Tse-Ling Teh

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Insurance Design in the Presence of Safety Nets

Tse-Ling Teh*

Abstract

Safety net assistance and insurance exist to manage risk and improve welfare. This shared goal may lead to crowding out. In a new approach, this paper analyzes the interaction of assistance with two dimensions of insurance design: level of coverage and types of risks covered. In a society of risk averse vulnerable individuals and risk neutral assistance providers, Pareto improvements in welfare are achieved through incompleteness in the types of risks covered. The results imply that safety nets promote demand for and the emergence of incomplete insurance. These results have a wide application to insurance markets where safety nets are available, including health care, disaster aid and social welfare.

1 Introduction

Safety net assistance exists to protect against hardship and can be found in health care, disaster aid, retirement pensions and social welfare. However, the presence of safety nets can lead to a Samaritan's Dilemma (Buchanan

*Columbia University in the City of New York, 420 W 118th St, New York, NY 10027, USA. Present address: London School of Economics and Political Science, Houghton St, London, WC2A 2AE, UK, t.teh1@lse.ac.uk.

1975). When safety net assistance provides protection against risk, individual demand for insurance against such risks may be limited (Coate 1995).[1] A commitment not to provide safety net assistance would lead to an alleviation of these inefficiencies. However, such a commitment may not be possible when faced with social need.

An alternative is to examine the way in which insurance is contracted. My paper explores two design aspects of insurance contracts. The first is the coverage level and the second is risk partitioning. The first is common to the literature (Coate 1995, Kaplow 1991, Lewis and Nickerson 1989), whilst the second is new. Risk partitioning is defined as partitioning states into those that are covered by the insurance contract and those that are not. For clarity I term an insurance with risk partitioning as incomplete insurance. An increasing level of incompleteness refers to more states excluded from coverage.[2] An example of incomplete insurance is an insurance contract that covers the destruction of a house in the event of a fire but not in the event of a flood. In this example, fire risk is in the set of covered risks and flood risk is in the set of risks that is not covered.

My findings demonstrate how incomplete insurance bridges the gap in insurance demand created by safety nets, by creating demand for incompleteness over completeness. The rationale of the result is driven by two factors. The first is that a safety net provides implicit subsidization of incomplete insurance but not complete insurance. The second is that there are decreasing marginal returns to completeness under full coverage. Each factor alone drives a wedge in the preference for incompleteness over completeness. When both factors feature in the insurance design, the effect on preferences is amplified.

These findings have implications for both the structure of insurance markets

[1] Examples of markets where insurance and safety net assistance coincide are: private health insurance and public health systems (Gruber and Simon 2008, Herring 2005, Rask and Rask 2000), and private disaster insurance and government disaster assistance (Brunette et al. 2013, Brunette and Couture 2008 Kunreuther et al. 1978).

[2] In contrast, complete insurance is where all states are covered by insurance.

and for the inception of insurance markets. Firstly, the findings demonstrate that the supply of incomplete insurance can be a demand driven phenomenon. Further, and somewhat surprisingly, incomplete insurance can increase welfare of both the vulnerable party and providers of the safety net. To my knowledge, this demand side reason for the existence of incomplete insurance has not been identified in the literature. The reasons for the development of incomplete products have tended to rest on the supply rather than the demand side. For example, incompleteness alleviates risks faced by the insurer associated with adverse selection and moral hazard (Doherty and Richter 2002), as well as covariant losses (Jaffee and Russell 1997). In contrast, these findings show that in the presence of a safety net, a potential assistance recipient will prefer incomplete insurance over complete insurance, generating demand.

Secondly, the findings offer a method to Pareto improve welfare in the face of the Samaritan's Dilemma. In situations where assistance crowds out insurance demand, an incomplete product improves welfare for both assistance recipients and providers. The introduction of incomplete insurance aims to complement existing assistance by allowing the transfer of some risk and generates demand when none would otherwise exist. This can be a particularly useful policy tool to complement assistance programs or in new markets. For example, in the face of emerging risks that are only recently quantifiable, such as environmental and climate change risks, insurance markets can be slow to develop. Incompleteness can reduce the risk borne by the insurance provider and increase demand for insurance, whilst not neglecting the benefits of assistance.

The consideration of risk partitioning is new to the literature on safety nets and insurance. Previous studies have focused on the interaction between the coverage level of insurance, and assistance (Coate 1995, Kaplow 1991, Lewis and Nickerson 1989).[3] Coate (1995) analyzes a market with indemnity (complete) insurance and shows that the possibility of assistance leads to an individual

[3]Incompleteness in the coverage level is also known as partial insurance. In this paper, full coverage is referred to as sufficient insurance, to differentiate it from the second dimension of incompleteness, risk partitioning.

either insuring their entire loss or not insuring at all. Lewis and Nickerson (1989) and Kaplow (1991) also examine the interaction of insurance and charity in the context of self-insurance and moral hazard, respectively. Lewis and Nickerson show that levels of self-insurance decrease under assistance availability. Whilst Kaplow shows moral hazard is generated by any positive amount of government assistance even if financed by lump-sum taxation. My analysis is distinguished from these existing models, by the additional examination of risk partitioning as a contractual component of the insurance design. By incorporating incompleteness through risk partitioning, this paper is the first to demonstrate that incompleteness has a large impact on insurance demand in the shadow of a safety net.

Section two of this paper provides a description of how the vulnerable party, donor and insurer are modeled. The timing of the model is also described, with an emphasis placed on the ex-post and safety net nature of assistance from the altruistic donor. Section three provides a summary of the insurance demand of the vulnerable party in terms of the two dimensions of incompleteness (coverage and risk partitioning) and the welfare impacts on the donor. These results are then extended to consider the how the safety net changes the value of insurance, whilst Section four concludes.

2 Model of the Interaction Between Assistance and Insurance

The model measures welfare in an expected-utility framework and is simplified to include a vulnerable party who is at risk (denoted by the subscript v) and a donor (assistance provider) who is not at risk (denoted by the subscript d).[4] The vulnerable party is risk averse, able to purchase insurance at an actuarially

[4] In Coate (1995) the vulnerable individual is termed the poor person and the donor is termed the rich person.

fair rate and receive assistance.

The donor provides assistance if it is of benefit to them and it is assumed that it is not of benefit to the donor to provide assistance if no risk materializes.[5] As in Coate (1995), the donor is risk neutral and empathetic towards the vulnerable party. The assumption of the donor as risk neutral is not strictly necessary in the model, however it simplifies the calculations without losing insight. Here the donor is imagined as a government, organization or rich individual, in these cases risk neutrality is not uncommon. The implications of the donor's utility function is that the donor prefers the vulnerably party to reach a safety net level of welfare. This is justifiable from a humanistic perspective, since such assistance has a moral foundation and can foster a stable society. Further examples are provided in Section three.

The new innovation in this model is the second dimension of incompleteness, established by partitioning risks into a set that is covered and a set that is not. The probability of a risk being excluded is denoted by γ, and represents the level of incompleteness in the insurance contract. For example, homeowners insurance is often contracted with a set of risks that are covered and a set that are not. In a standard contract, home damage due to fire and vandalism is often covered, but damage due to flood and earthquake are not. With the exclusion of flood and earthquake risk, home owners insurance is incomplete. The risks that are not covered are considered to be excluded and are expressed in the insurance contract through the exclusion clause.

Within the model, there are two probabilities of interest. The first probability is the probability of loss, denoted π. The second probability is the probability of claim exclusion, denoted $\gamma \in [0, 1)$.[6] The intersection of these probabilities creates three possible states: state one where there is no loss (probability $1-\pi$),

[5] In Coate (1995) there is a government that allows transfers from the rich to poor, to ensure that this is true.

[6] Note that γ is a conditional probability. That is conditional on a loss, the probability that the claim is excluded.

state two where there is a loss and a claim is paid (probability $\pi(1-\gamma)$) and state three where there is a loss and no claim is paid (probability $\pi\gamma$). Within these states of the world, it is assumed that the donor may provide assistance in state two and state three only. When $\gamma = 0$, the insurance is complete.

The order of decisions is important in determining the outcome of the model. For a fixed level of incompleteness (γ), the timing of decisions is as follows:

1. The vulnerable party chooses their level (z) of insurance coverage. This relates to the level of coverage, partial ($z < L$) or sufficient ($z = L$).

2. Nature chooses whether the risk occurs or not. That is, loss or no loss. In the case of a risk materializing, a cost of loss (L) is inflicted on the vulnerable party.

