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Article (Accepted version)
(Refereed)

Signaling to Dispersed Shareholders and Corporate Control[*]

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November 2014

Abstract

This paper analyzes how outsiders, such as bidders or activist investors, overcome the lack of coordination and information among dispersed shareholders. We identify the two basic means to achieve this goal. First, the outsider must relinquish private benefits in a manner that is informative about security benefits. We show under which conditions this is feasible and which acquisition strategies used in practice meet these conditions. Second, the outsider can alternatively use derivatives to drive a wedge between her voting power and her economic interest in the firm. Such unbundling of ownership and control, while typically considered a source of corporate governance problems, is an efficient response to the frictions dispersed ownership causes for control contestability. We also show that unbundling comes with costs and benefits for the bidder's incentives to improve firm value.

JEL Classifications: G32

Keywords: Tender Offers, Investor Activism, Signaling, Free-Rider Problem, Means of Payment, Unbundling Ownership and Control, Empty Voting

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1   Introduction

Dispersed shareholders do not coordinate their actions and individually have too little incentive to become informed and monitor managerial decisions. Such passivism is the origin of two fundamental corporate governance problems. Inside the firm, it allows "insiders" to extract private benefits at the expense of small shareholders, thus making the reduction of these benefits synonymous to good governance. Since Jensen and Meckling (1976), a commonly accepted principle for mitigating this problem is to tie control to ownership. Using this argument, the standard contractarian theory of corporate law supports allocating voting rights to shareholders in proportion to cash flow rights (one share-one vote) as economically efficient.[1] The violation of this principle is the reason why devices such as dual-class shares and pyramidal ownership structures are generally met with skepticism.

Outside the firm, the lack of coordination and information makes dispersed shareholders reluctant to sell their shares, which makes it difficult for "outsiders" who can enhance firm value, such as acquirers or activists, to gain control or influence over the firm. In this case, good governance aims to facilitate control transfers, and this may require that outsiders derive private benefits from control. Conceptually, this problem is manifested most purely in tender offers. Indeed, this is the context in which it has first been recognized: Grossman and Hart (1980) show that each individual shareholder is reluctant to tender because she wants to free-ride on the value improvement. Grossman and Hart (1981) show that they may not tender when the bidder might gain purely from being better informed.

The general insight to be drawn from the present paper is that the "outside" governance problem, despite having the same origin, calls for solutions that are the opposite of those to the "inside" problem. We analyze an outsider aiming to acquire control from shareholders that lack coordination and information and identify the two basic principles that shape her optimal strategies: First, to overcome the interaction of asymmetric information and collective action problem, the outsider must relinquish private benefits. But the reduction in her private benefits runs counter to good governance because it can frustrate efficient transfers of control. Second, this inefficiency is mitigated if the outsider can unbundle ownership and control. In fact, the optimal unbundling strategy implements the symmetric information outcome by fully preserving private benefits. Inverting this statement, this means that tying control to ownership to reduce private benefits exacerbates the governance problem dispersed ownership creates vis-à-vis outsiders – in direct contradiction to standard contractarian arguments.

Furthermore, by identifying these principles, we provide a prism through which one can "see" the common logic that underlies seemingly diverse strategies outsiders use to gain influence over firms in practice. To demonstrate this, we map existing models in the literature on tender offers with a privately informed bidder (Hirshleifer and Titman, 1990; Chowdry and Jegadeesh, 1994) into our generic framework and show that the proposed solutions all conform to the principle of relinquishing private benefits. Going further, we use the principle to identify novel signaling devices such as takeover leverage. The principles also clarify what

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[1]Easterbrook and Fischel (1983, 1991)'s article and book are the classic corporate law references for this argument.

does *not* work. For example, in contrast to results in the merger literature, cash-equity offers lack signaling power because they conform to neither of the two principles. The same principles apply when we adapt our framework to the situation of an activist investor who wants to gain influence on a corporate decision through open-market trades. In particular, the activist's optimal strategy is to acquire common shares in combination with derivatives that partly offset her equity interest in the firm, a strategy referred to in practice as empty voting. Empty voting – or more broadly, any activist strategy of exercising influence in excess of ownership (i.e., "unbundling") – is thus an implementation of our second general principle for overcoming the lack of coordination and information in public equity markets with dispersed owners.

We begin analyzing the governance problem between dispersed shareholders and outsiders in Section 2 with a simple tender offer game in which the bidder has exogenous private benefits and private information about the post-takeover value improvement. As regards separating equilibria in this setting, an impossibility result obtains: The bidder cannot reveal her type through the offer terms. (Pooling equilibria exist and are discussed in Section 3.2.) The interaction between asymmetric information and collective action problem is key to this result. Truthful revelation requires that high-valued bidders earn information rents. However, the dispersed shareholders' free-rider behavior precludes that these rents stem from gains in security benefits – and the private benefits are exogenous.

In Section 3, we introduce the possibility for the bidder to relinquish (part of) her private benefits. Two conditions must be satisfied for private benefits to be instrumental in signaling: The bidder must be able to commit to relinquish specific amounts of private benefits at the time of the offer, and doing so must be informative about the post-takeover security benefits. If these conditions are met, the bidder can reveal a low(er) post-takeover value by relinquishing (more) private benefits, and thereby succeed at a low(er) price. As we show, the signaling instruments proposed in the literature, toeholds and probabilistic tendering, work precisely because they operate on this principle. More generally, the principle allows us to identify signaling devices simply by relabeling variables of a specific tender offer game to match those in our generic framework. We explore three other sources of private benefits – diversion, debt, and bidder assets – and argue that of these only debt meets the conditions for a viable signal.

Relinquishing private benefits redistributes rents from the bidder to target shareholders. In a separating equilibrium, lower-valued bidders must give up relatively more private benefits. As a result, the equilibrium outcome typically exhibits inefficiency at the "bottom": Only bidders above a cut-off type make a bid, and a higher cut-off type amounts to less takeover activity. In pooling equilibria, the uniform price transfers rents from low-valued to high-valued bidders, since the former pay a premium but the latter buy at a discount. This redistribution among types can make a takeover unprofitable for some low-valued bidders. Since the price in a pooling equilibrium depends on the distribution of bidder types, so does the extent of the inefficiency. In a separating equilibrium, tender offers and inefficiency are solely determined by the incentive compatibility constraints, but not the type probabilities. Changes in the bidder type distribution can hence make pooling outcomes more or less efficient than separating ones, which precludes a general ranking of the two types of equilibria in terms of efficiency. By contrast, there is a clear-cut result regarding robustness: Only separating

equilibria survive the intuitive criterion.

It has been shown that the target firm's security-voting structure, such as dual-class shares or majority rules, affects the efficiency of pooling outcomes; though, the direction of the effect is ambiguous (At et al., 2011; Marquez and Yilmaz, 2012). We extend these results in Section 4. We first show that in the *separating* equilibria of Section 3.1 allowing control with less ownership unambiguously improves efficiency because the bidder can retain more private benefits. We then explore how the outcome changes if the *bidder* rather than the target firm can separate cash flow and voting rights by extending the contract space to securities other than cash and equity. As it turns out, the bidder can use call options to reveal her type without relinquishing any private benefits, thereby implementing the symmetric information outcome. By writing call options, the bidder assumes a negative financial interest in the firm that partly offsets the equity interest she has through voting shares. As a result, she acquires votes in excess of cash flow rights. Such unbundling remains optimal when we adapt our model to a setting with an activist investor who wants to buy shares from an uninformed market-maker to sway an upcoming shareholder vote.

Section 5 studies an extension in which the bidder exerts post-takeover effort to improve firm value. Introducing moral hazard to the framework does not undermine the optimality of using derivatives to help shareholders correctly infer the security benefits. Conditional on restoring symmetric information, the bidder then maximizes net private benefits rather than the value of her post-takeover block due to the shareholders' free-rider behavior (Burkart et al., 1998). In fact, she can implement the requisite effort level independent of the equity block she acquires through the same derivatives she uses to prevent shareholders from over-estimating the post-takeover security benefits. In particular, writing call options "caps" her incentives to increase security benefits once she is in control.

The impact of unbundling on equilibrium effort is in general ambiguous. On one hand, unbundling lets the bidder seize control with less ownership, which weakens her incentives. On the other hand, she need no longer forgo private benefits to reveal her type, which strengthens her incentives. When the negative effect dominates, unbundling creates a tension between outside governance and inside governance. For example, this tension arises whenever toeholds are the source of private gains. In this case a control-seeking outsider optimally decouples control from ownership to the extent that her incentives as a controlling insider decrease from a level commensurate with a majority stake to a level commensurate with the maximum toehold size. Where unbundling is feasible, this raises the question what an outsider for whom equity gains are the source of profit stands to win by wrestling a majority stake from the hands of free-riding shareholders relative to engaging in activism with the largest minority stake she can secretly accumulate in the open market.

