Willem H. Buiter and Nikolaos Panigirtzoglou

Overcoming the zero bound on nominal interest rates with negative interest on currency : Gesell's solution

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Overcoming the Zero Bound on Nominal Interest Rates with Negative Interest on Currency

Gesell's Solution

Willem H. Buiter[*][**]
Chief Economist and Special Counsellor to the President, European Bank for Reconstruction and Development, NBER, and CEPR and CEP.

Nikolaos Panigirtzoglou
Bank of England

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Abstract

An economy is in a liquidity trap when monetary policy cannot influence either real or nominal variables of interest. A necessary condition for this is that the short nominal interest rate is constrained by its lower bound, typically zero. The paper considers two small analytical models, one Old-Keynesian, the other New-Keynesian possessing equilibria where not only the short nominal interest rate, but nominal interest rates at all maturities can be stuck at their zero lower bound.

When the authorities remove the zero nominal interest rate floor by adopting an augmented monetary rule that systematically keeps the nominal interest rate on base money (including currency) at or below the nominal interest rate on non-monetary instruments, the lower bound equilibria are eliminated, thus allowing an economic system to avoid the trap or to escape from it. This rule will involve paying negative interest on currency, that is, imposing a 'carry tax' on currency, an idea first promoted by Gesell. The administration costs associated with a currency carry tax must be set against the benefits of potentially lower shoe-leather costs and lower menu costs which are made possible by the its introduction. There are also output-gap avoidance benefits from eliminating the zero lower bound trap.

Willem H. Buiter, European Bank for Reconstruction and Development, One Exchange Square, London EC2A 2JN, UK.
Tel: #44-20-73386805
Fax:#44-20-73386110/111
E-mail (office): buiterw@ebrd.com
E-mail (home): willembuiter@whsmithnet.co.uk
Home page: http://www.nber.org/wbuiter/

Nikolaos Panigirtzoglou, Bank of England, Threadneedle Street, London EC2R 8AH, UK.
Tel: #44-20-7601 5440
Fax: #44-20-7601 5953
E-mail: nikolaos.panigirtzoglou@bankofengland.co.uk

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Equation Section (Next)(I) Introduction

The liquidity trap used to be a standard topic in macroeconomic theory. The textbook treatment of liquidity traps was based on Hicks's [1936] interpretation of Keynes [1936], and involved the assumption that the demand for money balances would become infinitely responsive to its opportunity cost, proxied by the nominal interest rate, at some low level of that nominal interest rate.[1][2] In a liquidity trap, private agents would willingly absorb any amount of real money balances without changing their behaviour in any other respect. In most modern theories, the short riskless nominal interest rate on government debt is the opportunity cost of holding currency, and the floor on the short nominal interest rate is typically taken to be zero (the ‘zero lower bound’). The nominal yield on short government debt is then related to yields on other assets through a variety of equilibrium asset pricing relationships.

During the inflationary 1970s and 1980s, liquidity traps and the lower bound on short nominal interest rates ceased to be of concern to policy makers and to scholars other than economic historians and historians of economic thought. In the mainstream accounts of the monetary transmission mechanism, the liquidity trap was treated as a theoretical curiosum.[3] Since the late 1990s, there has been a revival of interest from scholars and from monetary policy makers in the liquidity trap in general, and the zero lower bound on the nominal rate of

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[1] The maturity of the interest rate was left rather vague. In later interpretations, the infinite interest elasticity of money demand involved a long nominal rate of interest; the demand for money becomes infinitely sensitive to the current value of this long nominal yield because of regressive (what we now call ‘mean reverting’) expectations about the future behaviour of short interest rates (see e.g. Tobin [1958] and Laidler [1993]).
[2] There was no presumption that this floor to the nominal rate of interest would be at zero.
[3] See e.g. Romer [2001], which covers the topic as half of an exercise at the end of the chapter 5, "Traditional Keynesian Theories of Fluctuations".

interest in particular.[4] As has so often been the case in monetary economics, scholarly interest was prompted by unfamiliar and unexpected empirical observations and by the needs of monetary policy makers who experienced or feared the loss of the standard instrument of monetary policy in economies with developed financial systems – the short nominal rate of interest.

The reality of the zero lower bound is an economic policy issue in Japan. The risk of the zero lower bound becoming a binding constraint on monetary policy has become a factor in Western Europe and the United States of America. Japan is in a protracted, ten-year old economic slump. Short nominal interest rates there are near zero. A number of observers have concluded that there is a liquidity trap at work (see e.g. Krugman [1998a,b,c,d; 1999], Ito [1998], McKinnon and Ohno [1999] and Svensson [2000]); for a view that liquidity traps are unlikely to pose a problem, see Meltzer [1999] and Hondroyiannis, Swamy and Tavlas [2000]).

In Euroland inflation, on the HIPC measure, averaged 1.1 percent per annum during 1999. The ECB's repo rate reached a trough of 2.5 percent during April 1999. At the time, this raised the question as to whether a margin of two hundred and fifty basis points provides enough insurance against a slump in aggregate demand. Today (mid-2002) the ECB's repo rate stands at 3.25 percent and inflation runs at around 2.0 percent per annum. While this appears to provide a reasonable cushion against the risk of getting stuck at the zero lower bound, the fear of deflation has not vanished completely.

[4] Recent theoretical analyses of liquidity traps include Wolman [1998], Buiter and Panigirtzoglou [1999], McCallum [2000, 2001], Cristiano [2000], Porter [1999], and Benhabib, Schmitt-Grohé and Uribe [1999a,b]. Recent empirical investigations of the issue include Fuhrer and Madigan [1997], Buiter and Panigirtzoglou [1999], Johnson, Small and Tryon [1999], Clouse, Henderson, Orphanides, Small and Tinsley [1999], Iwata and Wu [2001].

Finally, in the US too, with the Federal Funds rate in HI 2002 down at 1.75 per cent as a result of the recession and the events of 11 September 2001, the Fed has shown some concern about the possibility that monetary policy could become constrained by a lower bound on nominal interest rates. As early as the Fall of 1999, the Fed organised a conference to discuss the 'zero bound problem' and recently its staff have produced a thorough study of Japan's experience in the 1990s and the lessons this holds for preventing deflation (Ahearne et. al. [2002]).[5]

The terms 'monetary policy' and 'liquidity trap' means different things to different people. We shall offer definitions, but consider the concepts to be more important than the labels. Monetary policy changes the composition of the government's financial liabilities between monetary and non-monetary financial instruments for a given aggregate stock of financial liabilities (monetary plus non-monetary). Changes in the magnitude of the government's aggregate financial liabilities are the province of intertemporal fiscal policy. We consider an economy to be in a liquidity trap when monetary policy cannot stimulate aggregate demand through any channel.

A necessary condition for such monetary policy ineffectiveness is that monetary policy cannot influence the cost, availability or liquidity of funds to enterprises and consumers.[6] It is not sufficient, because monetary policy can be argued to operate also through channels like wealth effects or the real balance/Pigou effect. In contrast to an open

[5] The proceedings of the conference were published in the *Journal of Money, Credit and Banking* [2002]. Other studies by Federal Reserve Board staff members of monetary policy near the zero lower bound include Orphanides and Wieland [1998, 2000].

[6] More generally, a necessary condition for monetary policy ineffectiveness is that monetary policy cannot affect the joint distribution of real and nominal rates of return on financial and real assets. When monetary policy also works through channels other than rates of return (say, through the availability as well as the cost of credit, or through the exchange rate), a liquidity trap is operative only if these additional liquidity, credit or exchange rate channels of monetary transmission too are blocked. The exchange rate channel is discussed extensively in McCallum [2000,2002].

market purchase of public debt, which would be monetary policy according to our definition, a Friedman-style 'helicopter drop of money' would by us be considered a combination of fiscal and monetary policy, as it involves a capital transfer from the monetary authorities to the recipients.

The modern 'zero bound' argument assumes explicitly (and the traditional theories assumed implicitly) that the pecuniary (financial) rate of return on money is zero, an appropriate assumption for coin and currency, although not for the liabilities of private deposit-taking institutions that make up most of the broader monetary aggregates. The latter now typically have positive nominal returns. With the nominal rate of return on currency administratively fixed at zero, a floor to the spread between the non-monetary and monetary claims becomes a floor to the nominal yields on non-monetary financial instruments.

