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Discounting and the Representative Median Agent*

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Abstract

We derive a simple formula for the social discount rate (SDR) that uses the median, rather than average agent of the economy to reflect the consequences of consumption growth on income inequality. Under reasonable assumptions, the difference between the growth of median and mean incomes is used to adjust the wealth-effect in the standard Ramsey rule. In a plausible special case the representative agent has the median income. With inequality aversion elasticity of 2 (1.5, 1), the U.K. and U.S. SDR would be 1% (0.5%, 0.25%) lower than the standard Ramsey rule. This reflects two decades of inequality-increasing growth and implies greater weight placed on future generations in public appraisal.

JEL Classification: D31, D61, H43.

Keywords: Social Discount Rate, Income Inequality, Inequality Aversion, Cost Benefit Analysis

1 Introduction

The importance of income and consumption inequality has received great attention in the political and scientific debate in recent years (see for instance Stiglitz, 2012; Piketty, 2014), and its influence on public policy decisions is ongoing (Cingano, 2014). In this paper, we study the impact of inequality for long-term public policy evaluations by deriving a simple 'inequality-adjusted' social discount rate (SDR) for Cost Benefit Analysis (CBA)

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that takes into account the dynamics of income inequality. Under assumptions typically used in applied social discounting, the inequality-adjustment stems from an adjusted consumption growth rate, which uses the difference between growth in median and per capita incomes to reflect the distributional consequences of growth. This results in an inequality-adjusted wealth-effect in the standard Ramsey rule. Our SDR is a special case of previous work by Gollier (2015) and Emmerling (2011), and allows recent concerns about income inequality to be simply incorporated into the analysis of public projects through the discount rate.

Our inequality-adjusted SDR corresponds closely to current discounting practices and is easily operationalised since it only requires knowledge of the growth of median income. Indeed, a plausible special case defines the median agent of the income distribution as the representative agent. This corresponds to recent proposals to count inequality among the measures used to evaluate economic performance (Piketty, 2014; Aghion et al., 2013; OECD, 2017). Growth of median income is argued to better reflect the performance of society as a whole given its central location in the income distribution, whereas per-capita income growth is skewed by growth among the rich (ONS, 2016). For this reason data on the growth of median incomes are now routinely collected by government statistical agencies and international organizations. ~~For instance,~~ in 2013, global annual household income before taxes was $9,733, while median income was just $2,920, reflecting very different trajectories (Gallup, 2013). ~~The~~ inequality-adjusted SDR captures these disparities and is amenable to policy-makers. Our recommendations speak to recent policy debates where the choice of discount rate is pivotal, such as the analysis of climate change, natural capital valuation (Fenichel et al., 2016, 2017) and the appraisal of nuclear pow Notably, worsening inequality over time implies a lower discount rate, which increases the weight of future generations. Moreover, we provide an analytical foundation for replacing the representative agent by the median of the distribution, based on intergenerational distributional concerns, like those found in Andrews et al. (2017).

2 Discounting and the inquality-adjusted wealth effect

Assume an economy with a continuum of agents of type θ with cumulative distribution function $H(\theta)$, whose (unique) instantaneous felicity function is given by $U(c_t(\theta))$, where $c_t(\theta)$ is consumption of type θ at time t. All agents have the same pure rate of time preference δ. Using the standard expected utility framework, and due to the interchangeability of the orders of integration, inter-temporal well-being can be represented by the following social welfare function (SWF) (see also Gollier, 2015):

$$W_0 = \sum_{t=0} \exp(-\delta t) E \int_\theta U(c_t(\theta)) \, dH(\theta) \tag{1}$$

