

Emre Ozdenoren and Kathy Yuan

## Contractual externalities and systemic risk

Article (Accepted version)  
(Refereed)

**Original citation:** Ozdenoren, Emre and Yuan, Kathy (2017) *Contractual externalities and systemic risk*. [Review of Economic Studies](#), 84 (4). pp. 1789-1817. ISSN 0034-6527  
DOI: [10.1093/restud/rdw058](https://doi.org/10.1093/restud/rdw058)

© 2017 The Authors

This version available at: <http://eprints.lse.ac.uk/75998/>  
Available in LSE Research Online: November 2017

LSE has developed LSE Research Online so that users may access research output of the School. Copyright © and Moral Rights for the papers on this site are retained by the individual authors and/or other copyright owners. Users may download and/or print one copy of any article(s) in LSE Research Online to facilitate their private study or for non-commercial research. You may not engage in further distribution of the material or use it for any profit-making activities or any commercial gain. You may freely distribute the URL (<http://eprints.lse.ac.uk>) of the LSE Research Online website.

This document is the author's final accepted version of the journal article. There may be differences between this version and the published version. You are advised to consult the publisher's version if you wish to cite from it.

# Contractual Externalities and Systemic Risk

Emre Ozdenoren and Kathy Yuan\*

August 11, 2016

## Abstract

We study effort and risk-taking behaviour in an economy with a continuum of principal-agent pairs where each agent exerts costly hidden effort. Principals write contracts based on both absolute and relative performance evaluations (APE and RPE) to make individually optimal risk-return trade-offs but do not take into account their impact on endogenously determined aggregate variables. This results in contractual externalities when these aggregate variables are used as benchmarks in contracts. Contractual externalities have welfare changing effects when principals put too much weight on APE or RPE due to information frictions. Relative to the second best, if the expected productivity is high, risk-averse principals over-incentivise their own agents, triggering a rat race in effort exertion, resulting in over-investment in effort and excessive exposure to industry risks. The opposite occurs when the expected productivity is low, inducing pro-cyclical investment and risk-taking behaviours.

*JEL* classification codes: D86, G01, G30.

*Keywords:* Contractual externalities, relative and absolute performance contracts, pro-cyclical effort exertion and risk taking, many principal-agent pairs.

---

\*Ozdenoren is from the London Business School and CEPR; e-mail: [eozenoren@london.edu](mailto:eozenoren@london.edu). Yuan is from the London School of Economics, FMG and CEPR; e-mail: [K.Yuan@lse.ac.uk](mailto:K.Yuan@lse.ac.uk). We thank the Editor, Philipp Kircher, three anonymous referees, Ulf Axelson, Patrick Bolton, Peter Kondor, Thomas Mariotti, Ailsa Röell, Oguzhan Ozbas, Vania Stavrakeva, Gunter Strobl, and seminar participants at the Frankfurt School of Finance and Economics, HKUST, Hong Kong University, Imperial College, London School of Economics, London Business School, Shanghai Advance Institute of Finance (SAIF), European Finance Association Meeting 2013, and European Winter Finance Meeting 2014 for helpful comments, Oleg Rubanov for excellent research assistance, and ERC (617106) and ESRC (ES/K002309/1) for funding support.

# 1 Introduction

It is important to understand the sources of industry boom and bust cycles, especially in light of the recent episodes in the high-tech and the finance industries. In these situations, the excited anticipation of the arrival of a high productivity era associated with new technological breakthroughs leads to over-investment and excessive risk-taking in the corresponding industry. This “overheating” in economic activities often gives way to a subsequent crash where real investments and risks are substantially reduced. These pro-cyclical investment and risk-taking behaviours have significant social and economic consequences (eg, the recent great recession).

In this paper, we study a new mechanism based on frictions in contracting to explain pro-cyclical and potentially suboptimal risk-taking in the economy. Our model contributes to the contract theory literature by endogenising systemic risk creation within a multiple principal-agent framework. More broadly, it provides an equilibrium framework of decentralised contract choices of individual firms, setting an incentive-based foundation for studying aggregate implications of firm-level behaviour.

In the model, there are many firms in an industry. Each firm has a principal who owns a project and an agent who exerts costly hidden effort.<sup>1</sup> The return to effort of all agents is affected by an industry productivity shock. As a result, the level of industry risk faced by a firm is endogenous and is increasing in its agent’s effort choice.<sup>2</sup> Additionally, the project’s payoff is subject to idiosyncratic risk. Principals choose contracts to make risk-return trade-offs that are individually optimal. However, they do not take into account their impact on aggregate variables such as average effort in the industry. This results in contractual externalities when these aggregate variables enter the contracting problems as benchmarks. By investigating the conditions under which contractual externalities have welfare changing effects, our paper offers a new perspective on excessive risk-taking phenomenon over the boom-bust cycle.

---

<sup>1</sup>This contrasts with setups with one principal and many agents (eg, team incentives) or many principals and one agent (eg, common agency).

<sup>2</sup>In this paper we treat the correlated industry risk and the systemic/common risk as the same and use these terms interchangeably.

In our baseline model, each principal uses a contract based on both absolute and relative performance evaluations (hereinafter APE and RPE). By using industry average as the benchmark in RPE, principals are able to shield their agents from correlated industry risk but not from idiosyncratic risk. Hence, if they have to rely exclusively on RPE, principals encourage their agents to take on industry risk which they have to shoulder entirely. When feasible, principals would combine APE with RPE to improve risk sharing but the optimal weight on APE might be positive or negative. When principals care mostly about industry risk, they put positive weight on APE to expose their agents to industry risk and control ‘excessive’ industry risk taking. When principals care mostly about idiosyncratic risk, they put negative weight on APE to reduce agents’ idiosyncratic risk exposure.<sup>3</sup>

In this setting, we study how individually optimal contracting affects aggregate investment and risk-taking behaviour and its welfare implications using the baseline model as a building block. A key feature of our model is that the industry benchmark is endogenously determined because it is a function of the average managerial effort, an equilibrium outcome. Agents have incentives to match the industry benchmark to reduce their exposure to the industry productivity shock, which generates a feedback loop between individual and the industry average effort among agents. This feedback loop creates an externality in setting incentives among principals in the industry since principals take the industry benchmark as given and do not take into account the impact of their choices on it. To study the welfare impact of this externality we compare it with the second best where a planner maximises the sum of the payoffs of all the principals in the industry. When a principal gives stronger incentives to her agent, this leads to an increase in the industry average effort which has two effects. First, through the feedback loop, efforts of other agents, and hence expected outputs of all other firms increase. Second, higher efforts by other agents generate additional industry risks which are shouldered by all other principals. Since individual principals do not internalise these effects, relative to the second best, the first effect leads to too little while

---

<sup>3</sup>Negative weight on APE can be surprising since it reduces incentive for effort provision. However, this can be optimal when it is very costly to let the agents take on additional idiosyncratic risk (eg., when agents are quite risk averse relative to principals or idiosyncratic shocks are very volatile). Moreover, negative weight on APE does *not* mean that agents are punished for good performance. When APE and RPE are *combined*, agents are always rewarded for their own performance.

the second leads to too much effort provision. Despite this, in the baseline setup, where principals can use both APE and RPE to separate their agents' exposure to industry and idiosyncratic risks, contractual externalities do not have welfare impact, ie, the equilibrium outcome and the planner's solution coincide.

In reality, there are often frictions that restrict the principals' ability to separate the industry and idiosyncratic risks using APE and RPE. In these situations, contractual externalities have welfare changing implications because principals put too much weight on one contracting instrument relative to the other, triggering the feedback loop mentioned earlier. One situation where information frictions arise naturally about industry and firm level productivities is the arrival of a new technological innovation. In practice absolute performances of CEOs are often measured by their firms' individual stock prices and the industry benchmark corresponds to the industry stock index. The hype around the new technology often causes a run up in all stock prices in the industry, without revealing the underlying industry productivity, eg, the dot.com boom in 1990s. The hype washes out when comparing individual firms' stock prices with the industry stock index. Hence, this type of information friction does not affect RPE but makes APE a noisy contractual instrument. As a result, principals rely more heavily on RPE but less on APE, creating welfare changing effects of contractual externalities.

In the presence of such informational frictions, if the expected industry productivity is high, eg, during a boom, a principal, in order to reap the high productivity benefit, would like to elicit high effort from her agent by increasing incentives. Since APE is noisy, she relies more on RPE relative to the second best, which triggers a rat race among agents to exert effort and causes excessive industry risk exposure for principals. By contrast, if the expected industry productivity is low, eg, during a recession, the principal would like to reduce her agent's effort. Once again since APE is noisy, she reduces RPE instead. Relative to the second best, this triggers a race to the bottom to exert effort and generates too little industry risk. In this case, the planner can improve the total welfare by making RPE countercyclical: enforcing lower (higher) RPE during booms (busts). The model, therefore, offers some empirical predictions and policy guidance on managerial pay. For example, it predicts that excessive investment and risk-taking is more likely in an industry where principals are risk-

averse; the industry-wide productivity is expected to be high and volatile; and APE is noisy. This is more likely to be true in emerging industries or industries experiencing large technological shocks as opposed to mature ones where there is less uncertainty about the industry productivity. Hence, it is relatively more important to have close supervision of excessive risk-taking in industries with large positive technological shocks.

The mechanism described above leads to inefficiency in systemic but not in idiosyncratic risk taking. To show this, in Section 7.1, we let productivity shocks to be correlated rather than common across projects. As productivity shocks become less correlated, agents become less motivated to match the industry average effort since they cannot remove the exposure to the idiosyncratic component of the productivity shocks from their compensation by doing so. When the productivity shocks are completely idiosyncratic, there is no feedback loop between individual and industry average effort. This unique prediction on inefficient pro-cyclical systemic (as opposed to idiosyncratic) risk taking is well supported by the data.<sup>4</sup>

The structure of the paper is as follows. In section 2, we discuss the related literature. In section 3, we present the model. In section 4, we lay out agents', principals' and planner's optimisation problems. In section 5, we study the baseline case without any information frictions. We solve for the optimal linear contract under the equilibrium and the second best and compare the two. In sections 6, we study the case with information frictions and analyse the welfare impact of contractual externalities. In section 7, we extend the model to incorporate heterogeneity in the correlation of productivity shocks among firms, the degree of information frictions and firm sizes. Section 8 concludes.

---

<sup>4</sup>Hoberg and Phillips (2010) find that in the period of excessive risk taking, the common industry (rather than idiosyncratic) productivity shocks are volatile and difficult to predict, indicating the existence of information frictions, in their studies of industry boom-bus cycles. Bhattacharyya and Purnananda (2011) have documented between 2000 and 2006, the period of financial industry boom and excessive financial risk taking, idiosyncratic risks have dropped by almost half while systemic risks have doubled among US commercial banks.

## 2 Related Literature

Since the results in our paper hinge on the fact that contracts put some weight on the industry average, our paper is closely related to the literature on relative performance (starting with Holmström (1979); (1982)).<sup>5</sup> We contribute to this literature theoretically in several aspects. First we endogenise the relative benchmark by linking it with equilibrium outcomes. Second, we study contractual externalities among multiple principal-agent pairs and their welfare consequences. Our theoretical extension has many unique predictions on the use of APE and RPE in compensation contracts that match well with the data and our comparative statics produce many new testable implications.

Moreover, the theoretical results from our baseline model offer a potential resolution for conflicting findings in the empirical literature on RPE. Earlier empirical work has found that executives' compensations are very sensitive to industry performance.<sup>6</sup> These findings are interpreted as indirect evidence that little RPE is observed in practice, hence challenge the existing theory (Holmström 1979; 1982) which views the industry shocks as exogenous and unrelated to effort choices, and predicts that RPE will be used to make executives' compensations insensitive to such shocks. However, this interpretation conflicts with more recent empirical studies using a new source of data. Based on detailed disclosure data on executive compensation contracts, these studies find that a significant proportion of firms use some form of RPE.<sup>7</sup> Our results from the baseline model shed light on these seemingly

---

<sup>5</sup>There are several other strands of contracting literature that analyse interactions among multiple principals and agents. For example, the literature on rivalrous agency (Myerson (1982), Vickers (1985), Freshman and Judd (1986; 1987), Sklivas (1987), Katz (1991), among others)) has examined the case in which principals hire agents to compete on their behalf in an oligopolistic setup. Our paper differs in two aspects. First, in our model principals do not engage in direct competition and the interactions among agents arises endogenously via contracts that are based on correlated information. Second, our focus is different. We explore implications of contractual externalities for aggregate inefficiencies. Our model also differs from the literature on 'common agency' (Pauly (1974); Bernheim and Whinston (1986)) where multiple principals share a single agent. Lastly, our model is related to but different from models of (rank order) tournaments (Akerlof (1976); Lazear and Rosen (1981); Green and Stokey (1983); Nalebuff and Stiglitz (1983); and Bhattacharya and Mookherjee (1986)) with one principal and many agents.

<sup>6</sup>See Gibbons and Murphy (1990); Prendergast (1999); Aggarwal and Samwick (1999).

<sup>7</sup>See De Angelis and Grinstein (2010) and Gong et al. (2010).

conflicting findings since in our theory the sensitivity to industry risk can be desirable which can be achieved by using a combination of RPE and APE. Furthermore, our model makes new testable predictions on how structural economic variables affect the optimal mix of RPE and APE.

Our baseline model also sheds light on the related empirical observation that sensitivity of CEO compensation to industry shocks is asymmetric. CEOs are rewarded for good industry shocks but not punished for bad ones.<sup>8</sup> Literature has so far highlighted rent-seeking by CEOs as an explanation for these findings. We provide an alternative explanation based on optimal contracting. In our model, when the expected industry productivity is high, agents put more effort resulting in more industry risks. Principals respond by putting positive weight on APE to control industry risk taking, seemingly rewarding the agent for industry performance. By contrast, when the expected industry productivity is low, idiosyncratic risks become relatively more important. This results in lower weight on APE and makes agents' compensations less sensitive to industry risks during downturns. In fact, our baseline model makes the additional testable prediction that observed pro-cyclical pay sensitivity would be more pronounced in industries where industry shocks are large and principals are more risk-averse.

