# The Home Market Effect and Patterns of Trade Between Rich and Poor Countries

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**Abstract:**

This paper proposes a new theoretical framework for studying the patterns of trade between rich and poor countries by incorporating nonhomothetic preferences into the standard home market effect models of trade. It has a continuum of Dixit-Stiglitz monopolistic competitive sectors with iceberg trade costs. There are two countries, which may differ in their per capita labor endowment and the population size. Preferences across sectors are such that, as per capita income goes up, the households shift their expenditure shares towards higher-indexed sectors. In equilibrium, the Rich country, whose households achieve higher standard-of-living, runs a trade surplus in higher-indexed sectors through the home market effect, and hence becomes a net-exporter of high income elastic goods. The framework is flexible enough to allow for a variety of comparative statics. For example, a uniform productivity improvement causes the Rich to switch from a net exporter to a net importer in some middle sectors. The Rich gains relatively more (less) from such changes than the Poor when the goods produced in different sectors are substitutes (complements). The effects of globalization, captured by a reduction in the trade cost, are similar to those of uniform productivity improvements, except that it has additional effects of the terms of trade change when the two countries are unequal in size.

***Keywords:*** Home market effect, Nonhomothetic preferences, Implicitly additively separable CES, Log-supermodularity, Monotone likelihood ratio, Monotone comparative statics, Product cycles, Terms of trade effect, Leapfrogging

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# 1. Introduction

The standard models of international trade focus on the role of supply side differences across countries as determinants of the patterns of trade. For this reason, they typically assume that the consumers have homothetic preferences, which implies that the demand for every good has unitary income elasticity. This obviously makes these models ill-suited for explaining one of the well-known empirical regularities, i.e., rich countries tend to export products with high income elasticity and import those with low income elasticity, while poor countries tend to import products with high income elasticity and export those with low income elasticity. This is one of the motivations behind the recent works on models of trade with nonhomothetic preferences.[1]

However, simply adding the nonhomotheticity of preferences into these models would, *ceteris paribus*, only make rich countries import high income elastic goods. While this might be useful for explaining the patterns of trade in caviar, diamonds, and other goods whose locations of production are determined by Mother Nature, it would predict exactly the opposite of what is observed empirically for most manufacturing products. For this reason, virtually all existing models of trade with nonhomothetic preferences *postulate* that the rich (poor) countries have comparative advantages in high (low) income elastic goods. For example, in their Ricardian models of trade, Flam and Helpman (1987), Stokey (1991), Matsuyama (2000), and Fieler (2011), the technological superiority of rich countries are greater in the sectors that produce the goods with higher income elasticity. In their factor endowment models of trade, Markusen (1986) and Caron, Fally, and Markusen (2014), rich countries are relatively more abundant in the factors used relatively more intensively in producing goods with high income elasticity. Although empirically well-grounded, such correlations between the differences on the supply side and the demand side are *not causally linked* in these models. Instead, they hold by assumption. In other words, these models all suggest that rich countries export high income elastic goods *despite* their demand composition is more skewed towards high income elastic goods.

In this paper, we pursuit a different approach by developing a new theoretical framework for studying the patterns of trade between rich and poor countries under nonhomothetic

[1] Markusen (2013) contains a survey on a range of trade questions that require nonhomothetic preferences.

preferences, which suggests that the rich countries export high income elastic goods *because* their demand composition is more skewed towards high income elastic goods. Due to nonhomotheticity, the cross-country difference in the standard of living *causes* systematic cross-country differences in the demand composition, which in turn *causes* the supply-side differences, or the patterns of comparative advantage, through the "home market effect". As Krugman (1980) argued, when production is subject to economies of scale and trade costs are positive but not prohibitively high, a relatively large domestic market gives an advantage to its local firms, as it provides the basis from which they could export to other markets. In Krugman's (1980) model, labor is the only factor of production and there are two Dixit-Stiglitz monopolistically competitive sectors, $\alpha$ and $\beta$, each of which produces horizontally differentiated goods that can be exported with iceberg costs. There are two countries of equal size, A and B, where A is a nation of $\alpha$-lovers with the minority of $\beta$-lovers and B is a nation of $\beta$-lovers with the minority of $\alpha$-lovers. Furthermore, the two countries are mirror-images of each other in that the fraction of $\alpha$-lovers in A is equal to the fraction of $\beta$-lovers in B. In this setup, Krugman showed that proportionately more firms in A operate in $\alpha$ than in $\beta$ under autarky, while *disproportionately* more firms in A operate in $\alpha$ than in $\beta$ under trade. As a result, A becomes a net exporter in $\alpha$-sector and B becomes a net exporter in $\beta$-sector. This is because A's domestic market for $\alpha$ is relatively large and B's domestic market for $\beta$ is relatively large. He called this mechanism the home market effect. In Krugman's model, the cross-country differences in the demand composition are due to exogenous cross-country variations in tastes.

In our framework, instead, the cross-country differences in the demand composition are due to the nonhomotheticity of preferences. We also dispense with the mirror-image assumption of the Krugman model. [2] More specifically, there are two countries, which may differ in per capita labor endowment and the population size, and a continuum of Dixit-Stiglitz monopolistically competitive sectors, which produce differentiated goods that can be exported

[2]The mirror-image setup, while simplifying the demonstration of the home market effect, has some drawbacks. First, it greatly restricts the range of comparative static exercises that can be performed. Second, it leaves what is meant by "A's domestic market for $\alpha$ is relatively large" ambiguous. Is it relative to B's domestic market for $\alpha$? Or is it relative to A's domestic market for $\beta$? It turns out that the answer is "neither". What matters for the home market effect is that the market size for $\alpha$ relative to the market size for $\beta$ is larger in A than in B. This will be shown later in section 3. (See also footnote 14.)

with iceberg costs.[3] Preferences across sectors are *implicitly additively separable with constant elasticity of substitution* (CES).[4] This class of utility functions, proposed by Hanoch (1975) and recently used by Comin, Lashkari, and Mestieri (2015) in their closed economy model of structural change, has some advantages. First, it contains the standard homothetic CES as a special case. Second, it allows for any number of sectors with sector-specific income elasticity parameters, while keeping the constant elasticity of substitution across sectors as a separate parameter.[5] Third, income elasticity differences across sectors are independent of per capita income level.[6] Furthermore, with their income elasticity parameters being the only heterogeneity, the sectors can be indexed such that their income elasticities are increasing in the index. Then, a higher per capita income shifts the household's expenditure shares towards higher-indexed sectors, holding prices and product varieties available in each sector fixed.[7] Such a shift in expenditure shares causes some entries (exits) in the higher (lower)-indexed sectors, which reduces the effective relative prices of higher-indexed goods, thereby amplifying (diminishing) the shift in expenditure shares, when different sectors produce substitutes (complements). In equilibrium, the demand composition of the Rich country, whose households achieve a higher standard-of-living, is more skewed towards higher-indexed sectors than the Poor country. This translates into the Rich's comparative advantage in higher-indexed sectors

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[3]As in Dornbusch, Fischer, and Samuelson (1980), a continuum of sectors facilitates the characterization of the equilibrium and comparative statics.

[4]Preferences are *explicitly* additively separable if written as $u = \int f_s(c_s)ds$ and *implicitly* additively separable if written as $\int f_s(u;c_s)ds = 1$. Furthermore, implicitly additively separable preferences are CES if $\int \omega_s(u)(c_s)^{1-1/\eta}ds$ = 1. We further assume that $\omega_s(u)$ is isoelastic in $u$ (i.e., a power function of $u$) so that $\partial \log \omega_s(u)/\partial u$ depends on *s* but *not* on $u$, which enables us to define the sector-specific income elasticity parameters.

[5]As known from the work of Houthakker (1960), Goldman and Uzawa (1964) and others, the explicitly additive separability of preferences would impose the restriction that the ratio of the income elasticity and the price elasticity is constant across all the sectors. Deaton (1974) and Hanoch (1975) argued that there is a priori no reason for such a restriction to hold empirically. Notice that one of the implications of this restriction is that homotheticity implies CES and vice versa. In other words, nonhomothetic CES preferences *cannot* be expressed in an explicitly additively separable form. But they can be expressed in an implicitly additively separable form.

[6]This is in strong contract to the Stone-Geary preferences, which implies that income elasticity differences across sectors decline with per capita income. Comin, Lashkari, and Mestieri (2015) offer empirical evidence that income elasticity differences across sectors are roughly constant over a wide range of per capita income levels.

[7]Formally, the weights attached to different sectors in preferences satisfy $\partial^2 \log \omega_s(u)/\partial u \partial s > 0$, i.e., $\omega_s(u)$ is *log-supermodular* in ***u*** and ***s***. Then, with ***u*** as a shift parameter, the density function of the expenditure shares across sectors satisfy the monotone likelihood ratio (MLR), and its cumulative distribution function satisfies the first-order stochastic dominance (FSD). See Athey (2002) and Vives(1999; Ch.2.7) for log-supermodularity and monotone comparative statics and Costinot (2009) for the first applications to international trade.

through the home market effect. Although there are two-way flows of differentiated goods in each sector, there is a unique cutoff sector such that the Rich runs a trade surplus in the sectors above the cutoff and the Poor runs a trade surplus in the sectors below it. Thus, the Rich becomes a net exporter of the high income elastic goods, and the Poor becomes a net exporter of the low income elastic goods.

Our framework is flexible enough to allow for a variety of comparative statics. For example, a uniform productivity improvement causes the cut-off sector to move up. Thus, the Rich switches from a net exporter to a net importer in some middle sectors, generating something akin to product cycles.[8] The intuition behind this result is easy to grasp. As the world becomes richer, the households in both countries shift their spending towards higher-indexed sectors. Thus, the relative weights of the higher indexed sectors, in which the Rich runs a surplus, become higher. In order to keep the overall trade account between the two countries in balance, the Rich's sectoral trade account must deteriorate in each sector. This is why the Rich's sectoral trade balances switch from being positive to negative in some middle sectors. How welfare gains from such a change are distributed across the two countries depends on the elasticity of substitution across sectors; a uniform productivity improvement widens (narrows) the welfare gap between the Rich and the Poor when the goods produced in different sectors are substitutes (complements).

The effects of globalization, captured by a trade cost reduction, are similar to uniform productivity improvements, except there are additional terms of trade effects when the two countries differ in size, measured in the total labor supply. When the two countries are equal in size, the wage rates are always equalized across the countries and hence the terms of trade are not affected by a reduction in the trade cost. This means that the country with higher per capita labor endowment always has higher per capita income and achieves higher standard-of-living. And without causing any terms of trade change, the effects of globalization are isomorphic to those of uniform productivity improvements. A lower trade cost allows the households in both countries to have better access to the differentiated goods produced abroad. In particular, globalization through a trade cost reduction causes the Rich (Poor) to switch from a net exporter

[8]The existing models of international product cycles, such as Krugman (1979), Grossman and Helpman (1991) and others, rely on some types of technology diffusion from the Rich to the Poor. Here, product cycles occur as a consequence of the world economy getting richer.

(importer) to a net importer (exporter) in some middle sectors, again generating something akin to product cycles. And again, a globalization widens (narrows) the welfare gap between the Rich and the Poor when the goods produced in different sectors are substitutes (complements).

When the two countries are unequal in size, the factor price is lower in the smaller country, reflecting its disadvantage of being smaller in the world of aggregate increasing returns due to the product variety effect. Globalization reduces (but never eliminates) this disadvantage, and causes the factor prices to converge (but never completely equalize) and hence the terms of trade to change in favor of the smaller country.[9] This generates some additional effects. If the smaller country has lower per capita labor endowment-- which includes the case where the two countries have the equal population size--, this country has lower standard-of-living regardless of the trade cost. However, if the smaller country has higher per capita labor endowment, globalization can cause a leapfrogging due to such a terms-of-trade change. At a high trade cost, the households in the smaller country might have a lower standard of living in spite of their higher labor endowment, because they benefit less from the product variety effect due to their disadvantage of living in a small country. Globalization reduces this disadvantage enough so that they achieve a higher standard of living at a lower trade cost. In our setup, this leads to a reversal of patterns of trade. The smaller country with higher per capita labor endowment is a net exporter of the low income elastic goods at a higher trade cost, and a net exporter of the high income elastic goods at a lower trade cost.

The present paper is most closely related to Fajgelbaum, Grossman, and Helpman (2011). Their baseline model has two monopolistically competitive sectors, H & L, that produce indivisible products, which are horizontally differentiated within each sector and vertically differentiated across sectors. In addition, there is a third sector that produces the divisible numeraire good competitively, which pins down the terms of trade between the two countries. Each household consumes one unit of a particular product from either H or L. Building on the discrete choice model of consumer behaviors, they derive a nested logit demand system, with the property that the rich consumers are more likely than the poor to choose an H-product under the assumption that marginal utility of the numeraire good is higher when combined with an H-

[9] This terms of trade effect of globalization is not due to the nonhomotheticity. It exists even if the standard home market effect models with homothetic preferences, as will be shown in our extension of the Krugman model in section 3, which drops the assumption of the two countries being equal in size.

product. By creating differences in demand structures through nonhomothetic preferences, they generate the patterns of trade where the Rich becomes a net-exporter of the high income elastic, high-quality H goods, while the Poor becomes a net exporter of low income elastic, low-quality L goods. While highly elegant and original, their nested-logit demand system departs from those in the standard models of the home market effect in many dimensions. This makes it difficult to isolate the effects of nonhomotheticity. In contrast, our framework stays close to the standard models, which helps to isolate the effects of nonhomotheticity. Our framework also allows us to conduct a variety of comparative statics with any number of sectors and the terms of trade effect. Furthermore, the elasticity of substitution across sectors is a separate parameter from the sector-specific income elasticity parameters. This means that it encompasses both the case where different sectors produce goods that are substitutes and the case where they produce goods that are complements, which turns out to be important for evaluating how the gains from productivity improvement and globalization are distributed between Rich and Poor countries.[10] Needless to say, these comments should not be viewed as criticisms of the Fajgelbaum-Grossman-Helpman model. Clearly, the two models have quite different structures and are developed with quite different objectives in mind and complement each other.

