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Should Derivatives Be Privileged in Bankruptcy?

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Abstract

Derivatives enjoy special status in bankruptcy: They are exempt from the automatic stay and effectively senior to virtually all other claims. We propose a corporate finance model to assess the effect of these exemptions on a firm's cost of borrowing and its incentives to engage in efficient derivative transactions. While derivatives are value-enhancing risk management tools, seniority for derivatives can lead to inefficiencies: It transfers credit risk to debtholders, even though this risk is borne more efficiently in the derivative market. Seniority for derivatives is efficient only if it provides sufficient cross-netting benefits to derivative counterparties that provide hedging services.

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Derivative contracts enjoy special status under U.S. bankruptcy law: Derivative counterparties are exempt from the automatic stay, and—through netting, closeout, and collateralization provisions—they are generally able to immediately collect payment from a defaulted counterparty.[1] Taken together, these special provisions make derivative counterparties *effectively senior* to almost all other claimants in bankruptcy. The costs and benefits of this special treatment are the subject of a recent debate among legal scholars, policymakers, and regulators. Notably, this debate is characterized by considerable disagreement about the costs and benefits of the special bankruptcy treatment of derivatives, which is reflected in substantial differences in the bankruptcy treatment of derivatives across different jurisdictions.[2]

In this paper we provide the first formal analysis of the economic consequences of the privileged treatment of derivative contracts in bankruptcy. The fundamental observation underlying our analysis is that (effective) seniority for derivatives does not eliminate default risk—it transfers default risk from derivative counterparties to other claimholders, particularly creditors. The desirability of seniority for derivatives thus depends on whether default risk is more efficiently borne in the derivative market or in the debt market.

To address this question we extend the standard incomplete contracts framework in corporate finance,[3] in which debt contracts are insufficiently state-contingent, by introducing derivative contracts that allow the firm to arrange state-contingent transfers with a separate derivative counterparty. Specifically, derivatives allow for payments tied to publicly observable and verifiable events that are correlated with the firm's unobservable (or unverifiable) cash flow outcomes. Derivatives are supplied by derivative counterparties that are themselves subject to a moral hazard problem, which is mitigated by requiring the posting of costly collateral as in Biais, Heider, and Hoerova (2012). Within this framework, we characterize the conditions under which the current privileged bankruptcy treatment of derivatives is desirable (or undesirable).

Our baseline model considers a single firm that undertakes a positive NPV investment, which is optimally financed with debt. Cash flow from operations is risky, so that the firm does not always have sufficient funds to meet its debt obligations. As a result, the firm is exposed to default risk, which gives rise to a demand for derivatives as hedging tools. By allowing for transfers of cash from states of the world correlated with high-cash flow realizations to states correlated with low-cash flow realizations, derivative contracts help reduce—possibly even eliminate—the risk of default and inefficient early liquidation.[4]

The main novelty of our analysis is that it considers how the bankruptcy treatment of derivatives affects these hedging benefits. The conventional wisdom is that effective seniority for derivatives lowers a firm's cost of hedging and should thus be beneficial overall. We show that this argument is at best incomplete. Although the privileged treatment of derivatives reduces counterparty risk in derivative markets, it increases credit risk for the firm's creditors, who now face larger losses in default. In frictionless financial markets à la Modigliani and Miller, this transfer of risk between different claimants would have no effect on the firm's overall cost of capital. In our incomplete contracting framework, the priority ranking of debt relative to derivatives matters because it affects endogenous contractual frictions in derivative and debt markets.

A net cost of providing hedging services arises endogenously in our framework because derivative writers (counterparties) must post costly collateral to back up their promises. When a derivative contract moves against the derivative writer, it must post collateral to prevent it from engaging in risk-shifting actions that increase counterparty risk, as in Biais, Heider, and Hoerova (2012). This posting of collateral is costly because it means giving up other, more productive uses of the counterparty's capital. Thus, the priority ranking of derivatives relative to debt affects the net costs of hedging services because it affects the amount of costly collateral that providers of derivatives have to post.

Our analysis reveals that the impact of the priority ordering of derivatives on the overall deadweight costs of hedging depends on the interplay of three main effects. The first effect, which is commonly stressed by practitioners, is that once the firm has issued its debt it is (ex post) optimal to hedge default risk with a derivative that is senior to existing debt. That way, derivative writers get maximum protection against default by the firm on its derivative obligations, thereby reducing the stand-alone cost of the hedge.

Ex ante, however, the firm's creditors anticipate the resulting subordination of their claims to derivative counterparties, which leads to a second and countervailing effect: Creditors demand higher promised repayments to compensate for the higher credit risk they face. The higher required debt payments in turn increase the firm's demand for hedging, so much so that the benefits of seniority for derivatives are wiped out by the concomitantly higher collateral requirements for the derivative counterparty. In addition, when derivatives are senior and hedging positions are entered only after debt has been issued, the firm may have an incentive to dilute existing debtholders by over-hedging or by taking risky bets in derivative markets. Such ex post dilution is inefficient, unless it is strictly required to induce the firm's shareholders to undertake a value-increasing hedge (i.e., a hedge that is beneficial to shareholders and debtholders combined).[5,6]

The third effect arises when we extend our firm-level analysis to a multi-firm setting. When derivative counterparties deal with many firms, the cross-netting benefits to derivative writers from being a senior claimant in bankruptcy can make seniority for derivatives efficient, even when there is no gain from the special treatment of derivatives at the individual firm level. Specifically, when defaults by firms that use derivatives as hedging tools are imperfectly correlated, payments that senior derivative counterparties receive from defaulted firms can reduce their expected net liabilities in bad states. This, in turn, reduces the amount of collateral that derivative writers are required to post, which can reverse the benefits of junior derivatives at the firm level. Our analysis shows that seniority for derivatives is the efficient arrangement only when such cross-netting benefits for derivative writers are sufficiently large, which is the case when either cash flow risk or basis risk are (mostly) idiosyncratic.

Finally, we investigate how the seniority treatment of derivatives affects the possibility that the firm may default in high-cash flow states due to losses incurred on its derivative position. Our analysis shows that this outcome is unambiguously more likely when derivatives are senior. First, the combined debt and derivative repayments are larger when derivatives are senior, raising the prospect that the firm may be unable to meet its obligations even in the high-cash flow state. Second, under the current privileged bankruptcy treatment of derivatives, it may be in the counterparty's interest to make an inefficient collateral call that pushes the firm into bankruptcy. If the firm could impose a stay on collateral demands by derivative counterparties, it would be protected against such inefficient collateral calls (or runs on collateral).

To the extent that the favorable bankruptcy treatment of derivatives can lead to inefficiencies, a relevant question is whether firms can contractually "undo the law" in such cases. For example, firms may want to commit not to collateralize derivative contracts, thus stripping them of their effective seniority. However, debt covenants prohibiting the collateralization of derivatives are likely to be difficult to draft and costly to enforce (see Ayotte and Bolton (2011)). Enforcement constraints are likely to be especially severe for financial institutions: While it may be possible to shield physical collateral from derivative counterparties (for example, by granting collateral protection over plant and equipment to secured creditors), it is generally harder to shield unassigned cash from collateral calls by derivative counterparties in situations when a financial institution approaches financial distress. By the very nature of their business, financial institutions cannot assign cash as collateral to all depositors and creditors, because this would, in effect, erase their value added as financial intermediaries.

A number of legal scholars have taken up the question of the costs and benefits of the privileged treatment of derivatives (and, more generally, "qualified financial contracts") in bankruptcy.[7] We contribute to this debate by offering the first formal *ex ante* and *ex post* analysis of this issue.[8] A set of related studies on optimal corporate risk management (most notably those by Rampini and Viswanathan (2010, 2013) and Rampini, Sufi, and Viswanathan (2014)) also relies on models of state-contingent contracting subject to collateral constraints. However, in these models all financial contracts are fully collateralized, so that there is no default in equilibrium. In addition, these models allow the lender to bundle financing and risk management in a single state-contingent contract, which means that they are not well suited to studying the priority ranking of debt and derivatives in bankruptcy. In contrast, our working assumption, which is in line with the incomplete contracting literature, is that debt contracts cannot be written in an optimally state contingent form from the outset (whether directly or by bundling debt with a derivative contract), so that firms enter hedging transactions that appear suitable over time by using separate derivative contracts.[9] Finally, the dilution of debt contracts through senior derivatives is related to other forms of debt dilution that have been discussed in the corporate finance literature, whether through risk shifting (Jensen and Meckling (1976)), via the issuance of additional senior or short-term debt (Fama and Miller (1972), Diamond (1993a,b), and Brunnermeier and Oehmke (2013)), or by granting security interests to certain creditors (Bebchuk and Fried (1996)).

The remainder of the paper is organized as follows. Section I briefly summarizes the special status of derivative securities under U.S. bankruptcy law. Section II introduces the model. Section III analyzes a benchmark case without derivatives. Section IV discusses the effect of the bankruptcy treatment of derivatives in the case where the derivative has no basis risk. Section V extends the analysis to allow for basis risk and presents the main findings of our analysis. Section VI shows that, in a multi-firm setting, cross-netting benefits for counterparties can make seniority for derivatives efficient. Section VII discusses the effects of the bankruptcy treatment of derivatives on the firm's hedging incentives and on the incidence of strategic default. Section VIII offers some concluding remarks.

I. The Special Bankruptcy Status of Derivatives

In this section we briefly summarize the special status of derivatives in bankruptcy and explain why derivatives are often referred so as "super senior" claims.[10] Strictly speaking, derivatives are not senior in the formal legal sense.[11] However, derivative, swap, and repo counterparties enjoy certain rights that set them apart from regular creditors. While not formally senior, these rights make derivatives *effectively* senior to regular creditors, at least to the extent that they are collateralized.

The most important advantages a derivative, swap, or repo counterparty has relative to a regular creditor pertain to closeout, collateralization, netting, and the treatment of eve-of-bankruptcy payments, eve-of-bankruptcy collateral calls, and fraudulent conveyances. First, upon default, derivative counterparties have the right to terminate their contract with the firm and collect payment by seizing and selling collateral posted to them. This differs from regular creditors, who cannot collect payments when the firm defaults, because, unlike derivative counterparties, their claims are subject to the automatic stay. In fact, even if they are collateralized, regular creditors are not allowed to seize and sell collateral upon default, since their collateral, in contrast to the collateral posted to derivative counterparties, is subject to the automatic stay. Hence, to the extent that a derivative counterparty is collateralized at the time of default, collateralization and closeout provisions imply that the derivative counterparty is *de facto* senior to essentially all other claimants.[12]

Second, when closing out their positions with the bankrupt firm, derivative counterparties have stronger netting privileges than regular creditors. Because they can net offsetting positions, derivative counterparties may be able to prevent making payments to a bankrupt firm that a regular debtor would have to make, thereby strengthening the position of derivative counterparties vis-à-vis regular creditors in bankruptcy.[13]

Finally, derivative counterparties have stronger rights regarding eve-of-bankruptcy payments or fraudulent conveyances. For example, while regular creditors often have to return payments made or collateral posted to them within 90 days before bankruptcy, derivative counterparties are not subject to those rules. Any collateral posted to a derivative counterparty at the time of a bankruptcy filing is for the derivative counterparty to keep.

These factors taken together, the special bankruptcy treatment puts derivative counterparties in a much stronger position than regular creditors: To the extent that derivative contracts are collateralized, they are effectively senior to almost all other claims. In practice, this collateralization is usually ensured via regular marking-to-market and collateral calls. While for most of the remainder of the paper we will somewhat loosely refer to derivatives as being senior to debt, this should be interpreted in the light of the special rights and effective priority of derivative counterparties discussed in this section.

II. Model Setup

A. The Firm

We consider a firm that can undertake a two-period investment project. This firm can be interpreted as a nonfinancial firm undertaking a real investment project or as a financial institution investing in a risky loan or loan portfolio. The investment requires an initial outlay F at date 0 and generates cash flow at dates 1 and 2. At date 1, the project generates high cash flow C_1^H with probability θ and low cash flow $C_1^L < C_1^H$ with probability $1-\theta$. At date 2, the project generates (expected) cash flow C_2. Following the realization of the first-period cash flow, the project can be liquidated for a liquidation value L. We assume that $0 \leq L < C_2$, implying that early liquidation is inefficient. For simplicity (but without loss of generality), we normalize the firm's date 1 liquidation value to $L = 0$. After the realization of C_2, the firm is liquidated for a date 2 value of zero.

The firm has no initial funds and finances the project by issuing debt. The debt contract specifies the following terms: (i) the firm is to make a contractual repayment R at date 1; (ii) if the firm makes this contractual payment, it has the right to continue the project and collect the date 2 cash flow; (iii) if the firm fails to make the contractual date 1 payment R, the creditor has the right to discontinue the project and liquidate the firm. Liquidation can be interpreted as either an outright liquidation under a Chapter 7 cash auction, or a Chapter 11 reorganization. In the latter interpretation, L denotes the expected payment the creditor receives in a Chapter 11 reorganization. Both the firm and the creditor are risk neutral. The creditor has a cost of funds of 1, and the risk-free interest rate is normalized to zero.[14]

In financing the project, the firm faces a limited pledgeability problem similar to that in Bolton and Scharfstein (1990, 1996) and Hart and Moore (1994, 1998). More specifically, we assume that only the minimum date 1 cash flow C_1^L is verifiable and that all other cash flows can potentially be diverted by the borrower. This means concretely that, even if the high cash flow C_1^H obtains at date 1, the firm can claim to have obtained only the low cash flow and pay out C_1^L instead of R. We also assume that none of the date 2 cash flow can be committed to the lender. Finally, to make financing choices non trivial, we assume that $C_1^L < F$, such that the project cannot be financed with risk-free debt.