Our paper is also related to models that study excessive risk taking by allowing agents to choose effort and level of risk separately (See Diamond (1998); Biais and Casamatta (1999); Palomino and Prat (2003); and Makarov and Plantin (2010)). In our model an agent's effort choice and the riskiness of his project are tightly linked. This is because the productivity of effort is random and correlated across firms, and thus when an agent increases his effort, both the expected return and the systematic risk exposure of the project are higher. We view this feature of the model desirable when studying excessive risk taking from a social perspective, especially considering that episodes of over (under) investment at the industry and/or the economy level are often observed together with excessive (insufficient) risk taking. We acknowledge that in some cases agents can choose risk and return of the projects

---

<sup>8</sup>See Bertrand and Mullainathan (2001) and Garvey and Milbourn (2006).



separately, however in others agents have to choose a *portfolio* of risk and return together.<sup>9</sup> By treating the risk-return as a portfolio, our framework complements the understanding of sub-optimal risk-taking in the principal-agent framework. Importantly, we differ from this literature because agents in our framework take suboptimal amounts of *industry* rather than idiosyncratic risks which arises from contractual externalities as opposed to nonlinearities in payoff schedules.

The recent crisis has ignited an interest in macro and banking literature on excessive risk taking behaviour of banks. To our knowledge, only one other line of literature predicts excessive undertaking of systemic risks and the prediction is one-sided about booms. This literature studies the incentive for banks to take on excessive risk collectively anticipating bailouts in case of financial crisis (Acharya and Yorulmazer (2007); Acharya and Yorulmazer (2008); Farhi and Tirole (2012); and Acharya et al. (2011)).

There is a line of financial literature that shows career or reputational concerns can lead to herd like behavior among agents (eg., Scharfstein and Stein (1990); Rajan (1994); Zwiebel (1995); and Guerrieri and Kondor (2012)). For example, Rajan (1994) models the information externality across two banks where reputational concerns and short-termism induce banks to continue to lend to negative NPV projects. He derives a theory of expansionary (or liberal) and contractionary (or tight) bank credit policies which influence, and are influenced by other banks' credit policies and conditions of borrowers. However, his model does not examine whether banks correlate their lending to similar industries or not. Further, in his model the short-term nature of managerial decisions drives career concern and hence expansionary bank credit policies during the boom, whereas in our model it is the information frictions on systemic productivity shocks. By and large, the major difference between our paper and this line of research is that we study explicit rather than implicit incentives. This allows us to generate quite different and unique testable predictions and policy implications; eg, regulations on executive compensation over the business cycles.

More broadly, by studying equilibrium consequences of decentralised contract choices of individual firms for risk-taking, our paper is also connected with a growing literature on

---

<sup>9</sup>Put differently, agents may have to trade off investing effort in high-risk-high-return projects versus low-risk-low-return projects.

firm-macro dynamics (Khan and Thomas (2003); Bloom et al. (2007); Khan and Thomas (2008); House (2014); Bloom et al. (2014)). In this line of literature, the frictions considered include adjustment cost, fixed cost, or irreversibility of investment. Our model offers an alternative friction based on incentive provision to study macro implications of firm level behaviour.

### 3 Model

In this section, we describe our setup, information environment, and equilibrium definition.

#### 3.1 The Setup

There is a continuum of principals in an industry. Each principal owns a firm which in turn owns a project. There is also a continuum of agents who are able to obtain a fixed reservation utility in a competitive labor market. The principal hires an agent to work on the project and offers the agent a contract. Each principal, agent and project triplet is indexed by  $i \in [0, 1]$ . The principal's objective is to maximise her expected utility which is based on the expected final value of the project. Principals are potentially risk averse, and principal  $i$ 's utility over wealth is given by  $u_P(w_i) = -\exp(-r_P w_i)$  where  $r_P \geq 0$ .<sup>10</sup>

There are three dates  $t = 0, 1, 2$ . At  $t = 0$ , principal  $i$  offers agent  $i \in [0, 1]$  a contract. We assume that contracts are offered simultaneously. Agent  $i$  observes his contract and decides whether to accept or reject it. If he accepts the contract, he chooses hidden effort denoted by  $e_i$  on project  $i$ . Agent  $i$ 's effort is costly and the cost is specified as  $C(e_i) = e_i^2/2$ . We assume that agent  $i$ 's utility over wealth and effort is given by  $u(w_i, e_i) = -\exp(-r(w_i - C(e_i)))$  where  $r \geq 0$ . At  $t = 1$ , two payoff-relevant public signals about project  $i$  are revealed. One is about agent  $i$ 's performance and the other is about the average performance of all

---

<sup>10</sup>Note that we allow for risk-averse principals. In presence of contractual externalities risk-neutrality of principals is not an innocuous assumption. In reality, there are a number of reasons why principals might be risk-averse or act as if they are risk-averse. Banal-Estanl and Ottaviani (2006) have discussed these in detail, which include concentrated ownership, limited hedging, managerial control, limited debt capacity and liquidity constraints, and stochastic productions.

projects in the industry. We assume that these signals are contractible and determine agent  $i$ 's compensation. All agents are paid at time 1. At  $t = 2$ , the final values of all projects are realised and principals receive their payoffs. For simplicity we assume no discounting.

### 3.2 Production Technology

We assume that project  $i$  generates output  $V_i$ , which is a random function of agent  $i$ 's unobservable effort and two stochastic shocks,

$$V_i = V(e_i, \tilde{h}, \tilde{\epsilon}_i). \tag{1}$$

The randomness arises from a common random variable  $\tilde{h}$ , and a project-specific random variable  $\tilde{\epsilon}_i$ . We interpret  $\tilde{h}$  as a common productivity shock to all projects and  $\tilde{\epsilon}_i$  as an output shock specific to the individual project. In the rest of the paper, we refer to  $\tilde{h}$  as the industry productivity shock or the systemic shock as it cannot be diversified away. The important assumption is that  $\partial^2 V_i / (\partial h \partial e_i) \neq 0$ , ie, the state of nature that is common across agents, affects the productivity of effort. This specification is meant to capture the uncertainty about industry productivity after a technological innovation.

Our results are based on a linear specification where  $V_i = \tilde{h}e_i + \tilde{\epsilon}_i$ . In our model, the random variable  $\tilde{h}$  is normally distributed with mean  $\bar{h} > 0$  and variance  $\sigma_h^2$  (ie, precision  $\tau_h = 1/\sigma_h^2$ ). The random variable  $\tilde{\epsilon}_i$  is normally distributed with mean zero and variance  $\sigma_\epsilon^2$  (ie, precision  $\tau_\epsilon = 1/\sigma_\epsilon^2$ ).<sup>11</sup>

Note that in our specifications, the productivity shock enters multiplicatively with effort. When  $\sigma_h = 0$ , the specification for output in our model is standard. In the more general case where  $\sigma_h > 0$ , higher average effort generates a higher return, but since the productivity of effort is random it also leads to higher volatility. Here, we have in mind a broad interpretation of effort as choosing the scale of the project, eg, by devoting more resources (time, personnel, etc.) to it.<sup>12</sup>

---

<sup>11</sup>To show that the common component of productivity shocks is a key driver for our results, in Section 7.1, we analyse an alternative specification where the productivity shocks have both common and idiosyncratic components.

<sup>12</sup>Similar multiplicative function forms of productivity shocks and firm input choices have also been used

### 3.3 Information Structure

In our model principals receive contractible signals about the output of their individual projects, and the average output of the industry. We assume that the industry average reveals the industry productivity shock  $\tilde{h}$  with noise. The idea is that after a major technological innovation there is uncertainty about industry productivity and it is difficult to assess the realisation of this uncertainty through public signals such as industry stock price indices, which themselves are very noisy.

Specifically, the first contractible signal is a noisy signal of project  $i$ 's outcome, ie, agent  $i$ 's performance, given by

$$s_i = \tilde{h}e_i + \tilde{\epsilon}_i + \tilde{\zeta}, \quad (2)$$

where  $\tilde{\zeta}$  is an industry-wide noise normally distributed with mean zero and variance  $\sigma_{\tilde{\zeta}}^2$  (ie, precision  $\tau_{\tilde{\zeta}} = 1/\sigma_{\tilde{\zeta}}^2$ ).

The second is a noisy signal of the industry average project outcome, ie, the average performance of all agents, given by

$$s_I = \tilde{h}\bar{e} + \tilde{\zeta}, \quad (3)$$

where  $\bar{e} = \int_0^1 e_i di$  is the average effort of all agents. Note that since the signals about the projects' outcomes are correlated, the industry average output is observed with noise  $\tilde{\zeta}$ . Hence, the industry average reveals the industry productivity  $\tilde{h}$  with noise.

In this paper, we restrict attention to linear compensation contracts which is common in the theoretical literature on principal-agent models. We let  $q_i$  be a signal about the agent's performance relative to his peers given by,

$$q_i = s_i - s_I = \tilde{h}(e_i - \bar{e}) + \tilde{\epsilon}_i. \quad (4)$$

In a linear contracting environment any contract written on  $s_i$  and  $s_I$  can be written in terms of  $s_i$  and  $q_i$ , and vice versa. To provide better intuition, in the rest of the paper, we assume that the principals write contracts on the relative performance signals rather than the industry average signal.

---

to study firm dynamics with microeconomic rigidities in the macro literature. For example, Bloom et al. (2014) model the firm output as a triple multiplicative product of industry, idiosyncratic productivity shocks as well as firm's choices on capital and labor.

### 3.4 Equilibrium Definition

We assume that agent  $i$ 's linear compensation contract has three components. The first component is a fixed wage  $W_i$  and the other two components condition the agent's payment on the realisation of the two signals. Therefore, agent  $i$ 's total compensation is given by  $l_i q_i + m_i s_i + W_i$  where  $l_i$  and  $m_i$  measure the relative performance evaluation (RPE) and absolute performance evaluation (APE) components of the contract.

Now we are ready to specify agent  $i$ 's optimisation problem. We assume that agents' reservation utility is  $u(\bar{W})$ . Agent  $i$  accepts contract  $(l_i, m_i, W_i)$  if his expected utility from accepting the contract exceeds his reservation utility

$$E[u(l_i q_i + m_i s_i + W_i - C(e_i(l_i, m_i, W_i)))] \geq u(\bar{W}), \quad (5)$$

where  $e_i(l_i, m_i, W_i)$  is the optimal effort choice conditional on accepting the contract. That is,

$$e_i(l_i, m_i, W_i) = \arg \max_{e_i \geq 0} E[u(l_i q_i + m_i s_i + W_i - C(e_i))]. \quad (6)$$

We define an equilibrium of the model as follows.

**Definition 1** *An equilibrium consists of contracts  $(l_i^*, m_i^*, W_i^*)$ , effort choices  $e_i^*$ , where  $e_i^* = e_i(l_i^*, m_i^*, W_i^*)$  for each  $i \in [0, 1]$  and average effort  $\bar{e} = \int_0^1 e_i^* di$  such that given  $\bar{e}$ , the contract  $(l_i^*, m_i^*, W_i^*)$  solves principal  $i$ 's problem, ie, it maximises  $E[u_P(V_i - (l_i q_i + m_i s_i + W_i))]$ , subject to  $E[u((l_i q_i + m_i s_i + W_i) - C(e_i))] \geq u(\bar{W})$ , where  $e_i = e_i(l_i, m_i, W_i)$  (given in (6)).*

To study the potential externality in the economy, we also define the second best of the model. It is defined as the solution to the planner's problem where the planner maximises the sum of the utilities of all principals conditional on the incentive and individual rationality constraints for the agents. Formally,

**Definition 2** *A second-best solution consists of a contract  $(l^{SB}, m^{SB}, W^{SB})$  and effort choice  $e^{SB}$  where  $e^{SB} = e_i(l^{SB}, m^{SB}, W^{SB})$  and the contract solves the planner's problem, ie, it maximises  $\int_0^1 E[u_P(V_i - (l_i q_i + m_i s_i + W_i))] di$ , subject to  $E[u((l_i q_i + m_i s_i + W_i) - C(e_i))] \geq u(\bar{W})$ , where  $e_i = e_i(l_i, m_i, W_i)$  (given in (6)).*

Note that the planner's role is limited to coordinating the contracts written by principals. In particular, the planner must give agents incentives to accept the contract and exert the desired level of effort.<sup>13</sup>

We begin our analysis in section 4 by first solving the agents', principals' and planner's problems in the contractual environment discussed above. In section 5, we study a baseline case where  $\tau_\zeta = \infty$  where there is no information friction regarding the uncertain industry productivity shock,  $\tilde{h}$ . In section 6, we incorporate in the model an information friction by letting  $0 \leq \tau_\zeta < \infty$ . In these cases, APE is not fully informative and principals rely more on RPE as contracting instruments. We discuss how the equilibrium effort level compares with the second-best and present results on comparative statics.

## 4 Agents', Principals' and Planner's Problem

In this section we first solve agents' equilibrium effort choices for a given contract. We then use this solution to characterise principals' and the planner's choices of optimal contract.

### 4.1 Agents' Effort Choice

Given contract  $(l_i, m_i, W_i)$  agent  $i$ 's compensation is:

$$l_i q_i + m_i s_i + W_i = l_i \left( \tilde{h} (e_i - \bar{e}) + \tilde{\epsilon}_i \right) + m_i \left( \tilde{h} e_i + \tilde{\epsilon}_i + \tilde{\zeta} \right) + W_i. \quad (7)$$

Substituting (7) into agent  $i$ 's maximisation problem in (6) and computing the expectation, agent  $i$ 's problem can be stated as choosing  $e_i$  to maximize:

$$(l_i + m_i) \bar{h} e_i - l_i \bar{h} \bar{e} + W_i - C(e_i) - \frac{1}{2} r \left( (l_i (e_i - \bar{e}) + m_i e_i)^2 \frac{1}{\tau_h} + (l_i + m_i)^2 \frac{1}{\tau_\epsilon} + m_i^2 \frac{1}{\tau_\zeta} \right). \quad (8)$$

From (8) we see how a given incentive package shapes agent  $i$ 's exposure to various sources of risks. His risk exposure to the common productivity shock ( $\tilde{h}$ ) depends on (i) the power of the relative performance-based pay  $l_i$  times the difference between his effort and the average

---

<sup>13</sup>The second best contract pushes agents exactly to their reservation utilities. However, it would be misleading to think that the second-best contract favours the principals' since given CARA utilities and linear contracts, the solution also maximises the total surplus.

effort ( $e_i - \bar{e}$ ), and (ii) the power of absolute performance pay  $m_i$  times his effort  $e_i$ . His risk exposure to the common noise  $\tilde{\zeta}$  depends solely on the power of absolute performance-based pay while his risk exposure to the idiosyncratic noise  $\tilde{\epsilon}_i$  depends on the power of *total* performance-based pay. From this we can see that by matching the average effort in the industry, agent  $i$  is able to completely hedge his exposure to the industry risk that comes through his relative performance pay, although he might still be exposed to some industry risk that comes through his absolute performance pay. Taking the first-order condition and solving for  $e_i$ , we obtain agent  $i$ 's effort choice as

$$e_i = \frac{(l_i + m_i) \bar{h} + \frac{r}{\tau_h} l_i (l_i + m_i) \bar{e}}{1 + \frac{r}{\tau_h} (l_i + m_i)^2}. \quad (9)$$

Note that agent  $i$ 's effort is increasing in  $\bar{e}$ , the average effort exerted by all the other agents with a positive relative performance pay sensitivity. Thus, when the average effort increases, agent  $i$ 's best response is to increase his effort.

