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Predatory Short Selling

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Abstract. Financial institutions may be vulnerable to predatory short selling. When the stock of a financial institution is shorted aggressively, leverage constraints imposed by short-term creditors can force the institution to liquidate long-term investments at fire sale prices. For financial institutions that are sufficiently close to their leverage constraints, predatory short selling equilibria co-exist with no-liquidation equilibria (the *vulnerability region*), or may even be the unique equilibrium outcome (the *doomed region*). Increased coordination among short sellers expands the doomed region, where liquidation is the unique equilibrium. Our model provides a potential justification for temporary restrictions of short selling for vulnerable institutions and can be used to assess recent empirical evidence on short-sale bans.

JEL Classification: G01, G20, G21, G23, G28

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1. Introduction

The financial crisis of 2007-09 and the recent European sovereign debt crisis have led to a heated discussion on short selling financial stocks. For example, as financial stocks fell sharply in the spring, summer, and fall of 2008, a number of banks, most notably Bear Stearns, Lehman Brothers, and Morgan Stanley, blamed short sellers for their woes.[1] In response, the Securities and Exchange Commission (SEC) and a number of international financial regulators took measures against short selling; most significantly, some imposed temporary restrictions on the short selling of financial stocks, some even on short selling in general. In August 2011, when European banks were struggling because of losses due to the European sovereign debt crisis, market regulators in France, Spain, Italy and Belgium imposed temporary bans on short selling for some financial stocks.

On both occasions, the worry was that short selling was "predatory," in the sense that short sellers were attempting to bring down fundamentally solvent financial institutions by triggering self-fulfilling downward spirals. However, this line of argument is at odds with the consensus view in economics, which—broadly speaking—says that there is nothing wrong with short selling. In fact, most economists would argue that short selling is a valuable activity—short sellers help enforce the law of one price, facilitate price discovery, and enhance liquidity. Moreover, short sale restrictions may lead to overvaluation and bubbles, and reduce the ability of investors to hedge exposures.[2] In the light of these findings, is there any economic justification to impose restrictions on short selling the stock of financial institutions?

In this paper, we present a model of predatory short selling. We show that, even though short selling activity is beneficial during "normal times," at times of stress short sellers can, in fact, destabilize financial institutions through predatory short sales. Predatory short selling can occur in our model because financial institutions are subject to leverage constraints imposed by their short-term creditors and uninsured depositors. These leverage constraints capture a first-order difference between financial institutions and regular corporates: Relative to corporations that can match maturities of assets and liabilities, the business model of a financial institution almost necessarily involves maturity and liquidity mismatch (see, e.g., Diamond and Dybvig (1983) and Brunnermeier, Gorton, and Krishnamurthy (2013)), which exposes financial institutions to sudden withdrawal of funding in response declines in equity value. We show that, in the presence of such leverage constraints, predatory short sellers that temporarily depress the stock price of a financial

[1] See, for example, "Anatomy of the Morgan Stanley Panic," *Wall Street Journal*, November 24, 2008.

[2] For theoretical models on how short-sale constraints can lead to overvaluation, speculative trading, and bubbles, see, for example, Miller (1977), Harrison and Kreps (1978), Scheinkman and Xiong (2003), and Hong and Stein (2003). Diamond and Verrecchia (1987) show theoretically that a market with short-sale constraints incorporates information more slowly than a market in which short sales are not restricted. Empirical evidence that short sellers contribute to market efficiency and market quality can be found, among others, in Dechow, Hutton, Meulbroek, and Sloan (2001), Desai, Krishnamurthy, and Kumar (2006), Bris, Goetzmann, and Zhu (2007) , Chang, Cheng, and Yu (2007), Boehmer, Jones, and Zhang (2008), Saffi and Sigurdsson (2011), and Boehmer and Wu (2013). Short positions are important hedging tools in a number of common trading strategies (e.g., hedging options, convertible bonds, or market risk in long-short strategies).

institution can force the financial institution to sell long-term assets in order to repay debt to satisfy their leverage constraint. When long-term assets have to be unwound at a sufficient discount, the resulting losses for the financial institution allow predatory short sellers to break even on their short positions.

Our model implies that financial institutions can be vulnerable to attacks from predatory short sellers when their balance sheets are weak. For financial institutions that are sufficiently close to their leverage constraints, predatory short selling equilibria co-exist with no-liquidation equilibria (the *vulnerability region*), or may even be the unique equilibrium outcome (the *doomed region*). In the vulnerability region there are two stable equilibria. In one equilibrium, no predatory short selling occurs. In that case, the financial institution does not violate its constraint and can hold its long-term investments until maturity. In the second equilibrium, however, predatory short selling causes the financial institution to violate its leverage constraint, leading to a complete liquidation of its long-term asset holdings. In the doomed region, there is a unique stable equilibrium in which predatory short sellers force the financial institution to liquidate its entire long-term asset holdings.

Comparing a regime with short selling to one with short-sale restrictions shows that, during "normal times" when financial institutions are well capitalized, the fundamental value of the financial institution cannot be affected by the presence of predatory short sellers. In this region short sellers exclusively fulfill their beneficial roles of providing liquidity and preventing overvaluation and bubbles that may distort real investment decisions—confirming the consensus view that restricting short selling during normal times is likely to have undesired consequences. However, this changes once a financial institution enters the vulnerability region or the doomed region. Here, short sellers can force inefficient liquidation of the financial institutions' long-term assets, such that restrictions on short selling can potentially be welfare-enhancing.

By highlighting the possibility of multiple equilibria, our model underlines the important role of coordination in short selling attacks. Specifically, adding a large short seller (or, equivalently, a mass of small short sellers that can coordinate their actions) to our competitive benchmark model expands the doomed region, where a predatory short selling attack and full liquidation is the unique (and inefficient) outcome. This contrasts sharply with the situation where sellers are regular shareholders (rather than short sellers): In this case, adding a large shareholder (or a mass of coordinated small shareholders) increases the safety region, in which no liquidation is the unique equilibrium. Finally, we show that when there is a large short seller and a potential large support buyer, the doomed region depends on the relative strength (or ability to coordinate) of the large short seller and the support buyer.

Overall, our results provide a potential justification for temporary short sale restrictions for financial institutions at times when their balance sheets are weak. These restrictions should be temporary and targeted specifically at weak financial institutions because well-capitalized financial institutions are not susceptible to predatory short selling attacks, such that the only effect of a ban on short selling for those institutions would be a reduction in liquidity and market quality of their stock. Moreover, because our results are driven by a constraint on market leverage that is imposed by short-term creditors, it highlights the particular vulnerability of financial institutions—because of the maturity mismatch and liquidity mismatch inherent in their business models, financial institutions are subject to financial fragility in the form of creditor runs, which makes them vulnerable to predatory short sellers. Our model is less likely to apply to firms with more stable capital structures.

Finally, our analysis has implications on the disclosure of short positions. By facilitating coordination among short sellers, full and timely disclosure of short positions may in fact make it *easier* for short sellers to prey on vulnerable financial institutions.

The empirical evidence on the recent short-sale bans in the U.S. and Europe unambiguously documents reductions in market liquidity and market quality as a result of short sale bans. However, there is no strong empirical support for positive price effects of recent short sale bans, perhaps the main motivation of these bans. Using daily international data on recent short-sale bans around the world, Beber and Pagano (2013) document that the short-sale bans implemented during the financial crisis of 2007-2009 led to reductions in market liquidity and slower price discovery. They also document that short-sale constraints failed to support stock prices, except potentially those of large U.S. financial stocks. In similar spirit, Boehmer, Jones, and Zhang (2013), using intraday data on the 2008 short-sale ban in the U.S., document a deterioration of liquidity and market quality in response to the short-sale ban. Yet, their analysis also finds little evidence that short-sale bans supported prices: Only the largest financial institutions had (permanent) positive abnormal returns during 2008 short-sale ban, and Boehmer, Jones, and Zhang (2013) point out that the price effects of the short selling ban are hard to disentangle from the effects of the contemporaneous Troubled Asset Relief Program (TARP).[3]

Our results may help interpret the existing empirical findings on the effect of short selling bans, and may also be useful in the empirical design of future studies. First, when looking at price effects, our model suggests that financial institutions, rather than regular firms, should be particularly affected by short sale bans. Second, in our model, the ability of short sellers to prey on financial institutions depends crucially on the financial condition of the financial institution. Hence, a second cross-sectional prediction of our model is that in assessing the price effects of short-sale restrictions, one should control for leverage, liquidity mismatch, or similar variables that measure financial fragility. The prediction of our model is that it is vulnerable financial firms for which the price effects of short sale bans are likely largest. Third, our model highlights the importance of taking into account the potential multiplicity of equilibria when interpreting the empirical evidence. For example, if investors expect that, with some probability, there is a switch to the dominated equilibrium in which the bank goes bankrupt, the elimination of the bad equilibrium through temporary short selling bans when financial institutions enter the vulnerability region may lead to permanent positive price effects. In addition to the cross-sectional predictions above, a

[3] A number of other studies have investigated the effects of recent short-sale bans. For example, Marsh and Payne (2012) document a decrease in market quality in UK equity markets in response to the 2008 short selling ban. Using U.S. and European data, Lioui (2011) documents an increase in volatility in response to the 2008 short selling ban, but no effect on the price skewness. Autore, Billingsley, and Kovacs (2011) document that during the 2008 short sale ban in the U.S., stocks with a larger decline in liquidity also have poorer contemporaneous returns, consistent with the model of Amihud and Mendelson (1986). Harris, Namvar, and Phillips (2013) use a factor model to document price inflation in banned stocks as a result of the 2008 short selling ban in the U.S., particularly for firms without traded options. Their results suggest that price effects were temporary for stocks with negative pre-ban performance and permanent for firms with positive pre-ban performance. Kolasinski, Reed, and Thornock (2013) document that variations in short interest had larger price effect during the shorting ban, consistent with an increase in the informativeness of short sales in response to increased short sale restrictions.

novel prediction of our model is that the vulnerability of a financial institution to predatory short selling depends not only on its own balance sheet but also on the balance sheets (or funding conditions) of its large shareholders.

At a theoretical level, the potential justification for restrictions on short selling provided by our model is similar to that given in the literature on feedback effects from stock prices to firms' real decisions.[4] For example, Goldstein and Gümbel (2008) provide an asymmetric information model, in which a feedback loop to real investment decisions allows a short seller to make a profit even in the absence of fundamental information. Khanna and Mathews (2012) study the interaction between an uninformed speculator and an informed blockholder to a firm. They show that, under certain conditions, manipulation by an uninformed speculator is possible even in the presence of an informed blockholder, whose incentives are aligned with value maximization. A major difference between these papers and our analysis is the channel through which short selling can be profitable. In both Goldstein and Gümbel (2008) and Khanna and Mathews (2012), short sellers reduce price informativeness, thereby inducing the firm (whose manager learns from prices) to inefficiently distort its (future) investments, which makes the short position profitable. In our framework, price declines brought about by short sellers can trigger inefficient early liquidation of existing investments via the leverage constraint. In our view, this latter channel is particularly relevant for the recent discussion on short-sale bans, which has centered around financial institutions.A related feedback mechanism arises in Goldstein, Ozdenoren, and Yuan (2013). In their model, a provider of equity capital learns from the firm's stock price and provides less capital to the firm when he infers negative information from the firm's stock price.[5]

In terms of the focus on financial institutions, the paper closest to ours is Liu (2011). Liu develops a two-stage global games model: In the first stage, short sellers take positions. In the second stage, creditors decide whether or not to roll over their debt. As in Goldstein and Gümbel (2008), the presence of short sellers reduces price informativeness. The resulting increase in uncertainty about fundamentals reduces the value of short-term debt claims (due to their concave payoff) and can induce creditors to run in the second stage. Hence, a major difference to our paper is how short-sale attacks work: In our framework it is the reduction in the market value of equity that makes a short selling attack possible. In Liu (2011), it is the increase in price uncertainty (and not the price reduction) that leads to a creditor run. Liu's framework leads to policy prescriptions that are broadly in line with ours. First, as in our model, his framework implies that banks with weak fundamentals are prone to short-sale attacks. Second, Liu argues that more maturity mismatch makes short-selling attacks more likely. This is consistent with our model, where one can interpret the severity of the leverage constraint as a proxy for maturity mismatch (the more short term creditors the bank has, the more binding the run constraint).[6]

[4] For a comprehensive survey of this literature, see Bond, Edmans, and Golstein (2012).

