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Towards a behavioral theory of the exchange rate

Book section

Original citation:
Originally published in de Grauwe, Paul (2017) *Towards a behavioral theory of the exchange rate.* In: Cheung, Yin-Wong and Westermann, Frank, (eds.) International Currency Exposure. CESifo Seminar series. The MIT Press, Cambridge, USA, pp. 11-36. ISBN 9780262036405

Towards a behavioral theory of the exchange rate

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Keynote Lecture presented at the CESifo Venice Summer Institute organized by Yin-Wong Cheung and Frank Westermann on International Currency Exposure, San Servolo, Venice, 20-21 July. The lecture is based on research done with Marianna Grimaldi.

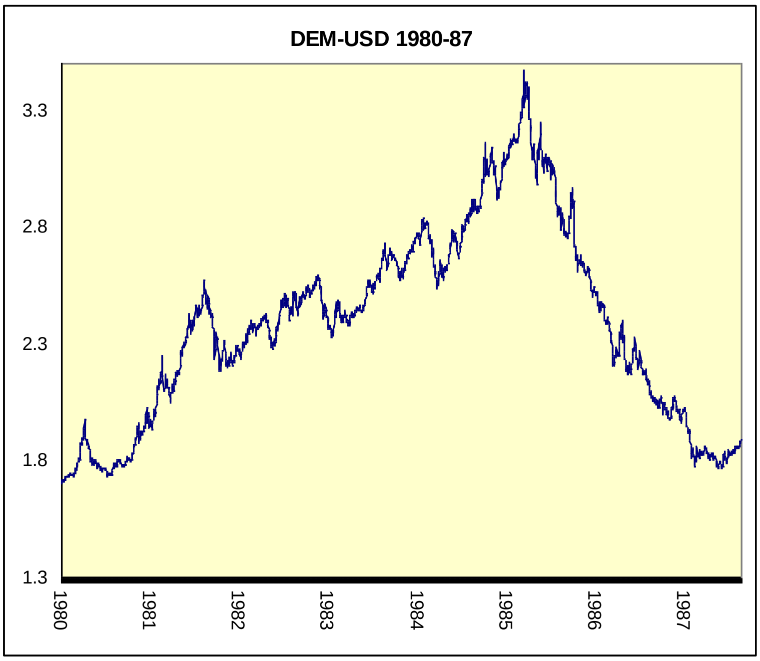
Introduction

Since the start of the rational-expectations revolution in the mid 1970s, macroeconomic analysis has been dominated by the assumption of the rational representative agent. This assumption has now become the main building block of macroeconomic modeling, so much so that macroeconomic models without a microfoundation based on the rationality assumption are simply no longer taken seriously.

The main ingredients of the rational representative agent model are well-known. First, the representative agent is assumed to continuously maximize his utility in an intertemporal framework. Second, the forecasts made by this agent are rational in the sense that they take all available information into account, including the information embedded in the structure of the model. This implies that agents do not make systematic errors in forecasting future variables. The great attractiveness of the rational-expectations model is that it imposes consistency between the agent's forecasts (the subjective probability distribution of future variables) and the forecasts generated by the model (the objective probability). Third, the model implies that markets are efficient, i.e., asset prices (including the exchange rate that will be the focus of our analysis in this chapter) reflect all relevant information about the fundamental variables that determine the value of the asset. The mechanism that ensures efficiency can be described as follows: when rational agents value a particular asset, they compute the fundamental value of that asset and price it accordingly. If they obtain new information, they will immediately incorporate that information in their valuation of the asset. Failure to arbitrage on that new information would imply that they leave profit opportunities unexploited. Rational agents will not do this.

The efficient-market implication of the model is important because it generates a number of predictions that can be tested empirically. The main empirical prediction of the rational representative agent model is that changes in the price of an asset must reflect unexpected changes (news) of the fundamental variables. The corollary of this prediction is that when there is no news about the underlying fundamentals the price cannot change. Thus, only if the fundamentals change unexpectedly should one observe movements in the asset price.

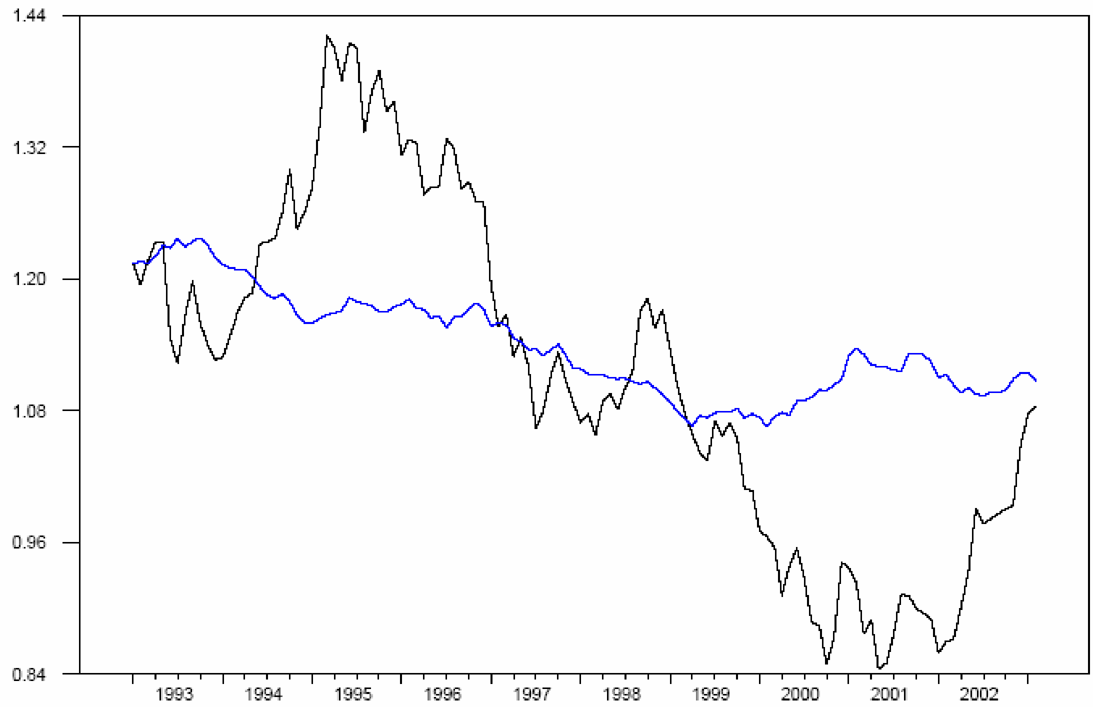
This prediction surely must be rejected. In Figure 1, we show the dollar exchange rate vis-à-vis the Deutsche mark (in the 1980s) and vis-à-vis the euro (during the late 1990s and early 2000s). Since 1980 dollar has been involved in bubble and crash scenarios more than half of the time. News models can only explain this by first a very long series of positive news during 1980-85 and 1995-2000 followed by long series of negative news (1985-88 and 2000-04). There is just not enough news to do the trick and as shown in De Grauwe and Grimaldi (2003) during the bubble fase the bad news about the dollar was systematicall negelected and during the crash fase the same happened with the good news.