In Section II we revisit an old proposal, which we attribute to Silvio Gesell, for removing the zero lower bound on the nominal rate of interest on non-monetary assets by paying negative nominal interest rates on base money – currency (including vault cash) and commercial bank balances (electronic bank reserves) held at the central bank.[7] Goodfriend [2000] contains an extensive discussion of the practicalities of paying negative interest on currency and electronic bank reserves by levying a "carry tax". Like him, we conclude that such a carry tax is feasible (simple for bank reserves, awkward for currency) and that it could be more efficient than a policy of minimising the risk of hitting the zero lower bound by keeping the inflation target and the inflation rate sufficiently high (see also Bryant [2000] and Freedman [2000]).

The administrative problems associated with paying negative interest on base money, that is, taxing the holding of base money, are due to the fact that one component of base money, coin and currency, are fiat *bearer bonds*. This means that the owner of the coins and currency is anonymous - the identity of the holder is unknown to the issuer, the central bank. It is difficult to tax an asset when the identity of its owner is unknown to the tax authority. A way must be found for the owner of the currency to reveal himself to pay the tax. The other component of the monetary base, (electronic) commercial bank reserves with the central bank poses no problems as regards the payment of negative (or positive) interest. Commercial bank reserves with the central bank are *registered* financial claims: the identity of the owner is known to the issuer. It is no more difficult for the central bank to pay negative (or positive) interest on commercial bank reserves with the central bank than it is for commercial banks to pay negative (or positive) interest on demand deposits or time deposits.

In Section III we use two standard small macroeconomic models to show how, with an exogenous nominal interest on currency (zero, say) and with the short nominal interest rate on non-monetary government debt determined by a simple Taylor rule, the zero lower bound on the nominal interest rate can become a binding constraint, and how under these conditions conventional monetary policy (working through changes in the current short nominal interest rate or by changes in current *and* anticipated future short nominal rates) becomes powerless. We then show how, by paying negative interest on currency, the zero lower bound constraint is eliminated and the associated zero lower bound trap ceases to exist. The first model is old-Keynesian, with a conventional IS curve and a backward-looking accelerationist Phillips curve. The second model is New-Keynesian, with a forward-looking IS curve and Calvo's version of the staggered, overlapping price setting model, which implies a forward-looking accelerationist Phillips curve.

[7] In principle, negative interest might have to be paid on coin as well. However, as pointed out in Porter [1999] and Goodfriend [2000], the carry cost of coins are sufficiently high that they would not be an attractive store of value for large sums.

When interest rates are constrained at their lower bound, expansionary fiscal policy, or any other exogenous shock to aggregate demand, is supposed to be at its most effective. This is obviously the case for our old-Keynesian model. Even our new-Keynesian model, which exhibits Ricardian equivalence or debt neutrality, has the property that aggregate demand is boosted temporarily by a temporary increase in public spending on goods and services. For this fiscal policy channel to be ineffective also, public spending on goods and services must be a direct perfect substitute for private spending on goods and services, say because public consumption is a perfect substitute for private consumption in private utility functions, and/or public sector capital is a perfect substitute for private capital in private production functions.[8]

The main results of the paper are the following:

First, the zero lower bound on the nominal rate of interest can be eliminated completely by the payment of a negative rate of interest on base money, that is, by imposing a carry tax on base money. By following a policy that maintains the short nominal interest rate on base money at or below the short nominal interest rate on non-monetary financial instruments, all real economic variables of the economic system, except for the stock of real money balances, but including the rate of inflation,[9] behave as they would if the nominal interest rate on base money were zero and there were no non-negativity constraint on the short nominal interest rate on non-monetary financial instruments.

Second, one component of base money, commercial banks' balances with the central bank, can pay negative interest easily and at little or no cost. The second component, currency, could pay negative interest but at some, probably significant, administrative cost. These administrative costs of paying a negative nominal interest rate on currency are distinct from the familiar 'shoe-leather cost' of managing money balances and from the menu costs of

[8]See Buiter [1977].

any non-zero rate of inflation. It is indeed possible both to eliminate the zero bound on the nominal interest rate by paying negative interest on currency and to eliminate 'shoe-leather costs' by closing the gap between the short nominal interest rate on non-monetary financial instruments and the nominal interest rate on currency, thus achieving satiation with real money balances.

Third, the formal models considered in the paper have the property that, when the nominal interest rate on non-monetary financial instruments is government by a Taylor rule and the nominal interest rate on base money is exogenous (say, zero), an increase in the target rate of inflation – a key parameter of the Taylor rule – will not help the economy escape from a situation where nominal interest rates at all maturities are at their lower (zero) bound.

(II) Paying negative interest on base money

The nominal rate of return on base money (coin, currency and commercial bank balances with the central bank) net of carry costs (costs of storage, taxes etc), sets a floor under the nominal rates of return net of carry costs on all other assets. The fundamental 'no arbitrage' axiom of finance theory provides the reason. No rational economic agent will hold a store of value that is (net) rate-of-return-dominated by another store of value. Base money is the most liquid store of value. Its advantages as medium of exchange and means of payment (often enhanced by its official status as legal tender) means that a rational economic agent will not hold any asset other than base money, unless that alternative store of value promises a pecuniary return at least as high as base money. If the nominal interest rate on bonds were to be below the nominal interest rate on base money by more than the carry cost

[9] Strictly speaking, real consumption, real output, the real interest rate, the nominal interest rate and the inflation rate. The stock of real money balances will depend on $i - i_M$.

differential, any rational private agent would wish to borrow an infinite amount by issuing bonds, in order to build up infinite holdings of base money.

Interest rates must of course be adjusted for carry costs to obtain the relevant net financial rates of return. Carry costs for coins are non-trivial. Storage costs for coins are sufficiently high to rule out their widespread use as a store of value if the nominal interest rate on bills and bonds were to go to zero. For that reason, coins will be ignored in what follows.

Storage cost for commercial bank reserves with the central bank (entries in an electronic ledger) are very low.[10] For simplicity, we can take them to be zero. The carry cost of currency, including vault cash is non-negligible, if one allows not just for the cost of physical storage, but also for the cost of insuring against loss, damage or theft. Carry costs on non-monetary government securities (bills or bonds) are low and are more like a fixed than a variable cost. Let i denote the instantaneous nominal interest rate on non-monetary government debt ('short bonds'), i_C the nominal interest rate on currency, i_R the nominal interest rate on commercial bank reserves with the central bank, γ the (instantaneous) marginal carry costs of bonds, γ_C the instantaneous marginal carry cost of currency and γ_R the instantaneous marginal carry cost on commercial bank reserves with the central bank. Then the superior liquidity of base money will set the following floor on the nominal interest rate on bonds:

$$i \geq \text{Max}\{i_C + \gamma - \gamma_C, i_R + \gamma - \gamma_R\} \qquad (1)$$

In the formal models of Section III, we omit explicit consideration of the three carry cost terms, γ, γ_C and γ_R. We also treat base money as if it were a homogeneous aggregate rather than the sum of two distinct components, currency and commercial bank balances with

[10] The main cost will be that of ensuring the security and integrity of the electronic balances. These costs are mainly overhead costs for the electronic ledger as a whole, and will be independent of the amount of reserves kept in any particular electronic ledger entry.

the central bank. Since $\gamma_C > \gamma \geq \gamma_R$, the true floor for the short nominal interest rate is likely to be slightly below zero if no nominal interest in paid on either base money component. For the purpose of this paper, these are matters of no real significance, so in what follows we have a single nominal interest rate on base money, i_M, that is, we set $i_M = i_C = i_R$ and we assume $\gamma = \gamma_C = \gamma_R = 0$. Equation (1) therefore becomes

$$i \geq i_M \tag{2}$$

The key message of this paper is that the zero bound on the nominal interest rate can be overcome, indeed eliminated and that zero bound traps can be avoided by paying a negative nominal interest rate on base money. This message is valid for any model in which the simple but fundamental 'no arbitrage' condition applies that rules out a negative pecuniary opportunity cost of holding base money. It holds for models with 'money in the direct utility function', as long as the marginal utility of money cannot become negative. It holds for models with 'money in the production function' as long as the marginal revenue product of money cannot become negative. It holds for 'shopping models' of money in which cash permits shoppers to economise on time or other valuable resources, as long as the marginal transaction cost savings or shopping cost savings of money cannot become negative. It also holds for cash-in-advance models.