3. Nature chooses how the loss is incurred, that is whether the insurance claim is paid (for example, does the loss fall within the exclusion?).

4. The donor decides how much assistance (τ) to provide.

5. The payoffs are concluded.

The main points of this sequence are that when the donor provides assistance they are aware of the level of income facing the vulnerable party and the state of the world, but when the vulnerable party chooses their level of insurance they are unaware of the future state of the world. In other words, the donor provides ex-post assistance and cannot commit to not providing assistance. In this model, the level of assistance depends on the individual's level of insurance and the empathy of the donor. This implies an endogenous form of limited liability, thereby taking into account a range of assistance levels.

The vulnerable party has an income level y_v and the donor has an income level y_d.

The vulnerable party has utility $u(.)$, where $u'(.) > 0$ and $u"(.) < 0$. Under incomplete insurance ($\gamma > 0$), the expected utility of the vulnerable party is defined as:

$$\begin{aligned} E\left[u_v^I\right] &= (1-\pi)u(y_v - \pi(1-\gamma)z) \\ &+\pi(1-\gamma)u(y_v - \pi(1-\gamma)z + z - L + \tau_a) \\ &+\pi\gamma u(y_v - \pi(1-\gamma)z - L + \tau_b) \end{aligned} \tag{2.1}$$

where τ_a and τ_b is the assistance provided in state two and three respectively.

The vulnerable party's welfare affects the donor's welfare at a weight of δ and assistance has a marginal cost of one.[7]

The welfare of the donor is defined as:

$$W_d^I = y_d - \tau + \delta u_v^I \tag{2.2}$$

where τ is the level of assistance, δ is the level of empathy for the vulnerable party by the donor, and u_v^I is the utility of the vulnerable party under incomplete insurance.

Under complete insurance the loss is fully covered by insurance. This implies $\gamma = 0$ and the expected utility of the vulnerable party becomes:

$$E\left[u_v^C\right] = (1-\pi)u(y_v - \pi z) + \pi u(y_v - \pi z + z - L + \tau_{a_c}) \tag{2.3}$$

Where τ_{a_c} is the amount of assistance provided by the donor when complete insurance is available.

[7]Note that δ need not be less than 1 since the vulnerable party's utility has not been scaled relative to the donor's utility.

And equivalently the welfare of the donor is:

$$W_d^C = y_d - \tau_{a_c} + \delta u_v^C \tag{2.4}$$

The complete model follows a similar framework to Coate (1995). However, since the focus of this paper is on welfare comparison between insurance types, the model has been simplified. In particular, the government and government transfer found in the Coate model have been removed.

The description of the complete and incomplete models in this section creates the framework for analysis in the following section. The complete model is derived directly from the incomplete model by setting $\gamma = 0$. It nonetheless represents the more commonly described model of insurance in the literature since it reflects an indemnity insurance product. However, constraining the insurance design to complete products limits the analysis of optimal contracts. The model presented here provides two dimensions of flexibility in determining the optimal contract, the first through the coverage level z, and the second by partitioning the type of risk γ.

3 Results

The results are separated into four subsections. The first subsection describes the construction of safety net assistance. The second considers the vulnerable party and their actions; whilst the third considers the welfare of donors. The results of the first three subsections provide the main theorem of the paper. Notably, the resolution of the vulnerable party's insurance choice proves that when complete insurance is first crowded out, any incomplete insurance product still appeals to the vulnerable party. This leads to the main theorem and implies that assistance encourages the supply of incomplete insurance. Within these subsections it is assumed that the insurer sets the insurance contract, and hence γ cannot be manipulated by the vulnerable party.[8] The fourth subsection considers how behavior changes with changes in the level of donor empathy. This reflects the impact of changing the safety net level and how this interacts with the vulnerable party's insurance decision.

The vulnerable party's dual goal of wealth maximization and protection from risk provides the intuition underlying the main theorem. Risk aversion creates a competition between reliance on the safety net and protection through insurance. Incomplete insurance supplies a cheaper, but less comprehensive form of protection than complete insurance and affects the vulnerable party's behavior through the implicit subsidization of premiums and the diminishing marginal returns to completeness. These two impacts reinforce each other to result in Lemma 1, that shows incomplete insurance is preferred to complete insurance at the point of indifference over complete insurance purchase.

3.1 Safety net assistance

The insurance demand of the vulnerable party depends on the anticipation of assistance from the donor. As described in the section above, a donor only provides assistance in the event of a loss. In other words, I assume the marginal benefit in the event of a loss is greater than the marginal cost, $\delta u'(y_v - L) > 1$. Second, it is assumed that donor does not wish to provide assistance to the vulnerable party if they are fully insured, and assistance is not provided at a level to replace insurance entirely, that is $\delta u'(y_v - E(L)) < 1$. This allows the proceeding analysis to focus on the interesting cases where safety net assistance is provided when there is a loss.

Proposition 1. *The donor provides assistance to ensure the vulnerable party has a target wealth level, w, corresponding to their level of empathy, δ, where w is defined by $\delta u_v'(w) = 1$.*

[8] Comparative statics are in Teh (2015).

Proof. See Appendix. □

Proposition 1 reflects that the donor is altruistic and concerned about absolute poverty rather than relative poverty. This means that regardless of the pre-loss income of a vulnerable party, if the vulnerable party's income falls below a threshold following a loss event the donor will provide assistance. Effectively, the donor does not take into account the pre-loss income of the vulnerable party. A donor with a different welfare function could consider the pre-loss income of the vulnerably party to vary the level of assistance.

Effectively, the donor aims to ensure a safety net level of assistance. Safety net assistance ensures that all citizens have at least a certain level of welfare. In this model, the focus is on a subsistence level of income or benefits to bring individuals up to a certain level of income. An example of an income based safety is the Australian Newstart allowance provides a base income to individuals seeking full time work based on their assets and fortnightly income (Social Security Act 1991). Another example is social housing in the United Kingdom, that provides low rent accommodation to citizens who are unable to afford private rents (effectively increasing income).[9] However, safety nets can also be in kind and justified by equality of opportunity, for example universal health care and education (Gasparini and Pinto 2006).

3.2 The vulnerable party's insurance demand

Proposition 2. *Under incomplete insurance, the vulnerable party optimizes by purchasing no insurance or more than sufficient insurance when assistance is anticipated.*

Proof. See Appendix. □

[9] Social housing became the responsibility of government in the Housing Act 1919; see Fitzpatrick and Pawson (2007) for more information.

The term sufficient insurance is used to describe insurance coverage equal to the level of loss, that is $z = L$.[10] More than sufficient insurance refers to the case where $z > L$. Under incomplete insurance a vulnerable party will optimize by purchasing no coverage or more than sufficient insurance ($z > L$). In situations where assistance crowds out insurance coverage, zero coverage will be purchased. On the other hand, if insurance coverage is not crowded out, the optimal level of coverage is more than sufficient coverage since in effect, assistance subsidizes the purchase of incomplete insurance on the margin. In the event of a loss, an individual improves their welfare by purchasing more than sufficient incomplete insurance in the state where there is a payment. If there is no payment, the vulnerable party receives assistance and this offsets the premium payment. As such, the expected net payment from more than sufficient insurance compensates above the increase in the amount paid in premium.

Although demand for more than sufficient insurance is generated by the incomplete insurance contract, it should be noted that this is a technicality rather than a driver of the results in the paper. The incomplete contract could be designed as a contingent premium, such that no premium is charged in the state with loss but no payment.[11]To distinguish this from the original contract, this insurance is termed the net of premium contract. This would lead to the vulnerable party optimizing by purchasing no insurance or sufficient insurance, as shown in Figure 3.2. Equivalent results for Lemmas 2 and 3, and Theorem 1 under a net of premium contract are possible, although the benefits of incomplete contracts are reduced. These results are shown in the Appendix. This shows that, although important, the subsidization of incomplete contracts created by assistance is not the sole driver of the results. One

[10]This is equivalent to the first dimension of incompleteness that considers the level of coverage.

[11]Equivalent results can be found leading to either sufficient or zero coverage under the incomplete product by altering the way the insurance premium is charged. If no premium is charged in the state with loss but no payment, then the actuarially fair premium would be set to $\tilde{P} = \frac{\pi(1-\gamma)}{1-\pi\gamma}z$. Thus, the limitation to sufficient insurance can equally be obtained by design of the insurance product.

type of contingent premium is found in long-term-care insurance, where premiums are paid only in the no loss state (Jaspersen and Richter 2015). However, the product described in Section 2 is more common and easily recognized as an insurance contract.

