Roberto Robatto, Balázs Szentes

On the biological foundation of risk preferences

Article (Accepted version)
(Refereed)

http://eprints.lse.ac.uk

Accepted Manuscript

On the biological foundation of risk preferences

Roberto Robatto, Balázs Szentes

Please cite this article in press as: Robatto, R., Szentes, B. On the biological foundation of risk preferences. *J. Econ. Theory* (2017), https://doi.org/10.1016/j.jet.2017.10.002

On the Biological Foundation of Risk Preferences[*]

Roberto Robatto[†]and Balázs Szentes[‡]

October 9, 2017

Abstract

This paper considers a continuous-time biological model in which the growth rate of a population is determined by the risk attitude of its individuals. We consider choices over lotteries which determine the number of offspring and involve both idiosyncratic and aggregate risks. We distinguish between two types of aggregate risk: *environmental variations* and *natural disasters*. Environmental variations influence the death and birth rates, while natural disasters result in instantaneous drops in population size.

Our main result is a utility representation of the evolutionary optimal behavior. The utility is additively separable in the two types of aggregate risk. The term involving environmental variations is a von Neumann-Morgenstern utility which induces the same attitude towards both idiosyncratic and aggregate risk. The term involving disasters cannot be interpreted as an expected utility maximization and induces less tolerance towards aggregate risk.

JEL classification: D81, D91

Keywords: Risk preferences; Evolution; Expected Utility

1 Introduction

Most models in economics take preferences as given and derive the choices induced by those preferences. This paper does just the opposite. We entertain the hypothesis that choice behaviours are genetically determined and shaped by natural selection. The underlying individual preferences are then merely the representations of those evolutionary optimal choice behaviours. We work from the basic premise that, in the long-run context of evolution, only the fastest-growing genes survive. As this paper focuses on risk preferences, we consider choices over lotteries that affect the reproductive value of individuals. Our main result is a utility representation of the optimal choice behaviour.

The cornerstone of our analysis is a characterization of the long run dynamics of a population that inhabits a risky environment. Two types of aggregate risk are present, which we refer to as environmental variations and natural disasters. Environmental variations influence the birth and death rates of individuals, determining the rate of increase of the population size. Meanwhile, natural disasters cause discrete drops in population size, affecting the level as opposed to the slope. The reproductive fitness of an individual is also subject to idiosyncratic risk; that is, conditional on an environmental variation or a natural disaster, the birth rate and the survival probability are still random variables. Our main result consists of the characterization of the asymptotic growth rate as a function of the ergodic distributions of the various types of risk. This function can then be interpreted as the utility representation of the evolutionary optimal choice behaviour over lotteries in an environment where the risk is determined by individuals' choices.

In order to better explain our contribution and to contrast our results with those in existence, we first describe Robson's (1996a) seminal paper on the evolution of risk preferences. Time is discrete and individuals live for one period. The number of offspring an individual has is determined by the realization of a lottery. A lottery, L, is described by a triple, $\left(\Omega, G, \{F(\cdot|\omega)\}_{\omega \in \Omega}\right)$, where Ω denotes the set of possible states of the world, G is the ergodic distribution on Ω, and $F(\cdot|\omega)$ is the distribution on the number of offspring if the state of the world is ω. Conditional on ω, the realization of $F(\cdot|\omega)$ is independent across individuals, so this element represents the idiosyncratic component of the risk. The distribution G represents the aggregate risk, since ω determines the distribution of reproductive values in the population. Robson (1996a) shows that the asymptotic growth rate of the population is

$$u(L) = \int \log\left(\int \eta dF(\eta|\omega)\right) dG(\omega). \tag{1}$$

Since the gene inducing the choice of the lottery L grows at rate $u(L)$ in the long run, Robson (1996a) interprets u as the utility representation of the evolutionary optimal behaviour. The main implication of (1) is that the evolved attitude towards risk depends strongly on whether the risk is idiosyncratic or aggregate. In fact, individuals will be relatively less tolerant of aggregate risk, compared to idiosyncratic risk.

In contrast to Robson (1996a), we consider a continuous time model, which allows us to distinguish between two types of aggregate risk: environmental variations and natural disasters. Recall that environmental shocks determine the rate of increase of the population size, while natural disasters cause instantaneous changes in the level. In discrete time, any aggregate shock necessarily results in a discrete change in the population size. It then becomes natural to ask whether the results obtained by Robson (1996a) apply only to natural disasters, or to aggregate risk in general. Our answer is that his results do not hold for environmental variations: the evolutionary optimal behaviour induces the same attitude towards both idiosyncratic risk and aggregate risk due to environmental variations.

