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Choice Deferral, Indecisiveness and Preference for Flexibility

Leonardo Pejsachowicz† and Séverine Toussaert‡

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Abstract

In a standard model of menu choice, we examine the behavior of an agent who applies the following Cautious Deferral rule: “Whenever in doubt, don’t commit; just leave options open.” Our primitive is a complete preference relation ≽ that represents the agent’s choice behavior. The agent’s indecisiveness is captured by means of a possibly incomplete (but otherwise rational) preference relation ≻. We ask when ≽ can be viewed as a Cautious Deferral completion of some incomplete ≻. Under the independence and continuity assumptions commonly used in the menu choice literature, we find that even the smallest amount of indecisiveness is enough to force ≽, through the above deferral rule, to exhibit preference for flexibility on its entire domain. Thus we highlight a fundamental tension between non-monotonic preferences, such as preferences for self-control, and tendency to defer choice due to indecisiveness.

Keywords: Incomplete preferences, preference for flexibility, choice deferral.

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1 Introduction

The primitive of the theory of choice among opportunity sets is a preference relation defined on a collection $X$ of subsets of a given space of alternatives. These subsets are interpreted as “menus” from which an alternative will be selected at some later (unmodeled) stage. With this dynamic interpretation in mind, Kreps [21] introduced a monotonicity property called “preference for flexibility,” which states that a decision maker (henceforth, DM) should weakly prefer a given menu to any proper subset of it. This property appears particularly appealing when the DM faces unforeseen contingencies and has become a fairly common postulate in the menu choice literature. Yet, there are many situations in life where an agent may strictly prefer smaller menus to larger ones, for instance if he suffers from temptation à la Gul and Pesendorfer [19] or if he anticipates regret as in Sarver [30]. Because they typically focus on a single psychological phenomenon, most models of menu choice allow for either preference for flexibility or commitment concerns, but not both. In this paper, we investigate the extent to which both concerns may coexist within a single framework, provided one imposes some discipline on the way those concerns may emerge.

We propose that one circumstance under which the DM may prefer flexibility over commitment is when he is unable to decide between two courses of action. Indeed, a large experimental literature starting with Tversky and Shafir [31] documents a higher tendency to defer choice when the available alternatives have conflicting attributes. We study the behavioral implications of imposing the rule “Whenever in doubt, don’t commit; just leave options open” in a standard menu choice environment. Our paper shows that this intuitive rule, which ties preference for flexibility to indecisiveness, may itself preclude the expression of any desire for commitment.

The idea of indecisiveness is of course not new in decision theory; it dates back to Aumann [2], and is usually modeled directly by dropping the assumption of completeness of the preference relation $\succeq$ that represents the tastes of the DM. Although rarely studied in this context, the assumption of incomplete preferences appears reasonable in the context of menu choice.

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1See for instance Dekel et al. [10, 11] and more recently Krishna and Sadowski [22].
2Many other phenomena have been modeled through non-monotonic preferences over menus; see Lipman and Pesendorfer [24] for a comprehensive review.
3Incomplete preferences have been studied in a variety of settings; see for instance, Peleg [29], Dubra et al. [13], Bewley [3]) or Ok et al. [28].
since the objects of comparison have a complex nature. At the same time, we rarely observe the tastes of the DM; instead, what we see are the choices he makes. Furthermore, assuming that the choice correspondence is non empty, then its revealed preference ≽ is necessarily complete.

We therefore take as our primitive a complete preference relation ⊳ on X, which represents the choice behavior of the agent. We assume that the choices of the DM reflect his tastes whenever those are defined, by requiring that ⊳ be a proper completion of some underlying incomplete preference ≽ representing the DM’s tastes. We connect the DM’s indecisiveness to his preference for flexibility by requiring that whenever two menus A and B cannot be compared by ≽ (denoted A ∇ B), then one should observe A∪B ≽ A,B. Intuitively, an indecisive DM will often seek to defer choice if he expects to be better informed in the future or if he needs additional time to contemplate a difficult decision. Under such circumstances, choosing not to commit to a given menu can be seen as a cautious attitude. We thus coin this behavioral property Cautious Deferral and call a completion consistent with it a Cautious Deferral completion.

