Transitory interest-rate pegs under imperfect credibility*

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Abstract

In this paper we show that the macroeconomic effects of a transient interest-rate peg can be significantly dampened when the peg is perceived to be imperfectly credible by the private sector. By doing so, we provide a solution to what has become known as the "forward guidance puzzle". This is the finding that pegging nominal interest rates to a specific value or path for an extended, yet finite, period of time in New Keynesian models generates macroeconomic responses that are implausibly large. This puzzle has been of interest because several central banks have implemented "forward guidance" which has been interpreted by some as a promise to hold the policy rate lower than had been previously expected: a so-called lower-for-longer (LFL) policy. The New Keynesian models that these central banks routinely use for policy analysis would predict that LFL policies generate very large effects.

The possibility that LFL policies might be imperfectly credible arises from their potential to be time inconsistent. Indeed, using an ad-hoc loss function for the central bank we show that it may have an incentive to renounce the LFL policy along the full commitment path. We examine cases in which the degree of imperfect credibility is exogenous and in which it is endogenously related to the state of the economy via the policymaker's incentive to renounce. Allowing for endogenous imperfect credibility tends to dampen the response of macroeconomic variables to an LFL policy announcement by more than under exogenous imperfect credibility.

KEY WORDS: New Keynesian model; monetary policy; zero lower bound
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1 Introduction

The macroeconomic effects of a monetary policymaker pegging the nominal interest rate to a specific value or path for a finite period of time can be significantly dampened when the peg is not perceived to be perfectly credible by the private sector. Allowing an announced interest rate peg to be imperfectly credible, therefore, provides a potential solution to what has become known as the "forward guidance puzzle"(Del Negro et al., 2012). This is the observation that, in New Keynesian models, the effects of a perfectly credible transitory interest rate peg – a common way to implement a policy strategy of "forward-guidance"– on variables such as inflation and GDP is widely thought to be implausibly large. In this paper, by modelling the effects of imperfect credibility, we are able to provide more plausible quantitative assessments of the impact of forward guidance-like policies using the class of medium-scale New Keynesian models, such as Christiano et al. (2005) or Smets and Wouters (2007), which have become the workhorse models of central banks around the world.

Our motivation for analysing the effects of imperfectly-credible interest-rate pegs follows the adoption of a strategy of "forward guidance" by a number of central banks internationally in the aftermath of the Great Recession.[1] The forward-guidance strategies implemented have entailed policymakers' providing greater information about future policy. The intention of this has been to influence private-sector expectations either to increase the effectiveness of policy, or provide greater stimulus to the economy. This has taken place at a time when conventional monetary loosening, via reductions in the short-term policy rate, has been infeasible on account of the constraint imposed by the effective lower bound (ELB).

There are a number of reasons why communications policy might be particularly important at the ELB. The infrequency of ELB episodes means that the private sector may have a less clear understanding of monetary-policy behaviour at the ELB than it does away from the lower bound, and so would benefit from extra guidance. In addition, the private sector may base its expectations for the policy at the ELB on the average of the central bank's past behaviour – as represented by a policy rule, adjusted for fact that interest rates cannot fall below the ELB. But this may not necessarily be consistent the policymaker's preferred strategy at the ELB. Communicating details about this strategy would, therefore, be beneficial.

A common prescription for monetary policy at the ELB is to hold rates at the ELB for a prolonged period. By lowering the expected path for the short-term nominal interest rate, the policymaker can reduce longer-term nominal interest rates. This leads to a reduction in longer-term real interest rates, stimulating demand in the short run, even

[1]For example, in the US, in late-2009 the Federal Open Market Committee (FOMC) made clear that it did not expect the Federal Funds Rate to rise for a considerable period. In April 2009, the Bank of Canada stated that: "Conditional on the outlook for inflation, the target overnight rate can be expected to remain at its current level until the end of the second quarter of 2010 in order to achieve the inflation target". See Bank of Canada (2009, p2). More recently, the FOMC has communicated that it would not raise interest rates until certain thresholds for unemployment or inflation had been crossed. Similarly, the Bank of England's Monetary Policy Committee (MPC) announced a form of forward guidance in August 2013 in which it stated its intention not to raise interest rates from their effective lower bound until the unemployment rate was below 7%, subject to inflation expectations' remaining anchored and risks to financial stability's being contained. See Monetary Policy Committee (2013). In February 2014, the MPC updated its guidance to communicate a qualitative assessment of both the appropriate path for the policy rate over the medium term and the factors that would influence that path. See Monetary Policy Committee (2014, p9 8–9).

though the short-term policy rate cannot be reduced below the ELB. Such a policy strategy has been promoted by Krugman (1998). Several authors (Eggertsson and Woodford, 2003; Jung et al., 2005) have demonstrated that optimal commitment policy in simple New Keynesian models has the feature that the policy rate remains at the ELB for longer than an optimal policy conducted under discretion.

In the context of the medium-scale New Keynesian models that are widely used in central banks, this type of monetary strategy at the ELB is often modelled as an announcement that the policymaker will deviate from its typical, systematic reaction function. Such policies have been labelled "Odyssean Forward Guidance" by Campbell et al. (2012, p3). Since these models are usually estimated over time periods that do not include ELB episodes, the estimated monetary policy rule can be thought of a description of policy in "normal times", away from the ELB. Therefore, to assess alternative strategies that policymakers might be interested in at the ELB, it is possible to consider short-lived deviations from an interest rate rule that is suitable for "normal times". These are implemented in the form of an interest-rate peg that lasts for an extended, yet finite, period of time. That is not to say that the systematic component of monetary policy, as represented by the estimated reaction function, has somehow changed. Instead it is that the simple rule that describes behaviour away from the ELB is an incomplete description of policy once the ELB is taken into consideration.

These types of policy experiments can be thought of as an approximation to a more fully worked out analysis of the monetary policymaker's problem, which would take account of its objective function and the constraints that it faces, including the effective lower bound. The main reason for considering deviations from an estimated rule is its relative tractability. In the medium-scale models considered by central banks, conducting an analysis of optimal policy, either under commitment or discretion, can lead to practical difficulties. This is particularly true for a optimal policy under discretion in a scenario constructed under the assumption that agents know the values of future the shock realisations ex ante (i.e. there are "anticipated" shocks).

A significant issue, however, in analysing forward-guidance policies as transient interest-rate pegs in medium-scale New Keynesian models is the "unreasonably large" responses of inflation and GDP they often generate, as discussed by Carlstrom et al. (2012) and others. This apparent pathology has been labelled the "forward-guidance puzzle"by Del Negro et al. (2012). In response to the puzzle, Del Negro et al. (2012) propose a solution, which involves restricting the response of longer-term interest rates to the announcement.

In this paper, by assuming the interest-rate peg may not be perfectly credible, we provide an alternative way to mitigate the "forward-guidance puzzle". Specifically, we allow for the fact that agents might place a non-zero probability on the policymaker's deviating from the peg and reverting to its policy rule earlier than had been announced. We motivate this by demonstrating that the large responses to fully credible announced deviations from the policymaker's usual reaction function give the policymaker an incentive to renege and so make it less likely that agents within the model will believe that the announced plans are in fact credible. To see this, suppose that the policymaker makes a fully credible announcement that the policy rate will be held lower for longer than would be implied by the usual reaction function. As we have noted, in many DSGE models such an announcement generates a large and immediate boom in output and inflation. If the effects of this announcement are sufficiently large, the policymaker may have an incentive to return to the usual reaction function earlier than initially announced. Doing so may deliver better macroeconomic outcomes than following through with the initial lower for longer policy. This is the familiar 'time inconsistency' problem in rational expectations models: the leverage over agents' expectations created by (credible) commitment generates strong incentives to deviate, bringing the assumption of credible commitment into question.

To implement an imperfectly-credible interest-rate peg we derive a general-purpose perfect-foresight algorithm. This solves for the equilibrium paths of the economy in response to the imposition of an interest-rate peg of an arbitrary (but finite) number of periods given a set of beliefs about the credibility of the announcement (measured in per-period probabilities).[2] We build the intuition for our algorithm and illustrate the effect of taking into account imperfect credibility of interest-rate pegs using a small New Keynesian model. We then go on to simulate the effect of an interest-rate peg – in the guise of what we refer to as a "lower-for-longer" policy – using COMPASS (a DSGE model used for forecasting and policy analysis at the Bank of England). Our objective is to quantify the effect of interest-rate pegs, or lower-for-longer (LFL) policies, under imperfect credibility in the sort of model routinely used for policy analysis in policy institutions. To do so, we construct a base scenario in which there is a large recession that has led policy rates to be cut to their effective lower bound. To stimulate the economy in response to the shock, we assume the policymaker announces a lower-for-longer policy to hold interest rates at the effective lower bound for three quarters longer than in the base case. We then compute the effects on key macroeconomic variables, such as inflation and output, under varying assumptions about the credibility of the policy.

We find that the macroeconomic effects of the LFL policy are significantly dampened even in cases where the private sector views the announced path as the most likely one. For example, the simulated effect of an LFL policy in which the central bank announces an intention to hold rates at the effective lower bound for three more quarters than the private sector had been expecting is roughly halved if the private sector attaches as little as a 5 percent per-period probability to a renouncement.

We also consider the effects of LFL policies under the assumption that their credibility – as measured by the probability of maintaining the peg – is endogenous to the state of the economy. We do so by modelling the probability of renouncement as a function of the policymaker's incentive to return to its usual reaction function more quickly, as discussed above. More specifically, we measure the incentive to renounce as the difference between the loss associated with macroeconomic outcomes if the peg is maintained and the loss associated with reversion to the rule. We then map the incentive to renounce into a probability of renouncement such that the larger is the incentive to renounce, the larger is the probability of renouncement. We find that accounting for the relationship between the credibility of the peg and the state of the economy can stabilise outcomes above and beyond incorporating imperfect credibility in an exogenous manner. This is because the trajectories for interest rates associated with the biggest boost to the economy are typically the same as those with the largest incentives to renounce. Therefore, when the probability of renouncement is allowed to be endogenous, relatively little probability is attached to these trajectories for interest rates, which works to dampen the macroeconomic effects of the LFL policy.

Our paper connects with several strands of the literature. Our focus is closely related

[2] In doing so, we depart from the standard timing assumption for an interest-rate peg under perfect credibility by assuming that the private sector makes its decisions before the policymaker sets the interest rate. The rationale for this timing assumption is to make the probability that the private sector attaches to reversion to the rule in period 1 meaningful.

to analysis of the effects of pre-announced policy paths in medium-scale DSGE models. Del Negro et al. (2012) document the “forward-guidance puzzle” that we investigate: perfectly credible promises to hold the policy rate fixed for prolonged periods appear to generate implausibly large effects. Similar results are also noted by Laséen and Svensson (2011) using the RAMSES DSGE model used for policy analysis and forecasting at the Riksbank.[3] Laséen and Svensson (2011) note that in some cases pre-announced paths for the policy rate can generate apparently counterintuitive results (for example, announcing a persistently lower path for nominal interest rates may *reduce* inflation). Coenen and Warne (2013) study lower for longer policies using stochastic simulations of the NAWM DSGE model used for policy analysis and forecasting at the European Central Bank.[4] They find that such policies have very powerful effects: promises to hold the policy rate at the zero lower bound for just two or three quarters longer than would be predicted by the monetary policy reaction function in the model can be sufficient to offset the deflationary risk caused by the zero bound.

Another strand of literature examines the behaviour of New Keynesian DSGE models under temporary interest rate pegs. Analytically, an experiment in which the interest rate is announced to be pegged to an arbitrary value or path is very similar to one in which policy is announced to deviate from a benchmark policy rule. Analyses of temporary interest rate pegs in stylised New Keynesian models include Blake (2012), Blake (2013), Carlstrom et al. (2012) and Levin et al. (2010). These papers provide insights into the determination of the responses of output and inflation to temporary interest rate pegs and the parameters that determine the strength of those responses. For example, Levin et al. (2010) show that, for a prototypical New Keynesian model, the behaviour of output and inflation when the interest rate is pegged is determined by the size of the unstable eigenvalue of the transition matrix mapping the vector of current inflation and output gap to the vector of next period's inflation and output gap. The size of the unstable eigenvalue is determined by the product of the slope of the New Keynesian curve and the interest elasticity of demand. These papers help us to understand the mechanics of the responses of larger-scale models that have broadly similar (though richer) specifications of aggregate demand and pricing behaviour.

Our paper is most closely related to several papers that explicitly account for the fact that policymaker's announced plans may not be fully credible. Blake (2013), Carlstrom et al. (2013) and Weale (2013) study cases in which there is a fixed probability each period that the monetary policymaker will exit a pre-announced monetary (and/or fiscal) policy plan. Bodenstein et al. (2012) consider policy at the zero bound under the ‘loose commitment’ framework developed by Roberds (1987), Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010). In that approach, there is a constant probability each period that the policymaker will abandon the pre-announced optimal commitment policy and reoptimise in a discretionary manner. Our approach extends the analysis of Blake (2013) and Carlstrom et al. (2013) by providing a general algorithm for large-scale models and we move beyond Bodenstein et al. (2012) by allowing for time-varying and potentially endogenous probabilities that the policymaker will revert to its usual policy rule earlier than announced. The cost of this additional flexibility and applicability is that we must make use of piecewise linear solution techniques so that, aside from the uncertainty associated with the period in which the policymaker reverts to its usual policy rule, we can only consider experiments under perfect foresight. In our view, this is a necessary sacrifice

[3]See Adolfson et al. (2007).

[4]See Christoffel et al. (2008).

to permit the investigation of imperfect credibility in the context of policy simulations using large-scale DSGE models similar to those used in central banks.

The remainder of the paper is organised as follows: Section 2 builds the intuition for our approach using a small New Keynesian model; Section 3 presents simulation results using a medium-scale DSGE model; Section 4 concludes.

2 Interest-rate pegs in a small New Keynesian model

In this section we explain our procedure for implementing interest-rate pegs under imperfect credibility using a small, textbook New Keynesian model. Our aim is to build intution about the approach and for the simulation results that follow in Section 3.

2.1 A small New Keynesian model

The New Keynesian model we consider is a fairly standard textbook model and is represented by equations (1)–(4). It comprises a New Keynesian Phillips curve, which relates current inflation, $\widehat{\pi}_t$, to expected inflation and the output gap, $\widehat{y}_t$, (1); a dynamic *IS* curve, which relates the expected change in the output gap to the gap between the ex-ante real interest rate, $\widehat{r}_t - \mathbb{E}_t\widehat{\pi}_{t+1}$, where $\widehat{r}_t$ is the nominal interest rate, and the natural rate of interest, $\widehat{r}^n_t$, (2); an exogenous process for the natural rate of interest, including the natural-rate shock, ε^n_t, (3); and a monetary-policy rule, where ε^r_t is the monetary-policy shock, (4).

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + \kappa\widehat{y}_t \tag{1}$$

$$\widehat{y}_t = \mathbb{E}_t\widehat{y}_{t+1} - \frac{1}{\sigma}\left(\widehat{r}_t - \mathbb{E}_t\widehat{\pi}_{t+1} - \widehat{r}^n_t\right) \tag{2}$$

$$\widehat{r}^n_t = \rho_n\widehat{r}^n_{t-1} + \varepsilon^n_t \tag{3}$$

$$\widehat{r}_t = \phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t + \varepsilon^r_t \tag{4}$$

In a perfect-credibility setting, the dynamics of the model would be governed by the rational-expectations solution of the model, which is given by (5). This relates current values of the vector of endogenous variables $x_t = \left[\begin{array}{cccc} \widehat{\pi}_t & \widehat{y}_t & \widehat{r}^n_t & \widehat{r}_t \end{array}\right]'$ to their lag, and the shocks $z_t = \left[\begin{array}{cc} \varepsilon^n_t & \varepsilon^r_t \end{array}\right]'$. The rational-expectations solution, (5), is written so that the vector of endogenous variables in the current period is a function of unanticipated monetary-policy and natural-rate shocks in the current period, and anticipated realisations of these shocks in future periods. The representation using anticipated shocks is essentially equivalent to a representation based on news shocks. In the perfect-credibility case, the implementation of an interest-rate peg would involve a sequence of anticipated monetary-policy shocks corresponding to the duration of the announced peg.

$$x_t = Bx_{t-1} + \sum_{s=0}^{H} F^s\Phi z_{t+s} \tag{5}$$

In our implementation of imperfectly-credible interest-rate pegs, the rational-expectations solution is still relevant, as will be discussed in the next sub-section.

2.2 A 1-period interest-rate peg

We formulate imperfect credibility as uncertainty over whether the policymaker will implement a pre-announced interest rate peg, or revert to setting interest rates in line with what would be implied by its policy rule instead. To shed light on our formulation of imperfect credibility, we consider an announcement by the policymaker of its intention to peg the interest rate at some arbitrary value, b, for one period.

This implementation relies on a departure from the usual timing assumption in New Keynesian models that the interest rate in the current period is known, having been set in line with the monetary-policy rule given the realisation of the shocks. Under our timing convention, we assume that the sequence of events is the following: the policymaker announces the peg at the end of period 0; the private sector responds in period 1 conditional on its belief about the probability that the policymaker will revert to the rule rather than stick to the announced policy; the policymaker then sets the interest rate after private-sector decisions have been made, either in line with the announcement or according to the policy rule.

Taken literally, the decision over whether or not the peg will be maintained is treated as a lottery in which the policymaker maintains the announced peg if the number drawn from a uniform distribution between 0 and 1 exceeds p. This type of approach is not dissimilar to the Calvo approach in modelling price setting as applied to modelling policymaker credibility in Bodenstein et al. (2012).[5] It is possible to conceptualise this as a decision by a policymaker who weighs up the costs and benefits of the policy. The costs and benefits in this decision are based on ad-hoc assumptions, rather than a deeper microeconomic foundation – we relax this assumption in a relatively loose way in Section 3.5. In particular, suppose that the 'benefit' of renouncement is exogenously given and scaled to lie between 0 and 1, and is denoted by p. Suppose also that the cost of renouncement (e.g. interpretable as a reputational cost for the policymaker), c, is randomly drawn from a uniform distribution between 0 and 1. If the cost of renouncement exceeds the benefit ($c > p$), the policymaker implements the peg as announced; if the cost of renouncement is less than the benefit ($c < p$), the policymaker renounces. The probability that the central bank will renounce the peg is, therefore, p. The private sector's belief about the probability the policy maker will renounce the peg is also p, consistent with rational expectations.