Proposition 3. *Under complete insurance, the vulnerable party optimizes by purchasing no insurance or sufficient insurance when assistance is anticipated.*

Proof. This can be easily shown by setting $\gamma = 0$ in the proof of Proposition 2. $\square$

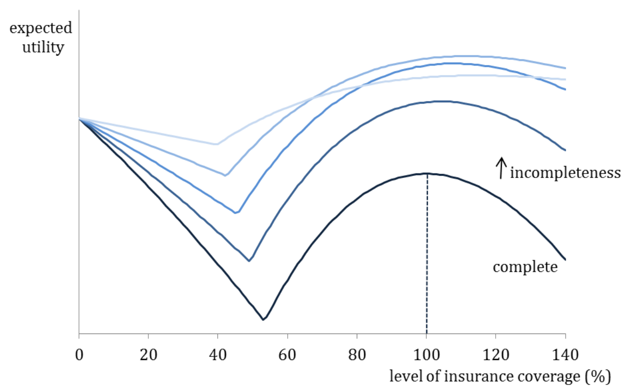

Figure 3.1: Expected utility under varying levels of insurance coverage

Figure 3.1 illustrates Propositions 2 and 3. The Figure shows the expected utility of the vulnerable party over varying levels of insurance coverage for different levels of incompleteness (γ). The darkest curve represents complete insurance $\gamma = 0$, and incompleteness increases (γ increases) as the curve becomes lighter. Each curve represents a different insurance product and the optimal level of coverage is found by finding the highest point on each curve. From Figure 3.1 (for this particular case), it can be seen that the optimum level of coverage is initially zero for complete insurance and some levels of incompleteness. However, for some levels of incompleteness, it becomes optimal to purchase insurance.

The initial downward trend in utility is caused by the crowding out of assistance by insurance purchase. The turning point in the curve is the point at which insurance no longer crowds out assistance and expected utility begins to increase with insurance coverage. The shape of the curve following this turning point represents the traditional response to insurance purchase. For complete insurance ($\gamma = 0$), the turning point represents the point when there is zero provision of assistance. Insurance coverage has completely crowded out assistance. For incomplete insurance ($\gamma > 0$), the turning point represents the point at which assistance has been crowded out in the second state. However, there is still charitable provision in the third state. The uninsurability of the third state, determines that assistance will be provided when incomplete insurance is purchased. Hence, incomplete insurance may provide higher expected utility across all levels of insurance coverage.

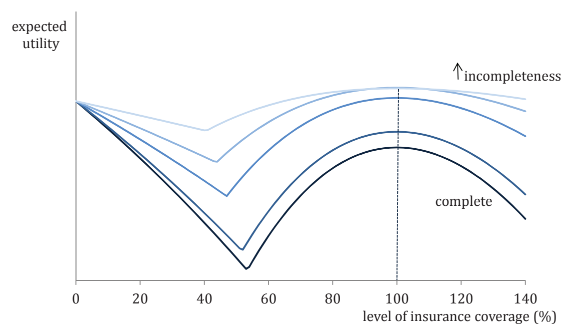

Figure 3.2: Expected utility under varying levels of insurance coverage (net of premium contract)

The combination of Propositions 1, 2, and 3, show that although the vulnerable party could choose any level of insurance, their optimum can only be at two levels of coverage under each product. This is due to the nature of the assistance provided by the donor, as a pure altruist. This simplifies the analysis of welfare under these products, as there are only three coverage levels that are of concern. These are no coverage, sufficient coverage in the case of complete insurance and more than sufficient coverage in the case of incomplete insurance.

The vulnerable party's insurance demand depends upon the type of insurance offered and the level of safety net assistance determined by the target wealth level in Proposition 1. On the one hand, the vulnerable party is risk averse and is therefore attracted to the protection of sufficient complete insurance. But on the other, incomplete insurance provides some protection and the possibility of using the safety net, making it a cheaper product at the expense of full protection. The vulnerable party's insurance decision balances these competing forces and will depend on the shape of an individual's utility curve.

Lemma 1. *When the level of assistance first precludes the purchase of complete insurance, the vulnerable party would purchase incomplete insurance.*

Proof. See Appendix. □

The vulnerable party's insurance decision as determined in Lemma 1 is driven by two factors intersecting with risk aversion. The first is the implicit subsidization of incomplete premiums by the safety net and the second is the decreasing marginal returns to completeness. Either of these factors alone leads to Lemma 1, however together the effects are reinforced.[12] The effect of implicit subsidization on the expected utility can be seen by the difference in Figures 3.1 and 3.2. Whilst the diminishing marginal returns to incompleteness effect is observed in 3.2.

Although the premium is assumed to be actuarially fair, the effective premium faced by the vulnerable party includes an absolute loading and subsidy created by assistance. Consider, the premium (P) charged by an insurer neglecting the safety net: $P = \pi(1-\gamma)z$. This premium is actuarially fair. However, for any level of coverage below $\bar{z}$, where $\bar{z} = \frac{w-(y_v-L)}{1-\pi(1-\gamma)}$, coverage directly crowds

[12] In the net-of-premium contract (with premium $\tilde{P}$), only the second factor is present.

out assistance. Thus, paying the premium for coverage up until level $\bar{z}$ acts as a premium loading that the vulnerable party must pay before they can receive some benefit. Because this loading does not depend on the level of coverage, above $\bar{z}$, it is in a sense an absolute premium loading that the vulnerable party must pay.

On the other side, the implicit subsidization of incomplete insurance by the safety net can be illustrated by considering the expected premium (P_E). When the vulnerable party suffers a loss, but does not receive a payoff, their wealth is topped up to w and this top up includes the premium. This means that the vulnerable party does not pay the premium if there is a loss but no payout. Thus, the expected premium payment is: $P_E = (1 - \pi\gamma)(\pi(1 - \gamma)z)$. For complete insurance, $\gamma = 0$ and $P = P_E$.

The effective premium can be rewritten as the sum of a relative and an absolute loading, along with a subsidy:

$$P_E = \underbrace{(1 - \pi\gamma)}_{\text{Subsidy}} \left[\underbrace{\pi(1 - \gamma)(z - \bar{z})}_{\text{Relative loading}} + \underbrace{\pi(1 - \gamma)\bar{z}}_{\text{Absolute loading}} \right] \tag{3.1}$$

Equation 3.1, shows the relative loading, the absolute loading, and the subsidy. The first term is the subsidy, which comes from the probability the premium is not paid. This subsidy reduces the relative and absolute loadings. The second term is the relative loading, this reflects the cost of insurance past the point of crowding out. The last term is the absolute loading, which reflects the premium that must be paid to get the level of payout up to the threshold.

Lemma 1 illustrates that when the vulnerable party is just indifferent between buying complete insurance and relying on assistance, then any incomplete insurance will be preferred by the vulnerable party. One of the main drivers of this result is the way incomplete insurance subsidizes the premium, as discussed above. However, Lemma 1 also holds without this subsidization. The reason for this is the diminishing marginal utility of a dollar.

In a standard insurance contract, an individual often chooses the level of coverage (how much of a payout is received in the case of loss). However, in incomplete insurance, the types of states covered by insurance can also be a choice variable. For example, the level of incompleteness can be determined through a choice of exclusions in an insurance contract. As with any other good, the more spent on insurance coverage (as in the number of states covered, or completeness) the higher the premium paid. For each additional slice of completeness, the marginal value of a dollar increases which lowers the relative value of additional insurance.

To make this clearer, consider the net of premium contract, with insurance premium $\tilde{P} = \frac{\pi(1-\gamma)}{1-\pi\gamma}z$, where z is the level of coverage. This premium removes the subsidization effect caused by assistance. It is easy to show that under the net of premium contract, for any γ, the optimal level of insurance is $z = L$ when the vulnerable party chooses to insure.[13] Also note that, the net of premium contract is the complete contract when $\gamma = 0$. Fix the level of insurance at $z = L$, and consider how the utility of the vulnerable party changes as γ changes. This change may be positive or negative, but the second derivative of this change is negative. This means utility is concave in γ, so there are decreasing marginal returns to completeness. A further implication of this is that if the vulnerable party is indifferent between complete insurance and no insurance, then by the concavity of utility, insurance must be optimal under any $\gamma \in (0, 1)$.

[13] This is shown in the Appendix under Theorem 2.