Source: De Grauwe and Grimaldi (2006)

Not only is there not enough positive and later negative news. Quite often the news and the exchange rate move in opposite direction. This is also confirmed by in a study of Ehrmann and Fratzscher (2005). These authors looked carefully at a whole series of fundamental variables (inflation differentials, current accounts, output growth) in the United States and in the Eurozone and computed an index of news in these fundamentals. We show the result of their calculation in Figure 3, together with the USD/euro rate during the period 1993–2003. It is striking to find that there is very little movement in the news variable, while the exchange rate is moving wildly around this news variable. In addition, quite often the exchange rate moves in the opposite direction to that expected if it were driven by fundamental news. For example, it can be seen from Figure 3 that during the period 1999–2001 the news about the euro was relatively more favorable than the news about the dollar and yet the euro declined spectacularly against the dollar.

Dollar-DM/euro exchange rate, market and fundamental, 1993-2003



Source: Ehrmann, M., and Fratzscher, M., Exchange Rates and Fundamentals: New Evidence from Real-time Data, forthcoming in Journal of International Money and Finance, 2004

Figure 3. USD/DM (euro) exchange rate, market and fundamental, 1993–2003.

This lack of correlation between news in the fundamentals and the exchange rate movements has been documented in many other studies. For example, using high-frequency data, Goodhart (1989) and Goodhart and Figliuoli (1991) found that most of the time the exchange rate moves when there is no observable news in the fundamental economic variables. This result was also confirmed by Faust et al. (2002). Thus, the empirical evidence that we now have is that the exchange rate movements are very much disconnected from movements in the fundamentals.

This finding for the foreign exchange market is consistent with similar findings in the stock markets (see Cutler et al. 1988; Shiller 1989; Shleifer 2000). It has led Obstfeld and Rogoff to identify this "disconnect puzzle" as the major unexplained empirical regularity about the exchange rates (Obstfeld and Rogoff 1996).

There is a need for an alternative approach. Such an alternative approach will be presented in this paper. It is based on the idea that agents have a limited capacity for understanding and processing the complex available information (bounded rationality, Simon(1955)). In order to cope with the uncertainty they use relatively simple behavioral rules (heuristics). This does not mean they are irrational. Because the world is so complex it is pointless to try to understand its full complexity Rationality in the model is introduced by assuming that agents are willing to learn. They follow a procedure that allows them to evaluate the simple rules (Brock and Hommes(1997), 1998)).

Two learning procedures are possible. One is sstatistical learning (see e;g. Evans and Honkapohja(2001); the is fitness learning. We follow the second procedure. In this fitness learning agents compare the rule they currently use to alternative rules. They decide to switch to the alternative if it turns out that this is more profitable. Profitability of the rule will be the fitness criterion (Brock and Hommes(1998)). This procedure is also a disciplining device: we have to avoid that all simple rules are possible; there must be a selection mechanism that only keeps the best rules. In this sense the procedure follows an evolutionary dynamics.

1 A behavioral model

In this section we develop a simple exchange rate model. As will be seen, the model can be interpreted more generally as a model describing any risky asset price. The model consists of three building blocks. First, utility maximising agents select their optimal portfolio using a mean-variance utility framework. Second, these agents make forecasts about the future exchange rate based on simple but different rules. In this second building block we introduce concepts borrowed from the behavioural finance literature. Third, agents evaluate these rules ex-post by comparing their risk-adjusted profitability. Thus, the third building block relies on an evolutionary economics.

1.1 The optimal portfolio

We assume agents of different types *i* depending on their beliefs about the future exchange rate. Each agent can invest in two assets, a domestic asset and foreign assets. The agents' expected utility can be represented by the following equation:

$$U(W_{i,t+1}) = E_t(W_{i,t+1}) - \frac{1}{2}\mu V_i(W_{i,t+1}) \quad (1)$$

where $W_{i,t+1}$ is the wealth of agent of type i at time $t+1$, E_t is the expectation operator , μ is the coefficient of risk aversion and V_i represents the conditional variance of wealth of agent i. The wealth of agents i is specified as follows:

$$W_{i,t+1} = (1+r^*)s_{t+1}d_{i,t} + (1+r)(W_{i,t} - s_i d_{i,t}) \quad (2)$$

where r and r^* are respectively the domestic and the foreign interest rates (which are known with certainty), s_{t+1} is the exchange rate at time $t + 1$, $d_{i,t}$ represents the holdings of the foreign assets by agent of type i at time t. Thus, the first term on the right-hand side of (2) represents the value of the foreign portfolio expressed in domestic currency at time $t + 1$ while the second term represents the value of the domestic portfolio at time $t+1$.

Substituting equation (2) into (1) and maximising the utility with respect to $d_{i,t}$ allows us to derive the standard optimal holding of foreign assets by agents of type i

$$d_{i,t} = \frac{(1+r^*)E_{i,t}(s_{t+1}) - (1+r^*)s_t}{\mu\sigma_{i,t}^2} \quad (3)$$

where $\sigma^2 = (1+r^*)^2 V_t^i(s_{t+1})$

The optimal holding of the foreign asset depends on the expected excess return (corrected for risk) of the foreign asset. The market demand for foreign assets at time t is the sum of the individual demands, i.e.:

$$\sum_{i=1}^{N} n_{i,t}\, d_{i,t} = D_t \quad (4)$$

where $n_{i,t}$ is the number of agents of type i in period t.

Market equilibrium implies that the market demand is equal to the market supply Z_t which we assume to be exogenous. The market supply arises from the current account position, assumed to be exogenous here. Thus,

$$Z_t = D_t \quad (5)$$

Substituting the optimal holdings (3) into the market demand (4) and then into the market equilibrium equation (5) and solving for the exchange rate s_t yields the market clearing exchange rate:

$$s_t = \left(\frac{1+r^*}{1+r}\right)\frac{1}{\sum_{i=1}^{N}\frac{w_{i,t}}{\sigma_{i,t}^2}}\left[\sum_{i=1}^{N} w_{i,t}\,\frac{E_t^i(s_{t+1})}{\sigma_{i,t}^2} - \Omega_t Z_t\right] \tag{6}$$

where

$$w_{i,t} = \frac{n_{i,t}}{\sum_{i=1}^{N} n_{i,t}}$$

$$\Omega_t = \frac{\mu}{(1+r^*)\sum_{i=1}^{N} n_{i,t}}$$

Note that the forecasts E_{it} are weighted by their respective variances σ^2 . When agent's i forecasts have a high variance the weight of this agent in the determination of the market exchange rate is reduced.