In order to state our representation theorem, let us describe our setup in detail. Time is

2

continuous, and population dynamics are determined by a lottery $L = (\Omega, G, \lambda, F_n(\cdot|\omega), F_e(\cdot|\omega))$. The set Ω denotes the set of states of the world and G is the ergodic distribution on Ω. The function $\lambda : \Omega \to \mathbb{R}$ is the arrival rate of a disaster. The c.d.f. $F_n(\cdot|\omega)$ denotes the distribution of the survival probability at the moment a natural disaster occurs. $F_e(\cdot|\omega)$ denotes the distribution of the net birth rate (birth rate minus death rate) of an individual conditional on ω. Our main result is that the asymptotic growth rate, $U(L)$, is

$$U(L) = \int \int \varepsilon dF_e(\varepsilon|\omega) \, dG(\omega) + \int \lambda(\omega) \log \left(\int \eta dF_n(\eta|\omega) \right) dG(\omega). \tag{2}$$

The two additive terms in the utility representation (2) have very different economic interpretations. The first term, which is associated with environmental variations, is a standard von Neumann-Morgenstern representation with Bernoulli utility as the identity function. This implies that the choice induced by U depends only on the expected value of the net birth rate; it doesn't matter whether the source of the risk is aggregate or idiosyncratic. The second term, which is associated with natural disasters, is analogous to (1) of Robson (1996a). Indeed, if $\lambda(\omega) \equiv 1$, which corresponds to the frequency of discrete changes in Robson (1996a), the two expressions coincide. Let us again point out that this term is not a von Neumann-Morgenstern representation of risk attitude. Interestingly, this term is formally identical to a smooth ambiguity-averse preference representation where the ambiguity is determined by G and the ambiguity aversion is determined by the logarithm function.[1]

We think that the discrete-time model of Robson (1996a) is useful to describe not only natural disasters but also environments in which risk cannot naturally be characterized in terms of birth and death rates, even in the absence of disasters. This may be exemplified by plant and animal species that reproduce periodically at particular times of the year. Consider an annual plant which reproduces only once in its lifetime and whose life cycle lasts one year. Although the plant might be exposed to various risks each day, this risk can intuitively be summarized using annual quantities, such as the probability that a seed survives until the reproductive season and the number of new seeds produced by the plant. As such, reproduction can be characterized in terms of *factors* rather than rates. The model of Robson (1996a) can be viewed as a description of the risk faced by such a plant in the case in which the plant always survives until the reproductive season and produces η seeds. Indeed, the appearance of the logarithmic function in the optimal choice criterion was first noticed in the context of annual organisms; see for example Cohen (1966). However, human reproduction occurs throughout the year rather than being confined to distinct breeding seasons.[2] Approximating human population dynamics by a discrete-time model requires us to group together all risks affecting the population in a given time period and to characterize reproductive values in terms of factors. Our continuous-time model allows us to describe risks in greater detail and to

---

[1] For an axiomatic characterization of smooth ambiguity averse preferences, see Klibanoff et al. (2005).

[2] Of course, risks faced by humans may be seasonal and this can be incorporated into the stochastic evolution of the states in our continuous model.

express reproductive values in terms of birth and death rates. In the absence of natural disasters, this leads to an expected-utility representation (see the first term in (2)), in contrast to Robson's non-expected utility representation in (1). We interpret our representation theorem as providing at least a partial evolutionary justification for von Neumann-Morgenstern preferences.

Numerous papers use evolution to explain preferences. The first is probably Becker (1976), who adopts an evolutionary argument to explain altruism. Overviews of the theories on biological foundations of economic behaviour can be found in Robson (2001 and 2002).

Robson (1996b) and Dekel and Scotchmer (1999) analyse optimal risk-taking strategic biological models, that is, an individual's reproductive value is determined by not only her lottery but also by the lotteries of others. Our model does not involves strategic interactions; individuals only solve decision problems. Indeed, it is not clear to us how one might identify preferences and strategic components from equilibrium behaviour.

The differences between the effects of idiosyncratic and aggregate risks on preferences are also emphasized by Robson and Samuelson (2009) in the context of time preferences. The authors consider an age-structured biological model and show that, if the effects of aggregate shocks on an individual's survival probability do not depend on the age of the individual, then aggregate risk slows down population growth. In Section A of our Online Appendix, we extend our results to an age-structured population. Among other things, we show that, if the effects of aggregate shocks on death rates are age-independent, the population growth is fully determined by the expected death rates, and there is no distinction between the aggregate and idiosyncratic components.

2 Model

Time is continuous and is indexed by $t \in \mathbb{R}_+$. At each moment, a continuum of individuals make up the population. The population dynamics are governed by the lottery

$$L = \left(\Omega, G, \{\lambda(\omega)\}_{\omega \in \Omega}, \{F_e(\cdot|\omega)\}_{\omega \in \Omega}, \{F_n(\cdot|\omega)\}_{\omega \in \Omega}\right). \tag{3}$$

$\Omega \subset \mathbb{R}$ is the set of possible states of the world. Let ω_t denote the state of the world at time t. The dynamic process $\{\omega_t\}_{t \in \mathbb{R}}$ is a Markov process with unique ergodic distribution G.[3] We assume that ω_t is almost surely continuous in t almost everywhere.