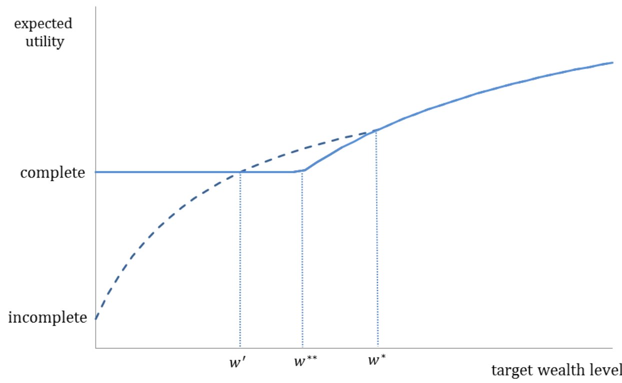

Figure 3.3: Comparison of the utility between complete and incomplete insurance

As before, the target wealth level w is defined as the minimum wealth that the donor would ensure the vulnerable party has, given their level of empathy δ.[14] Figure 3.3 shows the optimal utility at different target wealth levels for the vulnerable party under incomplete insurance (dashed curve) and complete insurance (solid curve). The incomplete insurance in the figure has a fixed level of incompleteness, γ and is compared to complete insurance, $\gamma = 0$. For a target wealth level of assistance above w^*, assistance crowds out all insurance in both incomplete and complete insurance. For the section (w^{**}, w^*), assistance crowds out complete insurance but not incomplete insurance (that is if both products are offered only the incomplete product will be purchased). For (w', w^{**}) assistance no longer crowds out insurance, however incomplete insurance provides higher utility as compared to the complete insurance. For a target wealth level of assistance below w' complete insurance is preferred.

Lemma 2. *The level of assistance to induce the purchase of complete insurance is less than the level of assistance to induce the purchase of incomplete insurance.*

Lemma 2 is implied by Lemma 1. Figure 3.3 indicates that complete insurance

[14]The target wealth level is a net level of wealth for the vulnerable party; after losses, income and assistance.

is purchased up to the point where assistance provides a target wealth level of w^{**} and incomplete insurance is purchased up to the point where assistance provides a target wealth level of w^*.

Combining Proposition 2, 3 and Lemma 1 provides the following summary of optimal behavior.

For a target wealth level w:

1. For $w > w^*$: the vulnerable party optimizes to have zero coverage under complete insurance ($z^C = 0$) and zero coverage under incomplete insurance ($z^I = 0$).

2. For $w^* > w > w^{**}$: the vulnerable party optimizes to have zero coverage under complete insurance ($z^C = 0$) and more than sufficient incomplete insurance ($z^I = \tilde{L}$).

3. For $w^{**} > w$: the vulnerable party optimizes to have sufficient coverage under complete insurance ($z^C = L$) and more than sufficient incomplete insurance ($z^I = \tilde{L}$).

This behavior is summarized in Table 3.1.

Table 3.1: Summary of Vulnerable Party Behavior

Target assistance level w	Complete insurance	Incomplete insurance
$w > w^*$	$z^C = 0$	$z^I = 0$
$w^* > w > w^{**}$	$z^C = 0$	$z^I = \tilde{L}$
$w^{**} > w$	$z^C = L$	$z^I = \tilde{L}$

An implication of Lemma 2 is the current format of the disaster insurance market. In numerous countries including the United States and Australia, homeowners insurance is designed so that natural disasters are excluded from generic insurance. The literature often points to these exclusions as supply driven. However, Lemma 2 suggests that such exclusions can be seen as both a result of demand in the market and the result of profit maximizing insurance companies. If insurance companies are aware of the existence of assistance in the face of natural disasters, it is more profitable to provide an incomplete insurance product, as this will have higher demand than a complete insurance product. Thus the specific exclusion of natural disasters from these insurance products can be seen as an example of the application of Lemma 2.

The following subsection considers the impact of incomplete insurance on donor's welfare. In order to assess donor welfare, I limit the amount of insurance a vulnerable party can purchase to sufficient insurance. More than sufficient insurance is equivalent to insuring more than the loss value. Opportunities to purchase such an insurance contract are rare. For this reason, the limit of insurance coverage to a sufficient level is implemented. This assumption however, is not material to the results of Lemma 1 and 2, as discussed this Section and proven in the Appendix. The subscript l in w_l indicates when the results require that the maximum possible level of coverage is equal to the loss value.

3.3 The donor's welfare

The donor's welfare depends upon the welfare of the vulnerable party and the cost of assistance, as defined in Equations 2.2 and 2.4.

Proposition 4. *Under either type of insurance, donor welfare is higher with sufficient insurance than with zero insurance.*

Proof. Under complete insurance, Proposition 4 holds since insurance directly compensates the vulnerable individual for losses, thereby reducing the amount of direct relief necessary from donors.

See Appendix for the proof under incomplete insurance. □

Proposition 4 illustrates that the transfer of some risk to the insurer, lessens the responsibility of the donor through assistance. The reduction of implicit risk born by the donor improves their welfare, as it reduces the cost of maintaining the safety net level of welfare for the vulnerable party.

Proposition 5. *Under incomplete insurance, when the vulnerable party buys insurance the donor's welfare is lowered by the allowance of more than sufficient insurance.*

Proof. See Appendix. □

Proposition 5 establishes that donor welfare strictly decreases if vulnerable parties are provided the opportunity to purchase more than sufficient insurance. The rationale behind this proposition is that in the event of a loss without a claim the donor is forced to compensate the vulnerable party for their loss of premium since they cannot commit to not providing assistance. When provided the opportunity to purchase more than sufficient insurance, vulnerable parties can exploit assistance to gain greater benefits in the state with a payout. The benefit of purchasing excessive insurance in this state is greater than the harm in other states.

Lemma 3. *Comparison of welfare under varying levels of assistance.*

(i) If $w > w_l^*$*, the welfare of the donor is independent of the type of insurance.*

(ii) If $w_l^* > w > w_l^{**}$*, the welfare of the donor is higher when incomplete insurance is available rather than complete.*

(iii) If $w_l^{**} > w$*, the welfare of the donor is higher when complete insurance is available rather than incomplete.*

where w_l^* *is the target wealth level where the vulnerable party ceases to purchase any insurance.*

and w_l^{**} is the target wealth level where the vulnerable party's optimal utility is achieved by purchasing incomplete insurance (limited to sufficient coverage)

Proof. See Appendix. □

Table 3.2: Summary of Donor welfare

Target assistance level w	Donor welfare
$w > w_l^*$	welfare equal under both contracts
$w_l^* > w > w_l^{**}$	welfare higher under incomplete insurance
$w_l^{**} > w$	welfare higher under complete insurance

In case (i), the vulnerable party has zero insurance coverage and relies entirely on assistance and thus the welfare is the same under either type of insurance. In case (ii), the vulnerable party has sufficient incomplete insurance but zero complete insurance, under Proposition 4 the purchase of insurance is welfare enhancing for the donor. In case (iii), the vulnerable party purchases sufficient insurance coverage. Under complete insurance the vulnerable party does not rely on assistance at all, whereas under incomplete insurance the vulnerable party still relies on assistance in the situation of no payout. Hence, the donor's welfare is higher when they do not need to provide assistance, that is, when complete insurance is purchased.

Theorem 1. *When the level of assistance first precludes the purchase of complete insurance, any type of incomplete insurance is welfare enhancing and Pareto improving.*

Theorem 1 follows directly from Lemma 3. Lemma 3 indicates that for $w > w_l^{**}$, welfare is higher under incomplete insurance. Since w_l^{**} is the level of assistance under which the individual chooses not to purchase complete insurance, Theorem 1 follows.

Theorem 1 determines that within a safety net framework, donor welfare is enhanced by the presence of an incomplete insurance product. The level of incompleteness of the product depends upon the extent of empathy, δ, the donor exhibits and the ratio of loss size to wealth of the vulnerable party.

One implication of Theorem 1 is that in markets where insurance competes with assistance, incomplete insurance products are likely to be more prevalent than complete insurance. Consider first insurance markets without assistance, for example life insurance and car insurance. Life insurance policies tend to have a single outright exclusion, the suicide clause. That can be explained as a way to minimize moral hazard.[15] It is not common for chronic illness to be excluded from life insurance. Rather these conditions may lead to a risk adjusted premium.[16] Similarly, there are few exclusions under car insurance.[17] This is curious because it is arguable that car insurers are susceptible to moral hazard and adverse selection, both of which can be combated through exclusion clauses. Yet, there are very few exclusion clauses under car insurance.

