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Aggregate Recruiting Intensity

By ALESSANDRO GAVAZZA, SIMON MONGEY, AND GIOVANNI L. VIOLANTE

We develop an equilibrium model of firm dynamics with random search in the labor market where hiring firms exert recruiting effort by spending resources to fill vacancies faster. Consistent with microevidence, fast-growing firms invest more in recruiting activities and achieve higher job-filling rates. These hiring decisions of firms aggregate into an index of economy-wide recruiting intensity. We study how aggregate shocks transmit to recruiting intensity, and whether this channel can account for the dynamics of aggregate matching efficiency during the Great Recession. Productivity and financial shocks lead to sizable procyclical fluctuations in matching efficiency through recruiting effort. Quantitatively, the main mechanism is that firms attain their employment targets by adjusting their recruiting effort in response to movements in labor market slackness.

(JEL D22, E24, E32, J23, J41, J63, M51)

A large literature documents cyclical changes in the rate at which the US macro-economy matches job seekers and vacant employment positions. Aggregate matching efficiency, measured as the residual of an aggregate matching function that generates hires from inputs of job seekers and vacancies, epitomizes this crucial role of the labor market. In fact, matching efficiency is a key determinant, over and above market tightness, of the aggregate job-finding rate, i.e., the speed at which idle workers are hired. Swings in the job-finding rate account for the bulk of unemployment fluctuations (Shimer 2012). Identifying the deep determinants of aggregate matching efficiency is therefore necessary to fully understand labor market dynamics.

The Great Recession represents a particularly stark episode of deterioration in aggregate matching efficiency. Our reading of the data, displayed in Figure 1, is that this decline contributed to a depressed vacancy yield, to a collapse in the job-finding...
rate, and to persistently higher unemployment following the crisis. Had match efficiency remained constant over this period, the job finding rate would have doubled, reducing peak unemployment by more than half.¹

A number of explanations have been offered for the decline in aggregate matching efficiency during the recession, virtually all of which have emphasized the worker side.² A shift in the composition of the pool of job seekers toward the long-term unemployed, by itself, goes a long way toward explaining the drop (Hall and Schulhofer-Wohl 2015); however, as documented by Mukoyama, Patterson, and Şahin (2014), workers’ job search effort is countercyclical and may compensate for

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¹See Figure B1 in the online Appendix for details of this counterfactual.
²A notable exception is the model in Sedláček (2014) that generates endogenous fluctuations in match efficiency through firms’ time-varying hiring standards.
compositional changes. Indeed, Hornstein and Kudlyak (2016) include both margins in their rich measurement exercise and conclude that they offset each other almost exactly, leaving the entire drop in match efficiency from unadjusted data to be explained. A rise in occupational mismatch shows more promise, but it can account for at most one-third of the drop and for very little of its persistence (Şahin et al. 2014).

The alternative view we set forth in this paper is that fluctuations in the effort with which firms try to fill their open positions affect aggregate matching efficiency. When aggregated over firms, we call this factor aggregate recruiting intensity. Our goal is to investigate whether this factor is an important source of the dynamics of aggregate matching efficiency, and to study the economic forces that shape how it responds to macroeconomic shocks.

Our main motivation is the empirical analysis of recruiting intensity at the micro level in Davis, Faberman, and Haltiwanger (2013)—henceforth, DFH—the first paper to rigorously use JOLTS data to examine which factors are correlated with vacancy yields at the establishment level. The robust finding of DFH is that establishments with a larger hiring rate (total hires per worker) fill their vacancies at a faster rate.3

One would therefore expect that, if an aggregate negative shock depresses growth rates of hiring firms, aggregate recruiting intensity, and, thus, aggregate match efficiency, declines since hiring firms use lower recruiting effort to fill their posted vacancies. We call this transmission channel, whereby the macro shock affects the growth rate distribution of hiring firms, the composition effect. Macro shocks also induce movements in equilibrium labor market tightness. When a negative shock hits the economy, job seekers become more abundant relative to vacancies, so firms meet workers more easily and can therefore exert less recruiting effort to reach a given hiring target. We call this second transmission channel the slackness effect, in reference to aggregate labor market conditions.

Both mechanisms seem potentially relevant in the context of the Great Recession. As evident from Figure 1, the data display a collapse in market tightness indicating the potential for a strong slackness effect. The figure also shows that the rate at which firms entered the economy fell dramatically in the aftermath of the recession. The dominant narrative is that the crisis was associated with a sharp reduction in borrowing capacity, and that start-up creation as well as young firm growth are particularly sensitive to financial shocks (Chodorow-Reich 2014; Siemer 2014; Davis and Haltiwanger 2015; Mehrotra and Sergeyev 2015). Combining this observation with the fact that young firms account for much of job creation and an even larger share of gross hires (Haltiwanger, Jarmin, and Miranda 2010) paves the way for a sizable composition effect.

Our approach is to develop a model of firm dynamics in frictional labor markets that can guide us to inspect the transmission mechanism of two common macroeconomic impulses, productivity and financial shocks, on aggregate recruiting intensity. The model is consistent with the stylized facts that are salient to an investigation of the interaction between macro shocks and recruiting activities: (i) it matches the

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3 The numerous exercises in DFH show that this finding is not in any way spurious. For example, by definition, an establishment that luckily fills a large amount of its vacancies will have both a higher vacancy yield and a higher growth rate. The authors show that luck does not drive their main result.
DFH finding that increases in hiring rates are realized chiefly through increases in vacancy yields rather than increases in vacancy rates; (ii) it allows for credit constraints that hinder the birth of start-ups and slow the expansion of young firms; and (iii) it is set in general equilibrium, since the recruiting behavior of hiring firms depends on labor market tightness, which fluctuates strongly in the data (Shimer 2005).

Our model is a version of the canonical Diamond-Mortensen-Pissarides random matching framework with decreasing returns in production (Cooper, Haltiwanger, and Willis 2007; Elsby and Michaels 2013; Acemoglu and Hawkins 2014). The model simultaneously features a realistic firm life cycle, consistent with its classic competitive setting counterparts (Jovanovic 1982; Hopenhayn 1992), and a frictional labor market with slack on both demand and supply sides. We augment this environment in three dimensions.

First, we allow for endogenous entry and exit of firms. This is a key element for understanding the effects of macroeconomic shocks on the growth rates of hiring firms, since it is well documented that young firms account for a disproportionately large fraction of job creation, grow faster than old firms, and are more sensitive to financial conditions.

Second, we introduce a recruiting intensity decision at the firm level: besides the number of open positions that they are willing to fill in each period, hiring firms choose the amount of resources that they devote to recruitment activities. This endogenous recruiting intensity margin generates heterogeneous job-filling rates across firms. In turn, the sum of all individual firms’ recruitment efforts, weighted by their vacancy share, aggregates to the economy’s measured matching efficiency.

Third, we introduce financial frictions: incumbent firms cannot issue equity, and a constraint on borrowing restricts leverage to a multiple of collateralizable assets, as in Evans and Jovanovic (1989).4

We parameterize our model to match a rich set of aggregate labor market statistics and firm-level cross-sectional moments. In choosing the recruiting cost function, we reverse-engineer a specification that allows the model to replicate DFH’s empirical relation between the job-filling rate and the hiring rate at the establishment level from the JOLTS microdata. Our parameterization of this cost function is based on a novel source of data, a survey of recruitment cost and practices based on over 400 firms that are representative of the US economy. Figure 2 gives a breakdown of spending on all recruitment activities in which firms engage in order to attract workers and quickly fill their open positions, as reported by the survey. Our hiring cost function is meant to summarize all such components.

We find that both productivity and financial shocks, modeled as shifts in the collateral parameter, generate substantial procyclical fluctuations in aggregate recruiting intensity. However, the financial shock generates movements in firm entry, labor productivity, and borrowing that are consistent with those observed during the 2008 recession, whereas the productivity shock does not. The credit tightening

4 Other papers that consider various forms of financial constraints in frictional labor market models include Wasmer and Weil (2004); Petrosky-Nadeau and Wasmer (2013); Eckstein, Setty, and Weiss (2014); and Buera, Jaef, and Shin (2015); though none of these models generate endogenous fluctuations in match efficiency. An exception is Mehrotra and Sergeyev (2013), where a financial shock has a differential impact across industries and induces sectoral mismatch between job seekers and vacancies.
accounts for approximately one-half of the drop in aggregate matching efficiency observed during the Great Recession through a decline in aggregate recruiting intensity. Notably, our model is consistent with a key cross-sectional fact documented by Moscarini and Postel-Vinay (2016): the vacancy yield of small establishments spiked up as the economy entered the downturn, whereas that of large establishments was much flatter. The reason is that the financial shock impedes the growth of a segment of very productive, already large, but relatively young, firms with much of their growth potential still unrealized. These firms drastically cut their hiring effort.

Our examination of the transmission mechanism indicates that the slackness effect is the dominant force: aggregate recruiting intensity falls mainly because the number of available job seekers per vacancy increases, allowing firms to attain their recruitment targets even by spending less on hiring costs. Surprisingly, the impact of the shock through the shift in the distribution of growth rates of hiring firms (in particular through the decline in firm entry and young-firm expansion) on aggregate recruiting intensity is quantitatively small. Two forces counteract the composition effect: (i) hiring firms are positively selected, more so in the recession than in steady state; and (ii) the rise in the abundance of job seekers, relative to open positions, allows productive firms, especially those that are financially unconstrained, to grow faster.

Figure 2. Breakdown of Spending on Recruiting Activities

*Notes:* Agencies/third-party recruiters: companies or third-party individuals that are paid to recruit candidates. *Job fairs/recruiting events:* events specifically held for the purpose of recruiting candidates or advertising a company’s employment brand to induce applications. *Job boards or search engine aggregators:* list purchases, licenses to databases (e.g., Monster, CareerBuilder). *Company websites:* websites that share information about a company’s mission and purpose (employment branding). *Professional networking sites:* websites that allow users to create a public profile and interact with other professionals in similar fields. *Professional associations:* groups of people seeking to further a particular profession, the interests of individuals engaged in that profession. *General social media:* websites and applications (e.g., Facebook, Twitter) that enable users to create and share content or to participate in social networking. *Campus recruiting:* recruitment of talent from colleges and universities. *Candidate pools:* lists or databases of applicants who have applied for an open position and remain eligible for hire. *Employee referrals:* candidates who are referred by a current employee and subsequently hired. 

*Source:* Bersin and Associates (O’Leonard, Krider, and Erickson 2015), Figure 15
In an extension of the model, we augment the composition effect with a sectoral component by allowing permanent heterogeneity in recruiting technologies across industries. As Davis, Faberman, and Haltiwanger (2013) document, construction and a few other sectors stand out in terms of their frictional characteristics by systematically displaying higher than average vacancy filling rates. In addition, these are the industries that were hit hardest by the crisis. In agreement with Davis, Faberman, and Haltiwanger (2012b), our measurement exercise concludes that, in the context of the Great Recession, the shift in the composition of labor demand away from these high-yield sectors played a nontrivial role in the decline of aggregate recruiting intensity.