Environmental variations.— The net birth rate of an individual at time t is $\varepsilon_t = b_t - \delta_t$, where b_t denotes her birth rate and δ_t denotes her death rate. The variable $\varepsilon_t \in \mathbb{R}$ is distributed according to the conditional c.d.f. $F_e(\varepsilon_t|\omega_t)$ and it is measurable in t almost surely. We allow the net birth

---

[3]In particular, whenever $\int_A 1 dG(\omega) > 0$ then for each $\omega' \in \Omega$, conditional on $\omega_0 = \omega'$:

$$\int I(\omega_t \in A) \, dt = +\infty,$$

almost surely. That is, conditional on any initial condition, the process spends an infinite amount of time in every positive G-measure set with probability one (see e.g. Glynn (1994) and Duffie and Glynn (2004)).

4

rate of an individual, ε_t, to change over time even if ω_t stays constant. We assume that $\mathbb{E}\left(\varepsilon|\omega\right)$ is a bounded function of ω, and $F_e\left(\varepsilon|\omega\right)$ is uniformly continuous in ω. The realizations of $\varepsilon_t's$ are assumed to be independent across individuals conditional on ω_t. We assume that the Law of Large Numbers holds, and hence, at time t the population grows at rate

$$r\left(\omega_t\right) \equiv \int \varepsilon dF_e\left(\varepsilon|\omega_t\right). \tag{4}$$

Natural disasters.— Natural disasters hit the population stochastically according to a non-homogenous Poisson process. The arrival rate of the process at time t is $\lambda\left(\omega_t\right) \in [0,\infty)$, where $\lambda : \Omega \to \mathbb{R}_+$ is a bounded, measurable function in $L^1\left(\Omega, G\right)$. If a natural disaster occurs at time t, an individual survives with probability $\eta_t \in [0,1]^4$, and $F_n\left(\cdot|\omega_t\right)$ denotes the distribution of η_t conditional on ω_t. We assume that $F_n\left(\eta|\omega\right)$ is uniformly continuous in ω. We also assume that $\mathbb{E}\left(\eta|\omega\right)$ is uniformly bounded away from zero; that is, the population never goes extinct. Again, the realization of η_t is independent across individuals conditional on ω_t. This assumption essentially implies that the survival probability η_t is re-drawn at each moment a natural disaster occurs, and hence, the distribution of η_t is indeed independent of the previous history of disasters. We appeal to the Law of Large Numbers once more and assume that the fraction of the population that survives a natural disaster is

$$j\left(\omega_t\right) \equiv \int \eta dF_n\left(\eta|\omega_t\right). \tag{5}$$

We further assume that, for each individual, ε_t and η_t are independent at each time t conditional on ω_t.

3   Results

In this section we analyze the speed of population growth. Let y_t denote the size of the population at time t and let y_0 be normalized to one. If there is a natural disaster at time t, y_t denotes the size of the surviving population. The basic difficulty is that, due to aggregate shocks, the population does not grow at a steady state rate. Nevertheless, it is possible to characterize a growth rate, g, such that if t is large enough, the size of the population is approximately the same as if it were growing at a constant rate g, that is,

$$y_t \approx e^{gt}. \tag{6}$$

Such a growth rate g is called the *continuously compounded growth rate* of the population and is formally defined in Section 3.2.

Next, we derive an expression for the law of motion of the population along a realized path of the random variables. In Section 3.2, we use this expression to prove our main result, which is a characterization of the continuously compounded growth rate in terms of the lottery (3).

---

[4]For our mathematical results to hold, we do not need η_t to be weakly less than one. We only make this assumption for the sake of interpretation.

3.1 Law of motion of the population

Let $N(t)$ denote the number of natural disasters which occurred between time zero and time t. In addition, let $\tau_i \in [0, t]$, $i \in \{1, ..., N(t)\}$, denote the arrival time of the ith natural disaster.

Proposition 1 *The size of the population at time t is given by:*

$$y_t = \exp\left[\int_0^t r(\omega_s)\, ds\right] \prod_{i=1}^{N(t)} j(\omega_{\tau_i}),\tag{7}$$

where $\prod_{i=1}^{N(t)} j(\omega_{\tau_i})$ is defined to be one if $N(t) = 0$.

A notable property of the expression on the right-hand side of (7) is that it is multiplicatively separable in natural disasters and environmental variations. To illustrate this observation, let us consider a population that grows at a constant rate for a unit amount of time. Suppose that this population is hit by a disaster and half of the population dies. Then, irrespective of the exact time of the disaster, the population at the end of the time period will be half as large as it would have been if the disaster had not occurred. Of course, this argument presumes that the disaster leaves the net birth rate of the surviving population unaffected. In our model, this assumption is satisfied because ε_t and η_t are independent conditional on ω_t.