[5] Khanna and Sonti (2004) and Ozdenoren and Yuan (2008) provide models with exogenous feedback from financial markets to real outcomes.

[6] More broadly, our paper also relates to the literature on market manipulation, and predatory trading. Allen and Gale (1992) provide a model in which a non-informed trader can make a profit if investors think the manipulator may be an informed trader. Other papers that consider manipulative trading strategies include Allen and Gorton (1992), Benabou and Laroque (1992), Kumar and Seppi (1992), Gerard and Nanda (1993), Chakraborty and Yilmaz (2004), and Brunnermeier (2005). Brunnermeier and Pedersen

The remainder of the paper is structured as follows. Section 2. gives a brief summary of regulatory response to short selling during the financial crises of 2007-2009 and the European sovereign debt crisis of 2011. Section 3. presents the model. Section 4. provides a discussion of the model's policy implications and empirical predictions. Section 5. concludes. All proofs are in the Appendix.

2. Recent regulatory response to short selling

As a result of the financial market turmoil in 2008, the SEC and a number of international financial market regulators put in effect a number of new rules regarding short selling. In July the SEC issued an emergency order banning so-called "naked" short selling[7] in the securities of Fannie Mae, Freddie Mac, and primary dealers at commercial and investment banks. In total 18 stocks were included in the ban, which took effect on Monday July 21 and was in effect until August 12.

On September 19 2008, the SEC banned *all* short selling of stocks of financial companies. This much broader ban initially included a total of 799 firms, and more firms were added to this list over time. In a statement regarding the ban, SEC Chairman Christopher Cox said, "The Commission is committed to using every weapon in its arsenal to combat market manipulation that threatens investors and capital markets. The emergency order temporarily banning short selling of financial stocks will restore equilibrium to markets. This action, which would not be necessary in a well-functioning market, is temporary in nature and part of the comprehensive set of steps being taken by the Federal Reserve, the Treasury, and the Congress." This broad ban of all short selling in financial institutions was initially set to expire on October 2, but was extended until Wednesday October 9, i.e., three days after the emergency legislation (the bailout package) was passed.

In addition to measures taken by the SEC, a number of international financial regulators also acted in response to short selling. On September 21 2008, Australia temporarily banned all forms of short selling, with only market makers in options markets allowed to take covered short positions to hedge. In Great Britain, the Financial Services Authority (FSA) enacted a moratorium on short selling of 29 financial institutions from September 18 2008 until January 16 2009. Also Germany, Ireland, Switzerland and Canada banned short selling of some financial stocks, while France, the Netherlands and Belgium banned naked short selling of financial companies.

International restrictions on short selling of financial stocks reappeared in 2011. In August of 2011, market regulators in France, Spain, Italy and Belgium imposed temporary restrictions on the short selling of certain financial stocks as European banks came under increasing pressure as part of the sovereign debt crisis in Europe. For example, both Spain and Italy imposed a temporary bans on new short positions, or increases in existing short positions, for a number of financial shares. France temporarily restricted short selling for 11 companies, including Axa, BNP Paribas and Credit Agricole.[8] On August 26, France,

(2005) provide a model in which a predatory trader can exploit another trader's need to liquidate.

[7] In a naked short-sale transaction, the short seller does not borrow the share before entering the short position.

[8] See Howard Mustoe and Jesse Westbrook, "Short Selling of Stocks Banned in France, Spain," *Bloomberg*, August 12, 2011.

Italy and Spain extended their temporary bans on short selling until at least the end of September.

Of course, measures against short selling are not exclusive to these recent episodes. In response to the market crash of 1929, the SEC enacted the uptick rule, which restricts traders from selling short on a downtick. In 1940, legislation was passed that banned mutual funds from short selling. Both of these restriction were in effect until 2007. Going back even further in time, the UK banned short selling in the 1630s in response to the Dutch tulip mania.

3. Model

We consider a simple model with three periods, $t = 0, 1, 2$. At $t = 0$, a financial institution has invested in X units of a long-term asset. The financial institution has also taken out demandable debt with face value D_0. We take both the initial position in the risky asset as well as the initial debt outstanding as given.

Most of our analysis focuses on the interim date $t = 1$. Seen from $t = 1$, the long-term asset is expected to pay off a deterministic amount R at $t = 2$. If needed, the long-term asset can be liquidated at $t = 1$, but early liquidation is subject to a discount; the liquidation value at $t = 1$ is given by δR, where $\delta < 1$. Hence, early liquidation is inefficient. For simplicity, we assume that the financial institution holds no cash, but the model could be straightforwardly extended to allow for cash holdings.

Leverage Constraint. Key to our analysis is that the financial institution is subject to a leverage constraint. Specifically, we assume that debt as a fraction of debt plus the market value of equity cannot exceed a critical amount $\gamma \in [0, 1]$:

$$\frac{D}{D+E} \leq \gamma. \tag{1}$$

This leverage constraint captures in a simple way a fundamental difference between financial institutions and regular corporations. Specifically, relative to corporations that can match maturities of assets and liabilities, the business model of a financial institution almost necessarily involves maturity and liquidity mismatch (see, e.g., Diamond and Dybvig (1983) and Brunnermeier, Gorton, and Krishnamurthy (2013)). Moreover, beyond the maturity mismatch that is inherent in their business model, financial institutions may have an additional incentive to take on significant maturity mismatch because of collective moral hazard (Farhi and Tirole (2012)) or because their inability to commit to longer-term financing leads to a maturity rat race (Brunnermeier and Oehmke (2013)).

In the presence of maturity mismatch, the leverage constraint (1) emerges because uninsured depositors and creditors of the bank withdraw their funding when the bank's market leverage exceeds γ at the interim date. While we do not model this formally, what we have in mind is that creditors use the market price of equity to update their expectations on the bank's prospects and refuse to roll over their loans whenever the financial institution's market leverage exceeds a threshold γ. One can thus think of the leverage constraint as a "run constraint," in the sense that creditors run on the bank following negative signals about the value of the bank's equity relative to its outstanding debt obligations (as in models of fundamental bank runs, such as Postlewaite and Vives (1987), Jacklin and Bhattacharya (1988), or Goldstein and Pauzner (2005)). This interpretation of the leverage constraint also highlights the connection to the literature on feedback effects of asset prices—the constraint captures, in reduced form, the feedback that arises when providers of capital learn from prices (as, for example, in Goldstein, Ozdenoren, and Yuan (2013)).

We formulate the leverage constraint in terms of market leverage. However, what ultimately is important for the model is that—independent of its particular form—the leverage constraint implies that when the financial institution's market value of equity falls below a certain value, the financial institution is forced to liquidate some of its long-term assets in order to repay creditors. As we will show below, in certain circumstances this feedback mechanism, triggered by stock price declines, can make the financial institution vulnerable to short sellers in the equity market.[9]

Equity Market. At date $t = 1$, the equity of the financial institution is traded in a financial market. This financial market is populated by two types of investors, a competitive fringe of passive long-term investors and active traders which act as short sellers. More generally, one could think of the short sellers as arbitrageurs that can take both long or short positions. However, since the main results of our paper revolve around the effects of short selling, we will refer to them as short sellers. We also assume that short sellers start with a zero position in the financial institution's equity. We discuss the case of regular sellers and differences between selling and short selling in Section 3.3.

The long-term investors are competitive and offer demand schedules to the short sellers. The long-term investors are thus not active traders themselves, they simply form a residual demand curve that short sellers can sell into. Upon observing the demand schedules offered by the long-term investors, short sellers decide whether to take a position in the stock. Short sellers are competitive and thus make zero profits in equilibrium.

We focus on the interaction in the equity market at the intermediate period, $t = 1$. At $t = 1$, the two types of players, long-term investors and short sellers, interact in the following way. Long-term investors choose the slope and intercept of a demand schedule that they offer to the short sellers. We denote the intercept by $\overline{P}$ and the slope by λ. Formally the long-term investors' action space is thus given by the pair $(\overline{P}, \lambda) \in \mathbb{R} \times \mathbb{R}^+$. Note that by assumption $\lambda > 0$, i.e., the residual demand curve for the stock is downward sloping. However, as we will argue below, the slope of the demand curve can be arbitrarily small.[10] Upon observing the demand schedules offered to them, the short sellers decide how much of the stock they want to sell short. Their action space is thus the size of their short position, $S \in \mathbb{R}$.[11] Given these ingredients, we can write price of the financial institution's equity at $t = 1$ as

$$\widetilde{P} = \overline{P} - \lambda S. \tag{2}$$

[9] This particular vulnerability of financial institution's is echoed in the SEC's justification of the 2008 short selling ban, which highlights the potential loss of confidence of trading counterparties in response to short selling.

[10] There are a number of ways to justify a downward-sloping demand curve. For example, our assumption may capture in reduced form that long-term investors are risk averse and need to be compensated for risk that hey hold in equilibrium. The downward sloping demand curve may also the be the result of information asymmetries (as in Kyle (1985)) that are not modeled explicitly here.

[11] While we focus on short positions, we do not rule out long positions, which can be taken by picking a negative S. Hence, short sellers in this model are not short sellers by assumption—rather, they take short positions to exploit the financial institution's constraint, which is not possible by taking long positions.

Equilibrium. The equilibrium amount of short selling will be determined by a zero profit condition, meaning that the stock price at $t = 1$, $\widetilde{P} = \overline{P} - \lambda S$ must be a rational prediction of the value of equity at $t = 2$, when the long-term investment pays off and equity investors receive their payoff. Denoting the payoff to equity holders at date $t = 2$ by P, the equilibrium condition is thus given by

$$\widetilde{P} = P. \tag{3}$$

Predatory Short Selling. We distinguish two types of short selling. In an equilibrium with *regular short selling*, short sellers are active, but their only effect is to ensure that the stock price coincides with the given fundamental value of the financial institution's equity. In other words, regular short selling ensures that the price is right, but does not affect the fundamental value of the firm. In an equilibrium with *predatory short selling*, on the other hand, the act of short selling reduces the fundamental value of the financial institution—it forces early liquidation of long-term assets at a loss. Through this feedback mechanism, predatory short selling reduces the value of equity and thus becomes self fulfilling. Hence, while regular short selling ensures that prices are right for a given fundamental and is thus beneficial, predatory short selling destroys fundamental value and is inefficient.

Note that, in our formulation, long-term investors act essentially as passive shareholders. Specifically, we rule out that current shareholders can meet the leverage constraint by recapitalizing the bank. This assumption reflects that recapitalization via issuing new equity may be hard in the midst of a short selling attack (in addition to the more general observation that issuing equity may be costly because of the usual asymmetric information considerations). Similarly, we rule out that the financial institution renegotiates its debt, for example through a debt for equity swap. This assumption seems reasonable given that a financial institution that faces dispersed short-term creditors (or depositors) will usually have a hard time renegotiating debt, because of the coordination issues inherent in renegotiating dispersed debt issues.[12]

3.1 BENCHMARK CASE WITHOUT LEVERAGE CONSTRAINT

We first solve a benchmark model without the leverage constraint. As we will see, in this setup short sellers serve a role in ensuring that the financial institution's equity is fairly priced (through regular short selling), but the short sellers' actions do not have any influence on the fundamental value of the financial institution. Hence, in the absence of the leverage constraint, predatory short selling cannot occur in equilibrium.

Lemma 1. *When financial institutions are not subject to the leverage constraint (1), predatory short selling does not occur in equilibrium.*

The intuition behind Lemma 1 is simple. Because absent the leverage constraint the fundamental payoff to equity holders is fixed at $XR - D_0$, only regular short selling can occur in equilibrium: short sellers may take a short position to ensure that the financial

[12] Potentially, the coordination problems in renegotiating debt could be mitigated if the financial institution issued some amount reverse convertible debt that can be converted into equity when the leverage constraint is binding. However, recall that in the Diamond and Rajan (2001) model of banking, it is precisely the inability to renegotiate debt that makes financing with dispersed short-term efficient for financial institutions.

institution's equity is valued correctly in cases where the long-term investors pick an intercept that exceeds fundamental value, i.e., $\overline{P} > XR - D_0$. However, there is no way for short sellers to affect the fundamental value of the financial institution's equity, which makes predatory short selling impossible.