The term  $r/\tau_h$  appears both in the denominator and the numerator of (9). In the denominator, this term captures the fact that higher effort results in higher industry risk and consequently agent's effort declines in this risk aversion and the volatility of the industry shock. More interestingly, the term  $r/\tau_h$  is also in the numerator of (9), capturing the fact that when  $r$  is higher or  $\tau_h$  is lower, the agent has a stronger incentive to match the average effort to hedge the industry risk. Through this second effect, for a given contract  $(l_i, m_i)$ , the agent's effort may increase with his risk aversion or the volatility of the industry productivity shock.

Similarly, the total performance-based pay,  $(l_i + m_i)$ , also has opposing effects on agent  $i$ 's effort choice. Increasing it makes agent  $i$  increase his effort because his pay becomes more sensitive to average productivity,  $\bar{h}$ , and the magnitude of his performance relative to the industry average  $\bar{e}$ . This is captured by the  $(l_i + m_i)$  term in the numerator of (9). At the same time, increasing  $(l_i + m_i)$  causes agent  $i$  to bear more industry risk by making the agent deviate more from the industry average. This increase in risk exposure induces him to lower his effort. This is captured by the  $(l_i + m_i)$  term in the denominator in (9). In addition, both effects become stronger as the industry risk  $1/\tau_h$  increases. As we show later, these two effects underly the externalities that principals have to face in designing the compensation

contracts.<sup>14</sup>

## 4.2 Principals' Choice of Optimal Contract

Now we turn to the principals' problem. Principal  $i$  chooses the contract terms  $(l_i, m_i, W_i)$  to maximise her expected utility,  $E[u_P(V_i - (l_i q_i + m_i s_i + W_i))]$  subject to agent  $i$ 's individual rationality constraint given by (5) and optimal effort choice  $e_i$  given by (9).

We proceed to solve the equilibrium contract terms  $(l_i, m_i, W_i)$ . Using (7) we obtain principal  $i$ 's final payoff as

$$V_i - (l_i q_i + m_i s_i + W_i) = \tilde{h}e_i + \tilde{\epsilon}_i - l_i \left( \tilde{h}(e_i - \bar{e}) + \tilde{\epsilon}_i \right) - m_i \left( \tilde{h}e_i + \tilde{\epsilon}_i + \tilde{\zeta} \right) - W_i. \quad (10)$$

Plugging (10) into principal  $i$ 's utility function, taking expectation and using agent  $i$ 's individual rationality constraint to substitute for  $W_i$ , we see that principal  $i$  chooses  $(l_i, m_i)$  to maximise

$$\begin{aligned} & \bar{h}e_i - C(e_i) - \frac{1}{2}r_P \left( (e_i - l_i(e_i - \bar{e}) - m_i e_i)^2 \frac{1}{\tau_h} + (1 - l_i - m_i)^2 \frac{1}{\tau_\epsilon} + m_i^2 \frac{1}{\tau_\zeta} \right) \\ & - \frac{1}{2}r \left( (l_i(e_i - \bar{e}) + m_i e_i)^2 \frac{1}{\tau_h} + (l_i + m_i)^2 \frac{1}{\tau_\epsilon} + m_i^2 \frac{1}{\tau_\zeta} \right) - \bar{W}. \end{aligned} \quad (11)$$

The above expression has an intuitive interpretation as it is principal  $i$ 's and agent  $i$ 's combined surplus. The first term is the expected output of the project, the second term is the cost of agent  $i$ 's effort, and the next two terms are the disutilities from the risk exposures of the agent and the principal respectively.

From the above expression, we see that APE and RPE play different roles in risk sharing between principals and agents. APE introduces agents to both industry and idiosyncratic risks. By contrast, when agents match each other's effort choices, RPE shields agents from industry risk, although it still exposes agents to idiosyncratic risk.

In this paper we will restrict attention to situations where the equilibrium is unique. Next proposition guarantees the existence of a unique equilibrium as long as the industry risk is not too large.<sup>15</sup>

---

<sup>14</sup>Note that in the limit, as the industry risk approaches zero, our model delivers the standard result where agent  $i$ 's effort is determined by his performance pay and the productivity of his effort, ie,  $(l_i + m_i)\bar{h}$ .

<sup>15</sup>When the industry risk is large, it is possible to construct examples of multiple equilibria. The multiplicity of equilibrium contracts is an interesting possibility that is worth studying further in future work.



**Proposition 1** *Given  $\bar{h}$ ,  $r$ ,  $r_P$ , there exists  $\bar{\tau}_h$  such that for all  $\tau_h > \bar{\tau}_h$  there exists a unique equilibrium contract which is symmetric.*<sup>16</sup>

Note that once the values of  $\bar{h}$ ,  $r$ ,  $r_P$  are fixed, Proposition 1 guarantees that there is a unique equilibrium for large enough  $\tau_h$  regardless of the values of  $\tau_\epsilon$  and  $\tau_\zeta$ .<sup>17</sup>

### 4.3 Planner's Problem

From Definition 2 we see that the planner chooses the contract terms  $l$  and  $m$  to maximise the sum of principals' utilities subject to incentive and participation constraints. Since principals' optimisation problems are identical, the planner's problem can be seen equivalently as maximising the utility of one of the principals taking into account that  $e_i^* = \bar{e}$ . That is, the planner internalises the impact of the contract terms on the industry average effort level  $\bar{e}$ . Thus, the planner chooses  $(l, m)$  to maximise

$$\begin{aligned} & \bar{h}e - C(e) - \frac{1}{2}r_P \left( e^2 (1-m)^2 \frac{1}{\tau_h} + (1-l-m)^2 \frac{1}{\tau_\epsilon} + m^2 \frac{1}{\tau_\zeta} \right) \\ & - \frac{1}{2}r \left( m^2 e^2 \frac{1}{\tau_h} + (l+m)^2 \frac{1}{\tau_\epsilon} + m^2 \frac{1}{\tau_\zeta} \right) - \bar{W}, \end{aligned} \quad (12)$$

where

$$e = \frac{(l+m)\bar{h}}{1 + \frac{r}{\tau_h}(l+m)m}. \quad (13)$$

In our model, industry benchmark is a function of the industry average effort. Since agents have incentives to match the industry benchmark to reduce their exposure to the industry productivity shock, this generates a feedback loop between individual and the industry average effort of the agents. By comparing equations (11) and (12), we observe that in the decentralised equilibrium principals do not internalise their choices of contract terms on the industry average effort while the planner does. As a result, this feedback loop creates externalities, which we term as contractual externalities, in the decentralised equilibrium where the principals do not take into account their impact on the industry benchmark. Comparing the decentralised equilibrium outcome with the second best allows us to investigate

---

<sup>16</sup>All proofs are in Appendix A.

<sup>17</sup>This allows us to fix  $\tau_h$  and perform comparative statics with respect to  $\tau_\epsilon$  and  $\tau_\zeta$  (without losing existence and uniqueness of the equilibrium).

the magnitude and the direction of these contractual externalities and perform comparative statics.

## 5 The Baseline Model

In this section, we study the baseline set up where  $\tau_\zeta \rightarrow \infty$  and the noise  $\tilde{\zeta}$  disappears. As we show below, in this case information friction regarding the uncertain industry shock  $\tilde{h}$  is absent. We begin our analysis by explicitly characterising the equilibrium in this baseline case.

**Proposition 2** *When  $\tau_\zeta$  approaches infinity, the optimal contract  $(l^*, m^*, W^*)$  is symmetric and unique. The total performance sensitivity  $a^* = l^* + m^*$  is the unique positive root to the following equation:*

$$\begin{aligned} & \bar{h}^2 \left( \frac{r}{\tau_h} a + 1 \right) \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right)^2 (a - 1) \\ & + \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} + \left( \frac{r}{\tau_h} \right)^2 a^2 \left( \frac{r_P}{\tau_h} + 1 \right) + 2 \frac{r_P}{\tau_h} \frac{r}{\tau_h} a \right)^2 \left( a \frac{r}{\tau_\epsilon} + (a - 1) \frac{r_P}{\tau_\epsilon} \right) = 0. \end{aligned} \quad (14)$$

Given  $a^*$  the contract term  $m^*$  is given by:

$$m^* = a^* - \frac{\frac{r}{\tau_h} \left( 1 + \frac{r}{\tau_h} (a^*)^2 \right) + \frac{r_P}{\tau_h} (a^* - 1) \left( \frac{r}{\tau_h} a^* + 1 \right)}{\left( \frac{r}{\tau_h} a^* + 1 \right) \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right)}. \quad (15)$$

The equilibrium contract of the baseline model features both APE and RPE, although the optimal weight on APE,  $m^*$ , might be positive or negative. Corollary 1 characterises the sign of APE in equilibrium.

**Corollary 1** *The weight on the absolute performance signal  $m^*$ , is positive (negative, zero) if*

$$\bar{h}^2 \frac{r_P}{\tau_h} \left( \frac{r_P}{\tau_h} + 1 \right) + \frac{r_P}{\tau_h} \frac{r_P}{\tau_\epsilon} \left( 1 + \frac{r}{\tau_h} \right) + \frac{r}{\tau_\epsilon} \left( \frac{r_P}{\tau_h} - \frac{r}{\tau_h} \right) \quad (16)$$

*is positive (negative, zero).*

It is interesting to note that when  $m^*$  is strictly positive, agents are rewarded for better industry performance. In contrast, in the single-agent relative performance model, under corresponding assumptions, agents would not be rewarded by what seems to be luck rather than effort.<sup>18</sup> The difference in the results is due to the fact that in our model the level of industry risk faced by the firm is endogenously determined and increasing in the agent's effort.

We can see from the principal's objective function in (11) that when the agents match the average effort in the industry, they do not face any industry risk through RPE. The only industry risk they face comes from APE. In this sense, like in Holmström (1982), RPE completely filters out the correlated risk or the luck component. At the same time, the fact that RPE shields them from the industry risk means that the agents do not consider the impact of their effort choice on their firm's exposure to the industry risk, potentially exposing their principals to it excessively. Therefore, different from the classical relative performance literature, our model finds that principals use APE to control and share risks with agents which RPE alone cannot achieve.

Specifically, APE plays two roles from risk-sharing perspective. First, by exposing agents to the industry risk, it reduces their incentive to take on industry risk. Second, it offsets agents' idiosyncratic risk exposure. The condition in Corollary 1 shows which of these forces prevails in equilibrium. Suppose principals are risk averse and fix  $r_P > 0$ . When average return to effort and/or industry risk is high, the optimal contracts puts a positive weight on APE (ie.,  $m > 0$ ) so that agents would internalise their tendency to take on too much industry risk. By contrast, when average return to effort and/or industry risk is low, the optimal contract puts a negative weight on APE (ie.,  $m < 0$ ) to reduce the agents' idiosyncratic risk exposure.<sup>19</sup>

---

<sup>18</sup>In the standard relative performance model principal observes two signals: a noisy signal of the agent's performance and a second signal that is uninformative about the agent's performance but correlated with the noise term of the first signal. The second signal could be the performance of other agents working on the project but could also be any other information correlated with the signal about the agent's performance. When the two signals are positively correlated, the second signal gets a negative weight. This is because when the second signal is higher, the principal learns that the noise in the first signal is likely to be high. Putting a negative weight on the second signal, allows the principal not to reward the agent for luck.

<sup>19</sup>The assumption of risk-neutrality of principals is not innocuous in the presence of contractual external-

This finding regarding the purposes of APE versus RPE in compensation contracts offers a unique explanation to various empirical puzzles. For example, the empirical phenomenon of “paying for luck” might be due to the fact that principals want to control agents’ excessive risk-taking tendency. This empirical fact is established by running a regression of executive pay on industry benchmarks. However, as we show, there might be a large amount of RPE in the compensation contracts (high  $l$ ) even when the pay is positively correlated with industry risk (high  $m$ ). This simple regression only reflects the net effect of APE and RPE and is no longer sufficient. Our model shows that principals’ usage of APE and RPE is more complex in the presence of both industry and idiosyncratic risks and a careful decomposition of the pay package to uncover this underlying cause of a particular mix of APE and RPE instruments is needed instead. Furthermore, based on Corollary 1, our model predicts the “pay for luck” phenomenon occurs more often in industries with volatile and high expected productivity, while in industries where expected productivity is low, and firm-specific risks are larger, our model finds that the sensitivity to industry risks is much lower, even turns negative, predicting an asymmetry in “paying for luck.” These are new testable implications.

Next we turn to the comparison of the the decentralised equilibrium and the planner’s solution in the baseline case.

