of (C3)) in the Japanese markets but idiosyncratic in the US and European ones. Some other shocks, quite on the contrary, are pervasive in all subpanels, and can be considered as worldwide or "global" shocks. By construction those shocks belong to the intersection of the common spaces of all three blocks. Moreover, the decomposition we consider allows also for shocks which might be pervasive in some panels only, e.g. local shocks belonging to the intersection of the US and European common spaces but not to the common space of Japan. A full characterization of the subspaces in which such block-structured common space decomposes, leads us to account for $(2^3 - 1) = 7$ non-overlapping, i.e. mutually orthogonal, parts. Thus the space of $\mathbf{Y}$ or $\mathbf{s}$ decomposes into seven orthogonal common subspaces plus one strongly idiosyncratic space as their orthogonal complement. The main advantage of such decompositions is that they are unique, since the projections of the variables in the joint panels $\mathbf{Y}$ or $\mathbf{s}$ onto orthogonal subspaces themselves are mutually orthogonal. On the contrary, uniqueness is lost when considering a decomposition involving the three common spaces (defined by US, Europe, and Japan) only, without taking into account their possible intersections.

Let us first give a more precise account of the Hallin and Liška (2011) approach while at the end of the section we compare this approach with other existing factor models with block structure. Since the dynamic analysis of volatility shocks is the main objectives of this study, we start by giving details for the block-wise factor analysis of the volatility panel $\mathbf{s}$. Similar decompositions also hold, with obvious changes, for the panel of returns $\mathbf{Y}$.

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[9]For example, the "space of $\mathbf{s}$" is the space containing all convergent (in quadratic mean) linear combinations of the form $\eta_t = \sum_{i=1}^{N} \sum_{k=0}^{\infty} a_{ik} s_{it-k}$, along with the limits of mean-square converging sequences of such linear combinations.

A GDFM representation (5) holds for the joint panel of volatilities **s**, but also for the three subpanels $\mathbf{s}^{\mathrm{US}}$, $\mathbf{s}^{\mathrm{JP}}$, and $\mathbf{s}^{\mathrm{EU}}$ separately. Considering, for example, an element s_i^{US} of the US subpanel, its GDFM decomposition in the joint panel **s** reads

$$s_{it}^{\mathrm{US}} = \chi_{\mathrm{US};it} + \xi_{\mathrm{US};it} = \sum_{j=1}^{Q^s} d_{ij}(L)\varepsilon_{jt} + \xi_{\mathrm{US};it}, \qquad i = 1,\ldots,N^{\mathrm{US}}, \tag{7}$$

the common component of which is driven by the same Q^s shocks as in (5). Hence, for the units belonging to the US subpanel, $\chi_{\mathrm{US};it} \equiv \chi_{it}$: call it the *jointly common* component. This decomposition therefore satisfies the GDFM conditions (C1)-(C6). The same s_i^{US}, when considered in the US subpanel $\mathbf{s}^{\mathrm{US}}$ only, admits another GDFM decomposition, with, say, q_{US}^s common shocks, of the form

$$s_{it}^{\mathrm{US}} = \chi_{\mathrm{US};it}^{\mathrm{US}} + \xi_{\mathrm{US};it}^{\mathrm{US}} = \sum_{j=1}^{q_{\mathrm{US}}^s} d_{ij}^{\mathrm{US}}(L)\varepsilon_{jt}^{\mathrm{US}} + \xi_{\mathrm{US};it}^{\mathrm{US}}, \qquad i = 1,\ldots,N^{\mathrm{US}}. \tag{8}$$

The common components $\chi_{\mathrm{US};it}$ and $\chi_{\mathrm{US};it}^{\mathrm{US}}$ can be identified by analyzing either the joint panel or the US subpanel alone, respectively. Note however that the vectors $\boldsymbol{\varepsilon} = (\varepsilon_1 \ldots \varepsilon_{Q^s})'$ in (7) and $\boldsymbol{\varepsilon}^{\mathrm{US}} = (\varepsilon_1^{\mathrm{US}} \ldots \varepsilon_{q_{\mathrm{US}}^s}^{\mathrm{US}})'$ in (8), need not be mutually orthogonal. Therefore, nothing can be said about the global or local nature of the shocks driving the common components in (7) and in (8) unless a finer decomposition is considered.

Since, in the presence of three blocks, the common space of **s** is made up of seven mutually orthogonal common subspaces, the jointly common component in (7) decomposes into the sum of seven orthogonal components:

$$\chi_{\mathrm{US};it} = \varphi_{\mathrm{US};it} + \psi_{\mathrm{US/EU,JP};it} + \psi_{\mathrm{US,EU/JP};it} + \psi_{\mathrm{JP,US/EU};it} \tag{9}$$
$$+ \zeta_{\mathrm{US/EU,JP};it} + \zeta_{\mathrm{US,EU/JP};it} + \zeta_{\mathrm{JP,US/EU};it},$$

where[10]

(i) $\varphi_{\mathrm{US};i}$ belongs to the intersection of the three subpanel common spaces; call it s_i^{US}'s *strongly common* component, in the sense that its dynamics is driven by shocks which are pervasive in all three blocks, that is by global shocks;

(ii) $\psi_{\mathrm{US/EU,JP};i}$ belongs to the common space of $\mathbf{s}^{\mathrm{US}}$ but to the idiosyncratic spaces of $\mathbf{s}^{\mathrm{JP}}$ and $\mathbf{s}^{\mathrm{EU}}$, $\psi_{\mathrm{US,EU/JP};it}$ to the common spaces of $\mathbf{s}^{\mathrm{US}}$ and $\mathbf{s}^{\mathrm{EU}}$, but to the idiosyncratic space of $\mathbf{s}^{\mathrm{JP}}$, etc.; those ψ's are called *weakly common* components, since their dynamics is driven by shocks which are pervasive only in US ($\psi_{\mathrm{US/EU,JP};i}$) or pervasive for both US and Europe but not for Japan or for both US and Japan but not for Europe ($\psi_{\mathrm{US,EU/JP};i}$ or $\psi_{\mathrm{JP,US/EU};i}$, respectively);

(iii) $\zeta_{\mathrm{US/EU,JP};it}$ belongs to the idiosyncratic space of $\mathbf{s}^{\mathrm{US}}$, but to the common spaces of $\mathbf{s}^{\mathrm{JP}}$ and $\mathbf{s}^{\mathrm{EU}}$, $\zeta_{\mathrm{US,EU/JP};i}$ to the idiosyncratic spaces of $\mathbf{s}^{\mathrm{US}}$ and $\mathbf{s}^{\mathrm{EU}}$, but to the common space of $\mathbf{s}^{\mathrm{JP}}$, etc.; those ζ's are called *weakly idiosyncratic* components, since their dynamics is driven by shocks which are not pervasive for US but are pervasive for Europe and/or Japan;

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[10]The strong/weak terminology used here is completely unrelated to, and should not be confused with, the concepts of "strong" and "weak" factors developed elsewhere in the literature.

(iv) $\xi_{\text{US};i}$ belongs to the intersection of the three subpanel idiosyncratic spaces and is called *strongly* idiosyncratic, since no pervasive shock drives its dynamics.

The common shocks driving the strongly common, weakly common, and weakly idiosyncratic components are then mutually orthogonal by construction.

Using the same notation as in (9), we can also consider the decompositions of the common and idiosyncratic components in (8) which are given by

$$\chi_{\text{US};it}^{\text{US}} = \varphi_{\text{US};it} + \psi_{\text{US}/\text{EU},\text{JP};it} + \psi_{\text{US},\text{EU}/\text{JP};it} + \psi_{\text{JP},\text{US}/\text{EU};it} \qquad \text{and} \qquad (10)$$

$$\xi_{\text{US};it}^{\text{US}} = \zeta_{\text{US}/\text{EU},\text{JP};it} + \zeta_{\text{US},\text{EU}/\text{JP};it} + \zeta_{\text{JP},\text{US}/\text{EU};it} + \xi_{\text{US};it},$$

respectively. The relation between decompositions (7), (8), (9), and (10), appears more clearly by writing

$$s_{it}^{\text{US}} = \underbrace{\overbrace{\varphi_{\text{US};it} + \psi_{\text{JP},\text{US}/\text{EU};it} + \psi_{\text{US},\text{EU}/\text{JP};it} + \psi_{\text{US}/\text{EU},\text{JP};it}}^{\chi_{\text{US};it}} + \overbrace{\zeta_{\text{JP},\text{US}/\text{EU};it} + \zeta_{\text{US},\text{EU}/\text{JP};it} + \zeta_{\text{US}/\text{EU},\text{JP};it} + \xi_{\text{US};it}}}_{\xi_{\text{US};it}^{\text{US}}}.$$

Similar decompositions hold, of course, with obvious modifications in the notation, for s_i^{JP} and s_i^{EU}.

To conclude, notice that the numbers q_{US}^s, q_{EU}^s q_{JP}^s satisfy the natural constraint

$$\max\left(q_{\text{US}}^s, q_{\text{JP}}^s, q_{\text{EU}}^s\right) \leq Q^s \leq q_{\text{US}}^s + q_{\text{JP}}^s + q_{\text{EU}}^s, \qquad (11)$$

where the lower bound is attained when one of the subpanel common spaces contains the other two, while the upper bound is attained when the three common spaces do not pairwise intersect. Repeated application of the Hallin and Liška (2007) criterion on the appropriate subpanels allows us to identify the numbers of shocks $q_{\text{US}}^s, q_{\text{EU}}^s, q_{\text{JP}}^s$, and Q^s as N^{US}, N^{EU} and N^{JP}, and therefore also N, all tend to infinity. By knowing the number of common shocks in each subpanel and in the joint panel, we can infer directly the numbers of common shocks in the intersections of the subpanel common spaces. In this way, we identify the numbers of global and local shocks in returns and volatilities such that (11) is satisfied.