1.2 The forecasting rules

We now specify how agents form their expectations of the future exchange rate and how they evaluate the risk of their portfolio. We start with an analysis of the rules agents use in forecasting the exchange rate. We take the view that individual agents are overwhelmed by the complexity of the informational environment, and therefore use simple rules to make forecasts. Here we describe these rules. In the next section we discuss how agents select the rules.

We assume that two types of forecasting rules are used. One is called a "fundamentalist" rule, the other a "technical trading" rule. Such a distinction between fundamentalists and chartists was first proposed by Frankel and Froot (1987); see also De Long et al. (1990). The agents using a fundamentalist rule, the "fundamentalists", base their forecast on a comparison between the market and the fundamental exchange rate, i.e. they forecast the market rate to return to the fundamental rate in the future. In this sense they use a negative feedback rule that introduces a mean reverting dynamics in the exchange rate. The speed with which the market exchange rate returns to the fundamental is assumed to be determined by the speed of adjustment in the goods market which is assumed to be in the information set of the fundamentalists (together with the fundamental exchange rate itself). Thus, the forecasting rule for the fundamentalists is :

$$E_t^f(\Delta s_{t+1}) = -\psi(s_{t-1} - s_{t-1}^*) \quad (7)$$

where s^* is the fundamental exchange rate at time t, which is assumed to follow a random walk and $0 < \psi < 1$. We assume that the fundamental exchange rate is exogenous.

The timing of the forecasts is important. When fundamentalists forecast the future exchange rate they use publicly available information up to period $t-1$. This implies that fundamentalists make their forecasts before the market clearing exchange rate s_t has been revealed to them . This assumption is in the logic of the model used here in which agents do not know the full model structure. As a result, they cannot compute the market clearing exchange rate of time t that will be the result of their decisions made in period t .

The timing assumption underlying the agents' forecasts in (7) allows us to derive the market clearing exchange rate in (6) as a unique price for which demand equals supply (see Brock and Hommes (1998)). An issue that arises here is how this timing assumption can be made consistent with the optimisation process described in the previous section. There we assumed that when computing their optimal holdings of foreign assets in period t, agents have information about the exchange rate in period t. The inconsistency is only apparent. The optimal holdings derived in equation (3) can be interpreted as a Marshalian demand curve in which an auctioneer announces a price, s_t. Agents then decide on their optimal holdings conditioned on this announced price. The auctioneer then collects the bids and offers, and computes the market clearing price. The latter is not in the information set of the agents when they make their forecasts for the exchange rate in period $t+1$.

The chartists are assumed to follow a positive feedback rule, i.e. they extrapolate past movements of the exchange rate into the future. The chartists' forecast is written as:

$$E_{c,t}(\Delta s_{t+1}) = \beta \sum_{h=1}^{H} \rho_h \, \Delta s_{t-h} \quad (8)$$

Here $E_{c,t}$ is the forecast made by the chartists using information up to time $t-1$, and β is the coefficient expressing the degree with which chartists extrapolate the past change in the exchange rate; we assume that $0 < \beta < 1$ to ensure dynamic stability. Thus, the chartists compute a moving average of the past exchange rate changes and they extrapolate these changes into the future exchange rate change H hange.

Thus, technical traders take into account information concerning the fundamental exchange rate *indirectly*, i.e. through the exchange rate itself. In addition, technical rules can be interpreted as rules that attempt to detect "market sentiments". In this sense the technical trader rules can be seen as reflecting herding behavior (see Mentkhoff(1997, 1998)).

1.3 Fitness of the rules

The next step in our analysis is to specify how agents evaluate the fitness of these two forecasting rules. The general idea that we will follow is that agents use one of the two rules, compare their (risk adjusted) profitability *ex post* and then decide whether to keep the rule or switch to the other one. Thus, our model is in the logic of evolutionary dynamics, in which simple decision rules are selected. These rules will continue to be followed if they pass some "fitness" test (profitability test). Another way to interpret this is as follows. When great uncertainty exists about how the complex world functions, agents use a trial and error strategy. They try a particular forecasting rule until they find out that other rules work better. Such a trial and error strategy is the best strategy agents can use when cannot understand the full complexity of the underlying model.

In order to implement this idea we use an approach proposed by Brock and Hommes(1997) which consists in making the weights of the forecasting rules a function of the relative profitability of these rules[1], i.e.

$$w_{c,t} = \frac{exp\left[\gamma\pi'_{c,t}\right]}{exp\left[\gamma\pi'_{c,t}\right]+exp\left[\gamma\pi'_{f,t}\right]} \tag{9}$$

$$w_{f,t} = \frac{exp\left[\gamma\pi'_{f,t}\right]}{exp\left[\gamma\pi'_{c,t}\right]+exp\left[\gamma\pi'_{f,t}\right]} \tag{10}$$

where $\pi'_{c,t}$ and $\pi'_{f,t}$ are the risk adjusted net profits computed by technical traders and fundamentalists who forecast the exchange rate in period t using information up to t-1.

Equations (9) and (10) can be interpreted as switching rules. When the risk

[1] This specification of the decision rule is often used in discrete choice models. For an application in the market for differentiated products see Anderson, de Palma, and Thisse (1992). The idea has also been applied in financial markets, by Brock and Hommes (1998) and by Lux (1998).

adjusted profits of the technical traders' rule increases relative to the risk adjusted net profits of the fundamentalists rule, then the share of agents who switches and use technical trader rules in period t increases, and vice versa. This parameter γ measures the intensity with which the technical traders and fundamentalists revise their forecasting rules. With an increasing γ agents react strongly to the relative profitability of the rules. In the limit when γ goes to infinity all agents choose the forecasting rule which proves to be more profitable. When γ is equal to zero agents are insensitive to the relative profitability of the rules. In the latter case the fraction of technical traders and fundamentalists is constant and equal to 0.5.

Thus, γ is a measure of inertia in the decision to switch to the more profitable rule As will be seen, this parameter is of great importance in generating bubbles[2].

2 Stochastic simulation of the model

Assuming the process of the fundamental exchange rate s^*_t as exogenously given, the system of the dynamic equations (6) - (10), some of which are high order equations, defines a high-dimensional nonlinear discrete- time model (for more detail see De Grauwe and Grimaldi(2006). The non-linear structure of our model does not allow for a simple analytical solution. As a result we have to use numerical simulation methods. One drawback of this approach is that we cannot easily derive general conclusions. We will compensate for this drawback in two ways: first by presenting sensitivity analyses of the numerical solutions (section 3), and second by characterizing the steady states within a simplified deterministic version of the model (section 4).