B. *The Derivative Counterparty*

Next, we introduce derivative contracts into the analysis. As with debt contracts, we do this in the simplest possible way. Formally, a derivative contract specifies a payoff that is contingent on the realization of a *verifiable* random variable $Z \in \{Z^H, Z^L\}$. This variable Z could, for example, be a financial index that is observable to both contracting parties and is verifiable by a court.[15]

A derivative contract of a notional amount X is a promise by the derivative counterparty (described in more detail below) to pay X to the firm if $Z = Z^L$, against a payment x that is payable from the firm to the derivative counterparty when $Z = Z^H$.[16] For simplicity, we assume that Z^L is realized with the same probability as the low cash flow C_1^L (i.e., $\Pr\left(Z = Z^L\right) = 1 - \theta$). Hence, a long position in the derivative pays off with the same unconditional probability with which the firm receives the low cash flow C_1^L. The derivative's usefulness in hedging is then determined by the correlation of the derivative payoff with the realization of the low cash flow. We capture this correlation with the parameter γ. Specifically, we assume that Z^L is realized conditional on $C_1 = C_1^L$ with probability γ:

$$\Pr\left(Z = Z^L | C_1 = C_1^L\right) = \gamma. \tag{1}$$

A derivative position of size $X = R - C_1^L$ therefore eliminates default in the low-cash flow state with probability γ.[17] If $\gamma = 1$ the derivative is a perfect hedge for the low-cash flow state. When

$\gamma < 1$, on the other hand, the derivative has basis risk and only imperfectly hedges the low-cash flow state and sometimes pays off in the high-cash flow state instead.[18]

When the firm enters a derivative position, the other side of the contract is taken by a derivative counterparty that is distinct from the original lender. This counterparty could be a financial institution, an insurance company, or a hedge fund providing hedging services to the firm. Generally, the provision of this type of insurance is not free of costs for the derivative counterparty. In particular, when faced with a notional exposure of X, the counterparty may face deadweight costs if it has to post collateral or set aside capital to fulfill capital requirements. We model these costs by building on the framework of Biais, Heider, and Hoerova (2012). The derivative counterparty has assets A on its balance sheet, which it can optimally invest for a gross return of 1, such that the counterparty has the same opportunity cost of funds as the creditor. However, when the counterparty enters into a derivative contract, it may have to post collateral in a margin account. This collateral account earns a return $\Gamma < 1$ that is strictly less than the counterparty's cost of funds. The margin requirement therefore induces a deadweight cost.

The derivative counterparty may be required to post collateral because of a moral hazard problem. Specifically, the derivative counterparty can take an unobserved action $a \in \{0, 1\}$ that alters the riskiness of its assets. When the counterparty chooses $a = 1$ (which can be interpreted as prudent risk management), the gross return on its assets is deterministic and given by 1. When the counterparty chooses $a = 0$ (which can be interpreted as shirking on risk management), the return on its assets is risky and equal to 1 only with probability $p < 1$. With probability $1 - p$ the gross return is equal to zero. However, choosing $a = 0$ generates a private benefit b (per unit of assets) to the counterparty. We assume that it is efficient for the counterparty to choose prudent risk management ($a = 1$):

$$1 > p + b. \tag{2}$$

However, when liabilities on existing derivative contracts build up, the counterparty may prefer to shirk and choose $a = 0$. In particular, suppose that, before choosing the action a, the counterparty and the firm learn more information about the odds they face on their derivative contract (X, x). For simplicity, assume they observe a signal $s \in \{s^L, s^H\}$ that is perfectly correlated with Z. When $s = s^H$, incentives are aligned. If the counterparty chooses $a = 1$, its payoff is given by

$A+x$, which is strictly higher than the payoff the counterparty would receive if it chose $a=0$, $A(p+b)+x$. However, incentives may not be aligned when $s=s^L$. If the counterparty chooses $a=1$, it now receives $A-X$, which may be lower than the payoff the counterparty receives if it chooses $a=0$, $p(A-X)+Ab$. It is therefore optimal for the counterparty to shirk on risk management whenever

$$b > \frac{(1-p)(A-X)}{A}. \tag{3}$$

When condition (3) holds, preserving the counterparty's incentives requires that the counterparty post an amount ζ of its assets as collateral in a margin account, such that the counterparty's incentive constraint is satisfied:

$$\zeta\Gamma + (A-\zeta) - X \geq p\left[\zeta\Gamma + (A-\zeta) - X\right] + (A-\zeta)b. \tag{4}$$

The minimum amount ζ that needs to be posted as collateral is then given by

$$\zeta = \frac{X - A\mathcal{P}}{\Gamma - \mathcal{P}}, \tag{5}$$

where we defined

$$\mathcal{P} \equiv 1 - \frac{b}{1-p}, \tag{6}$$

which can be interpreted as the counterparty's pledgeable income per unit of assets (unit pledgeable income).[19]

Derivatives have economic value in our setting. In particular, the derivative can be used to decrease the variability of the firm's cash flow at date 1, thereby effectively raising the verifiable cash flow the firm has available. From a welfare perspective this is beneficial because, by raising the low date 1 cash flow, the derivative allows the firm to reduce the probability of default at date 1. Hence, in the presence of derivatives as hedging instruments, the date 2 cash flow C_2 is lost less often. At the same time, collateral requirements for the counterparty create a deadweight cost of $1-\Gamma$ per unit of collateral posted. The counterparty has to post collateral ζ with probability $1-\theta$, such that, in expectation, it incurs a deadweight cost of $(1-\theta)(1-\Gamma)\zeta$. Using (5) and defining

$$\delta \equiv \frac{(1-\theta)(1-\Gamma)}{\Gamma - \mathcal{P}}, \tag{7}$$

we can rewrite the expected deadweight cost of collateral as $\delta(X - A\mathcal{P})$, which shows that the derivative counterparty faces linear deadweight costs for each dollar that the derivative obligation X exceeds the pledgeable part of its balance sheet.[20] In a competitive derivative market, these costs are passed on to the firm. Overall, derivatives increase surplus whenever the gains from reducing date 1 bankruptcy costs outweigh the deadweight cost of using derivatives.[21]

C. Seniority Treatment of Debt and Derivatives

We model the seniority of derivatives by first considering two extreme cases: first the case where derivatives are senior to debt and then the alternative extreme case in which derivatives are junior. The former situation is one where the required payment to the counterparty x is fully collateralized, and where cash collateral in the amount of x can be seized by the derivative counterparty in the event of a default on debt payments.[22] In the other extreme case, when derivatives are junior to debt, the payment x to the counterparty is not collateralized. Moreover, in this case the debt contract also specifies that it is senior to the derivative claim in bankruptcy. After exploring these polar cases, we will also consider the more general, intermediate case in which derivatives can be partially collateralized by assigning limited collateral $\overline{x} \leq x$ to the derivative counterparty. In this case, only the amount $\overline{x}$ can be seized by the derivative writer in the event of default. The remaining amount the firm owes to the derivative counterparty, $x - \overline{x}$, is then treated as a regular debt claim in bankruptcy. For simplicity we will assume that this remainder is junior to the claims of the debtholder.[23]

The treatment of derivatives in bankruptcy affects the pricing of the derivative (the payment x promised by the firm when $Z = Z^H$) in the following way: In the event that period 1 cash flow is C_1^L and that $Z = Z^H$, the firm is unable to meet all its financial obligations—it owes R to its creditor and x to the derivative counterparty, but $R + x > C_1^L$. The priority of debt relative to derivatives therefore affects the size of the payments the derivative counterparty and the lender can expect in this state of the world. We assume that when the derivative is senior to debt, the counterparty is guaranteed to receive x in the event that $Z = Z^H$ (i.e., we assume that $x \leq C_1^L$).[24] If, on the other hand, the derivative is junior to debt, then the counterparty will not receive any payment when $Z = Z^H$ and $C_1 = C_1^L$ since the creditor seizes all assets. This happens with probability $(1-\theta)(1-\gamma)$.

To clearly distinguish senior and junior contracts, in what follows we will denote the pricing terms of senior contracts by a superscript S and those of junior contracts by a superscript J.[25] When the derivative is senior to debt, the payment from the firm in the event that $Z = Z^H$ is the sure payment x^S, and the break-even condition for the counterparty is

$$\theta x^S - (1-\theta)\, X^S - (1-\theta)\,(1-\Gamma)\,\zeta^S = 0. \tag{8}$$

The term $(1-\theta)\,(1-\Gamma)\,\zeta^S$ reflects the expected deadweight cost of collateral that the counterparty is required to post when the derivative is senior.[26]

In comparison, when the derivative is junior to debt and $Z = Z^H$, the counterparty is paid only if the firm receives a high cash flow C_1^H. Hence, the counterparty receives x^J only with probability $\theta - (1-\theta)\,(1-\gamma)$. The break-even condition for the derivative counterparty is then given by

$$[\theta - (1-\theta)\,(1-\gamma)]\, x^J - (1-\theta)\, X^J - (1-\theta)\,(1-\Gamma)\,\zeta^J = 0, \tag{9}$$

where

$$\zeta^J = \frac{X^J - A\mathcal{P}}{\Gamma - \mathcal{P}}. \tag{10}$$

Comparing (5) and (10) shows that the bankruptcy treatment of derivatives affects required collateral and hence the counterparty's deadweight cost of providing insurance to the firm.

D. *Timing of Moves*

Implicit in our description of the model so far is the following assumption on the timing of moves. The firm enters the derivative contract after it has signed the debt contract with the creditor. Moreover, at the initial contracting stage, the firm and the creditor cannot condition the debt contract on a particular realization of Z. This assumption reflects the idea that at the ex ante contracting stage it may not be known which business risks the firm needs to or can hedge in the future and which derivative positions will be required to do so. Essentially, this assumption is in line with the literature on incomplete contracting and rules out a fully state-contingent contract at date 0 between the creditor and the firm that bundles financing and hedging.[27]

III. Benchmark: No Derivatives

We first describe the equilibrium in the absence of a derivative market. The results from this section provide a benchmark against which we can evaluate the effects of introducing derivative markets.

In the absence of derivatives, the firm always defaults if the low cash flow C_1^L realizes at date 1. Because $C_1^L < F$, the low cash flow is not sufficient to repay the face value of debt. Moreover, the date 2 cash flow C_2 is not pledgeable and, since the firm has no other cash it can offer to renegotiate with the creditor, it has no other option than to default when C_1^L is realized at date 1. The lender then seizes the cash flow C_1^L and shuts down the firm, collecting the liquidation value of the asset L. Early termination of the project leads to a social loss of $C_2 - L$, the additional cash flow that would have been generated had the firm been allowed to continue its operations.

If the high cash flow C_1^H realizes at date 1, the firm has enough cash to service its debt. However, the firm may nonetheless choose to default strategically. A strategic default occurs when the firm is better off defaulting on its debt at date 1 than repaying the debt and continuing operations until date 2, which is the case when the continuation value C_2 is sufficiently low. For most of our analysis we will assume that C_2 is high enough, such that the firm services its debt in the high state. We return to the issue of strategic default in Section VII.B.

Assuming that the firm repays its debt obligation in the high-cash flow state, the lender's break-even constraint (given competitive capital markets and the simplifying assumption that $L = 0$) is given by

$$\theta R + (1-\theta)\, C_1^L = F. \tag{11}$$

Given the break-even condition (11), we can then summarize the credit market outcome in the absence of derivatives as follows.

PROPOSITION 1: *In the absence of derivative markets and assuming that C_2 is sufficiently high* *that no strategic default occurs in the high-cash flow state, the equilibrium face value of debt is given by* $R = \left[F - (1-\theta)\, C_1^L\right] / \theta$. *The firm is shut down after the low-cash flow realization, and social surplus is equal to* $\theta \left(C_1^H + C_2\right) + (1-\theta)\, C_1^L - F$.

For the purpose of our analysis, the most important implication of Proposition 1 is that, in the absence of derivatives, the firm is always shut down if the low cash flow realizes at date 1. Such early termination leads to an inefficiency because the continuation value C_2 is lost. As we will show in the following section, derivatives can reduce this inefficiency by reducing the risk of default at date 1.

IV. Financing with Derivatives: No Basis Risk

In this section (and in Sections V and VI), we consider the firm's problem of optimal hedging when it can commit to selecting the ex ante optimal derivative contract. For a benchmark, we consider first the case where the derivative has no basis risk. This corresponds to the situation where $\gamma = 1$. When there is no basis risk, the firm can completely eliminate default risk by choosing an appropriate position in the derivative. As we will see, in this benchmark case, a firm that can commit to its derivative position always takes the socially optimal hedging position, and the priority ordering of the derivative relative to debt is irrelevant.

A firm that can commit to its derivative position always chooses the derivative position that maximizes the overall surplus: Both the creditor and the derivative counterparty just break even, and all remaining surplus is captured by the firm. The firm will thus choose to hedge whenever it is socially optimal to do so. Moreover, because the derivative is costly, when hedging is optimal the firm will always take the minimum position in the derivative that is needed to eliminate default. In this case, the priority ranking of debt relative to the derivative is irrelevant: Whenever the firm chooses to hedge, debt becomes risk free and default will never occur. But when there is never any default, the bankruptcy treatment of debt relative to derivatives does not matter.

We can see this more formally by comparing the costs and benefits from hedging in either regime. Eliminating default leads to a gain of $(1-\theta)\, C_2$, since now the firm can be kept alive even after the realization of C_1^L at date 1. The net cost of eliminating default is given by the deadweight cost that needs to be incurred in derivative markets. Since the derivative completely eliminates default when there is no basis risk, debt becomes safe, so that $R = F$, irrespective of the priority ranking of debt relative to derivatives. Hence, the deadweight cost of taking the required derivative position $X = F - C_1^L$ is given by

$$(1-\theta)\,(1-\Gamma)\,\zeta, \tag{12}$$

where

$$\zeta = \frac{F - C_1^L - A\mathcal{P}}{\Gamma - \mathcal{P}} \tag{13}$$

does not depend on the bankruptcy regime (hence no superscript). The firm chooses to hedge whenever the presence of derivatives raises surplus, which is the case when

$$(1-\theta)\,C_2 - (1-\theta)\,(1-\Gamma)\left(\frac{F - C_1^L - A\mathcal{P}}{\Gamma - \mathcal{P}}\right) > 0, \tag{14}$$

or, inserting (7),

$$(1-\theta)\,C_2 - \delta(F - C_1^L - A\mathcal{P}) > 0. \tag{15}$$

Hence, the hedging cost is linear in $F - C_1^L - A\mathcal{P}$, the difference in the counterparty's exposure $F - C_1^L$ and its total pledgeable income $A\mathcal{P}$. Condition (15) is satisfied whenever the continuation value of the firm C_2 is sufficiently large, or when the cost of hedging is sufficiently low (which is the case when the pledgeable income $A\mathcal{P}$ is sufficiently high).