By disentangling local and global components, we can study the amount of total variance of returns or volatilities accounted for by each of these components. Explained variances of the strongly and weakly common components over different periods provide information about (i) possible international financial contagion signalled by changes in the explained variance of the weakly idiosyncratic components and (ii) the role of interdependencies determined by strongly and weakly common components. Specifically, the explained variances are obtained by considering the eigenvalues of their spectral densities. So, for example, the variance explained by the strongly common component of US volatilities is given by

$$EV_{\varphi_{\text{US}}} = \frac{\sum_{j=1}^{q^\varphi} \int_{-\pi}^{\pi} \lambda_{j;\varphi_{\text{US}}}(\theta)\mathrm{d}\theta}{\sum_{j=1}^{N^{\text{US}}} \int_{-\pi}^{\pi} \lambda_{j;\varphi_{\text{US}}}(\theta)\mathrm{d}\theta},$$

where q^φ is the number of global shocks and $\lambda_{j;\varphi_{\text{US}}}(\theta)$ is the jth largest eigenvalue of the spectral density of $\varphi_{\text{US};it}$ at frequency θ.

Note that by looking at decompositions (7), (8), (9), and (10), we immediately see that strongly idiosyncratic terms are readily available from the estimation of the GDFM for the

joint panel (see (5)) and weakly idiosyncratic terms are obtained by subtracting subpanel common components $\chi_{\mathrm{US};it}^{\mathrm{US}}$ from joint panel common components $\chi_{\mathrm{US};it}$. Finally, by projecting $\chi_{\mathrm{US};it}^{\mathrm{US}}$ on the intersection of the three subpanel common spaces, we obtain the strongly common terms, while weakly common terms are the residuals of such projections. We refer to Hallin and Liška (2011) for details.

An analogous decomposition holds for the panels and subpanels of returns Y_i^{US}, Y_i^{JP}, and Y_i^{EU}, with block-specific numbers of shocks q_{US}, q_{JP}, and q_{EU} satisfying

$$\max\left(q_{\mathrm{US}}, q_{\mathrm{JP}}, q_{\mathrm{EU}}\right) \leq Q \leq q_{\mathrm{US}} + q_{\mathrm{JP}} + q_{\mathrm{EU}}, \tag{12}$$

where Q is the number of common shocks in (1). With obvious notation, we have the decomposition

$$Y_{it}^{\mathrm{US}} = \underbrace{\overbrace{\underbrace{\varphi_{\mathrm{US};it}^{Y} + \psi_{\mathrm{JP},\mathrm{US/EU};it}^{Y} + \psi_{\mathrm{US},\mathrm{EU/JP};it}^{Y} + \psi_{\mathrm{US/EU},\mathrm{JP};it}^{Y}}_{X_{\mathrm{US};it}^{\mathrm{US}}} + \underbrace{\zeta_{\mathrm{JP},\mathrm{US/EU};it}^{Y} + \zeta_{\mathrm{US},\mathrm{EU/JP};it}^{Y} + \zeta_{\mathrm{US/EU},\mathrm{JP};it}^{Y}}_{Z_{\mathrm{US};it}^{\mathrm{US}}}}^{X_{\mathrm{US};it}} + Z_{\mathrm{US};it}}.$$

Notice, however, that the block structure of returns, although of independent interest, is not needed for building and analyzing the volatility panel (Stage 2 of previous section).

An alternative factor-based approach exploiting the block-structure of the data is the hierarchical factor model considered in Kose et al. (2003), Moench and Ng (2011), and Moench et al. (2013). However, those authors propose a decomposition which is substantially different from the one illustrated in this section. First and foremost, local shocks, in their hierarchical approach, only can be specific to one given block: using our terminology, the intersections between the common spaces of two subpanels are assumed to be empty. This is in contrast both with the spirit of our structural analysis, in which we have no prior knowledge on where common shocks belong to, and with the empirical findings in next section, where both for returns and volatilities we find non-empty intersections of the US and European common subspaces and no global shocks in returns.[11]

Second, there is no method available for identifying the numbers of global and local shocks, and typically one global and one local factor for each block are assumed (see e.g. Moench et al., 2013), which is quite restrictive. As explained above, our approach instead relies on a data-driven identification of those numbers. Third, hierarchical models are formulated in a state space form, hence common shocks drive some latent factors, which in turn are loaded by the observable variables.[12] Therefore, as pointed out by Moench and Ng (2011), the responses of variables to shocks to local factors in a given block can differ only to the extent that their exposure to the block-level factors differs. On the contrary, in our analysis the impulse response functions are unconstrained, thus allowing for more heterogeneity in the dynamics induced by the common shocks.

Summing up, by imposing more assumptions, hierarchical factor models are nested into our more general and fully dynamic approach. On the other hand, all decompositions considered in this paper constitute *representation results*, not *assumptions*—meaning that, in sharp

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[11]Trivially, in the absence of a global shock, the hierarchical factor approach reduces to the factor analysis of unrelated, i.e. disjoint, blocks.

[12]Note also that in principle their model is formulated with dynamic loadings, while their applications, due to identification issues, only consider static loadings—i.e. their factors are loaded contemporaneously by the observable variables.

contrast with the hierarchical approach, their existence and uniqueness holds without any additional restrictions (besides the existence of a spectrum) on the data-generating process.

Clearly, allowing for such a general approach also has a price: the complexity of the decomposition increases with the number of blocks, since G blocks yield $(2^G - 1)$ common subspaces and, of course, one strongly idiosyncratic space as their orthogonal complement. Therefore, dealing with more than the three blocks considered in this paper might be computationally (but not theoretically) hard and is left for further research.

4 Common shocks in US, Europe, and Japan

In this section, we apply the methodology outlined in Sections 2 and 3 to analyze a three-block panel of stocks belonging to the US, European, and Japanese financial markets. Returns are computed from the daily closing prices of the constituents of three of the most popular stock market indices: Standard & Poor's 500, Standard & Poor's Europe 350, and Nikkei 225. The data is collected from December 31, 1999 to August 31, 2015, which corresponds to a sample size of $T = 4086$ trading days. In order to work with a balanced panel, only the constituents priced over the full sample are retained. We are then left with $N = 830$ stocks: $N^{\text{US}} = 336$ stocks in the US market (henceforth SP500), $N^{\text{EU}} = 293$ stocks in the European market (henceforth SPEU350), and $N^{\text{JP}} = 201$ stocks in the Japanese market (henceforth NKK225).[13]

In Sections 4.1 and 4.2, we present the block decomposition of the returns and volatility panels, respectively. In Section 4.3 we describe the dynamics of the estimated shocks with respect to the dynamic of the financial markets. Finally, in Section 4.4 we apply the block decomposition to the assessment of interdependencies and financial contagion.

4.1 Common shocks in returns

It is well known that stock returns are strongly cross-correlated, and this finding justifies a factor approach (see e.g. the empirical studies in Connor et al., 2006 and Sentana et al., 2008). In addition, there is also recent empirical evidence of little but nevertheless significant predictability in returns due to their (cross-)autocorrelations (see e.g. Rapach and Zhou, 2013, for a review of stock return predictability, Giovannelli et al., 2017 for an application of the GDFM to the forecasting of returns, and Wongswan, 2006, for a study based on high-frequency data).

These findings are confirmed when looking at the cross-correlations in our dataset, as shown in Figure 1, where we also highlight those correlations which are 5% significant according to confidence intervals; the latter are computed through wild-bootstrap, in order to account for conditional heteroskedasticity (Mammen, 1993, Gonçalves and Kilian, 2004). Most contemporaneous correlations are significant, but several of them stay significant even at lags 1 and 2. Strongly significant correlations are found between NKK225 stocks and lagged values of the others, reflecting the effect of time zone in Japan - i.e., the fact that events in US and European markets occur while the Tokyo Stock Exchange is closed, so that no Japanese reaction can take place until the next business day. The same can be said about the correlations between SPEU350 and lagged SP500.

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[13]The complete list of the stocks is available in a complementary Appendix to this paper.