We argue that our taxonomy of slackness and composition channels is useful for three reasons. First, it offers a useful heuristic lens for thinking about the complex, sometimes offsetting, firm-level forces that determine the dynamics of aggregate hiring in response to a macroeconomic shock. Second, when the slackness effect is dominant, as we conclude, firms’ recruiting efforts are very responsive to the availability of job seekers relative to vacant positions in the labor market. This, in turn, implies that any aggregate impulse that reduces labor market tightness will have an amplified impact on the aggregate job-finding rate through firms’ recruiting intensity decisions. Third, a strong slackness effect also implies that any policy intervention directed at raising unemployed workers’ search effort with the aim of accelerating their reentry into the employment ranks will, by lowering aggregate tightness, reduce firms’ recruiting effort, thus mitigating the original intent of the policy. By the same logic, the endogenous response of recruiting effort to labor market tightness reinforces the direct impact of subsidies targeted to hiring firms.

To the best of our knowledge, only two other papers have developed models of recruiting intensity. Leduc and Liu (2017) extend a standard Diamond-Mortensen-Pissarides model to one in which a representative firm chooses search intensity per vacancy. Without firm heterogeneity, they are unable to speak to the cross-sectional empirical evidence that recruiting intensity is tightly linked to firm growth rates, a key observation that we use to discipline our framework and assess the magnitude of the composition effect. Kaas and Kircher (2015) is the only other paper that focuses on heterogeneous job-filling rates across firms. In their directed search environment, different firms post distinct wages that attract job seekers at differential rates, whereas we study how firms’ costly recruiting activities determine differential job-filling rates. One would expect both factors to be important determinants of the ability of firms to grow rapidly. For example, from Austrian data, Kettemann, Mueller, and Zweimüller (2016) document that job-filling rates are higher at high-paying firms. However, after controlling for the firm component of wages, they remain increasing in firms’ growth rates, implying that wages are not the whole story: employers use other instruments besides wages to hire quickly.

Moreover, while they (and Leduc and Liu 2017) study aggregate productivity shocks, as we do as well, we further analyze financial shocks, a more natural choice if one’s attention is on the Great Recession. Finally, while aggregate recruiting intensity drops after a negative aggregate shock in both our model and theirs, the reasons for the drop fundamentally differ. Kaas and Kircher (2015) argue that the drop depends on recruiting intensity being a concave function of firms’ hiring policies, whose dispersion across firms increases after a negative shock. Our
decomposition of the transmission mechanism linking macroeconomic shocks and aggregate recruiting intensity allows us to infer that the main source of the drop is the increase in the number of available job seekers per vacancy, which allows firms to scale back their recruiting effort.

The rest of the paper is organized as follows. Section I formalizes the link between firm-level recruiting intensity and aggregate match efficiency. Section II outlines the model economy and the stationary equilibrium. Section III describes the parameterization of the model and highlights some cross-sectional features of the economy. Section IV describes the dynamic response of the economy to macroeconomic shocks, explains the transmission mechanism, and outlines the main results of the paper. Section V examines the robustness of our main findings. Section VI concludes.

I. Recruiting Intensity and Aggregate Matching Efficiency

We briefly describe how we can aggregate hiring decisions at the firm level into an economy-wide matching function with an efficiency factor that has the interpretation of average recruiting intensity. This derivation follows DFH. For much of the paper, we abstract from quits and search on the job, and thus in our baseline model there is no role for replacement hiring: gross hires always equal the net growth of expanding firms. We discuss the implications of this assumption in Section V.

At date $t$, any given hiring firm $i$ chooses $v_{it}$, the number of open positions ready to be staffed and costly to create, as well as $e_{it}$, an indicator of recruiting intensity. Let $v_{it}^{*} = e_{it}v_{it}$ be the number of effective vacancies in firm $i$. Integrating over all firms, we obtain the aggregate number of effective vacancies:

$$V_t^{*} = \int e_{it}v_{it} \, di.$$  

(1)

Under our maintained assumption of a constant returns to scale Cobb-Douglas matching function, aggregate hires equal

$$H_t = (V_t^{*})^{\alpha}U_t^{1-\alpha} = \Phi_t V_t^{\alpha}U_t^{1-\alpha},$$  

(2)

with $\Phi_t = \left(\frac{V_t^{*}}{V_t}\right)^{\alpha} = \left[\int e_{it}v_{it} \, di / V_t\right]^{\alpha}$, which corresponds to DFH’s generalized matching function. Therefore, measured aggregate matching efficiency $\Phi_t$ is an average of firm-level recruiting intensity weighted by individual vacancy shares, raised to the power of $\alpha$, the economy-wide elasticity of hires to vacancies. Finally, consistency requires that each firm $i$ faces hiring frictions, implying that

$$h_{it} = q(\theta_t^{*}) e_{it}v_{it},$$  

(3)

where $\theta_t^{*} = V_t^{*}/U_t$ is effective market tightness.\(^5\) Thus, $q(\theta_t^{*}) = H_t/V_t^{*} = (\theta_t^{*})^{\alpha-1}$ is the aggregate job-filling rate per effective vacancy, constant across all firms at date $t$.

\(^5\)Throughout, we are faithful to the notation in this literature and denote measured labor market tightness $V_{it}/U_{it}$ as $\theta_t$. 

II. Model

Our starting point is an equilibrium random-matching model of the labor market in which firms are heterogeneous in productivity and size, and the hiring process occurs through an aggregate matching function. As discussed in the introduction, we augment this model in three dimensions, all of which are essential to developing a framework that can address our question. First, our framework features endogenous firm entry and exit. Second, beyond the number of positions to open (vacancies), hiring firms optimally choose their recruiting intensity: by spending more on recruitment resources, they can increase the rate at which they meet job seekers. Third, once in existence, firms face financial constraints.

In what follows, we present the economic environment in detail, outline the model timing, and then describe the firm, bank, and household problems. Finally, we define a stationary equilibrium for the aggregate economy. Since our experiments will consist of perfect foresight transition dynamics, we do not make reference to aggregate state variables in agents’ problems. We use a recursive formulation throughout.

A. Environment

Time is discrete and the horizon is infinite. Three types of agents populate the economy: firms, banks, and households.

Firms.—There is an exogenous measure \( \lambda_0 \) of potential entrants each period, and an endogenous measure \( \lambda \) of incumbent firms. Firms are heterogeneous in their productivity \( z \in Z \), stochastic and i.i.d. across all firms, and operate a decreasing returns to scale (DRS) production technology \( y(z, n', k) \) that uses inputs of labor \( n' \in N \) and capital \( k \in K \). The output of production is a homogeneous final good, whose competitive price is the numéraire of the economy.

All potential entrants receive an initial equity injection \( a_0 \) from households. Next, they draw a value of \( z \) from the initial distribution \( \Gamma_0(z) \) and, conditional on this draw, decide whether to enter and become an incumbent by paying the setup cost \( \chi_0 \). Those that do not enter return the initial equity to the households.\(^6\) This is the only time when firms can obtain funds directly from households. Throughout the rest of their life cycle they must rely on debt issuance.

Incumbents can exit exogenously or endogenously. With probability \( \zeta \), a destruction shock hits an incumbent firm, forcing it to exit. Surviving firms observe their new value of \( z \), drawn from the conditional distribution \( \Gamma(dz', z) \), and choose whether to exit or continue production. Under either exogenous or endogenous exit, the firm pays out its positive net worth \( a \) to households. Those incumbents that decide to stay in the industry pay a per-period operating cost \( \chi \) and then choose labor and capital inputs.

The labor decision involves either firing some existing employees or hiring new workers. Firing is frictionless, but hiring is not: a hiring firm chooses both vacancies \( v \) and recruitment effort \( e \) with associated hiring cost \( \tilde{C}(e, v, n) \), which also depends

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\(^6\)Without loss of generality, we could have assumed that a fraction of the initial equity is sunk to develop the blueprint, i.e., attain the draw of \( z \), and in case of no entry, only the remaining fraction is returned to the financier.
on initial employment. Given \((e, v)\), the individual hiring function (3) determines current period employment \(n'\) used in production. To simplify wage setting, we assume that firms’ owners make take-it-or-leave-it offers to workers, so the wage rate equals \(\omega\), the individual flow value from nonemployment.

Firms face two financial constraints. First, the capital decision involves borrowing capital from financial intermediaries (banks) in intraperiod loans. Because of imperfect contractual enforcement frictions, firms can appropriate a fraction \(1/\varphi\) of the capital received by banks, with \(\varphi > 1\). To preempt this behavior, a firm renting \(k\) units of capital is required to deposit \(k/\varphi\) units of their net worth with the bank. This guarantees that, ex post, the firm does not have an incentive to abscond with the capital. Thus, a firm with current net worth \(a\) faces a collateral constraint \(k \leq \varphi a\).

The model requires both constraints. Without the equity constraint (nonnegative dividends), firms can arbitrarily obtain funds from households. The collateral constraint will still impose a maximum ratio of \(k\) to \(a\), but \(a\) can increase freely through raised equity \((d < 0)\), so \(k\) is in effect unconstrained. Without the collateral constraint, firms can arbitrarily increase \(k\) through debt while keeping dividends nonnegative. In both cases, the only limit is determined by the exit option (i.e., a negative continuation value).

**Banks.**—The banking sector is perfectly competitive. Banks receive household deposits, freely transform them into capital, and rent it to firms. The one-period contract with households pays a risk-free interest rate of \(r\). Capital depreciates at rate \(\delta\) in production, and so the price of capital charged by banks to firms is \((r + \delta)\).

**Households.**—A representative household has \(\bar{L}\) family members, \(U\) of which are unemployed. The household is risk-neutral with discount factor \(\beta \in (0, 1)\). It trades shares \(M\) of a mutual fund comprising all firms in the economy and makes bank deposits \(T\). It earns interest on deposits, the total wage payments that firms make to employed family members, and \(D\) dividends per share held in the mutual fund. Moreover, unemployed workers produce \(\omega\) units of the final good at home. Household consumption is denoted by \(C\).

Before describing the firm’s problem in detail, we outline the precise timing of the model, summarized in Figure 3. Within a period, the events unfold as follows: (i) realization of the productivity shocks for incumbent firms; (ii) endogenous and exogenous exit of incumbents; (iii) realization of initial productivity and entry decision of potential entrants; (iv) borrowing decisions by incumbents; (v) hiring/firing decisions and labor market matching; (vi) production and revenues from sales; (vii) payment of wage bill, costs of capital, hiring, and operation expenses, firm dividend payment/saving decisions, and household consumption/saving decisions.

To be consistent with our transition dynamics experiments in Section IV, it is useful to note that we record aggregate state variables as the measures of incumbent firms \(\lambda\) and unemployment \(U\), at the beginning of the period, between stages (i) and (ii). Moreover, even though the labor market opens after firms exit or fire, workers who separate in the current period can only start searching next period.
B. Firm Problem

We first consider the entry and exit decisions, then analyze the problem of incumbent firms.

Entry.—A potential entrant who has drawn $z$ from $\Gamma_0(z)$ solves the following problem:

$$\max \{ a_0, \mathbb{V}^i(n_0, a_0 - \chi_0, z) \},$$

where $\mathbb{V}^i$ is the value of an incumbent firm, a function of $(n, a, z)$. The firm enters if the value to the risk-neutral shareholder of becoming an incumbent with one employee $(n_0 = 1)$, initial net worth equal to the household equity injection $a_0$ minus the entry cost $\chi_0$, and productivity $z$ exceeds the value of returning $a_0$ to the household. Let $i(z) \in \{0, 1\}$ denote the entry decision rule, which depends only on the initial productivity draw, since all potential entrants share the same entry cost, initial employment, and ex ante equity injection. As $\mathbb{V}^i$ is increasing in $z$, there is an endogenous productivity cutoff $z^*$ such that for all $z \geq z^*$, the firm chooses to enter. The measure of entrants is therefore

$$\lambda_e = \lambda_0 \int_Z i(z) d\Gamma_0 = \lambda_0 [1 - \Gamma_0(z^*)].$$

Exit.—Firms exit exogenously with probability $\zeta$. Conditional on survival the firm then chooses to continue or exit. An exiting firm pays out its net worth $a$ to shareholders. The firm’s expected value $\mathbb{V}$ before the destruction shock equals

$$\mathbb{V}(n, a, z) = \zeta a + (1 - \zeta) \max \{ \mathbb{V}^i(n, a, z), a \}.$$  

We denote by $x(n, a, z) \in \{0, 1\}$ the exit decision.