Our main question is whether and when our complete preference ⊳ can be interpreted as a Cautious Deferral completion of some underlying incomplete (but unobservable) preference ≽. We ask this question in the standard framework of Dekel et al. [10, 11]; that is, we consider preference relations defined on menus of lotteries that satisfy the usual rationality assumptions of independence and continuity. Our answer highlights a strong connection between indecisiveness, Cautious Deferral and preference for flexibility: provided ⊳ exhibits some incompleteness, then any completion ⊳ that satisfies Cautious Deferral must exhibit preference for flexibility on its entire domain (i.e. including all menus that can be ranked by ⊳). Conversely, if ⊳ is a Cautious Deferral completion that violates preference for flexibility at some pair (A, B), then ⊳ must be complete. Thus, in the context of rational preferences, our main proposition delivers an impossibility result: under the restriction of Cautious Deferral, underlying incompleteness is incompatible with any desire for commitment.

The present paper speaks to a recent literature that studies completion and/or extension rules for incomplete preference relations in various contexts. For instance, Gilboa et al. [18], Kopylov [20] or Lehrer and Teper [23] investigate such rules in the Anscombe-Aumann framework, while Cerreia-Vioglio et al. [4] perform a similar exercise in a risky choice setting. We note that most of these papers adopt a weaker notion of completion than the one adopted in this paper, for they do not require the strict part of the incomplete relation to be preserved by its completion.
Our work also contributes to a literature that explores the connections between incompleteness on the one hand, and preference for flexibility or choice deferral on the other hand. Gerasimou [16] studies a model in which indecisiveness induces choice deferral, as captured by a possibly empty-valued choice correspondence. In menu choice environments, a one-to-one link between incomparability and strict preference for flexibility appears in various papers (Kreps [21], Arlegi and Nieto [1], Danan [7, 8]).\footnote{Kreps [21] studies a complete preference relation \(\succ\) on \(X\) that satisfies monotonicity (i.e. \(A \cup B \succ A, B\) for all \(A, B \in X\)). Key to his analysis is an auxiliary domination relation \(\hat{\succ}\) defined as \(A \hat{\succ} B\) if \(A \sim A \cup B\). Notice that \(\hat{\succ}\) is possibly incomplete and that, given monotonicity, \(A \hat{\succ} B\) if and only if \(A \cup B \succ A, B\). Furthermore, \(\succ\) is a proper completion of \(\hat{\succ}\).} This one-to-one link is used by Danan [8] to uniquely derive incomplete preferences from observed behavior. Unlike Danan [8], our main concern is not identification of \(\hat{\succ}\), but existence of an incomplete \(\hat{\succ}\), in an environment with more structure but with a weaker connecting condition.\footnote{We only require that incompleteness implies (weak) preference for flexibility (but not conversely). On the other hand, we work in an environment of menus of lotteries with the usual structural assumptions of independence and continuity, while Danan [8] considers a generic environment without this additional structure.}

The rest of the paper is organized as follows. In Section 2, we introduce our framework and completion rule, present our main result and discuss the degree of identification of \(\hat{\succ}\). Section 3 provides a brief discussion of the role played by each of our assumptions. Additional results can be found in an Online Appendix.

## 2 The Model

### 2.1 Preliminaries

We work in the standard menu choice framework of Dekel et al. [10, 11] (henceforth DLR). Let \(\Delta\) stand for the set of all probability distributions over a finite prize space \(Z\), with generic members \(a, b\). Let \(X\) be the set of all nonempty closed subsets of \(\Delta\) endowed with the Hausdorff topology, with generic members \(A, B\). Members of \(X\) are interpreted as menus from which the DM will make a choice at a later (unmodeled) stage. We refer to \(\lambda A + (1 - \lambda)B\) as the Minkowski mixture of two sets with weight \(\lambda \in (0, 1)\).