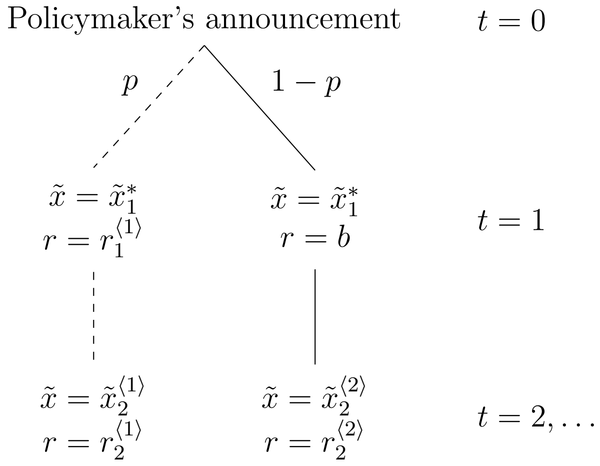
The timing assumption ensures that even if the policymaker follows through with their promise and sets the policy rate at the announced peg, the values of the endogenous variables in period 1 will be influenced by the possibility, captured in the private sector's expectations, that the policymaker might revert to the rule instead. The key point is that, under our timing assumption, when the private sector makes its economic decisions, it has uncertainty about what the value of the policy rate will be. This is how we depart from the equilibrium dynamics implied by (5), which would hold under perfectly credible policy. To the extent that some probability is attached to the policymaker reverting to the rule rather than following through with the peg, this will dampen the impact of the announced policy on the endogenous variables. In what follows, therefore, we use the expectations operator $\mathbb{E}_t^*$ to denote agents' expectations in period t over the policymaker's reverting to the rule or sticking with peg. The fact that the policymaker may either revert to the rule or stick with the peg in period 1 implies that there are two alternative trajectories

[5]One real world justification for this type of approach could be stochastic variation in the preferences of monetary policy committees as membership of those committees varies over time.

for the economy. We denote with a superscript $\langle 1 \rangle$ the variables along the trajectory in which the economy evolves conditional on the policymaker's having reverted to the rule in period 1. To denote variables on the other trajectory, in which the economy evolves conditional on the policymaker's having stuck with the peg, we use a superscript $*$.

It is possible to shed further light on the two alternative trajectories for the economy by considering a diagrammatic representation of the timing convention, Figure 1, in which we denote $\tilde{x}_t = \begin{bmatrix} \widehat{\pi}_t & \widehat{y}_t & \widehat{r}_t^n \end{bmatrix}'$. The figure makes clear that the economy can take two possible paths, consistent with the policymaker either setting policy in line with the peg, or reverting to the rule. Furthermore, it shows that in the case where the policymaker implements the peg, the dynamics of the endogenous variables are nevertheless affected by the non-zero probability the private sector places on the possibility that that the policymaker might have reverted to the rule in period 1.

Figure 1: Diagrammatic representation of a one-period interest rate peg under imperfect credibility



Notes: In period 0 the policymaker announces that policy rates will be held at b for a single period. In period 1 the private sector responds, basing its decisions on an intra-period expectation that nominal rates will be set in line with the policy rule, $r_1^{\langle 1 \rangle}$, with probability p, and set in line with the announced peg, b, with probability $1 - p$. This expectation for interest rates determines variables that depend on policy (i.e. inflation and the output gap) in the vector of non-interest rate endogenous variables, $\tilde{x}_1$. The policymaker then sets the interest rate. The dotted lines show the economy's trajectory if the policymaker reverts to the rule in period 1, while the solid lines show its trajectory when the policymaker sticks to its peg announcement. Because $\tilde{x}_1$ is determined before the interest rate is set in period 1, it takes the same value along both possible trajectories of the economy, $\tilde{x}_1^*$. From period 2 onwards, however, the trajectories diverge because they are conditioned on a different value for the interest rate in period 1.

The remainder of this sub-section outlines how we solve for the values of the endogenous variables along the equilibrium path in which the policymaker follows through with its announced peg. This is a specific application of the general case that is detailed in Appendix A. First, it is useful to write equations (1)–(4) as a system partitioned to separate out the policy rule from the non-policy equations, which we do in (6) and (7). In doing so, we assume that there is no persistence in the natural rate process, $\rho_n = 0$, to simplify the algebra:

$$\begin{bmatrix} \beta & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \widehat{\pi}_{t+1} \\ \mathbb{E}_t \widehat{y}_{t+1} \\ \mathbb{E}_t \widehat{r}_{t+1}^n \end{bmatrix} - \begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1 & \frac{1}{\sigma} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \widehat{\pi}_t \\ \widehat{y}_t \\ \widehat{r}_t^n \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{\sigma} \\ 0 \end{bmatrix} \widehat{r}_t = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \varepsilon_t^n \tag{6}$$

$$\widehat{r}_t = \begin{bmatrix} \phi_\pi & \phi_y & 0 \end{bmatrix} \begin{bmatrix} \widehat{\pi}_t \\ \widehat{y}_t \\ \widehat{r}_t^n \end{bmatrix} + \varepsilon_t^r \tag{7}$$

In period 1, therefore, it is possible to represent the non-policy structural equations from (6) as the system (8). Here, we have made the additional simplifying assumption that there is no realisation of a natural rate shock. In other words, the policymaker implements a peg starting from steady state. In doing so, we are able to isolate the impact of the imperfectly credible interest rate peg on the endogenous variables more cleanly.

$$\begin{bmatrix} \beta & 0 & 0 \\ \frac{1}{\sigma} & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbb{E}_1^* \widehat{\pi}_2 \\ \mathbb{E}_1^* \widehat{y}_2 \\ \mathbb{E}_1^* \widehat{r}_2^n \end{bmatrix} - \begin{bmatrix} 1 & -\kappa & 0 \\ 0 & 1 & \frac{1}{\sigma} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \widehat{\pi}_1^* \\ \widehat{y}_1^* \\ \widehat{r}_1^{n*} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{\sigma} \\ 0 \end{bmatrix} \mathbb{E}_1^* \widehat{r}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

It is clear from (8) that the private sector must form expectations around the value of the policy rate in the current period, $\widehat{r}_1$. In particular, agents place a probability of p on the possibility that the policymaker will revert to the rule in period 1, and a probability of $1 - p$ on the possibility that period-1 interest rates will be equal to b, consistent with the pre-announced policy:

$$\mathbb{E}_1^* \widehat{r}_1 = p r_1^{\langle 1 \rangle} + (1 - p)\, b \tag{9}$$

In the case where the policymaker reverts to the rule in period 1, the interest rate is simply given by the policy rule (where we have assumed there are no monetary policy shocks):

$$r_1^{\langle 1 \rangle} = \begin{bmatrix} \phi_\pi & \phi_y & 0 \end{bmatrix} \begin{bmatrix} \widehat{\pi}_1 \\ \widehat{y}_1 \\ \widehat{r}_1^n \end{bmatrix} \tag{10}$$

The private sector's period-1 expectation of endogenous variables in period 2 depends on whether the policymaker has reverted to the rule in period 1, or has stuck to the pre-announced policy. That is,

$$\mathbb{E}_1^* \tilde{x}_2 = p \tilde{x}_2^{\langle 1 \rangle} + (1 - p)\, \tilde{x}_2^{\langle 2 \rangle} \tag{11}$$

where in keeping with the notation convention, $\tilde{x}_2^{\langle 1 \rangle}$ is the vector of endogenous variables in period 2 conditional on the policymaker's reversion to the rule in period 1, and $\tilde{x}_2^{\langle 2 \rangle}$ is the vector of endogenous variables in period 2 in the case where the policymaker implements its announced peg in period 1 (and so reverts to the policy rule in period 2). If the policymaker reverts to the rule in period 1, the economy's dynamics are back in line with the rational expectations equilibrium. Since the model has no persistence, either endogenously or through the exogenous process for the natural rate, in the model's rational expectations equilibrium, $B = 0$. An implication of this is that $\tilde{x}_2^{\langle 1 \rangle} = \tilde{x}_2^{\langle 2 \rangle} = 0$, and hence $\mathbb{E}_1^* \tilde{x}_2 = 0$. In other words, when there is no persistence, even though the endogenous variables can take two alternative sets of values in period 1, from period 2 onwards, the two alternative trajectories depicted in Figure 1 collapse down to a single path.

It is now possible to solve for the endogenous variables in period 1 along the path in which the policymaker follows its pre-announced plan, $\tilde{x}_1^*$. This can be done by substituting (10) into (9) and using the fact that $\mathbb{E}_1^* \tilde{x}_2 = 0$. These can then be substituted into (8) to give:

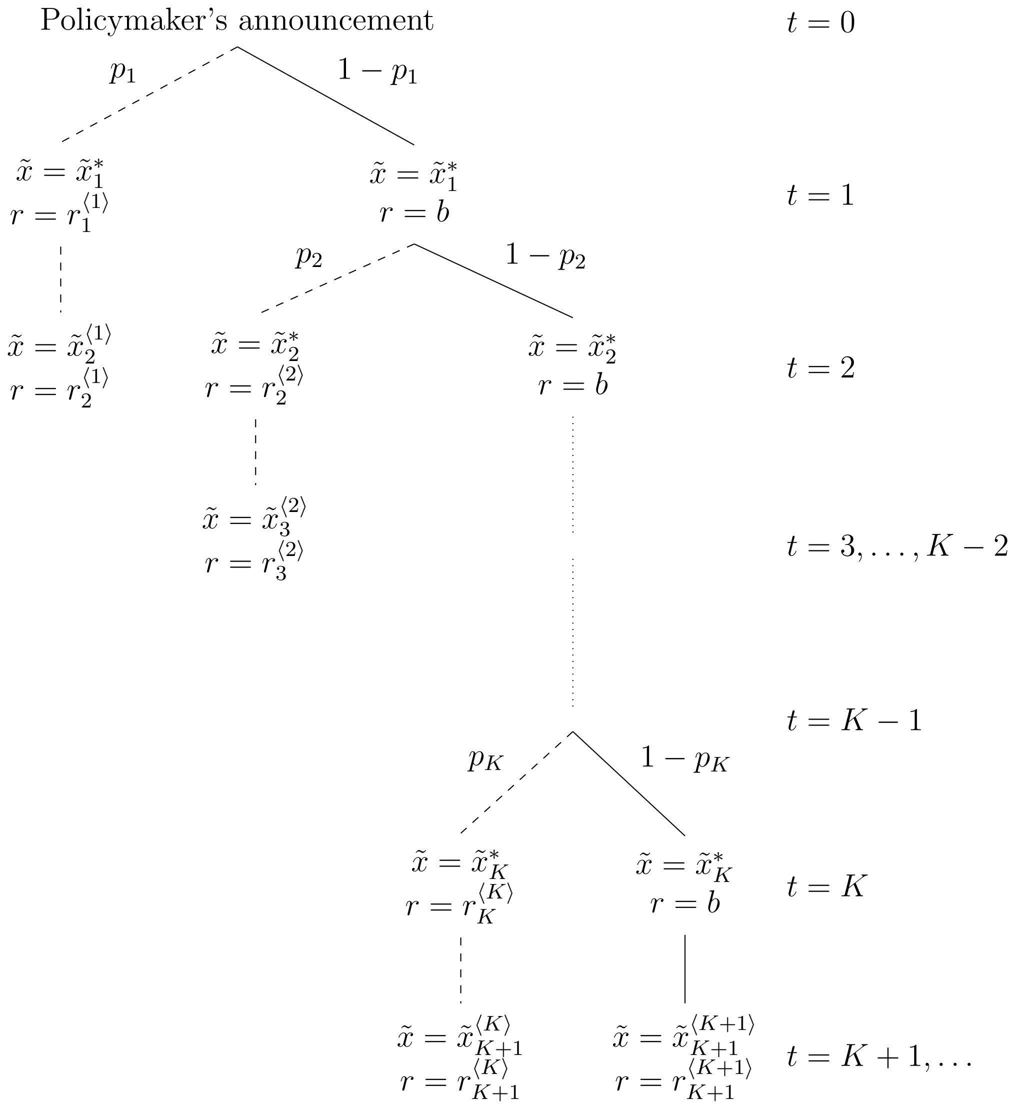
$$\begin{bmatrix} \widehat{\pi}_1^* \\ \widehat{y}_1^* \\ \widehat{r}_1^{n*} \end{bmatrix} = -\frac{(1-p)\, b}{\sigma + p\,(\phi_y - \kappa \phi_\pi)} \begin{bmatrix} \kappa \\ 1 \\ 0 \end{bmatrix} \tag{12}$$

This equation shows that the effect of the peg depends on its level, b; the credibility of the peg, $1-p$; the behaviour of the policymaker if the peg is not implemented, as shown by the coefficients in the interest rate rule, ϕ_π and ϕ_y; the slope of the Phillips curve, κ; the interest elasticity of aggregate demand, σ. As the credibility of the peg increases, $p \to 0$, its impact tends to the effects under perfect credibility, $\tilde{x}_1^* \to \begin{bmatrix} -\frac{b\kappa}{\sigma} & -\frac{b}{\sigma} & 0 \end{bmatrix}'$; while as the credibility of the peg decreases, $p \to 1$, its effects tend to zero, $\tilde{x}_1^* \to \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}'$.

2.3 A K-period interest-rate peg

Generalising the logic of a single-period interest-rate peg to a K-period promise – as will be considered in our simulation results in Section 3 – is conceptually straightforward and is depicted diagrammatically in Figure 2. In this case, the timing convention is unchanged insofar as the sequence of events in each period, $t = 1, 2, \ldots, K$, is such that the private sector chooses allocations and then the policymaker sets the policy rate. At the time the private sector makes its decisions, it is uncertain about the policy rate in that period, attaching a probability p_t to the possibility that the policymaker reverts to the rule in that period and sets $r_t = r_t^{\langle 1 \rangle}$, and a probability $1 - p_t$ to the possibility that the policymaker implements the peg and sets $r_t = b$. Since $\tilde{x}_t$ is determined before the interest rate is set in period t, it takes the same value, $\tilde{x}_t^*$, regardless of whether the policymaker reverts to the rule in period t or implements the peg. From period $t + 1$ onwards, however, the trajectories diverge because they are conditioned on a different value for the interest rate in period t. As is clear from Figure 2, there are now $K + 1$ trajectories: K trajectories in which the policymaker reverts to the rule earlier than announced; 1 trajectory in which the policymaker fully implements the announced peg. In Section 3 we use this framework to provide a quantitative assessment of lower-for-longer policy under imperfect credibility in a medium-scale New Keynesian model.

Figure 2: Diagrammatic representation of K-period interest rate peg under imperfect credibility



Notes: In period 0 the policymaker announces that policy rates will be held at b for K periods. In each period, $t = 1, 2, \ldots, K$, the sequence of events is such that the private sector responds, basing its decisions on an intra-period expectation that nominal rates will be set in line with the policy rule, $r_t^{\langle t \rangle}$, with probability p_t, and set in line with the announced peg, b, with probability $1 - p_t$. This expectation for interest rates determines variables that depend on policy (i.e. inflation and the output gap) in the vector of non-interest rate endogenous variables, $\tilde{x}_t$. The policymaker then sets the interest rate. The dotted lines show the economy's trajectory if the policymaker reverts to the rule in period t, while the solid lines show its trajectory when the policymaker implements the announced peg. Because $\tilde{x}_t$ is determined before the interest rate is set in period t, it takes the same value when the policymaker reverts to the rule in period t as if the policymaker implements the peg announcement, $\tilde{x}_t^*$. From period $t + 1$ onwards, however, the trajectories diverge because they are conditioned on a different value for the interest rate in period t. There are K trajectories in which the policymaker reverts to the rule earlier than had been announced and a single trajectory in which the announcement is fully implemented.

3 Simulating the effects of "lower-for-longer" policy

This section builds on the framework described in Section 2 by simulating the effect of lower-for-longer (LFL) policy in a medium-scale DSGE model. As discussed in Section 1, an LFL policy is defined as an announcement by the central bank that the policy rate will be maintained at the zero (or effective) lower bound for longer than agents had been expecting. It is one of many instruments that policymakers could turn to when confronted by the constraint of a zero lower bound and can be thought of as a very crude (and, therefore, not particularly realistic) form of forward guidance. Our aim is to quantify the effect of LFL policy both with and without perfect credibility in a model that is representative of the type of model used for policy analysis in policy institutions.

3.1 The Model

Throughout this section we use a variant of the Bank of England's model, COMPASS, which is used at the Bank for forecasting and scenario analysis. COMPASS is a medium-scale, New-Keynesian DSGE model similar to Christiano et al. (2005) or Smets and Wouters (2007). Models in this class share the same basic building blocks as the small New-Keynesian model described in Section 2, but have additional structure to generate more realistic dynamics in the models' endogenous variables. To add to their empirical congruence, these models are typically estimated, usually with Bayesian techniques, as COMPASS has been on UK data. The additional features that COMPASS has over and above those in the simple model from Section 2 include sticky wages as well as sticky prices, investment adjustment costs, habit formation in consumption, and a proportion of 'rule-of-thumb' consumers who consume the entirety of their labour income. Moreover, reflecting the UK economy's openness, COMPASS is a small open economy model, which implies developments in the world economy affect the dynamics of domestic variables, although the obverse is not true. For a complete description of COMPASS, its derivation, and its estimation, see Burgess et al. (2013).

3.2 Specification of monetary policy

In the version of COMPASS we use in this paper, we assume monetary policy is set so that the real interest rate tracks the flexible-price real rate, which we will refer to as the model's "natural rate" of interest.[6] This differs to the monetary-policy specification in the Burgess et al. (2013) version of COMPASS, in which interest rates follow a relatively standard Taylor rule.[7] Specifically, we assume monetary policy follows the rule described by equation (13). Here, the central bank adjusts the nominal interest rate, r_t, to ensure

[6] The concept of a "natural rate" of interest dates back to Wicksell (1898) and, as discussed in, for example, Woodford (2003) is a useful benchmark for monetary policy in New Keynesian models. In simple New Keynesian models like the one presented in Section 2, a policy that tracks the flexible-price interest rate is also consistent with stable inflation and no output gap. Recent evidence for the United States in Cúrdia et al. (2011) suggests that New Keynesian models may fit the data better (as measured by the marginal data density in Bayesian maximum likelihood estimation) if they are estimated with a policy rule that includes a flexible-price interest rate term.

[7] More precisely, the policy rule in Burgess et al. (2013) is:
$r_t = \theta_R r_{t-1} + (1-\theta_R)\left[\theta_\Pi\left(\frac{1}{4}\sum_{j=0}^{3}\pi_{t-j}\right) + \theta_Y \hat{y}_t\right] + \hat{\varepsilon}_t^R$, where r is the policy rate, π is quarterly consumer price inflation, $\hat{y}$ is the flexible-price output gap and $\hat{\varepsilon}^R$ is an exogenous policy disturbance. The estimated parameter values are: $\theta_R \approx 0.8$, $\theta_\Pi \approx 1.5$ and $\theta_Y \approx 0.125$.

the ex-ante real interest rate, $r_t - \mathbb{E}_t\pi_{t+1}$, tracks the flexible-price real interest rate, r_t^{flex} (subject to an exogenous disturbance, $\hat{\varepsilon}_t^R$). An inflation stabilisation term, $\iota\pi_t$, is also included to ensure that the model has a determinate, rational-expectations solution.[8]

$$r_t - \mathbb{E}_t\pi_{t+1} = r_t^{\text{flex}} + \hat{\varepsilon}_t^R + \iota\pi_t \tag{13}$$

We use this specification for monetary policy because it more realistically captures aspects of the monetary policy response to the financial crisis, an episode we use to motivate our lower-for-longer experiments. In response to the financial crisis, policy rates were cut rapidly to the ELB in the UK (and in other economies around the world), a fact that a rule that tracks the natural rate is able to generate. By contrast, the policy rule outlined in Burgess et al. (2013) – a relatively standard Taylor rule in which policy responds to deviations of (annual) inflation from target and the output gap,[9] with a role for interest rate smoothing – does not mimic the rapid and large cuts in policy rates witnessed in response to the financial crisis.[10] Nevertheless, the real-rate tracking rule still does not provide a complete description of monetary policy (particularly at the ELB), which motivates the investigation of deviations from that rule such as lower-for-longer policies.