PROPOSITION 2: *When the derivative has no basis risk ($\gamma = 1$) and the firm can commit to a derivative position it takes ex post:*

1. *The firm chooses the socially optimal derivative position.*
2. *The bankruptcy treatment of derivatives is irrelevant.*
3. *Hedging with derivatives raises surplus whenever*

$$(1-\theta)\,C_2 - (1-\theta)\,(1-\Gamma)\left(\frac{F - C_1^L - A\mathcal{P}}{\Gamma - \mathcal{P}}\right) > 0. \tag{16}$$

V. Financing with Derivatives: Basis Risk

We now consider the case where the derivative contract has basis risk ($\gamma < 1$). We begin by establishing a lemma about collateralization of the derivative position stating that, once the face value of debt is set, it is always optimal ex post to maximally collateralize the derivative contract when there is basis risk. The reason is that, once R is fixed, collateralization of the derivative contract makes hedging cheaper for the firm. Thus, suppose that the firm can choose to only partially collateralize derivatives by assigning only limited collateral $\overline{x} \leq x$ to the derivative counterparty, such that only the amount $\overline{x}$ can be seized by the counterparty in the event of default. The remaining amount that the firm owes to the counterparty, $x - \overline{x}$, is treated as a regular debt claim in bankruptcy. For simplicity, assume that this remainder is junior to the claims of the debtholder. Then the following lemma obtains.

LEMMA 1: *Once financing has been secured and the face value of debt R has been set, it is privately optimal for the firm and the derivative counterparty to fully collateralize the derivative position. This is because the cost of the derivative $x(\overline{x})$ is decreasing in the level of collateralization:*

$$\frac{\partial x(\overline{x})}{\partial \overline{x}} < 0. \tag{17}$$

Lemma 1 illustrates the conventional wisdom supporting the collateralization and effective seniority of derivatives: Collateralization and seniority for derivatives make hedging cheaper, which benefits the firm. According to this logic, it is often argued that full collateralization and the concomitant seniority of derivative contracts are optimal and that, conversely, reducing collateralization or making derivative contracts junior to debt is undesirable because it raises the cost of the derivative and thereby makes hedging more expensive for the firm.

However, changing the level of collateralization and hence the effective seniority of derivatives while holding the face value of debt constant is not the correct thought experiment. After all, in the event of default, debtholders and derivative counterparties hold claims on the same pool of assets. Accordingly, changing the effective seniority of derivatives must, in equilibrium, also have an impact on the pricing of the firm's debt. As we show below, once we allow the pricing of the firm's debt to adjust in response to the effective seniority of derivative contracts, the argument for full collateralization and effective seniority for derivatives is reversed. We show this by contrasting the two polar cases of senior derivatives and junior derivatives. These two cases contain essentially all the economic intuition for why, at the firm level, an arrangement where derivatives are junior is more efficient. One can easily extend the analysis to the intermediate case, in which derivatives can be partially collateralized.[28]

A. *Seniority of Derivatives over Debt*

As discussed in Section I, under the current special bankruptcy treatment, derivatives are effectively senior to debt claims. When derivatives are senior, the counterparty is guaranteed to receive the contractual payment x^S that is due when $Z = Z^H$ (as long as $x^S \leq C_1^L$).[29] For the counterparty to break even, the expected payments received, θx^S, must equal the expected payments made, $(1-\theta)\,X^S$, plus the expected deadweight cost of hedging, $(1-\theta)\,(1-\Gamma)\,\zeta^S$, which yields

$$\theta x^S = (1-\theta)\,[X^S + (1-\Gamma)\,\zeta^S], \tag{18}$$

where

$$\zeta^S = \frac{X^S - A\mathcal{P}}{\Gamma - \mathcal{P}}. \tag{19}$$

Substituting for ζ^S in (18) and rearranging, we obtain the following expression for the counterparty's break-even constraint:

$$\theta x^S = (1-\theta)X^S + \delta(X^S - A\mathcal{P}), \tag{20}$$

where, as before, we define

$$\delta \equiv \frac{(1-\theta)(1-\Gamma)}{\Gamma - \mathcal{P}}. \tag{21}$$

The face value of junior debt, R^J, in turn, is determined by the creditor's break-even condition. When derivatives are senior to debt and $x^S \leq C_1^L$, this break-even condition is given by

$$[\theta + (1-\theta)\,\gamma]\,R^J + (1-\theta)\,(1-\gamma)\left(C_1^L - x^S\right) = F, \tag{22}$$

which simply states that the expected payments received by the creditor must equal the initial outlay F. Note that the seniority of the derivative contract becomes relevant in the state when $C_1 = C_1^L$ and $Z = Z^H$, which occurs with probability $(1-\theta)\,(1-\gamma)$. In that case, the derivative counterparty is paid its contractual obligation x^S before the creditor can receive any payment.

When $\gamma < 1$ the derivative is only a partial hedge, because it sometimes does not pay off when $C_1 = C_1^L$ and sometimes pays off when $C_1 = C_1^H$. Nevertheless, hedging can still be valuable for the firm because it reduces the probability of default. The optimal derivative position for the firm (when it can commit to its hedging policy) is the one that just eliminates default when $C_1 = C_1^L$ and the derivative pays off. Default can then be avoided with any derivative position that satisfies

$$X^S \geq R^J - C_1^L. \tag{23}$$

Thus, by setting $X^S = R^J - C_1^L$ the derivative contract eliminates default in states when $C_1 = C_1^L$ and $Z = Z^L$ (i.e., with probability $(1-\theta)\gamma$). Increasing the derivative position beyond this level does not generate any additional surplus; it only increases the deadweight hedging cost and is thus inefficient. Because the derivative is an imperfect hedge, the firm defaults when $C_1 = C_1^L$ and $Z = Z^H$ (i.e., with probability $(1-\theta)(1-\gamma)$).

Substituting the expression for x^S from (20) into the creditor's break-even condition (22) and setting $X^S = R^J - C_1^L$, we obtain the following expression for the face value of debt:

$$[\theta + (1-\theta)\gamma] R^J + (1-\theta)(1-\gamma)\left[C_1^L - \frac{(1-\theta)\left(R^J - C_1^L\right) + \delta(R^J - C_1^L - A\mathcal{P})}{\theta}\right] = F. \tag{24}$$

Solving (24) for R^J, we then obtain the following characterization of the equilibrium when derivatives are senior to debt.

PROPOSITION 3: ***Senior derivatives.*** *Assume that derivatives are senior and that $x^S \leq C_1^L$. Under full commitment with respect to hedging policy, the optimal derivative position is given by*

$$X^S = R^J - C_1^L. \tag{25}$$

This leads to an equilibrium face value of debt

$$R^J = \frac{\theta F - (1-\theta)(1-\gamma)(1-\delta)C_1^L - \delta(1-\theta)(1-\gamma)A\mathcal{P}}{\theta - (1+\delta)(1-\theta)(1-\gamma)} \tag{26}$$

and a price of the derivative of

$$x^S = \frac{(1-\theta+\delta)\left(F - C_1^L\right) - \delta\left(\theta + (1-\theta)\gamma\right)A\mathcal{P}}{\theta - (1+\delta)(1-\theta)(1-\gamma)}. \tag{27}$$

To gain intuition about Proposition 3, it is useful to recall the special case in which derivatives provide a perfect hedge against cash flow risk at date 1 ($\gamma = 1$). In this case, debt becomes risk free ($R^J = F$) so that the optimal derivative position is given by $X^S = F - C_1^L$. When the derivative is not a perfect hedge ($\gamma < 1$), on the other hand, debt is risky even in the presence of derivatives ($R^J > F$) and the required derivative position increases to $R^J - C_1^L > F - C_1^L$.

The social surplus generated in the presence of derivatives depends on how effective derivatives are at hedging the firm's cash flow risk. When the derivative has more basis risk (lower γ), it is less effective as a hedging tool, such that the probability of continuation of the firm at date 1, which is given by $\theta + (1 - \theta)\,\gamma$, is lower. Moreover, higher basis risk increases the deadweight costs of hedging because the required derivative position, $R^J - C_1^L$, is larger.

COROLLARY 1: ***Social surplus under senior derivatives.*** *Under senior derivatives, when the firm chooses a derivative position of $X^S = R^J - C_1^L$, social surplus is given by*

$$\theta C_1^H + (1-\theta)\,C_1^L + [\theta + (1-\theta)\,\gamma]\,C_2 - F - (1-\theta)(1-\Gamma)\zeta^S. \tag{28}$$

Derivatives raise social surplus when the gain from the greater likelihood of continuation outweighs the deadweight cost of hedging:

$$(1-\theta)\,\gamma C_2 - (1-\theta)(1-\Gamma)\zeta^S > 0. \tag{29}$$

Given that the amount ζ^S that the counterparty has to post as collateral is decreasing in the counterparty's unit pledgeable income $\mathcal{P}$, derivatives are more likely to raise overall surplus when the counterparty is well capitalized and therefore has to post less (or no) costly collateral.

B. *Derivatives Junior to Debt*

We now consider the opposite case, in which derivatives are junior to debt. As before, the firm defaults at date 1 when it obtains a low cash flow C_1^L and $Z = Z^H$. This happens with probability $(1-\gamma)\,(1-\theta)$. When derivatives are junior, the lender now receives the entire cash flow C_1^L while the counterparty receives nothing. Of course, the greater default risk that the derivative counterparty is now exposed to is passed on to the firm in the form of a higher cost of insurance x^J. The counterparty's break-even constraint is then given by

$$x^J \left[\theta - (1-\theta)(1-\gamma)\right] = (1-\theta)\left[X^J + (1-\Gamma)\,\zeta^J\right], \tag{30}$$

where

$$\zeta^J = \frac{R^S - C_1^L - A\mathcal{P}}{\Gamma - \mathcal{P}}. \tag{31}$$

Note that because the counterparty only receives the promised payment x^J with probability $\theta - (1-\theta)(1-\gamma)$, rather than with probability θ, it requires a higher promised payment, other things equal. The senior creditor, on the other hand, now receives the entire cash flow in the default state, so that its break-even constraint becomes

$$\left[\theta + (1-\theta)\gamma\right] R^S + (1-\theta)(1-\gamma)\, C_1^L = F. \tag{32}$$

Given the less risky debt claim, the promised face value of debt required to raise F is lower and given by

$$R^S = \frac{F - (1-\theta)(1-\gamma)\, C_1^L}{\theta + (1-\theta)\gamma}. \tag{33}$$

As before, the optimal derivative position is such that $X^J = R^S - C_1^L$, and default occurs only when $C_1 = C_1^L$ and $Z = Z^H$ (i.e., with probability $(1-\theta)(1-\gamma)$). Using (30) and (33), we can then characterize the equilibrium under junior derivatives as follows.

PROPOSITION 4: ***Junior derivatives.*** *Assume that derivatives are junior and that $x^J \leq C_1^L$. Under full commitment with respect to hedging policy, the optimal derivative position is given by*

$$X^J = R^S - C_1^L. \tag{34}$$

This leads to an equilibrium face value of debt

$$R^S = \frac{F - (1-\theta)(1-\gamma)\, C_1^L}{\theta + \gamma(1-\theta)} \tag{35}$$

and price of the derivative of

$$x^J = \frac{(1-\theta+\delta)\left[F - C_1^L\right] - \delta\left[\theta + \gamma(1-\theta)\right] A\mathcal{P}}{\left[\theta - (1-\gamma)(1-\theta)\right]\left[\theta + \gamma(1-\theta)\right]}. \tag{36}$$

As with Proposition 3, we can use the results from Proposition 4 to obtain an expression for the net social surplus when derivatives are junior.

COROLLARY 2: ***Social surplus under junior derivatives.*** *Under junior derivatives, when the firm chooses a derivative position of $X^J = R^S - C_1^L$, social surplus is given by*

$$\theta C^H + (1-\theta) C_1^L + \left[\theta + (1-\theta)\gamma\right] C_2 - F - (1-\theta)(1-\Gamma)\zeta^J. \tag{37}$$

Derivatives raise social surplus when the gain from the greater likelihood of continuation outweighs the deadweight cost of hedging:

$$(1-\theta)\gamma C_2 - (1-\theta)(1-\Gamma)\zeta^J > 0. \tag{38}$$

As before, the required collateral ζ^J that the counterparty must post is decreasing in the counterparty's unit pledgeable income $\mathcal{P}$, so that condition (38) is more likely to be satisfied when the counterparty is well capitalized.

From the expressions for R^J and R^S in Propositions 3 and 4, it is immediate that $R^J \geq R^S$ and therefore that $\zeta^S \geq \zeta^J$. Indeed, from (19) and (31), we see that ζ^S and ζ^J differ only in the face values of the derivative liabilities for the counterparty $X^S = R^J - C_1^L$ and $X^J = R^S - C_1^L$. It follows immediately that the net social surplus under senior debt and junior derivatives in Corollary 2 is higher than the net social surplus under junior debt and senior derivatives in Corollary 1. This is the key economic observation emerging from our analysis: *When debt is senior, the sum of the equilibrium cost of debt and the hedging contract is lower than when derivatives are senior.* We summarize this result in the following proposition.