Figure 1: CROSS-CORRELATIONS OF RETURNS
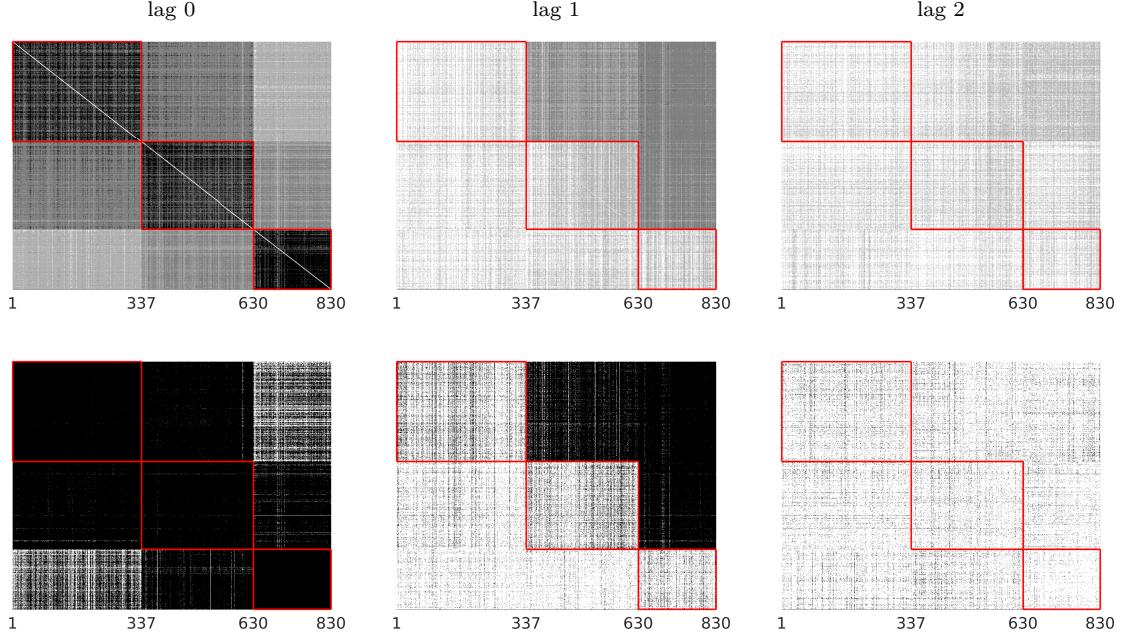
Each matrix is made of $N \times N$ cells, indexed by couples of stock; the ordering of the stocks is US (1-336), EU (337-629), JP (630-830). In the top panels we report correlations ρ_k between returns at lags $k = 0$, 1 and 2, using the following color code: white for $\rho_k < 0$; light-grey for $0 \leq \rho_k < 0.1$; dark-grey for $0.1 \leq \rho_k < 0.3$; black for $0.3 \leq \rho_k \leq 1$. In the bottom panels, still for lags $k = 0$, 1 and 2, the 5%-significant values according to wild-bootstrap based critical values are show in black.
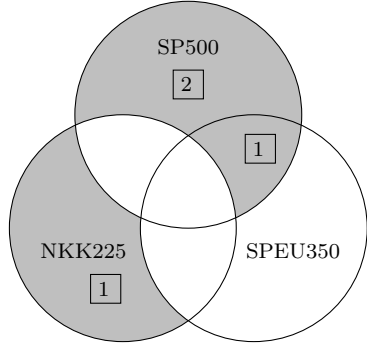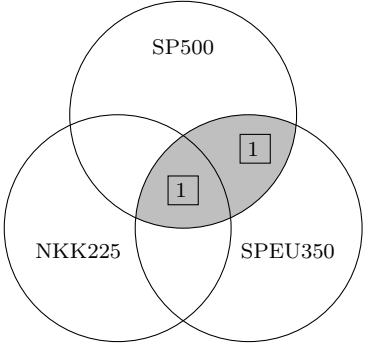
Before estimating the GDFM for the returns we employ the test proposed by Trapani (2016) for the null hypothesis $\mathsf{E}[Y_{it}^4] = \infty$ against the alternative of finite fourth moments. When applied to our panel of returns, this test rejects the null hypothesis at 1% significance level for all N returns considered.

The Hallin and Liška (2007) criterion indicates $Q = 4$ common shocks driving the dynamics of the joint panel of returns. In the top left panel of Table 1 we show the number of shocks (still obtained via the Hallin and Liška (2007) criterion) in each subpanel and in all pairwise unions. From these results we obtain the graphic in the bottom left panel, which shows the origin of the four common shocks. In the figure, empty common subspaces are white while the shaded ones are those in which some common shocks are present (their number is given in the square box). In particular, we find (note the absence of strongly common, global, shocks)

 (i) one shock, common in the US and Europe but idiosyncratic in Japan, driving the weakly common components of US and European returns and the weakly idiosyncratic components of Japanese returns;

 (ii) two local shocks, common only in the US, which are driving the weakly common components of US returns and the weakly idiosyncratic components of European and Japanese returns;

(iii) one local shock, common only in Japan, driving the weakly common components of Japanese returns and the weakly idiosyncratic components of US and European returns.

Therefore, using the notation of Section 3, the decomposition of returns, together with the

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Table 1: NUMBER OF COMMON SHOCKS

(sub)panels	Returns	Volatilities
SP500	3	2
NKK225	1	1
SPEU350	1	2
SP500 ∪ SPEU350	3	2
SP500 ∪ NKK225	4	2
SPEU350 ∪ NKK225	2	2
SP500 ∪ SPEU350 ∪ NKK225	4	2

percentages of variance explained by each component, reduces to

$$\mathbf{Y}^{\text{US}} = \underset{23.70\%}{\boldsymbol{\psi}^Y_{\text{US/EU,JP}}} + \underset{14.04\%}{\boldsymbol{\psi}^Y_{\text{US,EU/JP}}} + \underset{0.13\%}{\boldsymbol{\zeta}^Y_{\text{US,EU/JP}}} + \underset{62.13\%}{\mathbf{Z}_{\text{US}}},$$

$$\mathbf{Y}^{\text{EU}} = \underset{32.80\%}{\boldsymbol{\psi}^Y_{\text{US,EU/JP}}} + \underset{0.20\%}{\boldsymbol{\zeta}^Y_{\text{US,EU/JP}}} + \underset{1.68\%}{\boldsymbol{\zeta}^Y_{\text{EU,JP/US}}} + \underset{65.32\%}{\mathbf{Z}_{\text{EU}}},$$

$$\mathbf{Y}^{\text{JP}} = \underset{39.27\%}{\boldsymbol{\psi}^Y_{\text{JP/US,EU}}} + \underset{0.27\%}{\boldsymbol{\zeta}^Y_{\text{JP/US,EU}}} + \underset{1.12\%}{\boldsymbol{\zeta}^Y_{\text{EU,JP/US}}} + \underset{59.34\%}{\mathbf{Z}_{\text{JP}}}.$$

The various components, constituting the common component of the joint panel, account for (sum of the first three terms on the right hand side) about 35%-40% of the total variance within each market (strongly idiosyncratic terms in the three subpanels therefore explain between about 60%-65%). In Europe the US-European shock is, by far, the main term in the joint common component. Similarly, the local shock in Japan explains almost all the common variance. A more balanced situation is found in the US between the variance explained by the two local shocks and the variance explained by the US-European shock.

Finally, the absence of a global return shock, together with the very small role played by weakly idiosyncratic components, is consistent with the idea that the markets are not perfectly integrated, and are influenced by the so-called *home bias*, implying that domestic assets are largely overweighted in portfolios (see French and Poterba, 1991 and Tesar and Werner, 1994, for early references, and Coeurdacier and Rey, 2013 and Petzev et al., 2016, for more recent evidence). Note that, as pointed out in Section 3, ours are unique decompositions into orthogonal components which are related to the origins of the dynamic forces driving comovements; therefore, all international correlations due to pervasive shocks in the panel of returns are accounted for.

Figure 2: CROSS-CORRELATIONS OF VOLATILITIES
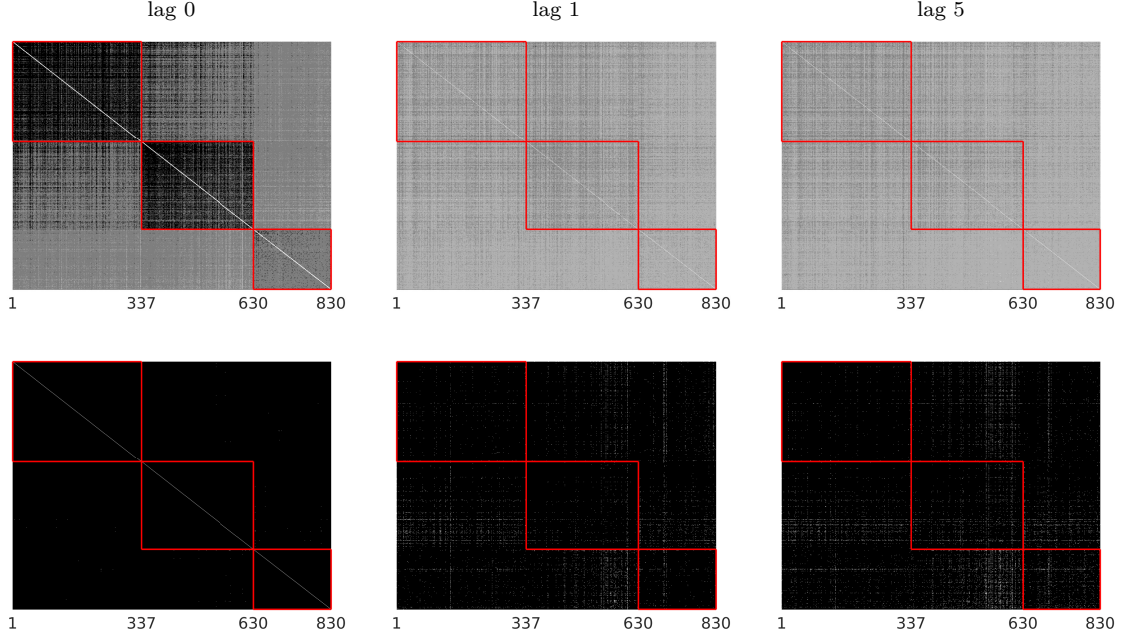
Each matrix is made of $N \times N$ cells, indexed by couples of stock; the ordering of the stocks is US (1-336), EU (337-629), JP (630-830). In the top panels we report correlations ρ_k between volatilities at lags $k = 0$, 1 and 5, using the following color code: white for $\rho_k < 0$; light-grey for $0 \leq \rho_k < 0.1$; dark-grey for $0.1 \leq \rho_k < 0.2$; black for $0.2 \leq \rho_k \leq 1$. In the bottom panels, still for lags $k = 0$, 1 and 5, the 5%-significant values according to wild-bootstrap based critical values are shown in black.