Lemma 1 thus highlights the beneficial role of regular short selling. When equity is overpriced relative to fundamental value, the ability to take short positions allows short sellers (or, more generally, arbitrageurs) to correct such overvaluation and make sure that equity is fairly priced. This is beneficial because overvaluation may lead to distorted investment incentives and, ultimately, misallocation of resources. This beneficial role of regular short selling is the main reason why unconditional short selling bans are undesirable.

For the remainder of the paper, we will focus on predatory short selling and put aside the beneficial role of short selling illustrated in Lemma 1. To do this, we focus on the case, in which the intercept chosen by long-term investors reflects the fundamental value of equity in the absence of short selling. For example, if in the absence of short selling the financial institution does not have to sell any assets at the interim date, the long-term investors pick $\overline{P} = XR - D_0$. Focusing on the case implies that there is no role for regular short sellers, since in the absence of leverage constraints short sellers would never take a position. This assumption thus allows us to restrict our analysis to predatory short selling. However, as we discuss below, focusing on the case in which long-term investors set the intercept $\overline{P}$ equal to the fundamental value of equity in the absence of short selling is essentially without loss of generality: With slight adjustments the analysis generalizes to arbitrary $\overline{P}$. The main difference between the two cases is that when $\overline{P}$ can deviate from fundamental value, short sellers may also serve a beneficial role by reducing overpricing.

3.2 INTRODUCING THE LEVERAGE CONSTRAINT

We now introduce leverage constraint. Recall that the leverage constraint (1) requires the financial institution to keep leverage (defined as debt divided by debt plus the market value of equity) below a critical level γ: $\frac{D}{D+E} \leq \gamma$. When the leverage constraint is violated at date $t = 1$, the financial institution must repay some of its debt to reduce leverage and thus has to liquidate some of the long-term asset holdings.

Denote the number of units of the long-term asset the financial institution has to sell at $t = 1$ by $\Delta X(S)$, where S is the position taken by short sellers. If at $t = 1$ the financial institution sells $\Delta X(S)$ units of the long-term asset to repay debt, this leads to an equity payout at time $t = 2$ of

$$P = \max\left[XR - D_0 - (1-\delta)R\Delta X(S), 0\right] \tag{4}$$

The reduction in equity value, $(1-\delta)R\Delta X(S)$, reflects the fact that the long-term asset can only be sold at a discount. Using equation (4), we can thus rewrite the equilibrium condition (3) as

$$\overline{P} - \lambda S = \max\left[XR - D_0 - (1-\delta)R\Delta X(S), 0\right]. \tag{5}$$

How much does the financial institution have to liquidate? In order to find potential equilibria, we need to determine how much the financial institution needs to liquidate at $t = 1$. First note that when, given a short position of S, the leverage constraint is not violated, the financial institution does not have to liquidate any of its long-term investments. In this case $\Delta X(S) = 0$. On the other hand, when the equity value at $t = 1$

(including the price effects of the short position S) is such that the constraint is violated,

$$\frac{D_0}{D_0 + \overline{P} - \lambda S} > \gamma, \tag{6}$$

the financial institution has to sell $\Delta X(S)$ units of the long-term asset and repay debt in order to satisfy the constraint. The amount the financial institution has to liquidate is then determined by the following condition:

$$\frac{D_0 - \Delta X(S)\delta R}{D_0 - \Delta X(S)\delta R + \overline{P} - \lambda S} = \gamma. \tag{7}$$

The numerator in (7) is the amount of debt remaining after liquidating $\Delta X(S)$ units of the long-term investment and thereby reducing outstanding debt by $\Delta X(S)\delta R$. The denominator contains remaining debt $D_0 - \Delta X(S)\delta R$ plus the market value of equity $\overline{P} - \lambda S$. Solving (7) for $\Delta X(S)$ yields the following result:

Lemma 2. *The amount of the long-term asset that the financial institution needs to liquidate under the leverage constraint (1) and in the presence of short sellers that take an aggregate short position S, is given by*

$$\Delta X(S) = \begin{cases} 0 & \textit{if } \frac{D_0}{D_0+\overline{P}-\lambda S} \leq \gamma \\ \min\left[\frac{(1-\gamma)D_0 - \gamma(\overline{P}-\lambda S)}{R\delta(1-\gamma)}, X\right] & \textit{if } \frac{D_0}{D_0+\overline{P}-\lambda S} > \gamma \end{cases} \tag{8}$$

Figure 1 illustrates what happens once we introduce the leverage constraint. In the illustration, absent short sales the leverage constraint is satisfied. However, a sufficiently large position by short sellers can force the financial institution to liquidate some of its long-term asset holdings. These forced sales reduce the financial institution's equity value equity because the long-term asset has to be sold at a discount to fundamental value (when sold at $t = 1$ it yields δR rather than R). Hence, the fundamental value of the financial institution's equity has a kink at the point where the leverage constraint becomes binding and forces the financial institution to sell assets. To facilitate comparison to the benchmark case discussed above, the dashed line indicates the fundamental value of equity in the absence of the leverage constraint.

Recall the equilibrium condition, $\widetilde{P} = P$. This condition implies that potential equilibria are intersections of the two lines in Figure 1, i.e., points where the price in the equity market at $t = 1$ rationally reflects the fundamental value of the equity of the financial institution at $t = 2$. Turn first to the top panel of Figure 1. We continue to assume that long-term investors choose the intercept $\overline{P}$ to be equal to fundamental value absent short sales, i.e., $\overline{P} = XR - D_0$.[13] Because the two lines only intersect once, the only equilibrium remains the one without short selling—even though the short sellers can drive down the fundamental value of the financial institution by forcing it to liquidate some of its long-term investments, they invariably lose money in the process. This is the case whenever the liquidation value of the long-term asset, which is parameterized by δ, is sufficiently large.

[13] As we show below, this assumption is not crucial in the sense that the equilibria are independent of the particular choice of $\overline{P}$ and λ. We focus on the case $\overline{P} = XR - D_0$ because it allows us to focus exclusively on predatory short selling. The main difference to the case $\overline{P} \neq XR - D_0$ is that now also the beneficial role of short selling (as discussed above) emerges.

In this case, the value destruction on response to a violation of the leverage constrained is small, such that predatory short selling is not profitable. The unique equilibrium is the one where $P = XR - D_0$. When $\overline{P} = XR - D_0$ this implies that the equilibrium amount of short selling is $S = 0$. More generally, when $\overline{P} \neq XR - D_0$, short selling can occur in equilibrium, but only in its beneficial role of ensuring that prices are equal to the fundamental $XR - D_0$.

The bottom panel of Figure 1, on the other hand, shows that when δ is sufficiently small, predatory short selling can emerge. In this case, in addition to the equilibrium without short selling, two further equilibria emerge. Both of these additional equilibria involve predatory short selling: Short sellers cause a decrease in the financial institution's equity value at $t = 1$, which forces the financial institution to liquidate long-term asset holdings to an extent that allows short sellers to break even. As is usually the case, the middle equilibrium is unstable (such that a small perturbation would lead to migration to either of the two stable equilibria).

Equilibrium prices are independent of λ and $\overline{P}$. One convenient feature of our model is that, as long as short sellers are not restricted in the number of shares they can short, the equilibrium prices and the equilibrium amount of the long-term asset that the financial institution needs to liquidate are independent of the particular $\overline{P}$ and λ chosen by the long-term investors. Hence, while there are many equilibria involving different combinations of $\overline{P}$, λ and S, these equilibria are isomorphic in terms of equilibrium prices and liquidation quantities. One implication of this feature of the model is that while setting the intercept $\overline{P}$ equal to fundamental value absent short selling allows us to focus exclusively on predatory short selling, equilibrium prices and the existence of predatory short selling equilibria do not depend on this assumption.

Lemma 3. *When short sellers are unconstrained in the size of the short position they take, the equilibrium prices and the amount that has to be liquidated by the financial institution is independent of λ and $\overline{P}$.*

This independence result is illustrated in Figure 2 for the case in which the leverage constraint is satisfied absent short selling. The top panel shows that when λ is decreased from 0.75 (dashed line) to 0.6 (solid line), the equilibrium amount of short selling changes, but equilibrium prices remain the same. The bottom panel shows that when, in addition, also the intercept $\overline{P}$ is increased from 32 to 34, the equilibrium prices again remain unchanged. In this case, the equilibrium in which $P = RX - D_0$ exhibits beneficial short selling, while the other two equilibria exhibit predatory short selling.

Lemma 3 is convenient since it allows us to classify equilibria by looking only at equilibrium prices and the amount of the long-term asset that has to be liquidated by the financial institution.

Overview of Equilibria. We are now in a position to summarize the equilibria in the equity market at $t = 1$. In the proposition, we focus on the case where the long-term asset is relatively illiquid, $\delta < \gamma$. This is the case depicted in the right panel of Figure 1. The (less interesting) case $\delta \geq \gamma$ is discussed in the appendix.

Proposition 1. *In the presence of the leverage constraint (1), when $\delta < \gamma$ we distinguish three regions.*

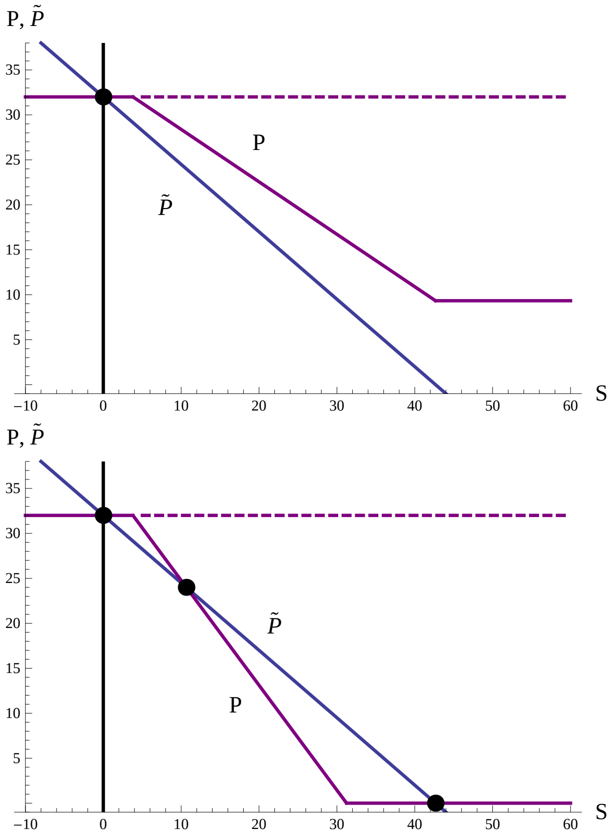

Fig. 1. **Introducing the leverage constraint.** When the leverage constraint is introduced (in this figure, $\gamma = 0.7$), a sufficiently large short position can force the financial institution to liquidate some of its long-term asset holdings. In the top panel, the loss to the financial institution from liquidating the long asset is not large enough to make a predatory short position profitable ($\delta = 0.75$). The only equilibrium is the one in which no predatory short selling occurs. In the bottom panel, on the other hand, we see that when the losses from liquidating the long-term asset are large enough, two predatory short selling equilibria emerge in addition to the equilibrium without predatory short selling ($\delta = 0.6$). The middle equilibrium is unstable. The remaining parameters in this figure are: $X = 10, R = 10, D = 68, \lambda = 0.75$

1. ***Safety region:*** *When the financial institution is sufficiently well capitalized,* $R > \frac{D_0}{\delta X}$*, there is a unique equilibrium in which the financial institution does not have to liquidate any of its long-term holdings. No predatory short selling can occur,* $\Delta X(S) = 0$*, and* $P = XR - D_0$*.*
2. ***Vulnerability region:*** *When* $\frac{D_0}{\gamma X} \le R \le \frac{D_0}{\delta X}$*, there are two stable equilibria and one unstable equilibrium.*
   a. *In one stable equilibrium, the financial institution does not liquidate any of its long-term holdings,* $\Delta X(S) = 0$*, and* $P = XR - D_0$*.*

