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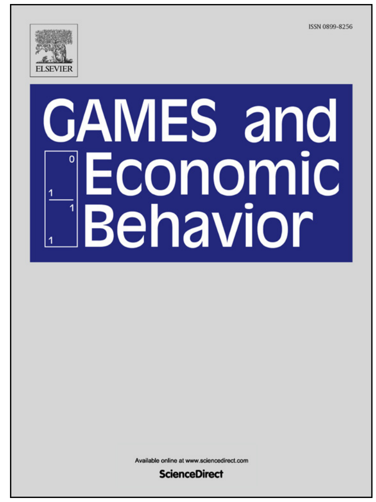

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Auctions with Selective Entry*

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Abstract

We consider auctions with entry based on a general analytical framework we call the Arbitrarily Selective (AS) model. We characterize symmetric equilibrium in a broad class of standard auctions within this framework, in the process extending the classic revenue equivalence results of Myerson (1981), Riley and Samuelson (1981) and Levin and Smith (1994) to environments with endogenous and arbitrarily selective entry. We also explore the relationship between revenue maximization and efficiency, showing that a revenue maximizing seller will typically employ both higher-than-efficient reservation prices and higher-than-efficient entry fees.

1 Introduction

Entry is a quantitatively and qualitatively important aspect of many real-world auction processes, but theoretical analysis of auctions with entry has primarily been limited to a few notable but restrictive special cases. Two paradigmatic examples in the literature are

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Samuelson (1985) (henceforth S), who proposes a simultaneous entry model in which potential bidders know their valuations *ex ante* but must incur a fixed cost to submit bids, and Levin and Smith (1994) (henceforth LS), who consider simultaneous entry under the alternative assumption that bidders learn their valuations after incurring the fixed cost. A common theme in this literature is that different assumptions on entry can produce very different practical and policy conclusions. For example, under the LS model a revenue-maximizing seller will set a zero reserve price and maximize social welfare, whereas in the S model revenue maximization requires a binding, socially inefficient reserve price. Hence while the existing literature contains many important insights on auctions with entry, it permits few overarching theoretical and policy conclusions.

This paper seeks to generalize several core results on auctions with entry to a framework we call the *Arbitrarily Selective (AS)* model. First suggested by Ye (2007) and subsequently explored by Marmer et al. (2013), Roberts and Sweeting (2013), Gentry and Li (2014), Bhattacharya and Sweeting (2015), and Lu and Ye (2015) among others, the AS model assumes that potential bidders receive imperfect signals of their valuations prior to entry, make simultaneous entry decisions based on these signals, then learn their valuations and submit bids. This structure imposes minimal *a priori* restrictions on pre-entry information, requiring only that higher signals lead bidders to expect stochastically higher post-entry valuations. It nests the LS model as a special case when signals and values are independent, and approaches the S model as the special case where values are determined by signals. The AS model thus represents an ideal basis for a general analysis of auctions with entry.

Motivated by these considerations, we extend the standard independent private values auction environment to accommodate endogenous and selective (AS) entry, focusing on a class of mechanisms we call *standard auctions with simultaneous entry* in the sense of Bhattacharya and Sweeting (2015).[1] For this class of auctions, we establish the following three

[1]Roughly, this class of auctions consists of mechanisms such that only the highest bidder has a positive probability of award, and the probability of award depends only on the highest bid. We borrow the label *standard auctions with simultaneous free entry* from Bhattacharya and Sweeting (2015), who compare auctions with free entry with a range of other mechanisms by which the seller might attempt to (explicitly or

results. First, we formally extend the classic revenue equivalence theorem of Myerson (1981), Riley and Samuelson (1981), and Levin and Smith (1994) to environments with endogenous and selective (AS) entry. Second, we characterize the efficient mechanism within the class of standard auctions with free entry, and show that the seller's revenue-maximizing auction will be inefficient in general. While Levin and Smith (1994) have long recognized that the congruence between revenue maximization and efficiency would fail when asymmetry among bidders or affiliated values are introduced, the latter result further clarifies the sense in which this congruence depends pivotally on the "knife edge" informational assumption of LS entry. Finally, we explore optimal reservation prices and entry fees under AS entry, showing a revenue-maximizing seller will typically prefer to set both positive in general.