The rest of the paper is organized as follows. Section 2 proposes and analyzes our framework for studying the home market effect where the cross-country differences in the demand composition across a continuum of differentiated sectors are endogenously derived under nonhomothetic preferences. For comparison, section 3 offers a home market effect model where the cross-country differences in the demand composition are due to the exogenous cross-country taste differences. This section might be of independent interest because it extends the Krugman (1980) model to the case of a continuum of sectors with general homothetic CES preferences without the mirror-image assumptions. Section 4 adds a competitive sector, which produces the numeraire good, into our framework. Hence, the framework presented in this section may be viewed as an extension of the Helpman and Krugman (1985, Ch.10) model of the

[10]The existing models can deal with just one of these two cases, due to the restriction imposed by the nonhomothetic preferences they used. In Flam and Helpman (1987), Stokey (1991), and Fajgelbaum, Grossman, and Helpman (2011), different sectors produce goods of different quality and lower prices of lower quality goods reduce the demand for higher quality goods. Thus, different sectors produce substitutes in these models. In contrast, in a hierarchical demand system of Matsuyama (2000), different sectors produce goods of different priority, and lower prices of necessities increase the demand for luxuries. Thus, different sectors produce complements.

home market effect, which has one competitive sector and one differentiated goods sector, to the case of a continuum of differentiated goods sectors with differential income elasticities. Section 5 concludes. The appendix offers two lemmas, which are used repeatedly in the analysis.

## 2. The Home Market Effect with Nonhomothetic Preferences

### 2.1 The Model

Imagine the world economy that consists of two countries, indexed by *j* or *k* = 1 or 2. Country *j* is populated by $N^j$ homogenous households. There is a single nontradeable factor of production, which shall be called labor. Each household in *j* supplies $h^j$ units of effective labor inelastically at the wage rate, $w^j$. Thus, the income (and the expenditure) of each household in *j* is $E^j = w^j h^j$ and the total labor supply is $L^j = h^j N^j$. The number of households, $N^j$, and its effective labor supply per household, $h^j$, are the only possible sources of heterogeneity across the two countries.

There is a continuum of monopolistic competitive sectors, indexed by $s \in [0,1]$, each of which produces a continuum of tradable differentiated goods, indexed by $\nu \in \Omega_s = \Omega_s^1 + \Omega_s^2$, where $\Omega_s^j$ (*j* = 1 or 2) are disjoint sets of differentiated goods in sector *s* produced in country *j* in equilibrium.

***Household Budget Constraints and Preferences:***

Let $c_s^k(\nu)$ denote per household consumption of variety $\nu \in \Omega_s$ and $p_s^k(\nu)$ the unit consumer price of variety $\nu \in \Omega_s$ in country *k* = 1 or 2. Then, with the per household expenditure, $E^k = w^k h^k$, the budget constraint of each household in *k* is written as:

$$(1) \qquad \int_0^1 \left[ \int_{\Omega_s} p_s^k(\nu) c_s^k(\nu) d\nu \right] ds \leq E^k = w^k h^k .$$

The preferences of each household have a two-tier structure. At the lower level, the consumption of differentiated varieties within each sector is aggregated by the usual Dixit-Stiglitz aggregator, $\tilde{C}_s^k$, $s \in [0,1]$, defined by:

(2) $$\tilde{C}_s^k \equiv \left[\int_{\Omega_s} \left(c_s^k(\nu)\right)^{1-\frac{1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}};\ \sigma > 1.$$

At the upper-level, these Dixit-Stiglitz aggregators are aggregated by the utility function, $\tilde{U}^k = U\left(\tilde{C}_s^k, s \in [0,1]\right)$, which are given *implicitly* by

(3) $$\int_0^1 (\beta_s)^{\frac{1}{\eta}} \left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}} \left(\tilde{C}_s^k\right)^{\frac{\eta-1}{\eta}} ds \equiv 1;\ \beta_s > 0 \text{ and } \eta \neq 1,$$

with $\varepsilon(s) > \eta$ for $0 < \eta < 1$ or $0 < \varepsilon(s) < \eta$ for $\eta > 1$, which implies $\left(\varepsilon(s) - \eta\right)/\left(1-\eta\right) > 0$. These parameter restrictions ensure that $\tilde{U}^k = U\left(\tilde{C}_s^k, s \in [0,1]\right)$ is globally monotone increasing and globally quasi-concave in $\tilde{C}_s^k$, $s \in [0,1]$. Without further loss of generality, we normalize $\varepsilon(s)$ such that $\int_0^1 \varepsilon(s) ds = 1$. In addition, it is assumed $\eta < \sigma$ so that differentiated goods are closer substitutes within each sector than across sectors.

The utility function (3) is *implicitly, additively separable with constant elasticity of substitution* (CES), a class of utility functions, introduced by Hanoch (1975). The standard homothetic CES preferences,

$$\tilde{U}^k \equiv \left[\int_0^1 (\beta_s)^{\frac{1}{\eta}} \left(\tilde{C}_s^k\right)^{1-\frac{1}{\eta}} ds\right]^{\frac{\eta}{\eta-1}},$$

is a special case of (3), where $\varepsilon(s) = 1$ for all $s \in [0,1]$. By letting $\varepsilon(s)$ dependent on *s*, this class of utility functions allows for the income elasticity to differ across sectors, while keeping the price elasticity, $\eta$, constant across sectors. In what follows, we assume that the sectors can be ordered such that $\varepsilon(s)$ is strictly increasing in *s*. Then, $(\beta_s)^{\frac{1}{\eta}} \left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}}$, the coefficient on the term $\left(\tilde{C}_s^k\right)^{\frac{\eta-1}{\eta}}$ in (3), is *log-supermodular* in *s* and $\tilde{U}^k$. By applying Lemma 1 (See Appendix), for $\hat{g}(s; \tilde{U}^k) = (\beta_s)^{\frac{1}{\eta}} \left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}}$, this implies that, as $\tilde{U}^k$ goes up, the household cares more about the higher-indexed goods in the sense that the density function of the weights attached to different sectors satisfies the monotone likelihood ratio (MLR) property and that its cumulative distribution function satisfies the first-order stochastic dominance (FSD) property.

***Household Maximization:***

Each household in $k$ maximizes $\tilde{U}^k = U\left(\tilde{C}_s^k, s \in [0,1]\right)$, where $\tilde{C}_s^k$ is defined by (2) and $U(\bullet)$ is defined implicitly by (3), subject to the budget constraint, (1). This maximization problem can be solved in two stages. At the first stage, each household chooses $c_s^k(\nu)$ for $\nu \in \Omega_s$ to:

$$\text{Maximize } \tilde{C}_s^k \equiv \left[\int_{\Omega_s} \left(c_s^k(\nu)\right)^{1-\frac{1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}},$$

$$\text{subject to } \int_{\Omega_s} p_s^k(\nu) c_s^k(\nu) d\nu \le E_s^k,$$

where $E_s^k$ is the household's expenditure in sector-s. The solution to this problem is well-known and given by:

$$(4) \qquad c_s^k(\nu) = \left(\frac{p_s^k(\nu)}{P_s^k}\right)^{-\sigma} C_s^k = \frac{\left(p_s^k(\nu)\right)^{-\sigma}}{\left(P_s^k\right)^{1-\sigma}} E_s^k, \quad \text{where}$$

$$(5) \qquad P_s^k \equiv \left[\int_{\Omega_s} \left(p_s^k(\nu)\right)^{1-\sigma} d\nu\right]^{\frac{1}{1-\sigma}}$$

is the Dixit-Stiglitz price index of differentiated goods in sector-s in country $k$, which the households treat as given, and $C_s^k$ is the maximized value of $\tilde{C}_s^k$, satisfying $E_s^k = P_s^k C_s^k$. At the second stage, each household choose $E_s^k = P_s^k C_s^k$ to:

$$\text{Maximize } \tilde{U}^k,$$

$$\text{subject to } \int_0^1 (\beta_s)^{\frac{1}{\eta}} \left(\tilde{U}^k\right)^{\frac{\varepsilon(s)-\eta}{\eta}} \left(C_s^k\right)^{\frac{\eta-1}{\eta}} ds \equiv 1 \text{ and } \int_0^1 P_s^k C_s^k ds = \int_0^1 E_s^k ds \le E^k.$$

The solution to this problem can be written in terms of the expenditure share of sector-s, $m_s^k$:

(6) $$m_s^k \equiv \frac{E_s^k}{E^k} = \frac{P_s^k C_s^k}{E^k} = \frac{\beta_s \left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta}}{\left(E^k\right)^{1-\eta}} = \frac{\beta_s \left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta}}{\int_0^1 \beta_t \left(U^k\right)^{\varepsilon(t)-\eta}\left(P_t^k\right)^{1-\eta} dt}, \text{ with } \int_0^1 m_s^k ds \equiv 1$$

where $U^k$ is the maximized value of $\tilde{U}^k$, which is given implicitly as a function of $E^k$ and the price indices, $P_s^k$, as follows:

(7) $$\left(E^k\right)^{1-\eta} \equiv \int_0^1 \beta_s \left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta} ds.$$ [11]

Recall the parameter restrictions that ensure the global monotonicity of the utility function, (3); $\varepsilon(s) > \eta$ for $0 < \eta < 1$ and $0 < \varepsilon(s) < \eta$ for $\eta > 1$. Thus, LHS of (7) is strictly increasing (decreasing) in $E^k$ if and only if RHS of (7) is strictly increasing (decreasing) in $U^k$. This implies that $U^k$ is strictly increasing in $E^k$.[12]

From Eq.(6), we could write the relative expenditure share of any two sectors as:

$$\log(m_s^k / m_{s'}^k) = \log(\beta_s / \beta_{s'}) + \left(\varepsilon(s) - \varepsilon(s')\right)\log(U^k) - (\eta - 1)\log(P_s^k / P_{s'}^k),$$

and the relative household demand curve as:

$$\log(C_s^k / C_{s'}^k) = \log(\beta_s / \beta_{s'}) + \left(\varepsilon(s) - \varepsilon(s')\right)\log(U^k) - \eta\log(P_s^k / P_{s'}^k).$$

This shows not only that the relative demand for a higher-indexed sector has higher income elasticity. It also shows that the slope of the Engel curve, $\partial\log(C_s^k / C_{s'}^k)/\partial\log(U^k) =$

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[11] If we define the aggregate price index, $P^k$, by $E^k \equiv P^k U^k$, $U^k$ could be interpreted as the real aggregate consumption per capita. Furthermore, eq.(6) could be written as:

$$C_s^k = \beta_s \left(\frac{P_s^k}{P^k}\right)^{-\eta}\left(U^k\right)^{\varepsilon(s)} = \beta_s \frac{\left(P_s^k\right)^{-\eta}}{\left(P^k\right)^{\varepsilon(s)-\eta}}\left(E^k\right)^{\varepsilon(s)}, \text{ where } \left(P^k\right)^{1-\eta} \equiv \int_0^1 \beta_s \left(U^k\right)^{\varepsilon(s)-1}\left(P_s^k\right)^{1-\eta} ds.$$

However, unlike the price indices of each sector, $P_s^k$, we cannot treat the aggregate price index, $P^k$, as fixed, when deriving the household demand, since the weights attached on sectors to construct this index depends on $U^k$.

[12] This can be also verified by partially differentiating (7) with respect to $E^k$ to obtain

$$\frac{\partial\log\left(U^k\right)}{\partial\log\left(E^k\right)} = \frac{\int_0^1 \beta_s \left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta} ds}{\int_0^1 \beta_s \left(\frac{\varepsilon(s)-\eta}{1-\eta}\right)\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta} ds} > 0.$$

$\varepsilon(s)-\varepsilon(s')$, is independent of the real aggregate consumption per capita (or the standard-of-living), $U^k$. Comin, Lashkari, and Mestieri (2015) offers the empirical evidence in support of the log-linear Engel curves implied by implicitly additively separable preferences with CES (against the Engel curves implied by the Stone-Geary preferences).

Notice also that $\beta_s\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta}$ is *log-supermodular* in $s$ and $U^k$. Hence, by applying Lemma 1 for $\hat{g}(s,U^k)=\beta_s\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{1-\eta}$, eq.(6) shows that, holding the price indices constant, the household with a higher $E^k$ (and hence a higher $U^k$) allocates larger shares of their expenditure towards higher-indexed goods in the sense that the density function of the expenditure share across sectors function satisfies the MLR property and that its cumulative distribution function satisfies the FSD property.