It should be noted that paying a negative nominal interest rate on base money is exactly the same thing as levying a carry tax on base money, a measure proposed and analysed in detail by Goodfriend [2000]. Indeed in earlier versions of this paper (Buiter and Panigirtzoglou [1999]), we pointed out that the way policy makers can achieve a negative (positive) net nominal yield on base money is by taxing (subsidising) base money. We called base money carrying a negative nominal interest rate *Gesell money* and the carry tax the

Gesell tax, after Silvio Gesell (1862-1930), a German-Argentine businessman and economist who was probably the best-known proponent of taxing currency (see Gesell [1949]).

The nominal interest rate floor at zero is not a technological, immovable barrier. It is the result of a political and administrative choice - the decision by governments or central banks to set the administered nominal interest rate on coin, currency and commercial bank reserves with the central bank at zero.[11]

There are two reasons why interest is not paid on coin and currency.[12] The first, and currently less important one in advanced industrial countries, has to do with the attractions of seigniorage (issuing non-interest-bearing monetary liabilities) as a source of government revenue in a historical environment of positive short nominal rates on non-monetary government debt.[13] The second, and more important reason why no interest (positive or negative) is paid on coin and currency, are the practical, administrative difficulties of paying a negative interest rate on bearer bonds. Significant costs are involved both for the state and for private agents. These costs are there because of a fundamental information asymmetry.

In order to pay a *positive* rate of interest on a financial instrument (security), it must be possible to for the issuer (and for a third party like a court of law) to determine unambiguously whether interest due on any particular quantity of that security has been paid by the holder (owner). Unless this can be established unequivocally, the security in question could be presented multiple times for payment, by the same or by different owners.[14] In order to pay a *negative* interest rate on a financial instrument, the issuer (and a third party like a

[11] Certain countries at certain times have paid positive nominal interest on commercial bank reserves with the central bank.
[12] From here on, 'currency' will be taken to include both coin and currency. There obviously are more severe technical problems with attaching coupons or stamps to coin than to currency notes.
[13]Of course, issuing negative interest-bearing monetary liabilities would be even more attractive, from a seigniorage point of view.
[14] Clipping coupons is a traditional way to ensure that interest is paid only once.

court of law) must be able to determine whether any particular quantity of that security has had the interest due on it paid by the owner to the issuer. The owner has to be able to establish unambiguously that interest due to the issuer has been paid. The issuer must create an incentive for the owner to reveal himself and pay the interest due to the issuer.

The problem with paying any kind of interest, positive or negative, on coin and currency is that they are bearer securities – or bearer bonds. A bearer bond is a debt security in paper or electronic form whose ownership is transferred by delivery rather than by written notice and amendment to the register of ownership. We shall refer to all securities that are not bearer bonds as *registered* securities. Bearer bonds are negotiable, just as e.g. money market instruments such as Treasury Bills, bank certificates of deposit, and bills of exchange are negotiable. Coin and currency therefore are bearer bonds issued by the central bank. They are obligations of the state, made payable not to a named individual or other legal entity, but to whoever happens to present it for payment - the bearer. The owners of the securities are anonymous, unknown to the issuer. If a security cannot be clearly linked to a known owner, it must instead be possible to ascertain from the security itself whether interest due has been paid. The security must be unambiguously 'marked' or identified when interest due is paid.

In order to provide appropriate incentives for the holders of currency to make a payment to the issuer, the issuer or its agents must be able to impose a sufficient penalty on anyone holding currency that cannot be unambiguously identified as current on all interest due. Confiscation without compensation would be one example of such a penalty. It would only work, of course, if the probability of apprehension is high enough. Gesell proposed physically stamping currency to provide evidence that negative interest had been paid - a negative interest rate analogue to the coupon-clipping solution for positive interest payments.

Special mechanisms for removing the zero nominal interest rate floor by taxing currency are not required for the other component of the monetary base: commercial banks' balances with the central bank. These balances are not bearer bonds, but registered securities, in the terminology of this paper. There is no practical or administrative barrier to paying negative nominal interest rates (whether market-determined or administered) on registered securities, including balances held in registered accounts, such as bank accounts. Positive interest payments or negative interest payments just involve simple book-keeping transactions, debits or credits, between accounts owned by known parties. With the identities of both issuer and holder (debtor and creditor) known or easily established, it is not difficult to verify whether interest due has been paid and received.

Removing the zero floor on nominal interest rates by taxing currency is not complicated by the existence of a private banking sector that issues demand and time deposits, certificates of deposits etc., and that operates in the interbank market (this point has been made effectively in Goodfriend [2000]). All bank deposits and all financial instruments traded in the Federal Funds market and in the interbank market are, in our terminology, *registered* financial instruments. There would be no need for the state to tax them in order to achieve negative nominal interest rates. Simple arbitrage would propagate the administratively imposed negative interest rates on currency and commercial bank balances with the central bank to repo rates, market-determined bank deposit rates, rates on financial instruments traded in the Federal Funds market or in the interbank market, and rates on private electronic or e-money, including 'money on a chip', internet accounts etc.[15]

There are costs associated with administering a carry tax on currency that will not be negligible even if one can come up with a slightly higher-tech (and tamper-proof) alternative

[15] It would be no harder for the Bank of England or the European Central Bank to impose a negative interest rate in their repo operations than it is to achieve a positive interest rate.

to physically stamping currency. These *carry tax administration costs* have to be set against the benefits of removing the zero floor to the nominal interest rate and the costs associated with the other method for reducing the likelihood of monetary policy being constrained by the zero bound. An example is a monetary policy that consistently produces sufficiently high nominal interest rates to reduce to a very low level the likelihood of the zero bound constraint becoming binding. The fact that high inflation leads to high nominal interest rates is good from the point of view of avoiding the zero lower bound, but it is costly from the point of view of Baily-Friedman-Allais-Baumol-Tobin shoe-leather cost of cash management (see Bailey [1956], Friedman [1969], Fischer [1981, 1994], Baumol and Tobin [1989]). A complete analysis should also consider menu costs, which are present whenever the inflation rate is non-zero and which increase with the frequency with which prices are changed, that is, with the absolute value of the rate of inflation.

There are costs to taxing currency other than carry tax administration costs, and benefits other than the avoidance of shoe-leather costs and menu costs. Taxing currency would be regressive, since only the relatively poor hold a significant fraction of their wealth in currency. Taxing currency would also, however, constitute a tax on the grey, black and outright criminal economies, which are heavily cash-based. In the case of the US dollar, with most US currency held abroad (one assumes mainly by non-US residents), it would represent a means of increasing external seigniorage.

(III) Taxing Currency, the Zero Lower Bound and the Liquidity Trap in Old- and New-Keynesian Models.

III.1 The zero bound equilibrium in an Old-Keynesian model

Consider a simple IS-LM model with an accelerationist Phillips curve. The following notation is used: y is real GDP, r is the short real rate of interest, f represents the exogenous or autonomous determinants of aggregate demand, $m \equiv M/P$ is the real value of the stock of currency (M is the nominal stock of currency and P the general price level); i is the short nominal interest rate, i_M is the nominal interest rate on currency, $\pi \equiv \dot{P}/P$ is the rate of inflation; $\bar{y}$ is the exogenous and constant level of real capacity output, π^* is the long-run target rate of inflation and ρ is the long-run real interest rate.

$$\begin{aligned} &y = -\alpha r + f \\ &y \geq 0;\ \alpha > 0 \end{aligned} \tag{3}$$

For $i \geq i_M$,

$$\begin{aligned} &\frac{m}{y} = \frac{\eta}{i - i_M} \\ &m \geq 0;\ \eta > 0 \end{aligned} \tag{4}$$

$$\begin{aligned} &\dot{\pi} = \beta(y - \bar{y}) \\ &\beta > 0;\ \bar{y} > 0 \end{aligned} \tag{5}$$

$$r \equiv i - \pi \tag{6}$$

$$\begin{aligned} i &= \delta + \pi^* + \gamma(\pi - \pi^*) && \text{if } \delta + \pi^* + \gamma(\pi - \pi^*) \geq i_M \\ &= i_M && \text{if } \delta + \pi^* + \gamma(\pi - \pi^*) \leq i_M \\ &\gamma > 1 \end{aligned} \tag{7}$$

$$\delta = \alpha^{-1}(f - \bar{y}) \tag{8}$$

and either

$$i_M = \bar{i}_M = 0 \tag{9}$$

or

$$\begin{aligned} i_M &= i - \nu \\ \nu &\geq 0 \end{aligned} \tag{10}$$

In this Old-Keynesian model, both the price level, *P,* and the rate of inflation, π, are assumed to be predetermined and output is demand-determined. Equation (3) is a standard IS curve, making aggregate demand a decreasing function of the real interest rate. The exogenous demand driver, *f*, includes the fiscal determinants of aggregate demand.