This study builds on and extends a substantial literature on auctions with endogenous entry. In addition to the studies cited above, notable early theoretical contributions to this literature include McAfee and McMillan (1987) and Smith and Levin (1996); the former explore a model of sequential entry where entry is interpreted as value discovery, the latter show that entry can lead a second-price auction to revenue dominate a first-price auction even when bidders are risk averse. In more recent work, Lu (2010) and Moreno and Wooders (2011) explore an extended version of the basic LS model in which bidders have private entry costs. Lu characterizes equilibrium, efficiency, and optimal auction design in this extended model, while Moreno and Wooders note that in the presence of private entry costs a revenue-maximizing seller will no longer achieve efficiency if ex ante entry fees are not allowed. Xu et al. (2013) study auctions with resale in a setting where bidders have either high or low entry costs and know their valuations before entry, showing that resale may introduce speculative motivations for entry, with ambiguous effects on efficiency and welfare. Finally, Bhattacharya and Sweeting (2015) explore the broader mechanism design implications of endogenous and selective (AS) entry. Bhattacharya and Sweeting (2015) show that the seller can often improve both revenue and efficiency by switching to one of several

implicitly) regulate entry. In Bhattacharya and Sweeting (2015), "free entry" means that potential bidders simultaneously and non-cooperatively decide whether to enter.

mechanisms which regulate entry in ways not permitted by the class of standard auctions with free entry. The current study complements these by providing a set of analytical results on optimal revenue and efficiency within the class of standard auctions with free AS entry.

Although our analysis is primarily theoretical, our investigation is motivated by a substantial empirical literature on auctions with entry. Earlier work in this literature has established the relevance of entry in a wide range of applications: Bajari and Hortacsu (2003) in online auctions, Hendricks et al. (2003) in outer continental shelf "wildcat" auctions, Li and Zheng (2009) and Krasnokutskaya and Seim (2011) in highway construction procurement auctions, and Li and Zheng (2012), Li and Zhang (2015), Athey et al. (2011) and others in timber auctions, to mention just a few. More recently, a smaller literature has developed exploring empirical properties of the AS model specifically: notable contributions to this literature include Marmer et al. (2013), Gentry and Li (2014), Roberts and Sweeting (2013), and Bhattacharya et al. (2014) explore specification testing, nonparametric identification, and empirical applications of the AS model respectively. This study provides a theoretical counterpart to this recent application-oriented work.

The rest of the paper is organized as follows. Section 2 outlines the structure of the AS model, and Section 3 characterizes equilibrium entry and payoffs under standard auction rules. In this section, we will also establish revenue equivalence in the class of auctions considered. Section 4 establishes that the seller's optimal auction will in general be inefficient, and Section 5 explores revenue-maximizing policies explicitly. Finally, Section 6 concludes. Technical proofs are relegated to an online appendix.

2 The Arbitrarily Selective (AS) model

We study an auction of a single indivisible good with endogenous entry. There is one seller facing N potential bidders who have independent private values for the good being sold. The seller and all potential bidders are risk-neutral. Timing of the auction game is as follows.

First, in Stage 1, each potential bidder i observes a private signal s_i of her (yet unknown) private value v_i, which falls in $\mathcal{V} = [0, \overline{v}]$, and all potential bidders simultaneously choose whether to enter the auction. Each entering bidder must pay an entry cost $c(> 0)$, which may be interpreted as the net of opportunity, learning, and bid preparation costs. The seller may charge an entry fee/subsidy e to each entrant.[2] In Stage 2, the n bidders who chose to enter in Stage 1 learn their true values v_i and submit bids for the object being sold. Auction outcomes (allocation and payments) are determined according to a standard auction mechanism M, which will be formulated in Definition 1 and is common knowledge to all potential bidders. Seller's value is $v_0 \in \mathcal{V} = [0, \overline{v}]$.

The value-signal structure and information structure of the *Arbitrarily Selective (AS)* entry model are further detailed in the following assumptions.

Assumption 1. *Each bidder i draws value-signal pairs (V_i, S_i) from a joint cumulative distribution $F(v, s)$ with density $f(v, s)$ satisfying the following properties:*

(i) The support of the random variable V_i is a bounded interval $\mathcal{V} = [0, \overline{v}]$, and the joint density distribution $f(v, s)$ is continuous.