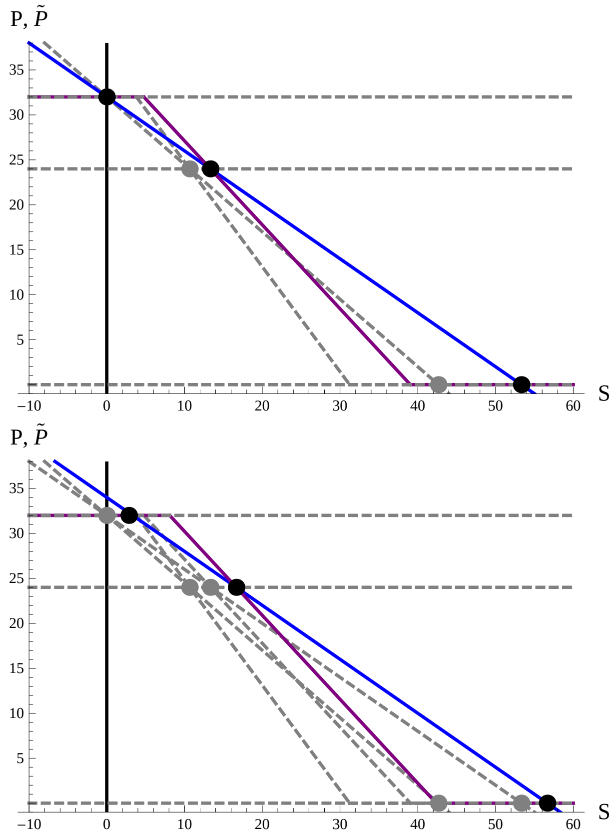

Fig. 2. **Equilibrium prices are independent of $\overline{P}$ and λ.** The top panel shows that when λ is decreased from 0.75 (dashed line) to 0.6 (solid line), the equilibrium amount of short selling changes, but equilibrium prices remain the same. The bottom panel shows that when in addition also the intercept $\overline{P}$ is increased from 32 to 34, the equilibrium prices again remain unchanged. In this case, the equilibrium in which $P = RX - D_0$ exhibits beneficial short selling, while the other two equilibria exhibit predatory short selling. The remaining parameters are $R = 10, X = 10, \delta = 0.6, \gamma = 0.7, D = 68$.

b. *In the other stable equilibrium, the financial institution is forced to liquidate its entire holdings of the long-term asset, i.e.,* $\Delta X(S) = X$ *and* $P = 0$.

c. *In the unstable equilibrium, the financial institution has to liquidate part of its equity holdings,* $\Delta X(S) = X\frac{\gamma - \frac{D_0}{XR}}{\gamma - \delta}$ *and* $P = \frac{1-\gamma}{\gamma-\delta}(D_0 - \delta X R)$

3. ***Doomed region:*** *When* $R < \frac{D_0}{\gamma X}$*, there is a unique stable equilibrium and an unstable equilibrium.*

a. *In the stable equilibrium, short sellers are active and the financial institution liquidates its entire holdings of the long-term asset,* $\Delta X(S) = X$ *and* $P = 0$.

b. *In the region* $\frac{D_0}{\gamma X} > R > \frac{D_0}{\delta X(1+\gamma)}$ *an unstable equilibrium exists, in which short sellers are not active and the financial institution liquidates part* *of the long-term asset holdings,* $\Delta X(0) = X\frac{\frac{D_0}{XR}-\gamma}{\delta-\gamma(1-\delta)}$ *and* $P = XR - D_0 - \frac{1-\delta}{\delta-\gamma(1-\delta)}\left[D_0 - \gamma XR\right]$.

Figure 3 illustrates the equilibria as a function of XR, the fundamental value of the financial institution's long-term asset holdings. As Proposition 1 points out, there are three regions of interest. First, when R is sufficiently high, short sellers cannot profitably force the financial institution to liquidate long-term asset holdings. In this region, the financially institution is sufficiently well-capitalized, such that the only equilibrium is the one in which the financial institution holds its long-term investments until maturity. We refer to this region as the *safety region.* In the safety region, short sellers solely fulfill the beneficial function of correcting the equity value of the financial institution when the long-term investors offer an intercept higher than the fundamental value of equity. One important implication from this region is that, when financial institutions are healthy one should not be concerned about predatory behavior by short sellers. Hence, our framework does not lend support to unconditional short selling bans.

However, when R drops sufficiently, there is a second region with multiple equilibria. In this region, the leverage constraint is satisfied if short sellers do not take predatory short positions. Hence, there still is an equilibrium without short selling and without liquidation by the financial institution. However, there are now also two equilibria in which short sellers take predatory short positions and force the financial institution to liquidate some or all of its long-term asset holdings. In this region, the financial institution is vulnerable to predatory short selling, even though absent short selling the leverage constraint is not binding. We thus refer to this region of multiple equilibria as the *vulnerability region.* This vulnerability region emerges only when $\delta < \gamma$, i.e., when the long-term asset held by the financial institution is sufficiently illiquid. This highlights the importance of liquidity mismatch (illiquid long-term assets financed with short-term credit) in facilitating predatory short selling.

Finally, there is a third region with two equilibria, on stable and one unstable. In the stable equilibrium, short sellers are active and force the financial institution to liquidate its entire asset holdings, such that the equity value of the financial institution is given by $P = 0$. In the unstable equilibrium, short sellers are not active and the financial institution liquidates part of its long-term asset holdings. Because in this region the unique stable equilibrium involves a complete liquidation of the financial institution, we refer to this region as the *doomed region.*

The effect of banning short selling. We are now in a position to compare outcomes under a regime in which short selling is allowed and a regime in which short selling is prohibited. When short selling is restricted, the financial institution only needs to liquidate at date $t = 1$ when the leverage constraint is violated *absent* predatory short sellers. Proposition 2 compares the two regimes, focusing on stable equilibria.

Proposition 2. *Consider again the case $\delta < \gamma$. The effect of banning short selling on equilibrium prices and the quantity of the long-term investment liquidated by the financial institution depends on the equilibrium region:*

1. ***Safety region:*** *When the financial institution is sufficiently well capitalized ($R > \frac{D_0}{\delta X}$), equilibrium prices and the amount the financial institution needs to liquidate coincide. In both cases, the unique equilibrium is $P = XR - D_0$ and $\Delta X = 0$.*

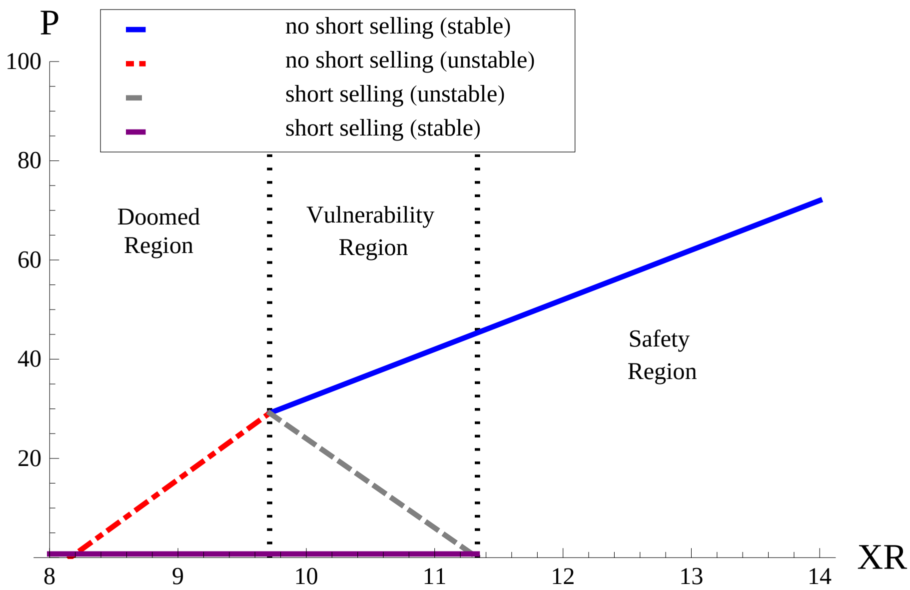

Fig. 3. **Overview of Equilibria.** This plot shows the equilibrium equity value of the financial institution as a function of R. For high values of R, there is a unique equilibrium without predatory short selling (*safety region*). Once R drops sufficiently low, there is a region with multiple equilibria (when $\gamma > \delta$). In this *vulnerability region* predatory short selling can emerge. The middle equilibrium is unstable. When R is so low that the leverage constraint binds in the absence of short selling, there is a stable equilibrium with predatory short selling and an unstable equilibrium without predatory short selling (the *doomed region*). The parameter values in this graph are $X = 10, \delta = 0.6, \gamma = 0.7, D = 68$.

2. ***Vulnerability region:*** *In the vulnerability region* ($\frac{D_0}{\gamma X} \leq R \leq \frac{D_0}{\delta X}$)*, when short selling is restricted no liquidation takes place and the unique equilibrium is given by* $P = XR - D_0$ *and* $\Delta X = 0$*. When short sellers are present, on the other hand, there is a second stable equilibrium, in which predatory short sellers force the financial institution to liquidate its entire long-term asset holdings:* $\Delta X = X$ *and* $P = 0$*.*
3. ***Doomed region:*** *In the doomed region* ($R < \frac{D_0}{\gamma X}$)*, when short selling is restricted the financial institution liquidates part of its long-term asset holdings as long as* $R > \frac{D_0}{\delta X(1+\gamma)}$*. In this region,* $\Delta X(0) = X \frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)}$ *and* $P = XR - D_0 - \frac{1-\delta}{\delta - \gamma(1-\delta)}[D_0 - \gamma XR]$*. When* $R \leq \frac{D_0}{\delta X(1+\gamma)}$*, the financial institution liquidates its entire long-term asset holdings even when short selling is restricted, and* $P = 0$*. When short sellers are present, in the doomed region the financial institution always liquidates its entire holdings and* $P = 0$*.*

Figure 4 illustrates the main differences between a regime with short selling (solid line) and a regime in which short selling is restricted (dashed line). First, note that when short

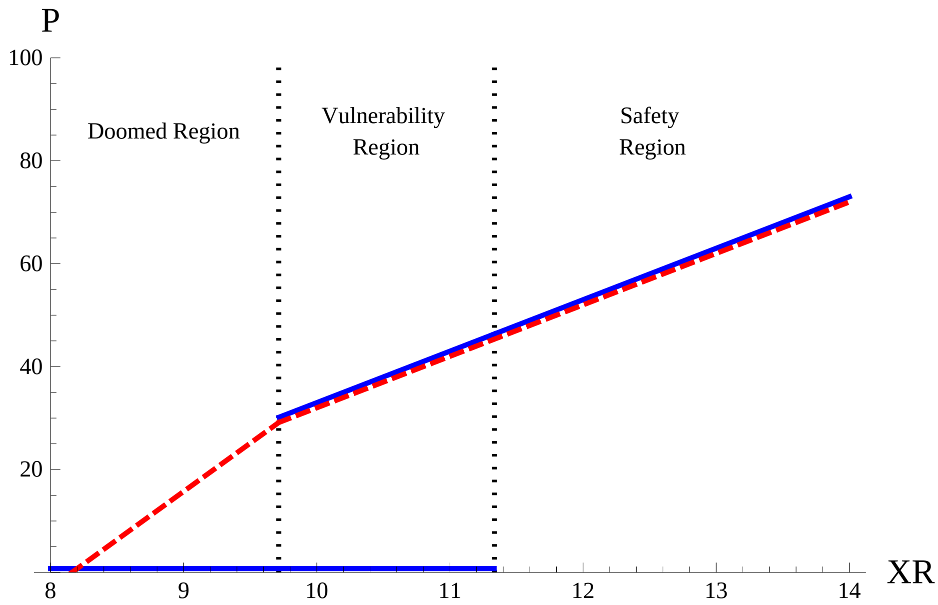

Fig. 4. **The effect of banning short selling.** This figure compares equilibria with and without short selling, focusing on stable equilibria. When short selling is allowed (solid line), there are multiple equilibria once the financial institution enters the *vulnerability region*. In one of the two stable equilibria, predatory short sellers force the financial institution to liquidate its entire long-term asset holdings. In the *doomed region*, in the unique stable equilibrium short sellers always force the financial institution to unwind its entire asset holdings. When short selling is not allowed (dashed line), the financial institution does not have to liquidate in the vulnerability region and $P = XR - D_0$. Moreover, in the doomed region, the financial institution only has to liquidate part of its long-term asset holdings when short selling is restricted (except when R is so low that the financial institution has to liquidate everything even in the absence of short selling). The parameter values in this example are $X = 10, \delta = 0.6, \gamma = 0.7, D = 68$.

sales are restricted, there is no vulnerability region—the financial institution only has to liquidate some of its long-term asset holdings if the leverage constraint is violated in the absence of temporary price movements caused by short sellers. When short selling is allowed, on the other hand, the vulnerability region emerges and there is a second equilibrium in which short sellers prey on the financial institution, forcing it to unwind its entire long-term asset holdings. Hence, in this region predatory short sellers can force a collapse of the financial institution, even though the financial institution would be sound in the absence of short selling.