Hire or Fire.—An incumbent firm $i$ with employment, net worth, and productivity equal to the triplet $(n, a, z)$ chooses whether to hire or fire workers to solve

$$\mathbb{V}^i(n, a, z) = \max \{ \mathbb{V}^h(n, a, z), \mathbb{V}^f(n, a, z) \}.$$
The two value functions $\mathbb{V}_f$ and $\mathbb{V}_h$ associated with firing ($f$) and hiring ($h$) are described below.

**The Firing Firm.**—A firm that has chosen to fire some of its workers (or to not adjust its work force) solves

$$
\mathbb{V}_f(n, a, z) = \max_{n', k, d} d + \beta \int_Z \mathbb{V}(n', a', z') \Gamma(dz', z)
$$

subject to

$$
n' \leq n,
$$
$$
d + a' = y(n', k, z) + (1 + r) a - \omega n' - (r + \delta) k - \chi,
$$
$$
k \leq \varphi a,
$$
$$
d \geq 0.
$$

Firms maximize shareholder value and, because of risk neutrality, use $\beta$ as their discount factor. The change in net worth $a' - a$ is given by revenues from production and interest on savings net of the wage bill, rental and operating costs, and dividend payouts $d$. The last two equations in (8) reiterate that firms face a collateral constraint on the maximum amount of capital they can rent and a nonnegativity constraint on dividends.

To help understand the budget constraint and preface how we take the model to the data, define firm debt by the identity $b \equiv k - a$, with the understanding that $b < 0$ denotes savings. Making this substitution reveals an alternative formulation of the model in which the firm owns its capital and faces a constraint on leverage. With state vector $(n, k, b, z)$, the firm faces the following budget and collateral constraints:

$$
d + k' - (1 - \delta)k = y(n', k, z) - \omega n' - \chi - rb + b' - b, \quad \text{Investment Operating profit Borrowing}
$$
$$
b/k \leq (\varphi - 1)/\varphi.
$$

This makes it clear that the firm can fund equity payouts and investment in capital through either operating profits or expanding borrowing/reducing saving.

**The Hiring Firm.**—The hiring firm additionally chooses the number of vacancies to post $v \in \mathbb{R}_+$ and recruitment effort $e \in \mathbb{R}_+$, understanding that, by a law of large numbers, its new hires $n' - n$ equal the firm’s job-filling rate $qe$ of each of its vacancies times the number of vacancies $v$ created: $n' - n = q(e^*) ev$\textsuperscript{7}. Note that the individual firm job-filling rate depends on the aggregate meeting rate $q$, which

\textsuperscript{7}The linearity of the individual hiring function in vacancies is one of the key empirical findings of DFH.
is determined in equilibrium and the firm takes as given, as well as on its recruiting effort $e$. The firm faces a variable cost function $C(e, v, n)$, increasing and convex in $e$ and $v$.

A firm’s continuation value depends on $n'$, not on the mix of recruiting intensity $e$ and vacant positions $v$ that generates it. As a result, one can split the problem of the hiring firm into two stages. The first stage is the choice of $n', k$, and $d$. The second stage, given $n'$, is the choice of the optimal combination of inputs $(e, v)$. The latter reduces to a static cost minimization problem:

$$\text{(9)} \quad C^*(n, n') = \min_{e,v} C(e, v, n)$$

subject to

$$e \geq 0, \quad v \geq 0, \quad n' - n = q(\theta^*) ev,$$

yielding the lowest cost combination $e(n, n')$ and $v(n, n')$ that delivers $h = n' - n$ hires to a firm of size $n$, and the implied cost function $C^*(n, n')$.

The remaining choices of $n'$, $k$, and $d$ require solving the dynamic problem

$$\text{(10)} \quad \mathcal{V}^h(n, a, z) = \max_{n', k, d} d + \beta \int_Z \mathcal{V}(n', a', z') \Gamma(dz', z)$$

subject to

$$n' > n,$$

$$d + a' = y(n', k, z) + (1 + r)a - \omega n' - (r + \delta) k - \chi - C^*(n, n'),$$

$$k \leq \varphi a,$$

$$d \geq 0.$$

The solution to this problem includes the decision rule $n'(n, a, z)$. Using this function in the solution to (9), we obtain decision rules $e(n, a, z)$ and $v(n, a, z)$ for recruitment effort and vacancies in terms of firm state variables.

Given the centrality of the hiring cost function $C(e, v, n)$ to our analysis, we now discuss its specification. In what follows, we choose the functional form

$$\text{(11)} \quad C(e, v, n) = \left[ \frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left( \frac{v}{n} \right)^{\gamma_2} \right] v,$$

with $\gamma_1 \geq 1$ and $\gamma_2 \geq 0$ being necessary conditions for the convexity of the maximization problem (9).

This cost function implies that the average cost of a vacancy, $C/v$, has two separate components. The first is increasing and convex in recruiting intensity per vacancy $e$. The idea is that, for any given open position, the firm can choose to spend resources on recruitment activities (recall Figure 2) to make the position more visible or the firm more attractive as a potential employer, or to assess more candidates per unit
of time, but all such activities are increasingly costly on a per-vacancy basis. The second component is increasing and convex in the vacancy rate and captures the fact that expanding productive capacity is costly in relative terms: for example, creating 10 new positions involves a more expensive reorganization of production in a firm with 10 employees than in a firm with 1,000 employees.

In online Appendix A, we derive several results for the static hiring problem of the firm (9) under this cost function and derive the exact expression for $C^* (n, n')$ used in the dynamic problem (10). We show that, by combining first-order conditions, we obtain the optimal choice for $e$:

$$e(n, n') = \left[\frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*) \frac{\gamma_2}{\gamma_1 + \gamma_2} \left( \frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}},$$

and, hence, the firm-level job-filling rate $f(n, n') \equiv q(\theta^*) e(n, n')$, as well as the optimal vacancy rate:

$$\frac{v}{n} = \left[\frac{\kappa_2}{\kappa_1} \left( \frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{-\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*) - \frac{\gamma_1}{\gamma_1 + \gamma_2} \left( \frac{n' - n}{n} \right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}}.$$

Equation (12) demonstrates that the model implies a log-linear relation between the job-filling rate and employment growth at the firm level, with elasticity $\gamma_2/(\gamma_1 + \gamma_2)$. This is the key empirical finding of DFH, who estimate this elasticity to be 0.82. In fact, one could interpret our functional choice for $C$ in equation (11) as a reverse-engineering strategy in order to obtain, from first principles, the empirical cross-sectional relation between the establishment-level job-filling rate and the establishment-level hiring rate uncovered by DFH. Put differently, micro-data sharply discipline the recruiting cost function of the model.8

Why does firm optimality imply that the job-filling rate increases with the growth rate with elasticity $\gamma_2/(\gamma_1 + \gamma_2)$? Recruiting intensity $e$ and the vacancy rate $(v/n)$ are substitutes in the production of a target employment growth rate $(n' - n)/n$ (see the last equation in (9)). Thus, a firm that wants to grow faster than another will optimally create more positions and, at the same time, spend more on recruiting effort. However, the stronger the convexity of $C$ in the vacancy rate ($\gamma_2$), relative to its convexity in effort ($\gamma_1$), the more an expanding firm finds it optimal to substitute away from vacancies and into recruiting intensity to realize its target growth rate. In the special case of $\gamma_2 = 0$, all the adjustment occurs through vacancies, and recruiting effort is irresponsive to the growth rate and to macroeconomic conditions, as in the canonical model of Pissarides (2000).

Figure 4 plots the cross-sectional relationship between employment growth and the vacancy rate (solid) and the vacancy-yield (dashed) in the model (black) and in the DFH data (gray), with the elasticity of the vacancy-yield to firm growth $\gamma_2/(\gamma_1 + \gamma_2) = 0.82$. Since the individual hiring function is linear in vacancies, the elasticity of the vacancy rate to firm growth equals $\gamma_1/(\gamma_1 + \gamma_2) = 0.18$.

8 Online Appendix A also shows that, once the optimal choice of $e$ is substituted into (11), $C$ can be stated solely in terms of the vacancy rate and becomes equivalent to one of the hiring cost functions that Kaas and Kircher (2015) use in their empirical analysis.
C. Household Problem

The representative household solves

\[
\mathbb{W}(T, M, D) = \max_{T', M', C > 0} \quad C + \beta \mathbb{W}(T', M', D')
\]

subject to

\[
C + \tilde{Q} T' + PM' = \omega \bar{L} + (D + P) M + T,
\]

where \( T \) are bank deposits, \( M \) are shares of the mutual fund composed of all firms in the economy, and \( D \) are aggregate dividends per share. The household takes as given the price of bank deposits \( \tilde{Q} \), the share price \( P \), and the price of the final good, which we normalize to 1. From the first-order conditions for deposits and share holdings, we obtain \( \tilde{Q} = \beta \) and \( P = \beta (P + D) \) which imply a time-invariant rate of return of \( r = \beta^{-1} - 1 \) on both deposits and shares. The household is therefore indifferent over portfolios.

Since firms make take-it-or-leave-it offers to workers (i.e., firms have all the bargaining power) and are competitive, they pay all their workers a wage equal to the individual’s flow value of nonemployment \( \omega \), which we interpret as output from home production. The total amount of resources available to households for consumption and saving as a result of market and home production is thus simply \( \omega \bar{L} \). Because of risk neutrality, the household is indifferent over the timing of consumption.

9 The initial equity injections into successful start-ups are treated as negative dividends, i.e., they are part of \( D \) every period.

10 If we let the wage be \( W \), then total resources from market and home production equal \( W(\bar{L} - U) + \omega U \). The term \( \omega \bar{L} \) in the household budget constraint follows from the fact that \( W = \omega \). This also explains why unemployment \( U \) is not a state variable in the household’s problem (14).
D. Stationary Equilibrium and Aggregation

Let $\Sigma_N$, $\Sigma_A$, and $\Sigma_Z$ be the Borel sigma algebras over $N$ and $A$, and $Z$. The state space for an incumbent firm is $S = N \times A \times Z$, and we denote with $s$ one of its elements $(n, a, z)$. Let $\Sigma_S$ be the sigma algebra on the state space, with typical set $S = N \times A \times Z$, and $(S, \Sigma_S)$ be the corresponding measurable space. Denote with $\lambda : \Sigma_S \rightarrow [0, \infty)$ the stationary measure of incumbent firms at the beginning of the period, following the draw of firm-level productivity, before the exogenous exit shock.

To simplify the exposition of the equilibrium, it is convenient to use $s \equiv (n, a, z)$ and $s_0 \equiv (n_0, a_0 - \chi_0, z)$ as the argument for incumbents’ and entrants’ decision rules.