In contrast, some typical exclusions from homeowners insurance include include earth movement, flood, violent uprisings and armed hostilities, nuclear radiation and ordinances by a government authority (Siemens et al. 2011). Apart from the final exclusion, this list of exclusions are emotive and it may be expected that assistance will be available if one's home is destroyed (Viscusi and Zeckhauser 2006). These exclusions class homeowners insurance as incomplete and is in line with the implications of Theorem 1. Compared to car and life insurance, homeowners insurance is more incomplete and this accords with perceived assistance availability.[18]

Therefore, existing insurance markets indicate product availability consistent with Theorem 1. Incomplete insurance is driven by the parameter γ , that represents the degree of incompleteness and links the probability of coverage

[15] There is mixed evidence of the level of adverse selection in life insurance (He 2009).

[16] In many cases, HIV is also not excluded (Association of British Insurers 2016).

[17] The Aviva 2016 product only lists two exclusions: if the type of car use is not covered, or the driver at the time of accident is not covered (Aviva Insurance Limited 2016).

[18] For example, van Asseldonk et al. (2002) finds continued belief in government disaster relief for farmers despite repeated statements that such relief is unavailable.

exclusion to premium levels. Theorem 1 illustrates that an incomplete insurance product can be designed to induce insurance purchase when complete insurance is first crowded out by assistance and that such a product is welfare enhancing.

3.4 The impact of the level of empathy on the value of insurance

This subsection considers the impact of changes in δ, donor empathy. As empathy increases, the target wealth level, w increases. The target wealth level, w, determines the safety net level and in turn determines the optimal level of insurance demand. As described in Lemma 2, for very high target wealth levels $w > w^*$, neither insurance product is purchased and so there is no impact on the value of insurance premiums. For very low target wealth levels $w < w^{**}$ the vulnerable party insures sufficiently and no safety net is required.

As expected, the change in the donor's empathy differentially affects the expected utility of the vulnerable party under complete and incomplete insurance. Under complete insurance, an increase in donor empathy increases the vulnerable party's expected utility until the point when insurance is purchased. Whereas in the case of incomplete insurance, the vulnerable party's expected utility increases at all levels of insurance coverage. The distinction is clear in Figure 3.4. For three levels of empathy it is visible that under complete insurance, expected utility varies only when no insurance is purchased, but under incomplete insurance expected utility increases across all coverage levels (incompleteness in this graph is set to $\gamma = 0.2$). The difference is due to the improving value of incomplete insurance as compared to complete insurance as the donor's empathy increases.

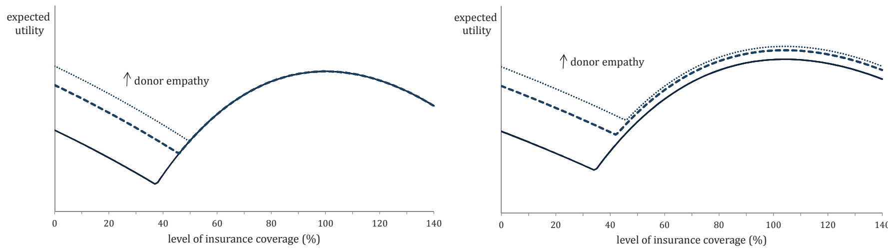

Figure 3.4: The effect of donor empathy on complete (on left) and incomplete (on right) insurance

4 Conclusion

The provision of assistance to those in need is an instinctive response from individuals, institutions and governments alike. Policy interventions such as social welfare, public health care, subsidized insurance, social housing, bailouts, disaster aid and public pensions all represent safety net assistance. The availability of a safety net prompts a Samaritan's Dilemma, and importantly acts as a quasi-subsidy on insurance contracts for these risks. This leads to the Pareto dominance of incomplete insurance over complete insurance, when safety nets are available. The implication of these novel findings is that incomplete insurance will have greater demand in markets where safety net assistance is available. Indicative evidence of this can be found in the contract structure of homeowners insurance as compared to car insurance.

One particular example that has not been touched on in this paper is that of government bailouts to companies and financial institutions that would otherwise collapse.[19] In providing assistance the government is creating an expectation of assistance in the future and has illustrated its inability to commit to not providing assistance. Whether these actions are considered right

[19]For example, car manufacturers Chrysler (1980), banks Citigroup (2008).

or wrong, they nevertheless create a form of the Samaritan's Dilemma. Although, the model presented in this paper is pared back to fundamentals, it provides the intuition that could lead to market-based solutions. For instance, the introduction of incomplete hedging/immunization mechanisms can encourage these entities to assume some of the risk that is currently being overlooked and improve the welfare of the donor (in this case, the government).

Finally, the paper has provided a transparent theoretical model to illustrate the welfare impact of a market with complete and incomplete insurance under the Samaritan's Dilemma. In doing so, I have abstracted by using expected utility and risk aversion to determine optimal actions of the vulnerable party. An extension of this research is to include broader methods for decision making under risk, such as generalized expected utility theory (Machina 1982), and decision weights (Starmer 2000) would provide valuable additional insights. The results would also garner additional value from an empirical test of the results through either data or experimental work.

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A Appendix

Proposition 1.

Proof. The donor's welfare function is $W_d = y_d - \tau + \delta u_v$. The donor will choose to provide assistance up to the point where the marginal cost is equal to the marginal benefit of providing assistance. This ensures that the amount of assistance will satisfy: $\delta u'(y_v(s) + \tau) = 1$ when$\tau > 0$ and $\delta u'(y_v(s) + \tau) \leq 1$ when $\tau = 0$, where $y_v(s)$ is the income of the vulnerable party in state s.

As previously, let w be defined by $\delta u'(w) = 1$. w is the minimum level of wealth the donor would ensure the vulnerable party has, given their level of care δ.

Now assistance τ is defined by:

$$\tau = \begin{cases} w - y_v(s) & if \quad y_v(s) < w \\ 0 & if \quad y_v(s) \geqslant w \end{cases}$$

This indicates that w is the lower bound on the amount of income a vulnerable party receives. □

Proposition 2. *Under incomplete insurance, the vulnerable party optimizes by purchasing no insurance or more than sufficient insurance when assistance is anticipated.*

Proof. The level of optimal insurance depends upon the level of anticipated assistance. First consider the assistance in state two of the world. In this case, the vulnerable party receives an insurance payment for the amount insured, but may also receive assistance. The donor will only provide assistance if it is of benefit to them. Thereby, the optimal level of assistance is,

$$\tau_a^* = argmax_{\tau_a \geq 0}\{y_d - \tau_a + \delta u(y_v + (1 - \pi(1 - \gamma))z - L + \tau_a)\}$$

Let w be defined by $\delta u'(w) = 1$. w is the target wealth level (the minimum level of wealth) that the donor would ensure the vulnerable party has, given their level of care δ. The target wealth level sets the donor's marginal utility of assistance gained equal to the marginal cost. At this point, the donor is indifferent between providing and not providing assistance. For notational convenience, the δ subscript will be removed since each w corresponds to a particular δ.

The optimal assistance level in state two of the world depends upon the extent of insurance coverage purchased by the vulnerable party, and is defined as $\tau_a^*(z) = max\{0, w - y_v + L - (1 - \pi(1 - \gamma))z\}$.

Next consider assistance in state three of the world. In this case even if the vulnerable party purchased insurance since it is incomplete, no payment is received despite being affected by the risk occuring.

Analogously, the optimal level of assistance is
$\tau_b^*(z) = max\{0, w - y_v + L + \pi(1 - \gamma)z\}$. Since $w - y_v + L + \pi(1 - \gamma)z \geq 0$, this can be simplified to $\tau_b^*(z) = w - y_v + L + \pi(1 - \gamma)z$. In state three, the vulnerable party always receives assistance and the amount depends upon the vulnerable party's insurance coverage. This comes about because of the

assumption made in section 3, that the donor will provide assistance when a loss occurs. Also, the donor is concerned with the net level of the vulnerable party's wealth, and hence will compensate for the loss of premium.

Based on these anticipated assistance levels, the optimal level of insurance coverage (z^*) for the vulnerable individual can be determined.

$$\begin{aligned} z^* = argmax_{z\geq 0}\{&\pi(1-\gamma)u(y_v + (1-\pi(1-\gamma))z - L + \tau_a^*) \\ &+\pi\gamma u(y_v - \pi(1-\gamma)z - L + \tau_b^*) \\ &+(1-\pi)u(y_v - \pi(1-\gamma)z)\} \end{aligned} \tag{A.1}$$

Consider z for $z \in [0, \frac{w-y_v+L}{1-\pi(1-\gamma)}]$.