Second, when the leverage constraint is violated even in the absence of short selling, the amount the financial institution has to liquidate is (weakly) smaller when short selling is restricted. This is the case because in the doomed region in the unique stable equilibrium short sellers force the financial institution to liquidate its entire portfolio. When no short sellers are present, on the other hand, the financial institution can in general satisfy the leverage constraint by selling only part of its long-term asset holdings, except when R

drops so low that the financial institution enters a "death spiral" (i.e, it has to liquidate all long-term asset holdings even when no short sellers are present). In the figure, this happens at the point where the dashed line meets the x-axis.

Of course, one caveat of the analysis above is that we have focused exclusively on the case in which, absent short selling, the equity of the financial institution is priced correctly. This allowed us to focus exclusively on characterizing the conditions under which *predatory short selling* can occur. More generally, the potential welfare costs of predatory short selling have to be weighed against the beneficial effects of regular short selling through the elimination of overvaluation and improvements in market quality and liquidity. As discussed above, in the safety region predatory short selling cannot occur and the only effect of short sellers is the elimination of mispricing. Clearly, in this region, a short-sale ban is not desirable. In the vulnerability region, on the other hand, the costs of potential predatory short selling have to be weighed against the potential benefits from regular short selling. The desirability of a potential short selling ban then depends on the relative size of these two effects. While formally our model does not deliver predictions on how large these two effects may be in practice, it does provide some informal guidance. For example, if one believes that a financial institution is only temporarily in the vulnerability region, a short selling ban may prevent the collapse of the financial institution, while the costs of temporary overvaluation of the financial institution's equity might be moderate. Similarly, in the doomed region one would have to weigh the benefits of controlled deleveraging that is possible in the absence of short sellers against the potential costs of overvaluation in this region.

3.3 REGULAR SELLING VERSUS SHORT SELLING

Up to now our analysis has focused exclusively on short selling. In this section, we contrast our results to those that would obtain if we replaced short sellers with regular sellers (i.e., investors who have an initial endowment of shares in the financial institution). This will sharpen the distinction between regular selling and short selling and highlight the important role important role of coordination.

The distinction between short selling and regular selling is particularly relevant in the vulnerability region where, as shown above, multiple equilibria Pareto-ranked equilibria are possible: In the Pareto-dominant equilibrium, the financial institution does not have to sell any of its long-term asset holdings and survives. In contrast, in the dominated equilibrium, the financial institution is forced to liquidate all long-term asset holdings and fails. The discussion in this section revolves around the following questions: Can the dominated equilibrium also emerge in a setting with regular sellers as opposed to short sellers? If yes, is there reason to believe that it is more likely to emerge as a result of short selling as opposed to regular selling?

Recall that in the competitive setup developed above, when all trades are executed at the final market clearing price, short sellers are indifferent between the two equilibria that are possible in the vulnerability region—they make zero profits in either. More generally, if short sellers can walk down the demand curve when establishing their short position (i.e., when not all trades are executed at the final price) they strictly prefer the equilibrium in which they collectively prey on the financial institution. This contrasts with situation that would arise if short sellers were regular sellers: While a setup with regular sellers instead of short sellers would lead to the same two equilibria in the vulnerability region, regular sellers strictly prefer the equilibrium in which they hold on to their shares and do not sell. Hence, with regular sellers, the dominated equilibrium can only emerge as a result of coordination failure—existing shareholders sell because they expect everyone else to sell, comparable to the dominated equilibrium in Diamond and Dybvig (1983). While this, of course, does not rule out the dominated equilibrium, it is a reasonable proposition that the dominated equilibrium is less likely to emerge through a pure coordination failure of regular sellers than through a (weakly) profitable attack by predatory short sellers.

In addition, as soon as we depart from the competitive benchmark and allow for some amount of coordination among short sellers or shareholders, the equilibrium regions differ depending on whether sellers are regular sellers or short sellers. Specifically, some amount of coordination among short sellers increases the doomed region where the unique equilibrium involves a complete liquidation of the financial institution. In contrast, some amount of coordination among regular sellers increases the safety region where the unique equilibrium involves no liquidation.

Formally, we model the degree of coordination by assuming that there is a large trader or, equivalently, a mass of small traders who can coordinate their actions. The large trader (or coordinated traders) internalize that their trading decision moves the share price. The remaining traders form a competitive fringe and take prices as given. In the short selling case, we assume that the large short seller can take a maximum short position of S^{MAX}. In the case of regular sellers, we assume that the large shareholder owns S^{MAX} shares in the financial institution, while the competitive fringe owns S_C^{MAX} shares. We also assume that if both the large shareholder and the competitive fringe sell their shares, the share price drops to zero and the financial institution has to liquidate all of its long-term asset holdings. Formally, this assumption requires that in the regular selling case $S^{\mathrm{MAX}} + S_C^{\mathrm{MAX}} = \widetilde{S}$, where $\widetilde{S}$ is defined by $\widetilde{P} = \overline{P} - \lambda\widetilde{S} = 0$. Note that in both the regular and the short selling case S^{MAX} proxies for the amount of coordination that is possible.

For simplicity and to reflect the role of large traders (such as George Soros) as first movers, we assume that the large trader moves first and that the competitive fringe moves after the large trader's order has been executed. However, as we describe in more detail below, the findings in Proposition 3 do not depend on the specific assumptions on move and execution order. For example, we could alternatively assume that the large trader and the competitive fringe submit their orders and are executed simultaneously, or that the execution order is random and traders submit limit orders. Both of these alternative setups would leave the equilibrium regions described in Proposition 3 unchanged (see footnotes 16 and 17 for more details).

Consider first the case of short sellers. The large short seller moves first and chooses $S \in [0, S^{\mathrm{MAX}}]$. The large short seller's trade is then executed at $P(S) = \overline{P} - \lambda S$. Then, the competitive fringe moves and chooses S_C. The orders of the competitive fringe are executed at $P(S + S_C) = \overline{P} - \lambda(S + S_C)$. Whenever the maximum short position of the large short seller, S^{MAX}, is sufficiently large to make the short sale profitable irrespective of the actions of other short sellers, the unique equilibrium involves predatory short selling. This is the case whenever $S^{\mathrm{MAX}} > S^*$, where S^* denotes the short position required to make a short sale profitable (for a formal definition of S^*, see equation (A7) in the appendix). This condition holds when the financial institution is sufficiently close to its leverage constraint. Hence, the presence of a large short seller expands the doomed region in which the unique equilibrium involves complete liquidation of the financial institution.

In contrast, in the case of regular sellers the presence of the large trader expands the safety region in which the unique equilibrium involves no liquidation by the financial institution: This is the case when the blockholder's decision not to sell his shares can ensure that no coordination failure occurs, which is the case when $S^{\text{MAX}} > \widetilde{S} - S^*$: Given that the large shareholder does not sell, the competitive fringe cannot profitably coordinate to sell because $\widetilde{P}(S_C^{\text{MAX}}) < P(S_C^{\text{MAX}})$. The unique best response is thus $S_C = 0$ and no liquidation is the unique equilibrium.[14] Solving for $S^{\text{MAX}} > S^*$ and $S^{\text{MAX}} > \widetilde{S} - S^*$ in terms of the parameters of the model yields the following proposition.

Proposition 3. *Assume that there is a large trader or, equivalently, a mass of small traders that can coordinate there actions up to a maximum of S^{MAX} shares.*

1. ***Short selling:*** *If traders who coordinate up to S^{MAX} are short sellers, the doomed region (with a unique short selling equilibrium) expands to $R \in \left[0, \min\left[\frac{D_0}{\gamma X} + \frac{\gamma-\delta}{(1-\delta)\gamma X}\lambda S^{MAX}, \frac{D_0}{\delta X}\right]\right)$.*
2. ***Regular selling:*** *If traders who coordinate up to S^{MAX} are regular sellers, the safety region (with a unique no-liquidation equilibrium) expands to $R \in \left(\max\left[\frac{D_0}{\delta X} - \frac{\gamma-\delta}{(1-\gamma)\delta X}\lambda S^{MAX}, \frac{D_0}{\gamma X}\right], \infty\right)$.*

The equilibrium regions in the presence of a large trader are illustrated in Figure 5. The top panel illustrates the expansion of the doomed region in the presence of a large short seller. The bottom panel illustrates the expansion of the safety region in the presence of a large shareholder in the regular selling case.

Proposition 3 shows that once a certain amount of coordination is possible, there is a sharp difference between short selling and regular selling: While in both cases coordination shrinks the parameter vulnerability region (the region with multiple equilibria), in the short selling case this happens via an expansion of the doomed region (in which predatory short selling is the unique equilibrium), whereas in the case of regular sellers this happens through an expansion of the safety region (where no liquidation is the unique equilibrium). In the extreme, the vulnerability region vanishes completely. In the short selling case, the financial institution is then liquidated as soon as short sellers have the ability to force the financial institution to violate its leverage constraint (i.e., when $R < \frac{D_0}{\delta X}$). In the case of

[14] The role of the large short seller or blockholder discussed here is similar to the role of large players in the literature on currency crises. See, in particular, Corsetti, Pesenti, and Roubini (2002) and Corsetti, Dasgupta, Morris, and Shin (2004) for a setting in which traders face a binary decision on whether or not to attack a currency.

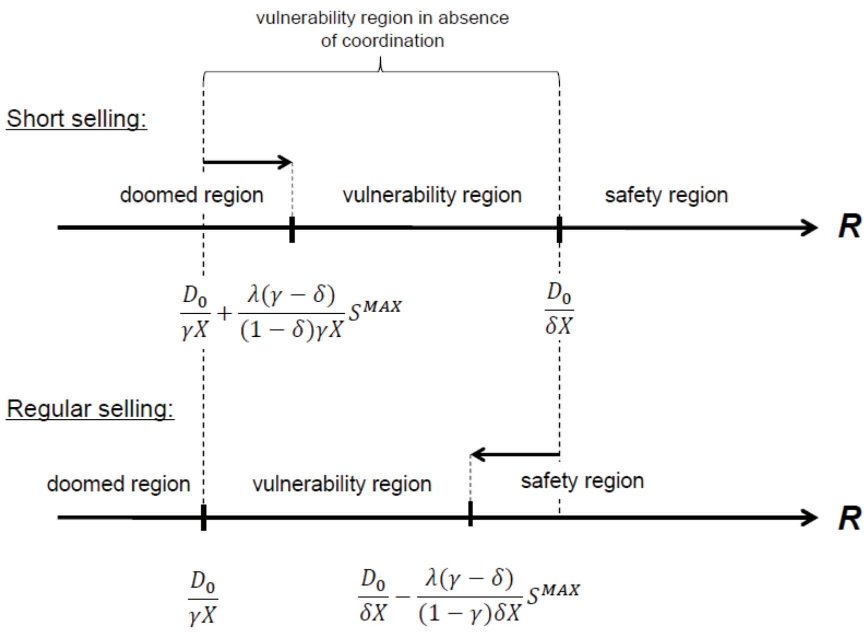

Fig. 5. **Equilibrium regions under short selling and regular selling.** The figure compares equilibrium regions under short selling and regular selling in the presence of a large trader (or a mass of small traders who can coordinate their actions) of size S^{MAX}. The parameter region for which the unique equilibrium involves liquidation of the financial institution is larger under short selling than under regular selling. Conversely, the parameter region for which the unique equilibrium involves no liquidation by the financial institution is larger under regular selling than under short selling.

regular sellers, on the other hand, no liquidation occurs unless the financial institution violates its leverage constraint in the absence of short sellers (i.e., when $R < \frac{D_0}{\gamma X}$).[15] [16] [17]

[15] One interesting implication of Proposition 3 is that the region in which the unique equilibrium involves predatory short selling depend on the slope of the demand curve λ, which we have taken as given here. This is the case when the position limit for the short seller are in terms of the maximum number of shares that can be shorted, S^{MAX}. Alternatively, if position limits are defined as "price impact" limits (which would be of the form S^{MAX}/λ), the region in which predatory short selling is the unique equilibrium is independent of the slope of the demand curve λ, thereby recovering the irrelevance property of Lemma 3.