A stationary recursive competitive equilibrium is a collection of firms’ decision rules $\{i(z), x(s), n'(s), e(s), v(s), a'(s), d(s), k(s)\}$, value functions $\{V, V^i, V^f, V^h\}$, a measure of entrants $\lambda_e$, share price $P$ and aggregate dividends $D$, wage $\omega$, a distribution of firms $\lambda$, and a value for effective labor market tightness $\theta^*$ such that: (i) the decision rules solve the firm’s problems (4)–(10), $\{V, V^i, V^f, V^h\}$ are the associated value functions, and $\lambda_e$ is the mass of entrants implied by (5); (ii) the market for shares clears at $M = 1$ with share price

$$P = \int_S V(s) \, d\lambda + \lambda_0 \int_Z i(z) \, V^i(s_0) \, d\Gamma_0$$

and aggregate dividends

$$D = \zeta \int_S a \, d\lambda + (1 - \zeta) \int_S \{[1 - x(s)] \, d(s) + x(s) \, a\} \, d\lambda - \lambda_0 \int_Z i(z) \, a_0 \, d\Gamma_0;$$

(iii) the stationary distribution $\lambda$ is the fixed point of the recursion:

$$\lambda(N \times A \times Z) = (1 - \zeta) \int_S [1 - x(s)] \, 1_{\{n'(s) \in N^e\}} \, 1_{\{a'(s) \in A\}} \, \Gamma(Z, z) \, d\lambda$$

$$+ \lambda_0 \int_Z i(z) \, 1_{\{n'(s_0) \in N^e\}} \, 1_{\{a'(s_0) \in A\}} \, \Gamma(Z, z) \, d\Gamma_0,$$

where the first term refers to existing incumbents and the second to new entrants; (iv) effective market tightness $\theta^*$ is determined by the balanced flow condition

$$\bar{L} - N(\theta^*) = \frac{F(\theta^*) - \lambda_e(\theta^*) \, n_0}{p(\theta^*)},$$

where $p(\theta^*)$ is the aggregate job-finding rate, $N(\theta^*)$ is aggregate employment,

$$N(\theta^*) = (1 - \zeta) \int_S [1 - x(s)] n'(s) \, d\lambda + \lambda_0 \int_Z i(z) n'(s_0) \, d\Gamma_0,$$

and $F(\theta^*)$ are aggregate separations,
\begin{equation}
F(\theta^*) = \zeta \int_S n d\lambda + (1 - \zeta) \int_S x(s) n d\lambda + (1 - \zeta) \int_S [1 - x(s)] (n - n'(s))^{-1} d\lambda,
\end{equation}

which include all employment losses from firms exiting exogenously and endogenously, plus all the workers fired by shrinking firms, which we have denoted by \((n - n'(s))^{-1}\). In equations (16)–(18), the dependence of \(\lambda_e, N,\) and \(F\) on \(\theta^*\) comes through the decision rules and the stationary distribution, even though, for notational ease, we have omitted \(\theta^*\) as their explicit argument.

The left-hand side of (16) is the definition of unemployment, labor force minus employment, whereas the right-hand side is the steady-state Beveridge curve, i.e., the law of motion for unemployment
\begin{equation}
U' = U - p(\theta^*) U + F(\theta^*) - \lambda_e(\theta^*) n_0
\end{equation}
evaluated in steady state. As in Elsby and Michaels (2013), the two sides of (16) are independent equations determining the same variable, unemployment, and combined they yield equilibrium market tightness \(\theta^*\). Note that equations (16) and (19) account for the fact that every new firm enters with \(n_0\) workers hired “outside” the frictional labor market (e.g., the firm founders).

Clearly, once \(\theta^*\) and \(\lambda\) are determined, so is \(U\) from either side of (16) and, therefore, \(V^*\). Finally, we note that measured aggregate matching efficiency, in equilibrium, is \(\Phi = (V^*/V)^\alpha\), where measured and effective vacancies are respectively
\begin{align*}
V &= (1 - \zeta) \int_S [1 - x(s)] v(s) d\lambda + \lambda_0 \int_Z i(z) v(s_0) d\Gamma_0,
V^* &= (1 - \zeta) \int_S [1 - x(s)] e(s) v(s) d\lambda + \lambda_0 \int_Z i(z) e(s_0) v(s_0) d\Gamma_0.
\end{align*}

Online Appendix C provides details on the computation of the decision rules and the stationary equilibrium.

### III. Parameterization

We begin with the subset of parameters calibrated externally, then consider those estimated within the model. The main problem we face in parameterizing the model is that the theory does not distinguish between firms and establishments. Ideally we would only use data on firms, since financial constraints apply at the firm level. However, JOLTS data are only available by establishment, as are other data sources we use in calibration. We are therefore forced to compromise: we use firm data whenever we have a choice, for example, when we use the Business Dynamics Statistics (BDS) data, and establishment data when we are limited. Data moments are averages over 2001–2007 unless otherwise specified.

\footnote{Entrant firms never fire, as they enter with the lowest value on the support for \(N, n_0\) normalized to 1.}

\footnote{Our computation showed that, typically, \(N(\theta^*)\) is decreasing in its argument and the right-hand side of (16) is always positive and decreasing. Thus, the crossing point of the left and right sides is unique, when it exists. However, an equilibrium may not exist. For example, for very low hiring costs, \(N(\theta^*)\) may be greater than \(L\). Conversely, for large enough operating or hiring costs, no firms will enter the economy. In this case, there is no equilibrium with market production (albeit there is always some home production in the economy).}
A. Externally Calibrated

The model period is one month. We set $\beta$ to replicate an annualized risk-free rate of 3 percent. Since the measure of potential entrants $\lambda_0$ scales $\lambda$ (see equation (15)), we choose $\lambda_0$ to normalize the total measure of incumbent firms to 1. We then fix the size of the labor force $\bar{L}$ so that, given a measure 1 of firms, and the model implied steady-state unemployment rate of 7 percent, the average firm size will be 22.6 (BDS).\(^{13}\) In line with empirical studies, we set $\alpha$, the elasticity of aggregate hires to aggregate vacancies in the matching function, to 0.5. Table 1 summarizes these parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor (monthly)</td>
<td>$\beta$</td>
<td>Annual risk-free rate</td>
<td>0.03</td>
</tr>
<tr>
<td>Mass of potential entrants</td>
<td>$\lambda_0$</td>
<td>Measure of incumbents</td>
<td>1</td>
</tr>
<tr>
<td>Size of labor force</td>
<td>$L$</td>
<td>Average firm size (BDS)</td>
<td>22.6</td>
</tr>
<tr>
<td>Elasticity of matching function wrt $V_t$</td>
<td>$\alpha$</td>
<td>JOLTS</td>
<td></td>
</tr>
</tbody>
</table>

B. Internally Calibrated

Table 2 lists the remaining 19 parameters of the model that are set by minimizing the distance between an equal number of empirical moments and their equilibrium counterparts in the model.\(^{14}\) It also lists the targeted moments, their empirical values, and their simulated values from the model. Even though every targeted moment is determined simultaneously by all parameters, in what follows we discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power.

We set the flow of home production of the unemployed $\omega$ to replicate a monthly separation rate of 0.03. We choose the shift parameter of the matching function (a normalization of the value of $\Phi$ in steady state) in order to replicate a monthly job-finding rate of 0.40. Together, these two moments yield a steady state unemployment rate of 0.07.

We assume a revenue function $y(z, n', k) = z \left[ (n')^{\nu} k^{1-\nu} \right]^\sigma$. We need not take a stand on whether $z$ represents demand or productivity shocks, or whether $\sigma < 1$ is due to DRS in production or downward-sloping demand.\(^{15}\) For simplicity, we will

---

\(^{13}\) The unemployment rate is $u = \bar{L}/N(\theta^*) - 1$, and with a unit mass of firms the average firm size is simply $N(\theta^*)$. Hence, for an unemployment rate of $u = 0.07$, $\bar{L}$ determines average firm size.

\(^{14}\) Specifically, the vector of parameters $\Psi$ is chosen to minimize the minimum-distance-estimator criterion function

$$f(\Psi) = (m_{\text{data}} - m_{\text{model}}(\Psi))' W (m_{\text{data}} - m_{\text{model}}(\Psi)),$$

where $m_{\text{data}}$ and $m_{\text{model}}(\Psi)$ are the vectors of moments in the data and model, and $W = \text{diag}(1/m_{\text{data}}^2)$ is a diagonal weighting matrix.

\(^{15}\) Given our class of frictions, the revenue function is sufficient. This would not be the case in alternative environments that endogenize components of revenue productivity, for example models with R&D, which affects productivity, or models with customer accumulation, which affects demand.
refer to the revenue function as if it were a production function: \( \sigma \) represents the span of control and \( z \) is total factor productivity.

We introduce a small degree of permanent heterogeneity in the scale parameter \( \sigma \). Specifically, we consider a three-point distribution with support \( \{ \sigma_L, \sigma_M, \sigma_H \} \), symmetric about \( \sigma_M \), leaving four unknown parameters: (i) the value of \( \sigma_M \); (ii) the spread \( \Delta \sigma \equiv (\sigma_H - \sigma_L) \); and (iii)-(iv) the fractions of low and high DRS firms \( \mu_L, \mu_H \). This heterogeneity allows us to match the skewed firm size distribution, with the parameters chosen to match the shares of total employment and total firms due to firms of size 0–49 and 500+ (BDS). Permanent heterogeneity in productivity could also be used to match these facts, but heterogeneity in \( \sigma \) also generates small old firms alongside young large firms, thus decoupling age and size, which tend to be too strongly correlated in standard firm dynamics models with mean reverting productivity.\(^{16}\) In other words, heterogeneity in \( \sigma \) captures the appealing idea that there exist some very productive businesses that are small simply because the optimal scale of production for many goods or services is small. This idea will turn out to be important for interpreting the response of firms to a macroeconomic shock.

Firm productivity \( z \) follows an AR(1) process in logs: \( \log z' = \log Z + \rho_z \log z + \varepsilon' \), with \( \varepsilon' \sim \mathcal{N}( - \theta_z^2 / 2, \sigma_z^2 ) \). We calibrate \( \rho_z \) and \( \sigma_z \) to match two measures of employment dispersion, one in growth and one in levels: the standard deviation of annual employment growth for continuing establishments in the US Census Bureau’s Longitudinal Business Database (Elsby and Michaels 2013) and the ratio of the mean size of the fourth to first quartile of the firm distribution (Haltiwanger 2011).\(^{17}\)

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\(^{16}\)See Elsby and Michaels (2013) and Kaas and Kircher (2015) for examples of the use of heterogeneity in permanent productivity.

\(^{17}\)Our estimates imply annual persistence of 0.91, and standard deviation of shocks of 0.27, within the range of estimates for revenue productivity processes as surveyed by Lee and Mukoyama (2015).