3.3 The lower-for-longer policy scenario

To illustrate the effects of a lower-for-longer policy under imperfect credibility in a quantitativey realistic setting, we construct a scenario in which the economic conditions are such that the policymaker might have a motivation for implementing a lower-for-longer policy. In other words, in response to shocks hitting the economy, the policy rate has hit and is constrained by the ELB.

The scenario we construct is loosely based on the behaviour of selected macroeconomic variables in the UK in the aftermath of the financial crisis of 2007/08. There are four features of the UK data in 2008/09 that we seek to capture in our scenario (Figure 3). First, as was the case in many advanced economies, the UK experienced a contraction in the level of activity. Second, and more specific to the UK economy, the level of productivity also fell.[11] Third, inflation was surprisingly resilient given the the fall in activity. Fourth, the Monetary Policy Committee of the Bank of England cut Bank Rate to the effective lower bound (ELB) of 0.5 percent.

To construct the scenario we assume the economy is initially at steady state and then subject it to a four-quarter sequence of unanticipated shocks. For reasons of parsimony, we consider sequences of innovations to only two of the model's shocks: the domestic risk-premium shock and labour-augmented productivity shock. Our choice of these shocks reflects the fact that, from the perspective of the model, they play an important role in explaining the macroeconomic dynamics over this period,[12] and a prior that these shocks

[8] The flexible-price interest rate is defined analogously to the concept of flexible-price output as described in footnote 9.

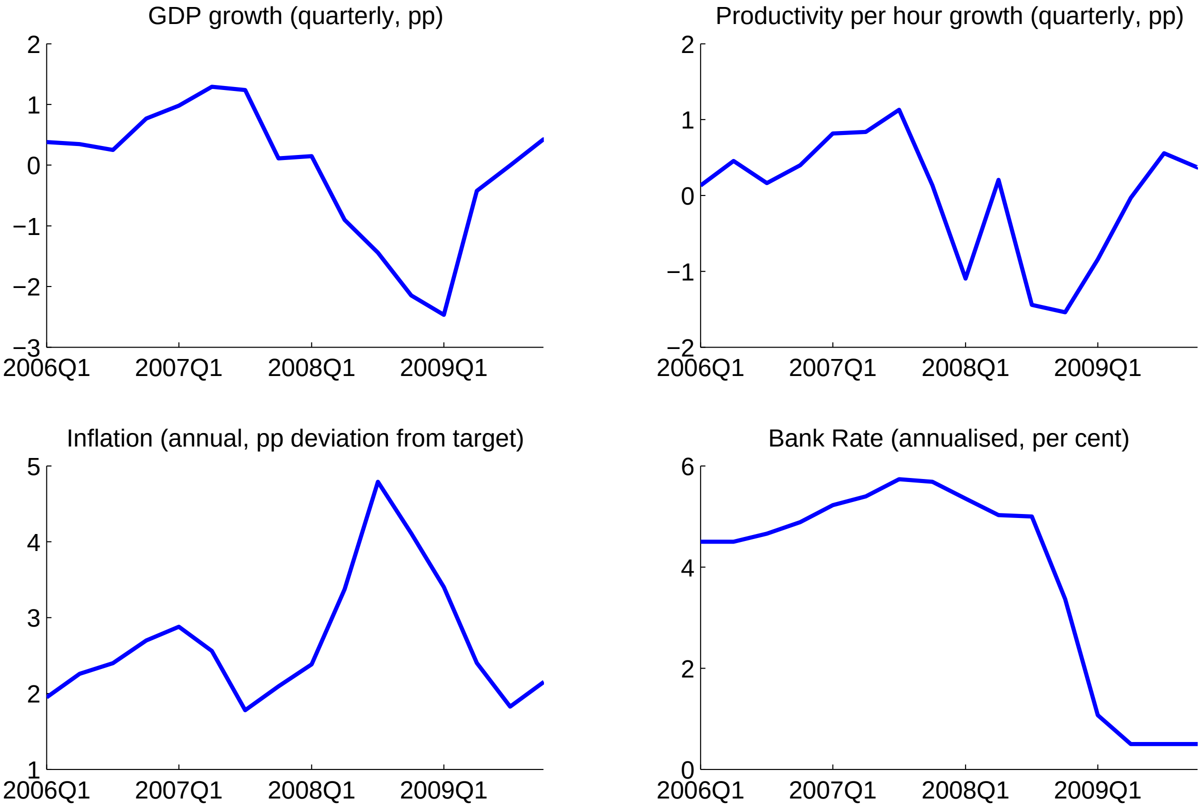
[9] Defined as the deviation of actual output from that which would prevail in a hypothetical economy without nominal rigidities, but which is otherwise identical: the so-called flexible-price output gap.

[10] In part, that is likely to reflect the precise parametrisation of the rule, which in turn depends on the sample of data on which the model is estimated. The sample of data used to estimate COMPASS (using Bayesian maximum likelihood) in Burgess et al. (2013) is 1993Q1-2007Q4 and so (deliberately) excludes the period in which the policy rate was cut to the ELB.

[11] See Hughes and Saleheen (2012) for a discussion of UK labour productivity over the financial crisis.

[12] See Burgess et al. (2013) Section 8.2.

Figure 3: Key features of the UK data to match in our scenario



Notes: Data are sourced as described in Burgess et al. (2013) with the exception of inflation, which is defined exclusive of the effects of changes in the rate of Value Add Taxation.

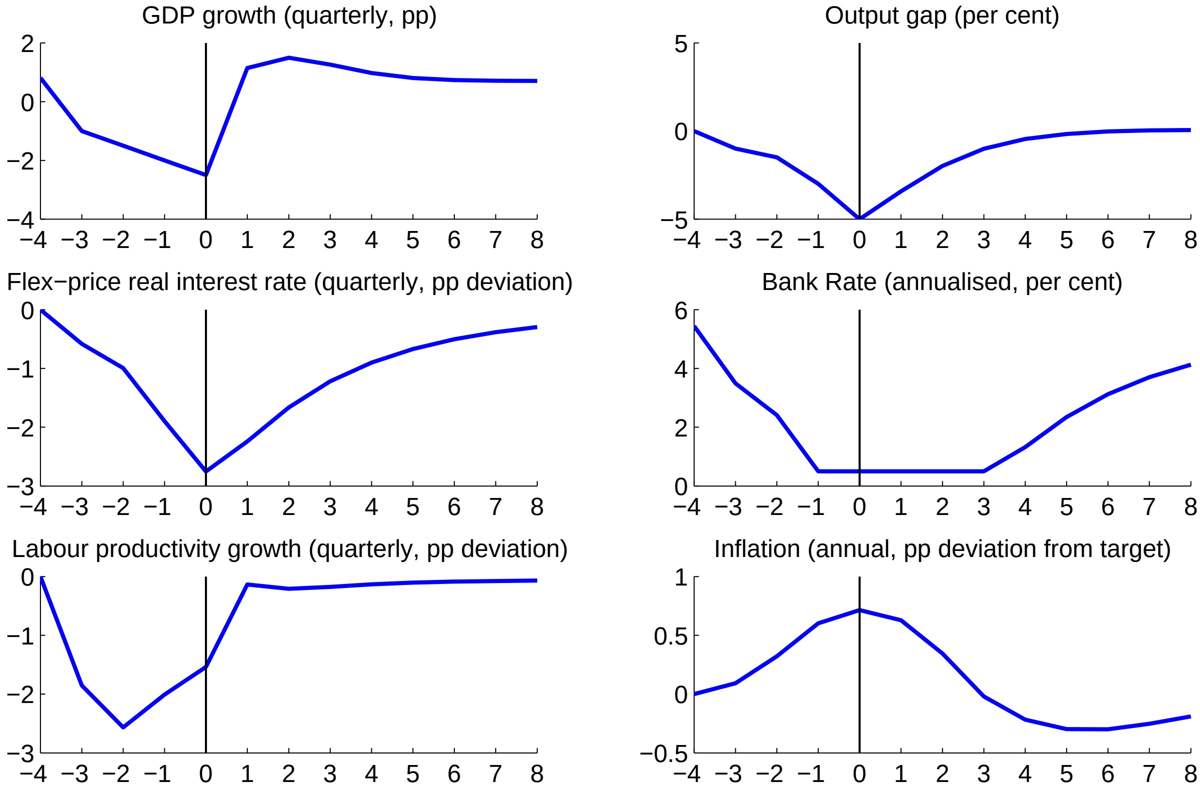
capture some features of the financial crisis.We calibrate the size of the shocks so that the model broadly matches the profiles for quarterly UK GDP growth and internal Bank of England estimates for the output gap between 2008Q2 and 2009Q1.

Figure 4 illustrates the results of this matching exercise for some of the model's key endogenous variables (up to the vertical lines) and shows the model's projections for those variables (right of the vertical lines). By construction, the paths for GDP growth and the output gap in periods -3 to 0 of the simulation approximately match those in the UK data between 2008Q2 and 2009Q1. The paths for the other variables are generated endogenously in response to the sequence of unanticipated domestic risk-premium and labour-augmented productivity shocks calibrated in the matching. As discussed above, the fall in UK activity was accompanied by a fall in productivity, which is qualitatively captured in the scenario as shown in the bottom left panel of Figure 4. Consistent with the discussion in Section 3.1, the flexible-price real interest rate falls substantially in response to the sequence of realised shocks. As a result, the policy maker cuts the interest rate, which hits the effective lower bound in period -1 of the simulation.[13,14] Moreover, policy

[13]This is slightly quicker than occurred in the data (Figure 3). This could reflect that the flexible-price interest rate rule does not adequately capture UK policymaker's behaviour over that period.

[14]It is necessary to account for the effect of policy hitting the ELB in the solution approach used to construct the simulation. To do so, we use a similar approach to that described in Section A.3. That is, we seek a fixed point in which a four-quarter set of unanticipated productivity and risk premium shocks and a minimal set of anticipated policy shocks implement the target values for output growth and the output gap, while ensuring that the interest rate does not fall below the ELB of 0.5 percent.

Figure 4: Baseline scenario



Notes: The x-axes refer to periods in the simulation with: period -4 corresponding to the steady-state starting position, periods -3 to 0 corresponding to the peiods in which the sequence of shocks is realised; periods 1-8 corresponding to a rational expectations forecast fomr the model with period 0 as initial conditions. The y-axes are labelled in each of the plot titles (with "pp deviation" meaning percentage point deviation from model steady state).

is projected to be constrained by the ELB for three additional quarters after the four-quarter sequence of shocks. This is similar to the duration of two quarters that market participants were expecting at the time of the May 2009 *Inflation Report*.[15] Finally, the scenario also broadly replicates the fact that inflation did not fall below target over this period despite weak activity.

3.4 Lower-for-longer policy under imperfect credibility

Our lower-for-longer policy experiment involves the policymaker announcing that policy rates will be held at the ELB for longer than agents had been expecting. In the context of the baseline scenario described above, we consider an announcement at the end of period 0 that the policymaker intends to hold the policy rate at the ELB for 6 quarters, rather than the 3 quarters that agents were expecting in the baseline scenario. This LFL experiment is a special case of the application of an interest-rate peg as described in the context of a small New Keynesian model in Section 2. In line with the discussion in Section 2, we assume that in each period the private sector makes their decisions prior to the policymaker's decision about the interest rate and that reversion to the policy rule is an absorbing state with no probability attached to the re-implementation of the peg. Given the announcement by the policymaker that rates will be held at the ELB for six quarters, this means that there are seven possible states of the world (which the private

[15] See Table 1 on p.43 of Monetary Policy Committee (2009).

sector must take into account): the one in which the policymaker fulfills its commitment to hold rates at the ELB for six quarters and one state for each of the six periods in which they may revert to the policy rule earlier than announced. The algorithm we use to compute the equilibrium paths is detailed in Appendix A and is a generalisation of the algebra used in Section 2.

3.4.1 Baseline results

In the first experiment we consider, the private sector attributes a 10 percent per-period probability that the policymaker will set the interest rate using the policy rule (and so a 90 percent per-period probability that they will continue to hold rates at the ELB). Denoting the per-period probability attributed to reversion to the policy rule as $\{p_t\}_{t=1}^{K}$, then the ex-ante probability of the policymaker having reverted to the policy rule in period i is given by:

$$\mathbb{P}^{\langle i \rangle} \equiv p_i \prod_{j=1}^{i-1} (1 - p_j) \tag{14}$$

for $1 < i \leq K$[16] and where $p_K \equiv 1$. Given a per-period probability of reversion to the rule of 10 percent, this implies that the ex-ante probability of the policymaker following through with the announcement by keeping rates at the ELB for six quarters is more likely than not and is given by: $\mathbb{P}^{\langle 7 \rangle} \equiv \prod_{j=1}^{6} (1 - p_j) \equiv 0.9^6 = 0.53$.[17]

Figure 5 illustrates the outcomes of the LFL policy described above for states of the world in which the policymaker renounces the LFL policy in period 1, 3, 5 and 7, and compares those paths to the baseline scenario described in Section 3.3. There are several things that are worth noting about the results. First, as would be expected, states of the world in which the policy rate is held at the ELB for longer are associated with larger increases in inflation, faster GDP growth (up to the point the peg is lifted) and a smaller (or positive) output gap. Second, this holds true in a comparison of the outcomes in the states of the world in which the policymaker reverts to the policy rule in periods 1 and 3 despite the fact that ex-post the policy rate rises more quickly in the latter.

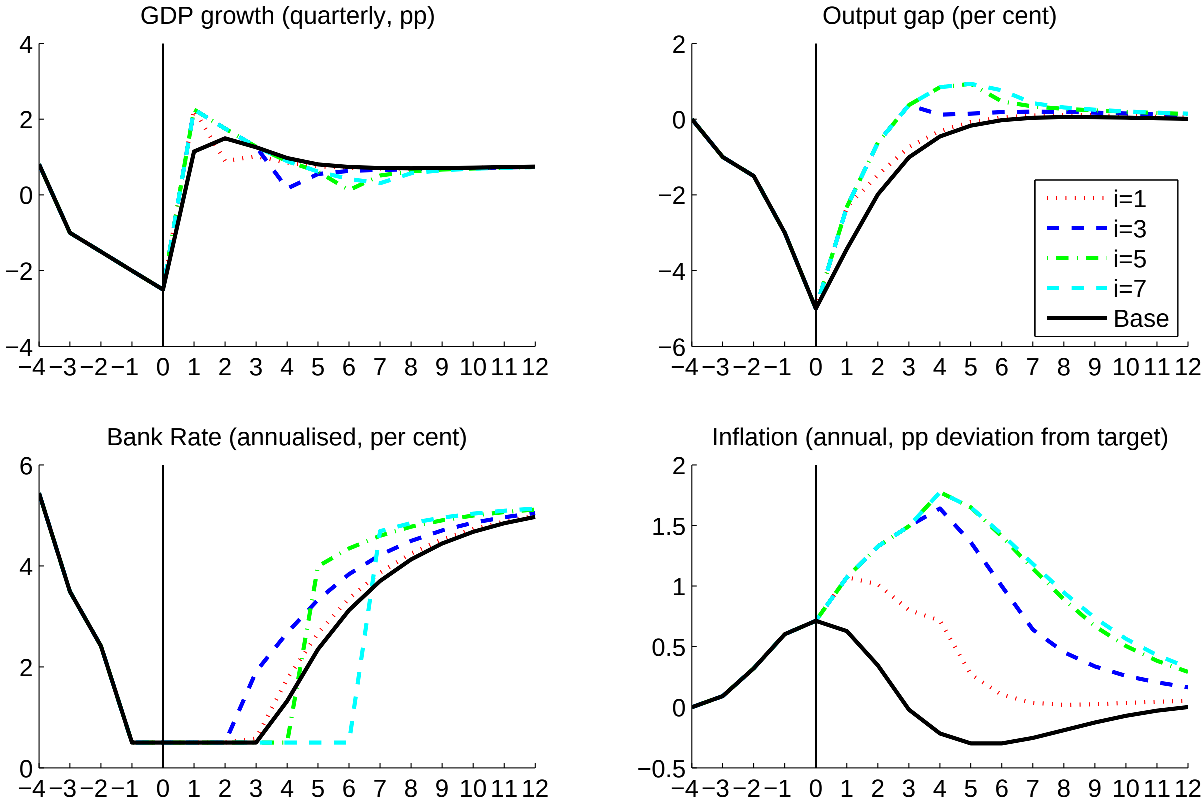
From a mechanical perspective, this reflects the fact that the ELB is still binding on reversion to the policy rule in period 1, but not period 3. In terms of the economics, this is brought about by the role of expectations. In periods 2 and 3, the expected path for the policy rate is lower in the state in which the policymaker reverts to the rule in period 3 than in the state in which reversion occurs in period 1, so activity and inflation are higher. And the additional stimulus imparted by this lower expected path for rates is sufficient to ensure that the interest rate is lifted off the ELB on reversion to the rule in period 3. Third, the outcomes for each possible state in which the policymaker reverts to the rule begin to converge as time progresses. This reflects the fact that the additional stimulus imparted by the policymaker in continuing to hold rates at the ELB is reduced as the terminal period for the announced peg gets closer.

It is straightforward to compute summary statistics for the LFL policy simulation. For example, Figure 6 shows the mean and swathe (the range of alternative outcomes

[16] The ex-ante probability that the policymaker reverts to the policy rule in period 1 is simply given by the per-period probability of reversion: $\mathbb{P}^{\langle 1 \rangle} \equiv p_1$.

[17] More generally, for an LFL policy announcement of K periods and a constant per-period probability of reverting to the policy rule of p, then the state of the world in which the policymaker follows through with its announcement will be more likely than not as long as: $p < 1 - \exp\left(\frac{\ln(0.5)}{K}\right)$.

Figure 5: Alternative states in a 6 period LFL policy simulation with a 10 percent per-period renouncement probability



Notes: Each path is associated with a particular renouncement period, i, with $i = 7$ referencing the state of the world in which the policymaker implements the LFL policy for the full six periods (as announced at the end of period 0).