By a preference \(\succ\) over menus, we mean a reflexive and transitive binary relation on \(X\). As usual, \(\succ\) (resp. \(\sim\)) denotes the asymmetric (resp. symmetric) part of this relation. Define the non-comparability relation \(\bowtie\) on \(X\) by \(A \bowtie B\) if and only if neither \(A \succ B\) nor \(B \succ A\). If \(\bowtie = \emptyset\), then \(\succ\) is
complete. We say that $\succeq$ is a proper completion of a preference relation $\succ$ if (i) $\succeq$ is complete; (ii) $\succeq \subseteq \succ$ and $\succeq \subseteq \succ$. In the following, we say that $\succeq$ has a (strict) preference for flexibility at a pair $(A, B)$ if $A \cup B \succ (\succ) A, B$. If $A \cup B \succ A, B$ for all $A, B \in X$, then $\succeq$ is said to be monotone. Finally, the DM has a desire for commitment at a pair $(A, B)$ if $A \succ A \cup B$ or $B \succ A \cup B$.

Throughout the paper, we will restrict attention to preferences that satisfy the following two axioms:

**Axiom 1** (Independence): For every $A, B, C \in X$ and $\lambda \in (0, 1)$

$$A \succ B \quad \text{if and only if} \quad \lambda A + (1 - \lambda) C \succ \lambda B + (1 - \lambda) C$$

**Axiom 2** (Continuity): $\succ$ is closed in $X \times X$.

Both are natural extensions of the standard expected utility axioms to a menu choice setting (see Dekel et al. [10, 11] or Gul and Pesendorfer [19] for a discussion), and are canonical in most of the menu choice literature. In the following, we refer to preferences satisfying A1 - A2 as rational preferences.

### 2.2 Cautious Deferral Completions

Our primitive is a complete and rational preference $\succ$ on $X$, which, as in the standard revealed preference approach, encodes the observed behavior of the agent. We wish to understand when such behavior is consistent with the choice deferral mechanism highlighted in the introduction, in which a subjective conflict leads the agent to postpone. This motivates the following definition:

**Definition**: A complete and rational preference $\succ$ on $X$ is a Cautious Deferral completion if there exists a rational preference $\succ\succ$ on $X$ such that:

1. (C1) $\succ$ is a proper completion of $\succ\succ$.
2. (C2) $A \succ\succ B$ implies $A \cup B \succ A, B$.

We interpret $\succ\succ$ as representing the comparisons the agent is confident about. Whenever the DM can say that $A$ is surely better than $B$, we expect him to choose $A$ over $B$ (and choose either when indifferent), which is the content

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*Footnote:* $X \times X$ is viewed as the product metric space induced by the Hausdorff metric on $X$. Note that continuity of $\succ\succ$ implies that $\succ\succ$ is open.
To understand \((C2)\), note that an incomplete preference \(\succ\) can always be viewed as the intersection of a collection of complete preferences, each representing a different evaluation criterion in the mind of the agent (see Ok [27], Section 1.4). Thus, the agent’s inability to compare menus through \(\succ\) can be seen as stemming from the conflict between the various considerations entering his evaluation of the problem.

When conflicted between two menus \(A\) and \(B\), what rule of conduct may the DM adopt? If the agent is constrained to choose from the feasible set \(\{A, B\}\), our representation imposes no constraint on what the final choice will be. But now suppose that his choice set is actually \(\{A, B, A \cup B\}\). Both introspection and empirical evidence suggest that the agent would seize this additional opportunity, meaning \(A \cup B \succ A, B\), as this leaves all options open at a later stage. A wide range of experimental studies in psychology and economics indeed document a link between decision conflict and tendency to differ choice (Tversky and Shafir [31], Dhar [12], Tykocinski and Ruffle [32], Costa-Gomes et al. [6], Danan and Ziegelmeyer [9]). Intuitively, an indecisive DM will often seek to defer choice if he expects to be better informed in the future or if he needs additional time to contemplate a difficult decision. Under such circumstances, choosing not to commit to a given menu can be seen as a cautious attitude. We thus coin \((C2)\) the Cautious Deferral rule.