PROPOSITION 5: ***Comparing surplus under junior and senior derivatives.*** *Relative to the situation without derivatives, hedging with junior derivatives raises surplus more than hedging with senior derivatives.*

Thus, the received wisdom that the seniority of derivatives is desirable (Lemma 1) reverses once one takes into account the transfer of risk from the derivative counterparty to creditors: Net social surplus is strictly higher under junior derivatives than under senior derivatives, except in two special cases. First, when the derivative is a perfect hedge ($\gamma = 1$) so that the firm never defaults, $R^S = R^J$, and seniority of the derivative contract is irrelevant. Second, when there is no deadweight hedging cost of hedging ($\zeta^S = \zeta^J = 0$), seniority is also irrelevant, as a result of a Modigliani-Miller type logic: Risk is transferred from debt markets to derivative markets, but net surplus remains unchanged.

VI. Multiple Borrowers and the Benefits of Cross-Netting

So far we have analyzed the economic effects of the privileged bankruptcy treatment of derivatives in the context of a single firm and a single counterparty. In this section, we reconsider the effects in a multiple-borrower context, in which derivative counterparties enter derivative contracts with many firms. The purpose of this analysis is to explore whether the senior treatment of derivatives may be efficient once diversification benefits from cross-netting by derivative counterparties are taken into account. To answer this question, we characterize how counterparties' balance sheets are affected by the bankruptcy treatment of derivatives when counterparties provide hedging services to multiple firms. We then ask how the change in balance sheet risk affects the counterparties' deadweight cost of providing insurance to firms.

We consider the market-wide effects of a change in the bankruptcy status of derivatives by analyzing the equilibrium interaction between a *representative derivative counterparty* that enters derivative contracts with many different *representative firms.* Formally, we consider a continuum of identical firms (with unit mass), each faced with the same setup cost F, identical potential cash flow realizations $\{C_1^L, C_1^H\}$ in period 1, and identical continuation values C_2. All firms are funded using the same debt contract and hedge their cash flow risk by entering a derivative contract with the representative derivative counterparty. The counterparty is subject to a free-entry zero-profit condition. The main difference to the preceding analysis is that the representative counterparty's balance sheet is now composed of its initial endowment of assets A and a continuum of derivative contracts (X, x), rather than a single derivative contract. While this is a stylized model of a derivative market—in practice, there is likely considerable heterogeneity across firms and counterparties—it has the virtue of simplicity and allows us to focus on the core issue: the potential cross-netting benefits to counterparties from being senior to creditors in bankruptcy.

Our analysis shows that the main determinant of the cross-netting benefits that derivative writers gain from seniority is the correlation structure of the shocks to which firms are exposed. Specifically, cross-netting benefits arise only if seniority for derivatives allows cross-netting that reduces the counterparty's net liability to firms in states where it has to post collateral. As we show below, for this to be the case, the basis risk of the derivative has to be (sufficiently) idiosyncratic. Intuitively, this requires that firms hedge their cash flow risk using different, imperfectly correlated derivatives.

For a concrete illustration of the distinction between idiosyncratic and systematic basis risk, consider firms that, during the recent housing boom, were seeking to hedge their exposure to the real estate market using derivatives. For these firms, basis risk would have been systematic if all (or most) firms had entered derivatives referenced to the same index (e.g., the ABX index). If, on the other hand, firms had chosen a number of different ways to hedge this risk (e.g., using a number of different indices), then at least some of the resulting basis risk would have been idiosyncratic.

As in the preceding analysis, the key question is how the bankruptcy status of derivatives affects the collateral requirements of the derivative counterparty. The main difference to the previous analysis is that this collateral requirement is now based on the derivative counterparty's net liability from *all* derivative contracts.[30] We first show that when basis risk is systematic (all firms use the same or similar derivative contracts), seniority for derivatives does not lead to cross-netting benefits for the counterparty. Hence, our results from the preceding single-firm analysis carry over, and making derivatives junior to debt is efficient. When basis risk is idiosyncratic, on the other hand, cross-netting benefits can be sufficiently large to make seniority for derivatives efficient. Specifically, when cash flow risk is systematic but basis risk is idiosyncratic, seniority for derivatives allows the counterparty to reduce its net obligation to firms and, thereby, the required collateral amount, reducing the deadweight cost of hedging.

A. Systematic Basis Risk

We first consider the case where basis risk is systematic. This corresponds to a situation where all firms use the same derivative to hedge their cash flow risk. Cash flow risk can be either systematic or idiosyncratic.

Consider first the case in which both cash flow risk and basis risk are systematic. When $Z = Z^L$ the derivative counterparty has a liability on *all* derivatives it has written, whereas when $Z = Z^H$ *all* derivatives have moved in favor of the counterparty. Because the counterparty's balance sheet is risky, it has to post collateral in the state where it has a liability to firms, thereby incurring deadweight costs. However, when both cash flow risk and basis risk are systematic, no cross-netting is possible and the analysis reduces to the single-firm setting discussed in Section V: Because under senior derivatives the required face value of debt is larger ($R^J > R^S$), the required notional derivative position is larger under senior derivatives than under junior derivatives ($X^S > X^J$). This leads to a higher aggregate net liability for the counterparty when $Z = Z^L$, so that the required collateral is higher under senior derivatives than junior derivatives ($\zeta^S > \zeta^J$). Hence, Proposition 5 applies, and deadweight costs are higher when derivatives are senior.[31]

Now consider the case in which basis risk is systematic but cash flow risk is idiosyncratic. If the derivative moves against the counterparty ($Z = Z^L$), the counterparty owes X^S when derivatives are senior and X^J when derivatives are junior. In this state, the counterparty has to post collateral. When the derivative moves in favor of the counterparty ($Z = Z^H$), then all firms owe a payment to the counterparty (x^S or x^J, depending on the seniority status of the derivative contract). In this state, the counterparty does not have to post collateral because it has a net inflow. If derivatives are senior, the counterparty receives x^S; if derivatives are junior, the counterparty receives θx^J, because only firms that receive the high cash flow (a fraction θ) can make the contractual payment x^J. The required derivative position X^i is equal to $R^i - C_1^L$, and x^i is set so that the counterparty breaks even in expectation taking into account expected costs of posting collateral, where the superscript $i = S, J$ denotes the bankruptcy ordering of debt and derivatives.

Recall that the amount of required collateral depends only on the size of the net liability in the state in which the counterparty owes a payment. As we can see from the cash flows above, when all firms use the same derivative, the seniority for derivatives favors the counterparty in the state where it has a net inflow because it affects how many firms make the contractual payment x^i to the counterparty. This is irrelevant for the amount of collateral that is required, which depends only on the counterparty's net liability in the state where $Z = Z^L$, such that the derivative moves against the counterparty. The derivative counterparty's net liability in that state is larger when derivatives are senior, $X^S > X^J$, because seniority for derivatives raises the face value of debt, $R^J > R^S$. This implies that more collateral is required when derivatives are senior ($\zeta^S > \zeta^J$) and junior derivatives are efficient.

These factors taken together show that if basis risk is systematic (all firms use the same derivative to hedge), no cross-netting benefits arise and junior derivatives are efficient.

B. *Idiosyncratic Basis Risk*

Now consider the situation where basis risk is idiosyncratic. This corresponds to a situation where firms use different derivatives to hedge their cash flow risk. As before, cash flow risk can be either systematic or idiosyncratic.

When both cash flow risk and basis risk are idiosyncratic, the priority ordering of debt and derivatives is irrelevant. By the law of large numbers, a fraction θ of the counterparty's derivative contracts moves in favor of the counterparty ($Z = Z^H$), while a fraction $1 - \theta$ of its derivative contracts requires the counterparty to make payments to firms ($Z = Z^L$), such that the counterparty's balance sheet is deterministic. Given that the counterparty does not face any balance sheet risk and has no net liability, it does not have to post collateral on any of its derivative contracts ($\zeta^S = \zeta^J = 0$). Hence, while the pricing of the derivative contract will differ depending on the priority ordering of derivatives relative to debt, no deadweight costs arise and the status of derivatives in bankruptcy is irrelevant.[32]

Cross-netting benefits arise when basis risk is idiosyncratic but cash flow risk is systematic. In this case, seniority for derivatives allows the derivative counterparty to cross-net exposures in the state in which it has a net liability, thereby reducing the amount of collateral it is required to post. As we show below, these cross-netting benefits outweigh the detrimental effects from higher required derivative positions at the firm level. Hence, when basis risk is idiosyncratic but cash flow risk is systematic, seniority for derivatives is efficient.

To see this, consider first the case when derivatives are senior. Because cash flow risk is systematic, either all firms receive the high cash flow or all firms receive the low cash flow. When all firms receive the low cash flow C_1^L, idiosyncratic basis risk implies that a fraction γ of firms receives a payoff X^S from the derivative position. For the remaining fraction $1-\gamma$ of firms, the derivative moves the wrong way, so that the firms owe a payment x^S to the derivative counterparty. Because they receive the low cash flow and their hedge moves the wrong way, these firms default. The senior counterparty then receives the full promised payment x^S (given the assumption that $x^S \leq C_1^L$) while junior creditors receive the remainder $C_1^L - x^S$. Conversely, when all firms receive the high cash flow C_1^H, a fraction $1-\frac{(1-\theta)(1-\gamma)}{\theta}$ of firms owes a payment x^S to the counterparty, while the counterparty is required to pay X^S to a fraction $\frac{(1-\theta)(1-\gamma)}{\theta}$ of firms.

When all firms receive the high cash flow, the counterparty's balance sheet is then given by

$$A+\left[1-\frac{(1-\theta)\,(1-\gamma)}{\theta}\right]x^S-\frac{(1-\theta)\,(1-\gamma)}{\theta}X^S, \tag{39}$$

assuming that no collateral has to be posted, which we verify below. In contrast, in the low-cash flow state, the counterparty's balance sheet is given by

$$\zeta^S\Gamma+A-\zeta^S+(1-\gamma)\,x^S-\gamma X^S, \tag{40}$$

where ζ^S is the amount of collateral the senior counterparty has to post so that the incentive constraint

$$\zeta^S\Gamma+A-\zeta^S+(1-\gamma)x^S-\gamma X^S \geq p\left[\zeta^S\Gamma+A-\zeta^S+(1-\gamma)x^S-\gamma X^S\right]+(A-\zeta^S)b \tag{41}$$

is satisfied. The minimum amount of collateral that aligns the counterparty's incentives is thus given by

$$\zeta^S=\frac{\gamma X^S-(1-\gamma)\,x^S-A\mathcal{P}}{\Gamma-\mathcal{P}}. \tag{42}$$

Equation (42) illustrates the crucial difference to the previous cases, particularly equation (5): Under idiosyncratic basis risk, seniority for derivatives allows the senior derivative counterparty to cross-net some of its obligation γX^S against payments $(1-\gamma)\,x^S$ it receives from defaulted firms, thereby reducing the counterparty's net liability in the low-cash flow state. The counterparty's ex-ante zero-profit condition is then given by

$$\theta x^{S} - (1-\theta)\, X^{S} - (1-\theta)\,(1-\Gamma)\,\zeta^{S} = 0. \tag{43}$$

Because the counterparty has a net liability in the low-cash flow state, this break-even condition can be satisfied only if the counterparty has a net gain in the high-cash flow state. This confirms, as we have assumed above, that no collateral has to be posted in the high cash flow state.

Now consider junior derivatives. In the high-cash flow state, no collateral has to be posted, and the derivative counterparty's balance sheet is given by

$$A + \left[1 - \frac{(1-\theta)\,(1-\gamma)}{\theta}\right] x^{J} - \frac{(1-\theta)\,(1-\gamma)}{\theta} X^{J}. \tag{44}$$

In the low-cash flow state, on the other hand, the counterparty's balance sheet becomes

$$\zeta^{J}\Gamma + A - \zeta^{J} - \gamma X^{J}, \tag{45}$$

where, as before, we can obtain the collateral requirement from the incentive constraint:

$$\zeta^{J} = \frac{\gamma X^{J} - A\mathcal{P}}{\Gamma - \mathcal{P}}. \tag{46}$$

The zero-profit condition for the counterparty is then given by

$$\left[\theta - (1-\theta)\,(1-\gamma)\right] x^{J} - (1-\theta)\, X^{J} - (1-\theta)\,(1-\Gamma)\,\zeta^{J} = 0. \tag{47}$$

Which of the two regimes is more efficient comes down to a comparison of ζ^{S} and ζ^{J}. On the one hand, under junior derivatives the required derivative position X^{J} is lower than under senior derivatives because the face value of senior debt R^{S} is lower. To the extent that the derivative position X^{J} is smaller, the counterparty has to post less collateral. On the other hand, when derivatives are junior, the counterparty obtains no income from firms that owe payments to the counterparty in the low-cash flow state—the entire cash flow of the defaulted firms goes to the senior creditor. This loss of income increases the required fraction of assets that the counterparty must post as collateral. It is a matter of algebra to verify that this latter effect dominates:

$$\gamma X^J \geq \gamma X^S - (1-\gamma)\, x^S. \tag{48}$$

This implies that the net liability for the derivative counterparty in the low-cash flow state is larger under junior derivatives than under senior derivatives, which implies that $\zeta^J \geq \zeta^S$. In this case seniority for derivatives is a more efficient arrangement.

We summarize the results on cross-netting benefits in the following Proposition.