Equation (4) is a standard LM curve, with the demand for currency proportional to real income and decreasing in the opportunity cost of holding currency, $i - i_M$. Note that, if $i < i_M$, currency would dominate non-monetary financial assets ('bonds') as a store of value. Portfolio holders would wish to take infinite long positions in currency, financed by infinite short positions in non-monetary securities. The rate of return on such a portfolio would be infinite. This cannot be an equilibrium. If $i = i_M$, currency and bonds are perfect substitutes as stores of value. In Section III.2, we derive a money demand function similar to (4) in the context of an optimising model of consumer behaviour. It is clear in such a model that, from the point of view of the optimal quantity of money, the Bailey-Friedman rule, $i = i_M$, will characterise the first-best equilibrium.

Equation (5) is a backward-looking accelerationist Phillips curve: the predetermined rate of inflation increases (decreases) when actual output is above (below) capacity output.

Equation (7) has the monetary authorities following a simplified Taylor rule for the short nominal interest rate on non-monetary financial claims, as long as this does not put the short nominal interest rate below the interest rate on currency. A standard Taylor rule for the short nominal bond rate which restricts the short nominal bond rate not to be below the short nominal rate on currency, would be

$$i = \delta + \pi^* + \gamma(\pi - \pi^*) + \rho(y - \bar{y}) \quad \text{if } \delta + \pi^* + \gamma(\pi - \pi^*) + \rho(y - \bar{y}) \geq i_M$$
$$= i_M \quad \text{if } \delta + \pi^* + \gamma(\pi - \pi^*) + \rho(y - \bar{y}) < i_M$$

$\gamma > 1;\ \rho > 0$

Equation (8) specifies that δ is the steady-state real interest rate in the normal case, when the zero lower bound is not binding. This normal long-run real interest rate is endogenous in our model.[16] The Taylor rule has the property that in the long run, when inflation is at its target level and output equals capacity output, the nominal interest rate equals the long-run real interest rate plus the target rate of inflation. In the short run, the nominal interest rate rises more than one-for-one with the actual rate of inflation. This means that the short real interest rate rises whenever the inflation rate rises, providing a stabilising policy feedback mechanism. The nominal interest rate also rises with the output gap.

For our purposes, all that matters is the responsiveness of the short nominal interest rate to the inflation rate. We therefore omit feedback from the output gap in what follows, that is, we set $\rho = 0$. The short nominal interest rate rule then simplifies to (7).

When the nominal interest rate on currency is exogenous (say zero), the behaviour over time of the economy is captured by the following switching differential equation:

$$\begin{aligned} \dot{\pi} &= \alpha\beta(1-\gamma)(\pi - \pi^*) && \text{if } \pi \geq \hat{\pi} \\ &= \alpha\beta\pi - \alpha\beta\bar{i}_M + \beta(f - \bar{y}) && \text{if } \pi \leq \hat{\pi} \end{aligned} \tag{11}$$

$$\hat{\pi} \equiv \gamma^{-1}[\bar{i}_M - \delta + (\gamma - 1)\pi^*] \tag{12}$$

When the inflation rate is above the critical value $\hat{\pi}$ (we shall refer to this as the '*normal zone*'), the lower bound on the nominal interest rate is not binding, and the Taylor

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[16] Substituting the definition of the real interest rate (6), the Taylor rule (7) and the definition of the steady-state real interest rate (8), into the IS equation (3), gives an aggregate demand equation that, because of the interest rate rule, is independent of *f*, the exogenous component of aggregate demand: $y = \bar{y} + \alpha(1-\gamma)(\pi - \pi^*)$. Nothing of importance for our purposes

rule is operative. When the inflation rate is at or below $\hat{\pi}$ (we shall refer to this as the *'lower bound zone'*), the lower bound constraint on the short nominal interest rate is binding and the short nominal interest rate is given by $i = i_M$. Figure 1 shows the behaviour of the model in these two regimes. There are two stationary equilibria: $\pi = \pi^*$ in the normal zone and $\pi = \pi^{**} \equiv \overline{i}_M - \delta$ in the lower bound zone. We assume that $\pi^* > \overline{i}_M - \delta$. If this is not the case, the monetary authorities will have parameterised their interest rate rules in such a way that the target rate of inflation (which is also the long-run rate of inflation in the normal zone) is below the long-run rate of inflation in the lower bound zone. This does not seem plausible, although the analysis of this case is straightforward.[17]

The normal steady state, $\pi = \pi^*$, is locally stable. The lower bound steady state, $\pi = \pi^{**}$, is locally unstable. Any initial rate of inflation above π^{**} will converge to the normal steady state $\pi = \pi^*$. If the initial rate of inflation were to be above π^{**} but below $\hat{\pi}$, the system would first move towards *B* along the divergent trajectory drawn with reference to π^{**}. When $\pi = \hat{\pi}$, it will switch to the convergent trajectory drawn with reference to π^*. Any initial rate of inflation below the lower bound steady state π^{**} would result in cumulative further disinflation and, sooner or later, increasing rates of deflation.

Note that, if the economy is in a deflationary spiral with $\pi < \pi^{**}$, raising the target rate of inflation, π^*, will not help. The lower bound solution trajectory *ABE* and the lower bound steady state inflation rate, π^{**}, are unaffected by π^*. Only the normal solution trajectory *DBC* is affected by an increase in the target rate of inflation: it shifts horizontally to the right by the same amount as the increase in π^*.

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depends on this assumed endogeneity of δ. With δ exogenous, equation (11) for $\pi \geq \hat{\pi}$ would become $\dot{\pi} = \alpha\beta(1-\gamma)(\pi - \pi^*) + \beta(f - \overline{y} - \alpha\delta)$

[17] From an initial rate of inflation above π^{**} (in the case where $\pi^{**} > \pi^*$) we would have a steadily *increasing* rate of inflation.

A simple modification (or amplification) of the Taylor rule that eliminates the lower bound problem completely is as follows. The exogenous own nominal interest rate assumption for money in (9) is replaced by equation (10). The Taylor rule for the short nominal interest rate on non-monetary financial instruments continues to be given, as before, by equation (7). The rest of the model is as before. Note, however, that there is now no restriction on the domain of the nominal interest rate function. Equation (10) ensures that the constraint that the short nominal interest rate on non-monetary instruments cannot fall below the nominal interest rate on currency never becomes binding. The lower bound zone in Figure 1 has been eliminated. For all values of π, the only solution now is the stable trajectory shown in Figure 1 as *DBC* through π^*, the map of $\dot{\pi} = \alpha\beta(1-\gamma)(\pi - \pi^*)$. There no longer is a lower bound steady state π^{**}.

Only the simplest kind of rule, maintaining a constant wedge (possibly zero) between the two interest rates is considered here, but it does the job of eliminating completely the lower bound constraint.[18] The rule for the two short nominal interest rates given in equations (7) and (10) may require the payment of non-zero (positive or negative) interest rates on base money. Through the carry tax on base money, the opportunity cost of holding base money, $i - i_M$, can be uncoupled from the nominal interest rate on non-monetary financial instruments, from the inflation rate and from the real rate of interest.

Finally, note that $\nu \equiv i - i_M = 0$, that is, a zero opportunity cost of holding currency, is part of the range of values for the wedge between the short nominal interest rate on non-monetary financial instruments and the nominal interest rate on currency that eliminates equilibria for which the lower bound constraint is binding. Satiation with real money

[18] Note that, because the opportunity cost of holding money, $i - i_M$, is positive and constant, the ratio of real money balances to consumption will also be constant in this model.

balances, the Bailey-Friedman optimal quantity of money rule, can now be achieved without the monetary authorities giving up control over the level of the nominal rate of interest, if a carry tax can be levied on currency.