Proof. We prove the proposition by induction with respect to the number of natural disasters, $N(t)$. Suppose first that $N(t) = 0$. Then, by (4), the law of motion of the population is described by the following differential equation between time zero and time t:

$$\dot{y}_\tau = y_\tau r(\omega_\tau).\tag{8}$$

The solution of this differential equation is[5]

$$y_t = \exp\left(\int_0^t r(\omega_s)\, ds\right),$$

which is just the statement of the proposition for $N(t) = 0$.

Suppose that the statement of the proposition is true for all t whenever $N(t) \leq k$ and let us assume that $N(t) = k + 1$. By the inductive hypothesis,

$$\lim_{\substack{\tau \to \tau_{N(t)} \\ \tau < \tau_{N(t)}}} y_\tau = \exp\left\{\int_0^{\tau_{N_t}} r(\omega_s)\, ds\right\} \prod_{i=1}^{N(t)-1} j(\omega_{\tau_i}).$$

At time $\tau_{N(t)}$ there is a natural disaster and, by (5), only a fraction $j(\omega_{\tau_{N(t)}})$ of the population survives. Hence,

$$y_{\tau_{N(t)}} = \exp\left\{\int_0^{\tau_{N_t}} r(\omega_s)\, ds\right\} \prod_{i=1}^{N(t)} j(\omega_{\tau_i}).\tag{9}$$

---

[5]The solution exists because r is continuous and bounded, see Coddington and Levinson (1955), Chapter 2.

The law of motion of the population for $\tau \in [\tau_{N(t)}, t]$ is again described by the differential equation (8) and initial condition (9). The solution is

$$y_t = y_{\tau_{N(t)}} \exp\left(\int_{\tau_{N(t)}}^{t} r(\omega_s)\,ds\right) = \exp\left\{\int_0^t r(\omega_s)\,ds\right\} \prod_{i=1}^{N(t)} j(\omega_{\tau_i}),$$

where the second equality follows from (9). ∎

3.2 Continuously compounded growth rate

Motivated by (6), we first provide a formal definition for the continuously compounded growth rate.

Definition 1 *We call the number $g \in \mathbb{R}$ the continuously compounded growth rate of the population if*

$$\lim_{t \to \infty} \frac{\log y_t}{t} = g$$

almost surely.

In order to see that the continuously compounded growth rate is indeed a useful object in the evolutionary context, consider two populations, y_t^1 and y_t^2, with corresponding compounded growth rates g_1 and g_2. We show that if $g_1 > g_2$ then, asymptotically and with probability one, y_t^1 is going to be infinitely large relative to y_t^2. To see this, note that, by Definition 1,

$$\lim_{t \to \infty} \frac{\log \frac{y_t^1}{y_t^2}}{t} = \lim_{t \to \infty} \frac{\log y_t^1}{t} - \lim_{t \to \infty} \frac{\log y_t^2}{t} = g_1 - g_2 > 0$$

almost surely. But this can only be the case if y_t^1/y_t^2 converges to infinity as t goes to infinity with probability one.

Next, we show that the continuously compounded growth rate exists, and characterize it in terms of the lottery, L. This is our main result.

Theorem 1 *For almost all $(\{\omega_t\}, \{N(t)\})_{t \in \mathbb{R}}$,*

$$\lim_{t \to \infty} \left(\frac{\log y_t}{t}\right) = \int \int \varepsilon \, dF_e(\varepsilon|\omega)\, dG(\omega) + \int \lambda(\omega) \log\left(\int \eta \, dF_n(\eta|\omega)\right) dG(\omega). \tag{10}$$

Let us explain the basic idea of the proof. By Proposition 1,

$$\lim_{t \to \infty} \frac{\log y_t}{t} = \lim_{t \to \infty} \frac{\int_0^t r(\omega_s)\,ds}{t} + \lim_{t \to \infty} \frac{\sum_{i=1}^{N(t)} \log j(\omega_{\tau_i})}{t}, \tag{11}$$

where the second term in the right-hand side is defined to be zero if $N(t) = 0$. Since ω_t is an ergodic process, both $r(\omega)$ and $\log j(\omega)$ are also ergodic. As a consequence, the right-hand side of (10) is the sum of the time averages of two ergodic variables. Birkhoff's Ergodic Theorem states that, under certain conditions, the time average of the realization of an ergodic variable converges

7

to the expected value of the variable with probability one, where the expectations are formed according to the ergodic distribution. In the proof, we argue that Birkhoff's Ergodic Theorem is applicable, and show that the time average of $r(\omega)$ and $\log j(\omega)$ converge to the first and second terms in the right-hand side of (10), respectively.