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### Table 2—Parameter Values Estimated Internally

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow of home production</td>
<td>( \omega )</td>
<td>1.000</td>
<td>Monthly separation rate</td>
<td>0.033</td>
</tr>
<tr>
<td>Scaling of match. function</td>
<td>( \delta )</td>
<td>0.208</td>
<td>Monthly job-finding rate</td>
<td>0.412</td>
</tr>
<tr>
<td>Prod. weight on labor</td>
<td>( \nu )</td>
<td>0.804</td>
<td>Labor share</td>
<td>0.627</td>
</tr>
<tr>
<td>Midpoint DRS in prod.</td>
<td>( \sigma_M )</td>
<td>0.800</td>
<td>Employment share ( n &lt; 50 )</td>
<td>0.293</td>
</tr>
<tr>
<td>High-mid spread in DRS</td>
<td>( \Delta \sigma )</td>
<td>0.047</td>
<td>Employment share ( n \geq 500 )</td>
<td>0.431</td>
</tr>
<tr>
<td>Mass: low DRS</td>
<td>( \mu_L )</td>
<td>0.826</td>
<td>Firm share ( n &lt; 50 )</td>
<td>0.945</td>
</tr>
<tr>
<td>Mass: high DRS</td>
<td>( \mu_H )</td>
<td>0.032</td>
<td>Firm share ( n \geq 500 )</td>
<td>0.004</td>
</tr>
<tr>
<td>SD of ( z ) shocks</td>
<td>( \varrho )</td>
<td>0.052</td>
<td>SD ann. emp. growth</td>
<td>0.450</td>
</tr>
<tr>
<td>Persistence of ( z ) shocks</td>
<td>( \rho_z )</td>
<td>0.992</td>
<td>Mean Q4 emp./median Q1 emp.</td>
<td>75.153</td>
</tr>
<tr>
<td>Mean ( z_0 \sim \exp (z_0^{-1}) )</td>
<td>( z_0 )</td>
<td>0.390</td>
<td>( \Delta \log z ): young versus mature</td>
<td>−0.246</td>
</tr>
<tr>
<td>Cost elasticity wrt ( e )</td>
<td>( \gamma_1 )</td>
<td>1.114</td>
<td>Elasticity of vac. yield wrt ( g )</td>
<td>0.816</td>
</tr>
<tr>
<td>Cost elasticity wrt ( v )</td>
<td>( \gamma_2 )</td>
<td>4.599</td>
<td>Ratio vac. yield: ( n &lt; 50/n \geq 50 )</td>
<td>1.242</td>
</tr>
<tr>
<td>Cost shifter wrt ( e )</td>
<td>( \kappa_1 )</td>
<td>0.101</td>
<td>Hiring cost ((100 +)/wage)</td>
<td>0.953</td>
</tr>
<tr>
<td>Cost shifter wrt ( v )</td>
<td>( \kappa_2 )</td>
<td>5.000</td>
<td>Vacancy share ( n &lt; 50 )</td>
<td>0.354</td>
</tr>
<tr>
<td>Exogenous exit probability</td>
<td>( \zeta )</td>
<td>0.006</td>
<td>Five-year survival rate</td>
<td>0.553</td>
</tr>
<tr>
<td>Entry cost</td>
<td>( \lambda_0 )</td>
<td>10.000</td>
<td>Annual entry rate</td>
<td>0.090</td>
</tr>
<tr>
<td>Operating cost</td>
<td>( \chi )</td>
<td>0.035</td>
<td>Share of job destruction by exit</td>
<td>0.199</td>
</tr>
<tr>
<td>Initial wealth</td>
<td>( \alpha_0 )</td>
<td>0.646</td>
<td>Start-up debt to output</td>
<td>1.462</td>
</tr>
<tr>
<td>Collateral constraint</td>
<td>( \varphi )</td>
<td>10.210</td>
<td>Aggregate debt to net worth</td>
<td>0.276</td>
</tr>
</tbody>
</table>
The initial productivity distribution for entrants $\Gamma_0$ is exponential. The mean $\overline{z}_0$ is chosen to match the revenue productivity gap between entrants and incumbents, specifically the differential between plants younger than age 1 and older than age 10 (Foster, Haltiwanger, and Syverson 2016).

We now turn to hiring costs. The cost function (11) has four parameters: the two elasticities $(\gamma_1, \gamma_2)$ and the two cost shifters $(\kappa_1, \kappa_2)$. From our discussion of equations (11) and (12), recall that the cross-sectional elasticity of the job-filling rate to employment growth, estimated to be 0.82 by DFH, is a function of the ratio of these two elasticities.\(^{18}\) The second moment used to separately identify the two elasticities is the ratio of vacancy yields at small ($n < 50$) and large ($n \geq 50$) establishments (JOLTS). Intuitively, increasing $\gamma_2$ makes vacancies expensive, but especially so for small firms, leading to relatively higher vacancy yields at small firms (less vacancies, more effort).

We use two targets to pin down the cost shift parameters. The first is the total hiring cost as a fraction of monthly wage per hire, a standard target for the single vacancy cost parameter that usually appears in vacancy posting models. We have a new source for this statistic. The consulting company Bersin and Associates runs a periodic survey of recruitment cost and practices at over 400 firms, all with more than 100 employees. Once the firms are reweighted by industry and size, the sample is representative of this size segment of the US economy. They compute that median annual spending on all recruiting activities (including internal staff compensation, agencies/third-party recruiters, job fairs and campus recruiting, job boards, employment branding, professional networking sites, social media, employee referral bonuses, travel to fly or to interview candidates, print/billboards advertisement) divided by the number of hires, at firms in 2011 was $3,479 (see Figure 3 in O’Leonard 2011).\(^{19}\) Given average annual earnings of roughly $45,000 in 2011, in the model we target a ratio of median recruiting cost to average monthly wage (in firms with more than 100 employees) of 0.928. The second target is the vacancy share of small ($n < 50$) establishments from JOLTS: $\kappa_2$ determines the size of hiring costs for small firms and, thus, the amount of vacancies they create.

The parameters $\chi$ and $\zeta$ have large effects on firm exit. The operating cost $\chi$ mostly affects the exit rates of young firms; therefore, we target the five-year firm survival rate which is approximately 50 percent (BDS). The parameter $\zeta$ contributes to the exit of large and old firms; hence, we target the fraction of total job destruction due to exit of 34 percent (BDS).\(^{20}\) To pin down the setup cost $\chi_0$, we target the annual firm entry rate of 10 percent (BDS).\(^{21}\)

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18 We cannot map $\gamma_2/(\gamma_1 + \gamma_2)$ directly into this value since in DFH, and in the model’s simulations for consistency, the growth rate is the Davis-Haltiwanger growth rate normalized in $[-2, 2]$. In practice, as seen in Table 2, the discrepancy between the structural and estimated parameters is very small. Moreover, DFH estimate the relationship between the job-filling rate and the gross hires rate rather than employment growth. In our model, the gross hires rate and rate of employment growth of hiring firms coincide, although this would not be the case in a model with replacement hires. We discuss this in Section V.

19 See the Notes of Figure 2 for a description of each of these components.

20 Unlike other moments used here from the BDS, job destruction by exit is only available by establishment exit, not firm exit.

21 When computing moments designed to be comparable to their counterparts in the BDS, we carefully time-aggregate the model to an annual frequency. For example, the entry rate in the BDS is measured as the number of age zero firms in a given year divided by the total number of firms. Computing this statistic in the model requires aggregating monthly entry and exit over 12 months. See online Appendix C for details.
The remaining two parameters are the size of the initial equity injection $a_0$ and the collateral parameter $\varphi$. To inform their calibration, we target the debt-output ratio of start-up firms computed from the Kauffman Survey (Robb and Robinson 2014) and the aggregate debt to aggregate net worth from the flow of funds accounts.

### C. Cross-Sectional Implications

We now explore the main cross-sectional implications of the calibrated model, at its steady-state equilibrium. Table 3 reports some empirical moments not targeted in the calibration and their model-generated counterparts. The fact that the ratio of dividend payments to profits in the model is close to its empirical value confirms that the collateral constraint is neither too tight nor too loose. The model can also replicate well the distribution of employment by establishment growth rate and firm age, neither of which was explicitly targeted. Figure 5 shows that the model is also able to satisfactorily replicate the observed distribution of hires and vacancies by size class.

In Figure 6, we plot the average firm size, job creation and destruction rates, fraction of constrained firms, and leverage (debt/saving over net worth, $b/a$) for firms from birth through to maturity. Panel A shows that $\sigma_H$ firms, those with closer to constant returns in production, account for the upper tail in the size and growth rate distributions. On average, though, firm size grows by much less over the life cycle, since these “gazelles,” as they are often referred to in the literature, are only a small fraction of the total ($\mu_H = 0.032$). This lines up well with the data: average firm size grows by a factor of 3.0 between ages 1–5 and 20–25 in the model and 3.1 in the data (BDS). Convex recruiting costs and collateral constraints slow down growth: most firms reach their optimal size around age 10, whereas $\sigma_H$ firms keep growing for much longer.

Panel B plots job creation and destruction rates by age and is a stark representation of the “up-and-out” dynamics of young firms documented in the literature (Haltiwanger 2012). Panel C depicts the fraction of constrained firms (defined as those with $k = \varphi a$ and $d = 0$) over the life cycle. In the model, financial constraints

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22 Robb and Robinson (2014) report $79,592 of average debt (credit cards, personal and business bank loans, and credit lines) and $54,994 of average revenue for the 2004 cohort of start-ups in their first year; see their Table 5. From the flow of funds 2005, we computed total debt as the sum of debt securities and loans of Nonfinancial Corporate (Table L.103) and Noncorporate Business (Table L.104), and total net worth as the sum of Nonfinancial Corporate (Table B.103) and Noncorporate Business (Table B.104) net worth.
bind only for the first few years of a firm’s life, when net worth is insufficient to fund the optimal level of capital. Panel D illustrates that leverage declines with age, and after age 10 the median firm is saving (i.e., \( b < 0 \)). Much like in the classical household “income fluctuation problem,” in our model firms have a precautionary saving motive because of the simultaneous presence of three elements: (i) a concave payoff function because of DRS, (ii) stochastic productivity, and (iii) the collateral constraint.
Panel A of Figure 7 shows that recruiting intensity and the vacancy rate are sharply decreasing with age. These features arise because our cost function implies that both optimal hiring effort and the vacancy rate are increasing in the growth rate, and young firms are those with the highest desired growth rates. Moreover, the stronger convexity of $C$ in the vacancy rate ($\gamma_2$), relative to its degree of convexity in effort ($\gamma_1$) implies that a rapidly expanding firm prefers to increase its recruiting intensity relatively more than vacancies to realize its target growth rate. Thus, young firms find it optimal to recruit very aggressively for the new positions that they open. As firms age, growth rates fall and this force weakens.

Panel B plots the fraction of total recruiting effort, vacancies, and hiring firms by age. It shows that, relative to the steady-state age distribution of hiring firms, the effort distribution is skewed toward young firms, whereas the vacancy distribution is skewed toward older firms. In the model, the age distribution of vacancies is almost uniform: young firms grow faster than old ones and, thus, post more vacancies per worker; however, they are smaller and, thus, they post fewer vacancies for a given growth rate. These two forces counteract each other and the ensuing vacancy distribution over ages is nearly flat. Figure 7 highlights that the JOLTS notion of a vacancy as “open position ready to be filled” is a good metric of hiring effort for old firms, for whom recruiting intensity is nearly constant, whereas it is quite imperfect for young firms aged 0–5, whose average recruiting intensity, as well as its variance, are much higher than those of mature firms.23

IV. Aggregate Recruiting Intensity and Macroeconomic Shocks

Our main experiment consists of studying the perfect foresight transitional dynamics of the model in response to a one-time, unexpected shock either to aggregate

23 Unfortunately, JOLTS does not report the age of the establishment, so there are no US data on vacancies and recruiting intensity by age that we can directly compare to our model. Kettemann, Mueller, and Zweimüller (2016) find that, in Austrian data, after controlling for firm fixed effects, job-filling rates are decreasing with firm age.
productivity $Z$ or to the financial constraint parameter $\varphi$. The economy starts in steady state and the path of the shock reverts back to its initial value, so the economy also returns to its initial steady state.$^{24}$

### A. Calibration of Aggregate Shocks

Let $X$ indicate either the productivity shock or the financial shock, depending on the experiment. The path for $\{X_t\}_{t=0}^T$ is such that $X_0 = X_T = \bar{X}$, and $(X_t - \bar{X}) = \rho_X(X_{t-1} - \bar{X})$ for $t \in \{1, \ldots, T-1\}$, where $\bar{X}$ is the value taken in steady state. We must provide values for $X_1$ and $\rho_X$. These two values are calibrated to replicate two features of the path for aggregate output described by Fernald (2015): the peak-trough drop and its half-life. $^{25}$ First, at the trough, GDP was around 10 percent below trend. Second, GDP returned to around 5 percent below trend three years after the trough. Figure B2 in the online Appendix shows the paths for output in the two experiments, which by construction are almost identical. Table 4 provides the details of this calibration exercise.$^{26}$

### B. Aggregate Dynamics

Figure 8 plots the dynamics of some key aggregate variables. We focus on three of the features of the data that arise in the model in response to the financial shock, but do not in response to the productivity shock.