(ii) For each bidder i, the conditional distribution of V_i is stochastically ordered in S_i: $s' \geq s$ implies $F(v|s') \leq F(v|s)$.

(iii) (V_i, S_i) are independent across bidders: $(V_i, S_i) \perp (V_j, S_j)$ for all $j \neq i$.

(iv) Without loss of generality, we normalize first-stage signals S_i to have a uniform marginal distribution on $[0, 1]$: $S_i \sim U[0, 1]$.[3]

The stochastic ordering condition in Assumption 1(ii) ensures that higher signals are "good news" in the sense of leading bidders to expect (weakly) stochastically higher distributions of valuations, but otherwise imposes minimal restrictions on the nature of selection.

[2] A positive (resp. negative) e is interpreted as an entry fee (resp. subsidy).

[3] If the marginal cumulative distribution function $G(\cdot)$ of S_i differs from that of a standard uniform distribution, one can work on alternative signals $\tilde{S}_i = G(S_i), \forall i$, which must follow a standard uniform distribution on $[0, 1]$.

In particular, the LS model corresponds to the "knife edge" case where $V_i \perp S_i$, while the S model is approached as the limiting case where S_i fully determines V_i.

Assumption 2. *Information structure:*

(i) Each bidder i observes own signal s_i prior to entry, but does not learn own value v_i until after entry.

(ii) The number of potential bidders N is known to all participants; the number of entrants n is either hidden until the auction concludes or revealed to all entrants before their bidding decisions are made.

Assumption 2(*ii*) ensures that the entrants' information on entry is symmetric among themselves, which entails a symmetric monotonic bidding strategy among entrants.

In the spirit of Riley and Samuelson (1981) and Levin and Smith (1994), we frame our analysis in terms of a general class of mechanisms we call *standard auctions*:

Definition 1. A *standard auction M* is any auction mechanism such that:

1. Mechanism rules are anonymous.
2. If award of the good is made, it is to the entrant submitting the highest bid.
3. The probability of award depends only on the highest bid, the award probability weakly increases with the highest bid.
4. For any symmetric distribution of values among entrants and any distribution of the number of entrants, there exists a unique symmetric strictly increasing bidding equilibrium.
5. An entrant with the lowest value gets non-negative finite expected payoff $\pi_M(0, n)$ for each number of entrants (i.e. n); $\pi_M(0, n)$ weakly decreases with n and is independent of the value distribution of entrants and the distribution of number of entrants.

This class of standard auctions covers most commonly used mechanisms: in particular, first-price, second-price and all-pay auctions with public or secret reservation prices.

As usual, we frame our analysis in terms of direct mechanisms. By the Revelation Principle, any mechanism has an equivalent truthful direct mechanism, therefore there is no loss of generality. By Assumption 2(ii) (information structure) and Definition 1 (standard auctions), the good can be awarded only to the entrant with the highest value. Let the *award rule* $\alpha_M(y)$ denote the probability that mechanism M results in a sale when the highest (truthfully reported) value among entrants is y. From Definition 1 (part 3), the award rule $\alpha_M(y)$ is weakly increasing in the maximum entrant value y.

3 Equilibrium

In this section, we characterize equilibrium entry behavior and ex ante information rents in the unique symmetric monotone equilibrium induced by standard auction M. We then apply these results to extend the classic revenue equivalence theorems of of Myerson (1981), Riley and Samuelson (1981) and Levin and Smith (1994) to settings with AS entry.

3.1 Stage 2: Entrant payoff for given entry threshold $\bar{s}$

Suppose that in Stage 1 each potential bidder chooses to enter if and only if $s_i \geq \bar{s}$. Then the (selected) cumulative value distribution function of a representative entrant is given by

$$F^*(v;\bar{s}) \equiv \frac{1}{1-\bar{s}} \int_{\bar{s}}^{1} F(v|s)\, ds, \tag{1}$$

where $F(v|s)$ stands for the a potential bidder's cumulative value distribution function conditional on signal s. $F^*(v;\bar{s})$ is stochastically increasing in $\bar{s}$ by Assumption 1(ii).