Proof. First, consider the time average of $r(\omega_t)$. Since the state of the world has an ergodic distribution and r is continuous, Birkhoff's Ergodic Theorem implies that

$$\lim_{t \to \infty} \frac{\int_0^t r(\omega_s)\, ds}{t} = \int r(\omega)\, dG(\omega)$$

almost surely[6]. Substituting the definition of the function r in (4), we obtain

$$\lim_{t \to \infty} \frac{\int_0^t r(\omega_s)\, ds}{t} = \int \int \varepsilon dF_e(\varepsilon|\omega)\, dG(\omega). \tag{12}$$

We now turn to the second expression on the right-hand side of (11) and rewrite it as

$$\frac{N(t)}{t} \frac{\sum_{i=1}^{N(t)} \log j(\omega_{\tau_i})}{N(t)}.$$

If $\int \lambda(\omega)\, dG(\omega) = 0$ then $\lambda = N(t) = 0$ almost surely, so (12) implies (10). In what follows we restrict our attention to the case in which $\int \lambda(\omega)\, dG(\omega) > 0$. We first show that

$$\lim_{t \to \infty} \frac{N(t)}{t} = \int \lambda(\omega)\, dG(\omega) \tag{13}$$

almost surely. To this end, assume that $t \in \mathbb{N}$ for simplicity and define $X_i = N(i) - N(i-1)$ for all $i \in \mathbb{N}$. Since $N(t)$ is a non-homogenous Poisson process, the random variables $X_1, X_2, \ldots$ are independent conditional on the realization of $\{\omega_t\}_{t \in \mathbb{R}_+}$, and $\mathbb{E}X_i = Var(X_i) = \int_{i-1}^i \lambda(\omega_\tau)\, d\tau$. By Kolmogorov's Strong Law of Large Numbers[7]

$$\lim_{t \to \infty} \frac{N(t) - \int_0^t \lambda(\omega_\tau)\, d\tau}{t} = \lim_{t \to \infty} \frac{\sum_{i=1}^t \left[X_i - \int_{i-1}^i \lambda(\omega_\tau)\, d\tau \right]}{t} = 0 \tag{14}$$

almost surely conditional on $\{\omega_t\}_{t \in \mathbb{R}_+}$. Finally, notice that Birkhoff's Ergodic Theorem implies that

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t \lambda(\omega_s)\, ds = \int \lambda(\omega)\, dG(\omega) \tag{15}$$

almost surely. From (14) and (15), the equation in (13) follows.

Next, we show that

$$\lim_{t \to \infty} \frac{\sum_{i=1}^{N(t)} \log j(\omega_{\tau_i})}{N(t)} = \int \log j(\omega) \frac{\lambda(\omega)}{\int \lambda(\omega')\, dG(\omega')} dG(\omega) \tag{16}$$

---

[6]See, for instance, Doob (1953), Chapter XI, for a version of Birkhoff's Ergodic Theorem for continuous-time processes.

[7]This theorem is indeed applicable since (See e.g. Feller (1968), Chapter X.7):

$$\sum_{i=1}^\infty \frac{Var[N(s_i) - N(s_{i-1})]}{i^2} = \lim_{i \to \infty} \frac{1}{i^2} \int_0^i \lambda(\omega_\tau)\, d\tau = \lim_{i \to \infty} \frac{1}{i} \int \lambda(\omega)\, dG(\omega) = 0 < \infty.$$

8

almost surely. Recall that τ_i denotes the time of the ith natural disaster. For all $i \in \mathbb{N}$, define $Z_i = \log j (\omega_{\tau_i})$. By (13) and $\int \lambda(\omega) \, dG(\omega) > 0$ it follows that there are infinitely many disasters with probability one, so these variables are well defined. The discrete time process $\{Z_i\}_{i \in \mathbb{N}}$ is also an ergodic Markov process, and its ergodic distribution, $\hat{G}$, is given by[8]

$$\int_{\{\omega : \log j(\omega) \in B\}} d\hat{G}(\omega) = \frac{\int_{\{\omega : \log j(\omega) \in B\}} \lambda(\omega) \, dG(\omega)}{\int \lambda(\omega') \, dG(\omega')}$$

for all Borel subset B of $(-\infty, 0]$. Therefore, Birkhoff's Ergodic Theorem implies that[9]

$$\lim_{n \to \infty} \frac{\sum_{i=1}^{n} Z_i}{n} = \int \log j(\omega) \, d\hat{G}(\omega)$$

with probability one. Substituting the definitions of Z_i and $\hat{G}$ into the previous equation yields (16). Finally, notice that, from (13) and (16), it follows that

$$\lim_{t \to \infty} \frac{\sum_{i=1}^{N(t)} \log j(\omega_{\tau_i})}{t} = \int \lambda(\omega) \log j(\omega) \, dG(\omega) = \int \lambda(\omega) \log \int \eta \, dF_n(\eta | \omega_t) \, dG(\omega) \qquad (17)$$

almost surely, where the second equality is just (5).