In Section 6, we interpret our framework to draw out parallels and differences between tender offers and investor activism as alternative methods to overcome the outside governance problem. In addition, we discuss novel predictions of our analysis for the use of equity as payment in takeovers, the impact of empty voting on firm value, and the relationship between takeover leverage and takeover returns.

Grossman and Hart (1981) and Shleifer and Vishny (1986), the first analyses of asymmetric information in tender offers, focus exclusively on pooling equilibria. Hirshleifer and Titman (1990) and Chowdry and

Jegadeesh (1994) are the only papers that construct separating equilibria in a tender offer game.[2] In Section 3, we derive the general "relinquishing-private-benefits" mechanism that these equilibria are examples of. We provide the first general analysis of the problem that a lack of information and coordination among target shareholders poses for outside bidders, which not only contextualizes existing results but provides a deeper insight into the nature of the problem and the solutions to overcome it. Our analysis in Section 5 of a setting that combines asymmetric information, the free-rider problem, and moral hazard is to our knowledge novel, though the interaction between the latter two frictions has been studied in Burkart et al. (1998).

Several papers show that cash-equity offers can overcome asymmetric information problems in mergers (Hansen, 1987; Berkovitch and Narayanan, 1990; Eckbo et al., 1990; Brusco et al., 2007; and Ferreira et al., 2007). These papers abstract from the free-rider problem, which plays a crucial role in undermining the signaling role of cash-equity offers in our setting.[3] Convertible securities as a means to overcome information asymmetries have been studied by Chakraborty and Yilmaz (2011) in the context of external financing.

Hu and Black (2006, 2008) were first to draw attention to empty voting and the possibility that it may be abused to pass poor corporate decisions.[4] Brav and Matthews (2011) formalize this concern and show how active investors may use empty voting to reduce firm value at the expense of noise traders in the secondary market. We abstract from noise trading and show that empty voting emerges endogenously as the activist's optimal response to free-riding by uninformed but rational investors (as manifested in the price impact) in the secondary market and enhances firm value. Two papers extend Grossman and Hart (1988)'s and Harris and Raviv (1988)'s insights on separating cash flow and voting rights in bidding *contests* to vote buying: Dekel and Wolinsky (2012) show that it can be privately optimal for the target shareholders to sell their votes to the less efficient bidder; Neeman and Orosel (2006) focus on elections (as opposed to asset purchases) and show that an election is certain to be "bought" by the efficient bidder only if the willingness-to-pay for winning is positively related with the ability to create value. Last, Esö et al. (2014) also consider elections and study vote trading in the presence of informed, uninformed, and preference-biased voters (shareholders).

2 Free-riding undermines signaling

A widely held firm faces a single potential acquirer, henceforth the bidder. If the bidder gains control, she can generate security benefits X. The bidder learns her type prior to making the tender offer, whereas target shareholders merely know that X is distributed on $\mathscr{X} = [0, \overline{X}]$ according to the continuously differentiable density function $g(X)$. The cumulative distribution function is denoted by $G(X)$. If the takeover does not materialise, the incumbent manager remains in control. The incumbent generates security benefits X^I that are

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[2]Marquez and Yilmaz (2008) reverse the information asymmetry and study a tender offer game in which target shareholders receive noisy private signals so that the bidder faces a winner's curse problem, as opposed to a signaling problem.

[3]Rather than reducing the information gap between bidder and target shareholders, signaling can also serve the purpose of deterring potential rivals, as in Fishman (1988, 1989), Bhattacharya (1990), and Liu (2008).

[4]Kalay and Pant (2009) argue that dispersed target shareholders can extract a higher price by assuming derivative positions prior to a pending bidding contest. This is different from empty voting, which is a strategy pursued by an activist who wants to gain control.

known to all shareholders and normalised to zero. Thus, we restrict attention to the case of value-improving bids.

In addition, control confers exogenous private benefits $\Phi(X) > 0$ on bidder type X. The private benefits are known only to the bidder and non-transferable.

Since the firm has a one share–one vote structure, a successful tender offer must attract at least 50 percent of the firm's shares. The tender offer is conditional, and therefore becomes void if less than 50 percent of the shares are tendered. In addition, the bidder can restrict the offer to a fraction $r \in [0.5, 1]$ of the shares. For simplicity, we assume that there are no takeover costs. Hence, the benchmark (full information) outcome is that all takeovers succeed.

The timing of the model is as follows. In stage 0, the bidder learns her type X. In stage 1, she then decides whether to make a take-it-or-leave-it, conditional restricted tender offer in cash (alternative means of payment will be considered later). If the bidder does not make a bid, the game moves immediately to stage 3. Otherwise, she offers to purchase a fraction r of the outstanding shares at a price rP.

In stage 2, the target shareholders non-cooperatively decide whether to tender their shares. Shareholders are homogeneous and atomistic.[5] In stage 3, the incumbent manager remains in control if the fraction of tendered shares β is less than 50 percent. Otherwise, the bidder gains control, extracts private benefits, and pays βP unless the offer is oversubscribed, in which case she pays rP, and tendering shareholders are randomly rationed.

There are two sources of frictions in this model: asymmetric information and the collective action problem. Both of them are crucial in the sense that neither of them alone creates any inefficiency. If target shareholders could observe the bidder's type, they would accept the offer whenever the price at least matches the security benefits under the bidder. The bidder would succeed and appropriate all her private benefits. Thus, the outcome would be efficient despite the collective action problem.

If target shareholders could coordinate their tendering decisions, they would accept the offer whenever the price at least matches the security benefits under the incumbent manager. The bidder would succeed and appropriate the entire value improvement from the takeover. Also, note that the tender offer game, if the bid price exceeds the security benefits under the incumbent, is de facto one of private contributions to a public good under complete information. As known from implementation theory, a central authority could thus implement the efficient outcome, in the case of tender offers simply by requiring unanimous participation.[6] In short, absent the coordination problem, the outcome is efficient despite the information asymmetry.

However, we assume that shareholders are atomistic, non-cooperative, and must participate voluntarily,

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[5]As in other tender offer models exploring the free-rider problem, we assume that the mass of outstanding shares is dispersed among an infinite number of shareholders whose individual holdings are both equal and indivisible. When either of these assumption is relaxed, Grossman and Hart's (1980) result that all the gains in security benefits go to the target shareholders becomes diluted (Holmström and Nalebuff, 1992).

[6]Given $P > X$, conditioning success on full participation does not eliminate failure as a Nash equilibrium but turns it into one requiring the play of weakly dominated strategies. It is then non-robust to equilibrium refinements (cf., e.g., Bagnoli and Lipman, 1989).

and that a successful tender offer requires only 50 percent of the shares. In this case, each shareholder tenders only if the offered price at least matches the expected security benefits. Since shareholders condition their expectations on the offer terms (r,P), a successful tender offer must satisfy the free-rider condition $P \geq \mathrm{E}(X|r,P)$. Now the bidder's private information matters, as the shareholders try to infer it from the offer terms.

We assume that shareholders — after observing a bid price P and updating their beliefs — tender unless the price is strictly lower than the expected post-takeover security benefits. This eliminates failure as an equilibrium outcome when the free-rider condition is strictly satisfied.[7] When the bid price exactly equals the expected post-takeover share value, the target shareholders are strictly indifferent between tendering and retaining their shares. That is, they are indifferent between these actions irrespective of their beliefs about the takeover outcome, so that the weak dominance criterion does not pin down a tendering strategy. The prevalent way of resolving the indeterminacy when $P = \mathrm{E}(X|r,P)$ is to assume that each shareholder tenders in this case, and hence the bid succeeds with certainty.[8] Alternatively, one can assume that strictly indifferent shareholders randomise, and that this leads to a probabilistic outcome.[9] Subsequently, we focus on deterministic outcomes, except in Section 3.1.5, which considers probabilistic outcomes.

Under the assumption that each shareholder tenders in case she is strictly indifferent, all shares ($\beta = 1$) are tendered in a successful takeover. Accordingly, a successful restricted bid is oversubscribed, and the bidder randomly selects the fraction r among all shareholders whose shares are purchased. The remaining $1 - r$ shareholders cannot sell and become minority shareholders.

As shown in Shleifer and Vishny (1986), Perfect Bayesian equilibria with a single pooling offer exist when bidders have exogenously given private benefits. We now show that no other equilibria exist in this setting and postpone the discussion of pooling equilibria to Section 3.2.