PROPOSITION 6: ***Comparison of Junior and Senior Derivatives with Multiple Borrowers.*** *In a multiple-borrower setting, seniority for derivatives is efficient only if it leads to sufficient cross-netting benefits in states where the derivative counterparty has a net liability. Specifically,*

1. *It is socially efficient to have debt senior to derivatives when basis risk is systematic (all firms use the same derivative to hedge).*
2. *It is socially efficient to have derivatives senior to debt when basis risk is idiosyncratic (firms use different derivative contracts to hedge) but cash flow risk is systematic.*
3. *The bankruptcy treatment of derivatives is irrelevant when both basis risk and cash flow risk are idiosyncratic.*

The general insight of Proposition 6 is that, unless the derivative counterparty's benefits from cross-netting are significantly sensitive to the priority ordering of derivatives and debt, the efficient allocation of risk is achieved by an ordering where debt is senior to derivatives. Our analysis reveals that cross-netting is most sensitive to the priority ordering of debt and derivatives when basis risk is idiosyncratic but cash flow risk is (mostly) systematic. In this situation, cross-netting benefits are achieved via the counterparty's ability to recover payments from failed firms. In all other situations, there are either no cross-netting benefits to be obtained (when both cash flow risk and basis risk are mostly systematic) or these benefits do not depend on the priority treatment of derivatives in bankruptcy (when both cash flow risk and basis risk are mostly idiosyncratic).[33]

VII. Strategic Hedging and Strategic Default

Our main analysis is cast in a static framework, in which the firm can commit ex-ante to an optimal hedging policy. In reality, it is difficult for firms to make such commitments and it is rare to see covenants in debt contracts restricting the firm's hedging options. When firms cannot commit to an optimal hedging policy, they strategically choose their hedging positions ex post to favor equityholders, potentially at the expense of creditors. The privileged treatment of derivatives in bankruptcy affects firms' incentives to enter into strategic hedging positions. In this section, we analyze the incentives created by this privileged treatment by considering, in turn, two key issues: (i) under-hedging and excess speculation, and (ii) default due to derivative losses and inefficient collateral calls.

A. *Hedging or Speculation?*

No basis risk: We begin by considering the special case where there is no basis risk ($\gamma = 1$). Recall that, in this case, under full commitment it is irrelevant for social efficiency whether derivatives are senior or junior. We now show that when the derivative is chosen ex post by the firm (and in the interest of its shareholders), then the privileged treatment of derivatives invites inefficient strategic behavior.

If the firm's shareholders cannot commit to a derivative position, their ex post incentives to hedge are too low. To see this, assume that creditors expect that the firm will optimally hedge, so that $R = F$. Taking the face value of debt R as given, it is in the firm's ex post interest to eliminate credit risk by choosing a derivative position of $X = F - C_1^L$ whenever

$$(1-\theta)\, C_2 - (1-\theta)\left[F - C_1^L\right] - \delta\left[F - C_1^L - A\mathcal{P}\right] > 0. \tag{49}$$

The first term in (49) is the benefit to the firm from being able to continue in the low-cash flow state. The second term in (49) is the actuarially fair cost of the derivative. The third term captures the deadweight cost of hedging. Comparing this condition to (15) we see that, under no commitment, the firm's incentives to hedge are strictly lower than is socially optimal. This is an illustration of the well-known observation that equityholders have suboptimal risk-taking incentives once risky debt is in place.

Assuming that the firm can only take long positions in the derivative, hedging incentives are independent of the bankruptcy treatment of derivatives. If, however, the firm can take short positions in the derivative, an additional effect emerges and the bankruptcy treatment matters: If the derivative contract is senior, the firm is able to dilute the creditor by taking a short position in the derivative. By doing so, the firm transfers resources that would usually accrue to the creditor in the default state into the high-cash flow state, in which they accrue to equityholders. Hence, when the derivative is senior to debt, a derivative that could function as a perfect hedge may well be deployed as a vehicle for speculation or risk shifting.

More formally, assume that $(1-\theta) C_2 - \delta \left(F - C_1^L - A\mathcal{P}\right) > 0$, so that it is socially optimal for the firm to hedge. When derivatives have seniority, we now have to compare the firm's payoff from hedging not only to the payoff from taking no derivative position, but also to the payoff from taking a short position in the derivative. It is straightforward to verify that the firm always (weakly) prefers taking a short position in the derivative to taking no position at all. Therefore, the firm will hedge in equilibrium only if the payoffs from hedging exceed the payoffs from taking a short position. Comparing these payoffs, we see that hedging is now privately optimal if, and only if,

$$(1-\theta) C_2 - (1-\theta) \left[F - C_1^L\right] - \delta \left[F - C_1^L - A\mathcal{P}\right] - \frac{(1-\theta)^2 C_1^L + \delta A\mathcal{P}}{1-\theta+\delta} > 0. \tag{50}$$

The additional term in this condition relative to condition (49) establishes that hedging is harder to sustain when short positions in the derivative are allowed. In addition, in cases where it is optimal not to hedge at all, the firm always takes an inefficient short position in the derivative.

PROPOSITION 7: *When the derivative has no basis risk* ($\gamma = 1$) *and the firm cannot commit to a derivative position when entering the debt contract:*

1. *The firm's private incentives to hedge are strictly less than the social incentives to hedge.*
2. *When only long positions in the derivative are possible, the bankruptcy treatment of derivatives does not matter for efficiency.*
3. *When the firm can take short "speculative" positions in the derivative, the bankruptcy treatment of derivatives matters: Under senior derivatives, the firm may choose to take a speculative position in the derivative to dilute its creditors. This is strictly inefficient and restricts*

the subset of parameters for which the efficient hedging position can be sustained.

Proposition 7 illustrates in the simplest possible setting another first-order inefficiency of seniority for derivatives: Rather than using derivatives as hedging tools, firms may strategically engage in risk shifting by speculating with senior derivatives. This is not possible when derivatives are junior to debt.[34]

Basis risk: Consider now the general case with basis risk ($\gamma < 1$). If C_2 is large enough that the firm finds it optimal to hedge, it would never want to take a derivative position that is smaller than $R - C_1^L$. Under senior derivatives the firm may, however, have an incentive to take a derivative position that strictly exceeds $R - C_1^L$, which is inefficient given the deadweight cost of hedging. To see this, assume that derivatives are senior and consider the firm's objective function with respect to hedging after it has already committed to a debt repayment of R. When it is privately desirable for the firm to minimize default, the firm's optimal derivative position X^{S^*} maximizes the firm's payoff subject to the constraint that $X^{S^*} \geq R - C_1^L$:

$$\max_{X^{S^*} \geq R - C_1^L} \theta \left[C_1^H - R + \frac{1-\theta}{\theta} (1-\gamma) X^{S^*} - \left[1 - \frac{1-\theta}{\theta} (1-\gamma) \right] x \left(X^{S^*} \right) \right] \\ + (1-\theta)\gamma \left[C_1^L + X^{S^*} - R \right] + \left[\theta + (1-\theta)\gamma \right] C_2, \tag{51}$$

where the promised payment to the counterparty $x\left(X^{S^*}\right)$ is determined by the protection seller's break-even constraint.

To see why the firm may take an inefficiently large derivative position, it is instructive to look at its marginal payoff from increasing its derivative position beyond the optimal size $X^S = R^J - C_1^L$:

$$\underbrace{1-\theta}_{\text{marginal derivative payoff}} - \underbrace{\left[1 - \frac{1-\theta}{\theta}(1-\gamma) \right]}_{\leq 1} \underbrace{\left[1 - \theta + \frac{(1-\theta)(1-\Gamma)}{\Gamma - \mathcal{P}} \right]}_{\text{marginal cost of derivative}} \gtrless 0. \tag{52}$$

The first term is the extra derivative payoff to the firm from increasing its derivative position by one unit. It is equal to $(1-\theta)$ because an increase in the derivative's notional value generates an additional dollar for the firm with probability $(1-\theta)$. The second term is the share of the marginal cost of an additional unit of the derivative that is borne by the firm's shareholders. The full marginal cost of an additional unit in notional derivative exposure is given by its actuarially fair marginal cost $(1-\theta)$ plus the marginal increase in the hedging cost $\delta = \frac{(1-\theta)(1-\Gamma)}{\Gamma-\mathcal{P}}$. However, this cost is borne by the firm's shareholders only in states in which they have a positive residual claim. In the default state, the marginal cost of the derivative is paid by the creditor, when the derivative is senior to debt. Thus, the firm's shareholders do not internalize the full cost of increasing the firm's derivative position when derivatives are senior and therefore may have an incentive to take a derivative position that is inefficiently large.

From (52) we can deduce that the firm's shareholders' optimal derivative position coincides with the socially optimal derivative position only when the derivative has relatively little basis risk: $\gamma \geq \overline{\gamma}$. When the derivative has significant basis risk, $\gamma < \overline{\gamma}$, the firm will enter a derivative position that is too large from a social perspective. Given that hedging costs are linear, when the firm chooses to increase the derivative position beyond the efficient level, it will increase its derivative position to the point where it completely expropriates the creditor in the default state. It then chooses a position X^{S^*} such that $x\left(X^{S^*}\right) = C_1^L$. This is summarized in the following proposition.

PROPOSITION 8: ***Senior derivatives may lead to inefficiently large derivative positions.*** *Assume that it is privately optimal for the firm to hedge default risk. When the firm cannot commit to a derivative position ex ante, the firm's shareholders' optimal derivative position coincides with the optimal derivative position only if $\gamma \geq \overline{\gamma}$, where*

$$\overline{\gamma} = 1 - \frac{\delta\theta}{(1-\theta)(1-\theta+\delta)}. \tag{53}$$

When $\gamma < \overline{\gamma}$ the firm chooses a derivative position that is inefficiently large and sets X^{S^*} such that $x\left(X^{S^*}\right) = C_1^L$, or

$$X^{S^*}_{\gamma<\overline{\gamma}} = \frac{\theta}{1-\theta+\delta} C_1^L. \tag{54}$$

The incentive to take inefficiently large derivative positions disappears when derivatives are junior to debt. To see this, consider the firm's ex post objective with respect to hedging with junior derivatives. The firm's surplus is unchanged relative to (51), except that the promised payment to the derivative $x(X^{S^*})$ is now determined by (30):

$$x\left(X^{S^*}\right) = \frac{(1-\theta)\,X^{S^*} + \delta\left(X^{S^*} - A\mathcal{P}\right)}{\theta - (1-\theta)\,(1-\gamma)}. \tag{55}$$

Differentiating (51) and (55) with respect to X^{S^*} then reveals that, under junior derivatives, the firm has no incentive to take an excessively large derivative position. Indeed, the marginal payoff from increasing the derivative position beyond $R^S - C_1^L$ is now given by $-\frac{(1-\theta)(1-\Gamma)}{A(1-\mathcal{P})} < 0$. This is intuitive: Under junior derivatives, the firm bears the full marginal cost of an additional unit of derivative exposure. Since the derivative is priced at actuarially fair terms net of the deadweight hedging cost, the firm cannot gain from increasing its derivative exposure beyond $R^S - C_1^L$.

PROPOSITION 9: ***Under junior derivatives there is no incentive to take excessively large derivative positions.*** *When derivatives are junior, the firm chooses the efficient derivative position whenever it is privately optimal for the firm to hedge.*

One implication of our analysis in this section is that, under the current privileged bankruptcy treatment for derivatives, firms may take derivative positions that are excessively large from a social perspective. This is true even though derivatives are fundamentally value-enhancing in our model as risk management tools. As shown above, the firm's shareholders' incentives to enter excessively large derivative positions is tightly linked to the basis risk of the derivative contract available for hedging. When basis risk is sufficiently small, the firm has no incentives to take excessively large positions. But when basis risk is large, the firm's shareholders have an incentive to take excessively large derivative positions, thereby diluting existing creditors. The derivative then becomes a vehicle for speculation rather than a hedging tool.

While this analysis suggests that the senior status of derivatives is an incentive for shareholders to speculate at the expense of creditors, there can also be situations where seniority of derivatives provides a crucial incentive for shareholders to hedge for the benefit of the firm as a whole. To illustrate this possibility we now consider situations where the firm's shareholders may have no incentive to hedge ex post. As is well known, once debt is in place the benefit to shareholders from hedging can be smaller than the total gain to the firm, as the firm's creditors also stand to gain from the hedge (see, for example, Smith and Stulz (1985)). This observation suggests that ex post hedging incentives for shareholders could be inefficiently low, or equivalently that shareholders may want to unwind existing hedges once debt is in place (due to risk-shifting considerations à la Jensen and Meckling (1976)).[35]

In this case, there could thus be advantages to having derivatives senior to debt, as then the costs as well as the benefits of hedging will be shared between debtholders and equityholders. In our setting, this is the case when C_2 is relatively low. Specifically, it can be shown that when derivatives are junior to debt it is optimal for the firm's shareholders to hedge if $C_2 > \bar{C}_2$, and when derivatives are senior to debt it is optimal for shareholders to hedge when $C_2 > \tilde{C}_2$. Depending on parameter values, it is possible that $\tilde{C}_2 < \bar{C}_2$, so that a region exists where the firm chooses to hedge ex post only when derivatives are senior to debt.[36]

B. Default Due to Derivative Losses and Inefficient Collateral Calls

Up to now we have assumed that the required debt and derivative payments are such that the firm meets all its obligations when it receives the high cash flow C_1^H and is required to make a payment on the derivative. While this helped simplify our analysis, this assumption is not without loss of generality. The reason is that the firm can make the required payment $R + x$ only if it has sufficient resources to do so. Moreover, even if there are sufficient resources in the firm, the firm may have an incentive to default strategically in states where payment is due both on debt and the derivative (recall that up to now we have assumed that C_2 is large enough that this incentive constraint is satisfied).

We now show that default due to derivative losses in the high state is more likely, the higher the level of collateralization (and thus effective seniority) of derivative contracts. Moreover, we show that this problem is exacerbated when the firm has no way of invoking a stay on inefficient collateral calls by the derivative counterparty.

B.1. Default Due to Derivative Losses

Suppose that the firm can choose to partially collateralize derivatives up to an amount $\overline{x} \leq x$. Recall that, in this case, only the amount $\overline{x}$ can be seized by the counterparty in the event of default. The remainder of the counterparty's claim against the firm $x - \overline{x}$ is treated as a regular debt claim in bankruptcy. For simplicity we assume that this remainder is junior to the debtholder's claim.

The reason that default in the high state is more likely when the level of collateralization $\overline{x}$ of the derivative is higher is that a higher level of collateralization of the derivative contract leads to a larger required *total* payment on debt and derivatives, $R(\overline{x}) + x(\overline{x})$, in states where the derivative moves against the firm. Intuitively, while more collateralization decreases the stand-alone cost of the derivative $x(\overline{x})$, this effect is more than outweighed by a concomitant increase in the face value of debt $R(\overline{x})$, such that the total required payment increases. However, a higher total required payment makes it more difficult to satisfy the constraint that the firm not default (either due to lack of resources or strategically) when it receives the high cash flow and the derivative moves in favor of the counterparty.