From equations (12) and (17), the statement of the theorem follows. ∎

As previously discussed, if the population corresponding to a particular gene has a higher continuously compounded growth rate than that of another population, it will eventually grow to be infinitely larger than that other population. Therefore, if the level of environmental risk is determined by choices made by individuals and that choice behaviour is genetic, the continuously compounded growth rate is the evolutionary optimal decision criterion. That is, only those genes which generate the largest continuously compounded growth rate survive in the long run. This leads us to interpret

$$U(L) = \int \int \varepsilon \, dF_e(\varepsilon | \omega) \, dG(\omega) + \int \log \int \eta \, dF_n(\eta | \omega) \, \lambda(\omega) \, dG(\omega) \qquad (18)$$

as the utility representation of the evolutionary optimal choice behaviour. One concern which might arise with respect to our analysis thus far pertains to our implicit assumption that the choice of a lottery is made once and for all and determines the growth rate of the gene forever. We have not yet demonstrated that the same utility criterion is used to solve individual choice problems if the overall risk is determined by a combination of various decisions. In Section B of our Online Appendix, we take our analysis one step further and formalize the claim that the function U can indeed be interpreted as a utility function of the evolutionary optimal choice behaviour in this context.

Interpretation.— The two terms on the right-hand side of (18) have very different behavioural implications. Observe that the lottery L can be viewed as the combination of two compound

---

[8]See Proposition 2 in Duffie and Glynn (2004).

[9]See Corollary 1 of Proposition 2 of Duffie and Glynn (2004).

lotteries: $\left(\{F_e\left(\cdot|\omega\right)\}_\omega, G\right)$ and $\left(\{F_n\left(\cdot|\omega\right)\}_\omega, G\right)$, which are associated with environmental variations and natural disasters, respectively. Equation (18) implies that a decision maker who maximizes U reduces the compound lottery $\left(\{F_e\left(\cdot|\omega\right)\}_\omega, G\right)$ to a simple one. That is, conditional on the risk associated with natural disasters, choices will be based only on the expected rate of reproduction. It is irrelevant whether environmental variations are aggregate or idiosyncratic. Therefore, the first term is simply a standard von Neumann-Morgenstern representation.

In sharp contrast, the second term does not correspond to expected utility maximization. In particular, the compound lottery $\left(\{F_n\left(\cdot|\omega\right)\}_\omega, G\right)$ is not reduced to a simple one in the utility function U. This term is formally identical to the representation of smooth ambiguity-averse preferences. In the context of ambiguity (Ω, G) corresponds to the subjective state space and beliefs, $F_n\left(\cdot|\omega\right)$ describes uncertainty, and ambiguity aversion is determined by the logarithmic function. Since the logarithmic function is concave, this representation implies that the decision maker with utility function U is less tolerant towards aggregate risk than towards idiosyncratic risk.

One notable feature of the function U is its additive separability in the risks due to environmental variations and natural disasters. This arises from the fact that y_t is multiplicatively separable in these two types of aggregate risk. As previously mentioned, this separability is due to our assumption that, for each individual, ε_t and η_t are independent conditional on ω_t. We could relax this independence assumption and still obtain an additive representation similar to (18). However, the c.d.f. F_e would be replaced by an ergodic distribution of the birth rates which would depend on, among other objects, F_n and λ. So, while the utility representation would still be additive, the term corresponding to environmental variations would depend on the risks faced due to disasters.

4 On the limit of the model of Robson (1996a)

Theorem 1 implies that environmental variations and natural disasters affect growth in very different ways. We should perhaps shed some additional light on this observation, and we shall make it the objective of this section. To this end, we revisit Robson's (1996a) discrete-time model and investigate the limit of a discrete-time model as the length of the time intervals shrinks to zero. Indeed, it seems quite reasonable to suspect that environmental variations might be approximated arbitrarily well with a sequence of *small* natural disasters. In the context of a binary example, we show that, in the limit, the growth rate induced by a lottery does not depend on the decomposition of the risk. In this sense, there is no conflict between our results related to continuous-time models and the limiting behaviour of discrete-time models.[10]

---

[10]In Section C of our Online Appendix, we also consider the reverse of this exercise. That is, we establish a mapping from our continuous-time models with only environmental variations to Robson's (1996a) discrete-time models. We make use of this mapping to provide a clear explanation for the divergence between the continuous and discrete models with respect to different types of risk: in the continuous model, individuals make no distinction between aggregate and idiosyncratic risks, while in the corresponding discrete model they do.