[16] If instead of sequential orders and execution we were to assume that orders are submitted and executed simultaneously, the resulting equilibrium regions would be identical to those in Proposition 3. The main difference is that, in the simultaneous-move game, it is the *threat* of the large short seller that eliminates the no-liquidation equilibrium. As

3.4 SUPPORT BUYING BY A LARGE TRADER

Next, we discuss the effect of adding investors who can step in to buy shares (recall that up to now the long-term investors that form the residual demand curve were assumed to be completely passive and thus never acted as active support buyers). To do this, consider the case in which both a large short seller (or a mass of short sellers who can coordinate) and a large support buyer (or a mass of traders who can coordinate to purchase stock in the financial institution) are present. We assume that the support buyer (this could, for example, be a blockholder or another large trader with a vested interest in the financial institution) can buy up to B^{MAX} *additional* shares to support the financial institution. For simplicity, we set the support buyer's initial endowment in shares of the financial institution to zero. As before, the large short seller can short a maximum of S^{MAX} shares. As in the previous subsection, we assume that the large traders (the short seller and the support buyer) trade first, followed by the competitive fringe.

In this case, the region in which predatory short selling is the unique equilibrium depends on the *relative strength* of the support buyer vis-à-vis the short seller. Specifically, starting from a conjectured no-liquidation equilibrium, the short seller's maximum position S^{MAX} must now be sufficiently large to make deviation profitable even if the support buyer purchases the maximum amount of shares B^{MAX}. If this is the case, the unique equilibrium involves predatory short selling. Similar to Proposition 3, we can then characterize the regions in which the unique equilibrium involves predatory short selling as follows:

Proposition 4. *Assume that there is a large short seller or, equivalently, a mass of small traders that can coordinate there actions up to a maximum of S^{MAX} shares. Assume also that there is a large support buyer (or a mass of small support buyers coordinate) who can purchase B^{MAX} additional shares to support the share price of the financial institution. Then the doomed region (with a unique short selling equilibrium) is given by* $R \in \left[0, \frac{D_0}{\gamma X} + \frac{\gamma-\delta}{(1-\delta)\gamma X}\lambda\left[S^{MAX} - B^{MAX}\right]^+\right)$.

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before, when $S^{\mathrm{MAX}} > S^*$, the large short seller has a strictly profitable deviation from a conjectured no-liquidation strategy profile. However, the large short seller cannot be part of a zero-profit short selling equilibrium with $S + S_C = \widetilde{S}$, because from any such equilibrium he would have an incentive to slightly reduce the size of his short position and make positive (instead of zero) profits. Hence, when $S^{\mathrm{MAX}} > S^*$ the unique equilibrium is a predatory short selling equilibrium in which the competitive fringe takes a short position of $S_C = \widetilde{S}$ in response to the *threat* of a short potions by the large short seller.

[17] Also a setup with random execution order in which traders can submit limit orders leads to the same equilibrium regions. Because there is a one-to-one mapping between the execution price and the order of execution, limit orders allow traders to effectively condition their sell orders on when they are executed. If the large trader is executed first, the analysis is identical to the one discussed in the text (the analysis in the text is a limiting case of the more general limit order setup: the probability that the large trader is executed first is one). In the case where the large trader is executed after the competitive fringe, the equilibrium regions remain the same but in the coordination failure equilibrium the large shareholder may sell slightly less if he anticipates that his order will be executed after the competitive fringe (and thereby at a lower price). See the appendix for more details.

Relative to Proposition 3, we thus see that the presence of a support buyer shrinks doomed region, in which full liquidation is the unique equilibrium. This is because the boundary of the doomed region is now determined by the relative strength of the large short seller and the support buyer, as captured by $S^{\text{MAX}} - B^{\text{MAX}}$. Moreover, under the interpretation that S^{MAX} and B^{MAX} reflect the extent to which multiple blockholders and multiple short sellers can coordinate, Proposition 4 highlights how, in addition to their sheer financial strength, the relative ability of blockholders and short sellers to coordinate becomes an important element in determining which equilibrium obtains.

4. Discussion

While the simple model presented above does not provide a full welfare analysis of short selling, our analysis generates a number of predictions that may help in devising a more differentiated regulatory approach to short selling. Moreover, the empirical predictions of our model may be helpful in interpreting existing empirical evidence as well as providing guidance for the design of future empirical studies on short selling.

4.1 REGULATORY IMPLICATIONS

Vulnerability of financial institutions to short sales. One of the main implications of our model is to highlight the potential vulnerability of financial institutions to predatory short selling. In our model, predatory short selling can emerge because of a leverage constraint that captures the run risk faced by financial institutions that inherently have significant maturity and liquidity mismatch. This run constraint allows short sellers to capitalize on financial weakness by forcing an institution to liquidate long-term investments, leading to a reduction in fundamental value. This reduction in fundamental value, in turn, allows short sellers to break even on their positions. Hence, the potential unwillingness of creditors to renew their funding leads to a fragility in financial institutions' funding structures that can potentially be exploited by predatory short sellers. Firms with more stable capital structures, on the other hand, should be less susceptible to the predatory behavior characterized in this paper.

Temporary short-sale bans. In terms of regulations that restrict short selling, our analysis implies that, while banning short selling during normal times is not desirable, it can make sense to restrict short selling of financial stocks temporarily when balance sheets across most financial institutions are weak: When banks are well-capitalized (and predatory short selling does not occur in equilibrium), short sellers merely carry out their beneficial role of enforcing the law of one price, providing liquidity and incorporating information into prices. Our model thus does not provide a justification for a *general ban* of short selling on the grounds of predatory behavior. However, when financial institutions' balance sheets are weak and they enter the vulnerability region, destabilizing predatory short selling can occur, leading to inefficient liquidation of long-term investments by vulnerable institutions. Hence, while we want to stress that our paper does not provide a full welfare analysis of short selling, this result provides a potential justification for temporary short sales restrictions to curb predatory behavior (if one believes that the costs of potential predatory short selling outweigh potential overvaluation and reduction in liquidity during a short selling ban).

Disclosure of short positions. A major result of our analysis is the role of multiplicity of equilibria in the vulnerability region. Because there are two stable equilibria in this region, coordination among short sellers is crucial in determining which of the two equilibria we end up in. This has implications for the disclosure of short positions. In particular, in addition to recent short-sale restrictions, a number of regulators have enacted tougher disclosure requirements for short positions. In the U.S., the SEC enacted a rule requiring institutional investors to publicly disclose their short positions on a weekly basis.[18]. In the UK, the FSA implemented a rule that requires that investors disclose on each day any short positions in excess of 0.25% of the ordinary share capital of financial companies at the end of trading the previous day.[19]. Since November 1, 2012, EU regulation requires short sellers to report to regulators is they intend to short sell more than 0.2% of a company's tradable shares and to publicly report short positions that exceed 0.5% of tradable shares.[20]

However, our analysis indicates that public disclosure of short positions can, in fact, be counterproductive. In particular, requiring public disclosure of all short positions may in fact *facilitate* coordination among predatory short sellers. When short sellers are required to publicly disclose positions, it may thus be more likely that we end up in the predatory equilibrium when in the vulnerability region. One way to capture this formally in our model is to consider an increase in the amount of shares up to which short sellers can coordinate, S^{MAX}. It follows directly from Proposition 3 that such an increase in coordination enlarges the doomed region, in which a predatory short selling attack is the unique equilibrium. This suggests that disclosure should either only be made to the regulator, or should be made public only with sufficient time delay.

Other regulatory interventions. Up to now, we have limited our discussion of potential regulatory interventions to restricting short sales. In this section, we briefly discuss other potential regulatory measures that may reduce the vulnerability of financial institutions to predatory behavior by short sellers.

First, the leverage constraint arises because short-term creditors may withdraw funding from the financial institution. This implies that interventions to increase the stability of the financial institution's financing may be desirable. For example, such an intervention could take the form of limiting liquidity mismatch, thus reducing financial fragility and increasing the resilience of the financial institution against short selling attacks. Another potential regulatory intervention that would help financial institutions fend off predatory short selling attacks is a requirement for financial institutions to hold more equity. Such a requirement would make it less likely that the financial institution enters the vulnerability region and thus reduces the financial fragility that can be exploited by short sellers. Alternatively (if additional equity capital is costly), a requirement to issue at least some amount of reverse convertible debt that converts into equity if the leverage constraint binds would allow the financial institution to reduce leverage without selling any of its long-term asset holdings.

Second, in our model a credible promise to recapitalize the financial institution could eliminate that "bad" equilibrium: If short sellers anticipate that the financial institution

[18] See SEC release 34-58785, available at http://www.sec.gov/rules/final/2008/34-58785.pdf.

[19] See "Implementing aspects of the Financial Services Act 2010" available at http://www.fsa.gov.uk/pubs/cp/cp1_18.pdf

[20] See "Europe's naked short selling ban leaves investors with skin in the game," *Reuters*, December 4, 2012.

never has to liquidate any of its long-term asset holdings, they will never attack, knowing that they cannot break even on their short positions. However, such an intervention effectively amounts to a government guarantee for the financial sector. In a richer model where financial institutions choose their investments, this would lead to substantial moral hazard concerns. It is thus not clear that such a guarantee would be desirable (at least if it is anticipated ex ante).

Third, to the extent that regulators use temporary short-sale bans to protect vulnerable financial institutions from predatory short selling attacks, this will likely increase the importance of prompt corrective action by regulators. Specifically, if short-sale bans make it harder for market participants to single out financial institutions that should be shut down or restructured (e.g., so-called zombie banks), then the regulator's role in identifying these institutions becomes all the more important.

Panic sales. While the focus of this paper is on short selling, another novel result of our analysis is that, in addition to short sales, regulators may also want to consider the possibility of destabilizing "panic sales" by current investors in the financial institution's equity. In particular, if current shareholders fear that other shareholders are likely to sell their holdings, such panic sales can become self-fulfilling for financial institutions that are subject to leverage constraints of the type described in our model. Hence, akin to the classic bank run problem described in Diamond and Dybvig (1983), our analysis implies that regulators may want to have an eye on financial market runs on the equity of for financial institutions that are subject to leverage constraints. As shown in Proposition 3, this concern is stronger the less likely it is that current shareholder can coordinate their actions.

4.2 EMPIRICAL PREDICTIONS

Existing empirical evidence. A number of recent papers empirically examine the effects of recent short selling bans in the U.S. and Europe. For example, Beber and Pagano (2013) use international data to document the effects of short sale bans across different markets. Boehmer, Jones, and Zhang (2013) provide a detailed investigation of the U.S. ban on short sales imposed by the SEC in September 2008. The main findings of these (and a number of other) studies is that short sale bans led to an unambiguous and significant reduction in liquidity, market quality, and the speed of price discovery (as measured, for example, by bid-ask spreads, effective spreads, or price impact measures). However, the results regarding the effect of short sale bans on prices—perhaps the main motivation for intervention—are much weaker. For example, Boehmer, Jones, and Zhang (2013) find significant abnormal excess returns only for the largest U.S. financial institutions and point out that those may have been caused by the Troubled Asset Relief Program (TARP), which was launched more or less at the same time. In fact, because their analysis suggests that the price effects were permanent, they conclude that the TARP may have been the more likely cause.