The simulations we perform are stochastic. Stochastic shocks occur in the model because the fundamental exchange rate is assumed to be driven by a random walk, i.e.

$$s_t^* = s_{t-1}^* + \varepsilon_t$$

We will assume that ε_t is normally distributed with mean equal to 0, and standard deviation equal to 0.1.

We present two examples of stochastic simulations that are quite typical for the kind of dynamics predicted by our model (see figure 4). The two upper parts of figure 4 present the simulated market and fundamental exchange rates obtained in two different simulation runs, using the same parameter configurations. The two lower parts present the corresponding shares of the chartists. The most striking

[2] The psychological literature reveals that there is a lot of evidence of a "status quo bias" in decision making (see Kahneman, Knetsch and Thaler(1991). This implies $\gamma < \infty$. Thus we set $0 < \gamma < \infty$.

features of these simulations are the following. First, it appears that the exchange rate is very often disconnected from the fundamental exchange rate. This means that the market exchange rate follows movements that are dissociated from the fundamental rate. This is especially obvious in the first simulation run (left panels), where we find that the exchange rate is disconnected from the fundamental most of the time. In the right hand panel there are many periods of disconnection, but these are less frequent. This leads to a second feature of these exchange rate movements. There appear to be two regimes. In one regime the exchange rate follows the fundamental exchange rate quite closely. These "fundamental regimes" alternate with regimes in which the fundamental does not seem to play a role in determining the exchange rate. We will call these "non-fundamental regimes". We will also call the latter ones "bubble regimes". The nature of the latter can be seen in the lower panels of Figure 4. Non-fundamental regimes are characterized with situations in which the chartists' weights are very close to 1. In contrast, fundamental regimes are those during which the chartists weights are below 1 and fluctuating significantly. These two regimes appear to correspond to two types of equilibria. Thus, a fundamental regime seem to occur when the exchange rate stays within the basin of attraction of a fundamental equilibrium. In such a regime the exchange rate movements stay very close to the fundamental exchange rate. Conversely, a non-fundamental regime seems to occur when the exchange rate moves within the basins of attraction around bubble equilibria. We will analyse the nature of these two equilibria in more detail in section 4.

We also note from Figure 4 that fundamental and non-fundamental regimes alternate in unpredictable ways. The left hand panels show a simulation during which bubble regimes tend to dominate, while the right hand panels show a simulation during which fundamental regimes are more frequent. The two simulations, however, were run with exactly the same parameters. The only difference is the underlying stochastic of the fundamental exchange rate.

3 Sensitivity Analysis

As mentioned earlier the numerical solutions are sensitive to the parameter values chosen. We illustrate this sensitivity by presenting simulations assuming different parameter values. Figure 5 shows the results of stochastic simulations of the model for different values of γ. It will be remembered that γ measures the sensitivity of the switching rule to risk adjusted profits. Thus when γ is high agents react strongly to changing profitabilities of the forecasting rules they have been using. Conversely when γ is small they do not let their forecasting rules depend much on these relative profitabilities. The results shown in Figure 5 are quite remarkable. We find that when γ is high, i.e. when agents are very sensitive to the relative profitability of the forecasting rules, the exchange rate tends to deviate strongly from the fundamental value most of the time. Thus, when γ is high the exchange rate seems to be attracted most of the time by non-fundamental equilibria. Conversely, when agents are not very sensitive to relative profitabilities (low γ) the exchange rate follows the fundamental rate closely, suggesting that it is then attracted by the fundamental equilibrium most of the time.

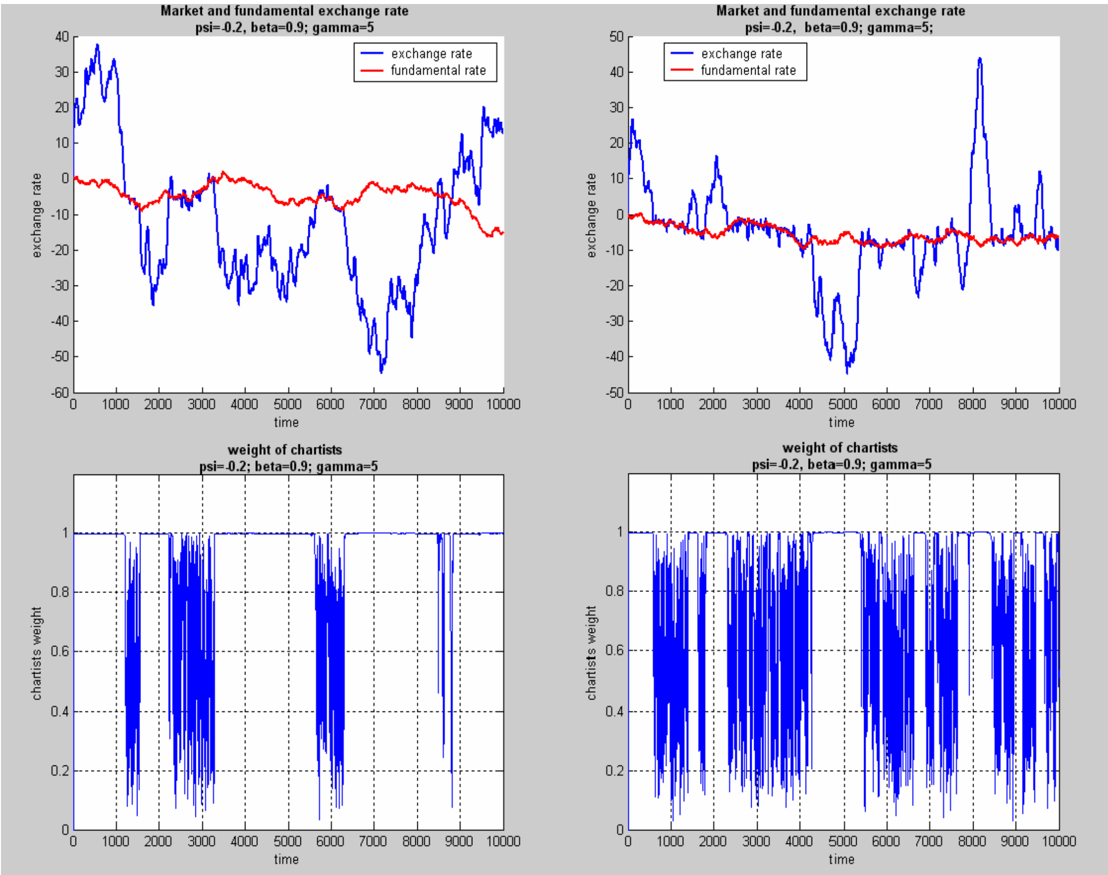


Figure 4: Prototype simulations in time domain

Another important parameter in the model is the degree of risk aversion. We performed a similar sensitivity analysis and present the results in Figure 6. We observe a remarkable phenomenon. When the degree of risk aversion is low the exchange rate remains very close to its fundamental value. As the degree of risk aversion μ increases the exchange rate starts to deviate increasingly from its fundamental value and the periods of disconnection tend to last longer. This suggests that when risk aversion is high the exchange rate seems to be attracted by non-fundamental equilibria. We will analyze this phenomenon in greater detail in the next section. Here we briefly discuss the intuition behind this result. This can be explained as follows. When agents who use fundamentalist rules are very risk avert, they will not be willing to use the profit opportunities that arise during bubbles. For example when the exchange rate increases relative to its fundamental, fundamentalists expect to be able to make profits in the future from selling the overpriced foreign currency If they are very risk avert, they may not be willing to do so. As a result, there is a failure of arbitrage[3]. This weakens the mean reverting forces in the model.