III.2 The zero bound equilibrium in a New-Keynesian model

The model presented in this subsection is, except for one simple but crucial modification – the explicit consideration of a carry tax on currency, or a negative interest rate on currency – the same as the new-Keynesian models of a closed economy analysed by McCallum [2000, 2001] and by Benhabib, Schmitt-Grohe and Uribe [1999a,b] (henceforth BSU). The 'IS curve' is forward-looking, through an Euler equation for private consumption growth. The inflation process is Calvo's [1983] forward-looking New-Keynesian Phillips curve with the general price level predetermined but the rate of inflation non-predetermined.

We model a closed endowment economy with a single perishable commodity that can be consumed privately or publicly, and with two stores of value, currency which can only be issued by the government, and risk-free non-monetary nominal debt (bonds). We use the following notation in addition to that already introduced in Section III.1: c is real private consumption and g is real government consumption. The model consists of equations (6), (7), (9) or (10) and:

$$\begin{aligned} &\dot{c} = (r-\delta)c \\ &c \geq 0;\ \delta > 0 \end{aligned} \tag{13}$$

and, for $i \geq i_M$,

$$\begin{aligned} &m = \left(\frac{\eta}{i - i_M}\right)c \\ &m \geq 0;\ \eta > 0 \end{aligned} \tag{14}$$

$$c + g = y \tag{15}$$

$$\begin{aligned} &\dot{\pi} = \beta(y - \bar{y}) \\ &\beta < 0 \end{aligned} \tag{16}$$

The derivation of the consumption Euler equation (13) and the money demand function (14) can be found in Appendix 1. The parameter δ (the long-run real interest rate of the old-Keynesian model) now has the interpretation of the household's pure rate of time preference.[19] It is a constant now. The derivation of (16), which is implied by Calvo's model of staggered, overlapping price setting, can be found in Appendix 2 (see Calvo [1983]). Note that $\beta < 0$: the short-run Phillips curve appears to have the 'wrong' slope. This is a paradox only until one realises that the inflation rate is forward-looking:

$$\pi(t) = -\beta \int_t^{\infty} [y(s) - \bar{y}] ds + \lim_{\tau \to \infty} \pi(\tau) .$$

In the New-Keynesian variant too, output is demand-determined, and the price *level*, *P,* is predetermined. However, the growth rate of the price level, the rate of inflation, π, is non-predetermined.[20]

The behaviour of the economy when the interest rate on currency is exogenous (that is, when i_M is given by equation (9)) can be summarised in two first-order differential equations in the two non-predetermined state variable c and π. The equation governing the behaviour of private consumption growth switches when the lower bound on the short nominal interest rate becomes binding.

$$\dot{\pi} = \beta(c + g - \bar{y}) \tag{17}$$

[19] The long-run real interest rate equals ρ in this New-Keynesian model also.

[20] Because in Calvo's model, the general *price level*, *P*, is a predetermined state variable, but its proportional rate of change, π, is a non-predetermined state variable, costless disinflation is possible. The sacrifice ratio is zero (see e.g. Buiter and Miller [1985]).

$$\dot{c} = (\gamma - 1)(\pi - \pi^*)c \quad \text{if } \pi \geq \hat{\pi}$$
$$= \left(\bar{i}_M - \pi - \delta\right)c \quad \text{if } \pi \leq \hat{\pi} \tag{18}$$

We can partition $c - \pi$ space into a *normal zone*, where the lower bound on the nominal interest rate is not a binding constraint (that is, where, as in the Old-Keynesian example, $\pi \geq \hat{\pi} \equiv \gamma^{-1}[\bar{i}_M - \delta + (\gamma - 1)\pi^*]$) and a *lower bound zone* where the constraint is binding (that is, where $\pi < \hat{\pi}$).

In the new-Keynesian model too there are two steady state inflation rates - the normal steady state with $\pi = \pi^*$ and lower bound steady state with $\pi = \pi^{**} = \bar{i}_M - \delta$. Again we assume that $\pi^* > \pi^{**}$. The steady state conditions are:

$$c = \bar{y} - g$$
$$r = \delta$$

$\pi = \pi^*$ Normal
or
$\pi = \pi^{**} = \bar{i}_M - \delta$ Lower bound

$i = \delta + \pi^*$ Normal
or
$i = \bar{i}_M$ Lower bound

$m = \left(\dfrac{\eta}{\delta + \pi^* - \bar{i}_M}\right)(\bar{y} - g)$ Normal
or
$m = +\infty$ Lower bound (19)

Note that steady-state household utility is higher in the lower bound equilibrium than in the normal equilibrium, unless $\delta + \pi^* = \bar{i}_M$, in which case utility is the same in both steady states.[21] Consumption is the same in both cases and in the lower bound steady state households are satiated with real money balances (see Appendix 1 for the details).

The equilibrium configuration near the lower bound steady state (Ω^L in Figure 2) is neutral and cyclical.

It is also possible to characterise the global dynamics of the model. From equation (17) and the normal version of equation (18) it follows that the integral curves in $c-\pi$ space in the normal zone ($c>0$ and $\pi \geq \hat{\pi}$) are given by:

$$\beta[c+(g-\bar{y})\ln c]=(1-\gamma)\pi^{*}\pi+\frac{(\gamma-1)}{2}\pi^{2}+k \tag{20}$$

where k is an arbitrary constant. Provided $(1-\gamma)^2\pi^{*2}+2(1-\gamma)\left(k-\beta[c+(g-\bar{y})\ln c]\right)\geq 0$, the integral curves in the normal zone are therefore given by

$$\pi=\pi^{*}\pm\frac{\sqrt{(1-\gamma)^2\pi^{*2}+2(1-\gamma)\left(k-\beta[c+(g-\bar{y})\ln c]\right)}}{1-\gamma}$$

From equation (17) and the lower bound version of equation (18) it follows that the integral curves in $c-\pi$ space in the lower bound zone (c > 0, $\pi \leq \hat{\pi}$) are given by

$$\beta[c+(g-\bar{y})\ln c]=(\bar{i}_M-\delta)\pi-\frac{1}{2}\pi^{2}+k \tag{21}$$

where k is again an arbitrary constant. Provided $(\bar{i}_M-\delta)^2+2\left(k-\beta[c+(g-\bar{y})\ln c]\right)\geq 0$, the integral curves in the lower bound zone are therefore given by:[22]

$$\pi=\pi^{**}\pm\sqrt{\pi^{**2}+2\left(k-\beta[c+(g-\bar{y})\ln c]\right)}\,.$$

The normal zone configuration is a *center*.[23] A linear approximation of the dynamic system at the normal steady state has two pure imaginary roots.[24] Some neighbourhood of the

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[21] In that case the two steady states coincide.

[22] Note that $\bar{i}_M-\delta=\pi^{**}$

[23] Anne Sibert provided the mathematical solution for the behaviour of the system in the two zones.

normal steady state is completely filled by closed integral curves, each containing the steady state in its interior. The lower bound zone configuration is a *saddlepoint*. A linear approximation at the lower bound steady state has one positive and one negative characteristic root.

On the boundary of the two regions (when $\pi = \hat{\pi}$) and at a given level of consumption, the slope of the integral curve in the normal zone is the same as the slope of the integral curve in the lower bound zone. This means that the center orbits of the normal zone and the saddlepoint solution trajectories of the lower bound zone merge smoothly into each other at the boundary between the two regions. Figure 2 shows the 'merged', global solution trajectories spanning the two zones. In all essential respects, this represents the model analysed by BSU [1999a] with $i_M = 0$.

As pointed out by BSU [1999a,b], there exists a plethora of solutions for this model, including some strange deflationary equilibria. A (two-dimensional) continuum of solutions exists even if we impose the usual *a-priori* restriction that explosively divergent solutions are ruled out, if these solutions at some point violate feasibility constraints. The multiplicity of non-explosive solutions in this model is due to the fact that both the inflation rate and consumption are non-predetermined state variables. This means that for neither state variable the boundary condition takes the form of an initial condition given by history. We impose the standard condition that discontinuous changes in the *level* of private consumption and the rate of inflation are permitted only at instants that news arrives. Except at such instants, solutions for c and π are required to be continuous functions of time.

The solutions that are permissible and 'well-behaved' are all orbits contained within the orbit (drawn with reference to the normal steady state Ω^N) that passes through (and just

[24] That is, two complex conjugate roots with zero real parts.