First, the debt-output ratio drops by a magnitude that is comparable to the data and recovers with a similar persistence.$^{27}$ Second, aggregate labor productivity endogenously increases by 1.5 percent, close to the 2 percent increase over 2008–2010 measured by McGrattan and Prescott (2012). Tighter financial frictions prevent the expansion of firms. With DRS and firms constrained further away from their frictionless optimal size, labor productivity increases. This is especially true for fast-growing high $\sigma$ firms, which have a large optimal scale of production and are most affected by the financial constraint. Third, entry declines by 24 percent,

<table>
<thead>
<tr>
<th>Shock:</th>
<th>$X_0$</th>
<th>$X_1$</th>
<th>$% \Delta X$</th>
<th>$\rho_X$</th>
<th>Drop in GDP at impact %</th>
<th>% $\Delta$ GDP after 3 years (half-life)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity</td>
<td>$Z_t$</td>
<td>1.00</td>
<td>0.96</td>
<td>-4</td>
<td>0.9764</td>
<td>-10.1</td>
</tr>
<tr>
<td>Financial</td>
<td>$\varphi_t$</td>
<td>10.21</td>
<td>2.55</td>
<td>-75</td>
<td>0.9903</td>
<td>-9.9</td>
</tr>
<tr>
<td>Data (Fernald 2014)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-9.7</td>
<td>-5.1</td>
</tr>
</tbody>
</table>

---

$^{24}$Online Appendix C provides details on the solution of the model along these transitional dynamics.

$^{25}$See Figure 5 in Fernald (2015). Output is filtered using a biweight kernel with a bandwidth of 48 quarters.

$^{26}$The monthly frequency of the model and slow transition of the distribution of firms back to steady state require solving the transition dynamics over more than 1,200 periods, which is computationally expensive. We therefore economize by setting a grid of evenly spaced values for $X_1$ and $\rho_X$ for each shock and choose those values that minimize the distance between our two data points and the model.

$^{27}$In the United States since 2008, the debt-output ratio drops by nearly 10 percent and six years later is still 5 percent below its pre-recession level. See footnote 22 for the construction of aggregate debt.
which, again, approximates its empirical counterpart of 22 percent. Specifically, young-firm values decline sharply, since a large fraction of them are constrained (recall Figure 6), leading to a decline in start-ups. Overall, we conclude that the differential responses of these three variables clearly identify a financial shock in the 2008 recession.

Figure 9 displays the dynamics of the labor market. In both experiments, the response is close to its empirical counterpart shown in Figure 1. The financial shock induces larger and more persistent responses in vacancies, unemployment, and the job-finding rate. Under both scenarios, the decline in aggregate recruiting intensity is sizable, but its magnitude and persistence are, again, larger under the financial shock: Φ falls by 29 percent at impact (25 percent under the productivity shock) and five years later it is still 10 percent below its initial value (5 percent under the productivity shock).

We conclude that, in the model, the financial shock, the more promising candidate to rationalize the Great Recession based on our discussion of Figure 8, can explain more than half of the observed decline in aggregate match efficiency (recall the empirical path in Figure 1). We should not expect the decline in Φ to account for the entire decline in match efficiency. As discussed in the introduction, there are other forces at work. In Figure B3 in the online Appendix, we show that changes in the composition of the pool of job seekers can account for around one-third of the decline in match efficiency.

At first sight, it may be surprising that the response of aggregate recruiting intensity is not too dissimilar across the two shocks, despite the entry of new firms, which

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28 Entry in the data is measured as the number of firms reporting an age of zero in the US Census Bureau’s Longitudinal Business Database. Since the survey is annual, the measure excludes firms that enter and exit within a year.

29 In the data, labor market variables move more slowly, but recall that we specified shocks that declined sharply on impact.
account for a disproportionate share of job creation, differing remarkably under the two experiments. In what follows, we explain this apparent puzzle.

C. The Transmission Mechanism

To understand how macroeconomic shocks transmit to aggregate recruiting intensity, we return to our expression for $\Phi_t$, using $\lambda_t^h$ to denote the distribution of hiring firms:

$$
\Phi_t = \left( \frac{V_t^*}{V_t} \right)^{\alpha} = \left[ \int e^{it\left(\frac{V_{it}}{V_t}\right)} d\lambda_t^h \right]^{\alpha}.
$$

Substituting the policy function for recruitment effort (12) into the equation above and taking log differences, we obtain

$$
\Delta \log \Phi_t = -\alpha \frac{\gamma_2}{\gamma_1 + \gamma_2} \Delta \log q(\theta_t^+) + \alpha \Delta \log \left[ \int g^{\gamma_1 + \gamma_2} v_{it} \left( \frac{V_{it}}{V_t} \right) d\lambda_t^h \right].
$$

We call the two elements of this equation the slackness and composition effect, respectively.

The Slackness Effect.—The slackness effect is the change in aggregate recruiting intensity $\Phi_t$, accounted for by firms’ changing effort in response to movements in labor market slackness $q(\theta_t^+)$, holding constant growth rates $g_{it}$, vacancies $v_{it}$, and the distribution of hiring firms $\lambda_t^h$. 
In a recession, equilibrium labor market slackness increases, as a spike in job separations increases the measure of unemployed workers, while the reduction in expected profitability reduces vacancy creation. This surge in slackness raises the probability $q(\theta^*_t)$ that any vacancy matches with a job seeker. Therefore, given the hiring technology $g_{it} = q(\theta^*_t) e_{it} v_{it}/n_{it}$, a hiring firm with a target growth rate $g_{it}$ reoptimizes its combination of recruiting inputs $e_{it}$ and $v_{it}$ and decreases both: a slack labor market makes it easier for employers to hire, so employers spend less to attract workers. Since recruiting effort is more sensitive than vacancies to $q(\theta^*_t)$ (recall the decision rules (12) and (13)), the slackness effect is always stronger on the effort margin and, in the aggregate, $V^*_t$ declines more than $V_t$ or, equivalently, $\Phi_t$ falls in recessions.

The Composition Effect.—We define the composition effect residually, thereby including the impact on aggregate recruiting intensity of changes in the distribution of growth rates $g_{it}$ and vacancy policies $v_{it}$ among all hiring firms. Figure 10 shows how these two components of aggregate recruiting intensity respond to the shocks. These figures reveal that the slackness effect (dashed line) is quantitatively the largest, accounting for almost all of the decline in aggregate recruiting intensity (solid line).

The large magnitude of the slackness effect was, perhaps, expected. Market tightness plunges and the elasticity of firm-level recruiting intensity with respect to $q$ is high, nearly 1.\textsuperscript{30} What is more surprising is that the composition effect is so small.

\textsuperscript{30} We chose to express the slackness effect as a function of $\theta^*_t$ because this is a sufficient statistic for aggregate labor market conditions in the firm’s hiring problem. One can also obtain an expression for the slackness effect that is a function of the more common measure of tightness $\theta$. Substituting the relationship $q(\theta^*_t) = q(\theta^*_t) \Phi_t^{-1/\alpha}$ in (21) and collecting the terms in $\Phi_t$ yields the alternative representation of the slackness effect $-\alpha [\gamma_2/(\gamma_1 + \gamma_2)] \Delta \log(q(\theta^*_t))$. The denominator is less than one and captures a “multiplier”: when
and, in particular, after a drop at impact, it becomes positive, i.e., it induces a small countercyclical movement in $\Phi_t$. We now explain this result.

**Inspecting the Composition Effect.**—A useful approach is to split the composition effect into two further elements, which we plot in Figure 11. The first is a *direct composition effect*: the response to the shock in a partial-equilibrium economy, keeping $\theta_t^*$ at its steady-state level, denoted $\overline{\theta^*}$. The second is the *indirect composition effect*: the response in an economy under the equilibrium path for $\theta_t^*$ induced by the shock while keeping $\varphi_t$ at its steady-state value $\overline{\varphi}$.

The direct effect reduces aggregate recruiting intensity on impact, since the drop in the collateral parameter lowers firm growth rates and reallocates hiring away from young, fast-growing firms that account for the bulk of recruiting intensity in the economy. Note that the direct component reverts rapidly toward zero. This is due to the fact that decline in $\varphi_t$ induces positive selection among the hiring firms. The fraction of firms hiring drops from 55 percent in steady state to 22 percent following $\Phi$ is low in the aggregate, firms exert less effort $e$. This alternative decomposition gives very similar results: if anything, the slackness effect is somewhat stronger.

31 We illustrate this decomposition only for the tightening of the collateral constraint. Results for the productivity shock are almost identical.
the shock. So these firms, both incumbents and entrants, have on average higher productivity and thus grow slightly more: a force that pushes aggregate recruiting intensity back up.

The indirect effect increases aggregate recruiting intensity on impact. As \( q(\theta^*) \) rises, growing firms can meet job seekers more easily, intertemporally substituting their planned hiring. As they grow more quickly, they exert more recruiting effort, pushing up aggregate recruiting intensity. Selection of hiring firms on productivity tempers this effect as well: the increase in \( q(\theta^*) \) reduces the average productivity of hiring firms, since some firms that did not hire in steady state do hire under higher \( q(\theta^*) \), thereby dampening aggregate recruiting intensity.

Overall, the direct and indirect components show large movements, but these movements offset each other and the composition effect remains small throughout the transition.

Figure 12 provides another way to appreciate why the slackness effect is bound to dominate the composition effect. Panel A describes the behavior of the (unweighted) distribution of firm growth rates. Relative to steady state \((t = 0)\), in the period following the shock \((t = 2)\), firing firms contract faster and hiring firms expand slightly faster (thus, the dispersion of growth rates increases, as we discuss in some detail below). Panel B shows how the slackness effect contributes to lower recruiting intensity at any given hiring rate (recall equation (12)). These two panels show that the choice of hiring firms to change their effort as market tightness varies over time dominates the compositional changes across growth rates in the pool of hiring firms.

The analysis in this section highlights the role of general equilibrium forces in the dynamics of aggregate recruiting effort of firms. A casual look at the micro-economic relationship between job-filling and hiring rates may induce one to conclude that economy-wide recruiting intensity declines after a negative macro shock because the shock curtails the speed at which hiring firms expand. Such a force is present and reflects the direct composition effect. But this logic ignores the adjustment of equilibrium market tightness that sets in motion the slackness effect and the indirect composition effect, the other essential, and quantitatively dominant, pieces of the transmission mechanism.