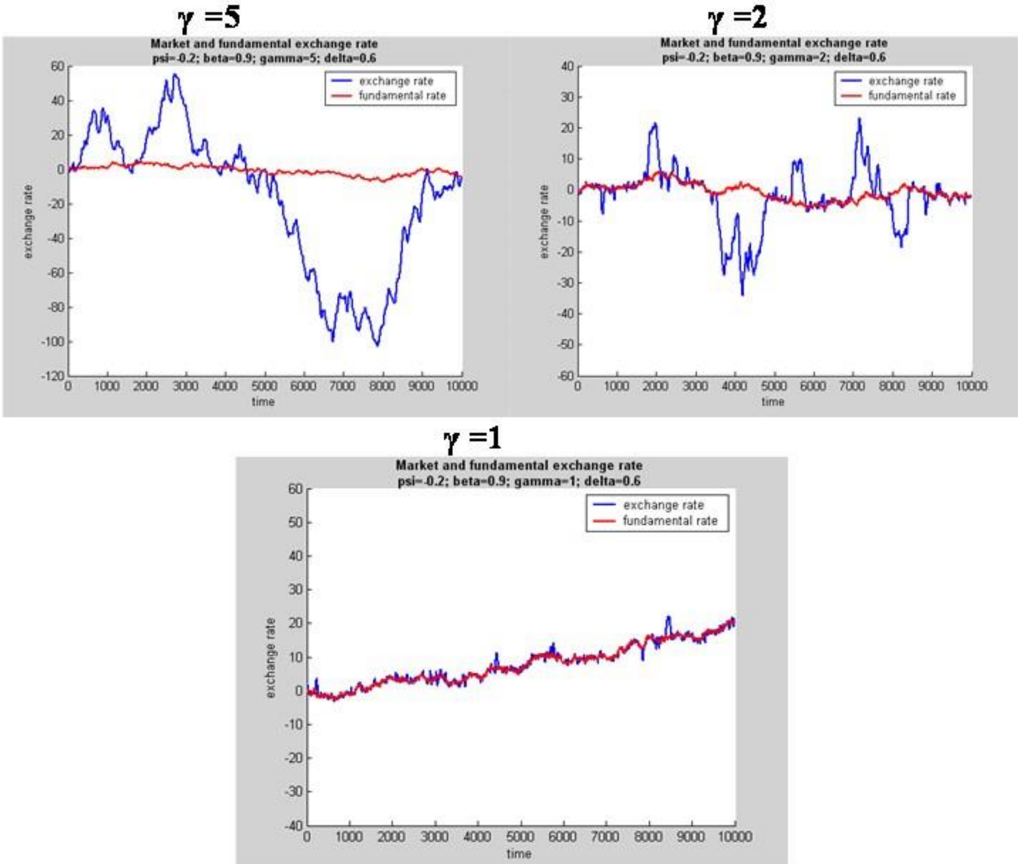


Figure 5: Sensitivity to switching parameter

3 There are other sources of failure of arbitrage that have been identified in the literature. For example, transaction costs or limits to borrowing can be reasons why arbitrage fails (see Shleifer (2000), Brunnermeier (2001)).