At a given point in time, we can use Atkinson (1970)'s concept of the equally distributed equivalent (EDE) level of consumption to rewrite this SWF: the EDE level of consumption at time t is defined as:

$$c_t^{ede} = U^{-1}\left(\int_\theta U\left(c_t\left(\theta\right)\right) dH\left(\theta\right)\right) \tag{2}$$

which clearly depends on the characteristics of $U\left(c\right)$, in particular aversion to inequality. The SWF can now be re-written simply as:

$$W_0 = \sum_{t=0} \exp\left(-\delta t\right) EU\left(c_t^{ede}\right) \tag{3}$$

We abstract from uncertainty and take the deterministic case: future consumption is certain, and assume that the costs and benefits of the public project are shared equally among the individuals of the economy. This is a natural assumption for the derivation of the discount rate, which focusses on the marginal impact of a policy on the population holding the income (or consumption [1]) distribution constant.[2] The standard derivation of the social discount rate then yields an inequality-adjusted SDR (Dasgupta (e.g., 2008, p 150)):

$$r^* = \delta + \eta g_t^{ede} \tag{4}$$

where $\eta \equiv \frac{-c_t^{ede} U''\left(c_t^{ede}\right)}{U'\left(c_t^{ede}\right)}$ is the elasticity of marginal utility with respect to consumption, and $g_t^{ede} = \frac{1}{t} log(c_t^{ede}/c_0^{ede})$ the annualized growth rate of EDE consumption between time 0 and t. The inequality-adjusted wealth-effect is given by the term ηg_t^{ede}. The standard Ramsey rule on the other hand is defined as $r' = \delta + \eta g_t^{pc}$, where g_t^{pc} is the growth of per capita consumption, and ηg_t^{pc} the wealth-effect. The difference between this and (4) depends therefore on the different wealth-effects, which we analyse in detail in the next section.

3 Discounting with representative median growth

Two commonplace assumptions, one theoretical and one empirical, lead to the simple inequality-adjusted SDR which is a special case of Gollier (2015) and Emmerling (2011). **Assumption 1** is a Constant Relative Risk Aversion (CRRA) utility function: $U\left(c_t\right) =$

[1]Note that in this paper we consider both distributions as equivalent, rationalizable for instance through a homogeneous savings rate.

[2]In the risk domain the equivalent assumption would be that the project does not affect the probability distribution of potential outcomes.

$(1-\eta)^{-1} c_t{}^{1-\eta}$. **Assumption 2** is log-normally distributed consumption: $c_t(\theta) \sim LN(\mu_t, \sigma_t^2)$. Assumption 1 is commonplace in applied theory and appears in practical applications of the Ramsey Rule (e.g., Arrow et al., 2014). Assumption 2 is strongly supported by empirical evidence (Pinkovskiy and Sala-i Martin, 2009). These assumptions lead to a convenient form for the EDE level of consumption.

The k^{th} raw moment of the log-normally distributed variable x (mean μ and variance σ^2) is given by $E\left[x^k\right] = \exp\left(k\mu + 0.5k^2\sigma^2\right)$. Therefore, if income at time t is log-normally distributed and preferences are CRRA, the EDE consumption level[3], c_t^{ede}, is:

$$c_t^{ede} = \exp\left(\mu_t + 0.5\left(1-\eta\right)\sigma_t^2\right) \tag{5}$$

This can be compared to average, per-capita consumption, $c_t^{pc} = \exp\left(\mu_t + 0.5\sigma_t^2\right)$ and median consumption, $c_t^{med} = \exp\left(\mu_t\right)$. The closed form expression for c_t^{ede} leads to an expression for its annualized growth, g_t^{ede}:

$$g_t^{ede} = \frac{1}{t}\left(\mu_t - \mu_0\right) + \frac{1}{t}\left(0.5\left(1-\eta\right)\Delta\sigma_{0,t}^2\right), \tag{6}$$

where $\Delta\sigma_{0,t}^2 = \sigma_t^2 - \sigma_0^2$ represents the change in the variance of log-consumption from time 0 to t. This growth rate can be compared with annualized average (per-capita) growth rate:

$$g_t^{pc} = \frac{1}{t}\left(\mu_t - \mu_0\right) + \frac{1}{t}\left(0.5\Delta\sigma_{0,t}^2\right) \tag{7}$$