The bidder's expected profit from a bid (r,P) is

$$\Pi(r,P) = q(r,P)\left[\Phi + r(X - P)\right],$$

where $q(r,P)$ denotes the success probability, which is equal to 1 for $P \geq \mathrm{E}(X|r,P)$, and 0 otherwise. In a separating equilibrium, the offer terms must be distinct across types that make a (successful) bid. This requires that each equilibrium offer satisfies the free-rider condition, $P(X) \geq X$, and the bidder's incentive

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[7]Given a conditional bid, a shareholder who believes the bid will fail is indifferent between tendering and retaining. Imposing this belief on all shareholders and breaking the indifference in favour of retaining supports failure as an equilibrium, irrespective of the offered price (Burkart et al., 2006). To avoid the co-existence of success and failure as equilibrium outcomes, it is typically assumed that shareholders tender their shares when they are indifferent (e.g., Shleifer and Vishny, 1986). Contrary to our assumption, this precludes failure as the equilibrium outcome for a conditional bid, and hence the existence of an equilibrium when the free-rider condition is violated.

[8]A common motivation for this approach is that the bidder could sway the shareholders by raising the price infinitesimally. Although this argument holds under full information, it does not apply in the asymmetric information setting, as even small price increases affect shareholders' expectations about the post-takeover security benefits.

[9]Judd (1985) shows that a continuum of independent and identically distributed variables can generate a stochastic aggregate outcome.

compatibility constraint

$$\Phi + r(X)[X - P(X)] \geq \Phi + \hat{r}(X - \hat{P})$$

for all $\hat{r} \in [0.5, 1]$ and $\hat{P} \in \mathbb{R}$ where $(\hat{r}, \hat{P})$ denotes a deviation offer.

Proposition 1. *In deterministic tender offer games with exogenous private benefits, no separating equilibrium exists.*

The proof for the inexistence of signaling equilibria is straightforward: Given that $P(X) \geq X$, a truthful bidder at best breaks even on the purchased shares, and her expected profit cannot exceed $\Phi(X)$. However, each type offering her actual security benefits cannot be an equilibrium outcome. If a type x were to succeed at a purchase price $r(x)x$, any type $X > x$ would mimic type x to acquire shares at a price below their true value X. This also holds if each type were to choose a different bid restriction $r(\cdot)$. Type X's profits are higher when buying $r(x)$ shares at a discount compared to buying $r(X)$ shares at their fair price, whether $r(x)$ is smaller or larger than $r(X)$. These arguments eliminate $P(X) = X$ combined either with a common r or a type-contingent $r(\cdot)$ as possible equilibria. They also rule out outcomes in which some types offer more than their true security benefits but less than the highest-valued type's security benefits. Successful offers with $P(x) \in (x, \overline{X})$ would be mimicked by bidders of type $X > P(x)$. Thus, a bidder can credibly signal her type only by offering a sufficiently large premium such that $P \geq \overline{X}$.

Revealing her type with an offer $P \geq \overline{X}$ is, however, not an attractive option for the bidder. She can instead make a bid $P = \overline{X}$ and restrict it to $r = 0.5$, the minimum fraction required to gain control. The less costly offer $(0.5, \overline{X})$ succeeds, since it satisfies the free-rider condition for all types (and any possible shareholder beliefs).

Even though the single-crossing condition holds, separation fails because of the free-rider problem.[10] The crucial role of the free-rider behaviour in eliminating separating equilibria can be explained in two ways. From the perspective of lower-valued types, the free-rider condition eliminates the possibility of producing a costly signal. Given that target shareholders extract all the gains in security benefits, the bidder cannot surrender (part of) these gains to signal her type. From the perspective of higher-valued types, the free-rider condition wipes out information rents. A bidder who at best breaks even on truthfully purchased shares always wants to mimic a lower-valued type.[11]

The inexistence result holds irrespective of whether private benefits are constant or an arbitrary — possibly type-contingent, stochastic — function of the bidder type. Indeed, the constraints in the bidder's maximisation problem are not affected by the non-transferable private benefits. They cancel out in the incentive compatibility constraint and are not part of the free-rider condition.

Also, note that letting bidders choose the fraction of shares that they acquire does not allow them to signal their type. The bid restriction merely limits the fraction of shares the bidder purchases for cash. This

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[10]For each fixed (r, P), $-\frac{\partial \Pi / \partial r}{\partial \Pi / \partial P}$ is strictly monotone in X.

[11]Proposition 1 mirrors the result in Nachman and Noe (1994) that competitive pricing among security issuers eliminates separating equilibria. The free-rider condition is our analogue to their competitive pricing assumption.

makes restricted bids in this setting equivalent to bids in which target shareholders are in part compensated through equity. Indeed, it is immaterial whether the bidder makes a partial bid for cash only or acquires all shares in exchange for some cash and $1 - r$ shares in the target firm under her control. Moreover, control requires that the partial bid be for at least half the shares or that the equity component not exceed the cash component in the cash-equity offer. By virtue of this equivalence, any separating equilibrium in cash-equity offers would also have to exist in restricted cash-only offers.

Corollary 1. *Introducing cash-equity offers into deterministic tender offer games with exogenous private benefits does not make separating equilibria feasible.*

Proposition 1 contrasts with results from bilateral merger models where cash-equity offers can reveal the bidder's type (Hansen, 1987; Berkovitch and Naranayan, 1990; Eckbo et al., 1990). Our basic framework differs in two key respects. First, target shareholders have no private information and face, instead, a collective action problem; that is, they are unable to coordinate their individual tendering decisions.[12] Second, the takeover is not undertaken to combine assets from two firms but to replace the incumbent managers. We explore the role of bidder assets later in the paper (Sections 3.1.4 and 3.4).

3   Informative exclusion

Since dispersed shareholders never tender unless they are offered at least the full post-takeover security benefits, relinquishing private benefits remains the only means for the bidder to reveal her type. To explore this possibility, we modify the model: Instead of $\Phi(X)$ being non-transferable, the bidder can now choose which fraction $1 - \alpha$ of the private benefits to relinquish. The foregone private benefits may or may not increase the post-takeover security benefits, depending on how the bidder chooses to relinquish the private benefits. To cover both cases, we denote the *actual* post-takeover security benefits with $K(\alpha, X)$, and refer to X as the *baseline* security benefits, that is, the security benefits when the bidder retains all private benefits. The bidder's payoff from a successful takeover can then be written as

$$\Pi(r, \alpha, P; X) = \alpha \Phi(X) + r[K(\alpha, X) - P]. \tag{1}$$

We first establish the existence of separating equilibria and then show how well-known variants of the tender offer game map into the present framework. Thereafter, we characterize pooling equilibria and discuss efficiency and robustness across both types of equilibria. A discussion of two-dimensional asymmetric information concludes the section.

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[12]In merger models, the shareholders' reservation price is typically the stand-alone value of the target firm and, if anything, private information of the shareholders, not of the bidder.

3.1 Separating equilibria

If a separating equilibrium exists, it can be implemented as a direct (truth-telling) mechanism. Let $\hat{X}$ denote a bidder's self-reported type. The bidder's problem can then be formulated as

$$\max_{\hat{X}} \Pi(\hat{X};X) = \alpha(\hat{X})\Phi(X) + r(\hat{X})[K(\hat{X};X) - P(\hat{X})],$$

$$\text{subject to } (a) \quad r(\hat{X}) \in [0.5, 1] \tag{2}$$
$$(b) \quad \alpha(\hat{X}) \in [0, 1]$$
$$(c) \quad P(\hat{X}) \geq E[K(\hat{X};X)]$$

for all $\hat{X} \in \mathscr{X}$, where (c) is the free-rider condition. With respect to K, we distinguish two cases. In one case, relinquishing private benefits increases the actual security benefits to $X + [1 - \alpha(\hat{X})]\Phi$. In the other case, the foregone private benefits do not accrue to the target shareholders, and the actual security benefits are therefore equal to the baseline security benefits X. We can thus define $K(\hat{X};X) = X + k[1 - \alpha(\hat{X})]\Phi(X)$ with $k = 1$ in one case and $k = 0$ in the other.

Under a separating offer schedule $\{r(\cdot), \alpha(\cdot), P(\cdot)\}$, the solution to this problem and hence to its first-order condition

$$r(\hat{X})P'(\hat{X}) + r'(\hat{X})P(\hat{X}) = \alpha'(\hat{X})\Phi(X) + r'(\hat{X})K(\hat{X};X) + r(\hat{X})K'(\hat{X};X) \tag{3}$$

must be $\hat{X} = X$ for all $X \in \mathscr{X}$. This and the free-rider condition (c) determine a relationship between $\alpha(\cdot)$ and $P(\cdot)$.