Cross-sectional predictions from our model. Our analysis may help interpret some of the findings of these recent empirical studies. Moreover, the empirical implications of our analysis could potentially be helpful in the design of future empirical studies of short selling bans on financial institutions. First, our model suggests that financial institutions, as opposed to other firms, should be particularly affected by short sale bans. Second, the ability of short sellers to prey on financial institutions depends crucially on the financial condition of the financial institution. Hence in assessing the price effects of short-sale restrictions, one should control for leverage, maturity mismatch, or similar variables that measure financial fragility. The cross-sectional prediction of our model is that it is vulnerable financial firms for which the price effects of short sale bans are largest. Third, our model highlights the importance of taking into account the potential multiplicity of equilibria when interpreting the empirical evidence. For example, if investors expect that, with some probability, there is a switch to the dominated equilibrium in which the bank goes bankrupt, the elimination of the bad equilibrium through a short selling ban may lead to a permanent price effect. This can be the case even if the ban itself is temporary: Investors now anticipate that regulators may impose another ban should financial institutions re-enter the vulnerability region. Hence, according to our analysis the permanent price increase documented by Boehmer, Jones, and Zhang (2013) could also be attributed to a short-sale ban that eliminates the likelihood of the dominated equilibrium.

The funding of blockholders. In addition to the cross-sectional predictions above, a novel prediction of our model is that the vulnerability of a financial institution to predatory short selling depends not only on its own balance sheet but also on the balance sheets (or funding conditions) of its large shareholders and other potential support buyers. Specifically, if large shareholders or other support buyers are important in fending off short selling attacks, their ability to do so depends on their own funding liquidity, as proxied by the parameter B^{MAX} in Proposition 4. Hence, our model makes the prediction that, in the cross-section, financial institutions with less well-capitalized blockholders, are more vulnerable to predatory short selling attacks. Moreover, in the time series, predatory short selling attacks are more likely to be successful when funding conditions for blockholders are tight on average, for example during financial crises. Finally, our model predicts that the vulnerability of financial institutions to predatory short selling depends on the ability of blockholders (and short sellers) to coordinate. Empirically, the number of blockholders may provide a proxy for their ability to coordinate.

Predictions on price skewness. In addition to cross-sectional predictions spelled out above, our model also predicts that large downward price movements can occur when a financial institution enters the vulnerability region or the doomed region. This means that, in our model, short selling increases negative skewness in equity prices, which is opposite to the prediction in Hong and Stein (2003), where *banning* short selling leads to negative skewness. Consistent with our prediction, Bris, Goetzmann, and Zhu (2007) find evidence that there is significantly less negative skewness in markets in which short selling is either not legal or not practiced. In addition, our model makes the testable prediction that, absent short-sale restrictions, negative skewness should be observed particularly for financial institutions with weak balance sheets (i.e., those that approach the vulnerability region or doomed region).

5. Conclusion

This paper provides a simple model of predatory short selling. Predatory short selling occurs when short sellers exploit financial weakness or liquidity problems of a financial institution. In our model, predatory short sales occur in equilibrium because the drop in equity valuation caused by short sellers leads non-insured depositors and short-term creditors to withdraw funding from the financial institution. Because of this effective leverage constraint, short sales can force the financial institution to liquidate long-term asset holdings at a discount. The resulting value reduction can allow short sellers to break even on their positions.

Our analysis shows that financial institutions can be vulnerable to attacks from predatory short sellers when their balance sheets are weak. For financial institutions that are sufficiently close to their leverage constraints. In the *vulnerability region* there are two stable equilibria. One of these stable equilibria does not involve short selling, while in the other predatory equilibrium short sellers force a complete liquidation of the financial institution's long-term asset holdings. In the *doomed region* there is a unique predatory equilibrium in which the financial institution liquidates its entire long-term asset holdings. The doomed region is larger, the better potential short sellers are able to coordinate their actions.

While our model does not develop a full welfare analysis of bans on short selling, the possibility of predatory short selling in the vulnerability region and the doomed region provides a potential justification for temporary short sale restrictions for financial institutions in those regions, although the benefits of such restrictions have to be weighed against the cost of preventing short sellers from performing their beneficial role of eliminating potential overvaluation.

Appendix

Proof of Lemma 1: Absent the leverage constraint, the fundamental value of the financial institution's equity is given by $XR - D_0$, irrespective of the short sellers' actions. This follows immediately from the fact that, in this case, the financial institution never has to liquidate early. Given the fixed fundamental value $XR - D_0$, competition among short sellers ensures that the share price is equal to the fundamental value $XR - D_0$. To see this, consider first the case in which the intercept chosen by long-term investors is larger than the fundamental value of equity, i.e., $\overline{P} > XR - D_0$. In this case, short sellers take a short position $S > 0$ and, because short sellers make zero profits in equilibrium, the equilibrium short position S is such that the share price is equal to fundamental, i.e., $\widetilde{P} = XR - D_0$. Now consider the case in which the intercept chosen by the long-term investors is below fundamental value. Analogously to before, short sellers now take a long position (they act, more generally, as arbitrageurs) to ensure that the equity if fairly priced. Finally, when the intercept chosen by the is equal to fundamental value, i.e., $\overline{P} = XR - D_0$, short sellers do not take a position in equilibrium, i.e., $S = 0$. Because in all of these cases the fundamental value of equity is fixed at $XR - D_0$, predatory short selling cannot occur. In all three cases, the equilibrium condition (3) can thus be rewritten as $\overline{P} - \lambda S = XR - D_0$.

Proof of Lemma 2: In the case that the constraint is violated, the result follows directly from solving (7) for $\Delta X(S)$. Combining this with the fact that no liquidation occurs when the constraint is satisfied and that the maximum amount that can be liquidated is X, yields the result.

Proof of Lemma 3: The result comes from the fact that, in equilibrium, a change in either $\overline{P}$ or λ will be exactly offset by a corresponding change in the equilibrium level of the short position S, such that the equilibrium condition $\widetilde{P} = P$ is satisfied. Equilibrium prices and the equilibrium amount that has to be liquidated by the financial institution thus remain unaffected.

Proof of Proposition 1: We first analyze the case $\delta < \gamma$, which is the case highlighted in the proposition. For completeness, we then also briefly discuss the cases $\delta > \gamma$ and $\delta = \gamma$, which are not discussed in the main text.

Intuitively, when $\delta < \gamma$, the P-curve is steeper than the $\widetilde{P}$-curve, as depicted in the bottom panel of Figure 1. We now consider the three regions in the proposition in turn. First, we compute the region in which no predatory short selling can occur (the safety region). Short sellers cannot break even on a predatory short position if, after forcing the financial institution to liquidate its entire long-term asset holdings, the fundamental equity value at $t = 2$ still exceeds the stock price that forces this maximum liquidation at $t = 1$. In this case, no matter how aggressive their selling, short sellers have to buy back at a higher price than they receive when shorting the stock at date $t = 1$.

To calculate the parameter region for which this is the case, assume that short sellers choose a short position $\overline{S}$ such that the entire portfolio of the financial institution is liquidated, i.e., $\Delta X(\overline{S}) = X$. This requires that

$$\frac{(1-\gamma)D_0 - \gamma(\overline{P} - \lambda\overline{S})}{R\delta(1-\gamma)} = X, \tag{A1}$$

which yields

$$\overline{S} = \frac{\overline{P}}{\lambda} + \frac{(1-\gamma)(\delta XR - D_0)}{\lambda\gamma}. \tag{A2}$$

This strategy cannot be profitable when the stock price that forces the financial institution to unwind all of its long-term asset holdings is smaller than the fundamental value of equity after such a liquidation, i.e.,

$$XR - D_0 - \lambda\overline{S} < \delta XR - D_0. \tag{A3}$$

Using (A2) to solve (A3) for R yields

$$R > \frac{D_0}{\delta X}, \tag{A4}$$

which is the expression defining the safety region in the proposition. When (A4) is satisfied, the financial institution is well capitalized, and the unique equilibrium is one without predatory short selling and $P = \widetilde{P} = XR - D_0$. Intuitively, in the safety region the liquidation value of the financial institution's long-term assets is higher than the face value of outstanding debt.

Second, consider the region in which $\frac{D_0}{\gamma X} \leq R \leq \frac{D_0}{\delta X}$ (vulnerability region). In this region, the leverage constraint is not violated in the absence of predatory short selling, since $\frac{D_0}{XR} \leq \gamma$. This means that there is still an equilibrium in which no predatory short selling occurs and $P = \widetilde{P} = XR - D_0$. However, now there is also an equilibrium in which short sellers force the financial institution to liquidate its entire asset holdings. In this equilibrium, by the zero profit condition, we have

$$\widetilde{P} = XR - D_0 - \lambda S = \max\left[\delta XR - D_0, 0\right] = P, \tag{A5}$$

where $S \geq \overline{S}$. Since we know that in this region $\delta XR - D_0 \leq 0$, the equilibrium price must be $P = 0$. In words, the financial institution has to liquidate all of its long-term assets, which are not sufficient to repay debt, and the equity value is zero. Both of these two equilibria are stable. Finally, there is a third, unstable equilibrium, in which only part of the financial institution's long-term asset holdings are unwound. Denote the amount of short selling in the unstable equilibrium by S^*. In this equilibrium, $\Delta X(S^*) < X$, such that we must have

$$\overline{P} - \lambda S^* = XR - D_0 - (1-\delta) R \Delta X(S^*). \tag{A6}$$

Substituting in for $\Delta X(S^*)$ from equation (8) and solving for S^* yields

$$S^* = \frac{\overline{P}}{\lambda} - \frac{(1-\gamma)(D_0 - \delta XR)}{\lambda(\gamma - \delta)}. \tag{A7}$$

Using this expression for S^*, we can determine the price in the unstable equilibrium as

$$P = \overline{P} - \lambda S^* = \frac{1-\gamma}{\gamma - \delta}(D_0 - \delta XR). \tag{A8}$$

Substituting into (A2) yields that the amount the financial institution has to liquidate in the unstable equilibrium is $\Delta X(S^*) = X \frac{\gamma - \frac{D_0}{XR}}{\gamma - \delta}$, as stated in the proposition.

Third, consider the region in which the leverage constraint is violated even in the absence of predatory short selling, $\frac{D_0}{XR} > \gamma$. In this region, long-term investors set the intercept $\overline{P}$ equal to the fundamental equity value in the absence of short selling: $\overline{P} = AR - D_0 - (1-\delta) R \Delta X(0)$, where $\Delta X(0)$ satisfies

$$\frac{D_0 - \delta R \Delta X(0)}{XR - (1-\delta) R \Delta X(0)} = \gamma, \tag{A9}$$

which yields $\Delta X(0) = X \frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)}$. When $\Delta X(0) = X \frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)} < X$, partial liquidation is possible and the equilibrium price is given by $P = XR - D_0 - (1-\delta) XR \frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)}$, which simplifies to $P = XR - D_0 - \frac{1-\delta}{\delta - \gamma(1-\delta)} [D_0 - \gamma XR]$, which is the expression in the proposition. In the presence of competitive short sellers, this is an unstable equilibrium: A perturbation that leads to a small decline in the financial institution's stock price triggers selling by short sellers and drives the stock price to zero, forcing a complete liquidation of the financial institution's long-term asset holdings. This full liquidation outcome is the unique stable equilibrium: Short sellers force the financial institution to sell its entire asset holdings, $\Delta X = X$. Because $\delta XR - D_0 \leq 0$, the equilibrium price must be $P = 0$. This is the doomed region.

To complete the proof, note that in the doomed region, short sellers (or, more generally, arbitrageurs) cannot profitably act as support buyers by pushing the stock price up such that the financial institution does not have to liquidate assets. Because at price $\widetilde{P} = XR - D_0$, the leverage constraint is violated, preventing liquidation by buying the stock requires driving up the price to a level that strictly exceeds the fundamental value $XR - D_0$. Since absent liquidation the value of equity is exactly $XR - D_0$, this cannot be an equilibrium.