'touches') the lower bound steady state Ω^L. It is shown as the shaded area in Figure 2. The highest rate of inflation for which there exists at least one solution that will not eventually land the system in the permanent lower bound zone is $\breve{\pi}$. Candidate solutions starting outside this orbit will explode. The explosive trajectories will sooner or later exhibit ever increasing rates of deflation (negative inflation) and consumption rising without bound. To the left of the vertical line NL through $\hat{\pi}$, the short nominal interest rate is at the lower bound, $i = i_M$. For trajectories that lie completely to the left of NL, the entire term structure of interest rates is stuck at i_M, that is, nominal interest rates at all maturities are at their lower bound. Note that for any initial rate of inflation, there always exist solutions that will cause the system to end up in the permanent lower bound zone.

With all nominal interest rates eventually at their lower bound, the rising rates of deflation characteristic of the explosive solution trajectories have ever rising real interest rates. From the consumption Euler equation, the growth rate of consumption will be rising steadily. This explosive growth of consumption does not run into a binding capacity constraint, because the capacity constraint in this model, $\bar{y}$, does not represent a strict upper bound to actual output. With either the Old-Keynesian or the New-Keynesian Phillips curve, actual output can exceed capacity output by any amount. This should be seen as a weakness of these models. Within the strict logic of the model, however, the hyper-deflationary solutions cannot be ruled out a-priori.

We saw in Section III.1 that, in the Old-Keynesian model, raising the target rate of inflation, π^*, had no effect on the behaviour of the economic system if it were to find itself with a rate of inflation below the lower bound steady-state rate of inflation ($\pi < \pi^{**} = \bar{i}_M - \delta$). A similar proposition applies in the New Keynesian model.

Note first that, from Figure 2, for any initial inflation rate below π^{**} there exists no solution that will not, sooner or later, end up in the permanent lower bound zone, with nominal interest rates at all maturities stuck at their lower bound.[25] Since π^{**} is independent of π^*, the range of *low* values of the initial inflation rate that will land the economy into this permanent lower bound zone is unaffected by the level of the target rate of inflation.

Second, note that the value of $\hat{\pi}$, the critical inflation rate below which the *short* (strictly the instantaneous) nominal interest rate is at its lower bound, is actually raised by an increase in the target rate of inflation.[26]

Of course, the normal steady-state rate of inflation also increases one-for-one with an increase in π^*, and $\breve{\pi}$, the highest inflation rate for which there exists at least one solution that does not end up in the permanent lower bound zone also increases as π^* increases.[27]

Can Expectational Stability be used to rule out certain rational expectations equilibria?

In a number of papers and books, Evans and Honkapohja [2001] have proposed that only those rational expectations equilibria that satisfy a condition or refinement called expectational stability or E-stability, should be considered admissible. A rational

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[25] The model has no trouble handling the experiment of, say, an unanticipated, immediate and permanent increase in π^*. We do not accept the argument that, since a change in π^* is a change in policy regime (in a parameter of the policy *rule*), rational expectations (assumed throughout) cannot reasonably be applied to analyse what transitional effects would occur in response to such a regime change. Once the economy has a rate of inflation so low that the system will eventually end up in the permanent lower bound zone (that is, a rate of inflation below π^{**}, the only thing a private agent must know in order to respond to a change in π^* in the manner we assume, is that π^{**} is independent of π^*. She does not have to know the new value of π^*, or even that it has changed. All she has to know is that π^* does not matter. If she believes that, it will not matter.

[26] $\dfrac{\partial \hat{\pi}}{\partial \pi^*} = \dfrac{\gamma - 1}{\gamma} > 0$.

[27] For the closed orbits centred on π^*, $\left.\dfrac{\partial \pi}{\partial \pi^*}\right|_{c=\bar{c}} = \dfrac{\pi}{\pi - \pi^*} > 0$ when $\pi = \breve{\pi}$.

expectations equilibrium is E-stable if it is locally asymptotically stable under least squares learning. McCallum [2001, 2002] also argues that 'non-fundamental' solutions, such as the explosively deflationary solutions of our model, are of dubious relevance because they are not 'adaptively learnable', whereas the well-behaved solutions are. 'Reasonable learning mechanisms', 'adaptive learning' and related notions and concepts are a major new area of economic enquiry. We cannot possibly hope to do it justice here and have to limit ourselves to the briefest possible expression of our reservations about the use of these refinements in dynamic macroeconomic models like ours.

Our objections to the proposition that rational expectations equilibria are not economically interesting if they are not E-stable are no less pertinent for not being original. There is only one way to be fully informed and rational. There are infinitely many ways of being 'reasonable', 'boundedly rational' etc. Any particular adaptive learning rule, such as the recursive least squares of Evans and Honkapohjah, is *ad-hoc* and *arbitrary*, unless there is empirical evidence, say from cognitive psychology, that empirically it is a reasonable representation of how agents tend to behave in the kind of circumstances described by the model. We have so far seen no evidence to this effect.

The adaptive learning of the 1990s does not appear to get us much beyond the adaptive expectations of the 1960s. The point of the 'rational expectations revolution' was to cut through the intractable knot of 'boundedly rational learning' by making a very strong equilibrium assumption.[28] We view the assumption of rational expectations as an appropriate, indeed essential, application of Occam's razor.

[28] There are some technical issues making the Evans-Honkapohja and McCallum E-stability test problematic. Our New-Keynesian rational expectations model is non-linear. Robust E-stability results only have been established for linear models. In addition, our adaptive

Eliminating the lower bound equilibria by paying a negative interest rate on currency.

As in the Old-Keynesian model, substituting equation (10) for equation (9) eliminates the lower bound equilibria, the lower bound steady state $\pi = \pi^{**}$ and indeed the whole lower bound zone ($\hat{\pi}$ does not exist). The entire state space (any value for π, any positive value for c) is covered by closed orbits. The saddlepoint trajectories have been eliminated. By keeping the short nominal interest rate on currency at or below the short nominal interest rate on non-monetary financial instruments, the behaviour of the model in $c-\pi$ space is the same as that of an economy with a constant nominal interest rate on currency (set at zero, say) for which the lower bound on the short nominal interest rate on non-monetary financial instruments is simply ignored.

IV. Conclusion.

To avoid the zero bound equilibrium trap, or to get out of one once an economy has landed itself in it, there are just two policy options. The first is to wait and hope for some positive shock (fiscal, private or external) to the effective demand for goods and services. The second option is to lower the zero nominal interest rate floor by taxing currency. If a rule were followed that kept the nominal interest rate on currency systematically at or below the nominal interest rate on non-monetary instruments, the economy could never end up in a zero bound equilibrium or in a liquidity trap. Such a rule would require the authorities to be able to pay interest, negative or positive, on currency, that is, to turn currency into 'Gesell money'.

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learners would have to master continuous time estimation in order to use recursive least squares in our model. These are minor issues, however.

The transactions and administrative costs associated with what amounts to periodic currency reforms would be non-trivial.[29] Such carry tax administration costs (currency conversion costs) could be reduced by lengthening the interval between tax assessments (conversions), but they would remain significant.

The cost of administering a 'carry tax' on base money has to be set against the costs of two alternatives. The first is the cost of keeping the nominal interest rate on currency at zero and risking ending up in a zero lower bound equilibrium. The second is the cost of keeping the nominal interest rate on currency at zero and pursuing a high nominal interest rate policy in order to minimise the risk of ending up in a zero lower bound equilibrium.[30] With a zero nominal interest rate on currency, the short nominal interest rate on non-monetary financial instruments is the opportunity cost of holding currency. The Allais-Baumol-Tobin view of the demand for money, which applies in our New-Keynesian model, implies that as the opportunity cost of money rises shoeleather costs go up and real resources are wasted in more frequent trips to the bank. Our simple example of a rule for paying interest on currency maintains a constant, non-negative wedge between the short nominal interest rate on bonds and the nominal interest rate on currency. Under this rule, the opportunity cost of holding currency is constant, and therefore independent of level of the nominal interest rate on bonds. By reducing the wedge to zero, the economy can be moved arbitrarily close to satiation with real money balances; a zero wedge is also an equilibrium.

A high nominal interest rate policy will be a policy of high anticipated inflation. The costs of anticipated inflation include the menu costs associated with any nonzero rate of

[29] Marking currency periodically, say by stamping it, in order to certify it 'current' as regards interest due, is logistically equivalent to replacing an existing currency by a new currency, that is, a currency reform.