Proposition 2. *In tender offer games where bidders can commit to relinquish any fraction of private benefits, a separating equilibrium exists if $\Phi(\cdot)$ is a non-decreasing function. All types above the cut-off type $X_S^c \in [0, \overline{X})$ make a bid, and higher types choose to relinquish less private benefits. The bid restriction is a redundant signal.*

Incentive compatibility requires that higher-valued bidders offer higher prices and retain a larger fraction of the private benefits. Bidders do not mimic lower-valued types because the gains from paying the lower price are offset by the loss in private benefits. Conversely, bidders refrain from mimicking higher-valued types because the gains from retaining more private benefits do not compensate for the higher price. Furthermore, since lower-valued types retain a smaller fraction of their private benefits, separating equilibria can have an *interior* cut-off type $X_S^c \in (0, \overline{X})$ whose retained private benefits $\alpha(X_S^c)\Phi(X_S^c)$ just equal her total takeover premium $r(X_S^c)[P(X_S^c) - K(X_S^c;X_S^c)]$. By contrast, the highest-valued type purchases shares at $P(\overline{X}) = \overline{X}$ and retains all private benefits, $\Phi(\overline{X})$, thus reaping her full information profit.

Being able to relinquish private benefits does not ensure the existence of separating equilibria. Proposition 2 identifies two further conditions. First, the bidder must be able to *commit* to relinquish the fraction of private benefits announced in the offer. Otherwise, the modified setting is de facto reduced to the case with

exogenous private benefits, since any bidder would opportunistically renege on the announced α and retain all private benefits. Second, forgoing private benefits must be *informative* such that the shareholders can infer the post-takeover security benefits. A sufficient condition for this is that $\Phi(\cdot)$ is non-decreasing. This ensures that relinquishing a given fraction of private benefits is more costly for higher-valued types, that is, the single crossing property holds.[13] As we show below, the assumption of $\Phi(\cdot)$ non-decreasing is satisfied in well-known variants of the tender offer game.

The bid restriction is a redundant signal because bidders reveal their type by relinquishing private benefits. This is done through the choice of α, and not of r. Therefore, bid restrictions, while affecting $\alpha(\cdot)$ and $P(\cdot)$ through (3), are a source of equilibrium multiplicity. For instance, separating equilibria can be supported for any uniform restriction, $r(X) = \bar{r} \in [0.5, 1]$. Another source of multiplicity is that the free-rider condition (c) is an inequality. Accordingly, shareholders accept prices that match or exceed the expected post-takeover security benefits, which implies that multiple price schedules can be supported as separating equilibria.

The above framework and Proposition 2 are cast in terms of the bidder relinquishing part of her private benefits. Equivalently, the analysis can be framed in terms of the bidder extracting part of the total value as private benefits. This is, in fact, the perspective taken in the literature building on Grossman and Hart (1980). Its focus is the means by which the bidder can exclude (minority) shareholders from part of the takeover value to overcome the free-rider problem. Identified *exclusion mechanisms*, such as toeholds, map into our abstract technology of relinquishing part of the private benefits.

3.1.1 Toehold acquisition

One exclusion mechanism, first studied by Shleifer and Vishny (1986), are equity stakes purchased prior to the tender offer (toeholds). Consider a target firm approached by a bidder. If the bidder gains control, she generates total post-takeover value $V \in \mathscr{V}$, which is private information to her. Suppose the bidder can purchase up to a fraction $\bar{t}$ of the target shares in the open market—for simplicity, at the price of $P = 0$— before making a tender offer. The upper bound $\bar{t}$ represents a mandatory disclosure rule that essentially prevents the bidder from acquiring further shares at prices below the takeover bid.

By acquiring a toehold $t \in (0, \bar{t}]$ prior to the bid, the bidder excludes t initial target shareholders from the takeover gains. Since these open market purchases do not affect the post-takeover value of the shares, the free-rider condition is $P \geq V$. The bidder's payoff from a successful takeover is

$$\Pi = tV + r(V - P)$$

with $r \in [0.5 - t, 1 - t]$.

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[13]When $\Phi(\cdot)$ is decreasing or non-monotonic, no general result obtains. One can find examples in which separation is feasible and others in which only pooling equilibria exist.

By defining $\alpha \equiv t/\bar{t}$, $X \equiv V$, and $\Phi \equiv \bar{t}V$, we can rewrite the bidder's payoff as

$$\Pi = \alpha\Phi(X) + r(X - P)$$

and the free-rider condition as $P \geq X$. Thus, the tender offer game with toehold acquisition is isomorphic to our generic framework for $k = 0$.

Importantly, toeholds as an exclusion mechanism satisfy the commitment and informativeness requirements: The bidder decides the size of the toehold prior to the tender offer, and the value of a toehold naturally increases with the security benefits, so that $\Phi'(X) = \bar{t} \geq 0$. Thus, by Proposition 2, toeholds are a viable signal and, in a separating equilibrium, bidders that acquire larger toeholds pay higher prices.

The signaling potential of toeholds has been analyzed within a probabilistic tender offer game by Chowdhry and Jegadeesh (1990). Our analysis shows that toeholds as a signal are but an example of the generic exclusion mechanism, and that they do not rely upon probabilistic outcomes.

3.1.2 Debt finance

Müller and Panunzi (2004) show that the bidder can use leverage to exclude target shareholders from (part of the) security benefits. The bidder sets up a shell company that issues debt backed by claims on the target assets. She then makes a tender offer for the target shares and, if the bid succeeds, merges the target firm with the shell company. This two-step process is referred to as a *bootstrap* acquisition. As before, the bidder generates a post-takeover value V. In addition, she can raise debt D up to a limit $\bar{D}$. To avoid bankruptcy issues, we impose a lower bound on the post-takeover share value $\underline{V} > \bar{D}$.

Raising debt against target assets lowers the post-takeover security benefits as part of the value is paid to the debtholders. This allows the bidder to acquire the target at a lower price, and hence to appropriate part of the takeover gains. By the same token, raising less debt means having to pay a higher price, and hence the free-rider condition is $P \geq V - D$. The bidder's payoff from a successful takeover is

$$\Pi = D + r(V - D - P).$$

By redefining $\alpha \equiv D/\bar{D}$, $X \equiv V - \bar{D}$, and $\Phi \equiv \bar{D}$, we can rewrite the bidder's payoff as

$$\Pi = \alpha\bar{D} + r[(V - \bar{D}) + (\bar{D} - D) - P] = \alpha\Phi + r[X + (1-\alpha)\Phi - P]$$

and the free-rider condition as $P \geq (V - \bar{D}) + (\bar{D} - D) = X + (1 - \alpha)\Phi$. Thus, the tender offer game with debt financing is isomorphic to our generic framework for $k = 1$.

Like toeholds, the debt is in place at the time of the bid, thereby committing the bidder to an exclusion level, and its value is non-decreasing in the security benefits, that is, $\Phi'(X) = 0$. Hence, debt financing is a viable signal and, in a separating equilibrium, bidders that raise more debt pay higher prices.[14]

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[14]Osano (2009) also analyzes the role of leverage in a tender offer game with private information but primarily focuses on pooling

3.1.3 Diversion

The exclusion mechanism introduced by Grossman and Hart (1980) is to let the bidder divert resources for private consumption once she is in control. Suppose the bidder chooses which fraction $\phi \in [0, \bar{\phi}]$ of the firm's total post-takeover value $V \in \mathcal{V}$ to divert as private benefits. The upper bound $\bar{\phi} < 1$ reflects exogenous constraints on diversion set by, e.g., shareholder protection laws. A successful bid results in private benefits ϕV and actual security benefits $(1 - \phi)V$. Consequently, the free-rider condition is $P \geq (1 - \phi)V$. The bidder's payoff from a successful takeover is

$$\Pi = \phi V + r[(1 - \phi)V - P].$$

By defining $\alpha \equiv \phi / \bar{\phi}$, $X \equiv (1 - \bar{\phi})V$, and $\Phi \equiv \bar{\phi}V$, we can rewrite the bidder's payoff as

$$\Pi = \alpha \bar{\phi} V + r[(V - \bar{\phi}V) + (\bar{\phi}V - \phi V) - P] = \alpha \Phi + r[X + (1 - \alpha)\Phi - P]$$

and the free-rider condition as $P \geq (V - \bar{\phi}V) + (\bar{\phi}V - \phi V) = X + (1 - \alpha)\Phi$. Thus, the tender offer game with diversion is isomorphic to our generic framework for $k = 1$. In a separating equilibrium, higher types extract a larger fraction of total value as private benefits and pay a higher price.