Let us consider discrete populations facing two different lotteries; one of which is purely idiosyncratic and the other purely aggregate. Robson (1996a) shows that the population facing the idiosyncratic lottery grows faster. In what follows, we show that as the length of a time interval approaches zero, the growth rates of the two populations converge.[11]

Suppose that time is discrete and indexed by $t \in \mathbb{N}$. Assume that $\Omega = \left\{ \omega^H, \omega^L \right\}$ and the ergodic distribution of ω is given by $\Pr\left(\omega^H\right) = \Pr\left(\omega^L\right) = 1/2$. Each individual lives for one period, and her number of offspring is the realization of a lottery. In each period t, the realization of the lottery is independent across individuals conditional on ω_t. We consider the following two lotteries

$$L_A = \left\{ \begin{array}{ll} H & \text{if } \omega = \omega^H, \\ L & \text{if } \omega = \omega^L \end{array} \right. \quad \text{and } L_I = \left\{ \begin{array}{ll} H & \text{with probability } 1/2, \\ L & \text{with probability } 1/2, \end{array} \right.$$

where $1 < L < H$. Note that the lottery L_A involves only aggregate risk and the lottery L_I involves only idiosyncratic risk. Also note that the risks induced by these two lotteries are the same from the viewpoint of a single individual; with probability one-half she produces H offspring, and with probability one-half she produces L. Let $g(L)$ denote the compounded growth *factor* of the population corresponding to the lottery $L (\in \{L_A, L_I\})$. The main result from Robson (1996a) implies

$$\log g\left(L_A\right) = \frac{\log H + \log L}{2} < \log\left(\frac{H + L}{2}\right) = \log g\left(L_I\right), \tag{19}$$

and hence, the population choosing L_I grows faster than the other.

In what follows, we shrink the length of the time intervals from one to Δ, and examine the consequences on the speed of population growth as Δ approaches zero. If the time intervals are downscaled, and in each period individuals reproduce according to the lotteries L_A and L_I, the populations grow faster, and they explode as Δ goes to zero. Therefore, in order to compare growth rates across different Δ's, the lotteries governing population dynamics must be redefined. We shrink the intensity of the shocks specified by the lotteries as Δ goes to zero. The idea of taking the limit this way is to spread the effect of the original lottery over many smaller time periods. We show that the per-unit-period growth rates of the populations corresponding the idiosyncratic and aggregate lotteries converge to the same value as Δ goes to zero. In other words, the distinction between idiosyncratic and aggregate risk disappears.

The intuition behind these observations can be explained as follows. If Δ is small, the change in the size of the population is also small within a Δ-long period. This means that the logarithmic function in Robson's (1996a) utility function, (1), can be approximated by a linear function arbitrarily well as Δ goes to zero. Therefore, the logarithmic function can be replaced by the linear one in the limit.

---

[11]We are able to prove the same result in the general model of Robson (1996a), but we believe that the binary example is sufficient to illustrate our point.

For each Δ, we define the following two lotteries:

$$L_A^\Delta = \begin{cases} H^\Delta & \text{if } \omega = \omega^H, \\ L^\Delta & \text{if } \omega = \omega^L \end{cases} \quad \text{and } L_I^\Delta = \begin{cases} H^\Delta & \text{with probability } 1/2, \\ L^\Delta & \text{with probability } 1/2. \end{cases}$$

Observe that if the realization of the lottery is constant within a unit of time, the induced growth factor does not depend on Δ. To see why, note that if an individual produces H^Δ offspring in each Δ-long period, she will have $\left(H^\Delta\right)^{1/\Delta} = H$ genetic copies after one unit of time. We maintain the assumption that the ergodic distribution of ω is given by $\Pr\left(\omega^H\right) = \Pr\left(\omega^L\right) = 1/2$.[12] Let $g_\Delta\left(L^\Delta\right)$ denote the compounded growth factor of the population corresponding to $L^\Delta \left(\in \left\{L_A^\Delta, L_I^\Delta\right\}\right)$ per Δ-long time period. Again, from Robson (1996a), it follows that

$$\log g_\Delta\left(L_A^\Delta\right) = \frac{\log H^\Delta + \log L^\Delta}{2} < \log\left(\frac{H^\Delta + L^\Delta}{2}\right) = \log g_\Delta\left(L_I^\Delta\right). \tag{20}$$

Next, we take Δ to zero and compare the growth factors per unit interval of the populations corresponding to L_A^Δ and L_I^Δ. Note that if the growth factor per Δ-long period is g_Δ then the growth factor per unit interval is $(g_\Delta)^{1/\Delta}$. Consider first the population governed by the aggregate lottery L_A^Δ. By (20),

$$\lim_{\Delta \to 0} \log g_\Delta^{1/\Delta}\left(L_A^\Delta\right) = \lim_{\Delta \to 0} \frac{\log H^\Delta + \log L^\Delta}{2\Delta} = \frac{\log H + \log L}{2}. \tag{21}$$