and the annualized growth rate of median consumption:

$$g_t^{med} = \frac{1}{t}\left(\mu_t - \mu_0\right). \tag{8}$$

Our main result is as follows. Combining the growth equations (6),(7) and (8), yields a simple expression for EDE growth in terms of inequality aversion, η, and mean and median growth:

$$g_t^{ede} = g_t^{pc} + \eta\left(g_t^{med} - g_t^{pc}\right) \tag{9}$$

This is an inequality-adjusted growth rate, where the Atkinson index is the measure of inequality considered. This follows because under Assumption 1 the Atkinson index is monotonic in $\sigma_{0,t}^2$, so $\Delta\sigma_{0,t}^2$ is synonymous with changes in inequality.[4] Under Assumption 2, a sufficient statistic for $\Delta\sigma_{0,t}^2$ is the difference between mean and median growth. Hence,

[3]For CRRA utility (2) becomes $c_t^{ede} = \left(\int_\theta c_t^{1-\eta}(\theta)\, dH(\theta)\right)^{\frac{1}{1-\eta}}$, the $(1-\eta)^{-1}\, th$ moment of c_t.

[4]Under Assumptions 1 and 2, the Atkinson inequality index is given by: $I(\eta) = 1 - \frac{c_t^{ede}}{c_t^{pc}} = 1 - \exp\left(-\frac{\eta}{2}\sigma_t^2\right)$.

$\left(g_t^{med} - g_t^{pc}\right)$ measures how much inequality has changed, and η scales this change to reflect the welfare consequences for societal welfare growth. Growth of the median agent g_t^{ede} therefore determines the wealth-effect for an inequality-averse planner. Inserting g_t^{ede} into the Ramsey rule in (4) yields our simple inequality-adjusted SDR:

$$r^* = \delta + \eta g_t^{pc} + \eta^2 \left(g_t^{med} - g_t^{pc}\right) \tag{10}$$

The term $\eta^2 \left(g_t^{med} - g_t^{pc}\right)$ is the inequality-adjustment to the wealth-effect in the SDR. This scales the change in inequality, $\left(g_t^{med} - g_t^{pc}\right)$, in η^2 because it multiplies inequality-adjusted growth, which scales in η, by the inverse of the intertemporal elasticity of substitution. The latter is simply η in the discounted Utilitarian framework.[5] The intuition to our inequality-adjustment is clear: if growth is inequality-increasing then $g_t^{med} < g_t^{pc}$, and an inequality averse society suffers a growth penalty which reduces the wealth-effect. The opposite is true when inequality decreases and $g_t^{med} > g_t^{pc}$. The size of the growth penalty depends on the extent of inequality aversion, η.

An interesting special case is when we apply $\eta = 1$ to (6):

$$g_t^{ede} = \frac{1}{t}\left(\mu_t - \mu_0\right) = g_t^{med}, \tag{11}$$

Here the appropriate wealth-effect reflects the growth of median income, not the average per-capita income: the agent with the median income becomes the representative agent.

4 Inequality adjustment in practice

Calibration of the inequality-adjusted SDR requires two components: a measure of societal inequality aversion, and the growth of median incomes. Estimates of inequality-aversion, η, vary from 0.8-2.0 (experiments on students), 0.4-2.5 (revealed in international transfers or progressive tax systems), to 1-4 (elasticity of marginal utility using risk-aversion or inter-temporal substitution) (Tol, 2010; Groom and Maddison, 2014). The UK Treasury and the Stern Review argue for $\eta = 1$ (H.M. Treasury, 2003; Stern, 2007). Others argue that, for normative reasons, $\eta = 2$ is a more suitable degree of inequality aversion (e.g., Dasgupta, 2008).