Separation is possible because the diversion technology satisfies the informativeness requirement: $\Phi'(X) = \bar{\phi} / (1 - \bar{\phi})$. It is, however, debatable whether bidders can, in practice, commit not to divert more once they are in control. On the one hand, the bidder is free to include in the offer provisions that limit her discretion, by either excluding certain types of post-takeover activities (e.g., asset transfers) or strengthening governance (e.g., independent directors). On the other hand, it is questionable to what extent such provisions effectively constrain extraction, let alone, fine-tune the bidder's extraction ability in a way that allows her to signal her type.

In the latter case a moral hazard problem arises: Without effective governance provisions, the controlling party chooses the level of private benefits opportunistically. The ownership stake of the controlling party may affect such discretionary private benefit extraction: A larger stake can reduce the bidder's *incentives* to engage in wasteful opportunism once she is in control (the "alignment" effect; Jensen and Meckling, 1976). At the same time, it could strengthen the bidder's *ability* to act opportunistically (the "entrenchment" effect; Morck et al., 1998). Either effect opens up the possibility that low-valued bidders reveal their type by committing to less private benefits *via* their choice of ownership stake. We show in the Appendix that this is indeed feasible in the "entrenchment" setting, but not in the "alignment" setting.

3.1.4 Bidder assets

Typically, tender offer models abstract from bidder assets other than cash. When the bidder owns assets, she could use claims on her assets to pay target shareholders, and the willingness to do so might reveal her

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equilibria. In fact, the signaling incentives we describe here do not arise under his model assumptions.

type. Suppose a bidder owns cash and a firm. If the bidder acquires control of the target firm, assets in both firms increase in value: target assets from 0 to X, and bidder assets from 0 to λX. The total value created by the takeover is thus $V = X + \lambda X$. The value increase in bidder assets represents the private benefits. The parameter $\lambda > 0$ is commonly known and the same for all bidder types.

The bidder makes a tender offer through her own firm, which assumes ownership of the tendered shares. If a fraction $\beta \geq 0.5$ of the target shares is tendered, the bidder firm has a post-takeover value of $\lambda X + \beta X$. The bidder wants to merge the firms ($\beta = 1$), and offers target shareholders a cash price C and $1 - s$ shares of the merged company. To have control of the merged firm, the bidder must retain $s \geq 0.5$.

Target shareholders tender only if the value of the cash-equity offer exceeds X, the value of a minority share in the target firm. The free-rider condition is thus $C + (1 - s)V \geq X$. The bidder's payoff from a successful merger is

$$\Pi = sV - C = s\lambda X + sX - C.$$

By defining $\alpha \equiv s$, $\Phi \equiv \lambda X$, and $r \equiv s$, and expressing the cash price in terms of the shares the bidder holds in the merged company, $C = sP$, we can rewrite the bidder's payoff as

$$\Pi = \alpha\Phi + r(X - P)$$

and the free-rider condition as $rP + (1 - r)(X + \Phi) \geq X$, which simplifies to $rP + (1 - r)\Phi \geq rX$. Clearly, this condition is satisfied for $P = X$. We can therefore map the merger game into our generic framework with $k = 0$ and the additional constraint $r = \alpha$. Since a separating equilibrium can be constructed for any $r(\cdot)$ in the generic framework, this constraint can indeed be satisfied (see Appendix).

In the separating equilibrium, the cash price C is inversely related to the shares s offered to target shareholders. Lower-valued bidders pay more in equity, which amounts to relinquishing a larger fraction of their private benefits λX.[15] Since the level of exclusion is set in the offer through the equity component s, the commitment requirement is fulfilled. The informativeness requirement is satisfied because $\Phi'(X) = \lambda$, but more fundamentally, because the bidder's information advantage is one-dimensional: It pertains only to a factor *common* to both firms, such that the value improvement of one firm is a sufficient statistic for the other. This seems unlikely to hold in practice, where bidders are bound to have private information about idiosyncratic factors as well. Indeed, the informativeness requirement cannot be satisfied in the case of two-dimensional private information (see Section 3.4).

3.1.5 Probabilistic outcomes

Hirshleifer and Titman (1990) show that a separating equilibrium can be constructed when target shareholders play probabilistic tendering strategies. To illustrate their result, we revisit the tender offer game with

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[15]Higher-valued bidders offering a larger cash component is also a result found in the means-of-payment literature (e.g., Eckbo et al., 1990). In these bilateral merger models, the result relies on two-sided asymmetric information. Here, only the bidder has private information.

non-transferable private benefits where the free-rider condition simply is $P \geq X$ (Section 2). Contrary to before, we assume that shareholders randomize their tendering decision if they are strictly indifferent after having observed the bid price P and updated their beliefs. This assumption generates probabilistic outcomes when the offered price exactly matches the expected security benefits. Otherwise, shareholders either always or never tender. Given an offer $P = E(X|r,P)$, the success probability q can lie anywhere in $[0,1]$, and the expected fraction of acquired shares γ can lie anywhere in $[0.5, 1]$. The bidder's expected payoff from a bid is therefore

$$\Pi = q\left[\Phi + \gamma(X - P)\right].$$

By defining $\alpha \equiv q$ and $r \equiv q\gamma$, we can rewrite this payoff as

$$\Pi = \alpha\Phi + r(X - P)$$

and the free-rider condition remains $P \geq X$. Thus, the probabilistic tender offer game is isomorphic to our generic framework for $k = 0$ and the additional constraint $r = \gamma\alpha$. As in the merger application, this additional constraint can be satisfied (see Appendix).

In equilibrium, a lower-valued bidder pays a smaller price but her bid is less likely to succeed. The higher failure rate protects her bid from being mimicked by higher-valued types. Importantly, this deterrence effect operates *exclusively* through the risk of losing *private* benefits. In fact, if $\Phi(\cdot) = 0$ or even if merely $\Phi(\overline{X}) = 0$, the signaling equilibrium breaks down.

Revealing bids with probabilistic outcomes are but another illustration of the common principle in Proposition 2. The specific feature is that bidders do not signal their type through relinquishing private benefits to the shareholders, but rather through "burning" private benefits by way of failure. Common to the other applications, the outcome is inefficient: While all types actually make a bid in equilibrium, bids do not always succeed. Furthermore, the bid restriction remains a redundant signal and there are multiple equilibrium schedules. Hirshleifer and Titman (1990) select a schedule where all types restrict their bid as much as possible ($r = 0.5$).

3.2 Pooling equilibria

In a pooling equilibrium, the subset of bidder types that make a bid submit the same offer (r_P, α_P, P_P). This pooling offer is individually rational for a bidder if

$$\alpha_P\Phi(X) + r_P[K(\alpha_P, X) - P_P] \geq 0. \tag{4}$$

Let $\mathscr{X}_{(r_P, \alpha_P, P_P)} \subseteq \mathscr{X}$ denote the set of bidder types for whom the participation constraint (4) is satisfied. The pooling offer must also satisfy the free-rider condition

$$P_P \geq E[K(\alpha_P, X)|X \in \mathscr{X}_{(r_P, \alpha_P, P_P)}]. \tag{5}$$

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Proposition 3. *There always exists a pooling equilibrium, in which all and only bidders $X \in \mathcal{X}_{(r_P, \alpha_P, P_P)}$ make a bid and offer the same contract (r_P, α_P, P_P).*

The equilibrium bid only reveals that the bidder belongs to the subset of types who profit from this bid. Among the successful bidder types, some are undervalued and some are overvalued. For some types, the mispricing may be so severe that a takeover at that price is unprofitable; hence, these types do not bid.

There exist multiple offers that satisfy (4) and (5) and hence constitute Perfect Bayesian Equilibria. Multiplicity not only arises from variations in α_P and r_P but also because the equilibrium price may exceed the conditional expectation of the security benefits, that is, the free-rider condition need not bind. In addition, there exist pooling equilibria with probabilistic outcomes, though they are Pareto-dominated.[16]

The existence of pooling equilibria neither requires transferable private benefits nor imposes constraints on the shape of $\Phi(\cdot)$. Depending on the shape of $\Phi(\cdot)$ and the bidder type distribution $G(\cdot)$, the set $\mathcal{X}_{(r_P, \alpha_P, P_P)}$ can consist of disjoint intervals. In particular, there need not exist a threshold type such that all and only types above the threshold make a bid. Furthermore, there can be equilibria in which different bidder sets offer different contracts. For example, there can be an equilibrium in which the highest types pool on one contract and the lowest types on another (see Appendix for details).