**Relationship with Kaas and Kircher (2015).**—In Kaas and Kircher’s model of competitive search, aggregate recruiting intensity can be expressed as an average of meeting rates in each market, where each meeting rate is a concave function of market tightness. In terms of our notation, \( \Phi_t^{KK} = \int q(\theta_m)(v_m/V) dm \), where \( m \) indexes markets. The authors find that, during productivity-driven recessions, the dispersion of tightness across markets increases, leading to a decline in \( \Phi_t^{KK} \). They ascribe the procyclicality of aggregate recruiting intensity chiefly to this mechanism.

A version of this mechanism is present in our model as well. An increase in the standard deviation of growth rates will have a negative effect on \( \Phi \), since the second

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32 Figure B4 in online Appendix B plots the employment-weighted kernel density function of the distribution of firm-level growth rates in the model. This reproduces well its data counterpart, Figure 5 in Davis, Faberman, and Haltiwanger (2012a).

33 This mechanism is explained in Kaas and Kircher (2015, pp. 3053–54).
term in (21) is concave in $g_t$ (i.e., $\gamma_2/(\gamma_1 + \gamma_2) < 1$). Note that this source of fluctuations in $\Phi_t$ will enter exclusively into the composition effect.

Is this mechanism quantitatively important in our model? Our calibrated model is well suited to answer this question since (i) we match the empirical standard deviation of growth rates (recall Table 2), and (ii) the financial shock generates an empirically reasonable increase in dispersion: a 46 percent increase in the standard deviation of growth rates, compared to a 39 percent increase in the data (Bloom et al. 2012). To gauge the importance of this mechanism, we compare our measure to one in which this curvature effect is absent, computing

$$\alpha \Delta \log \left[ \int g_t^{\gamma_2/(\gamma_1 + \gamma_2)} \left( \frac{V_{it}}{V_t} \right) d\lambda_t^h \right] - \alpha \Delta \log \left[ \int g_t \left( \frac{V_{it}}{V_t} \right) d\lambda_t^h \right].$$

The first term is the composition effect, and the second is its counterpart where we raise the integral (not the integrand) to the exponent. Following a financial shock, we find that, between $t = 0$ and $t = 2$, this measure equals $-4$ percent. Therefore,

34 This is a notable feature of our model in response to a financial shock which provides another over-identifying test. Even without a shock to the dispersion of firm-level productivity growth, we attain a significant increase in the dispersion of employment growth: as explained above, the shock adversely affects some firms, reducing their growth rates, whereas some other firms respond to the surge in labor market slackness by growing faster.

35 We are in effect computing $E[f(X)] - f(E[X])$, where $f(X) = X^{\gamma_2/(\gamma_1 + \gamma_2)}$ and the random variable $X$ is the hiring firm growth rate, which is distributed with a density $h(g) = \frac{V_{it}}{V_t} \lambda_t^h$. 

**Figure 12. Growth Rate Distribution and Recruitment Policies before and after the Shock**

Notes: (i) In period 0 the economy is in steady state, in period 1 the path for the shock is realized, and period 2 then follows. We choose period 2 rather than period 1 since due to the timing of the model, workers that are fired in period 1 do not enter the labor market as unemployed workers until period 2. (ii) The growth rate distribution is computed over bins of width 0.05.
this mechanism explains only around 15 percent of the decline in aggregate recruiting intensity generated by our model. Its contribution is limited by the fact that empirically $\frac{\gamma_2}{(\gamma_1 + \gamma_2)}$ is close to 1, so the degree of concavity of the integrand in the composition effect is small. We conclude that the key transmission mechanism of our model, the slackness effect, is different from that emphasized by Kaas and Kircher (2015).

### D. Cross-Sectional Dynamics

In addition to explaining the aggregate dynamics of the vacancy yield through recruiting intensity, our model also accounts for the cross-sectional dynamics of vacancy yields by size, as documented by Moscarini and Postel-Vinay (2016). We argue that the heterogeneity in the extent to which the financial friction binds across firms is key to understanding the latter.

We begin by splitting firms in the model into financially constrained firms and unconstrained firms. Panel A of Figure 13 shows that recruiting intensity dynamics differ markedly between the two types of firms. Among constrained firms, the financial shock causes a sharp drop in the growth rate and, therefore, in the recruiting intensity of those hiring. Unconstrained hiring firms, instead, increase their hiring in response to the surge in labor market slackness, choosing higher recruiting intensity. The constrained firms drive the direct component of the composition effect, whereas unconstrained firms drive the indirect component.

Turning to size, which is observable in JOLTS, panel B shows that, following the macro shock, the fraction of constrained firms rises significantly across all sizes but does so in particular among large firms. In the model, these are firms with a high span of control parameter $(\sigma_H)$ that are still far from their optimal size and growing quickly. Panel C illustrates that the vacancy yield of these large firms is flat: reduced recruitment effort due to the financial shock offsets the effect of a slacker labor market, which would usually lead to higher vacancy yields in a recession. Meanwhile, the vacancy yield of small firms increases, as they receive the full effect of a slacker labor market.

Panel D shows that this narrative implied by our model is borne out in the data and provides an over-identifying test of the model. During the Great Recession, the vacancy yields of small establishments increased substantially, whereas large establishments remained flat. It also confirms that our characterization of the labor market, comprising fast-growing high-scale firms responding directly to a macroeconomic shock, and small low-scale firms responding indirectly to market tightness, is a useful lens for thinking about hiring dynamics in the US labor market.

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36 Financially constrained firms in the model are firms for which both the collateral and dividend constraints bind.
In this section, we analyze the robustness of our main finding regarding how shocks are transmitted to aggregate recruiting intensity: a large slackness effect and a composition effect strongly tempered by its indirect component.

We start by examining three mechanisms that could lead to a larger composition effect: first, a lower elasticity of the vacancy filling rate to market tightness; second, permanent heterogeneity in vacancy filling rates across industries, for example, due to different recruiting methods; third, the inclusion of replacement hires and on-the-job search. Finally, we reflect on whether the strong offsetting force that counteracts the composition effect is only germane to financial frictions or would, possibly, survive under other mechanisms that may underlie the observed rich firm dynamics by age and size.

A. Alternative Calibration of the Matching Function

As is clear from equation (21), the magnitude of the composition effect is especially sensitive to the value of $\alpha$, the elasticity of hires with respect to vacancies. Figure 14 plots the response of aggregate recruiting intensity (panel A) and the composition effect (panel B) for three values of $\alpha$ in the neighborhood of existing estimates. In the range below 0.5, our baseline value, the total composition effect is small at impact and turns positive quickly as its indirect component takes over. However, for $\alpha = 0.7$, the composition effect becomes sizable at impact and remains negative for almost a year after the shock.

To understand this result, note that the strength of the indirect component of the composition effect (due to firm growth during a recession as labor market tightness falls) is determined by how much the meeting rate $\log(q_t) = -(1-\alpha)\log \theta_t^*$ rises.
in a downturn. A larger value of $\alpha$ mutes the increase in $q_t$, dampening the countercyclical indirect component, and inducing larger procyclical movements in the composition effect. A stronger composition effect implies a deeper drop in $\Phi_t$, as shown in panel A of Figure 14.37

B. Sectoral Heterogeneity in Recruiting Technology

Ours is a one-sector model of the aggregate economy in which all firms face the same recruiting technology. DFH document that different sectors of the economy display consistently different vacancy yields. To the extent that such discrepancies in vacancy yields stem from systematic differences in growth rates across sectors, then our model will capture these since we generate a realistic distribution of firm growth rates (Table 3).38 If, however, they are due to permanent characteristics of the recruiting technology across sectors, then a macro shock that changes sectoral composition of vacancies will affect aggregate match efficiency and should appear in the composition effect.39

In the context of the Great Recession, this point is especially relevant. The Construction sector is an outlier in terms of its frictional characteristics (its vacancy yield is about 2.5 times as large as in the economy as a whole), and it was hit particularly hard in the recession. One would therefore expect Construction to play a significant role in the national movement of aggregate recruiting intensity, despite its small share of employment (Davis, Faberman, and Haltiwanger 2012b).

37 Note that the slackness effect is not as sensitive to $\alpha$ because, as seen in equation (21), $\log q$, which contains the term $1 - \alpha$ is also multiplied by $\alpha$.

38 Indeed, DFH Figure B5 shows that the cross-sector variation in average hiring rates is strongly correlated with the cross-sector variation in vacancy yields.

39 We thank Steve Davis for suggestions that led to the inclusion of this section.
A fully specified multisector model is beyond the scope of this paper, but we can nevertheless estimate the size of this sectoral composition effect using the structure of our model and industry-level data on vacancy yields and vacancy shares from JOLTS. Suppose that the firm-level hiring technology in each sector \( s = 1, \ldots, S \) is subject to a sector-specific recruitment efficiency shifter \( \phi_s \),

\[
\tag{22} h_{ist} = \phi_s q(\theta_t^*) e_{ist} v_{ist},
\]

leading to a modified expression for aggregate recruiting intensity,

\[
\Phi_t = \left[ \int_i \phi_s e_{ist} v_{ist} dt \right]^\alpha, \tag{23}
\]

and the optimal choice of firm-level recruiting intensity,

\[
\tag{24} e_{ist} = \text{Constant} \times \phi_s \frac{\gamma_2}{\gamma_1 + \gamma_2} q(\theta_t^*) \frac{\gamma_2}{\gamma_1 + \gamma_2} g_{ist} \frac{v_{ist}}{V_t}.
\]

Firm-level recruiting intensity depends negatively on sector-specific efficiency since firms belonging to sectors with a high recruiting efficiency can use less effort to realize any desired growth rate.

To decompose aggregate recruiting intensity, we can again substitute the optimal policy (24) into (23) to arrive at

\[
\Phi_t = \text{Constant} \times q(\theta_t^*) \frac{\gamma_2}{\gamma_1 + \gamma_2} \times \left[ \sum_{s=1}^{S} \phi_s \frac{\gamma_1}{\gamma_1 + \gamma_2} \int_{i \in s} \left[ g_{ist} \frac{v_{ist}}{V_t} \right] dt \right]^\alpha.
\]

The effect we are trying to determine comes from the interaction of permanent differences in match efficiencies across sectors \( \phi_s \) and the sectoral composition of hiring firms given by the vacancy share \( v_{ist}/V_t \). Therefore, we assume that the distribution of growth rates and vacancies is identical within each sector and, thus, the integral term is constant across sectors. Under this assumption, we obtain a counterpart to our previous decomposition of aggregate recruiting intensity, with an additional term characterizing the sectoral composition effect:

\[
\tag{25} \Delta \log \Phi_t = -\alpha \frac{\gamma_2}{\gamma_1 + \gamma_2} \Delta \log q(\theta_t^*)
\]

\[
+ \alpha \Delta \log \left[ \int_i s^2 \frac{\gamma_2}{\gamma_1 + \gamma_2} \frac{v_{ist}}{V_t} dt \right] + \alpha \Delta \log \left[ \sum_{s=1}^{S} \phi_s \frac{\gamma_1}{\gamma_1 + \gamma_2} \frac{v_{ist}}{V_t} \right].
\]

\[\text{Sectoral composition effect}\]

\[\text{In what follows, we maintain the assumption that all firms hire in the same labor market. Accordingly, one could read our exercise as the counterpart to one conducted on the worker side, in which different groups of job seekers enter the same labor market but are weighted by some fixed level of search efficiency. For example, see Hall and Schulhofer-Wohl (2015) and Hornstein and Kudlyak (2016).}\]
Note that the exponent on $\phi_s$ is less than 1. A sector with a higher $\phi_s$ will be exogenously more productive in creating matches, increasing recruiting intensity with an elasticity of 1 with respect to its vacancy share; however, the firms in that sector will endogenously decrease effort with an elasticity of $\frac{\gamma_2}{\gamma_1+\gamma_2}$, leaving the net elasticity of $\frac{\gamma_1}{\gamma_1+\gamma_2}$.