In this interval assistance crowds out insurance purchase one for one in loss states and in the no loss state insurance purchase decreases utility. For z in this interval an increase in the purchase of insurance decreases utility, so the vulnerable party will not purchase insurance.

Consider z for $z \in (\frac{w-y_v+L}{1-\pi(1-\gamma)}, \tilde{L})$ where $\tilde{L} > L$.

In this interval charitable transfers $\tau_a^* = 0$. In the loss state with payment, increasing z improves utility. In the loss state without payment, assistance ensures a set level of utility. In the good state increasing insurance decreases utility. The improvement in utility in the loss state increases at a faster rate than it decreases in the good state. Thereby, an increase in insurance increases expected utility. Since an increase in z increases expected utility, it is evident that the vulnerable party will at least sufficiently insure. However, due to the incomplete nature of the insurance, it is of benefit to insure beyond sufficient insurance as shown by the first order conditions below.

Consider when $\gamma > 0$ and the insurance is incomplete.

The first order condition with respect to z is:

$$\begin{aligned}\frac{du}{dz} &= -\pi(1-\pi)(1-\gamma)u'\left(y_v-\pi(1-\gamma)z\right)\\ &\quad +\pi(1-\pi(1-\gamma))(1-\gamma)u'\left(y_v-L+(1-\pi(1-\gamma))z\right)\end{aligned}$$

The second order condition with respect to z is:

$$\begin{aligned}\frac{d^2u}{dz^2} &= \pi^2(1-\gamma)^2(1-\pi)u''\left(y_v-\pi(1-\gamma)z\right)\\ &\quad +\pi(1-\gamma)\left(1-\pi(1-\gamma)\right)^2u''\left(y_v-L+(1-\pi(1-\gamma))z\right)\\ &< 0\end{aligned}$$

for all $z > \frac{w-y_v+L}{1-\pi(1-\gamma)}$, due to the concavity of the utility function $u''(.) < 0$.

At sufficient insurance $\frac{du}{dz} > 0$.

$$\begin{aligned}&\pi(1-\gamma)\left[(1-\pi)\left(u'(y_v-L+(1-\pi(1-\gamma))z)-u'(y_v-\pi(1-\gamma)z)\right)\right]\\ &\qquad +\pi(1-\gamma)\left[\pi\gamma u'(y_v-L+(1-\pi(1-\gamma))z)\right] > 0\end{aligned}$$

for $z = L$.

$\tilde{L}$ is defined as the point at which $\frac{du}{dz} = 0$. Therefore, $\tilde{L} > L$, hence the optimal insurance for the vulnerable party is either no insurance or greater than sufficient insurance. □

Lemma 1. *When the level of assistance first precludes the purchase of complete insurance, the vulnerable party would purchase incomplete insurance.*

Proof. The donor will choose to provide assistance up to the point where the marginal cost is equal to the marginal benefit of providing assistance. This ensures that the amount of assistance will satisfy: $\delta u'(y_v(s) + \tau) = 1$, where $y_v(s)$ is the income of the vulnerable party in state s.

As previously, let w be defined by $\delta u'(w) = 1$. w is the minimum level of wealth the donor would ensure the vulnerable party has, given their level of care δ.

Now assistance τ is defined by:

$$\tau \quad = \quad \begin{cases} w - y_v(s) & if \quad y_v(s) < w \\ 0 & if \quad y_v(s) \geqslant w \end{cases}$$

This indicates that w is the lower bound on the amount of income a vulnerable party receives. Thus in every state of the world assistance will ensure that the vulnerable party has at least w.

From Proposition 2 and 3, the optimal insurance level for the vulnerable party under incomplete insurance is $\{0,\, \tilde{L}\}$ and under complete insurance $\{0,\, L\}$.

Under Jensen's inequality if $u'(.) > 0$, $u''(.) < 0$ and $L > 0$, $\gamma > 0$, $\pi > 0$.

$$u(y_v - \pi(1-\gamma)L) = u(\gamma y_v + (1-\gamma)y_v - \pi(1-\gamma)L) > \gamma u(y_v) + (1-\gamma)u(y_v - \pi L) \tag{A.2}$$

Pick w^{**} such that $u(y_v - \pi L) = (1 - \pi)u(y_v) + \pi u(w^{**})$. This equation equates the utility from sufficient insurance and no insurance. Thereby w^{**} is the amount of assistance in the loss state that makes the vulnerable party indifferent between purchasing sufficient and no insurance under complete insurance.

Consider w^{**} in the case of incomplete insurance.

Assume the vulnerable party purchases sufficient incomplete insurance, the expected utility is now:

$$\begin{aligned} E[u_v^I] &= (1-\pi)u(y_v - \pi(1-\gamma)L) \\ &\quad +\pi(1-\gamma)u(y_v - \pi(1-\gamma)L) \\ &\quad +\pi\gamma u(w^{**}) \end{aligned}$$

Using Equation (A.2) provides:

$$\begin{aligned} E[u_v^I] &> (1-\pi)[\gamma u(y_v) + (1-\gamma)u(y_v - \pi L)] \\ &\quad +\pi(1-\gamma)u(y_v - \pi(1-\gamma)L) + \pi\gamma u(w^{**}) \\ &= \pi(1-\gamma)u(y_v - \pi(1-\gamma)L) + (1-\gamma)(1-\pi)u(y_v - \pi L) \\ &\quad +\gamma u(y_v - \pi L) \\ &= \pi(1-\gamma)u(y_v - \pi(1-\gamma)L) + (1-\pi(1-\gamma))u(y_v - \pi L) \\ &> u(y_v - \pi L) \\ &= E[u_v^C]_{z=L} \end{aligned}$$

This indicates that the expected utility under incomplete insurance is higher than complete insurance at the same level of assistance. From above, the optimum level of insurance under incomplete insurance is $\{0, \tilde{L}\}$, where $\tilde{L} > L$. So it must be that at the optimum level of insurance $E[u_v^I] > u(y_v - \pi L)$ for w^{**}. Therefore, there is a level of assistance for which it is optimal to purchase incomplete insurance but not complete insurance. □

Proposition 4. *Under either type of insurance, donor welfare is higher with sufficient insurance than with zero insurance.*

Proof. Let w be the target wealth level in the loss state.

With zero insurance

$$\begin{aligned} W^I_{d,z=0} &= y_d - \pi(w - y_v + L) + (1-\pi)\delta u(y_v) + \pi\delta u(w) \\ &= y_d - \pi(w - y_v + L) + (1-\pi)\delta u(y_v) + (\pi - \pi\gamma)\delta u(w) + \pi\gamma\delta u(w) \end{aligned}$$

Using Jensen's inequality on the conditional expectation we have:

$$(1-\pi)y_v + \pi(1-\gamma)w = (1-\pi\gamma)x \tag{A.3}$$

where, $x = \frac{(1-\pi)y_v + \pi(1-\gamma)w}{1-\pi\gamma} = y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v)$, which implies

$$\begin{aligned} W^I_{d,z=0} &\leq y_d - \pi(w - y_v + L) + \pi\gamma\delta u(w) \\ &\quad +(1-\pi\gamma)\delta u\left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v)\right) \\ &= y_d - \pi(w - y_v + L) + \pi\gamma\delta u(w) \\ &\quad +(1-\pi\gamma)\delta u\left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v)\right) \\ &\quad +(1-\pi\gamma)\delta\left[u\left(y_v - \pi(1-\gamma)L\right) - u\left(y_v - \pi(1-\gamma)L\right)\right] \\ &\leq y_d - \pi(w - y_v + L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u\left(y_v - \pi(1-\gamma)L\right) \\ &\quad +(1-\pi\gamma)\left(\frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v) + \frac{\pi(1-\gamma)L(1-\pi\gamma)}{1-\pi\gamma}\right) \\ &\leq W^I_{d,z=L} \end{aligned}$$

Since $\delta u'(x) \leq 1$ when $x > w$. □

Proposition 5. *Under incomplete insurance, the donor's welfare is lowered by the allowance of more than sufficient insurance.*

Proof. From Proposition 2, it has been shown that at the optimal a vulnerable party would choose to insure more than sufficient incomplete insurance. Thus, by continuity, when restricted to only purchase up to sufficient insurance a vulnerable party would choose to purchase sufficient insurance.