**Proposition 3** *When  $\tau_\zeta$  approaches infinity, the effort choices and contracts coincide in equilibrium and in the planner’s solution.*

In other words, if the industry productivity shock is perfectly revealed, principals are able to completely counteract the impact of externalities among agents’ effort-taking through optimal contracting. To see this algebraically, let  $\tau_\zeta$  go to infinity, set  $a_i = l_i + m_i$  and  $c_i = l_i \bar{e}$ . Substituting these in (11) we can restate principal  $i$ ’s problem as choosing  $(a_i, c_i)$  to maximize:

$$\bar{h}e_i - \frac{1}{2}r_P \left( (e_i - a_i e_i + c_i)^2 \frac{1}{\tau_h} + (1 - a_i)^2 \frac{1}{\tau_\epsilon} \right) - C(e_i) - \frac{1}{2}r \left( (a_i e_i - c_i)^2 \frac{1}{\tau_h} + a_i^2 \frac{1}{\tau_\epsilon} \right) - \bar{W}$$

---

ities because it completely shuts down the first role played by APE. Risk neutral principals do not mind shouldering all the industry risk, so they do not need to reduce their agents’ incentive to take on industry risk. However, they are concerned about their agents’ idiosyncratic risk exposure since they have to compensate them for it. Hence, risk neutral principals always put negative weight on APE.

where agent  $i$ 's effort is given by

$$e_i = \frac{a_i \bar{h} + \frac{r}{\tau_h} a_i c_i}{1 + \frac{r}{\tau_h} a_i^2}$$

Note that stated this way principals' problems are completely separated and  $\bar{e}$  no longer plays a role. This is because principal  $i$  can completely eliminate the impact of the industry average effort  $\bar{e}$  by adjusting  $c_i$ . By redefining the principals' optimisation problem this way, we see that it coincides with the planner's problem and Proposition 3 is obvious. Intuitively, when information friction on industry risk is absent, principals can use the two contractual instruments – APE and RPE – to fine tune their agents' exposures to the two types of risks – industry and idiosyncratic – and undo the welfare effect of the contractual externality regardless of the industry average effort. The planner, therefore, has no role to play in this environment. Here we observe a parallel between the workings of contractual and pecuniary externalities. In general, pecuniary externality also does not have welfare changing effects except for conditions as established in Stiglitz (1982), Greenwald and Stiglitz (1986), Geanakoplos and Polemarchakis (1985), Arnott, Greenwald and Stiglitz (1994), and more recently Farhi and Werning (2013).<sup>20</sup>

In the following two sections, we extend the baseline model to  $0 \leq \tau_\zeta < \infty$ . In these cases principals receive noisy and correlated signals about absolute performances, and have to rely more on relative performance information. We illustrate how the resulting information friction restricts the principals' ability to separate the two types of risks, shapes the contracts and generates welfare changing effects.

---

<sup>20</sup>There is an explosion of the literature on the welfare effect of pecuniary externalities due to the growing interests in studying social inefficiency of booms-busts. This includes but not limited to the following: Krishnamurthy (2003); Caballero and Krishnamurthy (2001; 2003); Gromb and Vayanos (2002); Korinek (2010); Bianchi (2010); Bianchi and Mendoza, (2011); Stein (2012); Gersbach and Rochet (2012); He and Kondor (2013); Farhi and Werning (2013). Davila (2011) and Stavrakeva (2013) have nice summaries of this literature. Similar to pecuniary externalities, we show later that, with frictions, contractual externalities might have welfare changing effects.

## 6 Information Friction

Often major technological innovations make it extremely difficult to assess the productivity of an industry but it is still possible to evaluate an agent's performance relative to his peers. To capture this feature in the simplest way, we begin our analysis by allowing the industry-wide noise on APE to be extremely volatile, that is, by letting  $\tau_\zeta$  be zero.<sup>21</sup>

In this limiting case, principals do not have any information about  $\tilde{h}$  directly, and both signals  $s_i$  and  $s_I$  are uninformative by themselves. However, their difference  $q_i$  is informative because it is unaffected by the common noise  $\tilde{\zeta}$ . Consequently, principals can only assess how much better or worse their agents are performing relative to the average and have to base agents' compensation on this information alone. As a result,  $m_i^* = 0$ , that is, contracts do not include an absolute performance-based pay component. In section 7.2, we relax this assumption and study the intermediate case of  $0 < \tau_\zeta < \infty$ .

To solve her problem, principal  $i$  takes  $\bar{e}$  as given and chooses the optimal linear contract which we denote by  $l_i^*$ . The following proposition characterises the equilibrium contract and effort levels.

**Proposition 4** *When  $\tau_\zeta = 0$ , under the conditions in Proposition 1, a unique symmetric equilibrium contract exists and satisfies*

$$\frac{\bar{h}^2}{\frac{r}{\tau_h}(l^*)^2 + 1} (1 - l^*) \left( 1 - \frac{r_P}{\tau_h} l^* \right) - \frac{1}{\tau_\epsilon} (r l^* - r_P (1 - l^*)) = 0. \quad (17)$$

Moreover,  $l^* \in (0, 1)$  and the equilibrium effort level is  $e^* = \bar{e} = l^* \bar{h}$ .

---

<sup>21</sup>Intuitively, when there is a great uncertainty about the industry productivity, it is relatively easy to assess an agent's performance relative to his peers. That is, the information on the ranking of agents is more precise than the information on an agent's absolute performance level. Empirically, we observe that stock analysts are better at ranking stocks than pricing stocks (Da and Schaumburg (2011)). The finance literature is more successful in explaining cross-sectional equity returns while the equity premium remains a puzzle. Moreover, this information structure parsimoniously captures the tournament-like incentives that agents face in the real world. For example, the ranking of businesses, university programs, fund managers, doctors in different specialities, and even economists of different vintages, is prevalent when there is also (possibly quite noisy) information on their individual performance.

## 6.1 Equilibrium Properties

The expositional clarity of the equilibrium RPE ( $l^*$ ) in (17) allows us to explore further properties of contracts in this economy. To illustrate, we dissect the equilibrium condition (17) into terms that reflect the tradeoff between incentives and risk-sharing. To do so we define the *incentive provision* as the level of compensation when the sole purpose of the contract is to incentivise the agents to exert effort, and the *risk-sharing provision* as the level of compensation when the purpose of the contract is to allow risk sharing between principals and agents. The following corollary of Proposition 4 characterises the optimal contract in two limiting cases.

**Corollary 2** *When  $\tau_\epsilon$  goes to infinity, the optimal linear contract reflects only the incentive provision and is given by  $l_i^* = \min\{1, \tau_h/r_P\}$ . When  $\tau_\epsilon$  goes to zero, the optimal linear contract reflects only the risk-sharing provision and is given by  $l_i^* = r_P/(r_P + r)$ .*

Corollary 2 allows us to identify the terms in the equilibrium condition (17) that correspond to incentive and risk sharing provisions:

$$\bar{h}^2 \frac{1}{\underbrace{\frac{r}{\tau_h} (l_i^*)^2 + 1}_{\text{Cost of Unilateral Deviations in Incentive Provision}}} \underbrace{\left(1 - l_i^*\right) \left(1 - \frac{r_P}{\tau_h} l_i^*\right)}_{\text{Incentive Provision}} - \frac{1}{\tau_\epsilon} \underbrace{\left(r l_i^* - r_P (1 - l_i^*)\right)}_{\text{Risk-Sharing Provision}} = 0. \quad (18)$$

The magnitude of risk-sharing provision is standard and depends on the relative risk-aversions of principals and agents,  $r_P/(r_P + r)$ . The magnitude of the incentive provision has aspects unique to our model. In the standard moral hazard framework the magnitude of incentive provision is simply 1. This is because when there is no risk sharing concern, it is optimal to “sell the project” to the agent. A key insight of our model is that this intuition does not hold when there is endogenous risk creation by the agents and this risk is borne disproportionately by the principals. In fact, in our model, principals shoulder all industry risk in equilibrium and the amount of industry risk depends on agents’ effort choices.<sup>22</sup> Principals take into account the endogenous industry risk and their appetite for

<sup>22</sup>To see why this is this case, recall in equilibrium  $e_i^* = \bar{e}$ . This implies that each agent’s industry risk exposure in his compensation contract is zero in equilibrium (from (8)).

it and set  $l_i^* = \min\{1, \tau_h/r_P\}$  when there is no risk sharing concern. Thus, the magnitude of incentive provision is less than 1 when industry productivity is volatile or principals are risk averse enough.

The weights that the incentive and the risk-sharing concerns receive in the equilibrium contract are given by their coefficients in (18). The ratio of these two coefficients captures the relative importance of the two concerns.

The decomposition in (18) shows that industry and idiosyncratic risks affect the relative importance of incentive provision through different channels. Because idiosyncratic risks are shared, when  $1/\tau_\epsilon$  goes up, the importance of incentive provision relative to risk sharing declines. The impact of industry risk is more subtle. It affects the relative importance of incentive provision through the term  $(r(l_i^*)^2/\tau_h + 1)$ .<sup>23</sup> This term is affected by agent  $i$ 's risk aversion and captures his disutility from taking on additional industry risk when incentivised to work (potentially) more than the industry average.<sup>24</sup> Note that this cost is not incurred by agents in equilibrium. Nevertheless it plays a role in the determination of the equilibrium contract. This is because a principal, considering unilateral deviation from equilibrium, would take this cost into account.

Next we highlight comparative statics that are unique to our model with potentially new empirical implications.<sup>25</sup> In the standard moral hazard framework, the power of contracts increases in the marginal productivity of effort  $\bar{h}$  and the precision of idiosyncratic risk  $\tau_\epsilon$ . As the next proposition shows, in our model, this is not necessarily the case.

**Proposition 5** *If  $\tau_h/r_P < (>, =) r_P/(r_P + r)$ ,  $l^*$  decreases (increases, is constant) in  $\bar{h}$  and  $\tau_\epsilon$ .*

---

<sup>23</sup>This term appears in (9) when we solve agent  $i$ 's optimal effort (except that here  $m = 0$ ).

<sup>24</sup>Of course, in equilibrium, agent  $i$ 's industry risk exposure in his compensation contract is zero since agent  $i$  hedges industry risk by choosing  $e_i^* = \bar{e}$ . Since each principal takes other principals and agents behaviours as given, in her view, deviating from equilibrium choice and providing stronger incentives unilaterally would impose her agent to bear more risk and hence result in this additional cost of incentive provision.

<sup>25</sup>To test these implications, it is possible to obtain empirical proxies for the model parameters such as industry (marginal) productivity, industry risks and idiosyncratic risks, as well as risk aversions of the principals and agents. For example, one can use the proportion of institutional investors in the shareholder base of a firm as a proxy for (the inverse of) risk aversion of the firm.



To understand this proposition first note that the importance of incentive provision relative to risk sharing is increasing in  $\bar{h}$  and  $\tau_\epsilon$ . In the standard moral hazard framework, the magnitude of incentive provision is 1 and it always exceeds the magnitude of risk sharing provision  $r_P/(r_P + r)$ . Hence, when the relative importance of incentive provision increases, the power of the contract also increases. In contrast, in our model, as we explained above due to endogenous risk taking, it is possible to have the magnitude of incentive provision smaller than that of risk sharing provision. In this case, when the relative importance of incentive provision increases, the power of the contract decreases.

Comparative statics of the equilibrium contract  $l^*$  with respect to the principals' and the agents' risk aversion parameters,  $r_P$  and  $r$ , also provide new empirical implications. In the standard moral hazard setting, as the principal becomes more or the agent becomes less risk averse,  $l^*$  increases to provide better risk-sharing. The next proposition illustrates that in the present setting there are opposing effects which can dominate the direct effect of improved risk sharing.

**Proposition 6** *If  $\bar{h}$  or  $\tau_\epsilon$  are large enough,  $l^*$  decreases in  $r_P$ . If, in addition,  $\tau_h/r_P < r_P/(r_P + r)$ ,  $l^*$  increases in  $r$ .*

Both statements in Proposition 6 reflect the tradeoff between the two effects in (18). The first statement shows the comparative static with respect to the principal's risk aversion  $r_P$ . When  $\bar{h}$  or  $\tau_\epsilon$  are large, the incentive provision effect dominates the direct effect from the risk sharing. Since the principal needs to shoulder the entire industry risk, as she becomes more risk averse, the magnitude of incentive provision goes down. As a result,  $l^*$  decreases in  $r_P$ .

The intuition for the second statement in Proposition 6 is more subtle since agents' risk aversion  $r$  affects both the magnitude of the risk-sharing provision and the relative importance of risk-sharing versus incentive provision. Suppose the magnitude of risk sharing provision is larger than that of incentive provision, i.e.,  $\tau_h/r_P < r_P/(r_P + r)$ . As  $r$  increases, there are two opposing forces. First, the magnitude of risk-sharing effect decreases (even though it is still larger than that of incentive provision), which is the direct effect mentioned earlier. Second, the importance of risk-sharing relative to incentive provision increases and

hence the contract reflects more the relatively larger risk sharing effect. This statement shows that starting from a situation where the incentive effect dominates, increasing  $r$  results in a large shift towards risk-sharing, and hence the equilibrium power of the contract increases:  $l^*$  increases in  $r$ .

## 6.2 Comparison with the Second Best

Next, for the case  $\tau_\zeta = 0$ , we compare the equilibrium efforts and contracts with their second-best levels. Recall that second-best solves the problem of the planner who internalises the impact of the contracts on the industry average effort level  $\bar{e}$ . As we discussed in Section 4.3, the planner's problem can be viewed as maximising the objective function given in (12) subject to agents' effort choices given in (13). Since, when  $\tau_\zeta = 0$ , the planner optimally sets  $m = 0$ , from (13) we obtain  $e = \bar{h}$ . Plugging this into (12), the planner's problem becomes

$$\max_{l \geq 0} \left[ -\frac{1}{2} \bar{h}^2 l \left( l \left( 1 + \left( \frac{r_P}{\tau_h} \right) \right) - 2 \right) - \frac{1}{2} l^2 \frac{r}{\tau_\epsilon} - \frac{1}{2} (1-l)^2 \frac{r_P}{\tau_\epsilon} \right]. \quad (19)$$

The first-order condition of the problem is

$$\underbrace{\bar{h}^2 \left( 1 - l^{SB} \left( \frac{r_P}{\tau_h} + 1 \right) \right)}_{\text{Incentive Provision}} - \frac{1}{\tau_\epsilon} \underbrace{\left( r l^{SB} - r_P (1 - l^{SB}) \right)}_{\text{Risk-Sharing Provision}} = 0, \quad (20)$$

and the solution to the planner's problem is:

$$l^{SB} = \frac{\frac{r_P}{\tau_\epsilon} + \bar{h}^2}{\frac{r}{\tau_\epsilon} + \frac{r_P}{\tau_\epsilon} + \bar{h}^2 \left( \frac{r_P}{\tau_h} + 1 \right)}. \quad (21)$$

Like the optimal equilibrium contract, the second-best solution also reflects the incentive and risk-sharing provisions. From (20) we obtain the following limiting results for the second-best contract. When  $\tau_\epsilon$  goes to infinity, the second-best contract reflects only incentive provision and is given by  $l^{SB} = 1/(r_P/\tau_h + 1)$ . When  $\tau_\epsilon$  goes to zero, the second-best contract reflects only risk-sharing provision and is given by  $l^{SB} = r_P/(r_P + r)$ .