3.3 Efficiency and robustness

When separating equilibria and pooling equilibria co-exist, their relative efficiency and robustness is of interest.

Proposition 4. *The most efficient equilibrium can be either separating or pooling depending on the distribution of bidder types.*

A general efficiency ranking cannot be made because the bidder type distribution $G(\cdot)$ determines bid prices and the set of types making an offer in a pooling equilibrium but not in a separating equilibrium. We illustrate this point for the case of increasing private benefits $\Phi(\cdot)$ and $k = 0$. From Proposition 2, we know that in the separating equilibrium bids are made only by bidders above some cut-off type $X_S^c \in (0, \overline{X})$, who makes zero profit. For $k = 0$, the cut-off type is minimized by setting the bid restriction to $r(\cdot) = 1/2$ (as we show in Section 4.1 below) and pinned down by the equation

$$\int_{X_S^c}^{\overline{X}} \left\{ \Phi'(u) \left[1 - \int_u^{\overline{X}} \frac{1}{2\Phi(v)} dv \right] + 0.5 \right\} du = \Phi(\overline{X})$$

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[16]Consider a probabilistic pooling equilibrium in which the pooling offer with r and P succeeds with probability q. For type X, submitting this offer is individually rational if and only if $q\left[\Phi(X) + r(X - P)\right] \geq 0$. The sign of the left-hand side is independent of the takeover probability, so that changes in q leave the set of types for whom the participation constraint is satisfied, and hence shareholders' expectations about the post-takeover share value conditional on a bid, unaffected. Thus, if a pooling equilibrium offer can be supported under probabilistic outcomes, it can also be supported under deterministic outcomes. Given that all bidder types are value-improving, the probabilistic pooling outcomes are Pareto-dominated by the corresponding deterministic outcome.

which we obtain by substituting (13) into (14) for $P(X) = X$ (see the proof of Proposition 2 in the Appendix). This equation contains, except for X_S^c, only exogenous variables but, importantly, is independent of $G(\cdot)$.

Compare this to the pooling equilibria from Proposition 3. With $\Phi(\cdot)$ increasing, lower bidder types are strictly less inclined to bid, so the set of active bidders $\mathscr{X}_{(r_P, \alpha_P, P_P)}$ is a closed interval $[X_P^c, \overline{X}]$. In this case (first analyzed by Shleifer and Vishny, 1986), the most efficient equilibrium is one where bidders extract their full private benefits ($\alpha = 1$), shareholders receive the smallest acceptable price (the free-rider condition (5) strictly binds), and the least number of shares is traded ($r = 1/2$). Selecting this equilibrium and substituting the binding free-rider condition (5) into the participation constraint (4) pins down the cut-off type X_P^c. For $k = 0$, this gives

$$\Phi(X_P^c) + 0.5 \left[X_P^c - \int_{X_P^c}^{\overline{X}} g(u) u \, du \right] = 0$$

where the integral represents the average security benefits of the types above X_P^c and $g(\cdot)$ is the probability density function associated with $G(\cdot)$. Hence, we can move the cut-off type X_P^c by changing $G(\cdot)$. If we shift more probability mass to the types below (above) X_P^c, the left-hand side increases (decreases), in which case the cut-off type must decrease (increase) to restore the equality. In fact, we can choose a distribution function such that the cut-off type is arbitrarily close to the lower (upper) bound of $\mathscr{X}$. By contrast, such manipulations leave the cut-off type in the separating equilibrium, X_S^c, unaffected. For $X_S^c > 0$, there consequently exist $G(\cdot)$ such that X_P^c lies below or above X_S^c, making the pooling equilibrium more or less efficient than the separating equilibrium. The same reasoning applies to the comparison of the pooling equilibria in Proposition 3 with partially separating equilibria in which different bidder sets offer different contracts (see Appendix for details).

Proposition 4 also implies that the separating and pooling outcomes can in general not be Pareto ranked either. Moreover, even if one outcome is more efficient than another, it need not be Pareto dominant: In a pooling equilibrium, the highest bidder types always buy shares at a discount. Therefore, they always prefer a pooling equilibrium with $\alpha = 1$ even when a separating equilibrium is more efficient. At the same time, target shareholders earn information rents in a separating equilibrium and may hence prefer it over a more efficient pooling equilibrium.

While pooling and separating equilibria cannot be ranked in terms of efficiency, there is a clear-cut result with respect to their robustness.

Proposition 5. *Only the separating equilibrium survives the intuitive criterion.*

In any pooling equilibrium, some bidder types pay more, and others less, than their respective actual post-takeover security benefits. Given that $\Phi'(\cdot) \geq 0$, there always exists a deviating offer with a lower price in combination with a smaller quantity or smaller expected private benefits that is attractive only to the overpaying types. Such a deviation exists because forgoing private benefits, purchasing fewer shares, or failing with a higher probability is less attractive to higher-valued bidders. Under the intuitive criterion, the

deviating offer must be attributed only to the overpaying types and is therefore not rejected. This eliminates all pooling equilibria except for the degenerate case where only the highest-valued bidder makes a bid with positive probability.

3.4 Two-dimensional asymmetric information

Proposition 2 establishes that separation requires the bidder to relinquish private benefits in a manner which is *informative*. We now examine whether this is still feasible when the bidder has additional private information about her private benefits Φ, which is independent of the security benefits X. To this end, we consider two-dimensional bidder types, (X, Φ), that are continuously distributed on $[\underline{X}, \overline{X}] \times [\underline{\Phi}, \overline{\Phi}]$. The bidder is informed about both dimensions of her type. In contrast, the target shareholders neither know how much a particular bidder will improve the share value nor how much she values control. The setting is otherwise the same as in Section 3.1.

Proposition 6. *In tender offer games with two-dimensional bidder types, there exists no separating equilibrium in which a bidder's post-takeover security benefits are fully revealed.*

Signaling breaks down because the private information about Φ undermines the "credibility" of the exclusion level α as a signal. Since Φ is not a sufficient statistic of X, target shareholders cannot infer the level of security benefits from an offer conceding $1 - \alpha$ of the private benefits. The ambiguous relation between Φ and X "jams" the signal.[17]

Proposition 2 and Proposition 6 together imply that full revelation is feasible only if (knowing what the bidder knows about) the security benefits are a sufficient statistic of (what the bidder knows about) the private benefits. Whether or not this is an appropriate assumption depends on the specific exclusion mechanism. In case of toeholds or leverage, the assumption holds because the value of the private benefits is directly derived from the security benefits. This also applies to diversion if the scope for private benefit extraction is determined by target firm characteristics or the institutional environment, but is independent of bidder characteristics. By contrast, it is plausible to argue that bidder assets are at least to some extent unrelated to target assets, and that the bidder has additional private information about her own assets. Accordingly, Proposition 6 reinforces the conclusion from Corollary 1 that cash-equity offers are unlikely to be effective signaling devices in tender offers.

4 Unbundling control and ownership

At et al. (2011) and Marquez and Yilmaz (2012) show in similar settings that a target firm can influence the set of active bidder types in a pooling equilibrium by changing its security-voting structure. The specific

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[17]Pooling equilibria exist also in the case of two-dimensional bidder types. Indeed, the proof of Proposition 3 does not rely on any specific relationship between security benefits and private benefits.

applications they study are dual-class share structures and (supermajority) voting rules, both of which affect the fraction of cash flow rights a shareholder must own in order to exercise control, i.e., the degree to which control and ownership are separated in the firm.

In this section, we examine two complementary questions. First, we ask how the target firm's security-voting structure affects the set of active bidder types in *separating* equilibria. Second, we ask how the outcome changes if the *bidder* (rather than the target firm) separates cash flow and voting rights through the offer terms. Such unbundling is easily achieved by using securities other than cash and voting equity as means of payment.

4.1 Bid restrictions

We address the first question in the framework with one-dimensional bidder types and transferable private benefits (Section 3.1) by studying the impact of a constant bid restriction $r(\cdot) = r$ on the cut-off type in a separating equilibrium. We know that the bid restriction is a redundant signal in this setting. A supermajority rule that increases the number of votes required to pass a shareholder decision can be interpreted as an increase in r. Similarly, the introduction of non-voting shares can be interpreted as a decrease in r. Note that we retain the assumption that the bidder must pay target shareholders in cash or ordinary shares of the post-takeover firm, and the original parameter restriction $r \geq 0.5$. The following result is an afterthought to Proposition 2.