Taking the distribution into account when measuring national income has been a recommendation that goes back decades (Sen, 1976). More recently the LSE growth commission and the European Commission argued that "the median is better than the mean since it is reflective of progress in the middle of the income distribution." (Aghion et al., 2013;

[5] It is possible to use Kreps-Porteus preferences to separate fluctuation and inequality aversion and obtain a Ramsey-like formula: $r_t = \delta + \varepsilon g_t^{ede}$, where ε is the inverse of the elasticity of inter-temporal substitution, see also Emmerling (2011).

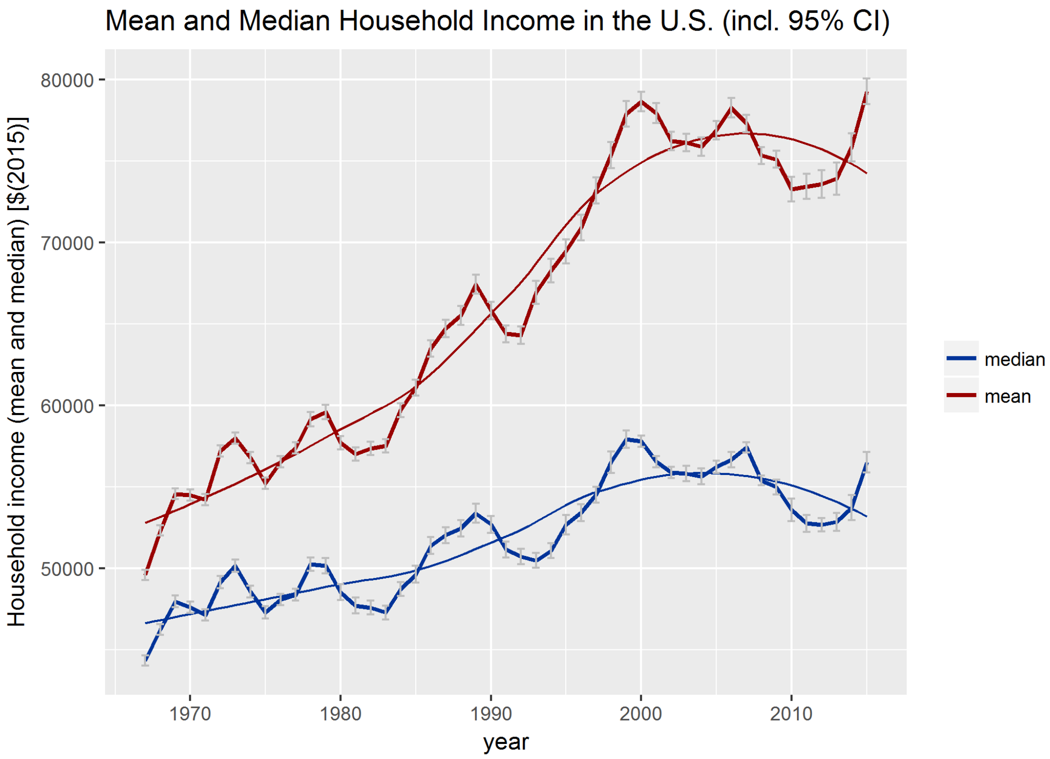

Figure 1: Household mean and median consumption growth in the U.S. (Source: Proctor et al. (2016))

European Commission, 2014). This sentiment was echoed by the Stiglitz report (Stiglitz, 2012).

We apply our obtained formula for the SDR to a set of 25[6] countries and quantify the inequality effect due to the difference between median and average growth. We use a recently created dataset by Max Roser et al. (2016). In Figure 1, we computed the growth rates of the average and median income per capita, using the longest available time period available. Based on these growth rates, we compute the inequality adjusted SDR for different countries and values of η.

In our sample of 25 countries, in 15 countries median growth fell short of average income, while the reverse was true in 10 (mostly middle income) countries. Thus, the effect of the inequality adjusted discount rate can go in both directions. In some high income countries, such as the U.K. and U.S., the growth adjustment leads to a reduction of 1% (0.5%, 0.25%) in the SDR for inequality aversion parameters of 2 (1, 0.5). For these economies, future costs and benefits should be discounted at a lower rate as they fall on more unequal, hence, from a social welfare perspective, worse-off generations.