***Iceberg Costs and Aggregate Demand for Differentiated Goods:***

The unit consumer price of each differentiated good, $p_s^k(\nu)$, $\nu\in\Omega_s^j$, depends on *k*, because of the (iceberg) trade costs; To consume one unit of $\nu\in\Omega_s^j$ in country *k*, $\tau_{jk}$ units need to be shipped from *j*. Thus, with the unit factory price, $p_s^j(\nu)$, $\nu\in\Omega_s^j$, $p_s^k(\nu)$ = $\tau_{jk}p_s^j(\nu)\geq p_s^j(\nu)$. Then, from (4) and (6), each household in *k* demands for $\nu\in\Omega_s^j$ by

$$\tau_{jk}c_s^k(\nu)=\tau_{jk}\ \beta_s\left(E^k\right)^{\eta}\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{\sigma-\eta}\left(p_s^k(\nu)\right)^{-\sigma}$$

$$=\tau_{jk}\ \beta_s\left(E^k\right)^{\eta}\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{\sigma-\eta}\left(\tau_{jk}p_s^j(\nu)\right)^{-\sigma}=\rho_{jk}\ \beta_s\left(E^k\right)^{\eta}\left(U^k\right)^{\varepsilon(s)-\eta}\left(P_s^k\right)^{\sigma-\eta}\left(p_s^j(\nu)\right)^{-\sigma},$$

where $\rho_{jk}\equiv\left(\tau_{jk}\right)^{1-\sigma}\leq 1$. Since there are $N^k$ households in *k*, the aggregate demand for $\nu\in\Omega_s^j$ can be expressed as:

(8) $$D_s(\nu)=A_s^j(p_s^j(\nu))^{-\sigma},$$

where

(9) $$A_s^j\equiv\sum\nolimits_k\rho_{jk}b_s^k;$$

(10) $$b_s^k\equiv\beta_s\left(E^k\right)^{\eta}\left(U^k\right)^{\varepsilon(s)-\eta}N^k\left(P_s^k\right)^{\sigma-\eta}=\beta_s\left(U^k\right)^{\varepsilon(s)}\left(P^k\right)^{\eta}N^k\left(P_s^k\right)^{\sigma-\eta}$$

where $A_s^j$ may be interpreted as the aggregate demand shift parameter for a variety produced in sector-*s* in country *j*; $b_s^k$ as the aggregate demand shift parameter for sector-*s* in country *k*; and

$\rho_{jk}$ is the weight attached to the aggregate spending by country *k* of varieties produced in country *j*. Eqs. (8)-(10) show that the demand curve for each variety has a constant price elasticity with its demand shift parameter, $A_s^j$, depending on the trade costs in a manner familiar in the standard Dixit-Stiglitz monopolistic competition models of trade. What is new is that the household utility level, $U^k$, has differential impacts on the demand shift parameters across sectors due to the nonhomotheticity of preferences.

For the remainder of this paper, we follow Krugman (1980) and others by assuming that $\tau_{11} = \tau_{22} = 1$ and $\tau_{12} = \tau_{21} = \tau > 1$, so that

(11) $\rho_{11} = \rho_{22} = 1$ and $\rho_{12} = \rho_{21} = \rho \equiv (\tau)^{1-\sigma} < 1.$

Thus, $\rho \in [0,1)$ measures how much each household spends on an imported variety relative to what it would spend in the absence of the trade cost; it is inversely related to $\tau$, with $\rho = 0$ for $\tau = \infty$ and $\rho \to 1$ for $\tau \to 1$.

***Production and Pricing By Monopolistically Competitive Firms:***

Each differentiated variety is produced by a monopolistically competitive firm**.** Producing one unit of each differentiated variety in sector-s requires $\psi_s$ units of labor, so that the marginal cost is equal to $w^j\psi_s$ for $\nu \in \Omega_s^j$. Eq. (8) shows that the price elasticity of demand for each variety is constant, $\sigma$. Since all the varieties in the same sector in the same country have the identical marginal cost, they all set the same price, given by:

(12) $$p_s^j(\nu) = \frac{w^j\psi_s}{1-1/\sigma} \equiv p_s^j \text{ for all } \nu \in \Omega_s^j,$$

and from (8), they are all produced by the same amount, given by:

(13) $$y_s^j \equiv A_s^j(p_s^j)^{-\sigma}.$$

By inserting (12) into (5),

(14) $$\left(P_s^k\right)^{1-\sigma} = \int_{\Omega_s}\left(p_s^k(\nu)\right)^{1-\sigma}d\nu = \sum_j\int_{\Omega_s^j}\left(\tau_{jk}p_s^k(\nu)\right)^{1-\sigma}d\nu = \sum_j V_s^j\rho_{jk}(p_s^j)^{1-\sigma}$$

where $V_s^j$ is the Lebesgue measure of $\Omega_s^j$, the equilibrium measure of varieties produced (and of active firms) in sector-*s* of country *j*.

***Free Entry Conditions and Distribution of Firms Across Sectors:***

This equilibrium measure, $V_s^j$, is determined by the free entry condition. To enter sector-*s*, all monopolistically competitive firms need to pay the setup cost per variety, $\phi_s$, in labor, and they have incentive to do so, as long as the profit is non-negative. Thus, in equilibrium, either a positive measure of firms (and varieties) enter, in which case they all make zero profit ($V_s^j > 0 \Rightarrow \pi_s^j \equiv p_s^j y_s^j - w^j\left(\psi_s y_s^j + \phi_s\right) = 0$), or no firms (and varieties) enter, because they would earn negative profit if they enter ($\pi_s^j < 0 \Rightarrow V_s^j = 0$). Using (13), this free entry condition can be written as the complementarity slackness condition:

$$V_s^j \geq 0\,;\ y_s^j = A_s^j (p_s^j)^{-\sigma} \leq (\sigma - 1)\phi_s / \psi_s\ .$$

In what follows, we use the following normalizations to keep the notation simple. First, let us choose the unit of each differentiated good in sector-s such that $\psi_s = 1 - 1/\sigma$. This implies

(15) $\quad p_s^j = w^j$ for all $s \in [0,1]$.

Second, let us choose the units of the measure of varieties in each sector, such that $\phi_s = 1/\sigma$. These two normalizations jointly imply that the free entry condition can be now written as:

(16) $\quad V_s^j \geq 0\,;\ y_s^j = A_s^j (w^j)^{-\sigma} \leq 1 \qquad$ for all $s \in [0,1]$ and $j$ = 1 and 2.

In other words, we choose the units such that each (active) firm sells its good at $p_s^j = w^j$, produce by $y_s^j = 1$, and hire labor by $\psi_s y_s^j + \phi_s = 1$ to break even in equilibrium. Furthermore, since each active firm hires labor by $\psi_s y_s^j + \phi_s = 1$, the labor demand by sector-s of country *j* is $V_s^j$. By integrating across sectors, the labor market clearing condition is given by $\int_0^1 V_s^j ds = L^j = h^j N^j$, which means that the distribution of firms across sectors can be written as:

(17) $\quad f_s^j \equiv \dfrac{V_s^j}{\int_0^1 V_t^j dt} = \dfrac{V_s^j}{L^j}$.

***Equilibrium Conditions:***

We are now ready to consolidate all the equilibrium conditions. From (9), (11), (16) and (17), the complementary slackness condition for free entry in each sector and in each country is given by:

(18) $f_s^1 \geq 0$; $(b_s^1 + \rho b_s^2)(w^1)^{-\sigma} \leq 1$; & $f_s^2 \geq 0$; $(\rho b_s^1 + b_s^2)(w^2)^{-\sigma} \leq 1$ for all s,

where $b_s^k$, given in (10), can be rewritten, by using the expenditure share, $m_s^k$, given in (6), in two different ways. First, by eliminating the terms $U^k$ from (6) and (10) and using $E^k N^k = w^k h^k N^k = w^k L^k$, we obtain

(19) $b_s^k = m_s^k (w^k L^k)(P_s^k)^{\sigma-1}$.

Second, by eliminating the terms $P_s^k$ from (6) and (10), we obtain

(20) $b_s^k = \left(\left(w^k h^k\right)^{\sigma} N^k\right)\left[\beta_s \left(U^k\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{1-\sigma}{1-\eta}\right)} \left(m_s^k\right)^{\left(\frac{\sigma-\eta}{1-\eta}\right)}$.

Next, from (11), (14), (15), and (17), the price index in each sector and in each country becomes:

(21) $(P_s^1)^{1-\sigma} = f_s^1 L^1 (w^1)^{1-\sigma} + \rho f_s^2 L^2 (w^2)^{1-\sigma}$; $(P_s^2)^{1-\sigma} = \rho f_s^1 L^1 (w^1)^{1-\sigma} + f_s^2 L^2 (w^2)^{1-\sigma}$ for all $s \in [0,1]$.

Finally, the market size distribution and the firm distribution across sectors must add up to one in each country.

(22) $\int_0^1 m_s^k ds \equiv 1$ for $k$ = 1 and 2.

(23) $\int_0^1 f_s^j ds \equiv 1$ for $j$ = 1 and 2.

## 2.2 Autarky Equilibrium

First, let us consider the case of autarky, $\rho$ = 0, where each differentiated good must be produced in the country of consumption. Then, there is a positive entry in each sector in each country. From (18), this implies $b_s^k = (w^k)^{\sigma}$ for all $s \in [0,1]$ and for $k$ = 1 and 2. Inserting this to (19), (20) and (21) yields

(24) $f_s^k = m_s^k = \left(\left(h^k\right)^{\sigma} N^k\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left[\beta_s \left(U_0^k\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$.

Subscript "0" is added here to indicate that $U_0^k$ is the equilibrium value of the utility level, or the standard-of-living, achieved in autarky ($\rho$ = 0). Note that eq. (24) shows that the firms are distributed proportionately with market sizes in autarky.

By integrating (24) across all the sectors and using (22) or (23), we can pin down $U_0^k$ as

$$\int_0^1 \left(\left(h^k\right)^{\sigma} N^k\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left[\beta_s \left(U_0^k\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} ds = 1,$$

which can be written more compactly as

(25) $$U_0^k = u\left(x_0^k\right) \quad \text{with } x_0^k \equiv \left(h^k\right)^{\sigma} N^k = \left(h^k\right)^{\sigma-1} L^k,$$

where $u(\bullet)$ is defined implicitly by

(26) $$(x)^{\left(\frac{1-\eta}{\sigma-\eta}\right)} \equiv \int_0^1 \left[\beta_s (u(x))^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} ds.$$

Lemma 2-i) in the appendix shows that $u(\bullet)$, defined in eq.(26), is a strictly increasing function. Thus, the utility level, or the standard-of-living, in autarky, increases with $x_0^k \equiv \left(h^k\right)^{\sigma} N^k = \left(h^k\right)^{\sigma-1} L^k$. It obviously increases in each household's labor endowment, $h^k$. It also increases with $N^k$. This is due to the familiar aggregate increasing returns to scale in the presence of "love for variety" and the fixed cost. Living in an economy with a larger population size is beneficial as it allows the households to share the fixed cost of adding more varieties of products to consume. Notice that the condition for $U_0^1 = u(x_0^1) < U_0^2 = u(x_0^2)$ can be expressed as $\left(h^1\right)^{\sigma-1} L^1 < \left(h^2\right)^{\sigma-1} L^2$, which may occur even if $h^1 > h^2$ when $L^1 < L^2$. In other words, the country with higher per capita labor endowment may have a lower standard-of-living when it is smaller. This is because those living in a small country has disadvantage in the presence of aggregate increasing returns.[13]

[13] This result does not contradict what we noted earlier, i.e., eq.(7) shows that the household's utility is increasing in per capita labor endowment, *holding the price indices given*. When comparing the two countries in equilibrium, the price indices differ across the two countries because the measure of varieties produced in each sector in each country is endogenously determined by the free entry condition.

Furthermore, plugging (25) and (26) into (24) yields the autarky equilibrium density of firms and market sizes across sectors as follows:

$$(27)\qquad f_s^k = m_s^k = \frac{\left[\beta_s\left(u(x_0^k)\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\left(x_0^k\right)^{\left(\frac{1-\eta}{\sigma-\eta}\right)}} = \frac{\left[\beta_s\left(u(x_0^k)\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\int_0^1\left[\beta_t\left(u(x_0^k)\right)^{(\varepsilon(t)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}dt}.$$

The numerator of (27) is *log-supermodular* in *s* and $x_0^k$. Thus, by applying Lemma 1 for $\hat{g}(s,x_0^k) = \left[\beta_s\left(u(x_0^k)\right)^{(\varepsilon(s)-\eta)}\right]^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$, eq.(27) shows that, for $U_0^1 = u(x_0^1) < U_0^2 = u(x_0^2)$, the households in country 2, whose standard-of-living is higher than those in country 1, spend relatively more on higher-indexed goods in the sense that $m_s^1/m_s^2$ is strictly decreasing in *s* (that is, the density functions of equilibrium market size distribution across sectors satisfies the MLR property) as well as in the sense that the cumulative distribution function for country 2 first-order stochastically dominates (FSD) the cumulative distribution function for country 1.

Notice the difference between the two expressions of $m_s^k$, eq.(6) and eq.(27), in particular how it depends on the household utility. Eq.(6) implies that, *holding the price indices given,* the relative market size of two sectors, $s > s'$, responds to an increase in $U^k$ as

$$\frac{\partial\log(m_s^k/m_{s'}^k)}{\partial\log\left(U^k\right)} = \varepsilon(s)-\varepsilon(s') > 0.$$

However, such a change in the relative market size causes some entries into higher-indexed sectors, and exits from lower-indexed sectors, which reduces the relative price indices of high-indexed goods, which amplifies (dampens) the shift in expenditure shares if different sectors produce substitutes (complements). Indeed, from eq. (24) or (27), it is easy to show that, *in equilibrium,* the relative market size of two sectors, $s > s'$, responds to an increase in $U^k$ as

$$\frac{\partial\log(m_s^k/m_{s'}^k)}{\partial\log\left(U^k\right)} = \left(\varepsilon(s)-\varepsilon(s')\right)\left(\frac{\sigma-1}{\sigma-\eta}\right),$$

where $(\sigma-1)/(\sigma-\eta)$ >(<) 1captures the amplifying (dampening) effect of endogenous entries and exits for $\eta$ >(<) 1.