By Definition 1, an entrant i with value v will win against potential bidder j in one of two events: either j does not enter, or bidder j enters but draws a value less than v. Let

$F_w^*(v;\bar{s})$ denote the joint probability of these events:

$$F_w^*(v;\bar{s}) = \bar{s} + (1-\bar{s}) \cdot F^*(v;\bar{s}).$$

Differentiating $F_w^*(v;s)$ with respect to $\bar{s}$ we obtain:

$$\frac{\partial}{\partial \bar{s}} F_w^*(v;\bar{s}) = 1 - F(v|\bar{s}) \geq 0.$$

Hence the distribution $F_w^*(v;\bar{s})$ is stochastically decreasing in $\bar{s}$, a fact we will reference repeatedly in the derivations below.

The form of the equilibrium bidding function will obviously depend on the payment rule of the mechanism M, which is not specified in Definition 1 as it covers a wide spectrum of standard auctions. Nevertheless, via standard arguments in mechanism design, we can characterize an entrant's expected Stage 2 *payoff* in any standard auction as follows.

Proposition 1. *For a given entry threshold $\bar{s}$, in any symmetric monotone Stage 2 bidding equilibrium of any standard auction mechanism M, the expected Stage 2 payoff of an entrant with value v is given by*

$$\pi_M(v;\bar{s},N) = \int_0^v \alpha_M(y) \cdot F_w^*(y;\bar{s})^{N-1} dy + \pi_M(0;\bar{s},N), \tag{2}$$

where $\pi_M(0;\bar{s},N) = \sum_{n=0}^{N-1} p(n;\bar{s},N-1)\pi_M(0,n+1)$ is an entrant's expected payoff if her value is 0, in which $p(n;\bar{s},N-1) = C_{N-1}^n(1-\bar{s})^n \bar{s}^{(N-1)-n}$ is the probability that an entrant faces n rivals in Stage 2 bidding competition.

Proposition 1 immediately follows Lemma 1 of Myerson (1981), which says that the derivative of expected payoff of a bidder with respect to their own value is simply the expected winning probability. In our environment, an entrant with value v wins with probability $\alpha_M(y) \cdot F_w^*(y;\bar{s})^{N-1}$ for given entry threshold $\bar{s}$.

3.2 Stage 1: Equilibrium entry threshold s^*

Given the Stage 2 payoff $\pi_M(v;\bar{s},N)$, we next characterize the symmetric Stage 1 equilibrium entry threshold s^*. Toward this end, consider the Stage 1 decision faced by potential bidder i with signal s_i facing $N-1$ potential rivals who enter according to $\bar{s}$. Bidder i's *ex ante* expected Stage 2 payoff if she enters is given by

$$\begin{aligned}\Pi_M(s_i;\bar{s},N) &= E_v[\pi_M(v;\bar{s},N)|s_i] = \int_0^{\bar{v}} f(v|s_i)\int_0^{v}\alpha_M(y)\cdot F_w^*(y;\bar{s})^{N-1}dy + \pi_M(0;\bar{s},N)\\ &= \int_0^{\bar{v}}\alpha_M(y)\cdot[1-F(y|s_i)]\cdot F_w^*(y;\bar{s})^{N-1}dy + \pi_M(0;\bar{s},N),\end{aligned}$$

where the second line follows from Proposition 1 and the third follows from integration by parts. The key properties of this ex ante profit function are stated in the following lemma.

Lemma 1. *Given entry threshold $\bar{s}$ and standard auction M,* ex ante *expected Stage 2 profit for an entrant with Stage 1 signal s_i is*

$$\Pi_M(s_i;\bar{s},N) = \int_0^{\bar{v}}\alpha(y)\cdot[1-F(y|s_i)]\cdot F_w^*(y;\bar{s})^{N-1}dy + \pi_M(0;\bar{s},N). \tag{3}$$

This function is weakly increasing in s_i for all $(\bar{s},N)$, strictly increasing in $\bar{s}$ for all (s_i,N), and strictly decreasing in N for all s_i and any $\bar{s}<1$.