Corollary 2. *In tender offer games where bidders can commit to relinquish any fraction of their private benefits and $\Phi(\cdot)$ is a non-decreasing function, the cut-off type $X_S^c \in [0,\overline{X})$ in the separating equilibrium increases with a constant bid restriction r for $k = 0$. For $k = 1$, the same is true if $\Phi'(X) < 1$.*

Smaller transaction sizes mitigate the asymmetric information problem: With fewer traded shares, a bidder gains less (in total) from paying a price below the post-takeover share value. This reduces the incentives to mimic low-valued bidders who, as a result, need to relinquish less private benefits to credibly reveal their type. This result holds for all but one of the applications reviewed in Section 3.1.[18] In particular, consider the two applications where $k = 1$. In the case of debt finance, we had $\Phi'(X) = 0$, and in the case of diversion, $\Phi'(X) = \bar{\phi}/(1 - \bar{\phi})$ which is smaller than 1 so long as the bidder cannot divert more than half of total firm value.

Of course, Corollary 2 also relies on the conditions that ensure existence of the separating equilibria derived in Section 3.1. However, as we show below, these conditions no longer pose a constraint when it is the bidder herself who can unbundle cash flow and voting rights.

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[18]The exception is the application with bidder assets (Section 3.1.4) where the bidder has to acquire the entire target to merge the assets, and then varies the fraction of post-merger equity transferred to target shareholders (as a way of relinquishing private benefits) depending on her type. By the nature of the solution, a constant "bid restriction" is impossible in this case.

4.2 Dual-class offers

In essence, Corollary 2 states that efficiency is decreasing in the fraction of ownership traded. This is due to the assumption that the gains from trade are contingent on the transfer of control, not the transfer of security benefits. While this is a stark assumption, it highlights why target shareholders who do not value control may be reluctant to sell it: The asymmetric information problem in the trade of security benefits contaminates the trade of control rights as long as the two are bundled.

The most straightforward way for the bidder to unbundle control and ownership is to make a dual-class security-exchange offer. The bidder offers to exchange each of the target's voting shares against a non-voting share. Shareholders accept the bid as it preserves their fraction of the cash flow rights. By construction, the bidder pays exactly the post-takeover security benefits. This replicates the full information outcome without revealing the bidder's type.

Despite resolving the asymmetric information problem, the dual-class offer is problematic because it leaves all cash flow rights with the shareholders. That is, the bidder has no equity interest in the firm after the takeover. On the one hand, this makes the offer equivalent to a simple replacement of management, which begs the question why a takeover is necessary in the first place. On the other hand, it makes the offer prone to abuse by value-decreasing bidders, since it does not require the bidder to put up any cash (Bebchuk and Hart, 2001). Cash payments put at least some (lower) bounds on the bidder's quality.

The latter point emerges when the parameters allow for value-decreasing bidder types. Let the security benefits under the incumbent temporarily be $X_I > 0$, instead of zero, thereby allowing for bidder types $X \in [0, X_I] \subset \mathcal{X}$ for whom $X + \Phi(X) < X_I$. In this setting, a dual-class exchange need not be efficient: Target shareholders might accept non-voting shares worth less than X_I and so concede control to a bidder who lowers *total* firm value. Suppose target shareholders would not knowingly accept such offers. It is straightforward to support a separating equilibrium in which any offer with a cash price less than X_I – including a dual-class exchange offer – fails because shareholders attribute it to the least valuable bidder type, and bids paying more than X_I in cash are constructed according to the separating schedule in Section 3.1. Clearly, any bidder willing to pay at least X_I in cash must be a value-increasing type.

This argument also carries over to a setting in which the bidder can choose to extract up to a fraction $\bar{\phi}$ of the post-takeover firm value V as private benefits (as in Section 3.1.3), so that security benefits are $(1 - \phi)V$. Here, target shareholders lose wealth if they accept a dual-class exchange offer from bidder types for whom $(1 - \phi)V < X_I$. Again, one can construct a separating equilibrium in which offers with a cash price below X_I are rejected, thereby eliciting only bids in which the cash component is high enough to imply that the value created by the bidder equals at least the security benefits under the incumbent.

4.3 Options

Dual-class exchanges are extreme in that the bidder does not acquire *any* cash flow rights. However, the problem is not that bidders are unwilling to buy any cash flow. Rather, it is that bidders are not – or only to

a limited extent – willing to pay for cash flows which they know do not exist. This problem is more suitably resolved by a cash offer that is combined with securities that leave (only) the "non-existing" cash flows to target shareholders. In the present setting, call options provide such a solution. It merely requires that every type $X \in \mathcal{X}$ purchases a target share in exchange for cash $P(X) = X$ and a (cash-settled) call option with an exercise price of $S(X) = X$.[19]

Proposition 7. *Offers with call options allow to implement the full information outcome.*

If a type X would succeed with an offer $x < X$, she would pay a cash price x for shares that are worth X. However, ex post she would not capitalize on this gain, as the target shareholders would exercise their options once the value improvement materializes. Conversely, the low-valued type does not mimic the high-valued type because she would pay X for shares that are worth x. Thus, the offer schedule is incentive-compatible, and every bidder succeeds irrespective of her private benefits. Indeed, the separating equilibrium is efficient and its existence does not rely on the shape of $\Phi(\cdot)$, i.e., a specific relationship between private benefits and security benefits.

Derivatives enable the bidder (i) to trade economic ownership void of voting rights and (ii) to issue non-linear contingent claims. The first step of the transaction consists of acquiring the target shares and stripping them of their votes. In the second step, the bidder re-issues some cash flow rights, restructured into claims that punish her for "lying" about the security benefits. The call options which are executed when the post-takeover security benefits are higher than professed penalize the pretense of low security benefits – ex post when the true value is observed. This offer amounts to the simplest solution to the asymmetric information problem: a bid price which is de facto contingent on the post-takeover share value.[20] The offer transfers, cash aside, only future claims but no actual future cash flows to target shareholders. This is an artifact of the assumption that the post-takeover security benefits X are deterministic (perfectly known by the bidder). Yet, the result carries over to a setting with stochastic cash flows (see Appendix).

It is worth comparing the control-cash flow allocation implied by Proposition 7 with those obtained in well-known capital structure models. In our setting, separating ownership and control as well as conceding the "upside" to non-controlling investors can improve efficiency. The former prevents that frictions in the cash flow trade spill over into the vote trade, while the latter allows bidders to effectively signal a low valuation. This contrasts with the conclusions from both external financing models, where the controlling party retains ownership and the "upside" to signal a high valuation (Leland and Pyle, 1977; Myers and Majluf, 1984), and moral hazard models, where retaining ownership and the "upside" improves the controlling party's incentives (Jensen and Meckling, 1976; Innes, 1990). In Section 5, we formally explore the tension between the signaling benefits and the potential moral hazard costs of unbundling within our takeover setting.

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[19]In a setting with value-decreasing bidder types, the analogous offer would (have to) be $P(X) = \max\{X, X_I\}$ and $S(X) = X$.

[20]Though, note that an offer in which the bidder pays (no cash upfront but) only a contingent price after implementing post-takeover changes is prone to exactly the same problems as a dual-class exchange offer.

4.4 Shareholder activism and empty voting

The insight that the separation of cash flow and voting rights mitigates asymmetric information problems is not confined to tender offers. This is perhaps best illustrated by adapting the present model to the case of an activist investor who has superior information about the value consequences of a shareholder proposal. Suppose the investor already owns a minority stake $\bar{t}$ in a firm and faces an uninformed market-maker in the secondary market. The investor knows that the proposal, if approved, increases the security benefits from currently 0 to X. As in Brav and Matthews (2011), the voting process is noisy and the investor can increase the probability that the proposal is approved by acquiring more voting shares. Let $q(b)$ denote the approval probability and b the fraction of voting shares bought in the open market with $q'(b) > 0$ and $q(0) = 0$.

To meet the investor's buy order b, the market-maker has to go short in the stock which exposes her to price risk. For a given b, the market-maker is willing to take the short side of the transaction if $P \geq q(b)X$, which is analogous to the free-rider condition in tender offer games. The problem is that, contrary to the activist investor, she does not know the true value of X. As in the tender offer game, the buyer (investor) must therefore convince the seller (market-maker) that the transaction price is adequate.