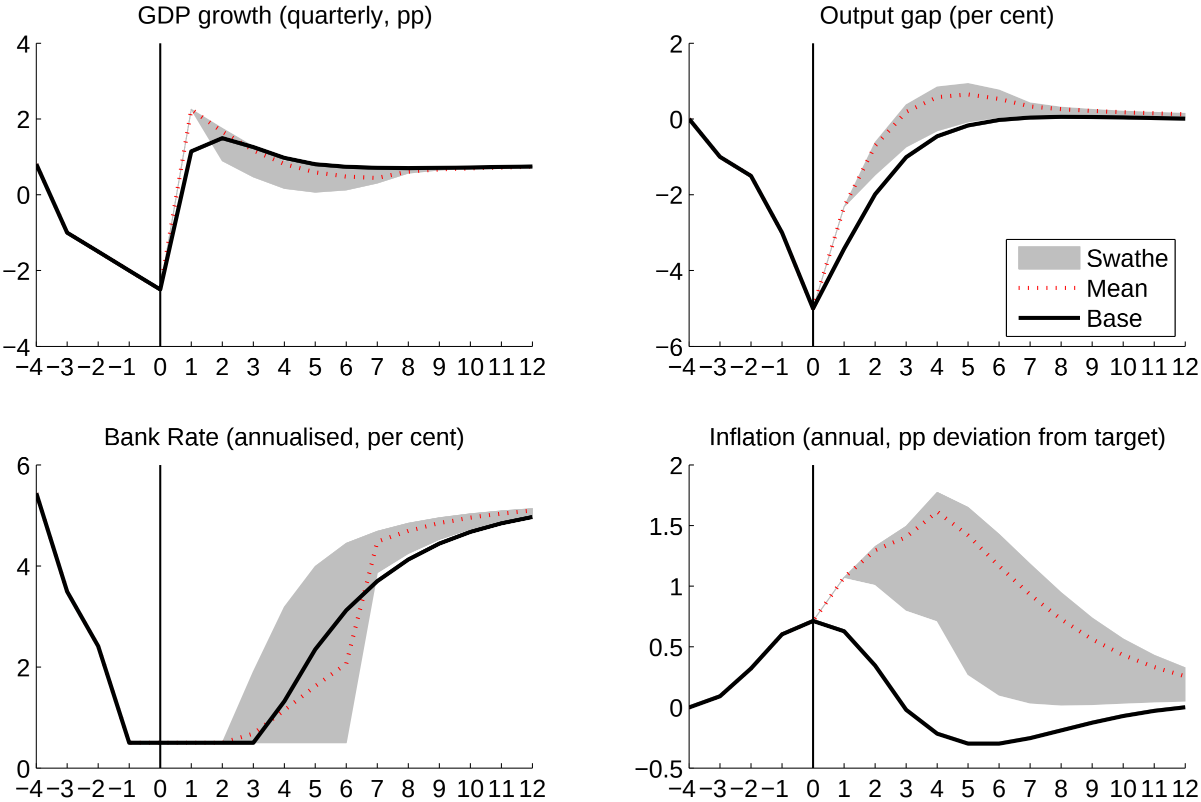
encompassed by the period-by-period minimum and maximum) from the alternative paths in the LFL policy simulation from Figure 5. It shows that the mean paths for activity and inflation tend to lie towards the tops of their respective swathes, highlighting the powerful effect on agents' expectations arising from the possibility that the peg will be maintained.

3.4.2 Sensitivity to varying the probability of renouncement

As the credibility of the LFL policy decreases – as measured by a rise in the probability of renouncement – there is a significant decrease in the policy's impact on macroeconomic variables. This is evident in Figure 7, which shows the economic effects of a six-period LFL policy for different probabilities of renouncement. Relative to the perfect credibility case (labelled as "p=0", meaning 0 percent probability of renouncement), the economic effects of the LFL policy are roughly halved for a per-period probability of reverting to the policy rule of somewhere between 5 and 10 percent. And for a larger per-period probability of 20 percent the economic effects are around one-tenth of the size.[18]

[18] In the 20 percent per-period probability case, the state of the world in which interest rates are held at the ELB for the full 6 quarters is no longer more likely than not, but it is still the most likely of all the possible states of the world (i.e. it is the modal path).

Figure 6: Mean and swathe of outcomes in a 6 period LFL policy simulation with a 10 percent per-period renouncement probability



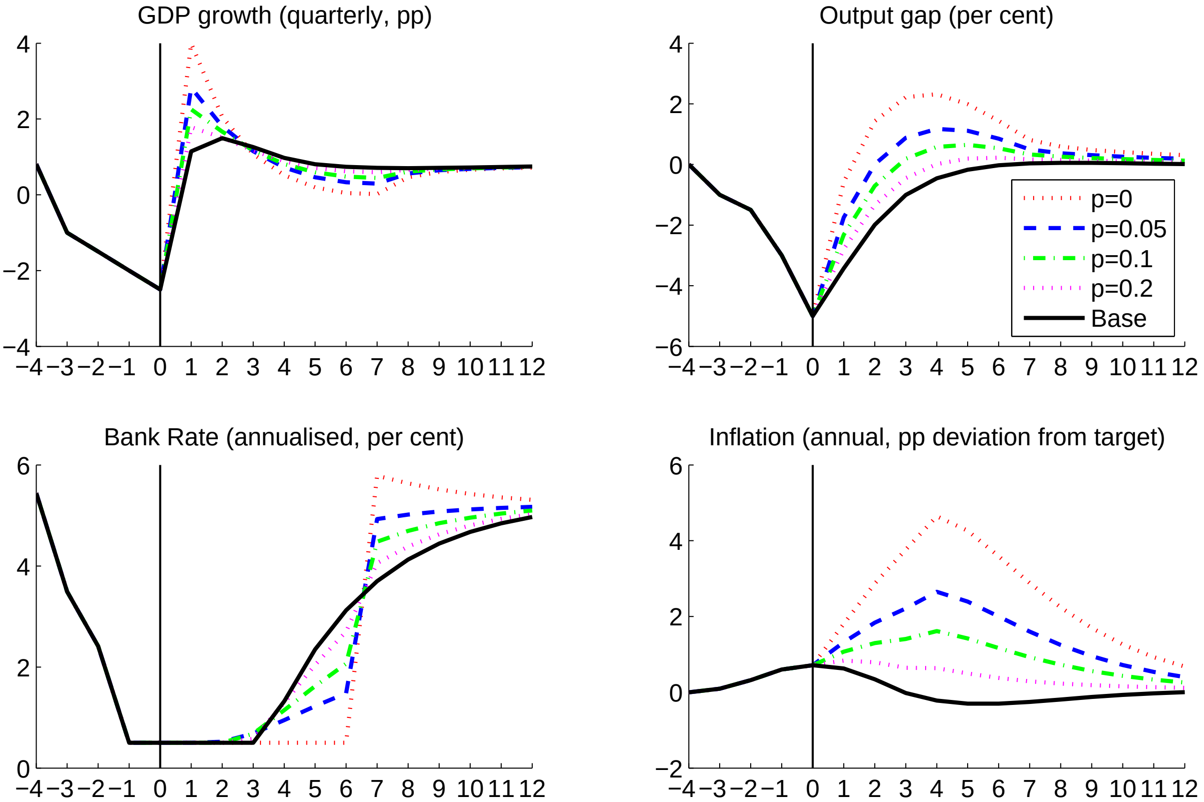
Notes: The mean is computed by weighting together the alternative paths generated by the LFL policy (as in Figure 5) by their probabilitis as defined in equation (14). The swathe is based on the per-period minimum and maximum ouctomes of all the alternative paths (regardless of their probabilities).

3.4.3 Impact on long-term interest rates

In comparison to the perfect credibility case, imperfectly credible LFL policies generate smaller – and more plausible – movements in long-term nominal interest rates (as computed using the expectations hypothesis for interest rates consistent with the model). We can, therefore, view imperfect credibility as a more structural – and endogenous – version of the mechanism underlying the solution to the forward guidance puzzle proposed by Del Negro et al. (2012). In that paper, the authors outline an algorithm for implementing LFL policies that directly constrains the ten-year interest rate from adjusting by more than a relatively small value – e.g. 10 basis points – in response to the announcement.[19] There are two motivations for this. First, the authors demonstrate that in forward-looking New-Keynesian models, aggregate demand is generally very sensitive to movements in long-term interest rates. Second, empirical evidence suggests that in response to forward-guidance announcements by central banks, long-term interest rates move by less than would be predicted in a model-based LFL policy simulation (under

[19] The Del Negro et al. (2012) algorithm finds the set of anticipated policy shocks over the K periods of the LFL policy that delivers the desired change in the ten-year spot rate, while minimising the weighted distance of the policy rate from the ELB in the first K periods and forward rates from their baseline values in the next $K+1\ldots40$ periods. So, in addition to specifying the desired change in the long rate, the policymaker or adviser must also specify a set of weights in order to implement the algorithm. One implication of this under-identification approach is that the policy rate is not exactly equal to the ELB in any of the K periods of the LFL policy.

Figure 7: Mean outcomes in 6 period LFL policy simulations with alternative assumptions about credibility



Notes: Each path (other than the one labelled "Base") shows the mean outcome of the six-period LFL policy under alternative assumptions about the credibility of the policy, as measured by the per-period renouncement probability, p.

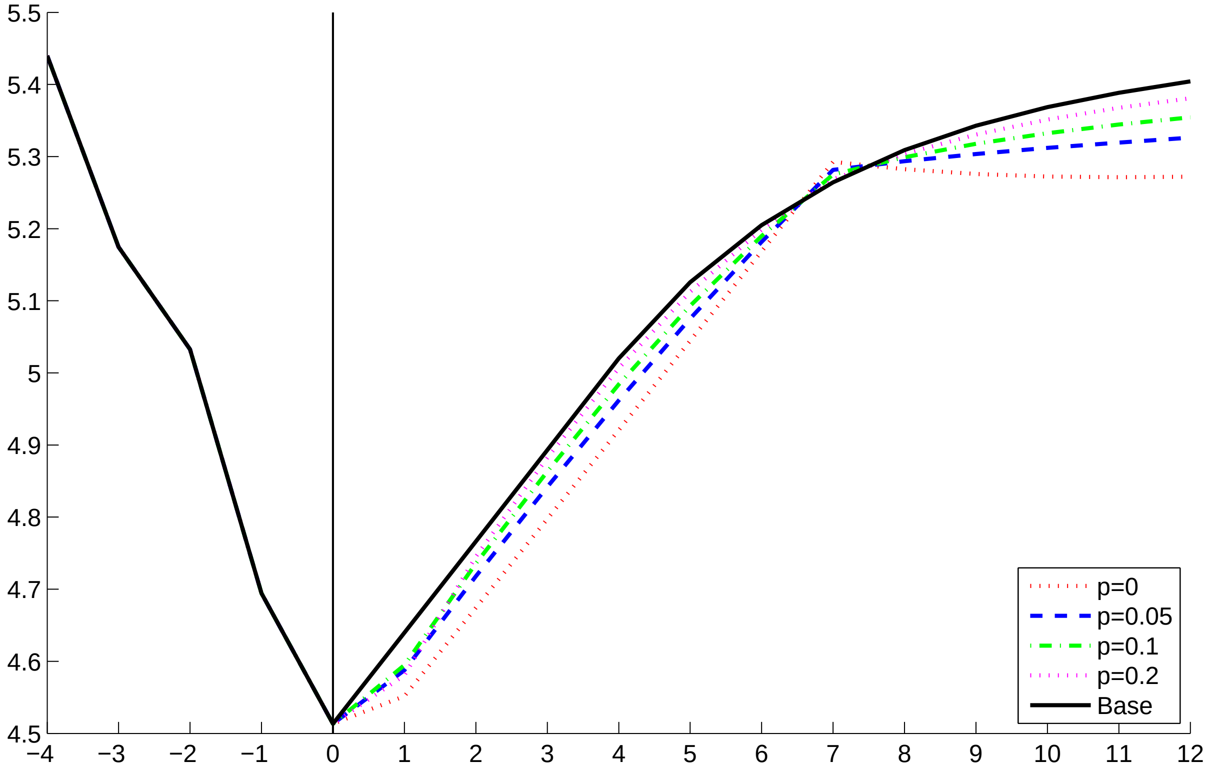
perfect credibility).

Allowing for imperfect credibility reduces the amount by which long-term interest rates fall in response to the LFL policy announcement without directly constraining the long rate, as in Del Negro et al. (2012). This can be seen in Figure 8, which shows the long-term interest rate falls by less relative to the baseline as the per-period probability of renouncement rises. Specifically, the figure shows the mean response of the ten-year (spot) interest rate for different values of the probability of renouncement (consistent with the simulations in Figure 7). Long-term interest rates fall by less under imperfect credibility because, in this case, agents attach some probability to policy being set in line with the rule, which typically implies higher interest rates than under the LFL announcement. That said, in period 7, when the peg is lifted, interest rates are slightly higher under perfect credibility. This is because the generally higher paths for rates under imperfect credibility dampen the impact on inflation and the output gap relative to perfect credibility. As a consequence, the policy rate does not need to rise as much when the peg is lifted.

3.5 Endogenising the credibility of the lower-for-longer policy

It is plausible to suppose that the credibility of the LFL policy, as measured by the probability of maintaining the peg, $1-p$, might be lower when the policymaker's incentive to abandon the peg is higher. In the analysis we have considered so far, the credibility

Figure 8: Mean 10 year interest rates outcomes in 6 period LFL policy simulations with alternative assumptions about credibility



Notes: Each path (other than the one labelled "Base") shows the mean 10 year spot rate (measured annualised in percentage points) computed using expectations consistent with each simulation in the six-period LFL policy under alternative assumptions about the credibility of the policy, as measured by the per-period renouncement probability, p, and consistent with Figure 7.

of the policymaker's LFL policy has been independent of the announcement itself and, relatedly, any incentive that the policymaker might have to renounce the policy based on economic outturns. Specifically, we have assumed that the 'benefit' from renouncement is exogenous, and given by p. The nature of the LFL announcement, however, might affect the incentives to renounce the policy, and hence, arguably, the credibility of the policy itself. In other words, the 'benefit' from renouncing the peg should, arguably, be endogenous to economic outturns via their influence over the policymaker's incentive to renounce.

The policymaker's incentive to renounce arises from the fact that the announcement of the LFL policy will, by itself, stimulate the economy. With the stimulus in place, the policymaker can then either set policy in line with the announcement or, in the case we consider here, revert to the rule. For example, with reference to Figure 7, announcing that a peg will be in place for 7 quarters delivers a substantial boost to the economy in the first few quarters after the announcement, particularly in the perfect credibility case. Once the economy has been stimulated, however, it is possible that the policymaker could achieve more 'favourable' outcomes by tightening (i.e. reverting to the rule) earlier than promised. This would, in effect, lock in the impact of the boom, but avoid the protracted overshoot in inflation and the output gap that would ensue if the policy were implemented as announced.[20]

[20]This is an example of the classic time-inconsistency problem. Analyses of time inconsistency are typically based on an explicit optimisation by the policymaker, in which the policymaker either optimises

It is important to be clear about what we mean by more 'favourable'. Specifically, we have in mind an objective function for the central bank that includes inflation and the output gap, both in terms of the squared deviations from their respective steady states (we provide a precise formulation below). This objective function is ad hoc insofar as it is not derived from the utility function of the representative agent. That said, we believe that it captures reasonably well the key ingredients of micro-founded loss functions, such as a focus on inflation stabilisation and as well as some concern for stabilising real variables. It is also broadly consistent with the mandates of real-world central banks.[21]

This incentive to abandon the peg after announcement is illustrated in Figure 9, which shows that the net benefit (in terms of loss utils) of reverting to the policy rule is positive at all points along the commitment path. That is, if, at each point in time, the private sector believes that the policymaker will implement the peg for the duration of the announcement, then the policymaker always has an incentive to leverage off those beliefs and revert to the rule rather than implementing the peg as had been expected. That incentive is strongest in the periods closest to the announcement. Intuitively, the policymaker can achieve a better ouctome by increasing rates earlier than promised, thereby avoiding the large overshoot of the inflation target and of the output gap associated with maintaining the peg. This suggests that the perfect credibility equilibrium incorporates beliefs that may not be consistent with reasonable expectations of how the policymaker might behave in the event that the outcomes generated by the policy were actually realised. This section builds on that logic to show how the probability that the private sector attributes to the policymaker reverting to the policy rule earlier than announced can be computed endogenously as a function of the incentive that the policymaker has to revert to the rule.

3.5.1 Algorithm for endogenous imperfect credibility

In keeping with the relatively ad-hoc nature of the policy experiment under consideration (as well as a desire to retain a full-information, rational-expectations equilibrium), we continue to treat the possibility of early reversion to the rule as a lottery. We relate the probability that the policymaker might abandon the peg to the net benefit (in terms of losses) of doing so. The net benefit of reverting to the rule in any period i, $\mathcal{B}^{\langle i \rangle}$, can be defined as:

$$\mathcal{B}^{\langle i \rangle} = \mathcal{L}_i^* - \mathcal{L}_i^{\langle i \rangle} \tag{15}$$

where $\mathcal{L}_i^{\langle i \rangle}$ represents the loss associated with economic outcomes in the event of reversion to the policy rule in period i and $\mathcal{L}_i^*$ represents the expected loss associated with continuing to implement the peg, which is defined as:[22]

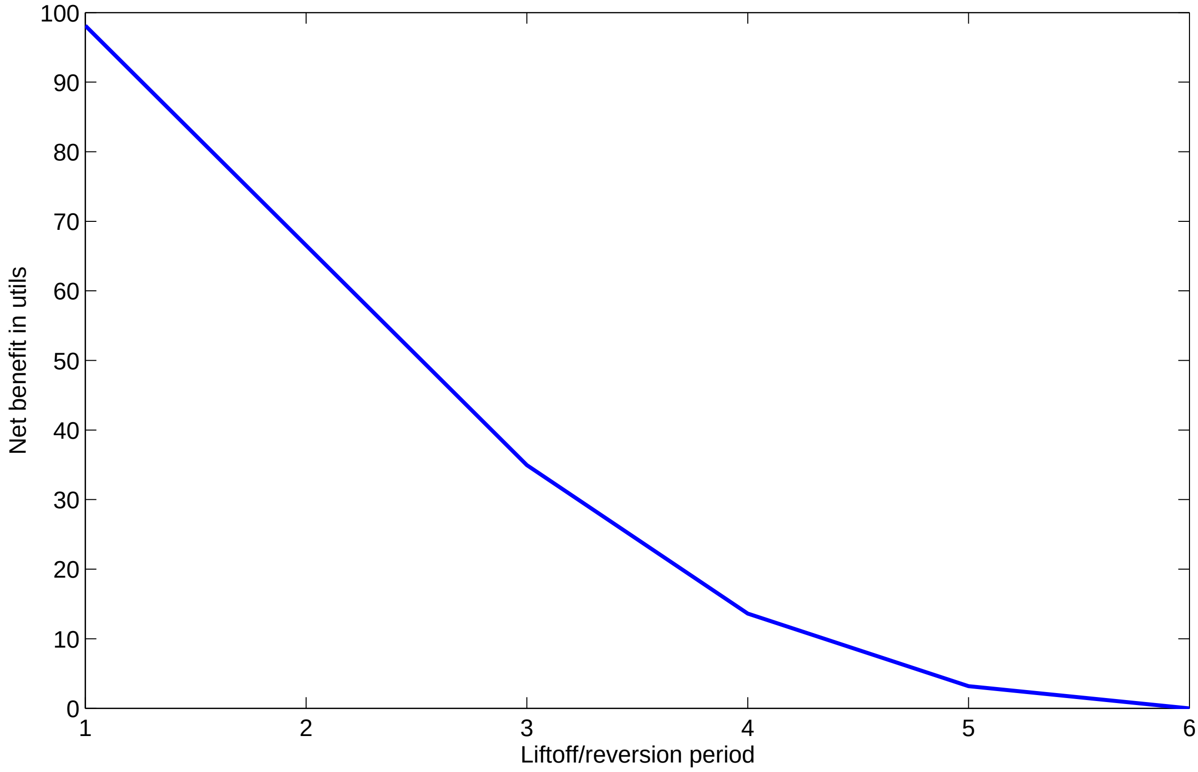
$$\mathcal{L}_i^* = \frac{1}{1 - p_i} \sum_{j=i+1}^{K+1} \mathbb{P}_i^{\langle j \rangle} \mathcal{L}_i^{\langle j \rangle} \tag{16}$$

once and for all at the start of time (optimal commitment), or optimises on a period-by-period basis (optimal discretion). This is distinct from the experiment we consider here, in which the policymaker is effectively choosing between commitment to a transient interest rate peg versus following a simple rule, which is itself a form of commitment.