4.2 Common shocks in volatilities

From the estimation of the GDFM of returns in Stage 1 of our methodology (Section 2.1), we obtain the residuals needed for building the panel of volatility proxies (4) as described in Stage 2 (Section 2.2). As a preliminary analysis, we look at the N distributions of the processes $e_i + v_i$ to check that effectively they do not have too much mass around zero. Following again the methodology by Trapani (2016), the null hypothesis $\mathsf{E}[s_{it}^4] = \infty$ is always rejected at 1% significance level in favour of the alternative of finite fourth moments. This shows that in our data large deviations in the left tail of the distribution of s_i, corresponding to cases in which $e_{it} + v_{it} \simeq 0$, are not an issue.[14] In view of these findings our volatility proxies are indeed well defined also empirically and can be used for the subsequent analysis.

Consistently with the fact that volatilities are know to display strong persistency (see e.g. Andersen et al., 2003), the cross-correlations of the estimated volatility panel stay significant for many lags, as shown in Figure 2 where again confidence bounds are computed via wild-boostrap to account for possible conditional heteroskedasticity in volatilities (see e.g. Corsi et al., 2008).[15]

The Hallin and Liška (2007) criterion indicates $Q^s = 2$ common shocks driving the dynamics of the joint panel of volatilities. In the top right panel of Table 1 we show the numbers of common shocks in each subpanel and their pairwise unions.[16] From these results we obtain

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[14] The distribution of s_i, not shown, is highly concentrated around zero for all $i = 1, \ldots, N$.

[15] Results are unaffected, though, if we use the classical confidence bands $\pm 1.96\sqrt{T^{-1}}$.

[16] In our application of the Hallin and Liška (2007) criterion we consider only the frequency band $\left[\frac{\pi}{252}, \frac{2\pi}{3}\right]$

the graph in the bottom right panel, which shows the origin of the two common shocks. In this figure, empty subspaces are white while the shaded ones are those in which some common shocks are present (their number is given in the square box). In particular, we find

(i) one global shock, common to the volatilities of all three stock markets, driving the volatility strongly common components of all stocks in the dataset;

(ii) one shock, common in US and Europe but idiosyncratic in Japan, which drives the weakly common components of US and European volatilities and the weakly idiosyncratic components of Japanese volatilities.

The corresponding decomposition of volatilities, along with the percentages of variance explained by each term, are

$$\mathbf{s}^{\text{US}} = \underset{17.21\%}{\boldsymbol{\varphi}_{\text{US}}} + \underset{7.31\%}{\boldsymbol{\psi}_{\text{US,EU/JP}}} + \underset{75.48\%}{\boldsymbol{\xi}_{\text{US}}},$$

$$\mathbf{s}^{\text{EU}} = \underset{15.95\%}{\boldsymbol{\varphi}_{\text{EU}}} + \underset{6.42\%}{\boldsymbol{\psi}_{\text{US,EU/JP}}} + \underset{77.63\%}{\boldsymbol{\xi}_{\text{EU}}},$$

$$\mathbf{s}^{\text{JP}} = \underset{12.97\%}{\boldsymbol{\varphi}_{\text{JP}}} + \underset{0.57\%}{\boldsymbol{\zeta}_{\text{JP/US,EU}}} + \underset{86.46\%}{\boldsymbol{\xi}_{\text{JP}}}.$$

Although the joint common components of volatilities (the sum of the first two terms on the right-hand side) explain less variance than in the panel of returns, the presence of a global volatility shock is a sign of higher interconnectedness. That strongly common shock explains about 17%, 16%, and 13% of US, European, and Japanese volatilities, respectively. The US-Europe shock explains 7% and 6% percent of volatilities in US and Europe, respectively, while it has very little impact in Japan. Summing up, these results show that (a) volatilities are strongly interconnected, thus confirming the view that risk premia have an international flavor (see e.g. Karolyi and Stulz, 2003, and references therein), and that (b) the common shocks driving volatilities are in general not of the same nature and origin as those driving returns (see also the results in Barigozzi and Hallin, 2015 for the Standard &Poor 100 stocks).

4.3   Common shocks dynamics

We now turn to a comparative study of the dynamics of the return and volatility common shocks in "stable" and "turmoil" periods. In what follows, we define the "turmoil" sample as the subsample corresponding to the period starting on August 9, 2007, when a press release by BNP Paribas mentioned a 'complete evaporation of liquidity in certain market segments of the US securitisation market', and terminating on July 26, 2012, when ECB president Mario Draghi voiced his defense for the Euro 'whatever it takes'. This period is characterized by the so-called Credit Crunch and it comprises the two major financial crises of the recent years: the Great Financial Crisis (2007-2009), originating in the US, and the European Sovereign Debt Crisis (2011-2012).[17]

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corresponding to oscillations with periods comprised between two years and three days, so that the spectral densities are not affected by proxies' noise and long memory in volatilities. We also checked that all volatilities can be considered stationary as their fractional integration coefficients are below 0.5.

[17]Although there is no clear consensus about the choice of the starting and ending days of that period, practitioners typically view them in mid 2007 and mid 2012, as we do: see, for example, `www.theguardian.com/business/2012/aug/07/credit-crunch-boom-bust-timeline`. Our results are robust with respects to alternative and similar choices of this sub-sample.

Figure 3: RETURN COMMON SHOCKS $(u_{1t}, \ldots, u_{4t})$
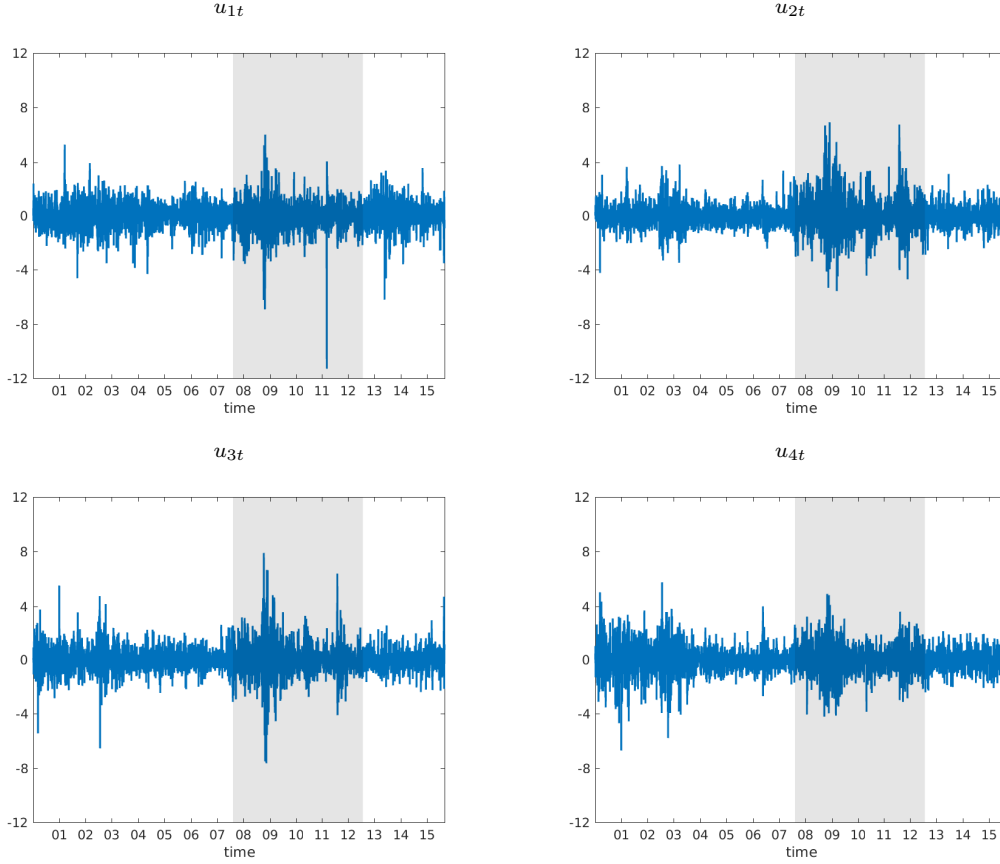
Shocks are standardized to have zero mean and unit variance. Shaded areas denote the turmoil subsample: August, 9 2007- July, 26 2012.