### 2.3 Main Result

The purpose of our paper is to understand the restrictions on choice behavior imposed by our cautious deferral mechanism. Our main result highlights the connection between indecisiveness, Cautious Deferral and preference for flexibility:

**Proposition 1**: Let \(\succ\) be a Cautious Deferral Completion on \(X\). Then either \(\succ\) is monotone, or \(\succ = \succ\).

In words, provided \(\succ\) exhibits some incompleteness, then any completion \(\succ\) that satisfies Cautious Deferral must exhibit preference for flexibility on its entire domain. Conversely, if \(\succ\) is a Cautious Deferral completion that violates set monotonicity, then \(\succ\) must be complete. As such, Proposition 1 delivers an impossibility result: in the context of rational preferences and under the restriction of Cautious Deferral, underlying incompleteness is incompatible with any desire for commitment.

To illustrate this impossibility result, suppose we wanted to model the behavior of an agent who suffers from temptation as in Gul and Pesendorfer
[19], but abides by the Cautious Deferral rule whenever in doubt. Assume that the DM’s behavior can be seen as the Cautious Deferral completion ≻ of some incomplete relation ≻ capturing the core preferences of the agent. It is reasonable to assume that a DM who may be tempted will favor commitment whenever he can clearly identify elements of temptation (i.e. $A \succ B$, and hence $A \succ B$, implies $A \succ A \cup B$), but will prefer not to commit when he is indecisive (i.e. $A \bowtie B$ implies $A \cup B \succ A$). Yet, this intuition runs to a difficulty in the presence of the standard rationality axioms. For, take any two menus $A$ and $B$ for which the DM can express a preference, say $A \succ B$. By our logic, it must be that $A \succ A \cup B$. However, if $\succ$ is also a Cautious Deferral completion of $\succ$, then Proposition 1 tells us that $\succ$ must be monotone, so that $A \cup B \succ A$. Therefore, $A \sim A \cup B$, implying that self-control motives must entirely disappear.

**Proof of Proposition 1.** Suppose that $\succ$ is a Cautious Deferral Completion of some underlying incomplete preference $\succ$ (i.e. with $\bowtie \neq \emptyset$), but $\succ$ is not monotone. Then there are menus $A, B, C, D$ such that $A \bowtie B$, $C \subset D$, and $C \succ D$. We show that Cautious Deferral must be violated. Since $\succ$ and $\bowtie$ satisfy continuity and independence, we can assume w.l.o.g. that $A, B, C$ and $D$ are closed convex sets and that $A \succ B$. In fact Independence implies that $A \sim co(A)$ where $co(A)$ is the convex hull of $A$. Moreover, since $C \succ D$ if $A \sim B$ then $A' = \alpha A + (1-\alpha)C \succ \alpha B + (1-\alpha)D = B'$ for any $\alpha \in (0, 1)$. As continuity implies incomparability is open, for $\alpha$ close enough to 1 we must have $A \bowtie B$. If $B \succ A \cup B$, then $A \succ A \cup B$ and, hence, Cautious Deferral is violated. So assume $A \cup B \succ B$. Let

$$\begin{align*}
E &= \frac{1}{2}C + \frac{1}{2}A \\
F &= \frac{1}{2}C + \frac{1}{2}B \\
G &= \frac{1}{2}D + \frac{1}{2}B \\
H &= \frac{1}{2}C + \frac{1}{2}co(A \cup B).
\end{align*}$$

Then $H \succ F \succ G$ and, hence, $F \sim I = \alpha G + (1-\alpha)H$ for some $\alpha \in (0, 1)$. Moreover, $F$ is contained in $I$. Since $\succ$ is a proper completion of $\bowtie$, either $F \bowtie I$ or $F \bowtie I$. If $F \bowtie I$, then $F \succ J = \beta G + (1-\beta)I \bowtie F$ and $F \subset J$ for any $\beta \in (0, 1)$, since the closed convex subsets of $\mathbb{R}^n$ form a mixture space.