[21] A practical issue is that derivation of a welfare-based loss function is typically intractable, given the size and complexity of the models used in central banks.

[22] The continuation losses are weighted by the probability of each possible state conditional on the policymaker not reverting to the rule in that period and so the probabilities are correspondingly scaled by the total probability of not reverting in that period, $1 - \mathbb{P}_i^{\langle i \rangle} \equiv 1 - p_i$.

Figure 9: Net benefit of abandoning the peg along the perfect credibility full commitment path



Notes: For each of the alternative early liftoff periods (measured on the x axis) the net beneift of abandoning the peg is computed using the definition in equation (15). That is, it is a comparison of the loss that would be realised by maintaining the peg with that which would be realised by lifting it conditional on the peg not having been lifted up to that point. Consistent with the discussion in Section 3.5.1, losses were computed as the sum of squared deviations of annual inflation from target and the output gap from zero across 40 periods (by which time the model is close to being back at steady state in all states of the world). Output and inflation deviations were weighted equally and the time preference rate of the policymaker was assumed to be 0.

where $\mathcal{L}_i^{\langle j \rangle}$ is the period i loss associated with reversion to the policy rule in period j (where $j > i$) and $\mathbb{P}_i^{\langle j \rangle}$ is the probability of reversion to the rule in period j conditional on the peg still being in operation in period i defined (for $j > i$) as:[23]

$$\mathbb{P}_i^{\langle j \rangle} \equiv p_j \prod_{t=i}^{j-1} (1 - p_s) \tag{17}$$

Throughout this section we assume that the policymaker computes losses using a quadratic function over deviations of annual inflation from target and the output gap from zero. This means that the loss associated with reversion to the rule in period, j, conditional on not already having done so by period i is defined as the sum of losses associated with outcomes while the peg is still in effect between periods i and j (denoted with a supersript $*$) and losses associated with the state following reversion the rule in

[23]This is a logical extension of the definition for the ex-ante probability of each state being realised after the peg has been announced as defined in equation (14).

period j (denoted with superscript $\langle j \rangle$):[24]

$$\mathcal{L}_i^{\langle j \rangle} = \sum_{t=i}^{j-1} \beta^{t-i} \left(\left[\sum_{s=t-3}^{t} \pi_s^* \right]^2 + \lambda \left[\hat{y}_t^* \right]^2 \right) + \sum_{t=j}^{\infty} \beta^{t-i} \left(\left[\sum_{s=t-3}^{t} \pi_s^{\langle j \rangle} \right]^2 + \lambda \left[\hat{y}_t^{\langle j \rangle} \right]^2 \right) \tag{18}$$

where $\sum_{t-3}^{t} \pi_t$ is the deviation of annual consumer price inflation from target, $\hat{y}_t$ is the output gap, β is the policmakers' discount factor and λ is the relative weight the policymaker places on output.

It is also necessary to make an assumption about the function we use to convert the net benefit of reverting to the rule in terms of losses, $\mathcal{B}^{\langle i \rangle}$, into the benefit of renouncement in the sense described in section 2.2, which, given our framework, is equal to the probability of renouncement. The function we choose for this mapping is an exponential distribution, as defined by (19). Our choice of the exponential distribution reflects a desire to incorporate several different ingredients into the analysis. Our overarching objective is to continue to capture a form of imperfect credibility; in other words, the policymaker's LFL policy is neither perfectly credible (i.e. $p = 0$), nor fully imperfectly credible (i.e. $p = 1$). This is qualitatively captured in a continuous distribution like the exponential. In this distribution, the probability of reversion to the rule rises as the net benefit of doing so increases. Moreover, the exponential distribution only has support on the positive real line, consistent with the idea that the policymaker would never abandon the peg if the net benefit of doing so were not positive (i.e. if they could achieve a better ouctome by continuing with the policy). Finally, the exponential distribution is parsimoniously parameterised with a single parameter, α, allowing us to vary how "committed" the policymaker is to the peg between the extremes of perfect credibility and fully imperfect credibility. A fully committed policymaker ($\alpha = 0$) will not revert to the rule even if the net benefit derived from reverting to the rule is positive (i.e. $p_i = 0$ always). Obversely, a policymaker without any ability to commit to the LFL policy ($\alpha = 1$) will revert to the rule whenever the net benefit from doing so is positive (i.e $p_i = 1$ for $\mathcal{B}^{\langle i \rangle} \geq 0$).[25]

$$p_i = \begin{cases} 1 - \exp^{-\left(\frac{1-\alpha}{\alpha}\right)\mathcal{B}^{\langle i \rangle}} & \mathcal{B}^{\langle i \rangle} \geq 0 \\ 0 & \mathcal{B}^{\langle i \rangle} < 0 \end{cases} \tag{19}$$

3.5.2 A 6-period LFL-policy with endogenous imperfect credibility

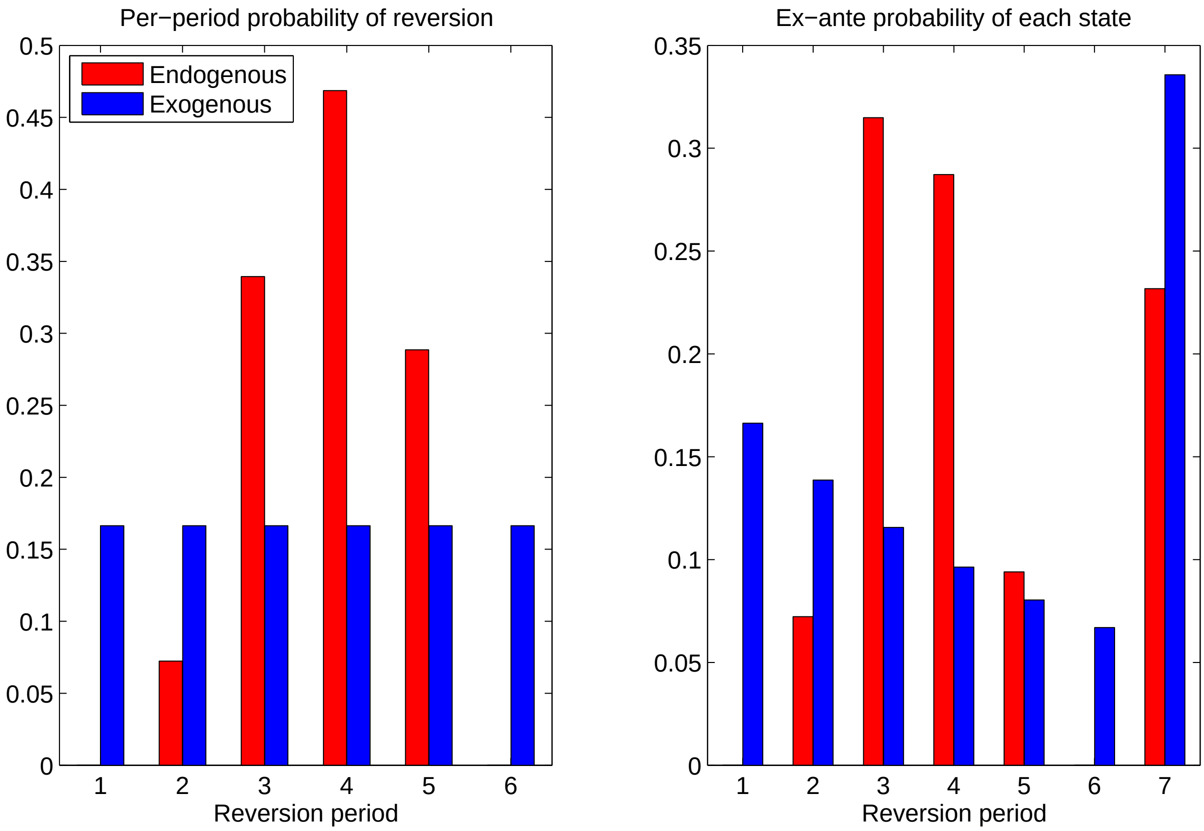
As a starting point for the analysis, we recompute the 6-period LFL policy introduced in Section 3.4 under exogenous probability of reversion to the rule with endogenous probabilities. To solve for the vector of endogenous probabilities, our computational procedure is to guess at a vector of time-varying numeric values for the per-period probability of reversion and iterate on those guesses until they converge. Each iteration requires the computation of the simulated paths as described in Appendix A, the computation of losses associated with those paths, and then the computation of updated probabilities using equations (15) through (19). In practice, the probabilities are not updated fully between iterations (and an update parameter of around one-quarter proved adequate to

[24] This is valid for $j > i$. For $j = i$ the loss id defined by only the second of the two summation terms.

[25] Note that this is delivered by the transformation $\frac{1-\alpha}{\alpha}$ such that $\alpha = 0$ delivers a standard exponential distribution "rate" parameter equal to ∞ and $\alpha = 1$ delivers a rate parameter equal to 0.

ensure robust and reasonably rapid convergence). As a starting point, we set the parameters in the policymaker's loss function – the discount factor, β, and the relative weight on output, λ – equal to one. And we set the credibility parameter, α, to 0.5, which is qualitatively consistent with an intermediate level of credibility. We explore the sensitivity of the results to both the credibility of the policymaker, α, and the weight they attach to output, λ, below.

Figure 10: Renouncement probabilities in a 6 period LFL policy simulation with endogenous and exogenous imperfect credibility



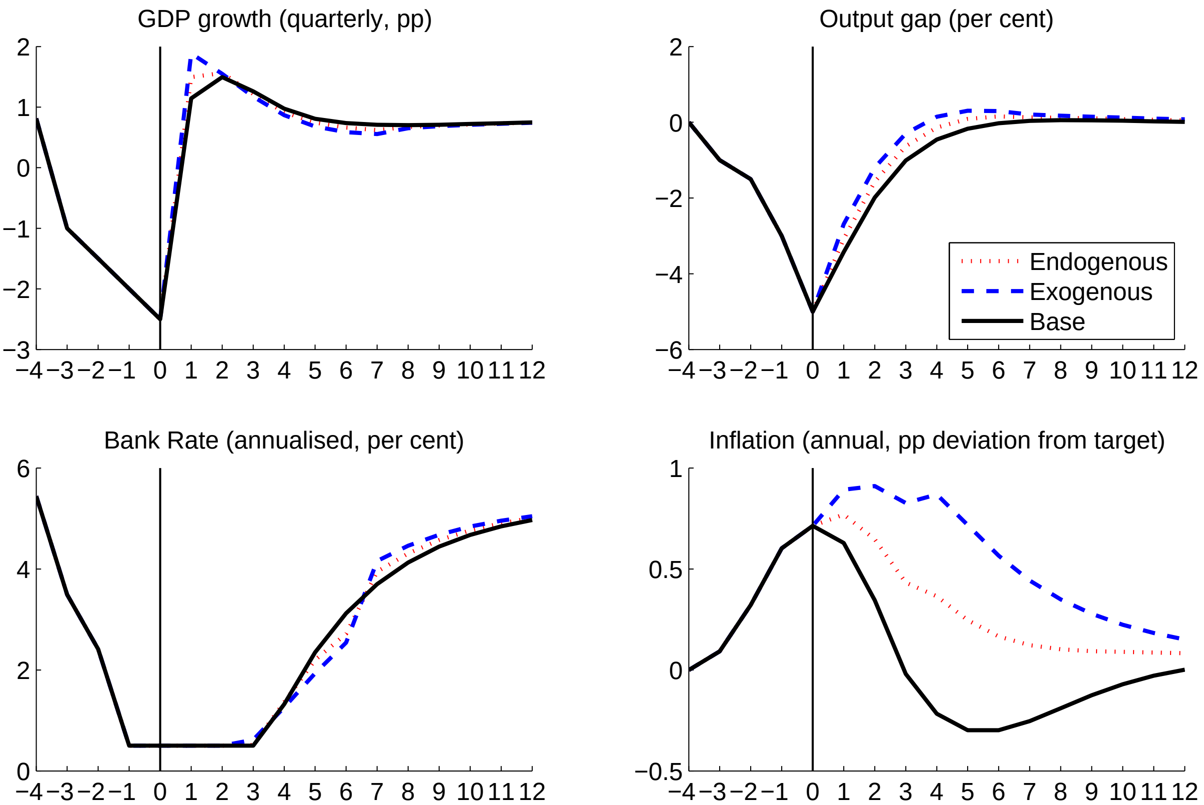
Notes: The constant exogenous per-period probability was set to match the expected duration of the LFL policy arising endogenously in the endogenous imperfect credibility simulation with the commitment parameter, α, set to 0.5. Note that consistent with the policy experiment being conducted here in which the LFL policy comes to an end with certainty in period 7, the per-period probability of reversion to the rule in period 7 is the same and equal to 1 in both cases (which is why the left plot only includes probabilities up to the period 6).

In the 6-period LFL policy, the probability of reversion to the rule is highest in periods 3, 4, and 5. This is shown in Figure 10, which compares the endogenous probabilities of reversion computed under the parameterisation described above with a set of constant and exogenous per-period probabilities set to deliver the same expected duration for the interest-rate peg.[26] The peak in the probability of reversion in the middle quarters of the LFL policy reflects the pattern of outturns in response to the policy scenario (as shown,

[26]The duration from the simulation with endogenous probabilities can be computed as $\sum_{i=1}^{K+1} \mathbb{P}^{\langle i \rangle}(i-1)$ with the probability of the policymaker renouncing in period i defined in equation (14). It is then straightforward to compute a constant per-period probability scalar that delivers the same duration using simple numerical search. In the example used here, the expected duration of the policy is 3.3 periods and the constant per-period probability of renouncement that delivers that expected duration is 0.17.

for example, in Figure 7) and can be understood with reference to the constant probability case. In this simulation, annual inflation peaks in period 4 and the output gap is close to being closed. This suggests that the credibility of continuing with the peg from period 4 would be low: the policymaker would be better off reverting to the rule and avoiding over-stimulating the economy by holding interest rates at their peg for the extra two quarters. In the endogenous probability case, the probabilities of renouncement are updated to reflect this fact. Another feature of the simulation under endogenous probability is that the per-period probabilities of reversion are approximately zero in periods 1 and 6. In period 1, this reflects the fact that the higher probability of renouncement in later periods is sufficient to eliminate the strong incentive to renounce that was evident in the full commitment simulation (Figure 9). While in the latter, it reflects that the additional stimulus imparted by one additional quarter of holding rates lower for longer is relatively small, which reduces the incentive of the policymaker to abandon the peg at that point (Figure 5).

Figure 11: Mean outcomes in 6 period LFL policy simulations with endogenous and exogenous imperfect credibility



Notes: By design the expected duration of the policy is the same in both simulations. The renouncement probabilities associated with these simulations are shown in Figure 10.

Endogenising the credibility of the LFL policy further dampens its effects relative to the exogenous imperfect credibility case. This is because, under endogenous imperfect credibility, policies that generate large macroeconomic effects, and hence a potentially larger incentive for the policymaker to renounce, are likely to be viewed as less credible – and, in turn, a lower probability weight is attached to their realisation. This is evident in Figure 11, which shows the mean paths when the probability of renouncement is both endogenously and exogenously determined, consistent with the probabilites shown in Figure 10. The impact of the LFL policy on the output gap and inflation is smaller for much of the simulation in the endogenous probability case relative to the exogenous case. This suggests that accounting for the relationship between the probability that the policymaker might renounce the peg and the state of the economy can dampen the effects of LFL policies over and above incorporating imperfect credibility in an exogenously-given manner.

Table 1: Sensitivity of the endogenous renouncement probabilities in 6 period LFL policy simulations to key parameters

	α								λ			
	0.01	0.1	0.25	0.5	0.75	0.9	0.99	0.999	0	0.5	1	2
Per-period probabilities												
Period												
1	0.00	0.00	0.00	0.00	0.00	0.13	0.17	0.06	0.34	0.00	0.00	0.00
2	0.00	0.00	0.00	0.07	0.24	0.27	0.16	0.04	0.38	0.23	0.07	0.00
3	0.00	0.00	0.20	0.34	0.36	0.30	0.11	0.03	0.43	0.38	0.34	0.25
4	0.00	0.39	0.52	0.47	0.35	0.22	0.05	0.01	0.39	0.44	0.47	0.50
5	1.00	0.77	0.50	0.29	0.15	0.08	0.01	0.00	0.16	0.23	0.29	0.39
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Ex-ante probabilities of each state												
Period												
1	0.00	0.00	0.00	0.00	0.00	0.13	0.17	0.06	0.34	0.00	0.00	0.00
2	0.00	0.00	0.00	0.07	0.24	0.24	0.13	0.04	0.25	0.23	0.07	0.00
3	0.00	0.00	0.20	0.31	0.27	0.19	0.08	0.02	0.18	0.29	0.31	0.25
4	0.00	0.39	0.41	0.29	0.17	0.10	0.03	0.01	0.09	0.21	0.29	0.37
5	1.00	0.46	0.20	0.09	0.05	0.03	0.01	0.00	0.02	0.06	0.09	0.15
6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.14	0.19	0.23	0.27	0.32	0.58	0.86	0.12	0.21	0.22	0.23

Notes: The table shows how the endogenous probabilities of renouncing the six-period LFL policy early vary by the policymaker's known commitment parameter, α, and the weight they place on output relative to inflation in the loss function, λ. In varying α, the output weight parameter is held fixed at its baseline value, $\lambda = 1$, and in varying λ, the commitment parameter is fixed at its baseline values, $\alpha = 0.5$.