Comparing this expression with the left-hand side of (19), we conclude that the population facing the aggregate lottery is unaffected by the time scaling. Consider now the population governed by the idiosyncratic lottery. Again by (20),

$$\lim_{\Delta \to 0} \log g_\Delta^{1/\Delta}\left(L_I^\Delta\right) = \lim_{\Delta \to 0} \frac{\log\left(\frac{H^\Delta + L^\Delta}{2}\right)}{\Delta}.$$

On the right-hand side, both the numerator and the denominator converge to zero. We apply L'Hopital's rule to obtain

$$\lim_{\Delta \to 0} \log g_\Delta^{1/\Delta}\left(L_I^\Delta\right) = \lim_{\Delta \to \infty} \frac{1}{\left(\frac{H^\Delta + L^\Delta}{2}\right)} \left(\frac{H^\Delta \log H + L^\Delta \log L}{2}\right) = \frac{1}{2}\left(\log H + \log L\right), \tag{22}$$

where the second equality follows from the observation that both H^Δ and L^Δ converge to one as Δ goes to zero. From (21) and (22), we conclude that, as Δ goes to zero, the population facing aggregate risk grows just as fast as the population facing only idiosyncratic risk.

In the limit, do these discrete models become continuous-time models with only environmental variations? Define h and l such that $H = e^h$ and $L = e^l$ and note that h and l are the rates corresponding to the factors H and L, respectively. The continuously compounded growth rates

---

[12]We emphasize that this does not mean that the state of the world switches more and more frequently as Δ goes to zero. One can assume, for example, that ω switches with only probability Δ in each Δ-long time period. Then the state of the world switches once per unit-period in expectation irrespective of Δ.

in (21) and (22) can then be written as $(h + l)/2$. In other words, the limits of L_A^Δ and L_I^Δ are lotteries involving only environmental variations according to which the net birth rate of an individual is h or l with equal probability. The limit of L_A^Δ induces only aggregate risk and the limit of L_I^Δ generates only idiosyncratic risk. By Theorem 1, this distiction, however, has no impact on the growth rate.

5   Conclusion

We believe endogenizing preferences to be an important research agenda, and that adopting the biological approach, as we have done, might prove to be a fruitful enterprise. It seems reasonable to hypothesize that evolution did not influence only physical traits but also shaped choice behaviours. An advantage of this approach is that it has strong predictions about the relationship between fertility and choices which, in principle, can be tested empirically. We do recognize that many choice problems faced by individuals in modern times were unlikely to be faced in evolutionary times. Yet, we hypothesize that preferences, at least in part, are hardwired and that many choices made today reflect the evolutionary optimal behaviour.

References

[1] Becker, G. (1976): "Altruism, Egoism, and Genetic Fitness: Economics and Sociobiology," *Journal of Economic Literature*, **14**, 817-826.

[2] Coddington, E. A. and N. Levinson (1955): *Theory of Ordinary Differential Equations*, McGraw-Hill.

[3] Cohen, D. (1966): "Optimizing Reproduction in a Randomly Varying Environment," *Journal of Theoretical Biology,* **12,** 119-129.

[4] Dekel, E. and S. Scotchmer (1999): "On the Evolution of Attitudes towards Risk in Winner-Take-All Games," *Journal of Economic Theory*, **87**, 1999, 125–143.

[5] Doob, J. L. (1953): *Stochastic Processes*, John Wiley and Sons.

[6] Duffie, D. and P. Glynn (2004): "Estimation of Continuous-Time Markov Processes Sampled at Random Time Intervals," *Econometrica*, **72**, 1773-1808.

[7] Feller, W. (1968): *An Introduction to Probability Theory and Its Applications*, Volume 1, John Wiley and Sons.

[8] Glynn, P. (1994): "Some Topics in Regenerative Steady-State Simulation," *Acta Applicandae Mathematicae*, **34**, 225-236.

[9] Klibanoff, P., M. Marinacci and Sujoy Mukerji (2005): "A smooth model of decision making under ambiguity", *Econometrica*, **73**, 1849-1892.

[10] Robson, A. (1996a): "A Biological Basis for Expected and Non-Expected Utility", *Journal of Economic Theory*, **68**, 397-424.

[11] Robson, A. (1996b): "The Evolution of Attitudes to Risk: Lottery Tickets and Relative Wealth", *Games and Economic Behavior*, **14**, 190-207.

[12] Robson, A. (2001): "The Biological Basis of Economic Behavior," *Journal of Economic Literature*, **39**, 11-33.

[13] Robson, A. (2002): "Evolution and Human Nature," *Journal of Economic Perspectives*, **16**(2), 89-106.

[14] Robson, A. and L. Samuelson (2009): "The Evolution of Time Preference with Aggregate Uncertainty," *The American Economic Review*, **99**(5), 1925-1953.