We are very grateful to an anonymous referee for offering this shorter proof.
under \( + \). Thus Cautious Deferral is violated. So assume \( \hat{F} \sim I \). We have \( E \bowtie F \) and, hence, \( K = \gamma E + (1 - \gamma) F \bowtie F \) and \( K \subset I \) for any \( \gamma \in (0, 1) \). Moreover, \( E \succ F \) and, hence, \( K \succ F \). It follows that \( K \bowtie I \) and \( K \succ I \), so Cautious Deferral is violated.

2.4 Identification of \( \hat{\succ} \)

Our Cautious Deferral rule draws a fairly weak link between indecisiveness and choice deferral. First, the rule does not require the DM to strictly prefer to postpone when unable to compare two menus, as it allows \( A \bowtie B \) and \( A \cup B \sim A \). Second, one can have pairs \((A, B)\) at which \( \succ \) exhibits strict preference for flexibility even though \( A \succ B \). Hence conflict in the underlying preference \( \hat{\succ} \) neither forcibly causes, nor exhausts all justifications for, postponement.

One natural question concerns what can be learned under our Cautious Deferral mechanism about the underlying relation \( \hat{\succ} \) that represents the tastes of the DM. Here we show by an example that not only one cannot hope to infer \( \hat{\succ} \) uniquely but, in fact, \( \hat{\succ} \) need not even be monotone.

**Example:** Consider the real maps \( W_1 \) and \( W_2 \) on \( X \) defined by
\[
W_1(A) := \max_{a \in A} w(a) - \frac{1}{2} \max_{a \in A} v(a)
\]
and
\[
W_2(A) := \max_{a \in A} v(a) - \frac{1}{2} \max_{a \in A} w(a),
\]
where \( w \) and \( v \) are continuous and affine functions on \( \Delta \) such that \( w \neq \alpha v + \beta \) for any \( \alpha > 0 \) and \( \beta \in \mathbb{R} \). Then, the relation \( \hat{\succ} \) defined as
\[
A \hat{\succ} B \quad \text{if and only if} \quad W_1(A) \geq W_1(B) \quad \text{and} \quad W_2(A) \geq W_2(B),
\]
is incomplete and satisfies Axioms 1-2. On the other hand, the preference relation \( \succ \) on \( X \) represented by \( W_1 + W_2 \) is a monotone proper completion of \( \hat{\succ} \). Hence, \( \hat{\succ} \) satisfies the cautious deferral rule, but \( \hat{\succ} \) is not monotone.

Finally, it should be noted that the complete preference \( \succ \) in the above example is also the proper completion of \( \hat{\succ} \) given by \( A \hat{\succ} B \) if and only if \( \max_{a \in A} w(a) \geq \max_{b \in B} w(b) \) and \( \max_{a \in A} v(a) \geq \max_{b \in B} v(b) \). This remark points to the weak identification of the underlying relation \( \hat{\succ} \) in our environment. In fact, it is impossible to uniquely identify an underlying
relation under our Cautious Deferral mechanism: since any monotone comple-
tion trivially satisfies the Cautious Deferral rule, a given monotone and
complete preference \( \succeq \) satisfying independence and continuity can be the
Cautious Deferral completion of any subrelation \( \succeq \) of \( \succeq \) that is proper (in
the sense that \( \succeq \subsetneq \succ \)) and satisfies independence and continuity.\(^8\)

3 Discussion

Since it is natural to expect agents to favor either flexibility or commitment
depending on the situation, our result puts into question either the validity
of the Cautious Deferral rule as a general rule of conduct, or the rationality
assumptions commonly made in the menu choice literature, continuity and
independence. We discuss each assumption below.