In the high-cash flow state, the firm defaults due to lack of resources when $R(\overline{x}) + x(\overline{x}) > C_1^H$. The firm defaults strategically whenever defaulting and pocketing $C_1^H - C_1^L$ yields strictly more than making the contractual payment $R(\overline{x}) + x(\overline{x})$ and collecting continuation value C_2. Hence, strategic default occurs when

$$C_1^H - C_1^L > C_1^H - [R(\overline{x}) + x(\overline{x})] + C_2, \tag{56}$$

which is the case when $R(\overline{x}) + x(\overline{x}) > C_1^L + C_2$.[37]

PROPOSITION 10: ***Default due to losses on the derivative position.*** *The firm meets its payment obligations when it receives the high cash flow but the derivative moves against the firm as long as*

$$R(\overline{x}) + x(\overline{x}) \leq \min\left[C_1^H, C_1^L + C_2\right]. \tag{57}$$

Condition (57) is harder to satisfy the higher the level of collateralization for derivatives $\overline{x}$:

$$\frac{\partial R(\overline{x})}{\partial \overline{x}} + \frac{\partial x(\overline{x})}{\partial \overline{x}} = \frac{\delta(1-\gamma)(1-\theta)}{[\theta - (1-\gamma)(1-\theta)][\theta + \gamma(1-\theta)]} > 0. \tag{58}$$

Proposition 10 establishes that the no-default constraint in the high-cash flow state (either due to lack of resources when $R(\overline{x}) + x(\overline{x}) \leq C_1^H$ or to strategic default when $R(\overline{x}) + x(\overline{x}) \leq C_1^L + C_2$) is harder to satisfy when the derivative is more highly collateralized and thereby more senior. In other words, the critical value of the setup cost F for which the firm is able to hedge without defaulting in the high-cash flow state is lower the higher is $\overline{x}$.

COROLLARY 3: *Derivatives can be used to hedge the low-cash flow state without causing default in the high-cash flow state as long as*

$$F \leq F(\overline{x}) = K_0 C_1^L + K_1 \min\left[C_1^H, C_1^L + C_2\right] + K_2 A\mathcal{P} - K_3 \overline{x}, \tag{59}$$

where K_0 ,K_1, and K_2 are positive constants, and

$$K_3 = \frac{(1-\gamma)(1-\theta)\delta}{\theta + \gamma(1-\theta) + \delta}. \tag{60}$$

Since $K_3 \geq 0$, $F(\overline{x})$ is decreasing in the level of collateralization $\overline{x}$.

B.2. Inefficient Collateral Calls

Finally, we extend the model to show how the exemption of derivatives from the automatic stay under Chapter 11 can lead to inefficient collateral calls by the derivative counterparty. To model collateral calls, we introduce into the model a working capital demand for the firm, which can also play the role of unassigned cash collateral. Specifically, suppose that the firm requires working capital or cash to be able to generate the date 2 cash flow C_2. Let $y = D - F$ be the amount of working capital in the firm, where D is the amount of funding the firm raises at date 0, and F is the amount it spends on fixed investment. Suppose also for simplicity that the firm can generate the second-period cash flow only if there is sufficient working capital in the firm:

$$C_2(y) = \begin{cases} V & \text{if} \quad y \geq \kappa \\ 0 & \text{otherwise} \end{cases}, \tag{61}$$

where $V > 0$ and $\kappa > 0$. Moreover, the working capital used to generate C_2 is spent by the firm before the realization of the date 1 cash flow, so that it is no longer available to make payments to the creditor or derivative counterparty at that point.

Consider briefly the outcome absent derivatives: If V is sufficiently large, it is optimal for the firm to hold sufficient working capital; that is, it is optimal to raise $D = F + \kappa$ at date 0 and to hold working capital $y = \kappa$. The payoff to the firm absent derivatives is then given by

$$\theta \left(C_1^H - R + V\right) \tag{62}$$

where R is determined by the following break-even condition:

$$\theta R + (1-\theta)\, C_1^L = F + \kappa. \tag{63}$$

Now consider the outcome in the presence of derivatives when the firm does not have the protection of a stay on collateral calls by the derivative counterparty. As we will show, this may then give rise to inefficient collateral calls on the firm. In particular, if the derivative moves against the firm, the counterparty to the derivative transaction may find it privately optimal to make a collateral call on the firm's cash in order to ensure full payment on the derivative, even if this reduces overall surplus because it drives the firm into default.

More formally, consider the following time line:

1. The firm writes a debt contract with the lender and borrows an amount $D = F + \kappa$.
2. The firm enters a derivative contract (x, X) with the counterparty. This contract involves basis risk γ.
3. The counterparty observes the realization of Z before the realization of the first-period cash flow; if $Z = Z^H$, the counterparty can initiate a procedure to collect x. If there is no stay, the counterparty can immediately make a collateral call on the cash available to the firm κ (and subsequently when C_1 realizes). In that case, the firm would be deprived of its working capital, with the consequence that $C_2 = 0$.
4. If the firm has working capital available, it spends it *before* the realization of C_1 and then receives $C_2 = V$ at date 2, provided that the firm is not liquidated before then.
5. First-period cash flow C_1 is realized. When $C_1 = C_1^H$ and payment is due on the derivative, the firm chooses whether or not to make the contractual payment $R + x$; when $C_1 = C_1^H$ and the firm receives a payment X from the derivative, or when $C_1 = C_1^L$ and the firm receives a payment X from the derivative, the firm chooses whether to repay R. When $C_1 = C_1^L$ and the firm must make a payment x on the derivative, the firm defaults.

6. If the firm continues to the second period and is able to use its working capital, it obtains V.

Given this timeline, the firm is exposed to inefficient collateral calls (effectively a run on working capital) if the firm cannot invoke an automatic stay against collateral calls from a derivative counterparty. To see this, suppose that the firm borrows $F + \kappa$ and takes out a derivative promising to pay $X = R - C_1^L$ when $Z = Z^L$, against a payment $x = [(1 - \theta + \delta) X]/\theta$ when $Z = Z^H$ (assuming that the derivative is senior to debt and that $x \leq C_1^L + \kappa$).

In this case, it is a best response for the derivative counterparty to make a collateral call immediately on the realization of Z^H. If it makes such a collateral call, the firm ends up with insufficient working capital, so that $C_2 = 0$. Because of the collateral call, the firm also chooses to strategically default when C_1^H is realized, because when $C_2 = 0$ running away with $C_1^H - C_1^L$ is strictly more profitable than making the required payment $R + x$ on the debt and the derivative: $C_1^H - C_1^L > C_1^H - R - x$. The derivative counterparty, however, is still able to fully recover its claim because $x \leq C_1^L + \kappa$. Note that, even though it is the collateral call that pushes the firm into default, it is privately optimal for the derivative counterparty to ask for collateral. Should it not make that collateral call, the derivative counterparty would lose access to κ and, in the case that C_1^L realizes, can only hope to receive a maximum amount C_1^L at date 1. Thus, making an immediate collateral call is strictly optimal for the counterparty if $C_1^L + \kappa \geq x > C_1^L$.

The collateral call by the counterparty is inefficient because it invariably leads to a loss of C_2 (and strategic default) in a state where, absent the collateral call, the firm would continue. Moreover, the collateral call is not needed for the derivative counterparty to break even. When V is sufficiently large, the firm's incentive to continue can support a high enough payment by the firm such that both the creditor and derivative counterparty break even in expectation. A stay on collateral calls by the counterparty would prevent this outcome.

VIII. Conclusion

This paper provides a tractable and transparent model to analyze the ex ante and ex post consequences of granting (effective) seniority to derivative contracts. The special treatment of derivatives in bankruptcy has been carved out with the main objective of providing stability to derivative markets. Over the years, exemptions for derivatives, swaps, and repo markets have been gradually extended, with the same stability objective in mind, largely as a result of a concerted push by ISDA to codify its "master agreements" (see Morgan (2008)). Remarkably, however, up to now there has been essentially no systematic analysis of the likely ex post and ex ante consequences of this special bankruptcy treatment. With the exception of a few law articles (most notably Edwards and Morrison (2005), who point to the potential destabilizing effect of the bankruptcy exemptions in the failures of LTCM and Enron), the general presumption was that the effect of the privileged bankruptcy treatment for derivatives was to strengthen derivative markets, to enhance financial stability, and that the effects on firms' cost of debt would be negligible. In contrast, our analysis suggests that, once ex post and ex ante effects of the bankruptcy treatment of derivatives are taken into account, the overall benefits of the special bankruptcy treatment for derivatives are no longer obvious.

Finally, some of the insights of our analysis may have policy relevance beyond the particular setting discussed in this paper. First, carefully taking into account ex ante consequences is likely similarly important with respect to the *Orderly Liquidation Authority* (OLA) for systemically important financial institutions, which was created as part of the Dodd-Frank Act of 2010. Under OLA, all *Qualified Financial Contracts* (QFCs), which include swaps, repos, and other derivatives, are transferred into a solvent bridge bank, such that counterparties are fully protected and therefore "prohibited from terminating their contracts and liquidating and netting out their positions" (see p. 9 in FDIC (2011)). While this responds to the potential ex post inefficiencies that can result from the exemption of the automatic stay under Chapter 11 bankruptcy (e.g., large-scale collateral liquidations), the treatment of QFCs under OLA may exacerbate ex ante distortions. For example, this solution may incentivize financial institutions to increasingly rely on QFCs as a source of funding and thereby substitute away from subordinated debt, which will be at risk of substantial haircuts under OLA.

Second, our analysis relates to the current debate on moving derivative contracts to clearinghouses. Similar to the transfer of risk to unsecured creditors highlighted in this paper, moving derivatives to clearinghouses reduces credit risk for those parties that are part of the clearinghouse, but *increases* credit risk for those that remain outside of the clearinghouse (see also Roe (2011a)). These repercussions should be taken into account when designing optimal clearinghouse arrangements and determining which contracts and market participants should be included in the clearinghouse.

IX. Appendix

Proof of Lemma 1: The steps needed to calculate the cost of the derivative as a function of the level of collateralization $\overline{x}$ are given below in the section characterizing the equilibrium under partial collateralization. As shown there, holding R fixed and assuming that $\overline{x} \leq C_1^L$, the counterparty's break-even condition implies that

$$x\left(\overline{x}\right) = \frac{\left(1-\theta\right)\left[R - C_1^L + \delta\left(R - C_1^L - A\mathcal{P}\right)\right] - \left(1-\theta\right)\left(1-\gamma\right)\overline{x}}{\theta - \left(1-\theta\right)\left(1-\gamma\right)}. \tag{64}$$

With R held fixed, this implies

$$\frac{\partial x\left(\overline{x}\right)}{\partial x} = -\frac{\left(1-\theta\right)\left(1-\gamma\right)}{\theta - \left(1-\theta\right)\left(1-\gamma\right)} < 0. \tag{65}$$

Hence, taking the face value of debt as given, the cost of the derivative is decreasing in the level of collateralization of the derivative as long as $\overline{x} \leq C_1^L$. When $\overline{x} > C_1^L$, a further increase in collateralization does not change the payoff of the derivative counterparty, such that in this region the cost of the derivative is unchanged.

Senior derivatives when $x^S > C_1^L$: Here we describe the equilibrium under senior derivatives when $x^S > C_1^L$, which we left out in the main text for space considerations. The main difference to the case discussed in the text is that the break-even conditions for the derivative counterparty and the creditor change. In particular, when $x^S > C_1^L$ the derivative counterparty receives the entire cash flow when the firm defaults. The break-even conditions for the creditor and the counterparty become

$$\begin{aligned} \left[\theta + \gamma\left(1-\theta\right)\right] R^J &= F & (66) \\ \left[\theta - \left(1-\theta\right)\left(1-\gamma\right)\right] x^S + \left(1-\gamma\right)\left(1-\theta\right) C_1^L &= \left(1-\theta\right)\left[X^S + \left(1-\Gamma\right)\zeta^S\right]. & (67) \end{aligned}$$

Inserting $X^S = R^J - C_1^L$ and solving (66) and (67) for R^J and x^S yields

$$R^J = \frac{F}{\theta + \gamma(1-\theta)} \tag{68}$$

$$x^S = \frac{(1-\theta+\delta)\left(F - [\theta+\gamma(1-\theta)]\, C_1^L\right) - \delta\,[\theta+\gamma(1-\theta)]\, A\mathcal{P}}{\theta\,[\theta+\gamma(1-\theta)]}. \tag{69}$$

Characterization of equilibrium under partial collateralization: This section contains the break-even conditions used to derive the equilibrium under partial collateralization. Under partial collateralization, the derivative counterparty is senior up to an amount $\overline{x}$. The remainder, $x - \overline{x}$, is junior to the creditor's claim. The required derivative position is given by

$$X(\overline{x}) = R(\overline{x}) - C_1^L. \tag{70}$$

The creditor's and counterparty's break-even conditions are given by

$$[\theta + (1-\theta)\gamma]\, R(\overline{x}) + (1-\theta)(1-\gamma)\left(C_1^L - \overline{x}\right) = F \tag{71}$$

$$[\theta - (1-\theta)(1-\gamma)]\, x(\overline{x}) + (1-\theta)(1-\gamma)\,\overline{x} = (1-\theta)\left[R(\overline{x}) - C_1^L + (1-\Gamma)\,\zeta(\overline{x})\right], \tag{72}$$

where

$$\zeta(\overline{x}) = \frac{R(\overline{x}) - C_1^L - A\mathcal{P}}{\Gamma - \mathcal{P}}. \tag{73}$$