[30] The need to trade off the administrative cost of a negative nominal interest rate policy against the opportunity cost of avoiding the zero bound has been emphasised by Goodfriend [2000, pp. 1017-8] and by Buiter and Panigirtzoglou [1999, pp 21-2 and 38-9].

inflation, as well as any further costs resulting from imperfect indexation in the private and public sectors. Empirically, higher inflation also tends to be more volatile and more uncertain inflation, which imposes further costs (see e.g. Fischer [1981, 1994]). It is not obvious that currency carry tax administration costs would necessarily exceed shoeleather costs, menu costs and the costs associated with non-zero output gaps. Paying negative interest on base money may turn out to be of more than academic interest.

Appendix 1. A Model of the Lower Bound Trap

We model a simple, closed endowment economy with a single perishable commodity that can be consumed privately or publicly.

Households

A representative infinite-lived, competitive consumer maximises for all $t \geq 0$ the utility functional given in (A1.1) subject to his instantaneous flow budget identity (A1.2), his solvency constraint (A1.3) and his initial financial wealth. We use the simplest money-in-the-direct-utility-function approach to motivate a demand for money even when it is dominated as a store of value. Instantaneous felicity therefore depends on consumption and real money balances. We define the following notation; c is real private consumption, y is real output, τ is real (lump-sum) taxes, M is the nominal stock of base money (currency), B is the nominal stock of short (zero maturity) non-monetary debt, i is the instantaneous risk-free nominal interest rate on non-monetary debt, i_M, is the instantaneous risk-free nominal interest rate on money (or the 'own' rate on money), P is the price level in terms of money, a is the real stock of private financial wealth, m is the stock of real currency and b the stock of real non-monetary debt.

$$\int_t^{\infty} e^{-\delta(v-t)}[\frac{1}{1+\eta}\ln c(v)+\frac{\eta}{1+\eta}\ln m(v)]dv$$

$$\eta > 0 \qquad \text{(A1.1)}$$

$$\delta > 0$$

$$\dot{M}+\dot{B} \equiv P(y-\tau-c)+iB+i_M M$$

$$\text{(A1.2)}$$

$$c \geq 0; M \geq 0$$

$$\lim_{v\to\infty} e^{-\int_t^v i(u)du}[M(v)+B(v)] \geq 0 \quad \text{(A1.3)}$$

$$M(0)+B(0)=\overline{A}(0) \quad \text{(A1.4)}$$

By definition,

$$a \equiv \frac{M+B}{P} \quad \text{(A1.5)}$$

The household budget identity (A2) can be rewritten as follows

$$\dot{a} \equiv ra + y - \tau - c + (i_M - i)m \quad \text{(A1.6)}$$

where r, the instantaneous real rate of interest on non-monetary assets, is defined by

$$r \equiv i - \pi \quad \text{(A1.7)}$$

and $\pi \equiv \frac{\dot{P}}{P}$ is the instantaneous rate of inflation.

The household solvency constraint can now be rewritten as

$$\lim_{v\to\infty} e^{-\int_t^v r(u)du} a(v) \geq 0 \quad \text{(A1.8)}$$

and the intertemporal budget constraint for the household sector can be rewritten as:

$$\int_t^\infty e^{-\int_t^v r(u)du}\left[c(v)+\tau(v)+[i(v)-i_M(v)]m(v)-y(v)\right]dv \leq a(t) \quad \text{(A1.9)}$$

The first-order conditions for an optimum imply that the solvency constraint will hold with equality. Also,

$$\dot{c} = (r-\delta)c \quad \text{(A1.10)}$$

and for $i \geq i_M$,

$$m = \left(\frac{\eta}{i-i_M}\right)c \quad \text{(A1.11)}$$

If $i < i_M$, currency would dominate non-monetary financial assets ('bonds') as a store of value. Households would wish to take infinite long positions in money, financed by infinite short positions in non-monetary securities. The rate of return on the portfolio would be infinite. This cannot be an equilibrium.

If $i = i_d$, currency and bonds are perfect substitutes as stores of value. This will, from the point of view of the household's utility functional, be the first-best equilibrium, characterised by satiation in real money balances. With the logarithmic utility function, satiation occurs only when the stock of money is infinite. Provided the authorities provide government money and absorb private bonds in the right (infinite) amounts, this can be an equilibrium.

There is a continuum of identical consumers whose aggregate measure is normalised to *1*. The individual relationships derived in this section therefore also characterise the aggregate behaviour of the consumers. The consumption function for our model is

$$c(t) = \frac{\delta}{1+\eta}\left[\frac{M(t)+B(t)}{P(t)} + \int_t^{\infty} e^{-\int_t^v [i(u)-\pi(u)]du}[y(v)-\tau(v)]dv\right] \qquad \text{(A1.12)}$$

Government

The budget identity of the consolidated general government and central bank is given in (A1.12). The level of real public consumption is denoted $g \geq 0$.

$$\dot{M} + \dot{B} \equiv iB + i_M M + P(g-\tau) \qquad \text{(A1.13)}$$

Again, the initial nominal value of the government's financial liabilities is predetermined, $M(0) + B(0) = \overline{A}(0)$.

This budget identity can be rewritten as

$$\dot{a} \equiv ra + g - \tau + (i_M - i)m \qquad \text{(A1.14)}$$

The government solvency constraint is

$$\lim_{v\to\infty} e^{-\int_t^v r(u)du} a(v) \leq 0 \qquad \text{(A1.15)}$$

Equations (A1.14) and (A1.15) imply the intertemporal government budget constraint:

$$\int_t^{\infty} e^{-\int_t^v r(u)du}\left[\tau(v)+[i(v)-i_M(v)]m(v)-g(v)\right]dv \geq a(t) \qquad \text{(A1.16)}$$

Government consumption spending is exogenous. To ensure that public consumption spending does not exceed total available capacity resources, $\bar{y}>0$, we therefore impose $g<\bar{y}$. With a representative consumer, this model will exhibit debt neutrality or Ricardian equivalence. Without loss of generality, we therefore assume that lump-sum taxes are continuously adjusted to keep the nominal stock of public debt (monetary and non-monetary) constant, $\dot{A}(t)=0,\ t\geq 0$, that is,

$$\begin{aligned}\tau &= g+ia+(i_M-i)m \\ &= g+i\frac{\bar{A}(0)}{P}+(i_M-i)m\end{aligned} \qquad \text{(A1.17)}$$

Appendix 2. Calvo's model of staggered price setting

Calvo's model views monopolistically competitive individual price setters as facing randomly timed opportunities for changing the nominal price of their product. The timing of opportunities to change the price is governed by a Poisson process with parameter $\lambda > 0$. There is a continuum of price setters distributed evenly on the unit circle. The parameter λ therefore measures not only the instantaneous probability of any price setter's contract being up for a change, but also the fraction of the population of price setters changing contract prices at any given moment. The model assumes perfect foresight. It implies that the (natural logarithm of) the current contract price, *w,* is a forward-looking moving average with exponentially declining weights of the logarithm of the (expected) future general price level, $p \equiv \ln P$, and of (expected) future excess demand, that is

$$w(t) = \lambda \int_t^\infty \{p(s) + \varphi[y(s) - \overline{y}]\} e^{-\lambda(s-t)} ds + e^{\lambda t} \lim_{\tau\to\infty} e^{-\lambda\tau} w(\tau) \qquad \text{(A2.1)}$$

$$\lambda, \varphi > 0$$

The current contract price *w(t)* is therefore a non-predetermined state variable.

The general price level, *p*, is a backward-looking, exponentially declining moving average of past contract prices.

$$p(t) = e^{-\lambda(t-t_0)} p(t_0) + \delta \int_{t_0}^t w(s) e^{-\lambda(t-s)} ds \qquad \text{(A2.2)}$$

$$t_0 \leq t$$

The (natural logarithm of the) general price level is therefore a predetermined state variable. From (A2.1) and (A2.2) it follows that the rate of inflation of the general price level, $\pi \equiv \dot{p}$, is given by

$$\pi(t) = \lambda\varphi \int_t^\infty [y(s) - \overline{y}] ds + k \qquad \text{(A2.3)}$$

where k is an arbitrary constant, which can be given the interpretation of the long-run rate of inflation, that is, $k = \lim_{\tau \to \infty} \pi(\tau)$. This implies the following 'quasi-accelerationist' Phillips curve

$$\dot{\pi}(t) = -\lambda\varphi[y(t) - \bar{y}] \tag{A2.4}$$

Thus in Section III.2, we have $\beta \equiv -\lambda\varphi < 0$.