3.5.2.1 Sensitivity analysis The results of the simulation above are sensitive to the values taken by parameters such as the relative weight the policymaker places on the output gap in its loss function, λ, and the parameter that controls the policymaker's credibility, α. Table 1 shows the sensitivity of the endogenous per-period renouncement probabilities and associated ex-ante probabilities of each state for alternative values of these two parameters.

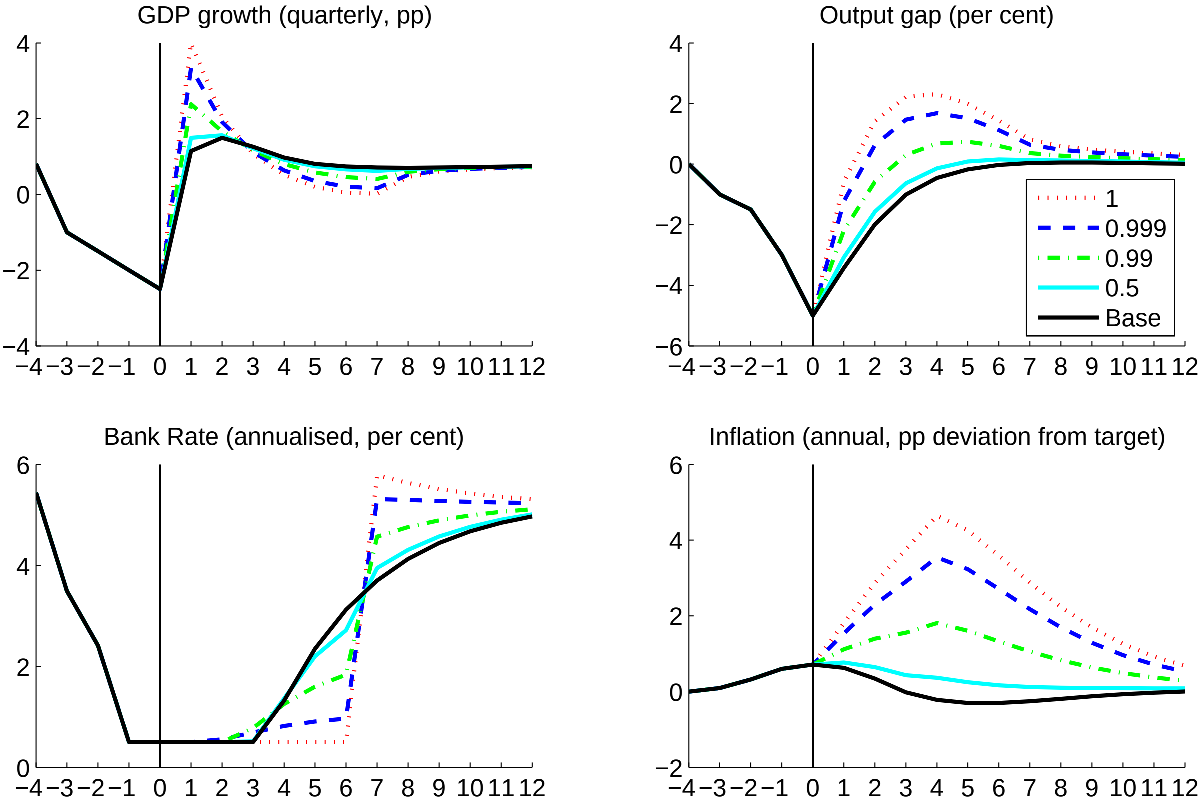
The results are sensitive to the value taken by λ on account of the trade-off between high inflation and a large negative output gap embodied in the scenario constructed in Section 3.3. This trade-off exists since the same policy action would not stabilise both variables at the same time – there is a tension between setting looser policy to close the output gap, and tighter policy to reduce inflation. As a result, a policymaker who places no weight on output in the loss function ($\lambda = 0$) is more likely to renounce the LFL policy earlier in order to raise interest rates and reduce inflation than a policymaker who is more concerned with stabilising the output gap (eg $\lambda = 2$).

As would be expected, the renouncement probabilities are very sensitive to the "commitment" level of the policymaker to the peg, α. The mass of outcomes converges towards centring on a particular period (in this case period 5) in the case of a policymaker known to be less committed to the peg (lower values of α).[27] By contrast, in the case of a policymaker known to be more committed to the peg (higher values of α), the mass of outcomes becomes polarised with a more even split between relatively early reversion to the rule in the first few periods and the LFL policy peg being maintained for the full six periods. This reflects that a policymaker who is more committed to the LFL policy is less likely to renounce the policy, which increases its macroeconomic effect (Figure 7) and so the incentive to renege (Figure 9).

3.5.2.2 Varying the policymaker's level of commitment It is possible to explore in more detail the effects of varying the policymaker's commitment level, α, on macroeconomic variables. Figure 12 shows the macroeconomic outcomes in response to the LFL policy and indicates that they are substantially reduced even in cases where the policymaker is operating very close to perfect credibility. This finding reflects the nature of the endogeneity: the more "committed" the policymaker is, the larger the impact the policy will have and so the greater the net benefit of renouncing the policy. However, Figure 12 also shows that there remain material differences between the policy outcomes, suggesting that the commitment of the policymaker and the associated credibility of the policy from the perspective of the private sector are key determinants of the impact of LFL policies in New Keynesian models.

[27] Interestingly, there is no rational-expectations equilibrium in the case of no commitment to the peg, $\alpha = 0$, and hence fully imperfect credibility (given the LFL policy under consideration and for the baseline parameterisation of the loss function). This follows from noting that the policymaker has an immediate incentive to abandon the peg after announcing the policy in the perfectly credible case (Figure 9). However, if the private sector knows with certainty that the policymaker will abandon the peg at the end of period 1, then the policy has no effect and the policymaker would choose to implement the peg for longer. It turns out that we could not find an equilibrium that simultaneously supports the policymaker reverting to the rule in one of the periods with certainty that does not leave the policymaker with an incentive to renege given those beliefs.

Figure 12: Mean outcomes in 6 period LFL policy simulations with alternative commitment parameterisations



Notes: The figure shows the mean outcomes of our six-period LFL policy with endogenous imperfect credibility for alternative parameterisations of the commitment parameter, α, with low values indicating a policymaker who is known to operate closer to discretion and higher values indicating a policymaker who is known to operate closer to commitment. Each simulation was computed under the baseline parameterisation for the loss function (i.e. with an equal weight on inflation and output). The associated renouncement probabilities are shown in Table 1.

4 Conclusions

The macroeconomic effects of lower-for-longer (LFL) policies depend crucially on the extent to which private agents view them as credible. The possibility that LFL policies might be imperfectly credible arises from their potential to be time inconsistent. In other words, the policymaker may have an incentive to abandon the peg after its announcement. By doing so, the policymaker may reap the benefits of improved economic outturns without paying the cost of the overshoot in inflation and output that comes later. Using an ad-hoc loss function defined over the squared deviations of inflation and the output gap from their steady states, we show the policymaker may indeed have an incentive to abandon the peg after it has been announced rather than implement it as promised.

In this paper, we model imperfect credibility as the private sector placing a non-zero probability on the policymaker abandoning the peg after it has been announced. To begin with we assume the degree of imperfect credibility is fixed and exogenous to the state of the economy. In this case, the impact on macroeconomic variables, such as inflation and GDP, will be significantly smaller than if the peg were perfectly credible. Allowing for the possibility of imperfect credibility can, therefore, mitigate the problems associated with the "forward-guidance puzzle" in model-based simulations of LFL policies. The puzzle is the observation that, when forward-guidance policies are simulated as a transient interest-rate peg in New Keynesian models, the effects are often implausibly large. In part, this finding reflects a relatively large fall in long-term interest rates to the announcement under perfect credibility. By contrast, when the interest rate peg is imperfectly credible, long rates fall by less, and hence the macroeconomic effects of the peg are smaller.

We then relax the assumption that the degree of imperfect credibility is exogenous and show that when it depends endogenously on the policymaker's incentive to renounce, it is possible the impact of an LFL policy on macroeconomic variables will be reduced further. This is because the interest rate trajectories along the LFL policy that generate large effects, and, in turn, a possibly greater incentive to renounce are less likely to be viewed as credible. As a result, agents will attach a lower probability the policy actually being implemented so that it will affect their behaviour less. Hence, the policy will have a smaller impact on macroeconomic variables.

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A Derivation of an algorithm for the imposition of interest rate pegs under imperfect credibility

This appendix details a general, perfect-foresight algorithm for implementing interest rate pegs under imperfect credibility in linear rational-expectations models. This algorithm was used to implement the "lower-for-longer" policy experiments described in Section 3. It is an extension of the Bank of England's "MAPS" MATLAB toolkit described in Burgess et al. (2013) and is an example of the type of toolkit development for monetary policy analysis that had been envisaged at the time Burgess et al. (2013) was published (as discussed in Section 6.2.6 of that paper).

A.1 Statement of the problem

The environment we consider is one in which the policymaker announces that they intend to peg the policy rate at some constant value for some abitrary, but finite number of periods (after which policy will be set according to the policy rule in the model). In a perfect foresight, perfect credibility setting, computing the equilibrium of the economy given known initial conditions (and a terminal condition pinned down by the policy rule in the model) is straightforward. In particular, the forward-looking linear class of models for which our algorithm applies can be written as follows:[28]

$$H^F \mathbb{E}_t x_{t+1} + H^C x_t + H^B x_{t-1} = \Psi z_t \tag{A.1}$$

where x is a vector of endogenous variables (both predetermined and non-predetermined), z is a vector of structural, orthogonal disturbances or shocks and the matrices H^F, H^C, H^B and Ψ are functions of the model's parameters. This model has a rational expctations solution of the form:

$$x_t = Bx_{t-1} + \sum_{i=0}^{\infty} F^i \Phi \mathbb{E}_t z_{t+i} \tag{A.2}$$

Given this RE solution, it is straightforward to implement an interest rate peg under perfect credibility using standard inversion techniques as, for example, described in Appendix C of Burgess et al. (2013) or using the algorithm outlined in Laséen and Svensson (2011).

The problem our algorithm solves is how to compute the equilibrium of the economy when the private sector attributes a non-zero probability to the policymaker deviating from the announced interest rate peg and reverting to set interest rates according to the policy rule earlier than had been announced.[29] More specifically, the imperfect credibility enviornment we consider is one in which the central bank makes an announcement at the end of period 0 that the interest rate will be pegged at some constant value, b, for K periods.[30] We suppose that the private sector attributes a probability, $\{p_t\}_{t=1}^{K}$, to the policymaker rerverting back to the policy rule earlier than had been announced (with $p_{K+1} = 1$ as in the standard perfect credibility case). So, the policy regime follows a

[28] Note that this does not exclude models with higher-order leads and lags, which can be nested in this more parsimonious description using lead and lag identities.

[29] Reversion to the policy rule on the peg being lifted early is a logical extension of the assumption in the perfect credibility setting that policy reverts back to the rule after the peg is lifted.

[30] Note that it is trivial to extend the analysis to allow for imperfect credibility around an arbitrary time-varying path (i.e. to add a time subscript to b).

two-state Markov process in which reversion to the rule is an absorbing state. In order to make the period 1 probability that the private sector attributes to reversion to the rule relevant, we alter the standard timing assumption by assuming that in each period the private sector makes their decisions *before* the outcome for the policy rate is revealed (see Figure 2 of Section 2).

In this environment, it is no longer possible to use the rational-expectations solution to compute the equilibrium paths of the model economy because expectations are a non-linear function of the state. The rest of this appendix details the derivation of an algorithm to solve for the equilibrium paths under perfect foresight.[31]

In what follows, we denote x_t^* as the vector of endogenous variables in any period $t = 1...K$ conditional on the policy maker having maintained the interest rate peg and $x_t^{\langle i \rangle}$ as the vector of endogenous variables in any period $t = 1...\infty$ conditional on the policymaker having already reverted to the policy rule in period $i = 1...K+1$ (which is valid for all $t \geq i$). Note that once the policymaker has reverted to the policy rule, it is straightforward to compute any path using the rational expectations solution to the model:[32]

$$x_t^{\langle i \rangle} = Bx_{t-1}^{\langle i \rangle} + \sum_{j=0}^{\infty} F^j \Phi z_{t+j}^{\langle i \rangle} \tag{A.3}$$

This means that solving for the equilibrium paths of the economy boils down to solving for the set of paths, $\{x_t^*\}_{t=1}^{K}$, and the outcomes on reversion to the rule, $\{x_i^{\langle i \rangle}\}_{i=1}^{K+1}$.

A.2 Derivation of the equilibrium paths

Without loss of generality, we proceed by partitioning the vector of endogenous variables, x, and shocks, z, as follows, where r denotes the policy rate variable, ε^r the policy shock, $\widetilde{x}$ the vector of endogenous variables excluding the policy rate and $\widetilde{z}$ the vector of shocks excluding the policy shock:[33]

$$x_t \equiv \begin{bmatrix} \widetilde{x}_t \\ r_t \end{bmatrix} \tag{A.4}$$

$$z_t \equiv \begin{bmatrix} \widetilde{z}_t \\ \varepsilon_t^r \end{bmatrix} \tag{A.5}$$

This partitioning implies comformable partitioning of the model solution matrices, so that we can write the rational-expectations solution in cases where the policymaker has

[31]The non-linearity inherent in the problem implies that global solution methods would need to be employed in order to extend our algorithm to a stochastic setting. In the application of interest rate pegs under imperfect credibility to medium-scale DSGE models of the sort described in Section 3, this would be a massive computational challenge.

[32]The superscript $\langle i \rangle$ applies to the vector of shocks in the event that reversion to the rule would violate the zero or effective lower bound. This is discussed further below.

[33]This partitioning is primarily for exposition. If the model isn't set up like this, a trivial re-ordering of variables can be undertaken to deliver the desired partitioning. But even that is not strictly necessary since it is straightforward to extract the relevant sub-matrices using the relevant indices.

already reverted to the policy rule (equation (A.3)) as:

$$\begin{bmatrix} \widetilde{x}_t^{\langle i \rangle} \\ r_t^{\langle i \rangle} \end{bmatrix} = \begin{bmatrix} B_{\tilde{x}\tilde{x}} & B_{\tilde{x}r} \\ B_{r\tilde{x}} & B_{rr} \end{bmatrix} \begin{bmatrix} \widetilde{x}_{t-1}^{\langle i \rangle} \\ r_{t-1}^{\langle i \rangle} \end{bmatrix} + \begin{bmatrix} S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{t+j} \\ S_r \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{t+j} \end{bmatrix} + \begin{bmatrix} S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} {\varepsilon^r}_{t+j}^{\langle i \rangle} \\ S_r \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} {\varepsilon^r}_{t+j}^{\langle i \rangle} \end{bmatrix} \tag{A.6}$$

where: $B_{\tilde{x}\tilde{x}}$ is the upper left block of the reordered B matrix containing loadings on the lagged set of endogenous variables that excludes the interest rate for that same set of variables with $B_{\tilde{x}r}$, $B_{r\tilde{x}}$ and B_{rr} defined analogously; $S_{\tilde{x}}$ is an $(n_x - 1) \times n_x$ selector matrix extracting the rows pertaining to $\widetilde{x}$ and S_r is a $1 \times n_x$ matrix extracting the row relevant for the nominal interest rate; $\Phi_{\tilde{z}}$ is constrcuted from Φ by removing the column loading on the monetary policy shock and Φ_{ε^r} is constructed from Φ by removing all columns other than that loading on the monetary policy shock.

In this perfect foresight setting, the non-policy shocks, $\{\widetilde{z}_t\}_{t=1}^{\infty}$, are known by agents at the start of period $t = 1$.[34] The policy shocks, $\{\{{\varepsilon^r}_t^{\langle i \rangle}\}_{t=i}^{\infty}\}_{i=1}^{K+1}$, are included in the event that reversion to the policy rule violates the effective lower bound in that agents' perfect-foresight expectations of the interest rate fall below the effective lower bound in any period on or after reversion to the rule.[35]

As discussed above, the objective is to compute the equilibrium in each period conditional on the policymaker not already having reverted to the rule, $\{x_t^*\}_{t=1}^{K}$, and the equilibrium in the period in which reversion does take place, $\{x_i^{\langle i \rangle}\}_{i=1}^{K+1}$. In order to compute $\{x_t^*\}_{t=1}^{K}$, the path for the endogenous variables conditional on the peg being maintained, we construct a "stacked-time" solution from the structural model equations with the equation governing the interest rate – the monetary policy rule – removed. As noted, it is more convenient to work with the partitioning in (A.4) and (A.5). To do so, we rearrange the structural model equations (A.1) to express them in terms of equations for the non-policy variables, $\widetilde{x}$, and non-policy shocks, $\widetilde{z}$, with the interest rate taken as exogenous for the duration of the interest rate peg announcement. Specifically, we have:[36]

$$\widetilde{H}_{\tilde{x}}^F \mathbb{E}_t^* \widetilde{x}_{t+1} + \widetilde{H}_{\tilde{x}}^C \widetilde{x}_t^* + \widetilde{H}_{\tilde{x}}^B \widetilde{x}_{t-1}^* + \widetilde{H}_r^F \mathbb{E}_t^* r_{t+1} + \widetilde{H}_r^C \mathbb{E}_t^* r_t + \widetilde{H}_r^B r_{t-1} = \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_t \tag{A.7}$$

where $\widetilde{x}_0^* = \widetilde{x}_0$. The matrices are constructed as follows: $\widetilde{H}_{\tilde{x}}^C$ is an $(n_x - 1) \times (n_x - 1)$ matrix constructed from H_C by removing the row associated with the policy rule and removing the column of loadings on the contemporaneous nominal interest rate (with the matrices $\widetilde{H}_{\tilde{x}}^F$ and $\widetilde{H}_{\tilde{x}}^B$ constructed analogously); $\widetilde{H}_r^C$ is an $(n_x - 1) \times 1$ matrix constructed from H_C by removing the row associated with the policy rule and extracting the column of loadings on the contemporaneous nominal interest rate (with the matrices $\widetilde{H}_r^F$ and $\widetilde{H}_r^B$ constructed analogously); $\widetilde{\Psi}_{\tilde{z}}$ is constructed from Ψ by removing the row corresponding to the policy rule and the column corresponding to the policy shock.

[34] In the application of this algorithm in Section 3 they take on zero values. We include them for completeness and there may be applications where they are a useful part of the story (eg for news shocks that were known by agents prior to the interest rate peg announcement).

[35] Note that this means they are state dependent since they depend on precisely when policy reverted to the rule.

[36] This is valid for any model in which the policy shock appears only in the monetary policy rule. It would be straightforward to extend the analysis to a model in which the policy shock drove a persistence forcing process that appeared in the rule.