Consider two levels of welfare:

$\tilde{W}_d^I$: The level of welfare when the vulnerable party is able to purchase more than sufficient insurance.

W_d^I: The level of welfare when the vulnerable party is limited to purchasing no more than sufficient insurance.

$$\begin{aligned}\tilde{W}_d^I &= y_d - \pi\gamma(w - y_v + L + \pi(1-\gamma)\tilde{L}) + \pi\gamma\delta u(w) \\ &\quad + \pi(1-\gamma)\delta u(y_v - L + (1-\pi(1-\gamma))\tilde{L}) + (1-\pi)\delta u(y_v - \pi(1-\gamma)\tilde{L})\end{aligned}$$

And

$$\begin{aligned}W_d^I &= y_d - \pi\gamma(w - y_v + L + \pi(1-\gamma)L) + \pi\gamma\delta u(w) \\ &\quad + \pi(1-\gamma)\delta u(y_v - \pi(1-\gamma)L) + (1-\pi)\delta u(y_v - \pi(1-\gamma)L)\end{aligned}$$

Taking the difference:

$$\begin{aligned}\tilde{W}_d^I - W_d^I &= \pi^2\gamma(1-\gamma)(L-\tilde{L}) + \pi(1-\gamma)\delta u(y_v - L + (1-\pi(1-\gamma))\tilde{L}) \\ &\quad + (1-\pi)\delta u(y_v - \pi(1-\gamma)\tilde{L}) - (1-\pi\gamma)\delta u(y_v - \pi(1-\gamma)L)\end{aligned}$$

$$
\begin{aligned}
\tilde{W_d^I} - W_d^I &= \pi^2\gamma(1-\gamma)(L-\tilde{L}) + \pi(1-\gamma)\delta u(y_v - L + (1-\pi(1-\gamma))\tilde{L}) \\
&\quad +(1-\pi)\delta u(y_v - \pi(1-\gamma)\tilde{L}) - (1-\pi\gamma)\delta u(y_v - \pi(1-\gamma)L) \\
&= \pi^2\gamma(1-\gamma)(L-\tilde{L}) \\
&\quad +\pi(1-\gamma)\delta\left[u(y_v - L + (1-\pi(1-\gamma))\tilde{L}) - \delta u(y_v - \pi(1-\gamma)L)\right] \\
&\quad -(1-\pi)\delta\left[\delta u(y_v - \pi(1-\gamma)L) - u(y_v - \pi(1-\gamma)\tilde{L})\right] \\
&< \pi^2\gamma(1-\gamma)(L-\tilde{L}) + \pi(1-\gamma)\delta u'\left(y_v - \pi(1-\gamma)L\right)\left[(1-\pi(1-\gamma))(\tilde{L}-L)\right] \\
&\quad -(1-\pi)\delta u'\left(y_v - \pi(1-\gamma)L\right)\left[\pi(1-\gamma)(\tilde{L}-L)\right] \\
&= \pi^2\gamma(1-\gamma)(L-\tilde{L}) + \pi^2\gamma(1-\gamma)(\tilde{L}-L)\delta u'\left(y_v - \pi(1-\gamma)L\right) \\
&< 0
\end{aligned}
$$

Noticing that the last line follows because $\delta u'\left(y_v - \pi(1-\gamma)L\right) \leq 1$ as $y_v - \pi(1-\gamma)L > w$, so
$\tilde{W_d^I} < W_d^I$. □

Lemma 3. *Comparison of welfare under varying levels of assistance.*

(i) If $w > w_l^*$, the welfare of the donor is the same.

(ii) If $w_l^* > w > w_l^{**}$, the welfare of the donor is higher when incomplete insurance is available rather than complete.

(iii) If $w_l^{**} > w$, the welfare of the donor is higher when complete insurance is available rather than incomplete.

Proof. (i) can be achieved directly by noticing that neither insurance product is desirable. The assistance level compensates the vulnerable party adequately to make insurance purchase unnecessary.

(ii) requires some manipulation of welfare functions and the utilization of

Jensen's inequality. Let w be the level of target wealth level in the loss state.

$$\begin{aligned} W_d^C &= y_d - \pi(w - y_v + L) + (1-\pi)\delta u(y_v) + \pi\delta u(w) \\ &= y_d - \pi(w - y_v + L) + (1-\pi)\delta u(y_v) + (\pi - \pi\gamma)\delta u(w) + \pi\gamma\delta u(w) \end{aligned}$$

$$\begin{aligned} W_d^I &= y_d - \pi\gamma(w - y_v + L + \pi(1-\gamma)L) + \pi\gamma\delta u(w) \\ &\quad + \pi(1-\gamma)\delta u\left(y_v - \pi(1-\gamma)L\right) + (1-\pi)\delta u\left(y_v - \pi(1-\gamma)L\right) \end{aligned}$$

Consider an insurance product as described in (A.3)

Using Jensen's inequality on the conditional expectation we have:

$$\begin{aligned} W_d^C &\leq y_d - \pi(w - y_v + L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u\left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v)\right) \\ &= y_d - \pi(w - y_v + L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u\left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v)\right) \\ &\quad +(1-\pi\gamma)\delta\left[u\left(y_v - \pi(1-\gamma)L\right) - u\left(y_v - \pi(1-\gamma)L\right)\right] \\ &= y_d - \pi(w - y_v + L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u(y_v - \pi(1-\gamma)L) \\ &\quad +(1-\pi\gamma)\left[\delta u\left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v)\right) - \delta u(y_v - \pi(1-\gamma)L)\right] \\ &\leq y_d - \pi(w - y_v + L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u\left(y_v - \pi(1-\gamma)L\right) \\ &\quad +(1-\pi\gamma)\delta u'(y_v - \pi(1-\gamma)L)\left(\frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v) + \pi(1-\gamma)L\right) \\ &\leq y_d - \pi(w - y_v + L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u\left(y_v - \pi(1-\gamma)L\right) \\ &\quad +(1-\pi\gamma)\left(\frac{\pi(1-\gamma)}{1-\pi\gamma}(w - y_v) + \pi(1-\gamma)L\right) \\ &= y_d - \pi\gamma(w - y_v + L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u\left(y_v - \pi(1-\gamma)L\right) \\ &\leq W_d^I \end{aligned}$$

The second and third inequalities use the fact that the donor does not provide assistance when net income is above the target wealth level, that is $\delta u'(t) \leq 1$ for $t > w$. In more detail, define
$A \equiv (1-\pi\gamma)\left[\delta u\left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w-y_v)\right) - \delta u(y_v - \pi(1-\gamma)L)\right]$. Under (ii), notice that $A \geq 0$, since $y_v - L \leq w$, that is the target wealth level is larger than the net income after loss. $\delta u\left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w-y_v)\right) - \delta u(y_v - \pi(1-\gamma)L)$ represents the difference between two points on the $\delta u(.)$ curve. Since $\delta u(.)$ is concave, this can be bounded by the tangent at the lowest point. In other words, $A \leq \delta u'(y_v - \pi(1-\gamma)L)\left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w-y_v) - y_v + \pi(1-\gamma)L\right)$. Further, as $y_v - \pi(1-\gamma)L > w$, it follows that $\delta u'(y_v - \pi(1-\gamma)L) \leq 1$ which implies $A \leq \left(y_v + \frac{\pi(1-\gamma)}{1-\pi\gamma}(w-y_v) - y_v + \pi(1-\gamma)L\right)$.

(iii) can be shown using Jensen's inequality.

$$
\begin{aligned}
W_d^I &= y_d - \pi\gamma\left(w - y_v + \pi(1-\gamma)L + L\right) \\
&\quad + \delta\left[(1-\pi\gamma)u\left(y_v - \pi(1-\gamma)L\right) + \pi\gamma u\left(w\right)\right] \\
&\leq y_d - \pi\gamma(w - y_v + \pi(1-\gamma)L + L) + \delta u\left((1-\pi\gamma)(y_v - \pi(1-\gamma)L) + \pi\gamma w\right) \\
&= y_d - \pi\gamma(w - y_v + \pi(1-\gamma)L + L) \\
&\quad + \delta u\left(y_v - \pi L + \pi\gamma(w - y_v + \pi(1-\gamma)L + L)\right) \\
&= y_d - \pi\gamma(w - y_v + \pi(1-\gamma)L + L) + \delta u\left(y_v - \pi L\right) \\
&\quad + \left[\delta u\left(y_v - \pi L + \pi\gamma(w - y_v + \pi(1-\gamma)L + L)\right) - \delta u(y_v - \pi L)\right] \\
&< y_d - \pi\gamma(w - y_v + \pi(1-\gamma)L + L) + \delta u\left(y_v - \pi L\right) \\
&\quad + \delta u'(y_v - \pi L)\pi\gamma(w - y_v + \pi(1-\gamma)L + L) \\
&< y_d + \delta u(y_v - \pi L) \\
&= W_d^C
\end{aligned}
$$

□

Theorem 2 below illustrates the identical main results for the net of premium contract. The net of premium contract removes the incentive for more than sufficient insurance coverage and the implicit subsidization of insurance by the safety net. Nonetheless, as described in the paper the results still hold due to the decreasing marginal returns to completeness. These results are formally derived below with results equivalent to Lemma 1 and 3 embedded in the proof.