The above amplification or dampening effect also affects the welfare impact of a change in $x_0^k$. From Lemma 2-ii) shown in the appendix, $d\log u(\lambda x)/d\log\lambda = \lambda x u'(\lambda x)/u(\lambda x) \equiv \zeta(\lambda x)$ is increasing (decreasing) in $x$ if $\eta$ >(<) 1. In words, welfare gains from a percentage increase in $x_0^k$ is higher (lower) at a higher $x$ if $\eta$ >(<) 1. This implies, among other things, that a uniform productivity improvement, $\partial h^1/h^1 = \partial h^2/h^2 > 0$, magnifies (reduces) the relative gap in the standard-of-living between the two countries, $U_0^2/U_0^1 = u(x_0^2)/u(x_0^1) > 1$, if different sectors produce substitutes (complements).

### 2.3 Trade Equilibrium and Patterns of Trade

In what follows, let us focus on the case $f_s^1 > 0$ and $f_s^2 > 0$ for all $s \in [0,1]$. Then, (18) is simplified to $b_s^1 + \rho b_s^2 = (w^1)^\sigma$ and $\rho b_s^1 + b_s^2 = (w^2)^\sigma$ and hence

(28) $$b_s^1 = \frac{(w^1)^\sigma - \rho(w^2)^\sigma}{1-\rho^2} \text{ and } b_s^2 = \frac{(w^2)^\sigma - \rho(w^1)^\sigma}{1-\rho^2} \quad \text{for all } s \in [0,1].$$

By inserting (28) into (19) and using (21), we obtain

(29) $$f_s^1 L^1 (w^1)^{1-\sigma} + \rho f_s^2 L^2 (w^2)^{1-\sigma} = \frac{(1-\rho^2)(w^1 L^1 m_s^1)}{(w^1)^\sigma - \rho(w^2)^\sigma}$$

$$\rho f_s^1 L^1 (w^1)^{1-\sigma} + f_s^2 L^2 (w^2)^{1-\sigma} = \frac{(1-\rho^2)(w^2 L^2 m_s^2)}{(w^2)^\sigma - \rho(w^1)^\sigma}$$

for all $s \in [0,1]$. Integrating these expressions across all sectors and using (22) and (23),

$$L^1 (w^1)^{1-\sigma} + \rho L^2 (w^2)^{1-\sigma} = \frac{(1-\rho^2)(w^1 L^1)}{(w^1)^\sigma - \rho(w^2)^\sigma},$$

$$\rho L^1 (w^1)^{1-\sigma} + L^2 (w^2)^{1-\sigma} = \frac{(1-\rho^2)(w^2 L^2)}{(w^2)^\sigma - \rho(w^1)^\sigma},$$

either of which can be rewritten as:

(30) $$\frac{L^1}{L^2} = \Lambda(\omega;\rho) \equiv (\omega)^{2\sigma-1}\frac{1-\rho(\omega)^{-\sigma}}{1-\rho(\omega)^\sigma},$$

where $\omega \equiv w^1/w^2$ is the relative factor price and $\Lambda(\bullet;\rho)$ is strictly increasing in $\omega \in (\rho^{1/\sigma}, \rho^{-1/\sigma})$ and satisfies $\lim_{\omega\to\rho^{1/\sigma}} \Lambda(\omega;\rho) = 0$, $\Lambda(1;\rho) = 1$, and $\lim_{\omega\to\rho^{-1/\sigma}} \Lambda(\omega;\rho) = \infty$.

Figure 1 illustrates eq.(30), which determines the (factor) terms of trade $\omega \equiv w^1/w^2$ as a function of the relative labor supply, $L^1/L^2$, for a given level of $0 < \rho < 1$. It shows that $\omega \equiv w^1/w^2$ is strictly increasing in $L^1/L^2$ and $\omega \equiv w^1/w^2 < 1$ if and only if $L^1/L^2 < 1$. Thus, the factor price is higher in the larger economy, which reflects the aggregate increasing returns to scale pointed out earlier.[14] It also shows the lower and upper bounds for the terms of trade, $\omega \in (\rho^{1/\sigma}, \rho^{-1/\sigma})$. The arrows indicate the effects of an increase in $\rho$. As shown, it flattens the graph, thereby causing a factor price convergence. This is because globalization, captured by a reduction in $\tau$ and hence an increase in $\rho$, reduces the smaller country's disadvantage.

In addition, combining (28) and (20) yields

$$(31) \qquad m_s^1 = \left(\frac{(1-\rho^2)(h^1)^\sigma N^1}{1-\rho(\omega)^{-\sigma}}\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left(\beta_s \left(U_\rho^1\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)},$$

$$m_s^2 = \left(\frac{(1-\rho^2)(h^2)^\sigma N^2}{1-\rho(\omega)^{\sigma}}\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left(\beta_s \left(U_\rho^2\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}.$$

Here, the subscript "$\rho$" is added to indicate that $U_\rho^k$, the equilibrium standard-of-living achieved in each country under trade, depends on $\rho$. By integrating (31) across all the sectors and using (22), we obtain

$$(32) \qquad U_\rho^1 = u\left(x_\rho^1\right), \qquad \text{with } x_\rho^1 \equiv \frac{(1-\rho^2)\left(h^1\right)^\sigma N^1}{1-\rho(\omega)^{-\sigma}} \equiv \frac{(1-\rho^2)x_0^1}{1-\rho(\omega)^{-\sigma}};$$

$$U_\rho^2 = u\left(x_\rho^2\right), \qquad \text{with } x_\rho^2 \equiv \frac{(1-\rho^2)\left(h^2\right)^\sigma N^2}{1-\rho(\omega)^{\sigma}} \equiv \frac{(1-\rho^2)x_0^2}{1-\rho(\omega)^{\sigma}},$$

[14] Note that eq.(30) implies $w^1L^1/w^2L^2 = \omega\Lambda(\omega;\rho) = \left((\omega)^\sigma - \rho\right)/\left((\omega)^{-\sigma} - \rho\right)$, which is strictly increasing in $\omega$ (hence also in $L^1/L^2$) and $w^1L^1/w^2L^2 < 1$ if and only if $\omega < 1$ (hence also if and only if $L^1/L^2 < 1$). Thus, the larger economy is larger regardless of whether it is measured in the total labor supply or in the aggregate GDP.

where $u(\bullet)$ is the same increasing function defined implicitly by (26). Note that the welfare effects of globalization on each country are summarized by a single index, $x_\rho^k$. Note also that the lower and upper bound on the terms of trade established earlier, $\omega \in (\rho^{1/\sigma}, \rho^{-1/\sigma})$, which can be seen in Figure 1, ensures gains from trade for both countries; $\omega < \rho^{-1/\sigma}$ implies $U_\rho^1 = u(x_\rho^1) >$ $U_0^1 = u(x_0^1)$ and $\omega > \rho^{1/\sigma}$ implies $U_\rho^2 = u(x_\rho^2) > U_0^2 = u(x_0^2)$.

Plugging (32) back into (31) and using the definition of $u(\bullet)$, given by (26), yields the equilibrium density function of the market size distribution across sectors in each country as follows.

(33) $$m_s^k = \frac{\left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\left(x_\rho^k\right)^{\left(\frac{1-\eta}{\sigma-\eta}\right)}} = \frac{\left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}}{\int_0^1 \left(\beta_t \left(u(x_\rho^k)\right)^{(\varepsilon(t)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)} dt} \quad \text{for } k = 1 \text{ and } 2.$$

Note that $\left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$ is *log-supermodular* in *s* and $x_\rho^k$. Hence, by applying Lemma 1 for $\hat{g}(s, x_\rho^k) = \left(\beta_s \left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$, it follows from eq. (33) that, for $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$, the households in country 2, whose standard-of-living is higher than those in country 1, spend relatively more on higher-indexed goods in the sense that $m_s^1 / m_s^2$ is strictly decreasing in *s* (that is, the density functions of the equilibrium market size distribution across sectors satisfies the MLR property) as well as in the sense that the cumulative distribution function for country 2 first-order stochastically dominates (FSD) the cumulative distribution function for country 1. In shorts, the country with higher standard-of-living has relatively larger domestic markets in higher-indexed sectors. The MLR property can also be seen by taking the ratio from (33) to obtain

(34) $$\frac{m_s^1}{m_s^2} = \left(\frac{x_\rho^1}{x_\rho^2}\right)^{\left(\frac{\eta-1}{\sigma-\eta}\right)} \left(\frac{u(x_\rho^1)}{u(x_\rho^2)}\right)^{(\varepsilon(s)-\eta)\left(\frac{\sigma-1}{\sigma-\eta}\right)}.$$

Clearly, this is strictly decreasing in $s$ if $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$ and strictly increasing in $s$ if $U_\rho^1 = u(x_\rho^1) > U_\rho^2 = u(x_\rho^2)$.

Unlike in autarky, the firm distribution in each country is no longer proportional to the market size distribution in that country. By solving (29) for $f_s^1$ and $f_s^2$ and using (30), we obtain the equilibrium density function of the firm distribution across sectors in each country as follows:

(35) $$f_s^1 = \frac{m_s^1 - \rho(\omega)^{-\sigma} m_s^2}{1-\rho(\omega)^{-\sigma}} > 0; \qquad f_s^2 = \frac{m_s^2 - \rho(\omega)^{\sigma} m_s^1}{1-\rho(\omega)^{\sigma}} > 0,$$

which requires $\rho(\omega)^{-\sigma} < m_s^1/m_s^2 < \rho^{-1}(\omega)^{-\sigma}$. Furthermore, the ratio of the two,

(36) $$\frac{f_s^1}{f_s^2} = \left(\frac{1-\rho(\omega)^{\sigma}}{1-\rho(\omega)^{-\sigma}}\right)\left(\frac{m_s^1/m_s^2 - \rho(\omega)^{-\sigma}}{1-\rho(\omega)^{\sigma} m_s^1/m_s^2}\right)$$

is increasing in $m_s^1/m_s^2$ and satisfies $f_s^1/f_s^2 > m_s^1/m_s^2 > 1$, $f_s^1/f_s^2 = m_s^1/m_s^2 = 1$, or $f_s^1/f_s^2 < m_s^1/m_s^2 < 1$.

Figure 2 illustrates eq.(34) and eq.(36) for the case of $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$. In this case, $m_s^1/m_s^2$ is strictly decreasing in $s$ and hence $f_s^1/f_s^2$ is also strictly decreasing in $s$. Furthermore, there is a unique cutoff sector, $s_c \in (0,1)$, such that $f_s^1/f_s^2 > m_s^1/m_s^2 > 1$ holds below the cutoff and $f_s^1/f_s^2 < m_s^1/m_s^2 < 1$ above the cutoff. Thus, disproportionately larger fractions of firms operate in lower-indexed sectors in the country with lower-standard-of-living, precisely because their domestic markets are relatively larger in lower-indexed sectors, which produce low income elastic goods. Likewise, disproportionately larger fractions of firms operate in the higher-indexed sectors in the country with higher standard-of-living, precisely because their domestic markets are relatively larger in the higher-indexed sectors, which produce high income elastic goods.

This disproportional effect of the market size distribution on the firm distribution under trade translates into the patterns of intra-sectoral trade across sectors, and the country with higher (lower) standard-of-living becomes a net exporter (importer) above the cutoff and a net importer (exporter) above the cutoff, as indicated in Figure 2. To see this, recall that the households in $k$ spend $b_s^k(p_s^k)^{1-\sigma} = \rho b_s^k(p_s^j)^{1-\sigma} = \rho b_s^k(w^j)^{1-\sigma}$ *per variety* produced in sector-s of country $j \neq k$. With the measure of varieties produced in this sector, $V_s^j$, the total gross export value from $j$ to $k$ in sector-s is $V_s^j \rho b_s^k(w^j)^{1-\sigma} = \rho f_s^j b_s^k(w^j)^{1-\sigma} L^j$. Thus, the net export value from 1 to 2 in sector-s is given by $NX_s^1 = -NX_s^2 = \rho\left(f_s^1 b_s^2(w^1)^{1-\sigma} L^1 - f_s^2 b_s^1(w^2)^{1-\sigma} L^2\right)$. Using (28), (30) and (35), this can be further rewritten as:

$$(37) \qquad NX_s^1 = -NX_s^2 = \frac{\rho w^2 L^2}{(\omega)^{-\sigma} - \rho}\left(m_s^1 - m_s^2\right) = \frac{\rho w^1 L^1}{(\omega)^{\sigma} - \rho}\left(m_s^1 - m_s^2\right).$$

Thus, $NX_s^1 = -NX_s^2 > 0$ for $s < s_c$ and $NX_s^1 = -NX_s^2 < 0$ for $s > s_c$ when $U_\rho^1 = u\left(x_\rho^1\right) < U_\rho^2 = u\left(x_\rho^2\right)$. This is due to the home market effect a la Krugman (1980), except that the cross-country difference in the market size distribution across sectors is due to nonhomothetic preferences in this model, not due to the exogenous cross-country variations in taste assumed in Krugman (1980).