The expectations operator, $\mathbb{E}_t^*$, denotes the expectations of the private sector conditional on information at the start of period t and conditional on the peg having been maintained in periods $1...t-1$. Note that the interest rate within each period is unknown by the private sector when they make their decisions (hence the use of $\mathbb{E}_t^* r_t$ in equation (A.7)). That is because the private sector form their expectations and make their decisions *before* the policymaker chooses whether to maintain the peg or to revert to the policy rule. Note also that this timing assumption means that $\widetilde{x}_t^{\langle t \rangle} \equiv \widetilde{x}_t^*$.

In period $t = 1$ (the first period in which the peg is active), equation (A.7) can be written as:

$$\widetilde{H}_{\tilde{x}}^F \mathbb{E}_1^* \widetilde{x}_2 + \widetilde{H}_{\tilde{x}}^C \widetilde{x}_1^* + \widetilde{H}_{\tilde{x}}^B \widetilde{x}_0 + \widetilde{H}_r^F \mathbb{E}_1^* r_2 + \widetilde{H}_r^C \mathbb{E}_1^* r_1 + \widetilde{H}_r^B r_0 = \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_1 \tag{A.8}$$

The stacked-time solution procedure relies on expectations being integrated out of the stacked equations.[37] We start by describing how we do that for period $t = 1$ and then generalise to all other periods. In order to compute the private sector's expectation of the outturn for the interest rate at the end of period $t = 1$, $\mathbb{E}_1^* r_1$, note that there are two possibilities: either the policymaker maintains the peg or they do not. This means that the expectation of the interest rate is a weighted average of the interest rate peg, b, and that which would prevail if the policymaker set the interest rate in line with the policy rule, where the weighting is determined by the probability that the private sector attributes to the policymaker reverting:

$$\mathbb{E}_1^* r_1 = p_1 r_1^{\langle 1 \rangle} + (1 - p_1)\, b \tag{A.9}$$

If the policymaker does not maintain the peg in period 1, then they take private sector decisions as given and set the interest rate as prescribed by the monetary policy rule:[38]

$$r_1^{\langle 1 \rangle} = \frac{\left[\begin{array}{c} \widehat{\Psi}_{\varepsilon^r} {\varepsilon^r}_1^{\langle 1 \rangle} - \widehat{H}_r^B r_0 - \widehat{H}_{\tilde{x}}^B \widetilde{x}_0 - \left(\widehat{H}_{\tilde{x}}^C + \widehat{H}_r^F B_{r\tilde{x}} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} \right) \widetilde{x}_1^* - \\ \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r \right) \left({\Xi^r}_2^{\langle 1 \rangle} + \widetilde{\mathbb{Z}}_2 \right) \end{array} \right]}{\widehat{H}_r^C + \widehat{H}_r^F B_{rr} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}r}} \tag{A.10}$$

---

[37]This is possible in a perfect foresight setting by integrating over the $K+1$ alternative states of the world. As briefly discussed above, the problem would be much harder in a stochastic setting.

[38]This can be derived in the following way. Partition the structural equations to include only the row corresponding to the interest rate (monetary policy) rule (noting that the partitioning of the shocks is only valid in models in which the only shock appearing in the monetary policy rule is the policy shock):
$\widehat{H}_{\tilde{x}}^F \mathbb{E}_1^{\langle 1 \rangle} \widetilde{x}_2^{\langle 1 \rangle} + \widehat{H}_{\tilde{x}}^C \widetilde{x}_1^* + \widehat{H}_{\tilde{x}}^B \widetilde{x}_0 + \widehat{H}_r^F \mathbb{E}_1^{\langle 1 \rangle} r_2^{\langle 1 \rangle} + \widehat{H}_r^C r_1^{\langle 1 \rangle} + \widehat{H}_r^B r_0 = \widehat{\Psi}_{\varepsilon^r} {\varepsilon^r}_1^{\langle 1 \rangle}$
where $\widehat{H}^B$, $\widehat{H}^C$, $\widehat{H}^F$ and $\widehat{\Psi}$ are constructed from H^B, H^C, H^F and Ψ by extracting the rows corresponding to the policy rule and partitioning by columns (to produce $\widehat{H}_{\tilde{x}}^C$, $\widehat{H}_r^C$ and $\widehat{\Psi}_{\varepsilon^r}$ etc) in an analogous way to that described in the main text for the non-policy rule equations. The expectations can be computed using the rational expectations solution to the model:
$\mathbb{E}_1^{\langle 1 \rangle} r_2^{\langle 1 \rangle} = B_{r\tilde{x}} \widetilde{x}_1^* + B_{rr} r_1^{\langle 1 \rangle} + S_r \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{2+j} + S_r \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} {\varepsilon^r}_{2+j}^{\langle 1 \rangle}$ and
$\mathbb{E}_1^{\langle 1 \rangle} \widetilde{x}_2^{\langle 1 \rangle} = B_{\tilde{x}\tilde{x}} \widetilde{x}_1^* + B_{\tilde{x}r} r_1^{\langle 1 \rangle} + S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{2+j} + S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} {\varepsilon^r}_{2+j}^{\langle 1 \rangle}$.
These expectations can be substituted into the partitioned, structural equation for the policy rule to give:
$\widehat{H}_{\tilde{x}}^F \left(B_{\tilde{x}\tilde{x}} \widetilde{x}_1^* + B_{\tilde{x}r} r_1^{\langle 1 \rangle} + S_{\tilde{x}} \widetilde{\mathbb{Z}}_2 + S_{\tilde{x}} {\Xi^r}_2^{\langle 1 \rangle} \right) + \widehat{H}_{\tilde{x}}^C \widetilde{x}_1^* + \widehat{H}_{\tilde{x}}^B \widetilde{x}_0 + \widehat{H}_r^F \left(B_{r\tilde{x}} \widetilde{x}_1^* + B_{rr} r_1^{\langle 1 \rangle} + S_r \widetilde{\mathbb{Z}}_2 + S_r {\Xi^r}_2^{\langle 1 \rangle} \right) +$
$\widehat{H}_r^C r_1^{\langle 1 \rangle} + \widehat{H}_r^B r_0 = \widehat{\Psi}_{\varepsilon^r} {\varepsilon^r}_1^{\langle 1 \rangle}$ where:
${\Xi^r}_2^{\langle 1 \rangle} = \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} {\varepsilon^r}_{2+j}^{\langle 1 \rangle}$ and
$\widetilde{\mathbb{Z}}_2 = \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{2+j}$
And this can be rearranged to give equation (A.10).

where $\widehat{H}^B_r$ is the element of the monetary policy rule row of H^B that loads on to the lagged policy rate (with $\widehat{H}^C_r$ and $\widehat{H}^F_r$ defined analogously), $\widehat{H}^B_{\tilde{x}}$ is the monetary policy rule row of H^B with the element that loads off the lagged interest rate removed (with $\widehat{H}^C_{\tilde{x}}$ and $\widehat{H}^F_{\tilde{x}}$ defined analogously), $\widehat{\Psi}_{\varepsilon^r}$ is the element of the monetary policy rule row of Ψ that loads on the monetary policy shock and where:

$$\Xi^{r\langle 1\rangle}_2 = \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r\langle 1\rangle}_{2+j} \tag{A.11}$$

$$\widetilde{\mathbb{Z}}_2 = \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{2+j} \tag{A.12}$$

Period 1 expectations for the vector of endogenous variables in period 2 can be defined using the same logic:

$$\mathbb{E}^*_1 \widetilde{x}_2 = p_1 \widetilde{x}^{\langle 1\rangle}_2 + (1-p_1)\, \widetilde{x}^*_2 \tag{A.13}$$

where:[39]

$$\widetilde{x}^{\langle 1\rangle}_2 = B_{\tilde{x}\tilde{x}} \widetilde{x}^*_1 + B_{\tilde{x}r} r^{\langle 1\rangle}_1 + S_{\tilde{x}} \left(\widetilde{\mathbb{Z}}_2 + \Xi^{r\langle 1\rangle}_2\right) \tag{A.14}$$

And, finally, period 1 expectations for the period 2 interest rate must take into account that there are two periods of uncertainty over the realisation of the interest rate (which reflects the timing assumption discussed above):

$$\mathbb{E}^*_1 r_2 = p_1 r^{\langle 1\rangle}_2 + (1-p_1)\, \mathbb{E}^*_2 r_2 \tag{A.15}$$

where:

$$r^{\langle 1\rangle}_2 = B_{r\tilde{x}} \widetilde{x}^*_1 + B_{rr} r^{\langle 1\rangle}_1 + S_r \left(\widetilde{\mathbb{Z}}_2 + \Xi^{r\langle 1\rangle}_2\right) \tag{A.16}$$

and:

$$\mathbb{E}^*_2 r_2 = p_2 r^{\langle 2\rangle}_2 + (1-p_2)\, b \tag{A.17}$$

with $r^{\langle 2\rangle}_2$ defined analogously to $r^{\langle 1\rangle}_1$ from equation (A.10) as (noting that $r^*_1 \equiv b$):

$$r^{\langle 2\rangle}_2 = \frac{\left[\begin{array}{c} \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle 2\rangle}_2 - \widehat{H}^B_r b - \widehat{H}^B_{\tilde{x}} \widetilde{x}^*_1 - \left(\widehat{H}^C_{\tilde{x}} + \widehat{H}^F_r B_{r\tilde{x}} + \widehat{H}^F_{\tilde{x}} B_{\tilde{x}\tilde{x}}\right) \widetilde{x}^*_2 - \\ \left(\widehat{H}^F_{\tilde{x}} S_{\tilde{x}} + \widehat{H}^F_r S_r\right) \left(\Xi^{r\langle 2\rangle}_3 + \widetilde{\mathbb{Z}}_3\right) \end{array}\right]}{\widehat{H}^C_r + \widehat{H}^F_r B_{rr} + \widehat{H}^F_{\tilde{x}} B_{\tilde{x}r}} \tag{A.18}$$

where:

$$\Xi^{r\langle 2\rangle}_3 = \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r\langle 2\rangle}_{3+j} \tag{A.19}$$

$$\widetilde{\mathbb{Z}}_3 = \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{3+j} \tag{A.20}$$

These definitions for expectations can be plugged into the original structural equation (A.8) and the result rearranged to put 'known' terms on the right hand side (those that

[39]This makes use of the timing assumption and associated definition that $\widetilde{x}^{\langle 1\rangle}_1 \equiv \widetilde{x}^*_1$.

agents take as given) and unknown terms (those as functions of $\widetilde{x}_1^*$ and $\widetilde{x}_2^*$ on the left hand side):[40]

$$\begin{aligned}
&\left(\widetilde{H}_{\tilde{x}}^C + p_1\left(\widetilde{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^F B_{r\tilde{x}} - \widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{\mathbb{H}}_{\tilde{x}}\right) - (1-p_1)\,p_2\widehat{\mathbb{H}}_r^{-1}\widetilde{H}_r^F\widehat{H}_{\tilde{x}}^B\right)\widetilde{x}_1^* \\
&+ (1-p_1)\left(\widetilde{H}_{\tilde{x}}^F - p_2\widehat{\mathbb{H}}_r^{-1}\widetilde{H}_r^F\widehat{\mathbb{H}}_{\tilde{x}}\right)\widetilde{x}_2^* \\
&= \widetilde{\Psi}_{\tilde{z}}\widetilde{z}_1 + p_1\left(\widetilde{\mathbb{H}}_r\left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r\right) - \left(\widetilde{H}_{\tilde{x}}^F S_{\tilde{x}} + \widetilde{H}_r^F S_r\right)\right)\left({\Xi^r}_2^{\langle 1\rangle} + \widetilde{\mathbb{Z}}_2\right) \\
&+ (1-p_1)\,p_2\widetilde{H}_r^F\left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r\right)\left({\Xi^r}_3^{\langle 2\rangle} + \widetilde{\mathbb{Z}}_3\right) \\
&- p_1\widetilde{\mathbb{H}}_r\widehat{\Psi}_{\varepsilon^r}{\varepsilon^r}_1^{\langle 1\rangle} - (1-p_1)\,p_2\widetilde{H}_r^F\widehat{\Psi}_{\varepsilon^r}{\varepsilon^r}_2^{\langle 2\rangle} \\
&+ \left(p_1\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}_{\tilde{x}}^B - \widetilde{H}_{\tilde{x}}^B\right)\widetilde{x}_0 + \left(p_1\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}_r^B - \widetilde{H}_r^B\right)r_0 \\
&+ (1-p_1)\left(p_2\widehat{\mathbb{H}}_r^{-1}\widetilde{H}_r^F\widehat{H}_r^B - \widetilde{H}_r^C - (1-p_2)\,\widetilde{H}_r^F\right)b
\end{aligned} \tag{A.21}$$

where:

$$\widehat{\mathbb{H}}_{\tilde{x}} = \widehat{H}_{\tilde{x}}^C + \widehat{H}_r^F B_{r\tilde{x}} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} \tag{A.22}$$

$$\widehat{\mathbb{H}}_r = \widehat{H}_r^C + \widehat{H}_r^F B_{rr} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}r} \tag{A.23}$$

$$\widetilde{\mathbb{H}}_r = \widetilde{H}_r^C + \widetilde{H}_r^F B_{rr} + \widetilde{H}_{\tilde{x}}^F B_{\tilde{x}r} \tag{A.24}$$

Assuming the policymaker does not revert to the policy rule at the end of period $t=1$, then in period $t=2$ we have (noting that $r_1^* = b$):

$$\widetilde{H}_{\tilde{x}}^F\mathbb{E}_2^*\widetilde{x}_3 + \widetilde{H}_{\tilde{x}}^C\widetilde{x}_2^* + \widetilde{H}_{\tilde{x}}^B\widetilde{x}_1 + \widetilde{H}_r^F\mathbb{E}_2^*r_3 + \widetilde{H}_r^C\mathbb{E}_2^*r_2 + \widetilde{H}_r^B b = \widetilde{\Psi}_{\tilde{z}}\widetilde{z}_2 \tag{A.25}$$

We can use analogous arguments to substitute out expectations and rearrange to give:

$$\begin{aligned}
&\left(\widetilde{H}_{\tilde{x}}^B - p_2\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}_{\tilde{x}}^B\right)\widetilde{x}_1^* \\
&+ \left(\widetilde{H}_{\tilde{x}}^C + p_2\left(\widetilde{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^F B_{r\tilde{x}} - \widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{\mathbb{H}}_{\tilde{x}}\right) - (1-p_2)\,p_3\widehat{\mathbb{H}}_r^{-1}\widetilde{H}_r^F\widehat{H}_{\tilde{x}}^B\right)\widetilde{x}_2^* \\
&+ (1-p_2)\left(\widetilde{H}_{\tilde{x}}^F - p_3\widehat{\mathbb{H}}_r^{-1}\widetilde{H}_r^F\widehat{\mathbb{H}}_{\tilde{x}}\right)\widetilde{x}_3^* \\
&= \widetilde{\Psi}_{\tilde{z}}\widetilde{z}_2 + p_2\left(\widetilde{\mathbb{H}}_r\left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r\right) - \left(\widetilde{H}_{\tilde{x}}^F S_{\tilde{x}} + \widetilde{H}_r^F S_r\right)\right)\left({\Xi^r}_3^{\langle 2\rangle} + \widetilde{\mathbb{Z}}_3\right) \\
&+ (1-p_2)\,p_3\widetilde{H}_r^F\left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r\right)\left({\Xi^r}_4^{\langle 3\rangle} + \widetilde{\mathbb{Z}}_4\right) \\
&- p_2\widetilde{\mathbb{H}}_r\widehat{\Psi}_{\varepsilon^r}{\varepsilon^r}_2^{\langle 2\rangle} - (1-p_2)\,p_3\widetilde{H}_r^F\widehat{\Psi}_{\varepsilon^r}{\varepsilon^r}_3^{\langle 3\rangle} \\
&+ \left(p_2\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}_r^B - \widetilde{H}_r^B + (1-p_2)\left(p_3\widehat{\mathbb{H}}_r^{-1}\widetilde{H}_r^F\widehat{H}_r^B - \widetilde{H}_r^C - (1-p_3)\,\widetilde{H}_r^F\right)\right)b
\end{aligned} \tag{A.26}$$

where ${\Xi^r}_4^{\langle 3\rangle}$ and $\widetilde{\mathbb{Z}}_4$ are defined analogously to ${\Xi^r}_3^{\langle 2\rangle}$ and $\widetilde{\mathbb{Z}}_3$.

The expression for $t=2$ reveals the generic form of the equations for periods $t =$

[40]Note that the anticipated policy shocks necessary to impose the ELB on reversion to the rule are technically unknown, baut are treated as known within the stacked-time algorithm. See Section A.3 for a description of how we find their values.