First, we analyze the common shocks. Those shocks are in general estimated up to an invertible linear transformation, as discussed in Sections 2.1 and 2.2, respectively. Inspection of the estimated common shocks of returns $(u_{1t}, \ldots, u_{4t})$ in Figure 3 reveals an exceptional and prolonged commonality due to many large common shocks during turmoil period. Although those shocks are orthornormal by construction they are not fully identified so we refrain to attach them any economic meaning but we limit ourselves to noticing that some of the main events influencing the markets under study are reflected in their respective dynamics. The first shock u_{1t} seems to capture some features of the Japanese economy, displaying a huge lonely spike on March 11, 2011, day of the Tohoku earthquake and tsunami (which witnessed the third steepest percentage fall in Nikkei's history), and the effects of fiscal stimulus, monetary easing, and structural reforms, consequences of Shinzō Abe's policies (the so-called 'Abenomics") are visible throughout 2013. The second and third shocks u_{2t} and u_{3t} both are capturing the high volatility of the 2011-2012 period associated to the European Sovereign Debt Crisis, but also the turbulence due the Great Financial Crisis. The fourth shock u_{4t} is characterized instead by a higher volatility in the the market downturn period between 2000 and 2003 which was related to the dot-com bubble burst, the 9/11 attacks, and the second Iraq war (a so-called "bear" market period).

In Figure 4 we similarly report (left-hand panels) the two log-volatility common shocks,

the strongly common or "global one" $\varepsilon_t^{*\text{global}}$ and the US-EU common one $\varepsilon_t^{*\text{US-EU}}$, identified in accordance with the restrictions described in the next section, along with (right-hand panels) their exponentials $\exp(\varepsilon_t^{*\text{global}})$ and $\exp(\varepsilon_t^{*\text{US-EU}})$, which are more readable (for simplicity, we also call them volatility shocks). Similarly to the return shocks in Figure 3, we find that the turmoil period witnesses some important extreme events. In particular, for the global shock $\varepsilon_t^{*\text{global}}$, we find four large spikes on

(i) January 4, 2008, a stock market fall following the release of the two-years highest figures in US unemployment rates;

(ii) March 23, 2009, a positive shock after US Treasury announced a bailout plan consisting in a 1 trillion USD purchase of toxic bank assets;

(iii) August 18, 2011, a stock market fall with an increase in the VIX index of 35% due to fears related to the European Sovereign Debt Crisis;

(iv) January 5, 2015, a stock market fall depicted by media as a global shock due to falling oil prices.

Minor spikes are observed also in the "bear" market period of 2000-2003. The US-Europe shock $\varepsilon_t^{*\text{US-EU}}$ displays the same volatility clusters during the Global Financial Crisis, the European Sovereign Debt Crisis, and "bear" market period, but are of low magnitude when compared with the global shocks. These facts are in line with previous findings in the literature about increasing volatility spillovers across markets during economic downturns (see e.g. Diebold and Yılmaz, 2009, 2012).

In order to compare the dynamics of these shocks with the observed market behavior, we also report in Figure 5 the daily returns and annualised realized volatilities of the aggregate market indexes (details about the realized volatility measures used are in Section 5.2).

4.4 Co-movements, contagion, and volatility spillovers

The results described in the previous sections indicate that the turmoil period has witnessed an unusual number of large shocks, both in returns and volatilities. While increased co-movements are likely to reflect the fact that financial agents in all markets react in a similar way when increased risk is perceived, the economics underlying such dynamics is still unclear. Two alternative theories are possible: interdependence and contagion.

Albeit various definitions can be found in the international finance literature, financial *contagion* is usually depicted as the spread of exceptionally adverse shocks originating abroad and transmitted into local financial markets in a way which has little to do with the international shock transmission observed in stable periods (see the surveys of Karolyi and Stulz, 2003; Karolyi, 2003, and references therein). That is, contagion does not necessarily take place if larger shocks are observed to hit markets around the globe but some sort of change must be observed in the transmission mechanism such that the usual international interdependence of stock markets cannot explain increased co-movements.

Much of the empirical evidence intended to distinguish contagion from interdependence, and volatility spillovers from efficient spread of information is based on the analysis of pairwise cross-correlation coefficients among market indices and their variations across time (see e.g. Calvo and Reinhart, 1996; Boyer et al., 1997; Forbes and Rigobon, 2002, the survey of Claessens et al. 2001 and references therein). However, Pesaran and Pick (2007) and Corsetti

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Figure 4: IDENTIFIED VOLATILITY SHOCKS $\varepsilon_t^{*\text{global}}$ AND $\varepsilon_t^{*\text{US-EU}}$
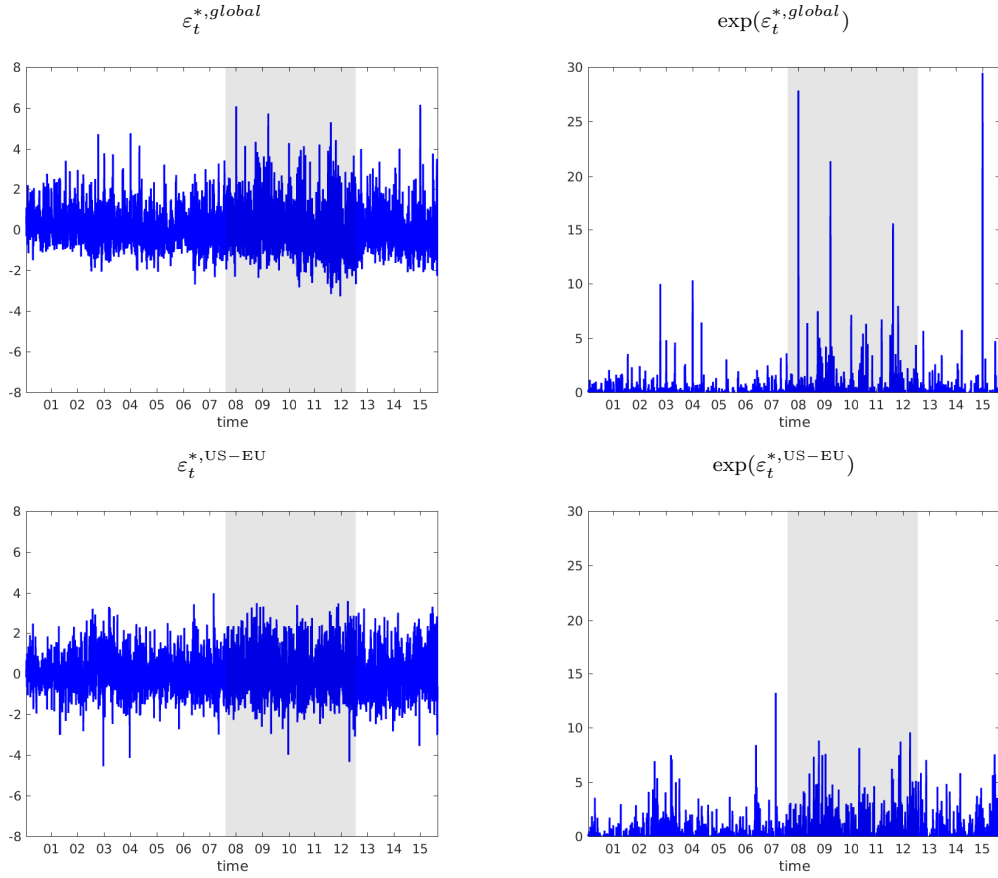
Shocks in the left panel are standardized to have zero mean and unit variance. Shaded areas correspond to the turmoil period August, 9 2007- July, 26 2012.

et al. (2005) raised some concerns about this approach because common and market specific components should be accounted for if unbiased estimates are to be constructed. Our analysis provides an ideal tool for addressing such issues, since it allows us to compare the behaviors of global and local volatility and return shocks during turmoils, and how they contribute to the increase in co-movements.

In order to shed light on whether financial contagion or just interdependence is to blame for increased co-movements in returns and volatilities, we report, in Table 2, results for three sub-samples in which the variance decompositions are performed in the stable period ending on August 8, 2007, the turmoil period, and the subsequent stable period starting on July 27, 2012. In Table 3, we also show the changes, from one period to the next, in the proportion of common variance explained by the strongly common shocks. For example, we compute the ratio of the variance of the strongly common component φ_{US} to that of $\chi_{\text{US}} = \varphi_{\text{US}} + \psi_{\text{US,EU/JP}}$ in stable and turmoil periods, respectively.

Results are obtained without re-computing the numbers of common shocks in each sub-sample. Indeed, if structural breaks are present, then, at worst, those numbers could be overestimated.[18] However, the cross-sectionally averaged overestimation error is known to

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[18]This result is well known in the static factor model case. If there are q factors and there is one change in the loadings, then, when determining the number of factors using the whole sample, we would find $2q$ factors