3.1 Cautious Deferral versus Cautious Avoidance

One could argue that the flip side of Cautious Deferral, requiring that
\( A \hat{\succ} B \) implies \( A, B \succ A \cup B \), might be a more appropriate assumption in the con-
text of temptation. Indeed, if the DM is indecisive between two menus
because of the respective temptations they contain, then committing to ei-
ther menu would seem to be a more cautious attitude than leaving options
open: by avoiding exposure to one of the temptations, the agent can in-
crease his chances of exerting self-control.\(^9\) We therefore coin this condition
Cautious Avoidance. Given the structure of our proof, it is easy to see that
a symmetric result would be obtained if Cautious Deferral were to be re-
placed by Cautious Avoidance, this time with the conclusion that nonempty
incomparability implies “negative” monotonicity (\( A \succeq B \) whenever \( A \subseteq B \)).

\(^8\)A tighter identification of \( \hat{\succeq} \) could be obtained by strengthening our connecting con-
tion to \( A \hat{\succeq} B \) if and only if \( A \cup B \succ A, B \). In this case, one can show that \( \hat{\succeq} \) can be
uniquely identified, which echoes the result of Danan [8]. See Online Appendix for more
details.

\(^9\)For instance, consider a DM who faces 3 options: broccoli \( b \), potato chips \( p \) and
chocolate cake \( c \). Options \( p \) and \( c \) are more tempting than \( b \), \( p \) as a salty craving and
\( c \) as a sweet craving, while \( b \) is healthier than both \( p \) and \( c \). Suppose the DM cannot
determine which craving is stronger, so that \( \{b, p\} \succeq \{b, c\} \). In this case, one would expect
the DM to choose either \( \{b, p\} \) or \( \{b, c\} \) over \( \{b, c, p\} \), since the latter set guarantees the
worst temptation. We thank John Stovall for providing this example.
3.2 Independence

The linearity imposed by the independence axiom plays an essential role in the proof of our main result, for it allows the consequences of the local interaction between indecisiveness and Cautious Deferral to spread globally. In the Online Appendix, we consider the natural relaxation of independence to Indifference to Randomization, which requires that \( A \sim \text{co}(A) \) for all \( A \in X \). We show that this relaxation, which breaks the linearity behind independence while preserving the convexity, allows Cautious Deferral to coexist with the non monotonicity of \( \succsim \). As such, our main result is another illustration of the technical power of the independence axiom.

While often considered a technical assumption, the independence axiom is particularly strong in environments of menu choice, for it relies on two premises: (i) the DM satisfies the standard independence axiom; (ii) he is indifferent as to the timing of the resolution of uncertainty. The second premise has been challenged in various papers for it precludes many interesting behavioral phenomena (Epstein et al. [14], Ergin and Sarver [15], Noor and Takeoka [26]). In this paper, we find that independence leaves no room for commitment concerns once Cautious Deferral is imposed.

3.3 Other structural assumptions

From the above discussion, one might be inclined to believe that our impossibility result is essentially driven by the independence axiom. However, the other ingredients of the model are necessary for the result to go through. First, the closed continuity of \( \succsim \) (implying \( \succ \) is open) ensures that incomparability, when present, will never be too “small”: for any \( A, B \in X \) such that \( A \not\succ B \), there will be two open neighbourhoods \( O_A \) of \( A \) and \( O_B \) of \( B \) such that \( C \not\succ D \) for \( C \in O_A \) and \( D \in O_B \). In the Online Appendix, we show that closed continuity cannot be replaced by the most common alternative for incomplete relations, open continuity. Finally, we show that our requirement that \( \succsim \) be a proper completion is not innocuous, as it cannot be weakened to only requiring that \( A \succsim B \) implies \( A \succ B \).
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