Bidder i will choose to enter whenever expected net profit from entry is positive:

$$\Pi_M(s_i;\bar{s},N) \geq c+e. \tag{4}$$

This fact in turn implies a break-even condition which must hold at any candidate *interior* equilibrium $s^*\in(0,1)$:

$$\Pi_M(s^*;s^*,N)\equiv c+e,$$

that is, a bidder drawing signal $S_i=s^*$ must be indifferent to entry when potential rivals also enter according to s^*. Noting that $\Pi_M(s_i;\bar{s},N)$ is increasing in $(s_i,\bar{s})$, we conclude:[4]

Proposition 2. *A symmetric entry equilibrium in the AS model is characterized by a signal threshold s^* such that only bidders with $s_i \geq s^*$ choose to enter. This signal threshold is uniquely determined as follows.*

- *If $\Pi_M(0;0,N) > c+e$, then $s^*=0$ and all potential bidders always enter.*
- *If $\Pi_M(1;1,N) < c+e$, then $s^*=1$ and no potential bidder ever enters.*
- *Otherwise, the signal threshold s^* satisfies the break-even condition*

$$\Pi_M(s^*;s^*,N) \equiv c+e, \tag{5}$$

  where $\Pi_M(\cdot;\cdot,\cdot)$ is defined as in Lemma 1.

Furthermore, considered as a function of $(N,c+e)$, the equilibrium threshold $s_N^*(c+e)$ satisfies the following monotonicity properties:

- *For any $N \geq 1$, $s_N^*(c+e)$ is continuous and weakly increasing in $c+e$, with strict monotonicity whenever $s_N^*(c+e) \in (0,1)$.*
- *For any c, we have $N' > N$ implies $s_{N'}^*(c+e) \geq s_N^*(c+e)$. If in addition $s_N^*(c+e) \in (0,1)$, then $s_{N'}^*(c+e) > s_N^*(c+e)$ and $s_{N'}^*(c+e) \in (0,1)$.*

Proposition 2 characterizes the unique symmetric entry equilibrium of the AS model under any standard auction with simultaneous entry.

[4] A formal proof is omitted to save space. Please refer to the proof of Proposition 2 of Gentry and Li (2014) for details. In particular, Assumption 1(i) guarantees continuity of $\Pi_M(\bar{s};\bar{s},N)$ in $\bar{s}$.

3.3 Information rent

Lemma 1 and Proposition 2 pin down the information rent of a first stage type $s_i(> s^*)$:

$$\begin{aligned}\Delta\pi_M(s_i, s^*) &= \Pi_M(s_i; s^*, N) - \Pi_M(s^*; s^*, N) \\ &= \int_0^{\bar{v}} \alpha_M(y) \cdot [F(y|s^*) - F(y|s_i)] \cdot F_w^*(y; s^*)^{N-1} dy \geq 0.\end{aligned}$$

Therefore, we have

$$\frac{\partial \pi_M(s_i, s^*)}{\partial s_i} = -\int_0^{\bar{v}} \alpha_M(y) \cdot F_{s_i}(y|s_i) F_w^*(y; s^*)^{N-1} dy \geq 0,$$

which says that the information rent is at least weakly increasing with the first stage type s_i, with strict inequality if and only if $F_{s_i}(y|s_i) < 0$ for some $y \in [0, \bar{v}]$.

Lemma 2. *The ex ante expected surplus of a potential bidder is*

$$\begin{aligned}\Pi_M(s^*) &= \int_{s^*}^1 \Delta\pi_M(s_i, s^*) ds_i \\ &= (1 - s^*) \int_0^{\bar{v}} \alpha_M(y) \cdot [F(y|s^*) - F^*(y; s^*)] F_w^*(y; s^*)^{N-1} dy. \qquad (6)\end{aligned}$$

Note that according to (6), bidders' information rent depends on e and $\pi_M(0, n)$ only through the entry threshold s^*. It is clear that $\Pi_M(s^*)$ would be zero if and only if our model reduces to the "knife edge" case of LS where $F(y|s^*) - F^*(y; s^*) = 0$. In Lemma 4, we will further study how the information rent changes with seller instruments including entry fees and reservation prices.

3.4 Revenue equivalence

We next extend the seminal revenue equivalence result of Myerson (1981), Riley and Samuelson (1981) and Levin and Smith (1994) to accommodate endogenous and selective (AS) entry. By definition, for a standard auction M with entry fee/subsidy e inducing equilibrium entry

s^*, expected seller revenue is the difference between social welfare and total bidder surplus:

$$R_M(s^*) = TS_M(s^*) - N\Pi_M(s^*),$$

where $TS_M(s^*)$ denotes expected total surplus generated, and $\Pi_M(s^*)$ is the expected *ex ante* equilibrium payoff for any given potential bidder at equilibrium, which we have identified in Lemma 2. Note that like bidders' information rent, social welfare TS_M depends on e and $\pi_M(0, n)$ only through their impacts on the entry threshold s^*.