It is also worth emphasizing that country 1 becomes a net exporter in sectors where $m_s^1 > m_s^2$ holds, which are not necessarily sectors where $m_s^1 w^1 L^1 > m_s^2 w^2 L^2$ holds. What determines the direction of net sectoral trade flows in a general equilibrium model of the home market effect is *not* the cross-country difference in the market size in each sector. What matters is the *cross-country difference in the demand compositions*, i.e., in the *cross-country difference in the market size distributions across sectors.*[15]

## 2.4 **Ranking the Countries**

Having established that the country with higher (lower) standard-of-living becomes the net exporter in higher (lower)-indexed sectors, our remaining task is to rank the two countries in

[15] The home market effect is often described simply as "relatively large domestic demand gives competitive advantages to exporting firms." To this, we have heard some IO people say something to the effect that the share of the domestic sale must be trivial for most exporting firms based in small economies like Denmark or Switzerland. The result here should explain why such a criticism is unwarranted. Even if the Swiss domestic market might be small relative to the EU market in every sector, Swiss domestic markets have to be larger in some sectors relatively to other sectors, when compared to the EU, as long as their demand composition differs from the EU. And that is what determines the patterns of comparative advantage in a general equilibrium model of the home market effect.

terms of the standard-of-living. This is simple when the two countries are in equal size, $L^1 = L^2 = L$. In this case, $\omega$ = 1 so that $x_\rho^k = (1+\rho)x_0^k = (1+\rho)(h^k)^\sigma N^k = (1+\rho)(h^k)^{\sigma-1} L$, and hence , $x_\rho^1 / x_\rho^2 = (h^1/h^2)^{\sigma-1} = (w^1 h^1 / w^2 h^2)^{\sigma-1}$ . Thus, the country with higher per capita labor endowment has higher standard-of-living. This country also has higher per capita income.

Generally, the condition under which Country 1 becomes the net-exporter of the lower income elastic goods and Country 2 becomes the net-exporter of the higher income elastic goods, $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$ or $x_\rho^1 < x_\rho^2$ can be written as:

$$\frac{1-\rho(\omega)^{-\sigma}}{1-\rho(\omega)^{\sigma}} > \frac{x_0^1}{x_0^2} = \left(\frac{h^1}{h^2}\right)^\sigma \frac{N^1}{N^2} = \left(\frac{h^1}{h^2}\right)^{\sigma-1} \frac{L^1}{L^2} \Leftrightarrow (\omega)^{2\sigma-1}\left(\frac{h^1}{h^2}\right)^{\sigma-1} < 1,$$

which can be further rewritten as:

$$\text{(38)} \quad \frac{L^1}{L^2} = \Lambda(\omega;\rho) < \Lambda\left(\left(\frac{h^1}{h^2}\right)^{\frac{1-\sigma}{2\sigma-1}};\rho\right) \equiv \tilde{\Lambda}\left(\frac{h^1}{h^2};\rho\right).$$

To understand this condition, it would be useful to compare it with the conditions under which Country 1 is poorer under autarky, $U_0^1 = u(x_0^1) < U_0^2 = u(x_0^2)$ and Country 1 has lower per capita income, $w^1 h^1 < w^2 h^2$, which can be written as:

$$\frac{L^1}{L^2} < \left(\frac{h^1}{h^2}\right)^{1-\sigma},$$

$$\frac{L^1}{L^2} = \Lambda(\omega;\rho) < \Lambda\left(\left(\frac{h^1}{h^2}\right)^{-1};\rho\right) \equiv \overline{\Lambda}\left(\frac{h^1}{h^2};\rho\right),$$

respectively. Figure 3 illustrates these conditions. The black curve depicts the graph of $L^1/L^2 = \tilde{\Lambda}(h^1/h^2;\rho)$ on which $U_\rho^1 = u(x_\rho^1) = U_\rho^2 = u(x_\rho^2)$ holds. It is downward-sloping, and $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$ holds below and to the left of this curve, and $U_\rho^1 = u(x_\rho^1) > U_\rho^2 = u(x_\rho^2)$ holds above and to the right of this curve. The red curve depicts the graph of $L^1/L^2 = (h^1/h^2)^{1-\sigma}$, on which $U_0^1 = u(x_0^1) = U_0^2 = u(x_0^2)$ holds. It is also downward-sloping and $U_0^1 = u(x_0^1) <$ $U_0^2 = u(x_0^2)$ holds below and to the left of this curve, and $U_0^1 = u(x_0^1) > U_0^2 = u(x_0^2)$ holds above and to the right of this curve. The blue curve depicts the graph of $L^1/L^2 = \overline{\Lambda}\left(h^1/h^2;\rho\right)$, on which $w^1h^1 = w^2h^2$ holds. It is also downward-sloping and $w^1h^1 < w^2h^2$ holds below and to the left of this curve, and $w^1h^1 > w^2h^2$ holds above and to the right of this curve. It is also easy to verify that $\overline{\Lambda}(1;\rho)$ = $\tilde{\Lambda}(1;\rho)$ = 1 and

$$\overline{\Lambda}\left(\frac{h^1}{h^2};\rho\right) < \tilde{\Lambda}\left(\frac{h^1}{h^2};\rho\right) < \left(\frac{h^1}{h^2}\right)^{1-\sigma} < 1 \qquad \text{for } \frac{h^1}{h^2} > 1;$$

$$\overline{\Lambda}\left(\frac{h^1}{h^2};\rho\right) > \tilde{\Lambda}\left(\frac{h^1}{h^2};\rho\right) > \left(\frac{h^1}{h^2}\right)^{1-\sigma} > 1 \qquad \text{for } \frac{h^1}{h^2} < 1,$$

as shown in Figure 3.

For $L^1/L^2$ = 1, all three curves intersect at $h^1/h^2$ = 1. Hence, $h^1/h^2$ < 1 implies $U_0^1 < U_0^2$, $U_\rho^1 < U_\rho^2$ and $w^1h^1 < w^2h^2$, while $h^1/h^2$ > 1 implies $U_0^1 > U_0^2$, $U_\rho^1 > U_\rho^2$ and $w^1h^1 > w^2h^2$. Thus, when the two countries are equal in size, comparing per capita labor endowment alone can determine which country becomes richer, as already pointed out. When the two countries are unequal in size, these three conditions diverge. To see this, consider the case of $h^1/h^2$ > 1. For $L^1/L^2 > \left(h^1/h^2\right)^{1-\sigma}$, $U_0^1 > U_0^2$, $U_\rho^1 > U_\rho^2$ and $w^1h^1 > w^2h^2$. Thus, when the country with higher per capita labor endowment is not too smaller or larger in size, it has higher standard-of-living both under autarky and under trade, and it becomes the net exporter of higher income elastic goods. It also has higher per capita income. For $L^1/L^2 < \left(h^1/h^2\right)^{1-\sigma} < 1$, however, the country with higher per capita labor endowment has lower standard-of-living in autarky. When the condition (38) holds, this country has lower standard-of-living and is the net-exporter of the lower income elastic goods. Notice that (38) is more stringent than $L^1/L^2 < \left(h^1/h^2\right)^{1-\sigma}$ < 1. In other words, for $\tilde{\Lambda}\left(h^1/h^2;\rho\right) < L^1/L^2 < \left(h^1/h^2\right)^{1-\sigma}$ < 1, the standard-of-living in this country is lower in autarky but higher under trade, because trade reduces this country's disadvantage of being smaller. Notice also that the condition (38) is less stringent than $L^1/L^2$ < $\overline{\Lambda}\left(h^1/h^2;\rho\right)$ < 1, the condition under which its per capita income becomes smaller. In other words, for $\overline{\Lambda}\left(h^1/h^2;\rho\right)$ < $L^1/L^2$ < $\tilde{\Lambda}\left(h^1/h^2;\rho\right)$ < 1, the standard-of-living in this country is lower even when its per capita income is still higher in this country. This can occur because this country benefits less from the variety effect due to its smaller size.

### 2.5 Comparative Statics

Having characterized the patterns of trade, we now turn to comparative static exercises.

#### 2.5.1 Uniform Productivity Improvement

First, consider the effects of a uniform productivity improvement. That is, labor productivity goes up at the same rate in all the activities in both countries. This can be captured by $\partial\log(h^1) = \partial\log(h^2) \equiv \partial\log(h) > 0$. This keeps $h^1/h^2$ and $L^1/L^2$ unchanged, with $\partial\log(L^1)$ = $\partial\log(L^2)$ = $\partial\log(h) > 0$. Therefore, $\omega = w^1/w^2$ is also unchanged, and so are $x_0^1/x_0^2$ and $x_\rho^1/x_\rho^2$, with $\partial\log(x_0^1) = \partial\log(x_0^2) = \partial\log(x_\rho^1) = \partial\log(x_\rho^2) = \sigma\partial\log(h) > 0$.

With $\partial\log(x_\rho^1) = \partial\log(x_\rho^2) > 0$, both $U_\rho^1 = u(x_\rho^1)$ and $U_\rho^2 = u(x_\rho^2)$ go up. With their standard-of-living improving, the households in both countries shift their expenditure shares towards higher-indexed sectors in the sense of both MLR and FSD. This can be seen from eq.(33) and applying Lemma 1 for $\hat{g}(s, x_\rho^k) = \left(\beta_s\left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)}$.

Even though $x_\rho^1$ and $x_\rho^2$ goes up at the same rate to keep $x_\rho^1/x_\rho^2$ unchanged, the standard-of-living in the two countries do not go up at the same rate. To see this,

$$\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(h)} = \frac{\partial\log(u(x_\rho^1)) - \partial\log(u(x_\rho^2))}{\partial\log(h)} = \sigma\left(\zeta\left(x_\rho^1\right) - \zeta\left(x_\rho^2\right)\right).$$

Hence, from Lemma 2-ii),

$$\text{(39)} \qquad \operatorname{sgn}\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(h)} = \operatorname{sgn}(\eta-1)\operatorname{sgn}(x_\rho^1 - x_\rho^2).$$

Thus, the standard-of-living goes up at a faster rate in the Richer country if $\eta > 1$ and in the Poorer country if $\eta < 1$. In words, welfare gaps widen (narrow) if the goods produced in different sectors are substitutes (complements) .

To see how the patterns of trade change, log-differentiate (34) to yield,

$$\frac{\partial \log(m_s^1/m_s^2)}{\partial \log(h)} = (\varepsilon(s)-\eta)\left(\frac{\sigma-1}{\sigma-\eta}\right)\frac{\partial \log(U_\rho^1/U_\rho^2)}{\partial \log(h)},$$

and then use (39) to obtain

$$\text{(40)} \qquad \operatorname{sgn}\frac{\partial \log(m_s^1/m_s^2)}{\partial \log(h)} = \operatorname{sgn}\left(\varepsilon(s)-\eta\right)\operatorname{sgn}(\eta-1)\operatorname{sgn}(x_\rho^1 - x_\rho^2) = \operatorname{sgn}(x_\rho^2 - x_\rho^1).$$

from Lemma 2-ii) and by recalling the parameter restriction, $\left(\varepsilon(s)-\eta\right)/(1-\eta) > 0$, that ensures the monotonicity of the upper-tier utility function.

Figure 4 illustrates this for $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$. In this case, the downward-sloping curve, $m_s^1/m_s^2$, shifts up, which causes the cutoff sector, $s_c$, to move up. As a result, the Rich's trade balances switch from net surpluses to net deficits in some middle sectors.[16] The intuition behind this result is easy to grasp. As the standard-of-living improves in both countries, the households everywhere shift their expenditure shares towards the higher-indexed sectors. In response, both countries reallocate their resources towards higher-indexed sectors. In other words, the relative weights of higher-indexed sectors, in which the Rich runs surpluses, go up and the relative weights of lower-indexed sectors, in which the Poor runs surpluses, go down. This means that, in order to keep the overall trade account between the two countries in balance, the Rich's sectoral trade account must deteriorate in each sector. This is why the Rich switches from being a net exporter to being a net importer in some middle sectors.

### 2.5.2 Globalization Without Terms of Trade Change

Next, consider the effects of globalization, captured by a trade cost reduction, or a higher $\rho = (\tau)^{1-\sigma}$. First, let us look at the case where the two countries are in equal size: $L^1 = L^2 = L$. In this case, the factor price is always equalized, $w^1 = w^2 = w$, or $\omega = 1$, independent of $\rho$, so that $x_\rho^k = (1+\rho)x_0^k = (1+\rho)\left(h^k\right)^\sigma N^k = (1+\rho)\left(h^k\right)^{\sigma-1}L$, and hence , $x_\rho^1/x_\rho^2 = x_0^1/x_0^2 = \left(h^1/h^2\right)^{\sigma-1}$, as noted earlier. That is, the country with higher standard-of-

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[16] For $U_\rho^1 = u(x_\rho^1) > U_\rho^2 = u(x_\rho^2)$, the *upward*-sloping curve, $m_s^1/m_s^2$, shifts *down*, which also leads to the cutoff sector, $s_c$, to move up. Either way, the Rich's trade balances must switch from net surpluses to net deficits in some middle sectors.

living is the one with per capita labor endowment and with higher per capita income.[17] Hence, the country whose households have higher per capita labor endowment is always a net exporter in higher-indexed sectors and a net importer in lower-indexed sectors, precisely because they have relatively larger expenditure shares in higher-indexed sectors, which causes disproportionately larger shares of firms to enter higher-indexed sectors due to the home market effect.