Although the second-best solution of (20) is similar to the equilibrium solution of (18) in reflecting both incentive and risk-sharing provisions, there are two important differences. First, the second best requires a lower magnitude of incentive provision than in equilibrium.<sup>26</sup>

---

<sup>26</sup>Since  $1/(r_P/\tau_h + 1) < \min\{1, \tau_h/r_P\}$ .

Second, there is no cost of unilateral deviations in incentive provision. That is, contractual externality has two opposing effects. Intuitively, the first effect arises because principals do not take into account the industry risk exposure of other principals in the industry. When principals are risk averse, they have to trade off incentivising their agents to work harder versus exposing themselves to more industry risks in their projects. Stronger the incentive they choose, higher the output they would expect, and larger the industry risk they are exposed. Their industry risk exposure is endogenously linked to the strength of the incentives they provide. When setting incentives, a principal optimally chooses her own risk-return tradeoff ignoring her impact on increasing other principals' industry risk exposure. In the second best, a planner sets incentives by taking into account the feedback loop between industry average and individual effort choices and consequences of industry risk exposure for other principals in the industry. This means, the second best requires weaker incentives for agents.

The second effect goes in the opposite direction and arises because each principal perceives a unilateral deviation from the industry average as being too costly. Recall, the cost of unilateral deviations in incentive provision is incurred in equilibrium when a principal, who takes the industry average effort  $\bar{e}$  as given, considers increasing incentives and making her agent work harder unilaterally. The principal realises that by doing so, her agent's effort would be above  $\bar{e}$  which imposes costly industry risk on the agent, and she has to compensate the agent for this risk. In the second best, this unilateral deviation cost disappears because planner can coordinate (dictate) incentive provision across all principals in the industry. Therefore, the relative importance of incentive provision is higher in second best.

To summarise, the externality in the model has two opposing effects on the performance-pay sensitivity in the contract. Compared with the second best, *the magnitude* of equilibrium incentive provision is larger because principals do not internalise the impact of their incentive provision on the average effort level and the industry risk exposure of other principals, consequently, provide too much incentive. However, *the relative importance* of equilibrium incentive provision is lower because principals perceive unilateral increases in incentive provision as too costly. The next proposition characterises which effect dominates and whether the equilibrium contract is more or less sensitive to performance than the second-best contract.

**Proposition 7** *The equilibrium contract is more sensitive to performance than the second-best contract, ie,  $l^{SB} < l^*$ , and consequently agents put more effort in equilibrium than the second best, ie,  $e^{SB} < e^*$ , if*

$$\bar{h}^2 \frac{r_P}{\tau_h} \left( \frac{r_P}{\tau_h} + 1 \right) + \frac{r_P}{\tau_h} \frac{r_P}{\tau_\epsilon} \left( 1 + \frac{r}{\tau_h} \right) + \frac{r}{\tau_\epsilon} \left( \frac{r_P}{\tau_h} - \frac{r}{\tau_h} \right) \quad (22)$$

*is positive. Similarly,  $l^{SB} > l^*$  and  $e^{SB} > e^*$ , if (22) is negative;  $l^{SB} = l^*$  and  $e^{SB} = e^*$ , if (22) is zero.*

Comparing Proposition 7 and Corollary 1, we note that (16) and (22) are identical. Hence, we immediately obtain the following result linking the usage and the sign of APE in the baseline model with the direction of inefficiencies that result from basing contracts on RPE alone.

**Proposition 8** *When  $\tau_\zeta = 0$  and contracts are based solely on RPE, the equilibrium contract is more (less, equally) sensitive to performance than the second-best contract if and only if the weight on the absolute performance signal,  $m^*$ , is positive (negative, zero) when informational friction is absent (ie, when  $\tau_\zeta = \infty$ ).*

Proposition 8 gives a different perspective regarding excessive/insufficient risk taking with only RPE. Recall from the discussion in section 5 that APE can be positive or negative in equilibrium depending on principals' desire to control their agents' excessive industry risk taking versus idiosyncratic risk exposures. When principals are more concerned about excessive industry risk taking, they use positive APE, ie, set  $m > 0$ , to incentivise their agents while letting them internalise the industry risk they generate. Otherwise, they use negative APE, ie, set  $m < 0$  to reduce the agents' idiosyncratic risk exposure. When principals would like to use positive APE but are constrained from doing so, they increase RPE instead to incentivise their agents, ie, set a larger  $l$ . This triggers a positive feedback loop between the industry average and agents' effort choices, causing excessive effort provision and risk taking in equilibrium relative to the second best. Similarly, when principals would like to use negative APE but are constrained from doing so, they lower RPE instead, triggering a negative feedback loop, this time causing insufficient equilibrium effort and risk-taking relative to the second best.

These results show a pro-cyclical pattern of incentive provision, effort choice and risk-taking in the economy. To see this, note that (22) is positive for a sufficiently large  $\bar{h}$  if principals are risk-averse. When  $\bar{h}$  is large, the incentive provision term gets a larger weight in equilibrium than in the second-best (shown as the coefficient in front of the incentive concern term in equations (18) and (20)). This means that when  $\bar{h}$  is large, ie, during the productivity boom, the contracting between principals and agents is more motivated by the incentive concern. During this time, the expected productivity of effort is very high, and principals would like to offer their own agents a contract with a high performance sensitivity. By doing so, they do not internalise the impact of their own incentive provision on increasing the industry average effort, and trigger a rat race. Since marginal productivity of effort is random in our model, an immediate consequence of this result is that there is excess risk-taking behaviour among agents in equilibrium. The planner, in this case, can improve the total welfare by enforcing lower performance-based pay sensitivities in agents' compensation contracts.

By contrast, when  $\bar{h}$  is low, eg, during downturns, (22) is likely to be negative.<sup>27</sup> In this case, since the expected productivity of effort is low, the incentive provision term gets a lower weight in equilibrium than in the second-best. The cost of providing incentives unilaterally becomes a major consideration for principals. Principals would like to free-ride on each other in incentive provision, offering their agents a contract with a low performance-pay sensitivity. By doing so, principals again do not internalise the impact of their own incentive-provision on increasing the industry average effort, and hence under-incentivise the agents relative to the second-best. This again triggers a race but this time causes a race to the bottom. There is insufficient effort- and risk-taking. In this case, the planner can improve the total welfare by enforcing contracts with higher performance based pay-sensitivities.

---

<sup>27</sup>More precisely, if agents are sufficiently risk averse and  $r > r_P > 0$ , (22) is positive if  $\bar{h}$  is above a cutoff and negative if below it.

## 7 Robustness and Extensions

### 7.1 Industry-wide vs. Idiosyncratic Variations in Productivity

In this section, we show that the excessive (insufficient) effort provision is related to the common/systemic rather than project-specific/idiosyncratic risk. To highlight the source of externality we consider the case where the productivity shock is correlated across firms in the industry. Specifically, we let  $V_i = \left(\alpha\tilde{h} + (1 - \alpha)\tilde{k}_i\right)e_i + \tilde{\epsilon}_i$  where  $\alpha \in [0, 1]$  and  $\tilde{k}_i$  is a project-specific random term which is independently and normally distributed across agents with mean  $\bar{k}$  and variance  $1/\tau_k$ . In this formulation, when  $\alpha = 0$  productivity shock is idiosyncratic, as  $\alpha$  increases it becomes more correlated, and when  $\alpha = 1$  it is common across firms in the industry.

As before, we assume that the two contractible signals are

$$s_i = \left(\alpha\tilde{h} + (1 - \alpha)\tilde{k}_i\right)e_i + \tilde{\epsilon}_i + \tilde{\zeta},$$

and

$$s_I = \left(\alpha\tilde{h} + (1 - \alpha)\bar{k}\right)\bar{e} + \tilde{\zeta},$$

where  $\bar{e}$  is the average effort exerted by the agents in the industry. The relative performance signal  $q_i$  is,

$$q_i = s_i - s_I = \alpha\tilde{h}(e_i - \bar{e}) + (1 - \alpha)\left(\tilde{k}_i e_i - \bar{k}\bar{e}\right) + \tilde{\epsilon}_i. \quad (23)$$

We can now write agent  $i$ 's compensation when absolute performance signals are not contractible (eg.,  $\tau_\zeta = 0$ ) as

$$l_i q_i + W_i = l_i \left(\alpha\tilde{h}(e_i - \bar{e}) + (1 - \alpha)\left(\tilde{k}_i e_i - \bar{k}\bar{e}\right) + \tilde{\epsilon}_i\right) + W_i. \quad (24)$$

Using (24), given a contract  $(l_i, W_i)$  and average effort  $\bar{e}$ , agent  $i$  chooses  $e_i$  to maximise

$$E(u((l_i q_i + W_i) - C(e_i))).$$

Plugging in  $q_i$  and computing the expectation in the above equation, the agent's problem can be restated as choosing  $e_i$  to maximise

$$l_i \left(\alpha\tilde{h} + (1 - \alpha)\bar{k}\right)(e_i - \bar{e}) + W_i - C(e_i) - \frac{1}{2}r \left(\alpha^2 l_i^2 \frac{1}{\tau_h} (e_i - \bar{e})^2 + (1 - \alpha)^2 (l_i e_i)^2 \frac{1}{\tau_k} + l_i^2 \frac{1}{\tau_\epsilon}\right).$$

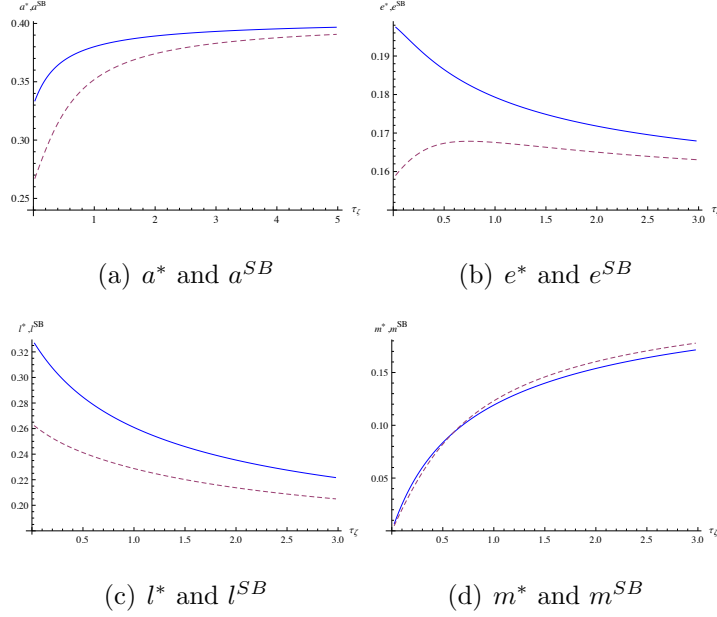


Figure 1: Noisy Industry Signal ( $\tau_\zeta$ ) and Excessive Effort: The solid and the long-dashed lines represent how the total performance sensitivity  $a$ , relative performance  $l$ , absolute performance  $m$ , and effort ( $e$ ) change with respect to the noise of the average industry performance signal ( $\tau_\zeta$ ) in equilibrium and in the planner's optimum, respectively. The parameters are fixed at  $r = 0.3$ ,  $r_P = 0.16$ ,  $\tau_\epsilon = 1$ ,  $\bar{h} = 0.6$ , and  $\tau_h = 0.05$ .

Taking the first-order condition and solving for  $e_i$ , we obtain agent  $i$ 's effort choice as

$$e_i = \frac{l_i (\alpha \bar{h} + (1 - \alpha) \bar{k}) + \frac{r}{\tau_h} \alpha^2 l_i^2 \bar{e}}{1 + \left( \alpha^2 \frac{r}{\tau_h} + (1 - \alpha)^2 \frac{r}{\tau_k} \right) l_i^2}. \quad (25)$$

As one would expect, effort decreases with idiosyncratic volatility ( $1/\tau_k$ ), but as in the main model, industry volatility ( $1/\tau_h$ ) has opposing effects. In particular, as the projects become more correlated agents have stronger incentive to match the average effort  $\bar{e}$  to shield themselves from industry volatility. Conversely, when  $\alpha = 0$ , and the productivity shock is only idiosyncratic, the feedback loop between the industry average effort and an individual agent's effort disappears. Therefore, earlier results on excessive (insufficient) effort provision arise when the productivity shocks are correlated across projects and become more pronounced as  $\alpha$  increases and the correlated component dominates across projects in the industry. Since industries differ in terms of correlations, intermediate cases where  $\alpha \in (0, 1)$  are important for taking the model to data.

## 7.2 Intermediate Cases of Information Friction

In sections 5 and 6, we derive closed-form solutions and explore the properties of the model with either no information frictions or with severe information frictions when only RPE is informative. These cases correspond to  $\tau_\zeta$  equal to infinity or zero. In this section, we look at the intermediate cases where  $0 < \tau_\zeta < \infty$ , that is, principals receive an informative but imperfect signal about absolute performances.

In these cases, the information friction does not eliminate, but nevertheless restricts principals' ability to use APE in contracts, causing externalities to prevail. Since a closed form solution is not possible, we provide two numerical examples, showing how the equilibrium and the second best incentives change with information friction, measured by the inverse of  $\tau_\zeta$ . One example is a case when contractual externalities cause excessive effort/risk taking relative to the second best; and the other is the opposite. In both examples, when  $\tau_\zeta$  increases, the impact of the endogenous contractual externality becomes smaller as principals' ability to span the risk space of the agents strengthens. Therefore, our numerical analysis indicates that when the noise  $\tilde{\zeta}$  becomes more precise, the impact of the externality weakens and the gap between the equilibrium and the second best narrows.