Solving (71) and (72) for $R(\overline{x})$ and $x(\overline{x})$, we obtain

$$R(\overline{x}) = \frac{F - (1-\theta)(1-\gamma)\left(C_1^L - \overline{x}\right)}{\theta + (1-\theta)\gamma} \tag{74}$$

$$x(\overline{x}) = \frac{(1-\theta+\delta)\left[F - C_1^L\right] - \delta\,[\theta+\gamma(1-\theta)]\, A\mathcal{P} - (1-\theta)(1-\gamma)\,[\theta - (1-\gamma)(1-\theta) - \delta]\,\overline{x}}{[\theta - (1-\gamma)(1-\theta)]\,[\theta+\gamma(1-\theta)]}. \tag{75}$$

Proof of Proposition 6: Following substitution of X^J, and $\gamma X^S + (1-\gamma)x^S$ for their equilibrium values, we can rewrite $\gamma X^J \leq \gamma X^S - (1-\gamma)\, x^S$ as

$$\frac{\theta(1-\gamma)\left[(1-\theta+\gamma\delta)\left(F - C_1^L\right) - \delta\left(\theta + (1-\theta)\gamma\right) A\mathcal{P}\right]}{[\theta + (1-\theta)\gamma]\,[\theta - (1-\theta)(1-\gamma) + \delta\theta(1-\gamma)]} \geq 0. \tag{76}$$

The denominator of this expression is positive. To see this, note that

$$\theta - (1-\theta)(1-\gamma) = \Pr\left[C_1 = C_1^H,\ Z = Z^L\right] \geq 0.$$

Hence, (76) is positive if

$$\begin{aligned} (1-\theta+\gamma\delta)\left(F - C_1^L\right) &\geq \delta\left(\theta + (1-\theta)\gamma\right) A\mathcal{P} && (77) \\ \frac{F - C_1^L}{\theta + (1-\theta)\gamma} &\geq \frac{\delta}{1-\theta+\gamma\delta} A\mathcal{P}. && (78) \end{aligned}$$

From the creditor's break-even condition under junior derivatives,

$$\theta + (1-\theta)\gamma R^S + (1-\theta)(1-\gamma) C_1^L = F, \tag{79}$$

we know that $R^S - C_1^L = \frac{F - C_1^L}{\theta + (1-\theta)\gamma}$. Substituting this into (78) and multiplying by γ, we obtain

$$\gamma\left(R^S - C_1^L\right) \geq \underbrace{\frac{\gamma\delta}{1-\theta+\gamma\delta}}_{<1} A\mathcal{P}. \tag{80}$$

As long as there are deadweight costs of hedging under junior derivatives (i.e., $\gamma\left(R^S - C_1^L\right) > A\mathcal{P}$), condition (80) is always satisfied. When $\gamma\left(R^S - C_1^L\right) \leq A\mathcal{P}$, no deadweight costs are incurred in either regime, and thus the bankruptcy ordering is irrelevant.

Proof of Proposition 7: The first two statements in the proposition follow directly from the discussion in the text. To derive equation (50), we need to compare the payoff to the firm from hedging, which is given by the NPV minus the deadweight cost of hedging,

$$\theta C_1^H + (1-\theta) C_1^L + C_2 - F - \delta(F - C_1^L - A\mathcal{P}), \tag{81}$$

to the payoff from entering a speculative short derivative position. (The deviation to a speculative short position is always more profitable for the firm than a deviation to taking no derivative position at all.) If, ex ante, creditors expect the firm to hedge and thus set $R = F$, the payoff to the speculative short derivative position is given by

$$\theta\left(C_1^H + X_{short} - F\right) + \theta C_2, \tag{82}$$

where X_{short} is the derivative position that fully expropriates the creditor in the default state (i.e., $x_{short} = C_1^L$). X_{short} can be determined from the counterparty's break-even condition

$$(1-\theta)\, C_1^L = \theta X_{short} + \frac{\theta}{1-\theta}\delta\left(X_{short} - A\mathcal{P}\right). \tag{83}$$

The firm chooses to hedge when (81) exceeds (82), which leads to equation (50).

Proof of Proposition 10: Assume that the firm receives the high cash flow C_1^H but has to make a payment $x\left(\overline{x}\right)$ on its derivative position. The firm will meet its total payment obligation $R\left(\overline{x}\right) + x\left(\overline{x}\right)$ under two conditions. First, the cash available to the firm must be sufficient, which is the case whenever

$$C_1^H - \left[R\left(\overline{x}\right) + x\left(\overline{x}\right)\right] \geq 0. \tag{84}$$

Second, the firm must have no incentive to default strategically. This is the case whenever

$$C_1^H - \left[R\left(\overline{x}\right) + x\left(\overline{x}\right)\right] + C_2 \geq C_1^H - C_1^L. \tag{85}$$

The left-hand side is the payoff from making the contractual payment and continuing, whereas the right-hand side is the payoff from declaring default, pocketing $C_1^H - C_1^L$, and letting the creditor and the derivative counterparty split C_1^H. Overall, the firm will thus meet its contractual obligations if

$$R\left(\overline{x}\right) + x\left(\overline{x}\right) \leq \min\left[C_1^H, C_1^L + C_2\right]. \tag{86}$$

Equation (58) follows from taking the derivatives of equations (74) and (75) and simplifying.

Proof of Corollary 3: The result follows from substituting (74) and (75) into (57) and simplifying. The constants not given in the main text are

$$K_0 = \frac{(1-\theta)(1-\gamma)[\theta-(1-\gamma)(1-\theta)]+1-\theta+\delta}{\theta+\gamma(1-\theta)+\delta}, \tag{87}$$

$$K_1 = \frac{[\theta-(1-\gamma)(1-\theta)][\theta+\gamma(1-\theta)]}{\theta+\gamma(1-\theta)+\delta}, \tag{88}$$

and

$$K_2 = \frac{\delta[\theta+\gamma(1-\theta)]}{\theta+\gamma(1-\theta)+\delta}. \tag{89}$$

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Notes

[1]Similarly, under FDIC receivership there is essentially no stay on derivative contracts. If not transferred to a new counterparty by 5 pm EST on the business day after the FDIC has been appointed receiver, derivative, swap, and repo counterparties can close out their positions and take possession of collateral. See, for example, Summe (2010, p. 66).

[2]For example, under current bank resolution law in the U.K. and Germany, closeout and netting provisions may not always be enforceable (see Hellwig (2011)).

[3]See, in particular, Grossman and Hart (1986), Bolton and Scharfstein (1990, 1996), Aghion and Bolton (1992), and Hart and Moore (1994, 1998).

[4]This result mirrors classic findings in the literature on corporate risk management, such as Smith and Stulz (1985) and Froot, Scharfstein, and Stein (1993).

[5]Under some parameter values (e.g., when the firm's continuation value is relatively low), the ability to dilute ex post is *necessary* to sustain a value-enhancing hedge: When the firm's continua-

tion value is low, the beneficiaries from a hedge are disproportionately the firm's debtholders. But when debt has seniority over derivatives, the costs of the hedge are mostly borne by shareholders. In such a situation, reversing the priority order so that derivatives are senior to debt can provide an efficient incentive for shareholders to hedge.

[6]Another related distortion produced by the privileged treatment of derivatives in bankruptcy is that firms have an incentive to *masquerade* debt as derivatives in order to protect creditors against dilution by derivatives. If debt can easily be dressed up as a swap and thereby obtains the same treatment as derivatives in bankruptcy, the overall effect of the exemption for derivatives is to hollow out the stability provided by the automatic stay. Although we do not explicitly model this distortion, it is likely to be another important unintended consequence of the special bankruptcy treatment of derivatives.

[7]See, in particular, Edwards and Morrison (2005), Bliss and Kaufman (2006), Lubben (2009), Roe (2011b), Skeel and Jackson (2012), and Duffie and Skeel (2012).

[8]Recent papers by Antinolfi, Carapella, Kahn, Mills, and Nosal (2012), Acharya, Anshuman, and Viswanathan (2012), and Auh and Sundaresan (2013) also offer an ex ante and ex post analysis of exemptions from the automatic stay, but with a specific focus on repo contracts. Oehmke (2014) provides a model of collateral fire sales that can occur after defaults in the repo markets. Infante (2013) explores the ex ante implications of collateral fire sales.

[9]In practice, there are examples of state-contingent contracts that bundle financing and risk management, for example, when hedging services are provided by the original lender. One advantage of the bundled contract is that the party providing the bundle internalizes transfers between the financing and hedging portions of the bundle. Although some banks bundle bank loans with derivative contracts, such contracting practice is the exception rather than the norm (see Cooper and Mello (1999)).

[10]The discussion in this section is kept intentionally brief and draws mainly on Roe (2011b). For more detail on the legal treatment of derivatives, see also Edwards and Morrison (2005) and Bliss and Kaufman (2006).

[11]As pointed out by Roe (2011b, p. 5), "The Code sets forth priorities in §§ 507 and 726, and those basic priorities are unaffected by derivative status."

[12]If after selling all the posted collateral a derivative counterparty still has a claim on the firm,

this remaining claim becomes a regular unsecured claim in Chapter 11. Hence, collateralization is key to the effective seniority of derivative contracts.

[13]The advantages from netting are best illustrated through a simple example. Suppose that a firm has two counterparties, A and B. The firm owes $10 to A. The firm owes $10 to B, and, in another transaction, B owes $5 to the firm. Suppose that when the firm declares bankruptcy there are $10 of assets in the firm. When creditor B cannot net its claims, it has to pay $5 into the firm. The bankruptcy mass is thus $15. A and B have remaining claims of $10 each, such that they equally divide the bankruptcy mass and each receive $7.5. The net payoff to creditor B is $7.5-$5 = $2.5. When creditor B can net its claim, it does not need to make a payment to the firm at the time of default. Rather, it now has a net claim of $5 on the bankrupt firm. As before, A has a claim of $10 on the firm. There is now $10 to distribute, such that A receives 2/3*$10 = $6.66 and creditor B receives 1/3*$10 = $3.33. Hence, with netting, B receives a net payoff of $3.33, while without netting it receives only $2.5.

[14]If the firm is a bank, then the above assumptions mean that beyond the minimum equity capital requirement, which we normalize to zero, the bank must raise the entire amount needed for the loan in the form of deposits. In what follows, when we interpret the firm as a bank, the *creditor* is a bank *depositor*, and R denotes the gross interest payment on deposits of size F.

[15]Verifiability of the realization of Z and the payment of the amount due under the derivative contract means that, in contrast to cash flows generated by the firm's operations, returns from derivative positions can be contracted on without commitment or enforceability problems.

[16]The derivative thus has payoffs that are equivalent to a swap contract, one of the most common derivatives used for hedging purposes in practice: It has value zero when entered and then moves in favor of the firm or the counterparty, depending on the realization of Z.

[17]Note that the binary cash flow setup of our paper means that there is no intensive margin for the hedging decision. Any derivative of size $X \geq R - C_1^L$ prevents default in the low-cash flow state with probability γ, while a derivative position of size $X < R - C_1^L$ is not an effective hedge. If the number of possible cash flow realizations is larger than two, such adjustment along the intensive margin is possible, but would not affect our main findings. We discuss this extension in the Internet Appendix available at http://www.afajof.org/details/page/3626901/Supplements.html.

[18]We have chosen the unconditional payoff probability of the derivative to coincide with the

probability that the low cash flow obtains (both are equal to $1-\theta$). This assumption is not necessary for the analysis. However, it has the convenient feature that when $\gamma = 1$, the derivative is a perfect hedge: It pays off if, and only if, the firm's cash flow is low.

[19]If the counterparty were not to post collateral, it would choose action $a = 0$ when it observes signal s^L and the incentive constraint is violated. This would result in a loss for the firm, which would only receive the promised payment X with probability $p < 1$. For simplicity, we assume that the derivative counterparty has to make its promises credible by posting collateral. Biais, Heider, and Hoerova (2012) also treat the case of endogenous counterparty risk. In our analysis, the main adjustment from allowing for this case would be that deadweight costs could take the form of either costly collateral or costly endogenous counterparty risk.

[20]When $X \leq A\mathcal{P}$, no collateral needs to be posted, such that no deadweight is incurred. Strictly speaking, the expression for the deadweight costs should thus be $\delta\,(X - A\mathcal{P})^+$. For notational simplicity, we suppress this detail in the remainder of the paper.

[21]While our discussion above highlights frictions in the derivative market, note that our model treats debt and derivative markets symmetrically: Imposing on the firm's creditor the same friction that we impose on the derivative counterparty would lead to no change in the model. The firm's creditor never has a net liability to the firm after entering the debt contract and thus never has to post collateral to preserve incentives.

[22]The cash the firm assigns as collateral to the derivatives margin account is obtained either from retained earnings or from the initial investment by the creditor. Retained earnings can be modeled by assuming that, after the firm sinks the setup cost F at date 0, the project first yields a sure return C_1^L at date 1^-. At that point it is still unknown whether the full period 1 return will be C_1^H or C_1^L; that is, the firm knows only that it will receive an incremental cash flow at date 1 of $\Delta C_1 = C_1^H - C_1^L$ with probability θ, and 0 with probability $(1-\theta)$. To hedge the risk with respect to this incremental cash flow, the firm can then take a derivative position by pledging cash collateral $x \leq C_1^L$. Alternatively, the cash collateral x can be obtained from the creditor at date 0 by raising a total amount $F + x$ from the creditor. Either way of modeling cash collateral works in our setup.

[23]In practice, such a claim could be classified in the same priority class as debt. We do not explicitly consider this case, since the pro rata allocation of assets to derivative counterparties and

debtholders that arises in this case considerably complicates the formal analysis, without yielding any substantive additional economic insights.

[24]The case where $x > C_1^L$ can be treated in an analogous way, but is omitted for brevity.

[25]For example, the face value of a junior debt contract is R^J, whereas the pricing terms and collateral requirement of a senior derivative contract are given by X^S, x^S, and ζ^S.

[26]We follow Biais, Heider, and Hoerova (2012) here by assuming that collateral must not be posted ex ante, and that the contract specifies a collateral requirement only in the event that the derivative contract moves against the counterparty: that is, only when signal s^L is observed. There would be no qualitative change to our analysis if we imposed the collateral requirement up front.