It is clear that under standard auction M and equilibrium entry s^*, we have

$$TS_M(s^*) = \int \{y\alpha_M(y) + v_0(1 - \alpha_M(y))\} d[F_w^*(y; s^*)^N] - N(1 - s^*)c.$$

By Proposition 2, any two mechanisms M_1 and M_2 having the same award rule and payoff of the lowest-value type must induce the same equilibrium entry s^* and information rents $\Pi_M(s^*)$. The conclusion that M_1 and M_2 are revenue equivalence then follows immediately:

Proposition 3 (Revenue Equivalence)**.** *Suppose standard auctions M_1 and M_2 implement the same award rule and render the same payoffs to the lowest-value type for each fixed n, and thus that they are revenue equivalent for each fixed n. Then for any entry fee/subsidy e, M_1 and M_2 are revenue-equivalent under AS entry.*

4 Efficiency versus revenue maximization

In this section, we study the relationship between social efficiency and revenue maximization in the class of standard auctions with simultaneous AS entry. We show that a revenue-maximizing seller will maximize social welfare only in the "knife edge" LS case: otherwise, the seller will generally prefer an inefficient mechanism. For current purposes, we assume that the allocation rule is fully described by a public reserve price $r \in [0, \bar{v}]$. In other words, an entrant with the highest value wins if and only if her value is above r. We focus on two policy instruments for the seller: a public reserve price r and an ex ante entry fee/subsidy e.

First consider social welfare. Fix an arbitrary entry threshold s; note that e and $\pi_M(0,n)$ can be chosen to induce this s without affecting welfare. Then for any r, total welfare is

$$TS(s,r) = v_0 F_w^*(r;s)^N + \int_r^{\bar{v}} y d[F_w^*(y;s)^N]dy - N(1-s)c, \tag{7}$$

which is clearly maximized at $r = v_0$ for all $s \in [0,1)$.

Now consider the entry threshold s_e maximizing $TS(s,v_0)$. Rearranging (7) via integration by parts produces the following equivalent representation for $TS(s,v_0)$:

$$TS(s,v_0) = \bar{v} - \int_{v_0}^{\bar{v}} F_w^*(y;s)^N dy - N(1-s)c. \tag{8}$$

This function is concave in s: $\frac{\partial TS(s,v_0)}{\partial s} = -N \int_{v_0}^{\bar{v}} F_w^*(y;s)^{N-1}[1 - F(y|s)]dy + Nc$, which decreases with s since $F_w^*(y;s)$ increases and $F(y|s)$ decreases with s. Social welfare is thus uniquely maximized at a threshold s_e satisfying the first-order condition[5]

$$-N \int_{v_0}^{\bar{v}} F_w^*(y;s_e)^{N-1}[1 - F(y|s_e)]dy + Nc \equiv 0. \tag{9}$$

As $\pi_M(0,n)$ does not affect $TS(s,r)$, we set $\pi_M(0,n) = 0, \forall n$. Then by Proposition 2, s_e must be the entry equilibrium when $e = 0$. We thereby obtain the following proposition, which generalizes the findings of Levin and Smith (1994) and Lu (2010) on ex ante efficient auctions when players must incur information costs to discover their values:

Proposition 4 (Efficiency)**.** *Within the class of standard auctions with simultaneous entry, social welfare is maximized in any ex post efficient auction M (which renders $\pi_M(0,n) = 0, \forall n$) with zero ex ante entry fee.*

We next show that in contrast to Levin and Smith (1994), this ex ante efficient auction is

[5] Without loss of generality, we assume s_e is an interior solution.

not revenue maximizing in general. Set $\pi_M(0,n) = 0$ and $r = v_0$. Now e is the seller's only policy choice. Let $s^*(e)$ denote the equilibrium entry threshold. By definition, seller revenue is the difference between social surplus and expected profits among potential bidders, which with slight abuse of notation we write as follows:

$$R(e) = TS(s^*(e), v_0) - N\Pi_M(s^*(e)).$$

The seller's optimal e^* satisfies

$$\frac{\partial R(e^*)}{\partial e} = \frac{\partial TS(s^*(e^*), v_0)}{\partial s}\frac{ds^*(e^*)}{de} - N\frac{d\Pi_M(s^*(e^*))}{de} = 0.$$