Figure 5: RETURNS AND REALIZED VOLATILITIES OF AGGREGATE MARKET INDEXES
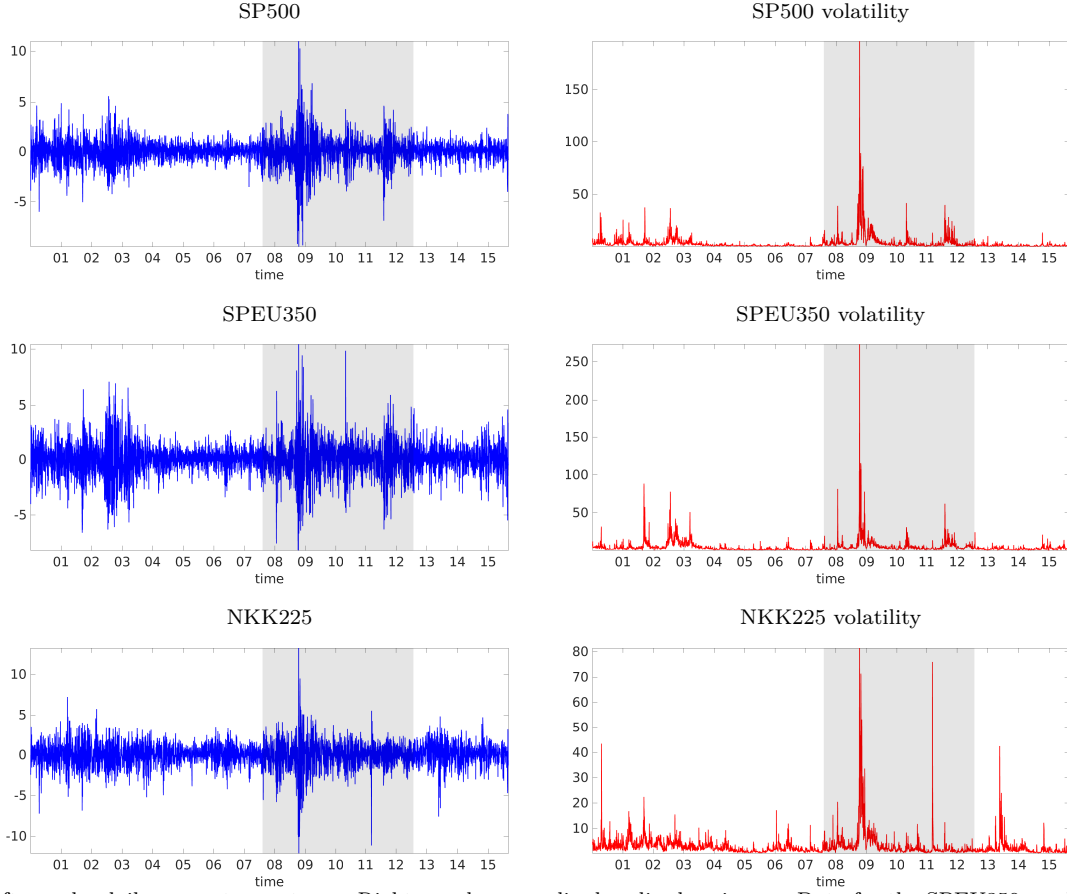
Left panels: daily percentage returns. Right panels: annualised realized variances. Data for the SPEU350 market come from the EUROSTOXX50 index. Shaded areas denote the turmoil subsample: August, 9 2007- July, 26 2012.

be asymptotically negligible as the cross-sectional dimension increases (see Corollary 2 in Forni et al., 2000 and also Proposition 1 in Onatski, 2015 for an analogous result in the static factor model case). Hence, our measures of explained variances, which are obtained via cross-sectional averaging, are likely to be unaffected by possible mis-specifications. This is confirmed by the results in a complementary Appendix to this paper, computed for different numbers of shocks.

As already mentioned, the presence of a global volatility shock implies pervasive volatility interdependence. In particular, for all subpanels the increase in commonality (namely, the variances of the common components $\chi_{\text{US};it}^{\text{US}}$, $\chi_{\text{EU};it}^{\text{EU}}$ and $\chi_{\text{JP};it}^{\text{JP}}$; see (10)) observed during the turmoil period is almost entirely due to the strongly common components φ_{US}, φ_{EU} and φ_{JP}, which are driven by the global (strongly common) shock $\varepsilon_t^{*\text{global}}$.

If the global shock were the unique cause of international volatility co-movements, there would be no room for contagion. However, the US-European shock $\varepsilon_t^{*\text{US-EU}}$ could be a source of contagion for the Japanese market. This is only partly true, as indicated by the constant,

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at most, and at least q of them. Similarly, if after a break a factor appears or disappears, we recover the maximum number of factors ($q+1$ or q respectively) when considering the whole sample for estimation of their number (see e.g. Corradi and Swanson, 2014). The same reasoning carries through to the present dynamic factor model case.

but small, increase in the variance explained by the weakly idiosyncratic Japanese component $\zeta_{\mathrm{JP/US,EU}}$. In general, the US-European shock explains a smaller amount of variance than the global one and its contribution in the second stable period is actually found to increase, rather than decrease, probably reflecting the fact that although the aftermath of the European Sovereign Debt Crisis has relieved fears, concerns nevertheless remain about the still shaky banking system in some of the Eurozone countries.

In the panel of returns, the only possible interdependence is between the US and European markets, induced by the weakly common component $\boldsymbol{\psi}^Y_{\mathrm{US,EU/JP}}$. The variance explained by this component in both markets increases considerably during the turmoil period, especially in Europe. On the other hand, this shock has almost no impact on the Japanese market. We also notice increased interdependence within the US and Japanese markets, as shown by the increase in commonality due to the local weakly common components $\boldsymbol{\psi}^Y_{\mathrm{US/EU,JP}}$ and $\boldsymbol{\psi}^Y_{\mathrm{JP/US,EU}}$, respectively. Finally, the US shock seems to have a small effect on the Japanese market during the turmoil period, as shown by the increased variance of $\boldsymbol{\zeta}^Y_{\mathrm{JP,EU/US}}$. Finally, the fact that Europe apparently becomes more interconnected to the US market after 2007 can be seen as a sign of the spread of the Great Financial Crisis from US to Europe; given the nature of the common shock that generates this co-movement, however, we cannot speak of contagion.

5 The dynamic effects of volatility shocks

We now focus, in Section 5.1, on the identification of volatility shocks and their dynamic effects while, in Section 5.2, we investigate the predictive power of these shocks in the analysis of realized measures of variances.

5.1 Impulse response functions

While the results in Section 4.4 are invariant within the class of moving average representations (6), an identification step is required for the analysis of impulse response functions to the two volatility shocks $\varepsilon^{*\mathrm{global}}_t$ and $\varepsilon^{*\mathrm{US\text{-}EU}}_t$. The moving average representations (6) of the common components $\boldsymbol{\chi}_t$ have the form

$$\boldsymbol{\chi}_t = \overline{\mathbf{D}}\left(L\right)\bar{\boldsymbol{\varepsilon}}_t$$

where $\overline{\mathbf{D}}\left(L\right) = \mathbf{D}\left(L\right)\mathbf{R}^{-1}$, $\bar{\boldsymbol{\varepsilon}}_t = \mathbf{R}\boldsymbol{\varepsilon}_t$ for an arbitrary invertible linear transformation $\mathbf{R}$.

As customary in the literature, we restrict our choice to orthogonal transformations, therefore in the present setting $\mathbf{R} \equiv \mathbf{R}(\omega)$ is a $Q^s = 2$-dimensional orthogonal matrix, which depends on a single angle $\omega \in [0, 2\pi]$. Identification of impulse responses requires to find a value for ω such that a given set of constraints is satisfied by $\mathbf{D}\left(L\right)\mathbf{R}'(\omega)$. To achieve just-identification we then need to impose only one restriction.

The time zones of the markets considered imply that when the US and European markets open the Japanese market is already closed. Therefore, no stock in the NKK225 subpanel should have a contemporaneous reaction to the US-Europe volatility shock. Thus, there exists in principle many possible equivalent restrictions—one for each NKK225 stock; in other words, we are in presence of $(N^{\mathrm{JP}} - 1)$ over-identifying restrictions. Rather than imposing one restriction on one single and arbitrarily chosen Japanese stock, leaving the remaining ones unrestricted, we take the rotation that is closest (in the mean square sense) to fulfilling all

Table 2: Explained variances in returns and volatilities: turmoil vs. stable periods

Returns

period	$\mathbf{Y}^{\mathrm{US}}$				$\mathbf{Y}^{\mathrm{EU}}$				$\mathbf{Y}^{\mathrm{JP}}$			
	$\psi^Y_{\mathrm{US/EU,JP}}$	$\psi^Y_{\mathrm{US,EU/JP}}$	$\zeta^Y_{\mathrm{US,EU/JP}}$	$\mathbf{Z}_{\mathrm{US}}$	$\psi^Y_{\mathrm{US,EU/JP}}$	$\zeta^Y_{\mathrm{US,EU/JP}}$	$\zeta^Y_{\mathrm{EU,JP/US}}$	$\mathbf{Z}_{\mathrm{EU}}$	$\psi^Y_{\mathrm{JP/US,EU}}$	$\zeta^Y_{\mathrm{JP/US,EU}}$	$\zeta^Y_{\mathrm{EU,JP/US}}$	$\mathbf{Z}_{\mathrm{JP}}$
Stable (2000 - 2007)	19.69	8.54	0.21	71.56	24.68	0.36	2.39	72.57	29.20	0.53	1.91	68.36
Turmoil (2007 - 2012)	29.83	23.34	0.35	46.48	43.96	0.55	2.66	52.83	50.34	0.51	2.71	46.44
Stable (2012 - 2015)	25.58	12.42	0.32	61.68	32.61	0.45	2.26	64.68	44.55	0.39	1.16	53.90

Volatilities

period	$\mathbf{s}^{\mathrm{US}}$			$\mathbf{s}^{\mathrm{EU}}$			$\mathbf{s}^{\mathrm{JP}}$		
	φ_{US}	$\psi_{\mathrm{US,EU/JP}}$	ξ_{US}	φ_{EU}	$\psi_{\mathrm{US,EU/JP}}$	ξ_{EU}	φ_{JP}	$\zeta_{\mathrm{JP/US,EU}}$	ξ_{JP}
Stable (2000 - 2007)	12.14	8.31	79.55	12.61	5.97	81.42	11.96	0.88	87.16
Turmoil (2007 - 2012)	24.80	6.57	68.63	22.89	6.21	70.90	16.91	1.28	81.81
Stable (2012 - 2015)	12.74	7.54	79.72	10.11	11.09	78.80	10.91	2.26	86.83

The first stable period is from January 17, 2000 to August 8, 2007, the turmoil period is from August 9, 2007 to July 26, 2012, the second stable period is from July 27, 2012 to August 31, 2015. All numbers are percentages of explained variance.