[27]For a more formal justification of this assumption, assume there is a continuum of Z-variables that may potentially be used to hedge the firm's business risk, but that at the ex ante contracting stage it is not yet known which of these potential Z-variables will be the relevant one from a risk management perspective. However, once the firm is in operation and learns more about its business environment, it can determine the relevant variable Z. This lack of knowledge on the relevant random variable Z ex ante would effectively prevent the firm from contracting on a particular derivative position or from making the debt contract contingent on the relevant Z-variable. Hence, it is more plausible that the firm chooses its derivative position after signing the initial debt contract. This assumption broadly reflects current market practice: Firms usually choose their derivative exposure for a given amount of debt only ex post. Moreover, in practice relatively few bonds or loans include direct restrictions on future derivative positions taken by the debtor. Nonetheless, we briefly discuss the optimal Z-contingent contract in footnote 34.

[28]To the extent that the incentive to collateralize ex post is undesirable, an important question is whether the firm can commit ex ante not to collateralize its derivative position ex post, for example via covenants that restrict such collateralization. Under current U.S. bankruptcy law this is difficult: If a breach of such a covenant is discovered in bankruptcy, the collateral has already left the firm and generally cannot be recovered by lenders (see Bjerre (1999)). Hence, such covenants would require significant monitoring.

[29]When $x^S > C_1^L$, the counterparty receives the entire cash flow C_1^L in the event that $Z = Z^H$ and $C_1 = C_1^L$. In the interest of brevity we focus on the first case, $x^S \leq C_1^L$, in the main text. The second case is covered in the appendix.

[30] In other words, we assume that the counterparty's obligations to one firm can be cross-netted with payments it receives from other firms. Indeed, in practice the amount of required collateral for OTC derivatives is a function of the soundness of the counterparty's balance sheet, which in turn depends on the cross-netting benefits the counterparty can rely on across its diverse contracts with different firms.

[31] The details for this case are available in the Internet Appendix available at http://www.afajof.org/details/page/3626901/Supplements.html.

[32] The details for this case are available in the Internet Appendix available at http://www.afajof.org/details/page/3626901/Supplements.html.

[33] This insight is related to Duffie and Zhu (2011), who show that clearing through a central clearing counterparty (CCP) is efficient if the benefits from multilateral netting across counterparties that arise under a CCP arrangement exceed the cross-product netting benefits that are lost by clearing trades on a CCP.

[34] This simple setting without basis risk is also useful to illustrate the situation where the firm and creditor are able to write a state-contingent contract based on the realization of Z at date 0. If the firm can commit not to take on additional derivative positions ex post, such a state-contingent contract (which can be viewed as a contract that bundles financing and hedging) makes the stand-alone derivative redundant: The optimal state-contingent contract would set $R\left(Z^L\right) = C_1^L$ to eliminate default in the low-cash flow state and $R\left(Z^H\right) = \left[F - (1-\theta)\, C_1^L\right]/\theta$ to guarantee that creditors break even. Since default would never occur, the priority ranking of derivatives relative to debt would be irrelevant. If, on the other hand, the firm cannot commit not to take further derivative positions, the priority ranking of the derivative relative to debt still matters. The reason is that, under no commitment, senior derivatives allow the firm to ex post undo the state-contingent contract agreed to at date 0, leading to the same inefficiency as in Proposition 7.

[35] This point is related to the beneficial role that ex post dilution can have in mitigating debt overhang (see, e.g., Stulz and Johnson (1985) and Diamond (1993b)).

[36] Note, however, that this situation arises in a region where hedging is less valuable in the first place, because C_2 is relatively small: $C_2 \in (\tilde{C}_2, \bar{C}_2)$.

[37] The strategic default condition (56) assumes that the firm can extract no surplus through renegotiation after a strategic default. This is the case if the creditor can commit not to renegotiate

with the debtor and always liquidates the firm after a strategic default. If, on the other hand, the creditor cannot commit not to renegotiate, then the firm can usually extract positive surplus $S > 0$ in renegotiation. In this case, the firm defaults strategically if $R(\overline{x}) + x(\overline{x}) > C_1^L + C_2 - S$. The assumption that the lender can commit not to renegotiate is not crucial for our analysis. A sketch of the analysis with renegotiation after a strategic default is provided in Appendix B of the NBER working paper version of this paper (NPER WP 17599).

Internet Appendix to
"Should Derivatives Be Privileged in Bankruptcy?"*

A. *Omitted Derivations of Results in Section 6*

This section contains a detailed discussion of the results presented in Sections 6.1 and 6.2 of the paper: the priority ranking in the presence of cross-netting when (i) basis risk and cash flow risk are systematic and (ii) basis risk and cash flow risk are idiosyncratic.

Basis Risk and Cash Flow Risk Are Systematic: Consider first the case in which both cash flow risk and basis risk are systematic. In this case, when the representative counterparty observes the signal s^H, it learns that all derivatives it has written will move against it, such that a significant liability is added to its balance sheet. To preserve the counterparty's incentives, it now must post collateral in a margin account, which leads to deadweight costs.

When derivatives are senior, the counterparty incurs an aggregate liability of X^S if all derivatives simultaneously move against it ($Z = Z^H$). In this case, which occurs with probability $1 - \theta$, the counterparty has to post an amount

$$\zeta^S = \frac{X^S - A\mathcal{P}}{\Gamma - \mathcal{P}} \tag{IA.1}$$

as collateral. When the derivative moves in favor of the counterparty, no collateral has to be posted. As a result, the ex ante zero-profit condition for the counterparty is given by

$$\theta x^S - (1-\theta) X^S - (1-\theta)(1-\Gamma)\zeta^S = 0, \tag{IA.2}$$

where the term $(1-\theta)(1-\Gamma)\zeta^S$ reflects the expected deadweight cost of collateral that the counterparty is required to post when derivatives are senior.

Now consider the junior derivatives. When $Z = Z^H$, the counterparty receives the payment x^J only if all firms obtain the high cash flow C_1^H (i.e., payment to the counterparty depends on the realization of the aggregate basis risk). Hence, the counterparty receives x^J only with probability

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$[\theta - (1-\theta)(1-\gamma)]$. When $Z = Z^L$, the counterparty incurs an aggregate liability of X^J and it must therefore post an amount

$$\zeta^J = \frac{X^J - A\mathcal{P}}{\Gamma - \mathcal{P}} \tag{IA.3}$$

of collateral. The ex ante zero-profit condition for the counterparty is then given by

$$[\theta - (1-\theta)(1-\gamma)]\, x^J - (1-\theta)\, X^J - (1-\theta)(\Gamma - 1)\, \zeta^J = 0. \tag{IA.4}$$

Again, the term $(1-\theta)(\Gamma-1)\,\zeta^S$ reflects the expected deadweight cost of collateral.

Not surprisingly, in this case the comparison of the junior and senior derivative regimes is analogous to the partial equilibrium analysis in the preceding section. Because $R^J > R^S$, the required notional derivative position is higher under senior derivatives than under junior derivatives: $X^S > X^J$. This leads to a higher aggregate net liability for the counterparty, such that the required collateral is higher under senior derivatives than junior derivatives, $\zeta^S > \zeta^J$. This leads to higher deadweight costs under senior derivatives. Essentially, when cash flow risk and basis risk are perfectly correlated across firms, seniority for derivatives does not generate any diversification benefits for the derivative counterparty. Hence, the only relevant efficiency consideration is to lower the size of the required derivative positions, which is achieved by making derivatives junior to debt in bankruptcy.

Basis Risk and Cash Flow Risk Are Idiosyncratic: Suppose now that both cash flow risk and basis risk are idiosyncratic. Then the payoff to the representative derivative counterparty is deterministic. By the law of large numbers, a fraction θ of the counterparty's derivative contract moves in favor of the counterparty ($Z = Z^H$), while a fraction $1-\theta$ moves out of favor ($Z = Z^L$). When derivatives are senior to debt, the balance sheet of the representative counterparty is thereby given by

$$A + \theta x^S - (1-\theta)\, X^S, \tag{IA.5}$$

assuming that the counterparty is not required to post collateral, which we verify below. Given that the counterparty cannot make strictly positive profits in equilibrium, the following zero-profit condition must hold:

$$A + \theta x^S - (1-\theta)\, X^S = A. \tag{IA.6}$$

The counterparty thus sets x^S such that $\theta x^S - (1-\theta)\,X^S = 0$. Because its balance sheet is deterministic, the counterparty never has a net liability, which implies that the counterparty's incentive constraint is always satisfied and no collateral has to be posted ($\zeta^S = 0$).

An analogous argument applies when derivatives are junior to debt. In this case, the balance sheet of the counterparty (again assuming that no collateral has to be posted) is given by

$$A + [\theta - (1-\theta)(1-\gamma)]\,x^J - (1-\theta)\,X^J, \tag{IA.7}$$

taking into account that the counterparty receives no payment from firms that receive C_1^L and owe x^S to the counterparty. The counterparty's zero-profit condition thus becomes

$$A + [\theta - (1-\theta)(1-\gamma)]\,x^J - (1-\theta)\,X^J = A. \tag{IA.8}$$

The counterparty thus sets x^J such that $[\theta - (1-\theta)(1-\gamma)]\,x^J - (1-\theta)\,X^J = 0$. Also in this case, because its balance sheet is deterministic, the counterparty never has a net liability, which implies that the counterparty's incentive constraint is always satisfied and no collateral has to be posted ($\zeta^J = 0$).

Because $\zeta^S = \zeta^J = 0$, no deadweight costs arise for the counterparty, irrespective of the relative priority of debt and derivative contracts. Hence, while the pricing of the derivative contract differs depending on the bankruptcy regime (equations (IA.6) and (IA.8)), the relative priority ranking of debt and derivatives does not affect aggregate surplus.

B. Robustness

To keep the analysis tractable we have considered the most stripped-down setting possible, with only two periods and two possible cash flow realizations at date 1. However, the main qualitative results hold in much more general settings.

Note first that it is not essential to restrict the number of possible cash flow outcomes to only two. As shown by Faure-Grimaud (2000) and Raith and Povel (2004), when the firm's cash flow at date 1 is drawn from a continuum of possible realizations, $C_1 \in [0, \bar{C}_1]$, with a probability density function $h(C_1)$, then the optimal financial contract also takes the form of a debt contract and has the following features. The firm can continue operating until date 2 if it repays the promised debt amount $B \leq C_2$ at date 1. If the firm repays an amount $\rho < B$, then it can continue to operate only with probability

$$\lambda(\rho) = 1 - \frac{B - \rho}{C_2}. \tag{IA.9}$$

This contract gives the firm an incentive to repay as much as it can ($\rho = C_1$) at date 1 whenever its cash flow realization falls short of its total obligation B. Note that this contract has the same general properties as the contract we derived in the case of two cash flow outcomes: It leads to efficient continuation in all cash flow states $C_1 \geq B$, and it results in inefficient liquidation (with positive probability) in all cash flow states $C_1 < B$. Moreover, the probability of inefficient liquidation increases in the repayment shortfall $(B - C_1)$. This result continues to hold when C_2 is also random (for details, see Faure-Grimaud (2000) and Raith and Povel (2004)).

As in the setting with only two possible cash flow realizations, it is straightforward to see that a derivative contract specifying payments contingent on a random variable Z that is negatively correlated with C_1 can improve on the outcome without derivatives. Consider, for example, the case in which Z is continuously distributed on the support $[\underline{Z}, \overline{Z}]$ and is perfectly negatively correlated with C_1, such that there is no basis risk. With a slight abuse of notation, let the support $[\underline{Z}, \overline{Z}]$ be the same as the support of cash flows $[0, \bar{C}_1]$. Then a derivative contract such that $X(C_1) = B - C_1$ for all $C_1 < B$ and $X(C_1) = -\alpha(C_1 - B)$ for all $C_1 \geq B$ (with $\alpha \in (0, 1)$) would provide perfect insurance to the firm as in the setting with only two possible cash flow realizations. To break even, the counterparty would set α such that

$$\alpha \int_B^{\bar{C}_1} (C_1 - B) dH(C_1) = \int_0^B (B - C_1) dH(C_1). \tag{IA.10}$$

In the presence of basis risk, the same general trade-off and qualitative result on the optimal priority ranking of debt and derivatives would obtain as in the case with two cash flow realizations, although the mathematical analysis would be considerably more involved. In particular, when the derivative counterparty and creditor have competing claims in a low-cash flow state ($C_1 < B$), then the effect of giving seniority to the derivative contract is to raise the promised debt repayment B and therefore the firm's demand for insurance via the derivative. As in the model with only two possible cash-flows, this raises the deadweight cost of hedging by increasing collateral requirements for the derivative counterparty.

Another simplification in our model is that the firm has a single investment of fixed size F. Our analysis can be extended to a situation where cash flows are an increasing concave function of initial variable investment I: $\{C_1^L(I), C_1^H(I), C_2(I)\}$. In this more general setting, there would be an additional cost of giving priority to derivatives over debt: By raising the cost of funding for the firm, it would also result in lower investment. Similarly, the model can be extended to introduce corporate taxes and a tax-shield benefit of debt, as is shown in Appendix B of the NBER working paper version of this article (NPER WP 17599). When the firm can obtain a tax-shield benefit of debt it will generally issue more debt claims than it needs in order to finance the setup cost F. One may then wonder whether having senior derivatives may actually not be a benefit to the firm, as the higher promised debt repayment it would entail would provide the firm with a higher tax shield. We show, however, that this benefit is always outweighed by the higher deadweight cost of hedging.

Finally, the analysis can be generalized to allow for costly outside equity financing. In our framework, the costs of outside equity could, for example, be auditing costs that arise when verifying realized cash flows. In this more general model, the firm may choose equity financing over debt financing if bankruptcy costs are too high. The costs of giving derivatives seniority over debt then no longer come only in the form of higher deadweight costs of hedging, but also from pushing the firm toward costly equity financing. The privileged bankruptcy treatment of derivatives would still be inefficient; only the specific form of the costly distortion would change.

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