Furthermore, in this case, the effects of globalization, a higher $\rho$, can be seen only by looking at $x_\rho^k = (1+\rho)x_0^k = (1+\rho)\left(h^k\right)^{\sigma-1}L$. Indeed, without causing any terms-of-trade change, the effects of a higher $\rho$ is isomorphic to a uniform productivity improvement, with $\partial\log(1+\rho) > 0$ equivalent to $(\sigma-1)\partial\log(h^1) = (\sigma-1)\partial\log(h^2) \equiv (\sigma-1)\partial\log(h) > 0$. Hence, by going through the analysis as done in the previous subsection, one can show that, in both countries, the standard-of-living improves (a higher $U_\rho^k$), and the households shift their expenditure shares towards higher-indexed sectors both in the sense of MLR and FSD. Furthermore, one can show:

$$\mathrm{sgn}\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(1+\rho)} = \mathrm{sgn}(\eta-1)\ \mathrm{sgn}(x_\rho^1 - x_\rho^2).$$

so that globalization causes the welfare gap between the Rich and the Poor to widen (narrow) if the goods produced in different sectors are substitutes (complements). One can also show:

$$\mathrm{sgn}\frac{\partial\log(m_s^1/m_s^2)}{\partial\log(1+\rho)} = \mathrm{sgn}(x_\rho^2 - x_\rho^1),$$

so that the cutoff sector moves up (see Figure 4). Thus, the Rich country, the country whose households have higher per capita labor endowment, switches from a net exporter to a net importer in some middle sectors, generating something akin to product cycles without any technology diffusion from the Rich to the Poor.

In summary, when the two countries are equal in size, globalization causes no terms-of-trade change. And without any terms-of-trade change, globalization is isomorphic to the effects of uniform productivity improvement, because it allows the households everywhere to have better assess to the varieties produced abroad,

[17] In this case, the two countries have the same aggregate GDP, but differ in GDP per capita.

### 2.5.3 Globalization With Terms of Trade Change: Possibility of Leapfrogging and Reversal of the Patterns of Trade

When the two countries are unequal in size, the factor price is lower in the smaller country, due to the disadvantage of being smaller in the presence of aggregate increasing returns. The larger the trade cost, the greater this disadvantage. Globalization reduces this disadvantage for the smaller country, thereby causing the terms of trade change in favor of the smaller country, as shown in Figure 1.

When the smaller country has lower per capita labor endowment, this country always has lower standard-of-living, regardless of the trade cost. However, when the smaller country has higher per capita labor endowment, it is possible that this country has lower standard-of-living at a high trade cost but higher standard-of-living at a low trade cost. This possibility is illustrated in Figure 5, which reproduces some parts of Figure 3. Below and to the left of the red curve, Country 1 has lower standard-of-living than Country 2 in autarky. Below and to the left of the black curve, Country 1 has lower standard-of-living than Country 2 under trade. Globalization, a higher ρ, rotates the black curve clockwise, as indicated by the arrows. As ρ approaches zero, the black curve converges to the red curve, which is invariant to the trade cost. As ρ approaches one, the black curve converges to the vertical line, $h^1/h^2 = 1$. Now, consider the case where Country 1 has higher per capita labor endowment, i.e., $h^1/h^2 > 1$ but it is sufficiently smaller so that $L^1/L^2 < (h^1/h^2)^{1-\sigma} < 1$. Thus, we consider the point, $(h^1/h^2, L^1/L^2)$, located to the right of the vertical line, $h^1/h^2 = 1$ and below the red curve. Then, with a sufficiently small ρ, the black curve passes above and to the right of this point, which means that Country 1 has lower standard-of-living. With a sufficiently large ρ, the black curve passes below and to the left of this point, which means the Country 1 has higher standard-of-living. Thus, closer to autarky, Country 1 is poorer due to its disadvantage of being smaller in the presence of aggregate increasing returns, hence running surpluses in lower-indexed sectors. Globalization reduces the disadvantage of being smaller, causing a factor price convergence, which makes it richer, hence running surpluses in higher-indexed sectors. This result thus suggests the possibility that some relatively small countries with relatively highly educated labor forces, which might initially have lower standard-of-living due to their remote locations and export relatively low income elastic goods, might benefit more from globalization and emerge as exporters of relatively high income elastic goods.

## 3. The Home Market Effect with Exogenous Taste Variations: A Comparison

In the model developed in the previous section, the cross-country differences in the demand composition behind the home market effect come from the nonhomotheticity of preferences. However, nonhomotheticity are not responsible for all the results. Some of them are due to the home market effect in general, regardless of the sources of the differences in the demand composition. To clarify which results are driven by the nonhomotheticity, let us modify the previous model, in which the upper-tier utility function is now given by the standard homothetic CES preferences, where the households in the two countries attach different weights on sectors. More specifically, the upper-level utility function, (3), is now replaced by:

(3') $$U^k\left(\tilde{C}_s^k, s\in[0,1]\right) \equiv \left[\int_0^1 (\beta_s^k)^{\frac{1}{\eta}}\left(\tilde{C}_s^k\right)^{1-\frac{1}{\eta}} ds\right]^{\frac{\eta}{\eta-1}}, \quad \beta_s^k > 0, \text{ normalized to } \int_0^1 (\beta_s^k)^{\frac{\sigma-1}{\sigma-\eta}} ds = 1.$$

Notice that, $\beta_s^k$, the weight on the Dixit-Stiglitz aggregator $\tilde{C}_s^k$, now depends on *k*. Furthermore, let us assume that the sectors can be ordered such that $\beta_s^1/\beta_s^2$ is strictly decreasing in *s*. That is, the households in country 1 put relatively more weights on the lower-indexed goods. All other features of the model are left unchanged. The Krugman (1980) model can be viewed as a limit case of the model in this section, where $\eta = 1$, $L^1 = L^2$, and $\beta_s^1/\beta_s^2 = \gamma > 1$ for $0 \le s < 1/2$; $\beta_s^1/\beta_s^2 = 1/\gamma < 1$ for $1/2 < s \le 1$.

By going through the analysis as in the previous section, one can show that eq. (30), which determines the terms of trade as a function of the relative country size; eqs. (35) and (36), which show how firm distributions are related to the market size distributions; and eq. (37), the expression for the net trade balances in each sector are not affected. The expressions for the standard-of-living, $U_\rho^k = u\left(x_\rho^k\right)$, as well as the definition of $x_\rho^k$ given in eq. (32), are also unaffected, except that the increasing function, $u(\bullet)$, defined in (26), is now simplified to:

(26') $$u(x) \equiv (x)^{\frac{1}{\sigma-1}}.$$

What changes significantly is the expressions of the market size distributions, eqs.(33) and (34) . They now become,

(33') $$m_s^k = \left(\beta_s^k\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)},$$

and

(34') $$\frac{m_s^1}{m_s^2} = \left(\frac{\beta_s^1}{\beta_s^2}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)},$$

which is strictly decreasing in *s*. This means that Figures 1, 2, and 3 remain valid in this case as well. In particular, there is a unique cutoff sector, $s_c$, such that country 1 is the net exporter in the sectors below the cutoff, while country 2 is the net exporter in the sectors above it, as shown in Figure 2.

Unlike (34), however, eq. (34') shows that the cross-country differences in the demand composition in this model depend entirely on the exogenous preference parameters. In particular, it is independent of $x_\rho^k$, and hence independent of $\rho$, $\omega$, $h^k$, $N^k$, and $L^k$. Thus, the cutoff sector, $s_c$, is also independent of these factors. Thus, neither a uniform productivity improvement nor globalization can shift the sectoral patterns of trade. In other words, the comparative static results shown in Figure 4 are entirely due to the nonhomotheticity of preferences. Also from (26'), the welfare gap between the two countries has much simpler expression,

$$\frac{U_\rho^1}{U_\rho^2} = \left(\frac{x_\rho^1}{x_\rho^2}\right)^{\frac{1}{\sigma-1}},$$

which means that the parameter changes that keep $x_\rho^1 / x_\rho^2$ unaffected, such as a uniform productivity change or globalization when the two countries are of equal size, do not affect the welfare gap. The possibility of globalization causing a leapfrogging when the smaller country has higher per capita labor endowment, illustrated in Figure 5, remains valid, even when the cross-country differences in the demand composition is exogenous. However, the result that such a leapfrogging also causes a reversal of the patterns of trade is entirely due to the nonhomotheticity of preferences, and cannot happen when the differences are due to the exogenous variations in taste.

## 4. Adding An Outside Goods Sector

Up to this point, we have followed Krugman (1980) to consider the case where all the goods are differentiated goods with iceberg trade costs, produced in Dixit-Stiglitz monopolistic competitive sectors. In another well-known model of the home market effect, Helpman and Krugman (1985, Ch.10), there are two sectors, only one of which is a Dixit-Stiglitz monopolistic competitive sector. The other sector is competitive and produces the homogeneous good that can be traded at zero cost, which pins down the terms of trade between the two countries. In this setup, they have shown a different form of the home market effect, i.e., the larger country becomes a net-exporter of the differentiated goods sector and a net-importer of the homogeneous good.

In this section, we add an outside goods sector into our framework. In doing so, our framework becomes an extension of the Helpman and Krugman (1985) model, where their unique differentiated goods sector is divided into a continuum of differentiated goods sectors with differential income elasticities. This also brings our framework closer to the Fajgelbaum-Grossman-Helpman model, which also pins down the terms of trade by the numeraire sector.

More specifically, we modify our framework of section 2 as follows. First, in addition to a continuum of monopolistic competitive sectors, there is an outside goods sector, which competitively produces the homogeneous good with constant returns to scale technology that converts one unit of labor into one unit of output. Furthermore, this good can be traded at zero cost, and hence sold at the same price in both countries. This allows us to choose the homogeneous good as the numeraire. Then, the household budget constraint is now written as, instead of (1),

$$(1') \qquad \tilde{C}_o^k + \int_0^1 \left[ \int_{\Omega_s} p_s^k(\nu) c_s^k(\nu) d\nu \right] ds \leq E^k = w^k h^k,$$

where $\tilde{C}_O^k$ denotes the numeraire consumption of household-*k*. Second, the preferences of each household now have a three-tier structure. The lower-tier aggregates all differentiated goods within each sector with a Dixit-Stiglitz aggregator, $\tilde{C}_s^k$, given in (2). The middle-tier aggregates a continuum of Dixit-Stiglitz aggregators with $\tilde{U}^k = U\left(\tilde{C}_s^k, s \in [0,1]\right)$, implicitly additively separable CES, given by (3). Then, the upper-tier defines the preferences over $\tilde{C}_O^k$ and $\tilde{U}^k = U\left(\tilde{C}_s^k, s \in [0,1]\right)$ by

$$\tilde{W}^k = (1-\alpha)\log \tilde{C}_O^k + \alpha \log(\tilde{U}^k), \text{ with } \alpha \in (0,1).$$

The structure is kept otherwise unchanged.

For a sufficiently small $\alpha > 0$, the numeraire sectors in both countries employ some labor, $L^j - \int_0^1 V_s^j ds > 0$. This pins down the wage rates of both countries at $w^j = 1$. This fixes the (factor) terms of trade at $\omega = 1$, independently of $\rho$. Furthermore, each household earns $h^k$ and spends $E^k = \alpha h^k$ on differentiated goods. The equilibrium conditions are otherwise unaffected. The equilibrium can be solved by following the steps analogous to those in section 2.

Under autarky, the household in each country achieves $W_0^k = (1-\alpha)\log((1-\alpha)h^k) + \alpha \log\left(U_0^k\right)$, where

(25') $\quad U_0^k = u(x_0^k), \quad \text{with } x_0^k \equiv \left(\alpha h^k\right)^\sigma N^k = \alpha\left(\alpha h^k\right)^{\sigma-1} L^k.$

Here $u(\bullet)$ is again defined by (26). However, notice that the definition of $x_0^k$ is now modified to $x_0^k \equiv \left(\alpha h^k\right)^\sigma N^k$, from $x_0^k \equiv \left(h^k\right)^\sigma N^k$, because each household spends only the fraction of their income, $\alpha h^k$, on differentiated goods. With this new definition of $x_0^k$, the distributions of the firms and market sizes across sectors have the same expressions with (27):

$$f_s^k = m_s^k = \frac{\left(\beta_s \left(u(x_0^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}}}{\left(x_0^k\right)^{\frac{1-\eta}{\sigma-\eta}}} = \frac{\left(\beta_s \left(u(x_0^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}}}{\int_0^1 \left(\beta_t \left(u(x_0^k)\right)^{(\varepsilon(t)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}} dt}.$$

Under trade equilibrium, $W_\rho^k = (1-\alpha)\log((1-\alpha)h^k) + \alpha \log\left(U_\rho^k\right)$, and

$$U_\rho^k = u(x_\rho^k), \quad \text{with } x_\rho^k \equiv (1+\rho)\left(\alpha h^k\right)^\sigma N^k = (1+\rho)x_0^k,$$

where the definition of $x_\rho^k$ reflects the fact that the terms of trade are now pinned down at $\omega = 1$. With this new definition of $x_\rho^k$, the market size distributions and their ratio have the same expressions with (33) and (34):

$$m_s^k = \frac{\left(\beta_s\left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}}}{\left(x_\rho^k\right)^{\frac{1-\eta}{\sigma-\eta}}} = \frac{\left(\beta_s\left(u(x_\rho^k)\right)^{(\varepsilon(s)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}}}{\int_0^1 \left(\beta_t\left(u(x_\rho^k)\right)^{(\varepsilon(t)-\eta)}\right)^{\frac{\sigma-1}{\sigma-\eta}} dt};$$

$$\frac{m_s^1}{m_s^2} = \left(\frac{x_\rho^1}{x_\rho^2}\right)^{\frac{\eta-1}{\sigma-\eta}} \left(\frac{u(x_\rho^1)}{u(x_\rho^2)}\right)^{(\varepsilon(s)-\eta)\left(\frac{\sigma-1}{\sigma-\eta}\right)}.$$

Note that $m_s^1/m_s^2$ is again strictly decreasing in s if and only if $x_\rho^1 < x_\rho^2$, which is now equivalent to $x_0^1 < x_0^2 \Leftrightarrow L^1/L^2 < \left(h^1/h^2\right)^{1-\sigma}$ because $\omega = 1$.