Table 3: Changes in explained variances in returns and volatilities: Turmoil vs. stable periods

Returns

from	to	$\mathbf{Y}^{\text{US}}$			$\mathbf{Y}^{\text{EU}}$			$\mathbf{Y}^{\text{JP}}$		
		$\psi^Y_{\text{US/EU,JP}}$	$\psi^Y_{\text{US,EU/JP}}$	$\zeta^Y_{\text{US,EU/JP}}$	$\psi^Y_{\text{US,EU/JP}}$	$\zeta^Y_{\text{US,EU/JP}}$	$\zeta^Y_{\text{EU,JP/US}}$	$\psi^Y_{\text{JP/US,EU}}$	$\zeta^Y_{\text{JP/US,EU}}$	$\zeta^Y_{\text{EU,JP/US}}$
Stable (2000 - 2007)	Turmoil (2007 - 2012)	+35.65	+52.04	+0.49	+70.29	+0.69	+0.98	+66.81	-0.06	+2.53
Turmoil (2007 - 2012)	Stable (2012 - 2015)	-7.94	-20.40	-0.06	-24.06	-0.21	-0.85	-10.81	-0.22	-2.89

Volatilities

| from	to	$\mathbf{s}^{\text{US}}$		$\mathbf{s}^{\text{EU}}$		$\mathbf{s}^{\text{JP}}$	
		φ_{US}	$\psi_{\text{US,EU/JP}}$	φ_{EU}	$\psi_{\text{US,EU/JP}}$	φ_{JP}	$\zeta_{\text{JP/US,EU}}$
Stable (2000 - 2007)	Turmoil (2007 - 2012)	+61.91	-8.51	+55.33	+1.29	+38.55	+3.12
Turmoil (2007 - 2012)	Stable (2012 - 2015)	-38.44	+3.09	-43.92	+16.77	-32.99	+5.39

The first stable period is from January 17, 2000 to August 8, 2007, the turmoil period is from August 9, 2007 to July 26, 2012, the second stable period is from July 27, 2012 to August 31, 2015. All numbers are changes in shares of explained variances relative to the explained variance of the joint common components ($\mathbf{X}_{\text{US}}$, $\mathbf{X}_{\text{EU}}$, $\mathbf{X}_{\text{JP}}$ for returns and χ_{US}, χ_{EU}, χ_{JP} for volatilities) in the previous sample.

Figure 6: Impulse response functions to the volatility shocks
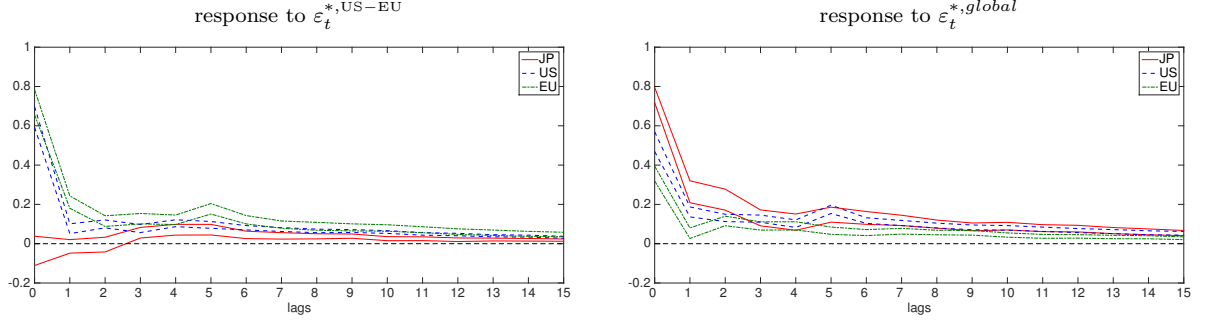
25th and 75th percentiles of impulse response functions to the identified volatility shocks in the three markets considered. Solid line: Japan; dashed line: US; dashed-dotted line: Europe.

those over-identification restrictions. That is, we look for the angle ω^* such that

$$\omega^* = \arg\min_{\omega \in [0,2\pi]} \sum_{i \in \text{NKK225}} \left[\mathbf{D}(0)\,\mathbf{R}'(\omega) \right]_{i,1}^2.$$

The identified impulse responses are then given by $\mathbf{D}^*(L) = \mathbf{D}(L)\mathbf{R}'(\omega^*)$ and the identified shocks, reported in Figure 4, are computed as

$$\boldsymbol{\varepsilon}_t^* = \begin{pmatrix} \varepsilon_t^{*,\text{US}-\text{EU}} \\ \varepsilon_t^{*,\text{global}} \end{pmatrix} = \mathbf{R}(\omega^*)\boldsymbol{\varepsilon}_t. \tag{13}$$

In Figure 6, for each market we show the 25-th and 75-th percentiles of the distribution across stocks of impulse response functions to the identified shocks - i.e. of the distribution of $[\mathbf{D}^*(L)]_{i,\cdot}$ for $i \in$ NKK225, SP500, SPEU350. All responses display strong within-market homogeneity: SP500 and SPEU350 volatilities have very similar responses to the global shock which, however, has a stronger impact on the NKK225 panel. On the contrary, the impact of the US-Europe shock is larger in the SPEU350 panel than in the SP500 one. Most impulse responses are positive and show a good amount of persistence, consistently with the idea that the dynamics of volatilities is characterized by long memory. Finally, the impulse responses of Japanese stocks to the US-European shock are almost null at impact, in agreement with the restriction imposed in our identification scheme.

5.2 The predictive power of volatility shocks

To conclude, we investigate, in this section, the explanatory power of the volatility shocks in predicting aggregated (market) realized variances. The aggregated realized variances RV_t^{US} and RV_t^{JP} of the SP500 and the NKK225 are readily available on line. The same information is not provided for the SPEU350, but it is available for the most popular indices for the European market, the EUROSTOXX50, the FTSE100 for the London Stock Exchange, and the DAX for the Frankfurt Stock Exchange (notation: RV_t^{EU1}, RV_t^{EU2} and RV_t^{EU3}, respectively).[19] Below, we consider predicting RV_t^{EU1}, RV_t^{EU2} and RV_t^{EU3} as a substitute for predicting the

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[19]As in Shephard and Sheppard (2010) the realized variances considered here are sums of squared 5-minute returns, that is

$$RV_t^{\cdots} = \sum_{0 \le t_{j-1,t} < t_{j,t} < 1} \text{ret}_{j,t}^2, \qquad \text{ret}_{j,t} := \mathcal{P}_{t+t_{j,t}} - \mathcal{P}_{t+t_{j-1,t}},$$

unavailable RV_t^{EU}. The generic notation $RV_t^{\cdots}$ is used in place of RV_t^{US}, RV_t^{EU1}, RV_t^{EU2}, RV_t^{EU3}, or RV_t^{JP}.

In Table 4, we report adjusted R^2 coefficients associated with various linear regressions of the realized variances $RV_t^{\cdots}$ with respect to present and lagged values of the common shocks $\boldsymbol{\varepsilon}^*$ obtained in (13). More precisely, the first row of Table 4 is about the projections

$$\widehat{RV}_t^{\cdots}(\text{GDFM}) := \text{Proj}\left(RV_t^{\cdots} | \boldsymbol{\varepsilon}_t^*, \ldots, \boldsymbol{\varepsilon}_{t-21}^*\right) \tag{14}$$

(which include the contemporaneous shock $\boldsymbol{\varepsilon}_t^*$—optimal fits or "nowcasts", thus, rather than genuine predictions or forecasts), the second row is about

$$\widehat{RV}_t^{\cdots}(\text{GDFM}) := \text{Proj}\left(RV_t^{\cdots} | \boldsymbol{\varepsilon}_{t-1}^*, \ldots, \boldsymbol{\varepsilon}_{t-22}^*\right) \tag{15}$$

(which do not include the contemporaneous shock $\boldsymbol{\varepsilon}_t^*$, hence have the nature of one-step ahead in-sample predictions), both considered as predictors of $RV_t^{\cdots}$. The maximum lag ($k=22$) has been chosen, for the sake of comparison, to be the same as in the HAR predictor described below. Results show that approximately 26%-34% of the total variation of the realized measures is explained by the volatility shocks in (14), a percentage that reduces to 25%-31% in (15) where the contemporaneous shock is not included.

For the sake of comparison, in the third row of Table 4, we show the adjusted R^2 values obtained from the Heterogeneous Autoregressive (HAR) model proposed by Corsi (2009), where the predictors take the form

$$\widehat{RV}_t^{\cdots}(\text{HAR}) := \text{Proj}\left(RV_t^{\cdots} | RV_{t-1}^{\cdots}, RV_{t-5}^{\cdots}, \ldots, RV_{t-22}^{\cdots}\right). \tag{16}$$

This model provides adjusted R^2 coefficients of approximately 37%-51%, thus comparable to those of $\widehat{RV}_t^{\cdots}(\text{GDFM})$ in (15).