Now we briefly consider the case $\delta > \gamma$. Intuitively, when $\delta > \gamma$, the P-curve is less steep than the $\widetilde{P}$-curve, as depicted in the left panel of Figure 1. The first thing to note is that now the vulnerability region disappears, since $\frac{D_0}{\gamma X} > \frac{D_0}{\delta X}$. In this case, as long as $R > \frac{D_0}{\gamma X}$, the financial institution is well capitalized and no predatory short selling occurs. As a result, liquidation only takes place when $R < \frac{D_0}{\gamma X}$. When $R < \frac{D_0}{\gamma X}$, the financial institution has to liquidate $\Delta X(0) = X \frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)}$, if it can satisfy the leverage constraint through a partial liquidation (i.e., $\Delta X(0) < X$). If $\Delta X(0) \geq X$, the financial institution has to liquidate its entire asset holdings. Because the P-curve is less steep than the $\widetilde{P}$-curve, short sellers cannot force a full liquidation if the financial institution can satisfy the constraint through a partial liquidation.

Finally, consider the knife-edge case when $\gamma = \delta$. In this case, the P and $\widetilde{P}$ curves have the same slope. This means that when $R > \frac{D_0}{\gamma X}$ predatory short selling cannot occur, while when $R < \frac{D_0}{\gamma X}$, the financial institution is forced to liquidate all its holdings and $P = 0$. When $R = \frac{D_0}{\gamma X}$, the equilibrium price can lie on any point on the interval $[0, XR - D_0]$.

Proof of Proposition 2: We again focus on the more interesting case $\delta < \gamma$, which is the one depicted in Figure 4. The stable equilibria in the presence of short selling follow directly from Proposition 1. The equilibrium prices when short selling is restricted are determined as follows. As before, assume that the long-term investors are rational, such that they correctly anticipate the $t = 2$ payoff $\overline{P} = XR - D_0 - (1 - \delta)R\Delta X(0)$. As long as the leverage constraint is not violated, $\frac{D}{XR} < \gamma$, the financial institution does not have to liquidate ($\Delta X(0) = 0$) and $P = XR - D_0$. When $\frac{D}{XR} > \gamma$, the financial institution has to liquidate an amount $\Delta X(0) = X\frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)}$, which can be derived as in (A9). When $\Delta X(0) = X\frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)} < X$, partial liquidation is possible and the equilibrium price is given by $P = XR - D_0 - (1 - \delta)XR\frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)}$, which simplifies to $P = XR - D_0 - \frac{1-\delta}{\delta - \gamma(1-\delta)}[D_0 - \gamma XR]$. When $\Delta X(0) = X\frac{\frac{D_0}{XR} - \gamma}{\delta - \gamma(1-\delta)} \geq X$, then the financial institution is forced to liquidate all its long-term asset holdings at $t = 1$ and $P = 0$ even in the absence of short sellers. Solving the above expression for R shows that full liquidation is required whenever $R \leq \frac{D_0}{\delta X(1+\gamma)}$.

Proof of Proposition 3: We consider the large short seller first. Recall that the large short seller moves first and chooses $S \in [0, S^{\text{MAX}}]$. The large short seller's trade is then executed at $P(S) = \overline{P} - \lambda S$. Then, the competitive fringe moves and chooses S_C. The orders of the competitive fringe are executed at $P(S + S_C) = \overline{P} - \lambda(S + S_C)$.

Conjecture a no-liquidation equilibrium. If $S^{\text{MAX}} < S^*$, the large short seller does not have a profitable deviation: For all $S \in [0, S^{\text{MAX}}]$ we have $\widetilde{P}(S) < P$, such that a short sale is unprofitable for the large short seller. Hence, the short seller chooses $S = 0$ and the no liquidation equilibrium is indeed an equilibrium: $S_C = 0$ is a best response for the competitive fringe.

Now assume that $S^{\text{MAX}} > S^*$. Now the short sale is strictly profitable for the large short seller, since for any $S \in (S^*, S^{\text{MAX}}]$ we have $\widetilde{P}(S) > P(S)$. Hence, the large short seller chooses $S > S^*$. This makes it optimal for the competitive fringe to choose $S_C = \widetilde{S} - S$ (such that the zero profit condition holds for the competitive fringe). Hence, when $S^{\text{MAX}} > S^*$ predatory short selling becomes the unique equilibrium. Inserting (A7) for S^* and solving for R yields

$$R < \frac{D_0}{\gamma X} + \frac{\gamma - \delta}{(1-\delta)\gamma X}\lambda S^{\text{MAX}}. \tag{A10}$$

Accordingly, the parameter region in which multiple equilibria are possible is given by $R \in \left[\frac{D_0}{\gamma X} + \frac{\gamma-\delta}{(1-\delta)\gamma X}\lambda S^{\text{MAX}}, \frac{D_0}{\delta X}\right]$. The region with a unique predatory short selling equilibrium (doomed region) expands to $R \in \left[0, \frac{D_0}{\gamma X} + \frac{\gamma-\delta}{(1-\delta)\gamma X}\lambda S^{\text{MAX}}\right)$.

Now consider the case of regular sellers. Recall that the large shareholder and the competitive fringe hold S^{MAX} and S_C^{MAX} shares, respectively, and that $S^{\text{MAX}} + S_C^{\text{MAX}} = \widetilde{S}$. The large shareholder moves first and chooses $S \in [0, S^{\text{MAX}}]$. This trade is executed at

$P(S) = \overline{P} - \lambda S$. Then the competitive fringe chooses $S_C \in [0, S_C^{\text{MAX}}]$. The orders of the competitive fringe are executed at $P(S + S_C) = \overline{P} - \lambda(S + S_C)$.

First consider $S^{\text{MAX}} < \widetilde{S} - S^*$. In this case, irrespective of the large shareholder's decision to sell, the competitive fringe can still cause a coordination failure by selling $S_C^{\text{MAX}} > S^*$ shares (in such a coordination failure equilibrium they will always sell the maximum amount). Anticipating this, the large shareholder will sell some or all of his shares at price $\widetilde{P} = \overline{P} - \lambda S$: $S = \arg\max S\left[\overline{P} - \lambda S\right] + (S^{\text{MAX}} - S)P(S_C^{\text{MAX}} + S)$

However, when $S^{\text{MAX}} > \widetilde{S} - S^*$, the large shareholder's decision not to sell rules out the coordination failure equilibrium. Given that the large shareholder does not sell, the competitive fringe cannot profitably coordinate to sell because $\widetilde{P}(S_C^{\text{MAX}}) < P(S_C^{\text{MAX}})$. The unique best response is thus $S_C = 0$ and no liquidation is the unique equilibrium. Using the conjectured equilibrium condition $\overline{P} - \lambda\widetilde{S} = 0$ we can rewrite the condition $S^{\text{MAX}} > \widetilde{S} - S^*$ as

$$S^{\text{MAX}} > \frac{XR - D_0}{\lambda} - \left[\frac{\overline{P}}{\lambda} - \frac{(1-\gamma)(D_0 - \delta XR)}{\lambda(\gamma - \delta)}\right]. \tag{A11}$$

Inserting $\overline{P} = XR - D_0$ and simplifying yields

$$S^{\text{MAX}} > \frac{(1-\gamma)(D_0 - \delta XR)}{\lambda(\gamma - \delta)}. \tag{A12}$$

Accordingly, the parameter region in which multiple equilibria are possible is given by $R \in \left[\frac{D_0}{\gamma X}, \frac{D_0}{\delta X} - \frac{\gamma - \delta}{(1-\gamma)\delta X}\lambda S^{\text{MAX}}\right]$. The region where no liquidation is the unique equilibrium (safety region) expands to $R \in \left(\frac{D_0}{\delta X} - \frac{\gamma - \delta}{(1-\gamma)\delta X}\lambda S^{\text{MAX}}, \infty\right)$.

The analysis is similar in the case with random execution and limit orders. The main insight is that limit orders effectively allow traders to condition on whether they are executed first or second. Hence, we can analyze these two cases in turn. When the large trader moves first, the equilibrium is exactly as just discussed. This leaves us to describe the equilibrium when the competitive fringe moves first. Consider the short selling case first. The competitive fringe makes zero profit in equilibrium, such that both $S_C = 0$ and $S_C = \widetilde{S}$ are possible in equilibrium. If the competitive fringe chooses $S_C = 0$, then the large short seller picks $S = \arg\max S[\overline{P} - \lambda S]$ if $S^{\text{MAX}} > S^*$ and $S = 0$ otherwise. If the competitive fringe picks $S_C = \widetilde{S}$, the large short seller chooses $S = 0$.

Now consider the case of regular sellers when the competitive fringe moves first. If $S^{\text{MAX}} > \widetilde{S} - S^*$ (or equivalently $S_C^{\text{MAX}} < S^*$), then no matter what the fringe does, it is optimal for the large trader to set $S = 0$. But then the competitive fringe optimally responds by setting $S_C = 0$ given that any $S_C \in (0, S_C^{\text{MAX}}]$ leads to strictly negative profits. Hence, the unique equilibrium is the no-liquidation equilibrium.

If $S^{\text{MAX}} > \widetilde{S} - S^*$, then there are multiple equilibria. In the "good" equilibrium, the competitive fringe chooses $S_C = 0$ and the large trader optimally responds by setting $S = 0$. In the "bad" equilibrium, the competitive fringe sets $S_C = S_C^{\text{MAX}} > S^*$. The large shareholder optimally responds with $S = \arg\max S[\overline{P} - \lambda(S_C^{\text{MAX}} + S)] + (S^{\text{MAX}} - S)P(S + S_C^{\text{MAX}}) > 0$. The optimal amount sold by the large shareholder is strictly less than S^{MAX}, since this would lead to a zero payoff.

Taken together, the setup with random execution and limit orders thus leads to exactly the same equilibrium regions as the setup in which the large trader moves first. The one difference is that in the coordination failure equilibrium in which the large shareholder moves second he may sell less aggressively, internalizing that he will be executed at a relatively lower price.

Proof of Proposition 4: The proof follows along similar lines to the proof of Proposition 3. We assume that the large short seller and support buyer simultaneously submit their orders and are executed first. The large short seller chooses a short position $S \in \left[0, S^{\text{MAX}}\right]$. The support buyer chooses a buy order $B \in \left[0, B^{\text{MAX}}\right]$. The orders of the large short seller and support buyer are executed at $\widetilde{P}(S-B) = \overline{P} - \lambda(S-B)$. After the large traders have moved, the competitive fringe chooses S_C and is executed at $\widetilde{P}(S-B+S_C) = \overline{P} - \lambda(S-B+S_C)$.

First, assume that $S^{\text{MAX}} > B^{\text{MAX}}$. The predatory short selling equilibrium is the unique equilibrium when, starting from a conjectured no-liquidation equilibrium, the short seller can effect a profitable short sale, irrespective of the action of the support buyer. When $S^{\text{MAX}} - B^{\text{MAX}} < S^*$, the support buyer's best response to any short position $S \in [0, S^{\text{MAX}}]$ is to lean against the short seller, making the short sale unprofitable. Hence, no liquidation remains an equilibrium. When $S^{\text{MAX}} - B^{\text{MAX}} > S^*$, on the other hand, any $S > S^* + B^{\text{MAX}}$ is a profitable deviation for the large short seller. In this case, no liquidation cannot be an equilibrium. In the unique liquidation equilibrium, the support buyer optimally chooses $B = 0$ (for any $B > 0$ he would lose money) and, by the zero profit condition, the competitive fringe chooses $S_C = \widetilde{S} - S$.

Now consider the case $S^{\text{MAX}} \leq B^{\text{MAX}}$. In this case, the support buyer can always profitably counter short sales by the large short seller, such that the upper boundary of the region in which the unique equilibrium involves short selling and liquidation is given by $R < \frac{D_0}{\gamma X}$. Taken together, the two cases imply that the region with a unique short selling equilibrium (doomed region) is given by $R \in \left[0, \frac{D_0}{\gamma X} + \frac{\gamma-\delta}{(1-\delta)\gamma X}\lambda \left[S^{\text{MAX}} - B^{\text{MAX}}\right]^+\right)$.

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