Some labor are now employed in the numeraire sector, so that the labor market clearing condition is no longer given by $\int_0^1 V_s^j ds = L^j$, and hence the share of sector-*s* in the firm distribution is no longer equal to $V_s^j / L^j$. Instead, by solving the free entry condition in each sector and in each country under the condition, $V_s^j > 0$, we obtain the measure of firms (and varieties produced) as follows:

$$V_s^1 = \frac{\alpha(m_s^1 L^1 - \rho m_s^2 L^2)}{1-\rho} > 0; \quad V_s^2 = \frac{\alpha(m_s^2 L^2 - \rho m_s^2 L^2)}{1-\rho} > 0,$$

which requires $\rho < m_s^1 L^1 / m_s^2 L^2 < 1/\rho$. From these expressions and $\int_0^1 m_s^k ds = 1$, we obtain,

$$\int_0^1 V_s^1 ds = \frac{\alpha(L^1 - \rho L^2)}{1-\rho} < L^1; \qquad \int_0^1 V_s^2 ds = \frac{\alpha(L^2 - \rho L^1)}{1-\rho} < L^2$$

from which the condition for $\alpha > 0$ that ensures a positive employment in the numeraire sector in each country is given by $\alpha < (1-\rho) Min\left\langle L^1/(L^1 - \rho L^2), L^2/(L^2 - \rho L^1)\right\rangle$. Using the above expressions, the firm distributions are

$$f_s^1 \equiv \frac{V_s^1}{\int_0^1 V_t^1 dt} = \frac{m_s^1 L^1 - \rho m_s^2 L^2}{L^1 - \rho L^2} > 0; \qquad f_s^2 \equiv \frac{V_s^2}{\int_0^1 V_t^2 dt} = \frac{m_s^2 L^2 - \rho m_s^1 L^1}{L^2 - \rho L^1} > 0,$$

so that

$$\frac{f_s^1}{f_s^2}=\left(\frac{L^2-\rho L^1}{L^1-\rho L^2}\right)\left(\frac{(m_s^1/m_s^2)L^1-\rho L^2}{L^2-\rho(m_s^1/m_s^2)L^1}\right),$$

which is strictly increasing in $m_s^1/m_s^2$ and satisfies $f_s^1/f_s^2>m_s^1/m_s^2>1$, $f_s^1/f_s^2=m_s^1/m_s^2=1$, or $f_s^1/f_s^2<m_s^1/m_s^2<1$.

The net trade balances in each sector, $NX_s^1=-NX_s^2\equiv V_s^1\rho b_s^2(w^1)^{1-\sigma}-V_s^2\rho b_s^1(w^2)^{1-\sigma}$, can now be rewritten as:

$$NX_s^1=-NX_s^2=\frac{\rho}{1+\rho}(V_s^1-V_s^2)=\frac{\alpha\rho}{1-\rho}(m_s^1L^1-m_s^2L^2)$$

Notice that its sign is no longer the same with the sign of $m_s^1-m_s^2$. Instead, it is the same with the sign of $m_s^1L^1-m_s^2L^2$. Thus, whether the country becomes a net-exporter or a net-importer is determined by the cross-country difference in the *absolute* market size in each sector, *not* the cross-country difference in the market size distributions, as was the case in section 2. This is because the active numeraire sectors in both countries, which pins down their wage rates and the terms of trade between the two, effectively turns this model into a partial equilibrium model. Furthermore, the trade account across all the differentiated goods sectors is given by:

$$\int_0^1 NX_s^1 ds=-\int_0^1 NX_s^2 ds=\frac{\alpha\rho}{1-\rho}(L^1-L^2).$$

Thus, instead of having a higher factor price, the larger country runs an overall surplus in the differentiated goods sectors, with a deficit in the numeraire good sector, which effectively reproduces the main result of the Helpman and Krugman (1985) model, which has one differentiated goods sector.

Figure 6 illustrates the patterns of trade for the case of $x_\rho^1<x_\rho^2$ , which is now equivalent to $x_0^1<x_0^2$ or to $L^1/L^2<\left(h^1/h^2\right)^{1-\sigma}$ due to $\omega=1$. For this case, $m_s^1/m_s^2$ is strictly decreasing in $s$. If $L^1$ and $L^2$ are not too different, there is a unique cutoff sector, $s_c\in(0,1)$ such that

$$NX_s^1=-NX_s^2=\frac{\alpha\rho L}{1-\rho}(m_s^1L^1-m_s^2L^2)>0 \qquad \text{for } s<s_c;$$

$$NX_s^1 = -NX_s^2 = \frac{\alpha\rho L}{1-\rho}(m_s^1 L^1 - m_s^2 L^2) < 0 \qquad \text{for } s > s_c.$$

However, if $L^1$ and $L^2$ are too different, the larger country, not necessarily the richer one, runs a surplus in all the differential sectors, with a deficit in the numeraire sector.

Assuming that the unique cutoff sector $s_c$ exists in the interior, the effects of a uniform productivity improvement are identical with those shown in section 2. Furthermore, without causing the terms of trade change, the effects of globalization are isomorphic to those of uniform productivity improvement, as can be seen from $x_\rho^k \equiv (1+\rho)\left(\alpha h^k\right)^\sigma N^k$. As productivity improves or trade costs fall, the world becomes richer. In response, the households in both countries shift their spending towards the higher-indexed in the sense that the density functions of the market size distributions before and after satisfy the MLR property and their cumulative distribution functions satisfy the FSD. Furthermore, one can show, following the same steps in Section 2.5,

$$\operatorname{sgn}\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(h)} = \operatorname{sgn}\frac{\partial\log(U_\rho^1/U_\rho^2)}{\partial\log(1+\rho)} = \operatorname{sgn}(\eta-1)\operatorname{sgn}(x_\rho^1 - x_\rho^2),$$

and

$$\operatorname{sgn}\frac{\partial\log(m_s^1/m_s^2)}{\partial\log(h)} = \operatorname{sgn}\frac{\partial\log(m_s^1/m_s^2)}{\partial\log(1+\rho)} = \operatorname{sgn}(x_\rho^2 - x_\rho^1).$$

Thus, these results cause the welfare gap between the rich and the poor to widen (narrow) if different sectors produce substitutes (complements). With these changes, the cutoff sector moves up, as shown in Figure 6, causing something akin to product cycles without any technology diffusions from the rich to the poor.

To summarize the results in this section, the effects of uniform productivity improvement are identical with those in section 2. Unlike in section 2, globalization cannot change the terms of trade even when the country sizes are different, because it is pinned down by the numeraire sector. Without the terms-of-trade change, the effects of globalization are isomorphic to those of uniform productivity improvements and as well as to those of globalization obtained for the case of the two equal size countries in section 2. However, without the terms of trade change, leapfrogging and a reversal of patterns of trade are no longer possible even if the two countries are unequal in size.

## 5. Concluding Remarks

Empirically, rich countries tend to export high income elastic goods and import low income elastic goods, while poor countries tend to export low income elastic goods and import high income elastic goods. Virtually all existing models of trade with nonhomothetic preferences *assume* that the rich (poor) countries *happen to have* comparative advantages in high (low) income elastic goods. With their sources of comparative advantage being unrelated to their demand compositions, these models suggest that rich countries export high income elastic goods *despite* they demand relatively more high income elastic goods. This paper offered our attempt to explain *why* the rich (poor) countries have comparative advantages in high (low) income elastic goods by building a theoretical framework, which incorporates nonhomothetic preferences into the standard general equilibrium models of trade with the home market effect. Under nonhomothetic preferences, the demand compositions in richer countries are more skewed towards the goods with higher income elasticity than those in poorer countries. In the presence of economies of scale in production and positive but non-prohibitive trade costs, such cross-country differences in the demand composition become sources of comparative advantage through the home market effect. In other words, rich countries export high income elastic goods *because* they demand relatively more high income elastic goods.

Although the intuition is simple, an attempt to capture it in a theoretical framework that is flexible enough to allow for a variety of comparative static exercises has been a challenge, because general equilibrium models with imperfect competition, economies of scale, positive trade costs, an arbitrary number of sectors, and nonhomothetic preferences could become quickly intractable. We have managed to keep it tractable by using nonhomothetic preferences that are implicitly additive separable CES, which implies that the weighs attached to different sectors in preferences satisfy log-supermodularity, which facilitate monotone comparative statics. It seems that this form of nonhomothetic preferences should find a wide range of applications.

## <u>Appendix</u>: Two Lemmas

This appendix offers two lemmas, which are used repeatedly in the analysis.

**Lemma 1:** For a positive value function, $\hat{g}(\bullet;x):[0,1] \rightarrow R_+$, with a parameter $x$, define a density function on [0,1] by $g(s;x) \equiv \dfrac{\hat{g}(s;x)}{\int_0^1 \hat{g}(t;x)dt}$, and its distribution function by $G(s;x) \equiv \int_0^s g(t;x)dt = \dfrac{\int_0^s \hat{g}(t;x)dt}{\int_0^1 \hat{g}(t;x)dt}$. If $\hat{g}(s;x)$ is *log-supermodular* in $s$ and $x$, *i.e.* $\dfrac{\partial^2 \log \hat{g}(s;x)}{\partial s \partial x} > 0$,

i) **Monotone Likelihood Ratio (MLR)**: $\dfrac{g(s;x_1)}{g(s;x_2)}$ is decreasing in $s$ for $x_1 < x_2$;

ii) **First-order Stochastic Dominance (FSD)**: $G(s;x)$ is decreasing in $x$.

**Proof:**[18]

i) With $\dfrac{g(s;x_1)}{g(s;x_2)} = \left[\dfrac{\int_0^1 \hat{g}(t;x_2)dt}{\int_0^1 \hat{g}(t;x_1)dt}\right]\left(\dfrac{\hat{g}(s;x_1)}{\hat{g}(s;x_2)}\right)$,

$$\frac{\partial}{\partial s}\log\left(\frac{g(s;x_1)}{g(s;x_2)}\right) = \frac{\partial}{\partial s}\log\left(\frac{\hat{g}(s;x_1)}{\hat{g}(s;x_2)}\right) = \frac{\partial \log \hat{g}(s;x_1)}{\partial s} - \frac{\partial \log \hat{g}(s;x_2)}{\partial s} = -\int_{x^1}^{x^2} \frac{\partial^2 \log \hat{g}(s;\xi)}{\partial s \partial x} d\xi < 0.$$

ii) Let $\psi(s) \equiv \dfrac{\int_0^s \hat{g}_x(t;x)dt}{\int_0^s \hat{g}(t;x)dt}$. Then, $\dfrac{1}{G}\dfrac{\partial G}{\partial x} = \psi(s) - \psi(1) = -\int_s^1 \psi'(t)dt < 0$, because

[18]The results in this lemma are not new. For example, they were used in Matsuyama (2013, 2014) without proof. Furthermore, ii) follows immediately from i). Indeed, they are special cases of more general properties of log-supermodularity known in the literature: see, e.g., Athey (2002) and Vives (1999; Ch.2.7). Nevertheless, we offer here a simpler and more direct (although less elegant) proof without the machinery of lattice theory under the differentiability assumption for the sake of the completeness.

$$\psi'(s)=\frac{\hat{g}_x(s;x)\int_0^s\hat{g}(t;x)dt-\hat{g}(s;x)\int_0^s\hat{g}_x(t;x)dt}{\left[\int_0^s\hat{g}(t;x)dt\right]^2}=\frac{\int_0^s\left[\hat{g}_x(s;x)\hat{g}(t;x)-\hat{g}(s;x)\hat{g}_x(t;x)\right]dt}{\left[\int_0^s\hat{g}(t;x)dt\right]^2}$$

$$=\frac{\int_0^s\left[\frac{\partial\ln\hat{g}(s;x)}{\partial x}-\frac{\partial\ln\hat{g}(t;x)}{\partial x}\right]\hat{g}(s;x)\hat{g}(t;x)dt}{\left[\int_0^s\hat{g}(t;x)dt\right]^2}=\frac{\int_0^s\left[\int_t^s\frac{\partial^2\ln\hat{g}(z;x)}{\partial x\partial z}dz\right]\hat{g}(s;x)\hat{g}(t;x)dt}{\left[\int_0^s\hat{g}(t;x)dt\right]^2}>0. \quad \textbf{Q.E.D.}$$

**Lemma 2:** For $\eta\neq 1$, define $u(x)$ implicitly by $x^{\frac{1-\eta}{\sigma-\eta}}\equiv\int_0^1\left[\beta_s(u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}}ds$. If $(\varepsilon(s)-\eta)/(1-\eta)>0$ and $\varepsilon(s)$

i) $u(x)$ is increasing in $x$

ii) $\zeta(x)\equiv\dfrac{xu'(x)}{u(x)}$ is decreasing in $x$ if $\eta<1$ and increasing in $x$ if $\eta>1$.