The key distinction between (A2.4) and the old-style backward-looking accelerationist Phillips curve is that in (A2.4), the rate of inflation, $\pi \equiv \dot{p}$, is, unlike the price level, p, a non-predetermined or 'forward-looking' state variable.

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Figure 1

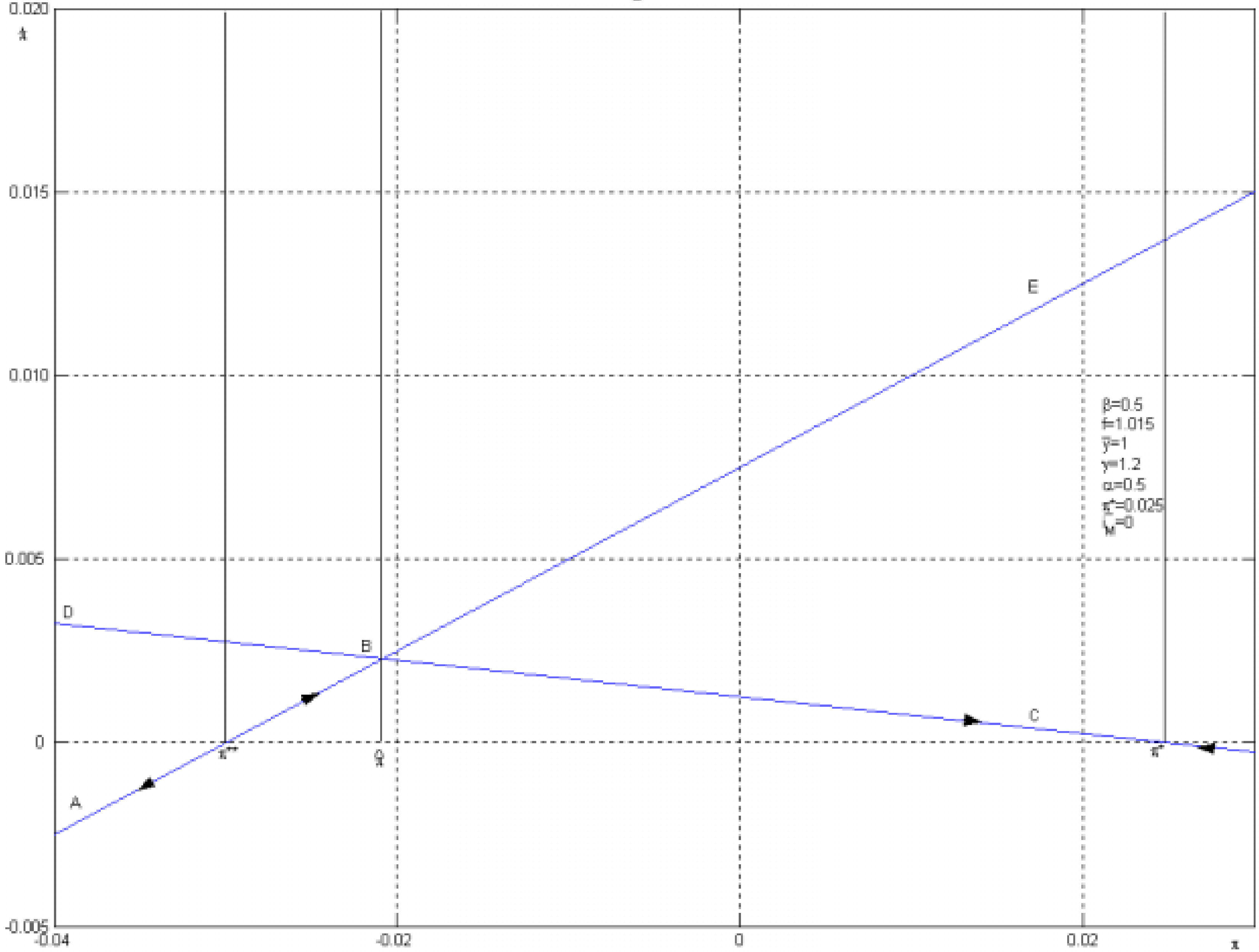

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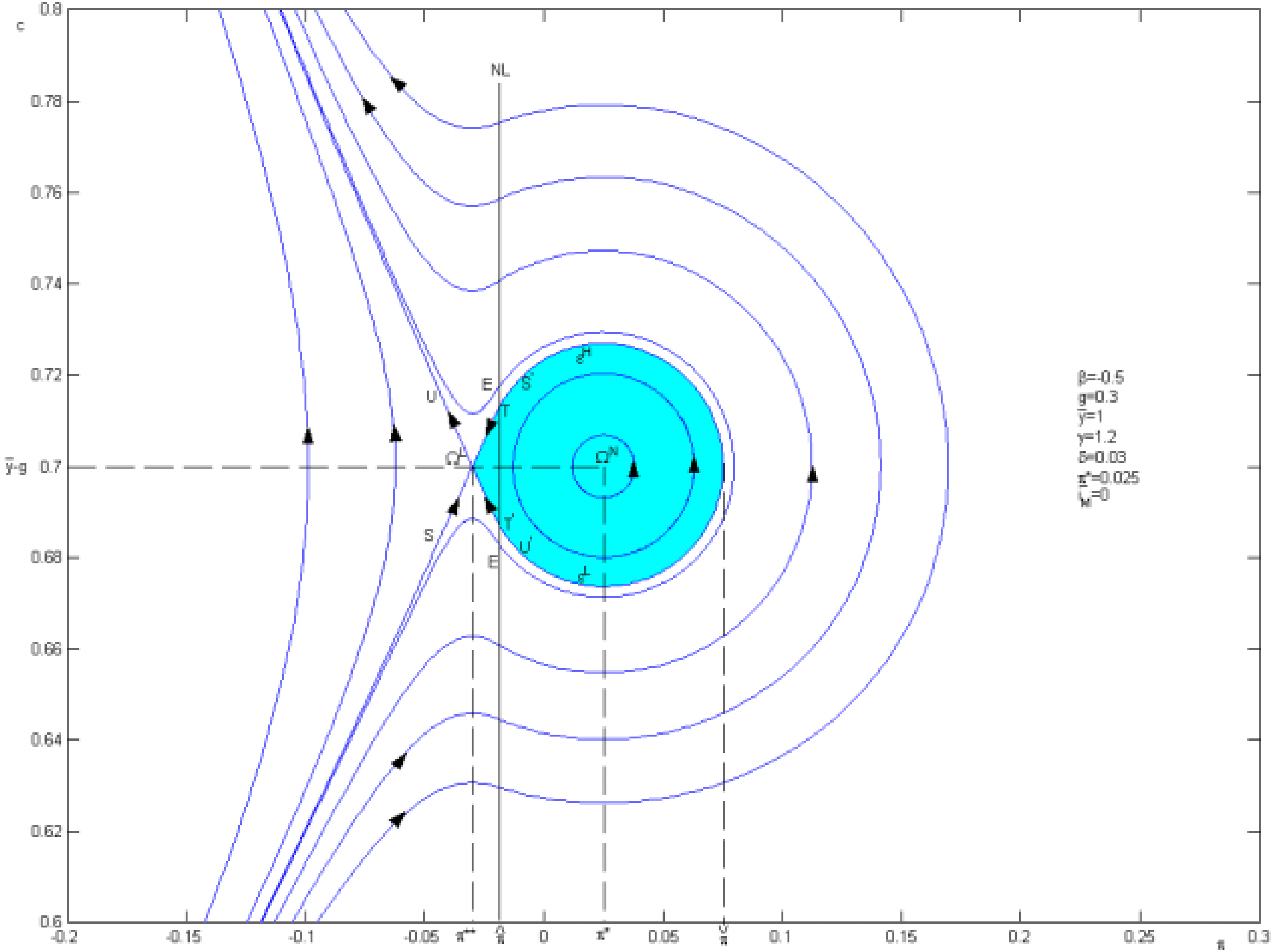
Figure 2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