It is then natural to ask wether augmenting the HAR models with our estimated common shocks adds significant extra information improving on the prediction of realized variances. We thus performed the factor-augmented HAR (FHAR) regressions

$$\widehat{RV}_t^{\cdots}(\text{FHAR}) := \text{Proj}\left(RV_t^{\cdots} | RV_{t-1}^{\cdots}, RV_{t-5}^{\cdots}, \ldots, RV_{t-22}^{\cdots}, \boldsymbol{\varepsilon}_{t-1}^*, \ldots, \boldsymbol{\varepsilon}_{t-22}^*\right), \tag{17}$$

which considers both common volatility shocks, and the global factor-augmented HAR (GFHAR) regressions

$$\widehat{RV}_t^{\cdots}(\text{GFHAR}) = \text{Proj}(RV_t^{\cdots} | RV_t^{\cdots}, RV_{t-5}^{\cdots}, \ldots, RV_{t-22}^{\cdots}, \varepsilon_{t-1}^{*,\text{global}}, \ldots, \varepsilon_{t-22}^{*,\text{global}}), \tag{18}$$

which augments HAR by the global (strongly common) volatility shock only. The adjusted R^2 coefficients for these augmented models are shown in the fourth, fifth rows of Table 4, respectively.

Last, we also consider the Bayesian global factor-augmented HAR (BGFHAR) regressions, in which we estimate the global volatility shock according to the hierarchical factor model by Moench et al. (2013). In so doing, it should be remarked that such Bayesian method cannot be considered as a fully fledged alternative to our approach, since volatility proxies

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where $\mathcal{P}_{t+t_{j,t}}$ are stock price indexes at times $t_{j,t}$ of day t, where $t_{j,t}$ are times of trades such that subsequent prices are taken at 5-minute intervals. All realized variances used in this paper are available at `http://realized.oxford-man.ox.ac.uk/data`. Data mnemonics are `SPX2.rv`, `N2252.rv`, `FTSE2.rv`, `STOXX50E.rv`, `GDAXI2.rv`, respectively (Heber et al., 2009).

are still required from our first step estimation, but it provides just as an alternative method to estimate the global shock once a panel of volatility proxies is given. Indeed, in our exercise we estimate a hierarchical factor model on **s** — the panel of proxies (4) obtained as defined in Section 2. The adjusted R^2 coefficients for these last augmented model are shown in the sixth row of Table 4. Note also that, as explained in Section 3, by construction the hierarchical factor model cannot identify the US-Europe volatility shock and so no comparison in that respect is possible.

Finally, a numerical assessment of relative predictive accuracies can be obtained by means of MSE ratios such as

$$\text{Rel-MSE}_{\text{FHAR/HAR}}^{\cdots} := \sum_{t=1}^{T} \left(\widehat{RV}_t^{\cdots}(\text{FHAR}) - RV_t^{\cdots} \right)^2 / \sum_{t=1}^{T} \left(\widehat{RV}_t^{\cdots}(\text{HAR}) - RV_t^{\cdots} \right)^2, \qquad (19)$$

$\text{Rel-MSE}_{\text{GFHAR/HAR}}^{\cdots}$, and $\text{Rel-MSE}_{\text{BGFHAR/HAR}}^{\cdots}$, similarly defined. We also report p-values for the test of equal predictive ability by Giacomini and White (2006). Results are in the mid and bottom panels of Table 4.

Overall, we find that the volatility shocks do add predictive power to that of the plain HAR models for all aggregate realized variance measures considered, and that both common shocks do help.[20] In particular, while the GFHAR is relatively more accurate than the HAR by approximately 2%, the FHAR improves over the HAR by 4%-6-%. Note also that the BGFHAR is as accurate as our GFHAR. Let us stress again that the hierarchical approach cannot retrieve local shocks comparable to ours, hence no analogous of FHAR can be defined with the hierarchical approach.

Finally, if we turn to a recursive out-of-sample forecasting exercise, the predictive power of our estimated shocks is partly reduced. Still, when predicting the turmoil sub-sample (top panel of Figure 5), we get significant results for the FTSE100 realized variance with the FHAR, GFHAR, and BGFHAR, and for the DAX realized variance with the GFHAR. When predicting the most recent stable sub-sample (bottom panel of Figure 5) the FHAR helps in predicting both the EUROSTOXX and DAX realized variances, while the GFHAR helps in predicting the DAX realized variance. In this case the global shock retrieved with the BGFHAR is never significant.

6 Conclusions

We propose a novel approach for the analysis of interactions between international stock markets based on the nonparametric estimation of a two-step general dynamic factor model for a large panel of returns and volatilities. Our analysis is conducted on a large panel of stock returns composed by the constituents of three of the major indices for the US, European and Japanese stock markets.

To the best of our knowledge, this is the first paper able to shed light on many important aspects of financial markets dynamics at the same time, such as the nature and origin of volatility shocks, interdependence, contagion, and the prediction of aggregated realized volatilities.

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[20]The contributions of the volatility shocks are jointly 5%-significant in all regressions. The results are quite insensitive to variations in the maximal lag, provided that it is large enough. Further results are available upon request.

Table 4: RESULTS FOR IN-SAMPLE PREDICTION OF REALIZED VARIANCES

	SP500	EUROSTOXX50	FTSE100	DAX	NKK225
In-sample fit:					
adj-R^2 (GDFM) (14)	0.328	0.284	0.285	0.337	0.262
In-sample predictive regressions:					
adj-R^2 (GDFM) (15)	0.277	0.255	0.263	0.309	0.248
adj-R^2 (HAR) (16)	0.476	0.376	0.375	0.513	0.515
adj-R^2 (FHAR) (17)	0.493	0.406	0.405	0.529	0.539
adj-R^2 (GFHAR) (18)	0.487	0.389	0.389	0.520	0.533
adj-R^2 (BGFHAR)	0.486	0.390	0.389	0.521	0.521
Relative MSEs:					
Rel-MSE $_{\text{FHAR/HAR}}$	0.957	0.941	0.941	0.954	0.939
Rel-MSE $_{\text{GFHAR/HAR}}$	0.975	0.975	0.973	0.980	0.957
Rel-MSE $_{\text{BGFHAR/HAR}}$	0.975	0.972	0.973	0.978	0.981
Giacomini-White p-values:					
FHAR/HAR	0.03	0.02	0.00	0.00	0.00
GFHAR/HAR	0.05	0.08	0.02	0.02	0.00
BGFHAR/HAR	0.05	0.10	0.01	0.01	0.01
FHAR/GFHAR	0.03	0.02	0.01	0.00	0.00
FHAR/BGFHAR	0.08	0.01	0.01	0.00	0.00
GFHAR/BGFHAR	0.92	0.75	0.94	0.71	0.02

In particular, we find no evidence of global shocks in returns, thus reflecting the well known *home bias* phenomenon in equity markets, while the volatility panel shows a greater level of *interconnectedness* across markets. When focussing on the financial turmoils of the past decade (from 2007 to 2012) we also observe a large increase in return and volatility commonalities but no evidence in favour of financial contagion. In this sense, the turmoil period we analyzed has witnessed an efficient spread of information through the stock markets rather than irrational fears.

Moreover, we find that the impulse response functions of volatilities to the shocks, identified via natural timing restrictions, are highly homogeneous within each market but show significant differences across markets, thus showing different levels of interconnectedness. Finally, albeit our volatility shocks are estimated from a panel of daily closing prices, so ignoring any information regarding intra-day volatility, we find that these shocks add useful information to heterogeneous autoregressive (HAR) forecasts of the realized variances in most of the markets and samples considered.

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Table 5: RESULTS FOR OUT-OF-SAMPLE PREDICTION OF REALIZED VARIANCES

	SP500	EUROSTOXX50	FTSE100	DAX	NKK225
Turmoil: August 9, 2007 – July 26, 2012					
Relative MSEs:					
Rel-MSE $_{\text{FHAR/HAR}}$	0.977	0.976	0.974	0.973	0.960
Rel-MSE $_{\text{GFHAR/HAR}}$	0.988	0.991	0.989	0.990	0.982
Rel-MSE $_{\text{BGFHAR/HAR}}$	0.950	0.947	0.955	0.958	0.966
Giacomini-White p-values:					
FHAR/HAR	0.18	0.29	0.07	0.12	0.15
GFHAR/HAR	0.20	0.26	0.07	0.09	0.16
BGFHAR/HAR	0.13	0.22	0.05	0.15	0.06
FHAR/GFHAR	0.21	0.34	0.16	0.19	0.23
FHAR/BGFHAR	0.20	0.26	0.24	0.40	0.77
GFHAR/BGFHAR	0.15	0.23	0.09	0.22	0.39
Stable: July 27, 2012 – August 31, 2015					
Relative MSEs:					
Rel-MSE $_{\text{FHAR/HAR}}$	0.900	0.920	0.960	0.940	0.930
Rel-MSE $_{\text{GFHAR/HAR}}$	0.930	0.960	0.980	0.970	0.970
Rel-MSE $_{\text{BGFHAR/HAR}}$	0.970	0.970	0.970	0.970	0.970
Giacomini-White p-values:					
FHAR/HAR	0.58	0.07	0.38	0.05	0.17
GFHAR/HAR	0.61	0.12	0.42	0.06	0.26
BGFHAR/HAR	0.55	0.26	0.38	0.29	0.19
FHAR/GFHAR	0.51	0.07	0.37	0.09	0.13
FHAR/BGFHAR	0.60	0.04	0.58	0.03	0.23
GFHAR/BGFHAR	0.66	0.89	0.39	0.88	0.74

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