The graphs in Figure 1 illustrate the intuition in the case where equilibrium effort level exceeds the second best. In this case, (22) is positive indicating that, without information friction, principals would like to use positive APE ( $m^* > 0$ ). As this noise becomes smaller (ie.,  $\tau_\zeta$  gets larger), the information friction on using APE is less constraining, principals increase the equilibrium sensitivity to APE ( $m^*$ ) to give agents a positive exposure to the industry risk and better control agents' excessive correlated risk-taking. This is shown in Figure 1(d). Correspondingly, this switch to the usage of APE in contracts leads to the sensitivity to relative performance ( $l^*$ ) to drop in equilibrium, as shown in Figure 1(c). However, the total performance sensitivity in the equilibrium contract ( $a^* = l^* + m^*$ ) increases since principals are able to use both contractual instruments, APE and RPE, more effectively as  $\tau_\zeta$  increases. Interestingly, agents reduce effort in equilibrium because they are now exposed to the industry risk through absolute performance pay, as shown in Figure 1(b). Further, as expected, when  $\tau_\zeta$  gets larger, the information friction gets smaller, the impact of the externality weakens, the gap between the equilibrium and the second best narrows.



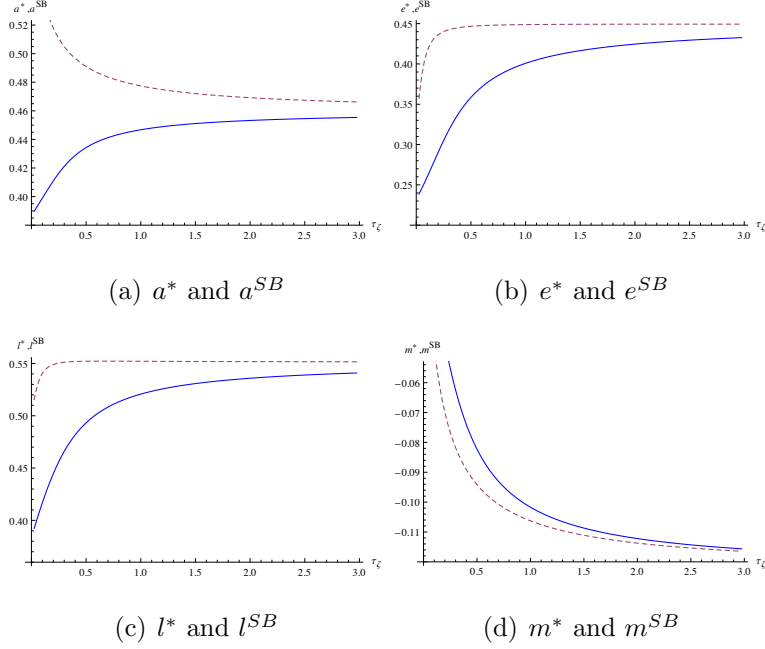


Figure 2: Noisy Industry Signal ( $\tau_\zeta$ ) and Insufficient Effort: The solid and the long-dashed lines represent how the total performance sensitivity  $l$ , relative performance  $l$ , absolute performance  $m$ , and effort ( $e$ ) change with respect to the noise of the average industry performance signal ( $\tau_\zeta$ ) in equilibrium and in the planner's optimum, respectively. The parameters are fixed at  $r = 0.3$ ,  $r_P = 0.01$ ,  $\tau_\epsilon = 1$ ,  $\bar{h} = 0.6$ , and  $\tau_h = 0.05$ .

The graphs in Figure 2 illustrate the intuition in the case where equilibrium effort level falls below the second best. In this case, (22) is negative indicating that, without information friction, principals would like to use negative APE ( $m^* < 0$ ). This happens when principals are less risk-averse and/or idiosyncratic risks are relatively larger than the industry risks. In these occasions, principals would not mind taking over a large portion of idiosyncratic risks from their agents to lower contracting costs, which implies negative absolute performance pay sensitivity. The noise in the industry output signal,  $\tilde{\zeta}$ , constrains principals' ability to do so. When  $\tau_\zeta$  gets larger, this constraint gets less binding, which explains the increased use of APE (that is, the contract is more negatively related to absolute performance) in Figure 2(d). This in turns makes it less costly for principals to use RPE since agents bear less firm-specific risks, resulting in higher usage of relative performance pay as in Figure 2(c). The total performance sensitivity in the equilibrium contract ( $a^*$ ), just as in the previous case, also increases in  $\tau_\zeta$ . Again when the noise in absolute performance signals becomes smaller, principals are less constrained to use both contractual instruments, APE and RPE, to span

the risk space agents are facing, as shown in Figure 2(a). Interestingly, agents increase their effort in equilibrium because they are less exposed to the idiosyncratic risk through the lowered absolute performance sensitivity and more incentivised to take on correlated industry risk through increased relative performance sensitivity, which is shown in Figure 2(b). Further, as expected, when  $\tau_\zeta$  gets larger, the information friction gets smaller, the impact of the externality weakens, the gap between the equilibrium and the second best narrows. These numerical examples show that the insights of the models are robust and endogenous contractual externalities exist when there exists some form of the information friction in the contracting environment.

### 7.3 Heterogeneity in Firm Sizes

In many industries the distribution of firm sizes is heterogeneous. Hence, incorporating heterogeneity into our main model can bring further realism and empirical relevance. In Appendix B, we consider a variation of the original setup with one large and a continuum of fringe firms. We show that the insights on contractual externalities are robust but heterogeneity in firm sizes introduces additional effects.

In this extension the outcome of a specific firm's project has a large impact on the industry benchmark. We call this firm the large firm. In addition, we allow the large firm to face a different level of idiosyncratic noise in its project and its principal and agent to have different risk aversion parameters from the fringe firms. Contract terms are the same as in the original setup, ie, contracts are based on each firm's own signal as well as the industry benchmark. For the fringe firms, our previous results on contractual externalities obtain because the fringe firms do not internalise their impact on the industry benchmark (but now the large firm has a non-negligible weight on the benchmark). Interestingly, there are some new effects. First, the fringe firms do not internalise the impact of their contractual choices on the large firm. This might lead the fringe to push the large firm into suboptimal investment and risk-taking. Second, we find a new feedback loop that introduces an additional source for contractual externalities. Although the large firm filters out its agent's effort from the industry average when solving its problem, its agent's effort affects the efforts of the fringe firms' agents (through its impact on the industry average), which in turn affects the effort

of its own agent. Finally, the existence of a large non-negligible firm presents interesting contracting issues for small firms. Since small firms use only the industry average and their own firm's payoff signal in their compensation contracts, they are unable to filter out the idiosyncratic noise in the large firm's project. As a result, the idiosyncratic noise in the large firm's project affects the contracting problem of the fringe firms and their agents' effort decisions, hence the endogenous industry-wide volatility. In this case, externalities will have welfare impact even when the industry-wide signal noise is absent. This problem will be dampened if the fringe firms include systemically important firms' payoff signals in their contracts. This is potentially interesting to examine empirically in future work.

## 8 Conclusion

In this paper, we study how information frictions in contracting leads to pro-cyclical and potentially excessive risk-taking in the economy. In our model, principals set contracts to make individually optimal risk-return trade-offs ignoring their impact on contracting benchmarks such as average effort in the industry. This results in contractual externalities. In our baseline model, absolute performance signals do not have any correlated noise and contracts are based on both APE and RPE. We show that by shielding agents from it, RPE encourages agents to take industry risk which the principals must shoulder. Despite this, principals use the two contractual instruments to tailor their agents' exposures to the industry and idiosyncratic risks and eliminate the welfare impact of contractual externalities.

However, in presence of information frictions contractual externalities have welfare changing effects. For example, when there is a high level of uncertainty about industry productivity, relative performance information is likely to be more precise and principals put more weight on RPE in contracting. Over-reliance on RPE may set off the ratchet effect in effort choices among agents. For example, risk-averse principals are eager to provide more powerful incentives during booms, causing the industry average effort to be high, triggering a rat race among agents to exert excessive effort, which results in excessive systemic risk exposure in the economy, relative to the second-best. During recessions, the opposite might happen: The incentive provision is too weak and the equilibrium level of effort is lower than the second

best.

Besides theoretical contributions, our results offer a novel explanation, based on frictions in managerial compensation, to the boom-bust cycle of investment and risk-taking observed in industries that experience new but uncertain productivity shocks. These episodes are abundant in recent years. For example, following the introduction of the Internet, the dot.com industry has been flooded with investment which is subsequently reduced. Similarly, following the innovations in the financial products such as asset-based securities, the financial industry has expanded significantly followed by a sharp contraction. Compensation regulations such as enforcing counter-cyclical performance pay could improve the total welfare.

More broadly, the baseline model in the paper offers a equilibrium framework of decentralised contract choices of individual firms, setting an incentive foundation for studying macro-firm dynamics. There are potentially a host of interesting extensions that incorporate various types of market structures and frictions. These possible extensions can introduce additional realism to our analysis and, more importantly, generate new insights about the inter-firm dynamics and implications for macro aggregates. For example, given that the uncertainty is modelled as normal distribution, the baseline model is well suited to study asset pricing implications for aggregate (and endogenous) market risks when combined with an equity market. The model can also be extended to study how the industry structure effects the contractual arrangements with a small number of large firms, and how idiosyncratic noises of the large firms may potentially affect small firms' investment decisions and hence become non-diversifiable industry risks.

## References

- ACHARYA, V., M. PAGANO, AND P. VOLPIN (2011): "Seeking alpha: Excess risk taking and competition for managerial talent," New York University, University of Naples Federico II, London Business School and CEPR Working Paper.
- ACHARYA, V. AND T. YORULMAZER (2007): "Too many to fail: An analysis of time-inconsistency in bank closure policies," *Journal of Financial Intermediation*, 16, 1–31.

- (2008): “Cash in the market pricing and optimal resolution of bank failures,” *Review of Financial Studies*, 21, 2705–2742.
- AGGARWAL, R. K. AND A. A. SAMWICK (1999): “Executive compensation, strategic competition, relative performance evaluation: Theory and evidence,” *The Journal of Finance*, 54, 1999–2043.
- AKERLOF, G. (1976): “The economics of caste and of the rat race and other woeful tales,” *Quarterly Journal of Economics*, 90, 599–617.
- ARNOTT, R., B. C. GREENWALD, AND J. E. STIGLITZ (1994): “Information and economic efficiency,” *Information Economics and Policy*, 6, 77.
- BANAL-ESTANOL, A. AND M. OTTAVIANI (2006): “Mergers with product market risk,” *Journal of Economics & Management Strategy*, 15, 577–608.
- BERNHEIM, B. D. AND M. WHINSTON (1986): “Common agency,” *Econometrica*, 54, 923–942.
- BERTRAND, M. AND S. MULLAINATHAN (2001): “Are CEOs rewarded for luck? The ones without principals are,” *The Quarterly Journal of Economics*, 116, 901–932.
- BHATTACHARYA, S. AND D. MOOKHERJEE (1986): “Portfolio choice in research and development,” *Rand Journal of Economics*, 17, 594–605.
- BHATTACHARYYA, S. AND A. PURNANANDAM (2011): “Risk-taking by banks: What did we know and when did we know it?” Working paper.
- BIAIS, B. R. AND C. CASAMATTA (1999): “Optimal leverage and aggregate investment,” *Journal of Finance*, 54, 1291–1323.
- BIANCHI, J. (2010): “Credit externalities: Macroeconomic effects and policy implications,” *American Economic Review*, 100, 398–402.
- BIANCHI, J. AND E. G. MENDOZA (2011): “Overborrowing, financial crises and macroprudential policy,” IMF Working paper 11/24.

- BLOOM, N., S. BOND, AND J. V. REENEN (2007): “Uncertainty and investment dynamics,” *Review of Economics Studies*, 74, 391–415.
- BLOOM, N., M. FLOETOTTO, N. JAIMOVICH, I. SAPORTA-EKSTEN, AND S. J. TERRY (2014): “Really uncertain business cycles,” Stanford University Discussion Papers.
- CABALLERO, R. J. AND A. KRISHNAMURTHY (2001): “International and domestic collateral constraints in a model of emerging market crises,” *Journal of Monetary Economics*, 48, 513–548.
- (2003): “Excessive dollar debt: Financial development and underinsurance,” *Journal of Finance*, 58, 867–894.
- DA, Z. AND E. SCHAUMBURG (2011): “Relative valuation and analyst target price forecasts,” *Journal of Financial Markets*, 14, 161–192.
- DAVILA, E. (2011): “Dissecting fire sales externalities,” Harvard University Working Papers.
- DE ANGELIS, D. AND Y. GRINSTEIN (2010): “Relative performance evaluation in CEO Compensation: Evidence from the 2006 disclosure rules,” Cornell University Working Paper.
- DIAMOND, P. A. (1998): “Managerial incentives: On the near linearity of optimal compensation,” *Journal of Political Economy*, 34, 931–957.
- FARHI, E. AND J. TIROLE (2012): “Collective moral hazard, maturity mismatch and systemic bailouts,” *American Economic Review*, 102, 60–93.
- FARHI, E. AND I. WERNING (2013): “On the inefficiency of financial market equilibria in macroeconomics models with nominal rigidities,” Harvard Working paper.
- FERSHTMAN, C. AND K. L. JUDD (1986): “Strategic incentive manipulation in rivalrous agency,” Working paper.
- (1987): “Incentive equilibrium in oligopoly,” *American Economic Review*, 77, 927–940.

- GARVEY, G. T. AND T. T. MILBOURN (2006): “Asymmetric benchmarking in compensation: Executives are rewarded for good luck but not penalized for bad,” *Journal of Financial Economics*, 82, 197–225.
- GEANAKOPOLOS, J. AND H. M. POLEMARCHAKIS (1985): “Existence, regularity, and constrained suboptimality of competitive allocations when the asset market is incomplete.” Cowles Foundation for Research in Economics Discussion Papers 764.
- GERSBACH, H. AND J.-C. ROCHET (2012): “Aggregate investment externalities and macroprudential Regulation,” CEPR Discussion Papers 8764.
- GIBBONS, R. AND K. J. MURPHY (1990): “Relative performance evaluation for chief executive officers,” *Industrial and Labor Relations Review*, 43, 30–51.
- GONG, G., L. Y. LI, AND J. Y. SHIN (2010): “Relative performance evaluation and related peer groups in executive compensation contracts,” Pennsylvania State University and University of Illinois at Urbana-Champaign Working Paper.
- GREEN, J. R. AND N. L. STOKEY (1983): “A comparison of tournaments and contracts,” *The Journal of Political Economy*, 91, 349–364.
- GREENWALD, B. C. AND J. E. STIGLITZ (1986): “Externalities in economies with imperfect information and incomplete Markets,” *The Quarterly Journal of Economics*, 101, 229–264.
- GROMB, D. AND D. VAYANOS (2002): “Equilibrium and welfare in markets with financially constrained arbitrageurs,” *Journal of Financial Economics*, 66, 361–407.
- GUERRIERI, V. AND P. KONDOR (2012): “Fund managers, career concerns and asset price volatility,” *American Economic Review*, 102, 1986–2017.
- HE, Z. AND P. KONDOR (2013): “Inefficient investment waves,” NBER Working Papers.
- HOBERG, G. AND G. PHILLIPS (2010): “Real and financial industry booms and busts,” *Journal of Finance*, 65, 45–86.
- HOLMSTRÖM, B. (1979): “Moral hazard and observability,” *Bell Journal of Economics*, 10, 74–91.