[6] We include all countries of the original dataset of Max Roser et al. (2016) for which at least a time series of ten years could be used to estimated the average growth rates.

Table 1: Mean/Median growth rates and impact on SDR $\eta^2(g_{med} - g_{pc})$

country	period	g_pc	g_med	$\eta = 1$	$\eta = 2$
Australia	1981-2010	1.61	1.45	-0.16	-0.62
Austria	1994-2004	1.07	1.20	0.13	0.52
Belgium	1985-2000	3.18	2.41	-0.77	-3.07
Canada	1981-2010	1.01	0.96	-0.04	-0.18
Czech Republic	1992-2010	3.28	3.10	-0.17	-0.69
Denmark	1987-2010	0.97	0.94	-0.03	-0.13
Estonia	2000-2010	5.69	6.38	0.69	2.76
Finland	1987-2010	1.95	1.70	-0.25	-1.02
France	1978-2010	1.13	1.28	0.14	0.57
Germany	1984-2010	0.89	0.83	-0.06	-0.25
Greece	1995-2010	2.07	2.32	0.25	1.00
Hungary	1991-2012	0.19	0.25	0.06	0.25
Ireland	1987-2010	3.61	3.73	0.12	0.48
Israel	1986-2010	1.93	1.79	-0.14	-0.57
Italy	1986-2010	1.27	1.26	-0.01	-0.03
Luxembourg	1985-2010	3.08	2.98	-0.11	-0.42
Netherlands	1993-2010	1.62	1.78	0.16	0.62
Norway	1979-2010	2.60	2.72	0.11	0.45
Poland	1992-2010	2.07	1.83	-0.23	-0.94
Slovak Republic	1992-2010	2.61	2.41	-0.20	-0.79
Slovenia	1997-2010	2.53	2.58	0.05	0.19
Spain	1980-2010	2.21	2.36	0.15	0.61
Sweden	1981-2005	1.89	1.68	-0.21	-0.85
United Kingdom	1979-2010	2.37	2.15	-0.22	-0.88
United States	1979-2013	0.77	0.49	-0.28	-1.11

5 Conclusion

We develop a simple policy rule to allow CBA guidelines on discounting to take into account concerns about inequality in the evaluation of public projects. Under two simple and defensible assumptions (isoelastic utility and log-normal income distribution) we derive an 'inequality-adjusted' SDR that corrects the wealth effect for changes in inequality. The adjustment to average (per capita) growth is simple to implement since it is proportional to the difference between the growth of median and average incomes. The inequality adjusted SDR is reduced if inequality is increasing so that median income growth lags behind average growth, and vice versa.

Growth has been inequality increasing in some countries, and inequality reducing in others. In the U.S., annual average (per capita) growth has been driven by growth in the upper tail of the income distribution. Mean incomes grew at an annual rate of 1% since 1970, while the median income increased by only 0.3% per annum. Similar figures are true for the UK. With an inequality aversion parameter of 1 (1.5, 2), the UK and U.S. SDRs would be approximately 0.25% (0.5%, 1%) lower than the standard Ramsey rule. The inequality adjustments to the SDR are therefore of consequence, indicating a higher (lower) weight on future generations in countries where inequality is expected to rise (fall). This outcome is particularly important for public policies and investments which have intergenerational consequences. These include energy efficiency investments or regulations that require the valuation of the social cost of carbon, the appraisal of nuclear power or projects expected to deliver long-lived health outcomes such as the eradication of disease.

Our approach also provides a welfare theoretical basis for focussing on median incomes as a measure of economic performance. In one plausible case, the agent with the median income become the representative agent. Our inequality-adjusted growth rate therefore has wider applications beyond discounting and CBA.

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