$2, \ldots, K-1$, which is:

$$
\begin{aligned}
&\left(\widetilde{H}_{\tilde{x}}^{B} - p_t \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_{\tilde{x}}^{B}\right) \widetilde{x}_{t-1}^{*} \\
&+ \left(\widetilde{H}_{\tilde{x}}^{C} + p_t \left(\widetilde{H}_{\tilde{x}}^{F} B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^{F} B_{r\tilde{x}} - \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}}\right) - (1-p_t)\, p_{t+1} \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^{F} \widehat{H}_{\tilde{x}}^{B}\right) \widetilde{x}_t^{*} \\
&+ (1-p_t) \left(\widetilde{H}_{\tilde{x}}^{F} - p_{t+1} \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^{F} \widehat{\mathbb{H}}_{\tilde{x}}\right) \widetilde{x}_{t+1}^{*} \\
&= \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_t + p_t \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widehat{H}_r^{F} S_r\right) - \left(\widetilde{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widetilde{H}_r^{F} S_r\right)\right) \left(\Xi^{r\langle t\rangle}_{t+1} + \widetilde{\mathbb{Z}}_{t+1}\right) \\
&+ (1-p_t)\, p_{t+1} \widetilde{H}_r^{F} \left(\widehat{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widehat{H}_r^{F} S_r\right) \left(\Xi^{r\langle t+1\rangle}_{t+2} + \widetilde{\mathbb{Z}}_{t+2}\right) \\
&- p_t \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle t\rangle}_{t} - (1-p_t)\, p_{t+1} \widetilde{H}_r^{F} \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle t+1\rangle}_{t+1} \\
&+ \left(p_t \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_r^{B} - \widetilde{H}_r^{B} + (1-p_t) \left(p_{t+1} \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^{F} \widehat{H}_r^{B} - \widetilde{H}_r^{C} - (1-p_{t+1}) \widetilde{H}_r^{F}\right)\right) b
\end{aligned} \tag{A.27}
$$

where:

$$
\Xi^{r\langle i\rangle}_{t} = \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r\langle i\rangle}_{t+j} \tag{A.28}
$$

$$
\widetilde{\mathbb{Z}}_{t+s} = \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{t+s+j} \tag{A.29}
$$

If period $t = K$ is reached with the peg having been maintained, then rates will revert to the policy rule in period $K+1$ with certainty. The solution in this period is then given by:[41]

$$
\begin{aligned}
&\left(\widetilde{H}_{\tilde{x}}^{B} - p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_{\tilde{x}}^{B}\right) \widetilde{x}_{K-1}^{*} + \left(\widetilde{H}_{\tilde{x}}^{C} + \widetilde{H}_{\tilde{x}}^{F} B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^{F} B_{r\tilde{x}} - p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}}\right) x_K^{*} \\
&= \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_K + p_K \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widehat{H}_r^{F} S_r\right) - \left(\widetilde{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widetilde{H}_r^{F} S_r\right)\right) \left(\Xi^{r\langle K\rangle}_{K+1} + \widetilde{\mathbb{Z}}_{K+1}\right) \\
&- p_K \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle K\rangle}_{K} - (1-p_K) \left(\widetilde{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widetilde{H}_r^{F} S_r\right) \left(\Xi^{r\langle K+1\rangle}_{K+1} + \widetilde{\mathbb{Z}}_{K+1}\right) \\
&+ \left(p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_r^{B} - \widetilde{H}_r^{B} - (1-p_K) \widetilde{\mathbb{H}}_r\right) b
\end{aligned} \tag{A.30}
$$

unless $K = 1$, in which case we have the following:

$$
\begin{aligned}
&\left(\widetilde{H}_{\tilde{x}}^{C} + \widetilde{H}_{\tilde{x}}^{F} B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^{F} B_{r\tilde{x}} - p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}}\right) \widetilde{x}_1^{*} \\
&= \widetilde{\Psi}_{\tilde{z}} \widetilde{z}_1 + p_1 \left(\widetilde{\mathbb{H}}_r \left(\widehat{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widehat{H}_r^{F} S_r\right) - \left(\widetilde{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widetilde{H}_r^{F} S_r\right)\right) \left(\Xi^{r\langle K\rangle}_{2} + \widetilde{\mathbb{Z}}_{2}\right) \\
&- p_1 \widetilde{\mathbb{H}}_r \widehat{\Psi}_{\varepsilon^r} \varepsilon^{r\langle 1\rangle}_{1} - (1-p_1) \left(\widetilde{H}_{\tilde{x}}^{F} S_{\tilde{x}} + \widetilde{H}_r^{F} S_r\right) \left(\Xi^{r\langle 2\rangle}_{2} + \widetilde{\mathbb{Z}}_{2}\right) \\
&+ \left(p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_r^{B} - \widetilde{H}_r^{B}\right) r_0 + \left(p_1 \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_{\tilde{x}}^{B} - \widetilde{H}_{\tilde{x}}^{B}\right) \widetilde{x}_0 - (1-p_1) \widetilde{\mathbb{H}}_r b
\end{aligned} \tag{A.31}
$$

[41]This follows from noting that:
$\mathbb{E}_K^{*} \widetilde{x}_{K+1} = p_K \widetilde{x}_{K+1}^{\langle K\rangle} + (1-p_K)\, \widetilde{x}_{K+1}^{\langle K+1\rangle}$ where:
$\widetilde{x}_{K+1}^{\langle K\rangle} = B_{\tilde{x}\tilde{x}} \widetilde{x}_K^{*} + B_{\tilde{x}r} r_K^{\langle K\rangle} + S_{\tilde{x}} \left(\widetilde{\mathbb{Z}}_{K+1} + \Xi^{r\langle K\rangle}_{K+1}\right)$ and
$\widetilde{x}_{K+1}^{\langle K+1\rangle} = B_{\tilde{x}\tilde{x}} \widetilde{x}_K^{*} + B_{\tilde{x}r} b + S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{K+1+j} + S_{\tilde{x}} \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r\langle K+1\rangle}_{K+1+j}$.
$\mathbb{E}_K^{*} r_K = p_K r_K^{\langle K\rangle} + (1-p_K)\, b$ where $r_K^{\langle K\rangle}$ is defined analogously to $r_1^{\langle 1\rangle}$ and $r_2^{\langle 2\rangle}$.
$\mathbb{E}_K^{*} r_{K+1} = p_K r_{K+1}^{\langle K\rangle} + (1-p_K)\, r_{K+1}^{\langle K+1\rangle}$ where:
$r_{K+1}^{\langle K+1\rangle} = B_{r\tilde{x}} \widetilde{x}_K^{*} + B_{rr} b + S_r \sum_{j=0}^{\infty} F^j \Phi_{\tilde{z}} \widetilde{z}_{K+1+j} + S_r \sum_{j=0}^{\infty} F^j \Phi_{\varepsilon^r} \varepsilon^{r\langle K+1\rangle}_{K+1+j}$.

To solve for the equilibrium path conditional on the peg being maintained, $\{x_t^*\}_{t=1}^K$, we stack the equations derived above to form the following system:

$$\mathbb{J}\widetilde{X}^* = \mathbb{C} \tag{A.32}$$

where:

$$\widetilde{X}^* \equiv \begin{bmatrix} \widetilde{x}_1^* \\ \vdots \\ \widetilde{x}_K^* \end{bmatrix}$$

It is straightforward to form the matrix $\mathbb{J}$ by collecting the loadings on the $\widetilde{x}$ terms from the left hand sides of the equations outlined above as follows:

$$\mathbb{J} \equiv \begin{bmatrix} J_1^C & J_1^F & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ J_2^B & J_2^C & J_2^F & 0 & \dots & 0 & 0 & 0 & 0 \\ & & & & \vdots & & & & \\ 0 & 0 & \dots & J_t^B & J_t^C & J_t^F & 0 & \dots & 0 \\ & & & & \vdots & & & & \\ 0 & 0 & 0 & 0 & \dots & 0 & J_{K-1}^B & J_{K-1}^C & J_{K-1}^F \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & J_K^B & J_K^C \end{bmatrix}$$

where:

$$\begin{aligned} J_t^B &= \widetilde{H}_{\tilde{x}}^B - p_t \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{H}_{\tilde{x}}^B \\ J_t^C &= \widetilde{H}_{\tilde{x}}^C + p_t \left(\widetilde{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^F B_{r\tilde{x}} - \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}} \right) - (1-p_t)\, p_{t+1} \left(\widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{H}_{\tilde{x}}^B \right) \\ J_t^F &= (1-p_t) \left(\widetilde{H}_{\tilde{x}}^F - p_{t+1} \widehat{\mathbb{H}}_r^{-1} \widetilde{H}_r^F \widehat{\mathbb{H}}_{\tilde{x}} \right) \\ J_K^C &= \widetilde{H}_{\tilde{x}}^C + \widetilde{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}} + \widetilde{H}_r^F B_{r\tilde{x}} - p_K \widehat{\mathbb{H}}_r^{-1} \widetilde{\mathbb{H}}_r \widehat{\mathbb{H}}_{\tilde{x}} \end{aligned}$$

and where J_1^C, J_1^F, J_2^B, J_2^C, J_2^F, J_{K-1}^B, J_{K-1}^C, J_{K-1}^F and J_K^B can be written as specific cases of the general period t case defined above. It is also straightforward to form the vector $\mathbb{C}$ by collecting the terms from the right hand sides of the equations above:

$$\mathbb{C} \equiv \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_t \\ \vdots \\ C_{K-1} \\ C_K \end{bmatrix}$$

where:

$$
\begin{aligned}
C_1 &= \widetilde{\Psi}_{\tilde{z}}\widetilde{z}_1 + p_1\left(\widetilde{\mathbb{H}}_r\left(\widehat{H}^F_{\tilde{x}}S_{\tilde{x}} + \widehat{H}^F_r S_r\right) - \left(\widetilde{H}^F_{\tilde{x}}S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\right)\left(\Xi^{r\langle 1\rangle}_{2} + \widetilde{\mathbb{Z}}_2\right) \\
&+ (1-p_1)\,p_2\widetilde{H}^F_r\left(\widehat{H}^F_{\tilde{x}}S_{\tilde{x}} + \widehat{H}^F_r S_r\right)\left(\Xi^{r\langle 2\rangle}_{3} + \widetilde{\mathbb{Z}}_3\right) \\
&- p_1\widetilde{\mathbb{H}}_r\widehat{\Psi}_{\varepsilon^r}\varepsilon^{r\langle 1\rangle}_{1} - (1-p_1)\,p_2\widetilde{H}^F_r\widehat{\Psi}_{\varepsilon^r}\varepsilon^{r\langle 2\rangle}_{2} \\
&+ \left(p_1\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}^B_{\tilde{x}} - \widetilde{H}^B_{\tilde{x}}\right)\widetilde{x}_0 + \left(p_1\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}^B_r - \widetilde{H}^B_r\right)r_0 \\
&+ (1-p_1)\left(p_2\widehat{\mathbb{H}}_r^{-1}\widetilde{H}^F_r\widehat{H}^B_r - \widetilde{H}^C_r - (1-p_2)\,\widetilde{H}^F_r\right)b
\end{aligned}
\tag{A.33}
$$

$$
\begin{aligned}
C_t &= \widetilde{\Psi}_{\tilde{z}}\widetilde{z}_t + p_t\left(\widetilde{\mathbb{H}}_r\left(\widehat{H}^F_{\tilde{x}}S_{\tilde{x}} + \widehat{H}^F_r S_r\right) - \left(\widetilde{H}^F_{\tilde{x}}S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\right)\left(\Xi^{r\langle t\rangle}_{t+1} + \widetilde{\mathbb{Z}}_{t+1}\right) \\
&+ (1-p_t)\,p_{t+1}\widetilde{H}^F_r\left(\widehat{H}^F_{\tilde{x}}S_{\tilde{x}} + \widehat{H}^F_r S_r\right)\left(\Xi^{r\langle t+1\rangle}_{t+2} + \widetilde{\mathbb{Z}}_{t+2}\right) \\
&- p_t\widetilde{\mathbb{H}}_r\widehat{\Psi}_{\varepsilon^r}\varepsilon^{r\langle t\rangle}_{t} - (1-p_t)\,p_{t+1}\widetilde{H}^F_r\widehat{\Psi}_{\varepsilon^r}\varepsilon^{r\langle t+1\rangle}_{t+1} \\
&+ \left(p_t\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}^B_r - \widetilde{H}^B_r + (1-p_t)\left(p_{t+1}\widehat{\mathbb{H}}_r^{-1}\widetilde{H}^F_r\widehat{H}^B_r - \widetilde{H}^C_r - (1-p_{t+1})\,\widetilde{H}^F_r\right)\right)b
\end{aligned}
\tag{A.34}
$$

$$
\begin{aligned}
C_K &= \widetilde{\Psi}_{\tilde{z}}\widetilde{z}_K + p_K\left(\widetilde{\mathbb{H}}_r\left(\widehat{H}^F_{\tilde{x}}S_{\tilde{x}} + \widehat{H}^F_r S_r\right) - \left(\widetilde{H}^F_{\tilde{x}}S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\right)\left(\Xi^{r\langle K\rangle}_{K+1} + \widetilde{\mathbb{Z}}_{K+1}\right) \\
&- p_K\widetilde{\mathbb{H}}_r\widehat{\Psi}_{\varepsilon^r}\varepsilon^{r\langle K\rangle}_{K} - (1-p_K)\left(\widetilde{H}^F_{\tilde{x}}S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\left(\Xi^{r\langle K+1\rangle}_{K+1} + \widetilde{\mathbb{Z}}_{K+1}\right) \\
&+ \left(p_K\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}^B_r - \widetilde{H}^B_r - (1-p_K)\,\widetilde{\mathbb{H}}_r\right)b
\end{aligned}
\tag{A.35}
$$

and where C_2 and C_{K-1} can be written as specific cases of the general period t case defined above. Finally, the special case of $K = 1$ is as follows:

$$
\begin{aligned}
C_1 \equiv C_K &= \widetilde{\Psi}_{\tilde{z}}\widetilde{z}_1 + p_1\left(\widetilde{\mathbb{H}}_r\left(\widehat{H}^F_{\tilde{x}}S_{\tilde{x}} + \widehat{H}^F_r S_r\right) - \left(\widetilde{H}^F_{\tilde{x}}S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\right)\left(\Xi^{r\langle K\rangle}_{2} + \widetilde{\mathbb{Z}}_2\right) \\
&- p_1\widetilde{\mathbb{H}}_r\widehat{\Psi}_{\varepsilon^r}\varepsilon^{r\langle 1\rangle}_{1} - (1-p_1)\left(\widetilde{H}^F_{\tilde{x}}S_{\tilde{x}} + \widetilde{H}^F_r S_r\right)\left(\Xi^{r\langle 2\rangle}_{2} + \widetilde{\mathbb{Z}}_2\right) \\
&+ \left(p_1\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}^B_r - \widetilde{H}^B_r\right)r_0 + \left(p_1\widehat{\mathbb{H}}_r^{-1}\widetilde{\mathbb{H}}_r\widehat{H}^B_{\tilde{x}} - \widetilde{H}^B_{\tilde{x}}\right)\widetilde{x}_0 - (1-p_1)\,\widetilde{\mathbb{H}}_r b
\end{aligned}
\tag{A.36}
$$

With the solution for the case in which the policymaker maintains the peg in hand, it is straightforward to recover the solution for the $K+1$ paths in which the policymaker could revert to the rule.[42] As noted above, the private sector's decisions are unaffected by the policymaker's decision to maintain the peg or otherwise *within each period*, reflecting that the policymaker sets the interest rate at the end of the period after private sector decisions have been made. It follows that we can define:

$$
x^{\langle t\rangle}_t \equiv \begin{bmatrix} \widetilde{x}^*_t \\ r^{\langle t\rangle}_t \end{bmatrix} \tag{A.37}
$$

which is valid for all periods $t = 1...K$ and in which the interest rate on reverting to the rule can be defined as follows (which is just the general case of the period $t = 1$ and $t = 2$

[42] And it is clearly trivial to define the complete vector of endogenous variables in states where the peg has been maintained as: $x^*_t \equiv \begin{bmatrix} \widetilde{x}^*_t \\ b \end{bmatrix}$.

definitions from above):

$$r_t^{\langle t\rangle} = \frac{\left[\begin{array}{c} \widehat{\Psi}_{\varepsilon^r}\varepsilon^{r}{}_t^{\langle t\rangle} - \widehat{H}_r^B b - \widehat{H}_{\tilde{x}}^B \widetilde{x}_{t-1}^* - \left(\widehat{H}_{\tilde{x}}^C + \widehat{H}_r^F B_{r\tilde{x}} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}\tilde{x}}\right)\widetilde{x}_t^* - \\ \left(\widehat{H}_{\tilde{x}}^F S_{\tilde{x}} + \widehat{H}_r^F S_r\right)\left(\Xi^{r}{}_{t+1}^{\langle t\rangle} + \widetilde{\mathbb{Z}}_{t+1}\right) \end{array}\right]}{\widehat{H}_r^C + \widehat{H}_r^F B_{rr} + \widehat{H}_{\tilde{x}}^F B_{\tilde{x}r}} \tag{A.38}$$

In the case where the policymaker maintains the peg and period $t = K+1$ is reached without the policymaker having already reverted to the rule, then the interest rate reverts back to the rule with certainty, in which case the following definitions apply:

$$x_{K+1}^{\langle K+1\rangle} \equiv \left[\begin{array}{c} \widetilde{x}_{K+1}^{\langle K+1\rangle} \\ r_{K+1}^{\langle K+1\rangle} \end{array}\right] \tag{A.39}$$

where:

$$\widetilde{x}_{K+1}^{\langle K+1\rangle} = B_{\tilde{x}\tilde{x}}\widetilde{x}_K^* + B_{\tilde{x}r}b + S_{\tilde{x}}\left(\widetilde{\mathbb{Z}}_{K+1} + \Xi^{r}{}_{K+1}^{\langle K+1\rangle}\right) \tag{A.40}$$

$$r_{K+1}^{\langle K+1\rangle} = B_{r\tilde{x}}\widetilde{x}_K^* + B_{rr}b + S_r\left(\widetilde{\mathbb{Z}}_{K+1} + \Xi^{r}{}_{K+1}^{\langle K+1\rangle}\right) \tag{A.41}$$

Having computed the vectors $x_i^{\langle i\rangle}$ for periods $i = 1...K+1$, it is then straightforward to compute the paths for the endogenous variables in any future period, $x_t^{\langle i\rangle}$ for any $t > i$, using the rational expectations solution to the model in equation (A.3).

A.3 Ensuring the zero or effective lower bound is not violated on reversion to the rule

Note that a feature of the solution approach outlined above is that any monetary policy shocks necessary to impose a zero or effective lower bound on reversion to the rule were taken as given by agents in the model. These shocks are not known ex-ante since their values depend on the state of the economy prior to reversion, which is the vector that the solution procedure solves for. In order to incorporate the possibility that the interest rate might violate the lower bound on reversion, we employ the following algorithm:

1. Form the system (A.32) under the assumption that the lower bound does not bind in any period and any state in which the policymaker can revert to the rule. This means setting $\varepsilon^{r}{}_t^{\langle i\rangle} = 0, \forall i = 1...K+1, t \geq i$.

2. Compute the equilibrium paths in the model economy using the algorithm outlined above.

3. Check whether the assumption made in step 1 is valid by computing the expected path of the policy rate conditional on reversion to the rule in each state $i = 1, \ldots, K+1$. If none of the projections for the policy rate falls below the zero or effective bound, then stop. Otherwise, iterate as follows:

   (a) Conditional on the equilibrium non-renege paths, compute the shocks required to enforce the lower bound $(\{\{\varepsilon^{r}{}_t^{\langle i\rangle}\}_{i=1}^{K+1}\}_{t=i}^{\infty})$ using the algorithm described below and use this set to update the guess for the set of anticipated policy shocks necessary to enforce the lower bound in all states of the world.[43]

[43]We use a simple heuristic that weights the previous guesses and the new shock values equally.

(b) Re-compute the equilibrium non-renege paths conditional on the updated guess using the algorithm in Section A.2.

(c) If the distance between the previous and current guesses for the equilibrium paths are small enough and the lower bound is respected in all states and time periods, then stop. Otherwise go back to 3a.

We implement step 3a using a similar approach to that described in Holden and Paetz (2012). Specifically, for each state of the world, i, we find the minimal set of shocks, $\{\varepsilon^{r\langle i\rangle}_t\}_{t=i}^{\infty}$, that ensures that the ELB constraint is not violated in any period following reversion to the rule. As described in Holden and Paetz (2012), an efficient way to find that set of shocks is to write the problem as a quadratic programming problem (with the constraint that the anticipated or "news" shocks must be positive - i.e. the bound can only be violated from below) and to use well-established and efficient algorithms to solve it (and we use MATLAB's "quadprog" routine).[44]

[44]Note that an alternative approach that would eliminate the iterative approach described in the algoirthm above would be to cast the whole problem as a quadratic programming problem and to simulatenously solve for the non-renege equilibrium paths and any anticipated policy shocks necessary to ensure that the ELB contsraint is not violated in any state of the world. This is a more elegant solution to the problem (and is likely to be faster computationally), but requires more algebra and would be more difficult to implement (increasing the risk of bugs).