Proof: Differentiating the definition yields

$$\left(\frac{1-\eta}{\sigma-\eta}\right)x^{\frac{1-\eta}{\sigma-\eta}-1}=\int_0^1\left(\frac{\sigma-1}{\sigma-\eta}\right)\left[\beta_s(u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}-1}\beta_s(\varepsilon(s)-\eta)(u(x))^{(\varepsilon(s)-\eta-1)}u'(x)ds$$

$$=\left(\frac{u'(x)}{u(x)}\right)\int_0^1\left(\frac{\sigma-1}{\sigma-\eta}\right)\left[\beta_s(u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}}(\varepsilon(s)-\eta)ds$$

$$\frac{1}{\zeta(x)}=\left(\frac{\sigma-1}{x^{\frac{1-\eta}{\sigma-\eta}}}\right)\int_0^1\left[\beta_s(u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}}\left(\frac{\varepsilon(s)-\eta}{1-\eta}\right)ds\;=\;(\sigma-1)\int_0^1\left(\frac{\varepsilon(s)-\eta}{1-\eta}\right)dF(s;x)>0,$$

where $F(s;x)\equiv\dfrac{\int_0^s\left[\beta_t(u(x))^{(\varepsilon(t)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}}dt}{\int_0^1\left[\beta_t(u(x))^{(\varepsilon(t)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}}dt}$ is a cdf.

First, with $\zeta(x) \equiv \dfrac{xu'(x)}{u(x)} > 0$, $u(x)$ is increasing. Furthermore, it implies $\left[\beta_s(u(x))^{(\varepsilon(s)-\eta)}\right]^{\frac{\sigma-1}{\sigma-\eta}}$ is log-supermodular in s and x. Hence, from ii) of Lemma 1, $F(s;x^1) > F(s;x^2) \Leftrightarrow x^1 < x^2$.

If $\eta < 1$, $\dfrac{\varepsilon(s)-\eta}{1-\eta}$ is increasing in s, so that $\int_0^1 \left(\dfrac{\varepsilon(s)-\eta}{1-\eta}\right) dF(s;x)$ is increasing in x, hence $\zeta(x) \equiv \dfrac{xu'(x)}{u(x)}$ is decreasing in x. Likewise, if $\eta > 1$, $\dfrac{\varepsilon(s)-\eta}{1-\eta}$ is decreasing in s, so that $\int_0^1 \left(\dfrac{\varepsilon(s)-\eta}{1-\eta}\right) dF(s;x)$ is decreasing in x, hence $\zeta(x) \equiv \dfrac{xu'(x)}{u(x)}$ is increasing in x. **Q.E.D.**

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Figure 1: (Factoral) Terms of Trade Determination: $L^1/L^2 = \Lambda(\omega;\rho)$

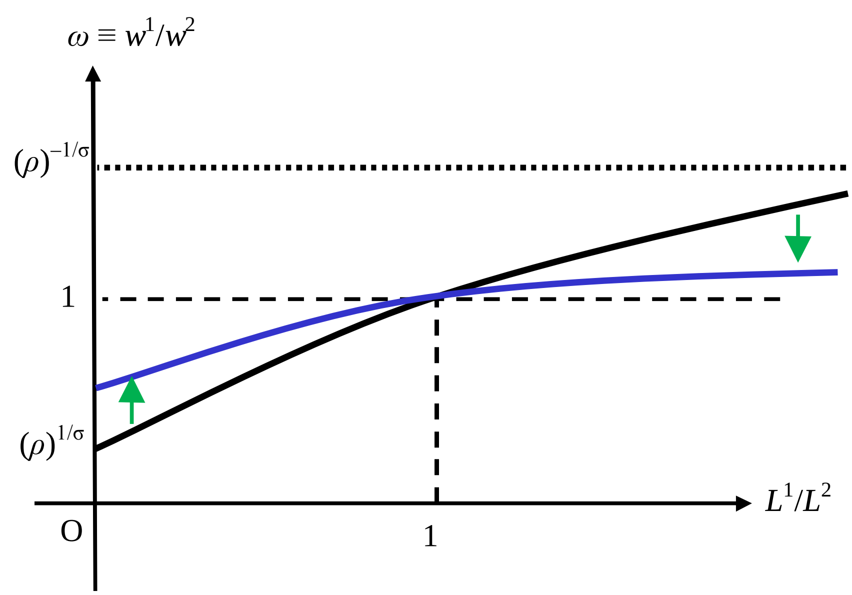


Figure 2: Home Market Effect and Patterns of Trade: for $U_\rho^1 = u(x_\rho^1) < U_\rho^2 = u(x_\rho^2)$

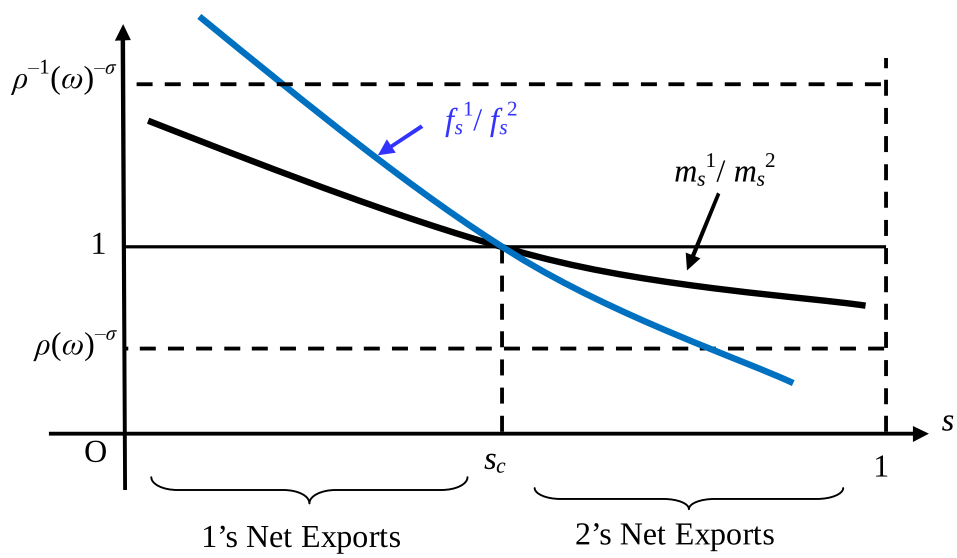

Figure 3; Ranking the Countries

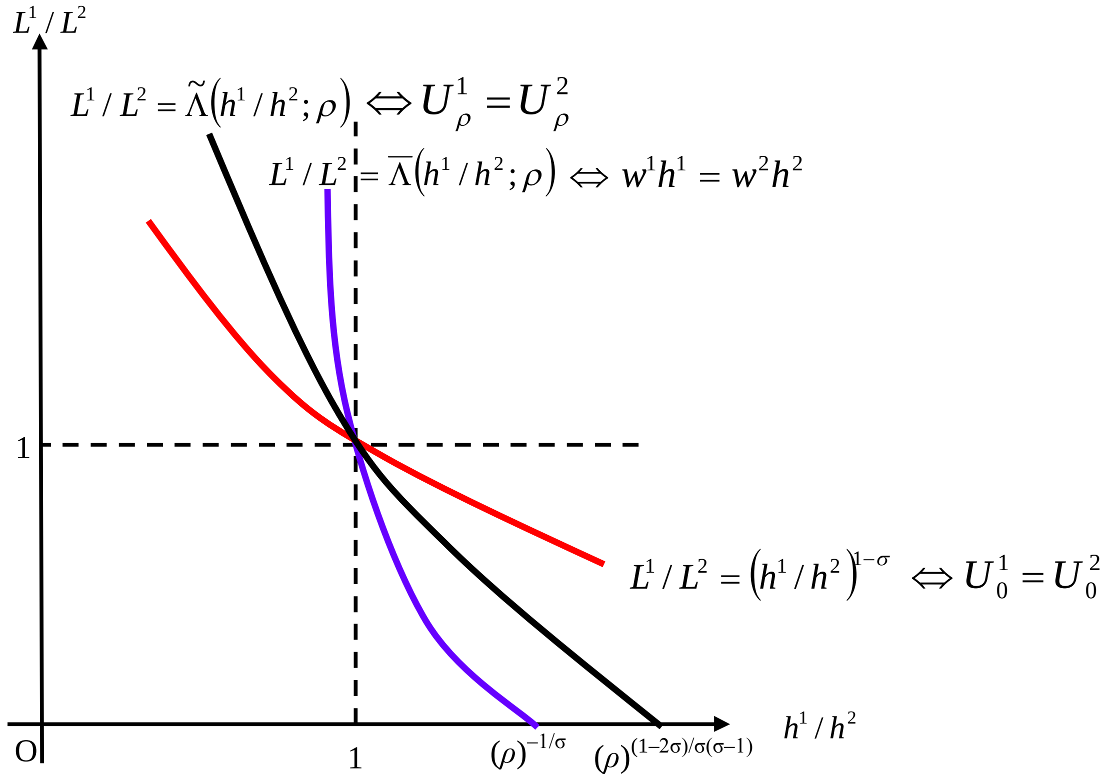


Figure 4: The Effect of An Uniform Productivity Improvement and Globalization (when the two countries are in equal size)

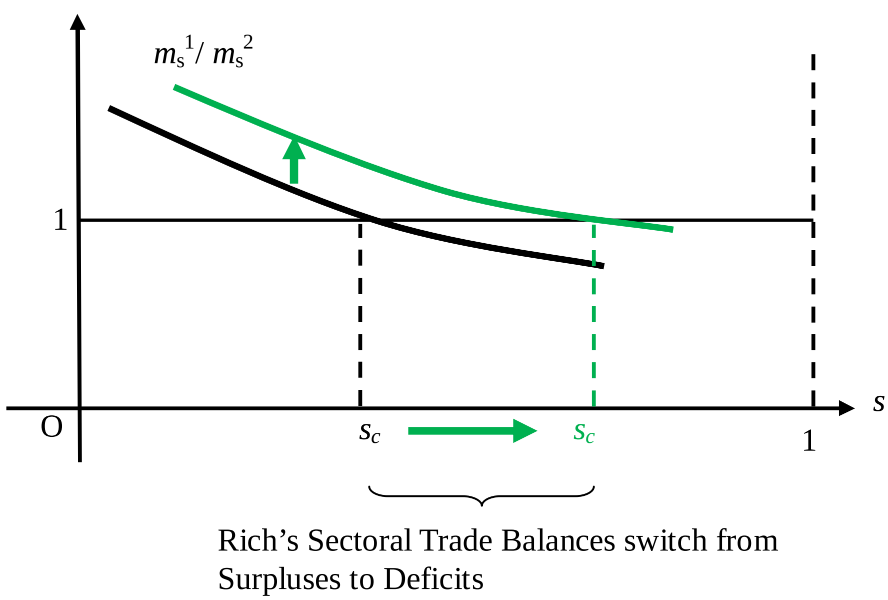

Figure 5: Possibility of Leapfrogging and Reversal of Patterns of Trade

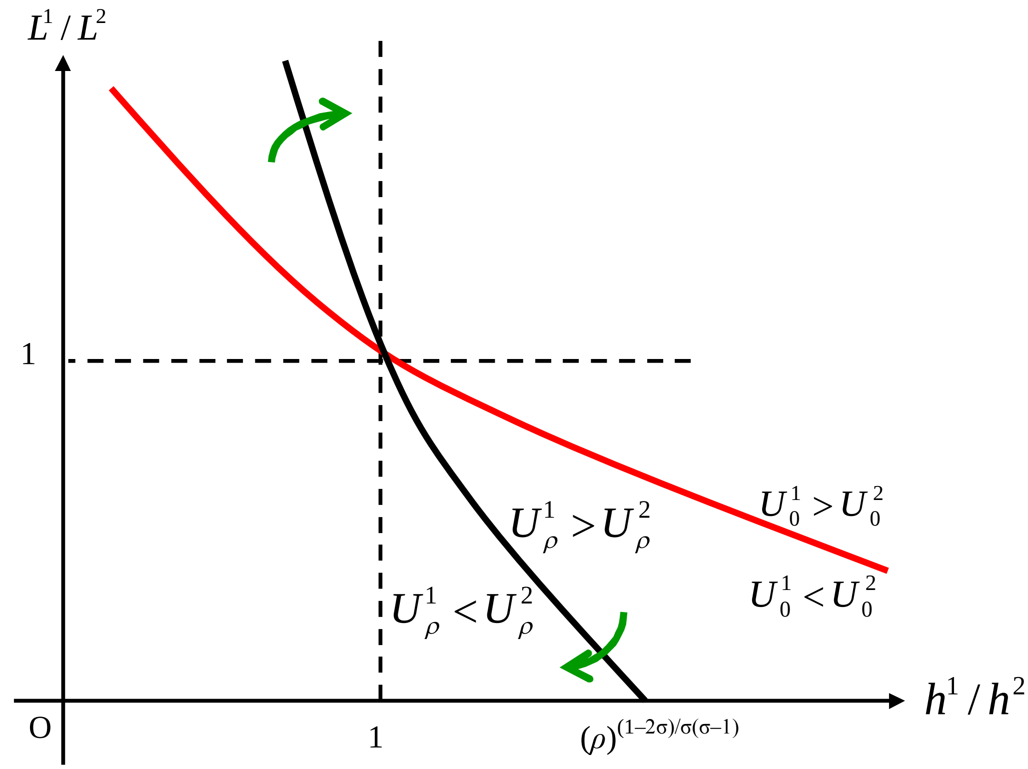


Figure 6: Home Market Effect and Patterns of Trade with An Outside Sector