- (1982): “Moral hazard in teams,” *Bell Journal of Economics*, 13, 324–340.
- HOUSE, C. (2014): “Fixed costs and long-lived investments,” *Journal of Monetary Economics*, 68, 86–100.
- KATZ, M. L. (1991): “Game-playing agents: Unobservable contracts as precommitments,” *RAND Journal of Economics*, 22, 307–328.
- KHAN, A. AND J. THOMAS (2003): “Nonconvex factor adjustment in equilibrium business cycle models: do nonlinearities matter?” *Journal of Monetary Economics*, 50, 331–360.
- (2008): “Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics,” *Econometrica*, 76, 395–436.
- KORINEK, A. (2010): “Hot money and serial financial crises,” University of Maryland Working Papers.
- KRISHNAMURTHY, A. (2003): “Collateral constraints and the amplification mechanism,” *Journal of Economic Theory*, 111, 277–292.
- LAZEAR, E. P. AND S. ROSEN (1981): “Rank-order tournaments as optimum labor contracts,” *The Journal of Political Economy*, 89, 841–864.
- MAKAROV, I. AND G. PLANTIN (2010): “Rewarding trading skills without inducing gambling,” London Business School Working Paper.
- MYERSON, R. (1982): “Optimal coordination mechanisms in generalized principal-agent problems,” *Journal of Mathematical Economics*, 10, 67–81.
- NALEBUFF, B. J. AND J. E. STIGLITZ (1983): “Prizes and incentives: Towards a general theory of compensation and competition,” *The Bell Journal of Economics*, 14, 21–34.
- PALOMINO, F. AND A. PRAT (2003): “Risk taking and optimal contracts for money managers,” *Rand Journal of Economics*, 34, 113–137.
- PAULY, M. (1974): “Overprovision and public provision of insurance,” *Quarterly Journal of Economics*, 93, 541–562.



- PRENDERGAST, C. (1999): “The provision of incentives in firms,” *Journal of Economic Literature*, 37, 7–63.
- RAJAN, R. G. (1994): “Why bank credit policies fluctuate: A theory and some evidence,” *The Quarterly Journal of Economics*, 109, 399–441.
- SCHARFSTEIN, D. S. AND J. C. STEIN (1990): “Herd behavior and investment,” *American Economic Review*, 80, 465–479.
- SKLIVAS, S. D. (1987): “The strategic choice of managerial incentives,” *RAND Journal of Economics*, 18, 452–458.
- STAVRAKEVA, V. (2013): “Welfare implications of the structure of the banking sector in a small open economy,” Harvard University Working Papers.
- STEIN, J. (2012): “Monetary policy as financial stability regulation,” *The Quarterly Journal of Economics*, 127, 57.
- STIGLITZ, J. E. (1982): “The Inefficiency of the stock market equilibrium,” *Review of Economic Studies*, 49, 241–261.
- VICKERS, J. (1985): “Delegation and the theory of the firm,” *Economic Journal*, 95, 138–147.
- ZWIEBEL, J. (1995): “Corporate conservatism and relative compensation,” *Journal of Political Economy*, 103, 1–25.

## Appendix A

### Proofs

In the proofs we drop the subscript  $i$  when there is no room for confusion.

#### Proof of Proposition 1

Note that

$$e_i = \frac{(l_i + m_i) \bar{h} + \frac{r}{\tau_h} l_i (l_i + m_i) \bar{e}}{1 + \frac{r}{\tau_h} (l_i + m_i)^2} = (l_i + m_i) \bar{h} + o(1/\tau_h).$$

Let  $a_i = l_i + m_i$ . Substituting this into (11) we obtain:

$$\max_{a_i, b_i} a_i (\bar{h})^2 - \frac{(a_i \bar{h})^2}{2} - \frac{1}{2} \left( a_i^2 \frac{r}{\tau_\epsilon} + m_i^2 \frac{r}{\tau_\zeta} \right) - \frac{1}{2} \left( (1 - a_i)^2 \frac{r_P}{\tau_\epsilon} + m_i^2 \frac{r_P}{\tau_\zeta} \right) + o(1/\tau_h) - \bar{W}.$$

For sufficiently large  $\tau_h$  this function is concave in  $(a_i, m_i)$  and has a unique maximum  $(a^*, m^*)$  which is identical for all principals. From this we solve for the unique  $l^* = a^* - m^*$ .

#### Proof of Proposition 2:

Let  $a = l + m$ . The principals' objective can be rewritten as

$$\bar{h}e - \frac{1}{2}e^2 - \frac{1}{2} \left( (ae - l\bar{e})^2 \frac{r}{\tau_h} + a^2 \frac{r}{\tau_\epsilon} \right) - \frac{1}{2} \left( ((1-a)e + l\bar{e})^2 \frac{r_P}{\tau_h} + (1-a)^2 \frac{r_P}{\tau_\epsilon} \right), \quad (\text{A1})$$

and the optimal level of effort as a function of contract terms and the average effort can be rewritten as

$$e = \frac{a\bar{h} + \frac{r}{\tau_h} la\bar{e}}{1 + \frac{r}{\tau_h} a^2}.$$

Next substituting for the effort level in the objective function of (A1) we obtain

$$\begin{aligned} & \left( \bar{h} \left( \frac{a\bar{h} + \frac{r}{\tau_h} la\bar{e}}{1 + \frac{r}{\tau_h} a^2} \right) - \frac{1}{2} \left( \frac{a\bar{h} + \frac{r}{\tau_h} la\bar{e}}{1 + \frac{r}{\tau_h} a^2} \right)^2 - \frac{1}{2} \left( \left( a \left( \frac{a\bar{h} + \frac{r}{\tau_h} la\bar{e}}{1 + \frac{r}{\tau_h} a^2} \right) - l\bar{e} \right)^2 \frac{r}{\tau_h} + a^2 \frac{r}{\tau_\epsilon} \right) \right) \\ & - \frac{1}{2} \left( \left( (1-a) \left( \frac{a\bar{h} + \frac{r}{\tau_h} la\bar{e}}{1 + \frac{r}{\tau_h} a^2} \right) + l\bar{e} \right)^2 \frac{r_P}{\tau_h} + (1-a)^2 \frac{r_P}{\tau_\epsilon} \right). \end{aligned} \quad (\text{A2})$$

For a given  $a$  the above function is negative quadratic in  $l$ . Thus for a given  $a$  principals' objective function is maximised at  $l(a)$  which is given by

$$l(a) = \frac{\bar{h}a \left( \frac{r}{\tau_h} \left( \frac{r}{\tau_h} a^2 + 1 \right) - \frac{r_P}{\tau_h} (1-a) \left( \frac{r}{\tau_h} a + 1 \right) \right)}{\bar{e} \left( \frac{r}{\tau_h} \left( \frac{r}{\tau_h} a^2 + 1 \right) + \frac{r_P}{\tau_h} \left( \frac{r}{\tau_h} a + 1 \right)^2 \right)}. \quad (\text{A3})$$

Substituting (A3) for  $l(a)$  in (A2) we reduce principals' problem to choosing  $a$  to maximize

$$\frac{1}{2}\bar{h}^2 a \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right) \frac{a \left( \frac{r}{\tau_h} - 1 \right) + 2}{\frac{r}{\tau_h} + \frac{r_P}{\tau_h} + \left( \frac{r}{\tau_h} \right)^2 a^2 + \frac{r_P}{\tau_h} \left( \frac{r}{\tau_h} \right)^2 a^2 + 2 \frac{r_P}{\tau_h} \frac{r}{\tau_h} a} - \frac{1}{2} a^2 \frac{r}{\tau_\epsilon} - \frac{1}{2} (1-a)^2 \frac{r_P}{\tau_\epsilon}.$$

Taking the derivative with respect to  $a$ , we obtain

$$\frac{-\bar{h}^2 \left( \frac{r}{\tau_h} a + 1 \right) \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right)^2 (a-1)}{\left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} + \left( \frac{r}{\tau_h} \right)^2 a^2 + \frac{r_P}{\tau_h} \left( \frac{r}{\tau_h} \right)^2 a^2 + 2 \frac{r_P}{\tau_h} \frac{r}{\tau_h} a \right)^2} - a \frac{r}{\tau_\epsilon} - (a-1) \frac{r_P}{\tau_\epsilon}.$$

Note that the above function starts as positive and crosses to negative once. Thus the objective function is maximised at  $a^*$  that solves

$$\begin{aligned} H(a) = & - \bar{h}^2 \left( \frac{r}{\tau_h} a + 1 \right) \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right)^2 (a-1) \\ & - \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} + \left( \frac{r}{\tau_h} \right)^2 a^2 \left( \frac{r_P}{\tau_h} + 1 \right) + 2 \frac{r_P}{\tau_h} \frac{r}{\tau_h} a \right)^2 \left( a \frac{r}{\tau_\epsilon} + (a-1) \frac{r_P}{\tau_\epsilon} \right) = 0. \end{aligned}$$

Note that  $a^* \in (0, 1)$ . In equilibrium

$$\bar{e} = \frac{a^* \bar{h} + \frac{r}{\tau_h} a^* l(a^*) \bar{e}}{1 + \frac{r}{\tau_h} (a^*)^2}.$$

Plugging for  $l(a^*)$ , we obtain

$$\bar{e} = \frac{\bar{h} a^* \left( \frac{r}{\tau_h} a^* + 1 \right) \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right)}{\frac{r}{\tau_h} \left( \frac{r}{\tau_h} (a^*)^2 + 1 \right) + \frac{r_P}{\tau_h} \left( \frac{r}{\tau_h} a^* + 1 \right)^2}.$$

Using the above to substitute for  $\bar{e}$  in (A3), we obtain

$$l(a^*) = \frac{\frac{r}{\tau_h} \left( 1 + \frac{r}{\tau_h} (a^*)^2 \right) + \frac{r_P}{\tau_h} (a^* - 1) \left( \frac{r}{\tau_h} a^* + 1 \right)}{\left( \frac{r}{\tau_h} a^* + 1 \right) \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right)}. \quad (\text{A4})$$

Thus

$$m^* = a^* - l(a^*) = a_i^* - \frac{\frac{r}{\tau_h} \left( 1 + \frac{r}{\tau_h} (a_i^*)^2 \right) + \frac{r_P}{\tau_h} (a_i^* - 1) \left( \frac{r}{\tau_h} a_i^* + 1 \right)}{\left( \frac{r}{\tau_h} a_i^* + 1 \right) \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right)}.$$

**Proof of Corollary 1:**

We continue to use the notation in the proof of Proposition 2. Using (A4),

$$m^* = a^* - l(a^*) > 0 \Leftrightarrow l(a^*) < a^* \Leftrightarrow \frac{r_P}{\tau_h} \left(1 + \frac{r}{\tau_h} a^*\right) > \frac{r}{\tau_h} (1 - a^*) \Leftrightarrow a^* > \left( \frac{\frac{r}{\tau_h} - \frac{r_P}{\tau_h}}{\frac{r}{\tau_h} \left(1 + \frac{r_P}{\tau_h}\right)} \right).$$

Note

$$H \left( \frac{\frac{r}{\tau_h} - \frac{r_P}{\tau_h}}{\frac{r}{\tau_h} \left(1 + \frac{r_P}{\tau_h}\right)} \right) = \frac{1}{\frac{r}{\tau_h} \left( \frac{r_P}{\tau_h} + 1 \right)^3} \left( \frac{r}{\tau_h} + \frac{r_P}{\tau_h} \right)^2 \left( \frac{r}{\tau_h} + 1 \right)^2 \left( \frac{r_P}{\tau_h} \frac{r}{\tau_\epsilon} - \frac{r}{\tau_h} \frac{r}{\tau_\epsilon} + \bar{h}^2 \left( \frac{r_P}{\tau_h} \right)^2 + \bar{h}^2 \frac{r_P}{\tau_h} + \frac{r_P}{\tau_h} \frac{r_P}{\tau_\epsilon} + \frac{r_P}{\tau_h} \frac{r_P}{\tau_\epsilon} \frac{r}{\tau_h} \right).$$

Thus  $l(a^*) \leq a^*$  if and only if

$$\left( \bar{h}^2 \frac{r_P}{\tau_h} \left( \frac{r_P}{\tau_h} + 1 \right) + \frac{r_P}{\tau_\epsilon} \frac{r_P}{\tau_h} \left( \frac{r}{\tau_h} + 1 \right) \right) \geq \frac{r}{\tau_\epsilon} \left( \frac{r}{\tau_h} - \frac{r_P}{\tau_h} \right)$$

which is equivalent to (16).

### Proof of Proposition 3

Proof is given in the text following the statement of Proposition 3.

### Proof of Proposition 4

From Proposition 1, we know the existence part holds.

We use (9) to plug in for  $e$  in the principals' problem (11) (where  $m$  is set to zero given that the signal  $s_i$  is uninformative) and take the derivative of the objective function with respect to  $l$  to find the first-order condition as a function of  $\bar{e}$ .