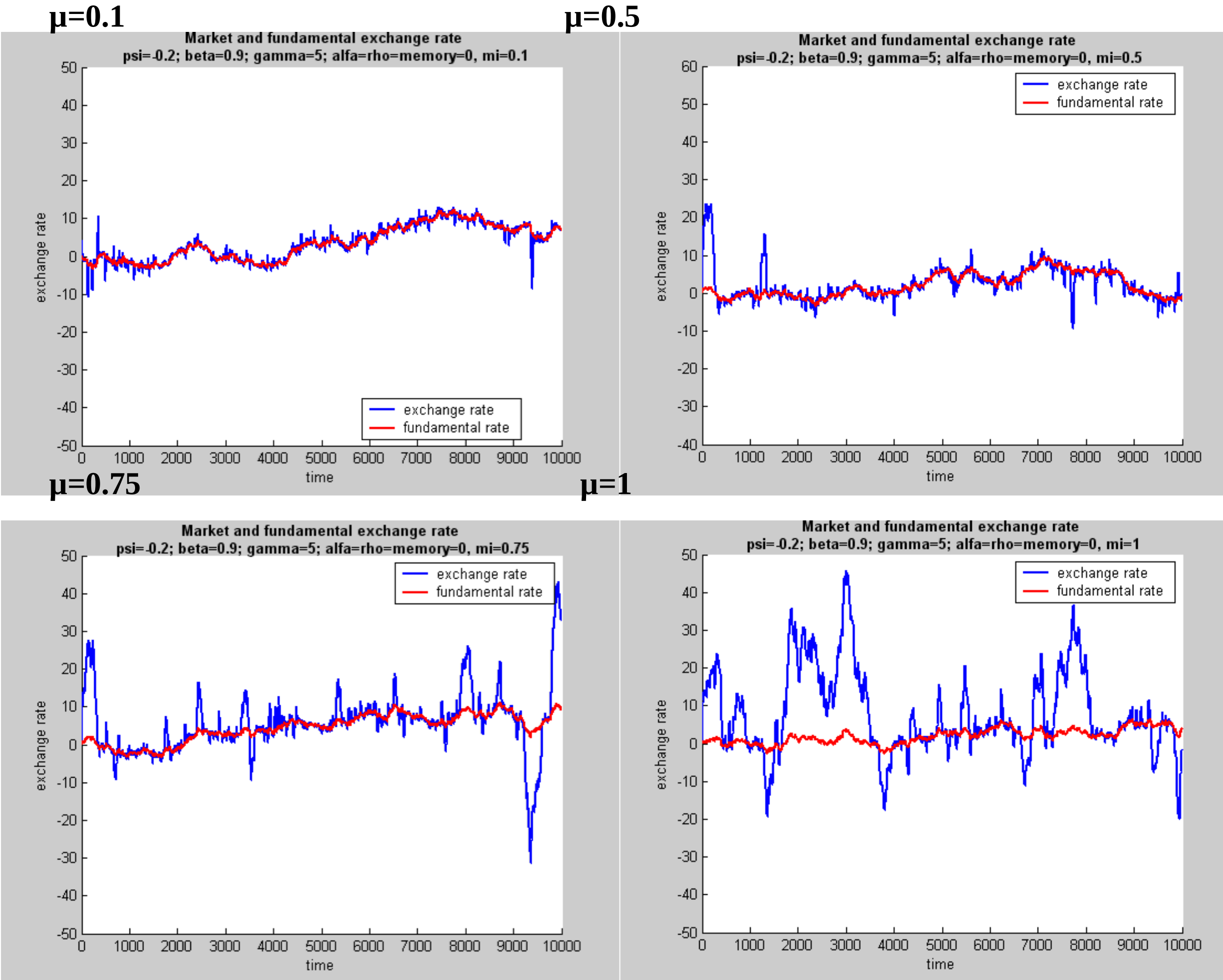


Figure 6: Sensitivity to risk aversion

4 Numerical analysis of deterministic dynamics

We now examine the dynamics of the deterministic part of the model, obtained by assuming a constant fundamental, which we normalize to zero[4]. The strong non-linearities make an analytical study of the model impossible. Therefore, we use simulation techniques which we will present in this and the following sections. We select "reasonable" values of the parameters, i.e. those that come close to empirically observed values. In appendix we present a table with the numerical values of the parameters of the model and the lags involved.

In figure 7 we show the long-run behavior of the exchange rate for different initial conditions. On the horizontal axis we set out the different initial conditions. These are initial shocks to the exchange rate in the period before the simulation is started . The vertical axis shows the fixed-point solutions corresponding to these different initial conditions. These were obtained from simulating the model over 10000 periods. We found that after such a long period the exchange rate had stabilized to a fixed point (a fixed attractor). We find numerically two types of fixed point solutions. First, for small disturbances in the initial conditions the fixed point solutions coincide with the fundamental exchange rate. As mentioned earlier, we call these solutions the *fundamental equilibria*. Second, for large disturbances in the initial conditions, the fixed point solutions diverge from the fundamental. These are the *non-fundamental* (*bubble) equilibria*. The larger is the initial shock (the noise) the farther the fixed points are removed from the fundamental exchange rate. The border between these two types of fixed points is characterised by discontinuities. This has the implication that in a neighbourhood of the border a small change in the initial condition (the noise) can have a large effect on the solution. The different nature of these two types of fixed point attractors can also be seen from an analysis of the technical traders' weights that correspond to these different fixed point attractors. We show these technical traders' weights as a function of the initial conditions in figure 8.

We find, first, that for small initial disturbances the technical traders' weight converges to 50% of the market. Thus when the exchange rate converges to the fundamental rate, the weight of the technical traders and the fundamentalists are equal to 50%. For large initial disturbances, however, the technical traders' weight comes close to 1. Thus, when the technical traders take over most of the market, the exchange rate converges to a bubble attractor . The meaning of a bubble attractor can now be understood better. It is an exchange rate equilibrium that is reached when the number of fundamentalists has become sufficiently small (the number of technical traders has become sufficiently large) so as to eliminate the effect of the mean reversion dynamics.
It is important to see that these bubble attractors are fixed point solutions. Once we reach them, the exchange rate is constant. The technical traders'

[4] This is equivalent to interpreting s_t as the deviation of the market exchange rate from the constant fundamental.

expectations are then model consistent, i.e. technical traders who extrapolate the past movements, forecast no change. At the same time, since the fundamentalists have all but left the market, there is no force acting to bring back the exchange rate to its fundamental value. Thus two types of equilibria exist: a fundamental equilibrium where technical traders and fundamentalists co-exist, and a bubble equilibrium where the technical traders have almost crowded out the fundamentalists. In both cases, the expectations of the agents in the model are consistent with the model's outcome.

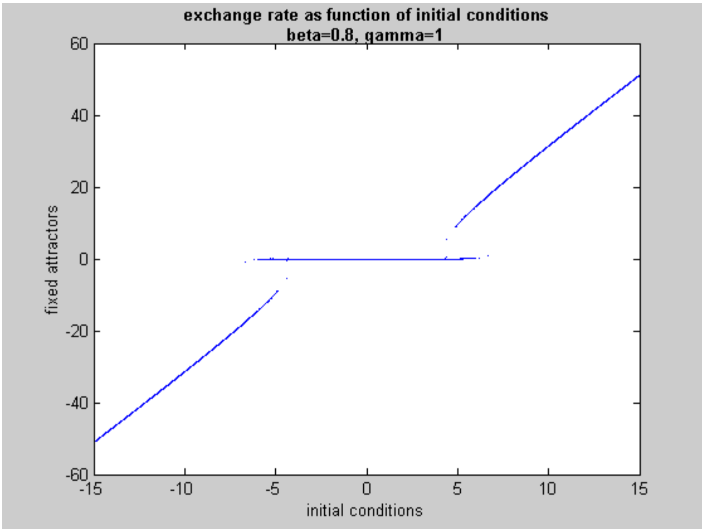


Figure 7: fundamental and bubble equilibria

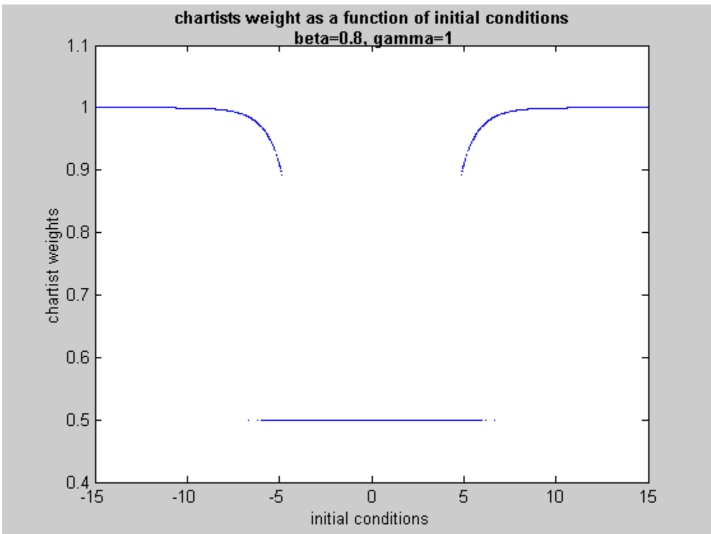


Figure 8: Chartist weights

5 Sensitivity analysis of deterministic model

In this section we perform a sensitivity analysis of the deterministic model. This will allow us to describe how the space of fundamental and bubble equilibria is affected by different values of the parameters of the model. In this section we concentrate on three parameters, i.e.μ, (the coefficient of risk aversion), β (the extrapolation parameter of technical traders) and γ (the sensitivity of technical traders and fundamentalists to relative profitability).