Theorem 2. *When the level of assistance first precludes the purchase of complete insurance, any type of net of premium incomplete insurance is welfare enhancing and Pareto improving.*

Theorem 2 is the equivalent statement of Theorem 1 for net of premium incomplete insurance. The net of premium product is one in which the premium is returned when there is a loss, but not payout. That is, in the state with probability $\pi\gamma$, there is a return of premium. The actuarially fair rate of insurance for this product is $\frac{\pi(1-\gamma)}{1-\pi\gamma}z$. Theorem 2 highlights the second intuition of the results, that incomplete insurance provides more value for the insured via the curvature of their utility function, not only through the subsidization of insurance. That is this reflects the decreasing marginal returns to completeness.

The donor's welfare function is as previous and assistance is in the form of a target wealth level.

Proof. The vulnerable party's insurance demand is governed by:

$$\begin{aligned} z^* = \quad argmax \quad & \left\{(1-\pi)u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}z\right) + (1-\gamma)\pi u\left(y_v + (1-\frac{\pi(1-\gamma)}{1-\pi\gamma})z - L + \tau_a^*\right)\right. \\ & \left. +\gamma\pi u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}z - L + \tau_b^*\right)\right\} \end{aligned}$$

So then the levels of net assistance are

$$\tau_a^* = argmax\left\{y_d - \tau_a + \delta u\left(y_v + \tfrac{1-\pi}{1-\pi\gamma}z - L + \tau_a\right)\right\}$$

$\tau_a^* = max\left[0, w - y_v + L - \frac{1-\pi}{1-\pi\gamma}z\right]$

and $\tau_b^* = max\left[0, w - y_v + L + \frac{\pi(1-\gamma)}{1-\pi\gamma}z\right]$

For $z \in \left[0, \frac{(w-y_v+L)(1-\pi\gamma)}{1-\pi}\right]$ there is a one to one crowd out effect so zero insurance coverage is optimal.

For $z \in \left(\frac{(w-y_v+L)(1-\pi\gamma)}{1-\pi}, L\right)$ there are increasing benefits from insurance so that the optimal insurance coverage is sufficient, that is $z^* = L$, this is shown through first order conditions

$$E\left[u_v\right] = (1-\pi)u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}z\right)+(1-\gamma)\pi u\left(y_v + (1 - \frac{\pi(1-\gamma)}{1-\pi\gamma})z - L\right)+\pi\gamma u\left(y_v - L\right)$$

First order condition is:

$$\frac{du_v}{dz} = -(1-\pi)\left(\frac{\pi(1-\gamma)}{1-\pi\gamma}\right)u'\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}z\right)+(1-\gamma)\pi\left(1 - \frac{\pi(1-\gamma)}{1-\pi\gamma}\right)u'\left(y_v + (1 - \frac{\pi(1-\gamma)}{1-\pi\gamma})z - L\right)$$

For $z = L$, $\frac{du_v}{dz} = 0$. Since $u''(.) < 0$ this is the optimum. The optimal level of coverage is no insurance ($z = 0$) or sufficient insurance ($z = L$).

Analogous to Theorem 1. I will begin by showing results equivalent to Lemma 1 and Lemma 3.

Begin with the new expected utility for the net of premium contract, at the optimal level given insurance is purchased.

Let w^{**} be defined as the level of assistance in the loss states that makes the vulnerable party indifferent between purchasing sufficient and no insurance under complete insurance. That is, $u(y_v - \pi L) = (1 - \pi)u(y_v) + \pi u(w^{**})$.

$$E[u_v^I] = (1 - \pi\gamma)u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right) + \pi\gamma u\left(w^{**}\right)$$

Note that using Jensen's inequality:

$$
\begin{aligned}
u(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L) &= u\left(\frac{(1-\pi)\,\gamma}{1-\pi\gamma}y_v + \frac{(1-\gamma)}{1-\pi\gamma}y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right) \\
&> \frac{(1-\pi)\,\gamma}{1-\pi\gamma}u(y_v) + \frac{(1-\gamma)}{1-\pi\gamma}u(y_v - \pi L)
\end{aligned}
$$

So now by substituting for $u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right)$, we have

$$
\begin{aligned}
E[u_v^I] &> (1-\pi\gamma)\left[\frac{(1-\pi)\,\gamma}{1-\pi\gamma}u(y_v) + \frac{(1-\gamma)}{1-\pi\gamma}u(y_v - \pi L)\right] + \pi\gamma u\left(w^{**}\right) \\
&= \gamma\left[(1-\pi)\,u(y_v) + \pi u\left(w^{**}\right)\right] + (1-\gamma)u(y_v - \pi L) \\
&= \gamma u(y_v - \pi L) + (1-\gamma)u(y_v - \pi L) \\
&= E[u_v^C]_{z=L}
\end{aligned}
$$

This is the analogy to Lemma 1.

Now turn to the donor's welfare.

First note, under the net of premium contract the welfare of the donor is now:

$$
\begin{aligned}
W_d^{I_2} &= y_d - \pi\gamma(w - y_v + L) + \pi\gamma\delta u(w) + \pi(1-\gamma)\delta u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right) \\
&\quad +(1-\pi)\delta u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right) \\
&= y_d - \pi\gamma(w - y_v + L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right)
\end{aligned}
$$

Previously $W_d^I = y_d - \pi\gamma(w - y_v + L + \pi(1-\gamma)L) + \pi\gamma\delta u(w) + (1-\pi\gamma)\delta u(y_v - \pi\,(1-\gamma)\,L)$

Note that $W_d^{I_2} \geq W_d^I$

Since

$$
\begin{aligned}
W_d^{I_2} - W_d^I &= \pi^2\gamma(1-\gamma)L - (1-\pi\gamma)\left[\delta u(y_v - \pi\left(1-\gamma\right)L) - \delta u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right)\right] \\
&> \pi^2\gamma(1-\gamma)L - (1-\pi\gamma)\delta u'\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right)\frac{\gamma\pi^2(1-\gamma)}{1-\pi\gamma} \\
&> 0
\end{aligned}
$$

i) is clear and ii) follows since $W_d^{I_2} \geq W_d^I \geq W_d^C$.

iii) In this case, the vulnerably party purchases sufficient insurance under both the complete and incomplete product. So then, the assistance provider does not need to provide assistance in the case of complete insurance since the target wealth level is below what the individual would receive net of insurance payment.

$$
\begin{aligned}
W_d^{I_2} &= y_d - \pi\gamma\left(w - y_v + L\right) + \delta\left[(1-\pi\gamma)u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right) + \pi\gamma u\left(w\right)\right] \\
&= y_d - \pi\gamma\left(w - y_v + L\right) + \delta(1-\pi\gamma)u\left(y_v - \frac{\pi(1-\gamma)}{1-\pi\gamma}L\right) + \delta\pi\gamma u\left(w\right) \\
&\leq y_d - \pi\gamma\left(w - y_v + L\right) + \delta u\left((1-\pi\gamma)y_v - \pi(1-\gamma)L + \pi\gamma w\right) \\
&= y_d - \pi\gamma\left(w - y_v + L\right) + \delta u(y_v - \pi L) \\
&\quad + \left[\delta u\left(y_v - \pi L + \pi\gamma(w - y_v + L)\right) - \delta u(y_v - \pi L)\right] \\
&< y_d - \pi\gamma\left(w - y_v + L\right) + \delta u(y_v - \pi L) + \delta u'(y_v - \pi L)\pi\gamma(w - y_v + L) \\
&< y_d + \delta u(y_v - \pi L) \\
&= W_d^C
\end{aligned}
$$

□