In equilibrium,  $\bar{e} = l\bar{h}$ . Therefore any equilibrium must solve for the first-order condition and  $\bar{e} = l\bar{h}$ . To find an equilibrium we plug  $\bar{e} = l\bar{h}$  in the first order condition. After simplifying we find the equilibrium condition:

$$\bar{h}^2 \frac{(l-1) \left( l \frac{r_P}{\tau_h} - 1 \right)}{\frac{r}{\tau_h} l^2 + 1} - \left( \frac{r}{\tau_\epsilon} l - \frac{r_P}{\tau_\epsilon} (1-l) \right) = 0.$$

The next lemma is useful in proving the comparative statics results:

**Lemma 1** *Let*

$$\Psi(l) = \bar{h}^2 \frac{(l-1) \left( l \frac{r_P}{\tau_h} - 1 \right)}{\frac{r}{\tau_h} l^2 + 1}. \quad (\text{A5})$$

*Suppose there is a unique equilibrium, then the following are true. (i)  $\Psi(l) - \left( \frac{r}{\tau_\epsilon} l - \frac{r_P}{\tau_\epsilon} (1-l) \right)$  crosses zero from above at  $l^* \in (0, 1)$  where  $\Psi$  is given in (A5). (ii) If  $\tau_h/r_P > r_P/(r+r_P)$  then  $l^* < \frac{\tau_h}{r_P}$ , otherwise  $l^* > \frac{\tau_h}{r_P}$ .*

### Proof of Lemma 1

Part (i) follows from  $\Psi(0) + \frac{r_P}{\tau_\epsilon} > 0$ ,  $\Psi(1) - \frac{r}{\tau_\epsilon} < 0$  and uniqueness. The proof of (ii) is immediate if  $\frac{\tau_h}{r_P} > 1$ . So suppose  $\frac{\tau_h}{r_P} < 1$ . Note  $\Psi(l) - \left( \frac{r}{\tau_\epsilon} l - \frac{r_P}{\tau_\epsilon} (1-l) \right)$  is above  $\Psi(l)$  for  $l < r_P/(r+r_P)$  and is below  $\Psi(l)$  for  $l > r_P/(r+r_P)$ . This and the fact that there is a unique equilibrium  $l^*$  prove part (ii).

### Proof of Proposition 5

Equilibrium  $l^*$  solves

$$\Psi(l^*) - \left( \frac{r}{\tau_\epsilon} l^* - \frac{r_P}{\tau_\epsilon} (1-l^*) \right) = 0$$

where  $\Psi$  is given in (A5). We write  $\Psi(l^*(\bar{h}), \bar{h})$  to make the dependence of  $\Psi$  and  $l^*$  on  $\bar{h}$  explicit. We use similar notation for other parameters, eg,  $\Psi(l^*(\tau_h), \tau_h)$ .

Taking the total derivative of the equilibrium condition with respect to  $\bar{h}$  we obtain

$$\frac{\partial l^*(\bar{h})}{\partial \bar{h}} = \frac{-\frac{\partial \Psi}{\partial \bar{h}}}{\frac{\partial \Psi(l^*(\bar{h}), \bar{h})}{\partial l} - \left( \frac{r}{\tau_\epsilon} + \frac{r_P}{\tau_\epsilon} \right)}.$$

Denominator is negative by Lemma 1 (i). By Lemma 1 (ii),

$$\frac{\partial \Psi(l^*, \bar{h})}{\partial \bar{h}} = -2\bar{h} \frac{(1-l^*) \left( \frac{r_P}{\tau_h} l^* - 1 \right)}{\frac{r}{\tau_h} (l^*)^2 + 1} \geq 0$$

if  $\frac{\tau_h}{r_P} \geq \frac{r_P}{\tau_\epsilon} / \left( \frac{r}{\tau_\epsilon} + \frac{r_P}{\tau_\epsilon} \right)$  which proves part (i) for  $\bar{h}$ . Proof for the result on  $\tau_\epsilon$  is entirely analogous.

## Proof of Proposition 6

To prove the first statement, we take the total derivative of the equilibrium condition with respect to  $r_P$  to obtain

$$\frac{\partial l^*}{\partial r_P} = -\frac{\frac{\partial \Psi}{\partial r_P} + \frac{1}{\tau_\epsilon}(1-l^*)}{\frac{\partial \Psi}{\partial l} - \left(\frac{r}{\tau_\epsilon} + \frac{r_P}{\tau_\epsilon}\right)}$$

Denominator is negative by Lemma 1 (i). Moreover,

$$\frac{\partial \Psi}{\partial r_P} + \frac{1}{\tau_\epsilon}(1-l^*) = \bar{h}^2 \frac{(l-1) \left(\frac{l}{\tau_h}\right)}{\frac{r}{\tau_h} l^2 + 1} + \frac{1}{\tau_\epsilon}(1-l^*)$$

Hence  $\frac{\partial l^*}{\partial r_P} < 0$  if

$$\bar{h}^2 \tau_\epsilon > \left(\frac{r}{\tau_h} (l^*)^2 + 1\right) \frac{\tau_h}{l^*} \quad (\text{A6})$$

On the right hand side of (A6) only  $l^*$  depends on  $\bar{h}$  or  $\tau_\epsilon$ . We know that  $l^*$  takes a value between  $\frac{r_P}{r+r_P}$  and  $\frac{\tau_h}{r_P}$ . Hence the r.h.s. is bounded in  $\bar{h}$  and  $\tau_\epsilon$ . As a result (A6) holds if  $\bar{h}$  or  $\tau_\epsilon$  are large.

To prove the second statement, we take the total derivative of the equilibrium condition with respect to  $r$  to obtain:

$$\frac{\partial l^*(r)}{\partial r} = \frac{-\frac{\partial \Psi}{\partial r} + \frac{1}{\tau_\epsilon} l^*}{\frac{\partial \Psi}{\partial l} - \left(\frac{r}{\tau_\epsilon} + \frac{r_P}{\tau_\epsilon}\right)}.$$

Denominator is negative by Lemma 1 (i). Thus  $\frac{\partial l^*(r)}{\partial r} > 0$  iff

$$\frac{\partial \Psi(l^*, r)}{\partial r} - \frac{1}{\tau_\epsilon} l^* = \bar{h}^2 \frac{(1-l^*) \left(\frac{r_P l^*}{\tau_h} - 1\right) (l^*)^2}{\left(\frac{r}{\tau_h} (l^*)^2 + 1\right)^2 \tau_h} - \frac{1}{\tau_\epsilon} l^* > 0.$$

From Lemma 1 (ii),  $\frac{\tau_h}{r_P} < \frac{r_P}{\tau_\epsilon} / \left(\frac{r}{\tau_\epsilon} + \frac{r_P}{\tau_\epsilon}\right)$  implies  $l^* > \frac{\tau_h}{r_P}$ . Hence, the above inequality holds if and only if

$$\bar{h}^2 \tau_\epsilon > \frac{\left(\frac{r}{\tau_h} (l^*)^2 + 1\right)^2 \tau_h}{(1-l^*) \left(\frac{r_P l^*}{\tau_h} - 1\right) l^*}. \quad (\text{A7})$$

On the right hand side of (A7) only  $l^*$  depends on  $\bar{h}$  or  $\tau_\epsilon$  and  $\frac{\tau_h}{r_P} \leq l^* \leq \frac{r_P}{r+r_P}$ . Hence the right hand side is bounded in  $\bar{h}$  and  $\tau_\epsilon$ . As a result (A7) holds if  $\bar{h}$  or  $\tau_\epsilon$  are large.

## Proof of Proposition 7

Proof follows from plugging  $l^{SB}$  in the equilibrium condition (17) and checking whether its value is positive (in which case  $l^{SB} < l^*$ ) or negative (in which case  $l^{SB} > l^*$ ).

## Appendix B

### One Large and a Continuum of Fringe Firms

This section contains a variation of the main model with one large and a continuum of fringe firms. We use the subscript  $L$  for variables relating to the large firm. The value of the large firm's project is  $V_L = \tilde{h}e_L + \tilde{\zeta}$  and signal is  $s_L = \tilde{h}e_L + \tilde{\epsilon}_L + \tilde{\zeta}$  where  $e_L$  is the effort choice of the large firm's agent,  $\tilde{\epsilon}_L$  is normal with mean zero and precision  $\tau_L$ . We index the fringe firms by  $i \in [0, 1]$ . We define the value of fringe firm  $i$ 's project  $V_i$  and its signal  $s_i$  exactly as in the main model. We denote the risk aversion parameters of the large firm's agent and principal by  $r^L$  and  $r_P^L$  respectively. The remaining parameters ( $\tau_h, \tau_\epsilon, \tau_\zeta, r, r_P$ ) are also defined as in the main model. Hence, we allow the large firm to be different from the fringe firms both in terms of the variance in its idiosyncratic noise and its risk aversion parameters.

We define the industry average signal as:

$$s_I = \rho s_L + (1 - \rho) \int_{i \in [0,1]} s_i d_i = \tilde{h}(\rho e_L + (1 - \rho)\bar{e}_F) + \rho \tilde{\epsilon}_L + \tilde{\zeta}$$

where  $\bar{e}_F$  denotes the average effort of the agents who work for the fringe firms and  $\rho \in [0, 1]$  is the impact of the large firm on the industry average signal. The parameter  $\rho$  captures the size of the large firm relative to the fringe.

We define the signal about the agent's performance relative to industry average as  $q_L = (s_L - s_I)/(1 - \rho)$  for the large firm and  $q_i = s_i - s_I$  for fringe firm  $i$ . Thus,  $q_L = \tilde{h}(e_L - \bar{e}_F) + \tilde{\epsilon}_L$  and  $q_i = \tilde{h}(e_i - \bar{e}) + \tilde{\epsilon}_i - \rho \tilde{\epsilon}_L$  where  $\bar{e} = \rho e_L + (1 - \rho)\bar{e}_F$ . When using RPE the large firm's principal compares the performance of its agent with only the average performance of the fringe firms' agents since it filters out its own performance from the average. In contrast, since the fringe firms are atomistic and ignore their impact on the average, they compare the performance of their agents both with the large firm's agent and the average performance of

other fringe firms' agents. In addition, the RPE signal for the fringe firms is affected by the large firm's idiosyncratic noise.

The large firm uses a linear contract in the form  $l_L q_L + m_L s_L + W_L$  and the fringe firm  $i$  uses a contract in the form  $l_i q_i + m_i s_i + W_i$ . Given the contract  $(l_i, m_i, W_i)$  and average effort  $\bar{e} = \rho e_L + (1 - \rho) \bar{e}_F$  agent  $i$ 's optimal effort choice  $e_i$  is given by:

$$e_i = \frac{(l_i + m_i) \bar{h} + \frac{r}{\tau_h} l_i (l_i + m_i) \bar{e}}{1 + \frac{r}{\tau_h} (l_i + m_i)^2}. \quad (\text{B1})$$

Similarly, given the contract  $(l_L, m_L, W_L)$  and  $\bar{e}_F$  agent  $L$ 's optimal effort choice  $e_L$  is given by:

$$e_L = \frac{(l_L + m_L) \bar{h} + \frac{r^L}{\tau_h} l_L (l_L + m_L) \bar{e}_F}{1 + \frac{r^L}{\tau_h} (l_L + m_L)^2}. \quad (\text{B2})$$

Finally, given  $\bar{e} = \rho e_L + (1 - \rho) \bar{e}_F$  principal  $i$  chooses  $(l_i, m_i)$  to maximize:

$$\begin{aligned} & \bar{h} e_i - C(e_i) - \frac{1}{2} r_P \left( (e_i - l_i (e_i - \bar{e}) - m_i e_i)^2 \frac{1}{\tau_h} + (1 - l_i - m_i)^2 \frac{1}{\tau_\epsilon} + \rho^2 l_i^2 \frac{1}{\tau_L} + m_i^2 \frac{1}{\tau_\zeta} \right) \\ & - \frac{1}{2} r \left( (l_i (e_i - \bar{e}) + m_i e_i)^2 \frac{1}{\tau_h} + (l_i + m_i)^2 \frac{1}{\tau_\epsilon} + \rho^2 l_i^2 \frac{1}{\tau_L} + m_i^2 \frac{1}{\tau_\zeta} \right) - \bar{I} \end{aligned} \quad (\text{B3})$$

subject to (B1) and, given  $\bar{e}_F$  principal  $L$  chooses  $(l_L, m_L)$  to maximize:

$$\begin{aligned} & \bar{h} e_L - C(e_L) - \frac{1}{2} r_P^L \left( (e_L - l_L (e_L - \bar{e}_F) - m_L e_L)^2 \frac{1}{\tau_h} + (1 - l_L - m_L)^2 \frac{1}{\tau_L} + m_L^2 \frac{1}{\tau_\zeta} \right) \\ & - \frac{1}{2} r^L \left( (l_L (e_L - \bar{e}_F) + m_L e_L)^2 \frac{1}{\tau_h} + (l_L + m_L)^2 \frac{1}{\tau_L} + m_L^2 \frac{1}{\tau_\zeta} \right) - \bar{I} \end{aligned} \quad (\text{B4})$$

subject to (B2).

From this analysis we make the following observations.

First, comparing (B3) with the principals' problem (11) in our main model, we see that the two are similar except that the fringe firms' agents have an incentive to match  $\bar{e} = \rho e_L + (1 - \rho) \bar{e}_F$  which is influenced by the large firm's agent's effort. In addition, the large firm's idiosyncratic risk enters fringe firms' problems through the term  $\rho^2 l_i^2 (1/\tau_L)$ . In contrast, we see from (B4) that the large firm's agent has an incentive to match the fringe firms' agents' average effort  $\bar{e}_F$ .

Second, the fringe firms do not internalise the impact of their contractual choices on the large firm. This might lead the fringe to push the large firm into suboptimal investment and risk-taking.



Third, although the large firm filters out its own agent's effort from the industry average when it is solving its own problem, its agent's effort indirectly affects the efforts of the fringe firms' agents (due to its impact on  $\bar{e}$ ) and through this channel affects the effort of its own agent. This creates a feedback loop that introduces an additional source for contractual externalities. If the large firm provides stronger incentives to its agent, than this increases the efforts of the fringe firms' agents. Since the large firm's agent has an incentive to match  $\bar{e}_F$ , this leads to an increase in the large firm's agent's effort.

Fourth, we see from (B3) that the idiosyncratic noise of the large firm enters the fringe firms' contracting problem playing a role similar to the industry-wide signal noise affecting optimal choice of contracts and effort. In this case, externalities will have welfare impact even when the industry-wide signal noise is absent (ie, when  $\tau_\zeta = \infty$ .)