5.1 Sensitivity with respect to β

We show the result of a sensitivity analysis with respect of β in figure 9, which is a three-dimensional version of figure 7. The attractors (i.e. the fixed point solutions of the exchange rate) are shown on the vertical axis. The initial conditions are shown on the x-axis and the different values for μ on the z-axis. Thus, the two-dimensional figure 7 is a 'slice' of figure 9 obtained for one particular value of β (0.8 in figure 7).

We observe that for sufficiently low values of β we obtain only fundamental equilibria whatever the initial conditions. As β increases the plane which represents the collection of the fundamental equilibria narrows. At the same time the space taken by the bubble equilibria increases, and these bubble equilibria tend to increasingly diverge from the fundamental equilibria. Thus as the extrapolation parameter increases, smaller and smaller shocks in the initial conditions will push the exchange rate into the space of bubble equilibria. Put differently, as β increases, the probability of obtaining a bubble equilibrium increases.

Note also that the boundary between the fundamental and the bubble equilibria is a complex one. The boundary has a fractal dimension.

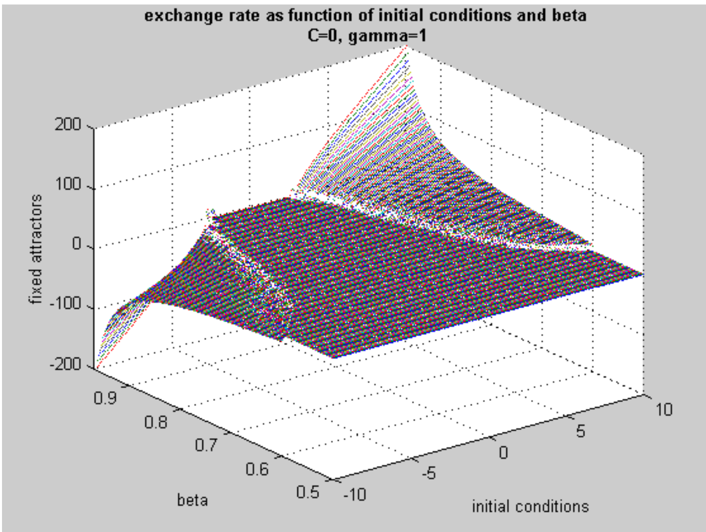


Figure 9: Sensitivity to β

5.2 Sensitivity with respect to γ

The parameter γ is equally important in determining whether fundamental or bubble equilibria will prevail. We show its importance in figure 10, which presents a similar three-dimensional figure relating the fixed attractors to both the ini- tial conditions and the values of γ. We find that for $\gamma = 0$ or close to 0, all equilibria are fundamental ones. Thus, when agents are not sensitive to changing profitability of forecasting rules, the exchange rate will always converge to the fundamental equilibrium whatever the initial condition. As γ increases, the space of fundamental equilibria shrinks. With sufficiently high values of γ, small initial disturbances (noise) are sufficient to push the exchange rate into a bubble equilibrium. Put differently, as γ increases, the probability of obtaining a bubble equilibrium increases. Finally, as in the case of β, we also observe that the boundary between the bubble and fundamental equilibria is complex.

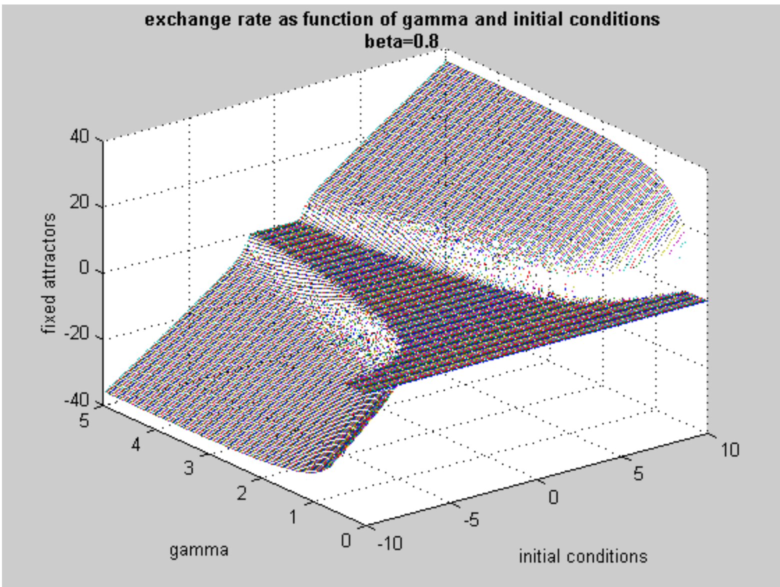


Figure 10: Sensitivity to γ

5.3 Sensitivity with respect to μ

Finally, we study the sensitivity of the equilibria with respect to the coefficient of risk aversion, μ. Figure 11 shows the results. We find that when the agents become more risk averse the space of fundamental equilibria shrinks while the space of non-fundamental equilibria becomes larger. This result forms the basis for understanding the stochastic simulations that uncovered that bubbles are larger and more likely to occur when agents are more risk averse. The intuition can now be understood better. When fundamentalists are willing to take large risks they will use the profit opportunities that arise when a bubble develops. As a result they will tend to move the exchange back towards the fundamental. This reinforces the mean reverting forces in the model thereby eliminating bubbles. Conversely, when these fundamentalists are not willing to take risks, they will not use the profit opportunities during a bubble. As a result, they will not sell when the exchange rate is overvalued (or buy when the exchange rate is undervalued), thereby eliminating the mean revering dynamics in the model. Thus in this interpretation bubble (non-fundamental) equilibria emerge because of a failure to arbitrage which itself is the result of excessive risk aversion from the part of fundamentalists.