Computing the last term in (25) requires data on vacancy shares by sector, readily available from JOLTS and data on sectoral match efficiency. Under our assumptions, (22) and (23) imply

$$\frac{\gamma_1}{\phi_s^{\gamma_1+\gamma_2}} = \frac{H_{st}/V_{st}}{H_{kt}/V_{kt}}$$

where match efficiency of the sector $k$ is normalized to 1 without loss of generality.\footnote{To estimate $\phi_s$, we use ratios of average sectoral vacancy yields from 2005 to 2006. We take Professional Business Services as the normalizing sector, since its average vacancy yield of 1.50 is the sectoral median.}

Using data on all 11 two-digit industries from JOLTS, we plot the sectoral component $\phi_s^{\gamma_1+\gamma_2} V_{st}$ for the largest 7 sectors in panel A of Figure 15 and the total sectoral composition effect in panel B. We find that this component generates an additional 4 percent drop in aggregate recruiting intensity during the Great Recession, mostly due to the decline in the vacancy shares of Construction, Manufacturing, and Hospitality and Leisure. Even though adding this mechanism shifts the decomposition more toward the composition effect, we tentatively conclude that it does not modify our conclusion that the slackness channel is dominant. Obviously, the economy has a lot more structural heterogeneity than that implied by our coarse partition into 11 industries. Incorporating additional relevant sources of heterogeneity remains an open area for future research.

**C. Replacement Hiring and On-the-Job Search**

In our baseline model, we have abstracted from replacement hiring associated with quits and search on the job, two related and prominent features of labor markets. We now assess to what extent these omissions could affect our conclusions.

**A Larger Composition Effect with Quits and Replacement Hiring?**—We have solved the model under a range of values for an exogenous quit rate between 1 and 3 percent per month, and found our results to be quantitatively very robust. The reason is that in our model, as in the data, the bulk of hires are made by firms with positive net hiring rates. However, the assumption of a quit rate that is invariant in the cross section and over time is stylized. Such an assumption might lead us to understate the composition effect.

In the cross section, the data show that among shrinking firms quits are especially high, but at the same time some of these firms display positive gross hiring rates.\footnote{For example, a negative productivity shock leads a firm to fire some of its worst workers. Meanwhile, some of its best workers quit to find a more productive match, leaving the firm to replace some of these quits with new hires.}
An adverse aggregate shock pushes many firms into this position of negative net growth but positive replacement hiring. In our model, these firms contribute to the determination of aggregate recruiting intensity before the shock (because they hire) but not after the shock (because downsizing firms do not hire). If the model allowed for replacement hires, some of these firms would instead contribute to aggregate recruiting intensity even after the shock, and do so with a lower gross hiring rate and, therefore, a lower recruiting intensity.\footnote{More specifically, they would be part of the integral in the second term of (21).} In our analysis, this shift would be captured by a more negative composition effect.

In the aggregate, the data show that quits are strongly procyclical, falling sharply during a recession. The average gross hiring rate therefore also declines, because of a reduction in replacement hires, and as a consequence, recruiting intensity decreases across all hiring firms. In our analysis, this result would also be captured by a more negative composition effect.

A Smaller Slackness Effect with On-the-Job Search?—When a large portion of job seekers are employed, the response of market tightness to spikes in layoffs to unemployment, such as those following financial and productivity shocks, would be smaller. This mechanism has the potential to weaken the slackness effect.

![Figure 15. Sectoral Composition Effect](image)

Notes: Panel A plots the sectoral components from the last term in equation (25) in the seven largest industries. In panel B, the log of the sectoral composition effect is normalized to zero in January 2008.
In an economy where firms engage in take-it-or-leave-it offers to risk-neutral workers, modeling search on the job is relatively simple once it is assumed that (i) firms commit to not responding to the poaching competitor when an employed worker receives an outside offer, and (ii) the worker, who is indifferent between staying and going, quits. In addition, we make the following minimal amendments to the model: (i) all employed workers search with a relative search intensity of \( s \) determining the effective units of search of an employed worker relative to an unemployed worker (whose intensity is normalized to 1); and (ii) the matching function is modified to take the total measure of effective search units \( S_t = U_t + s N_t \) as an input, where \( N_t = \bar{L} - U_t \) is the measure of employed workers. The firm-level hiring technology remains \( h_{it} = q(\theta^*_t) e_{it} v_{it} \), but the law of motion for firm employment is now

\[
n_{it+1} = n_{it} + h_{it} - f_{it} - sp(\theta^*_t) n_{it},
\]

where \( p(\theta^*_t) \) is the job-finding rate of the unemployed. By constant returns to scale in the matching functions, \( sp(\theta^*_t) \) is the job-finding rate of employed workers. As a result, the law of motion for unemployment becomes

\[
U_{t+1} = U_t + F_t - \left[ \frac{U_t}{U_t + s N_t} \right] H_t,
\]

where \( U_t / (U_t + s N_t) \) is the fraction of total hires that come from unemployment.

In choosing a value for on-the-job search intensity, note that \( s \) is equal to the ratio of employment-employment (EE) to unemployment-employment (UE) transition rates. Following Fujita and Moscarini (2017) and, thus, excluding recalls and workers on temporary layoffs from UE, we obtain \( s = 0.09 \) for the prerecession period.

What is the impact of on-the-job search on the slackness effect? Consider an increase in the firing rate due to a negative macro shock. In the baseline model, the monthly firing rate is \( F_t / N_t = 0.03 \). Suppose that this ratio were to spike in a recession, doubling. Without on-the-job search, the mass of effective search units increases nearly one for one, by 0.03. In the model with on-the-job search, \( S_t = (1 - s) U_t + s \bar{L} \), so although the number of unemployed workers rises by 0.03, the measure of total job seekers increases by \( (1 - s) \times 0.03 = 0.027 \). Therefore, labor market tightness falls by less and the slackness effect is somewhat weakened, as expected, but this correction is quantitatively small. The reason is that, although the stock of employed workers is large, their average search intensity is low relative to that of the unemployed. Moreover, if one were to also allow \( s \) to vary over the cycle and match the data, then the relative intensity of the employed would be countercyclical.\(^4\) This slackens the labor market further, thus making the total effect of on-the-job search on the dynamics of market tightness even smaller.

We acknowledge that ours is only a back-of-the-envelope calculation and that a thorough analysis would require a more satisfactory representation of on-the-job search behavior (i.e., one where workers are not indifferent between staying and quitting).

\(^4\) Figure B5 in online Appendix B documents the cyclicality of the relative search intensity of the employed.
moving). Frictional models of the labor market with both a realistic firm size distribution induced by DRS in production and a rich job ladder whereby high-productivity, high-wage firms can poach workers more easily from other firms, and thus the vacancy filling rate is increasing in the firm type because it is further up the ladder, have not yet been fully developed. Whether such class of models has novel forces at work relative to those emphasized here remains to be established.

D. Alternative Frictions

One of the main insights of our analysis is that financially unconstrained firms unravel the response of constrained firms and mitigate the composition effect of a macro shock on aggregate recruiting intensity. A natural question is whether this result is specific to models where the key source of firm dynamics over the life cycle is a financial friction? Frictions of a different nature may underlie the observed rich up-and-out dynamics of young firms: how robust is our insight to these generalizations?

Without fully solving alternative models, offering a precise answer to this question is challenging. However, we conjecture that our result is more general than it may appear at first sight. Any successful model of firm dynamics combined with labor market frictions would feature the following minimal set of ingredients: (i) heterogeneity across firms induced by idiosyncratic shocks, and (ii) a friction, over and above hiring costs tied to search/matching, that slows down growth for young firms over the life cycle and that, interacted with shocks, generates up-and-out dynamics. In our setup, (i) is generated by productivity shocks and (ii) by financial market imperfections.

A common property of any such model is that firms escape the friction conditional on surviving long enough. Consider, for example, two popular alternative sources of life-cycle dynamics: learning and customer capital. If the friction is tied to learning about one’s own productivity, after enough time in the market, much of the fixed individual productivity effect will be revealed. If the friction is tied to the necessity of attracting a customer base, after enough time in the market, the firm would have built a demand for its product. However, in both cases, even these older unconstrained firms are still subject to shocks that change their optimal size. Therefore, there will always be a share of firms in the economy that experience positive shocks and are gradually (because of the hiring costs) reaching their new higher employment target. When a negative aggregate shock hits the economy, many of these firms would still want to hire. As in our model, they would then take advantage of the slack labor market by growing even faster and exerting even more recruiting effort, counteracting the direct component of the composition effect.

While this logic suggests that the composition effect would remain relatively small across a wider range of firm dynamics models, ours remains a conjecture that can be verified only by explicitly solving and plausibly parameterizing these models.

45 Promising environments are those developed by Lentz and Mortensen (2008) with random search and Schaal (2017) with directed search. Even though they do not study the determinants of and the transmission of shocks to recruiting intensity, as we do, their frameworks lend naturally to addressing such questions as well.
VI. Conclusions

The existing literature on the cyclical fluctuations of aggregate match efficiency has focused almost exclusively on explanations involving the worker side of the labor market, such as occupational mismatch, shifts in job search intensity of the unemployed over the cycle, and compositional changes among the pool of job seekers. In this paper, we have shifted the focus to the firm side and, building on the microeconomic evidence in Davis, Faberman, and Haltiwanger (2013), developed a macroeconomic model of aggregate recruiting intensity.

The model, parameterized to replicate a range of cross-sectional facts about firm dynamics and hiring behavior, is able to explain about one-half of the collapse in aggregate match efficiency during the Great Recession through a sharp decline in firms’ recruiting intensity. Our analysis of the transmission mechanism points toward the importance of general equilibrium forces: aggregate recruiting intensity declined mainly because the number of available job seekers per vacancy increased (i.e., labor market tightness declined), making it easier for firms to achieve their recruitment targets without having to spend as much on recruitment costs. Changes in the within-sector composition of the pool of hiring firms, for example, due to the fall in new firm entry that is well matched by the model, did not play a large role. The shift in sectoral composition, in particular the bust in Construction and other sectors with structurally high job-filling rates, instead contributed to the measured deterioration in aggregate recruiting effort.

Besides its contribution to understanding the determinants of movements in match efficiency, and thus the job-finding rate (a key object for labor market analysis), our theory has broader implications for macroeconomics. First, as, for example, Faberman (2014) discusses, making progress in understanding how firms’ hiring decisions respond to macroeconomic conditions is important since job creation policies that fail to recognize the determinants of employers’ recruitment effort may fall short in achieving their goal. Our model predicts that subsidizing firm hiring (abstracting from the offsetting effects of higher tax rates) will increase the average firm growth rate and induce a rise in recruiting intensity, whereas a subsidy to workers’ job search that decreases market tightness will induce a decline in recruiting intensity, through the slackness effect discussed in the paper. Second, a richer model of employer recruiting behavior can lead to better estimates of the marginal cost of labor and, therefore, result in improved measures of the labor wedge and of the relative importance of labor and product market wedges (Bils, Klenow, and Malin 2018). In this respect, our model suggests that the price of labor faced by firms may be more procyclical than what would appear from naïvely using wages as a proxy.

REFERENCES