Recall that when $e = 0$, $\frac{\partial TS(s^*(0),v_0)}{\partial s} = 0$ as $e = 0$ induces efficient entry. We thus have

$$\frac{\partial R(0)}{\partial e} = -N\frac{d\Pi_M(s^*(0))}{ds}s^{*\prime}(0). \tag{10}$$

When entry is selective, neither $\frac{d\Pi_M(s^*(0))}{ds}$ nor $s^{*\prime}(0)$ in the RHS of (10) will be zero in general. Note that an interior s^* strictly decreases with total entry costs $c + e$. The monotonicity of Π_M with respect to s will be revealed by Lemma 4. Hence the seller's optimal policy need not correspond to the social optimum. We state this result formally as a lemma:

Lemma 3. *In general, a revenue-maximizing seller does not maximize social welfare.*

Intuitively, when potential bidders have no private ex ante information that is correlated to their ex post values, bidder surplus will be identically zero for all (e, r), so social welfare and seller revenue coincide and a revenue-maximizing seller will maximize total surplus. In contrast, when entry is strictly selective, bidder surplus is positive and decreasing in the entry threshold s^* as will be revealed by Lemma 4. Therefore, a revenue-maximizing seller will need to induce distortion to capture part of this additional surplus.

5 Revenue-maximizing auctions

Finally, we consider revenue-maximizing choices of the seller's policy variables e and r. Setting $\pi_M(0,n) = 0$ as above, observe that we may rewrite both total social welfare $TS(s,r)$ and potential bidders' information rent $\Pi(s,r)$ as functions of s and r as follows:

$$\begin{aligned} TS(s,r) &= \bar{v} + (v_0 - r)F_w^*(r;s)^N - \int_r^{\bar{v}} F_w^*(y;s)^N dy - N(1-s)c, \\ \Pi(s,r) &= (1-s)\int_r^{\bar{v}} [F(y|s) - F^*(y;s)]F_w^*(y;s)^{N-1} dy. \end{aligned}$$

The next lemma establishes several useful properties of $TS(s,r)$ and $\Pi(s,r)$:

Lemma 4. *(i) Social welfare $TS(s,r)$ is maximized uniquely at $r = v_0$, $\forall s$.*
(ii) For all $r \geq v_0$, $\frac{\partial TS(s,r)}{\partial s} \leq 0$.
(iii) Bidder information rent $\Pi(s,r)$ decreases with r, s.

Recalling that $R(s,r) = TS(s,r) - N\Pi(s,r)$. Using the above properties of $TS(s,r)$ and $\Pi(s,r)$, in the following proposition, we will establish that a revenue-maximizing seller will generally set both e and r positive.

Proposition 5 (Optimal entry fee and reserve)**.** *The optimal entry fee e^* must be nonnegative and the optimal reserve r^* must weakly exceed v_0. Furthermore, if entry is strictly selective in the sense that $s' > s$ implies $F(y|s') < F(y|s)$ for some $y \in [v_0, \bar{v}]$, then the following statements hold:*
(i) If the seller may set both e and r freely, then $e^* > 0$ and $r^* > v_0$;
(ii) If the reserve is constrained efficient ($r = v_0$), then the constrained optimum $e^* > 0$;
(iii) If the entry fee is constrained zero ($e = 0$), then the constrained optimum $r^* > v_0$.

6 Concluding remarks

In this paper, we study auctions with endogenous participation within the general Arbitrarily Selective (AS) entry model. We allow a broad class of standard auctions in our analysis and characterize symmetric equilibrium for this class of auctions. We find that the classic revenue equivalence results of Myerson (1981), Riley and Samuelson (1981) and Levin and Smith (1994) extend to environments with endogenous and arbitrarily selective entry. We further show that a revenue maximizing seller will typically employ both nontrivial reservation prices and positive entry fees, with revenue maximization inducing efficient entry only in the knife edge case of nonselective entry. These observations in turn illustrate the importance of accounting for selection in policy design and welfare analysis.

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