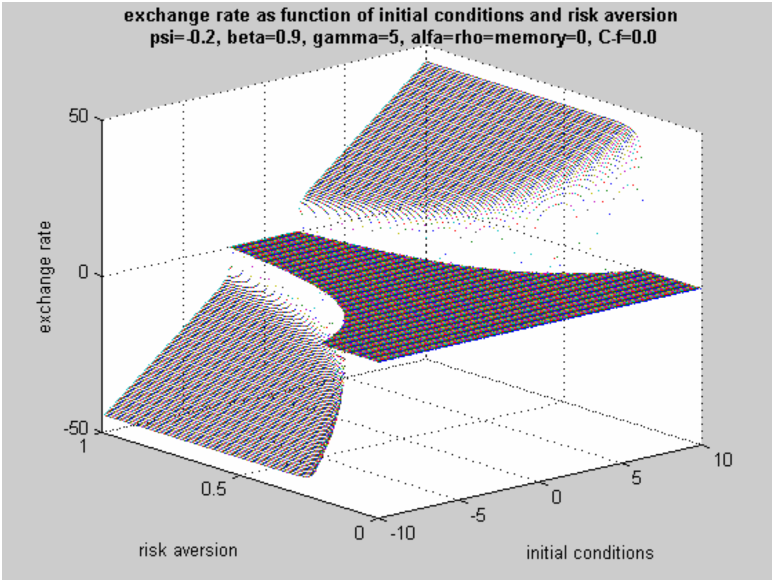


Figure 11: Sensitivity to μ

6 Why crashes occur

The model makes clear why bubbles arise in a stochastic environment. It may not be clear yet why bubbles are always followed by crashes. Here again shocks in the fundamental are of great importance. In order to analyse this issue we performed the following experiment. We fixed the initial condition at some value (+5) that produces a bubble equilibrium (for a given parameter configuration). We then introduced permanent changes in the fundamental value (ranging from -10 to +10) and computed the attractors for different values of β. We show the results of this exercise in figure 12. On the x-axis we show the different fundamental values of the exchange rate, while on the y-axis we have the different values of β. The vertical axis shows the attractors (exchange rate solutions). The upward sloping plane is the collection of fundamental equilibria. It is upward sloping (45%) because an increase in the fundamental rate by say 5 leads to an equilibrium exchange rate of 5. For low values of β we always have fundamental equilibria. This result matches the results of figure 9 where we found that for low β's all initial conditions lead to a fundamental equilibrium.
The major finding of figure 12 is that when permanent shocks in the fundamental are small relative to the initial (temporary) shock, (+5) we obtain bubble equilibria. The corollary of this result is that when the fundamental shock is large enough relative to the noise, we obtain a fundamental equilibrium. Thus if an initial temporary shock has brought the exchange rate in a bubble equilibrium, a sufficiently large fundamental shock will lead to a crash. In a stochastic environment in which the fundamental rate is driven by a

random walk (permanent shocks), any bubble must at some point crash because the attractive forces of the fundamental accumulate over time and overcome the temporary dynamics of the bubble.

The interesting aspect of this result is that the crash occurs irrespective of whether the fundamental shock is positive or negative. Since we have a positive bubble, it is easy to understand that a negative shock in the fundamental can trigger a crash. A positive shock has the same effect though. The reason is that a sufficiently large positive shock in the fundamental makes fundamentalist forecasting more profitable, thereby increasing the number of fundamentalists in the market and leading to a crash (to the new and higher fundamental rate). Put differently, while in the short run, technical traders exploit the noise to start a bubble, in the long run when the fundamental rate inexorably moves in one or the other direction, fundamentalists forecasting becomes attractive.

It is also interesting to note that as β increases, the size of the shocks in the fundamental necessary to bring the exchange rate back to its fundamental rate increases. In a stochastic environment this means that bubbles will be stronger and longerlasting when β increases.

In conclusion, it is worth noting that shocks in fundamentals both act as triggers for the emergence of a bubble and as triggers for its subsequent crash. The intuition can be explained as follows. When the exchange rate is in a fundamental equilibrium, an unexpected and permanent increase in the fundamental, sets in motion an upward movement of the ex- change rate towards the new fundamental. This is the result of the action by fundamentalists. This upward movement, however, also makes extrapolative forecasting (technical trading) increasingly profitable and can lead to a bubble.

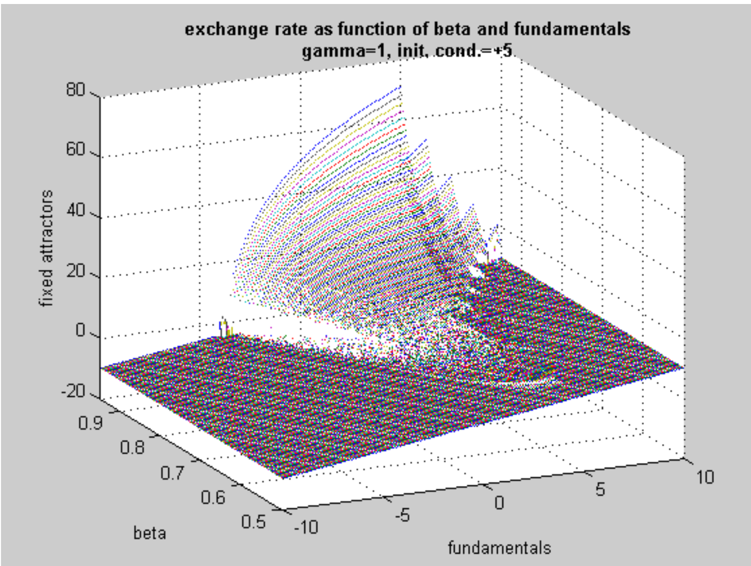


Figure 12: Why crashes occur

When the exchange rate is in a bubble equilibrium, a large enough (positive or negative) shock in the fundamental strenghtens the hand of fundamentalists' forecasting, and attracts agents towards this forecasting rule. This then leads to a crash.

As in the case of the bubble, the prediction of the timing of the crash is made difficult because of the fuzziness (complexity) of the border between bubble and fundamental equilibria (figure 12). Thus, although crashes are inevitable, their exact timing is unknown. The remarkable aspect of this result is that it is obtained in a deterministic model. For further analysis of the implications of the fractal nature of the border between fundamental and bubble equilibria see De Grauwe and Grimaldi(2006). There it is shown that this feature leads to sensitivity to initial conditions.

7 Conclusion

The world we have modelled is one in which agents do not understand its complexity. Therefore they use simple rules of behaviour which they check ex post (fitness criterion). This is the way to introduce discipline into the model. In such a world we get a very different dynamics compared to rational expectations world.

We find that there are bubble equilibria that attract the asset prices. They will be reached as a result of shocks which makes extrapolating forecasting profitable. It is also a world where there is sensitivity to initial conditions, or the importance of trivial events (see De Grauwe and Grimaldi(2006) on this).

Once in a bubble equilibrium one can stay there for a long time … or for a very short time. As a result, the exchange rate is disconnected from fundamentals very often. This feature has been called one of the main puzzles in the behaviour of exchange rates. The behavioural model presented here makes clear that this does not have to be a puzzle.

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Appendix: Numerical values of the parameters used in the base simulation

In the following table we present the numerical values of the model. In the first column we listed the parameters of the model, in the second column we present the numerical values in the base simulations. The last column indicates whether or not we have performed a sensitivity analysis on these numerical values. If not, we use the same numerical value in all simulations.

Table 1: Numerical values of parameters

Parameters	values	sensitivity analysis
ψ	0.2	No
ρ	0.6	No
β	0.8	Yes
θ	0.6	No
γ	1	Yes
μ	1	Yes
$H = K$ (lags)	5	No
r and r^*	0	No