One can construct a separating schedule with "market" orders b or "limit" orders (b,P). In either case, the r-P-schedule has to meet the same incentive compatibility constraints. The bidder's payoff from a successful activist campaign is

$$\Pi = q(b)\bar{t}X + q(b)bX - bP.$$

By defining $\Phi \equiv \bar{t}X$ and $\hat{P} \equiv P/q(b)$, we can rewrite the bidder's payoff as

$$\Pi = q(b)\left[\Phi + b\left(X - \hat{P}\right)\right]$$

and the free-rider condition as $\hat{P} \geq X$. This has the same structure as in the probabilistic tender offer game (Section 3.1.5) except that (i) the per-unit price $\hat{P}$ is normalized by the success probability and (ii) there is an intrinsic relationship between $q(b)$ and b. Following the same procedure as before, we further define $\alpha \equiv q(b)$ and $r \equiv q(b)b$ to get $\Pi = \alpha\Phi + r\left(X - \hat{P}\right)$, and can then apply Proposition 2 with the additional constraint $r = \alpha q^{-1}(\alpha)$, where $q^{-1}(\cdot)$ is the inverse of $q(\cdot)$. Because r is a redundant signal, the additional constraint can in principle be satisfied. To give one example, if $q(b) = \frac{b}{.5-\bar{t}}$, the constraint is isomorphic to the one in the probabilistic tender offer game, $r = \gamma\alpha$, with $\gamma = .5 - \bar{t}$.

In a separating equilibrium characterized by Proposition 2, $\alpha \equiv q(b)$ is increasing in X. The underlying intuition is the same as in the probabilistic signaling equilibrium. When the potential value improvement is small, the investor buys fewer shares at a lower price, and the proposal is less likely to be approved. The high failure rate justifies the low price, as it prevents mimicking from higher-valued types. This deterrence effect operates through the risk of forgoing the value improvement in the initial stake $\bar{t}$. Conversely, the investor is keen on buying more voting shares when larger value improvements are "at stake" in the shareholder vote.

As in Proposition 7, the activist could also unbundle cash flow and voting rights to implement the sym-

metric information outcome. In principle, the investor could offer to buy $(0.5 - \bar{t})$ voting shares at the price $P = X$ and simultaneously enter a derivative contract with which she goes short in $(0.5 - \bar{t})$ call options with strike price X. This would allow the investor to purchase sufficient voting shares to ensure that the proposal is accepted. The derivatives position of the activist investor represents a "bet" against the firm. This negative exposure is a prerequisite for buying the shares at the fair price, i.e., for acquiring the attached voting rights at no (additional) cost. The additional voting rights attained in this way are void of ownership and therefore referred to as empty voting. A transaction to the same effect is to borrow voting shares to register more votes on the record date but to return the shares before the actual vote (Christofferson et al., 2007).[21]

5   Moral hazard cost of signaling

We have alluded to the reservation that unbundling, while useful for signaling, may be problematic in the presence of moral hazard. To examine this question, we consider an extension of our framework in which the design of the tender offer affects not only the resolution of asymmetric information during the bid but also the bidder's incentives to improve firm value after the takeover.

Suppose the bidder, once in control, generates total firm value $V = V(e; \theta)$ at private cost $C(e)$, where $e \geq 0$ denotes her chosen effort level and θ her productivity. The bidder can extract up to $B(e; \theta)$ in private benefits, in which case the security benefits are $X(e; \theta) = V(e; \theta) - kB(e; \theta)$. As before, we include the parameter $k \in \{0, 1\}$ to account for the possibility that private benefit extraction may (or may not) reduce security benefits. All functions are twice continuously differentiable. We assume that firm value increases in effort and productivity ($V_e, V_\theta > 0$); the marginal return to effort weakly decreases in effort but weakly increases in productivity ($V_{ee} \leq 0$, $V_{e\theta} \geq 0$); and the cost of effort is increasing and convex ($C_e, C_{ee} > 0$). Furthermore, baseline private benefits and security benefits are positively related ($\partial B / \partial X \geq 0$), as in all the applications of Section 3.

The bidder knows her type θ and subsequent effort choice e, while target shareholders only know that θ is continuously distributed on $\Theta = [\underline{\theta}, \overline{\theta}]$ according to the distribution function $H(\theta)$. As before, the security benefits under the incumbent management are known to all shareholders and normalised to zero.

To explore the extent to which moral hazard undermines the optimality of unbundling, we must analyze a setting in which the bidder can both use derivatives as means of payment (as in Section 4) and forgo private benefits (as in Section 3), rather than being confined to one or the other signaling device. Hence, a bid $\mathscr{C} = (\beta, r, P, \mathscr{S})$ specifies a fraction β of private benefits retained, a bid restriction r, a cash price P, and a payment $\mathscr{S}$ contingent on post-takeover security benefits. The bidder's payoff from a successful takeover is

$$\Pi(\mathscr{C}, e) = \beta B(e; \theta) - C(e) + rK(e; \theta) - rP - r\mathscr{S}(K)$$

where $K = X(e;\theta) + (1-\beta)kB(e;\theta)$. In a separating equilibrium, the bidder chooses $\mathscr{C}$ to maximize this payoff subject to the non-mimicking constraints, the free-rider condition

$$rP + r\mathscr{S}(K) \geq rK(e;\theta),$$

and the post-takeover incentive constraint

$$\beta \frac{\partial B(e;\theta)}{\partial e} + r\frac{\partial K(e;\theta)}{\partial e} - r\frac{\partial \mathscr{S}(K)}{\partial K}\frac{\partial K(e;\theta)}{\partial e} = \frac{\partial C(e)}{\partial e}. \qquad (6)$$

Chosen after the takeover, the bidder's effort maximizes the sum of her net private benefits, acquired "long" controlling equity position, and "short" position as per the contingent payment.

Proposition 8. *With costly post-takeover effort, bidders use unbundling to implement the full information outcome, and each bidder type maximizes net private benefits.*

Introducing moral hazard does not overturn the optimality of unbundling; nor does unbundling preclude post-takeover effort provision. The intuition behind this result is as follows: Conditional on effort, the bidder issues call options to resolve the information asymmetry. Conversely, conditional on symmetric information, she chooses offer terms that maximize her private returns to effort. Any gain in security benefits she brings about is extracted by the free-riding shareholders, as shown by Burkart et al. (1998). To limit such extraction, the bidder uses call options to "cap" her exposure to security benefits, thereby diluting post-takeover incentives. Unbundling thus allows her to simultaneously counteract "overworking" and "overestimation" of security benefits – the two problems that arise from the interaction of the shareholders' free-riding behavior with moral hazard and asymmetric information. Indeed, it turns out that the optimal contract not only restores full information but also reduces the post-takeover incentive constraint (6) to

$$\frac{\partial B(e;\theta)}{\partial e} = \frac{\partial C(e)}{\partial e}. \qquad (7)$$

That is, the bidder uses her short call option position to neutralize the incentives induced by her controlling equity stake to such an extent that her optimal post-takever effort equates marginal cost to marginal private benefit.

The above analysis may suggest that restrictions on unbundling improve incentives to create value. If so, unbundling entails a trade-off between the outside governance problem (facilitating control transfers) and the inside governance problem (mitigating moral hazard). So consider the contracting environment of Section 3 where the means of payment are restricted to cash and common equity ($\mathscr{S} = 0$). The bidder chooses a more limited contract, $\mathscr{C} = (\beta, r, P, 0)$, to maximize her payoff

$$\Pi(\mathscr{C}, e) = \beta B(e;\theta) - C(e) + rK(e;\theta) - rP$$

subject to non-mimicking constraints, the free-rider condition, and the post-takeover incentive constraint, which now becomes

$$\beta \frac{\partial B(e;\theta)}{\partial e} + r \frac{\partial K(e;\theta)}{\partial e} = \frac{\partial C(e)}{\partial e}. \tag{8}$$

We show in the proof of Proposition 9 that a separating equilibrium exists under reasonable assumptions on $B(\cdot;\cdot)$ and $X(\cdot;\cdot)$.

Proposition 9. *Unbundling comes with costs and benefits for post-takeover incentives.*

The comparison of (7) and (8) reveals that the absence of unbundling has two countervailing effects: On one hand, the bidder can no longer "cap" her incentives by using derivatives to offset the equity interest she has through $rK(e;\theta)$, which strengthens post-takeover incentives. On the other hand, to signal her type, she must now relinquish private benefits by lowering β, with the opposite effect on incentives. This effect is more pronounced for lower-valued bidders who must relinquish larger fractions of their private benefits, which can make the net impact on incentives asymmetric across bidder types. More generally, the net impact depends not only on the type distribution $H(\theta)$ but also on assumptions regarding the source of private gains, which determine the relative sizes of B and K and institutional restrictions on r and β.

Nonetheless, there are cases in which the impact is unambiguous. For example, with toeholds (details in the proof of Proposition 9), the incentive constraint (8) without unbundling becomes

$$[\beta(\theta)\bar{t} + r(\theta)] \frac{\partial V(e;\theta)}{\partial e} = \frac{\partial C(e)}{\partial e},$$

where $\beta(\cdot)$ and $r(\cdot)$ denote equilibrium schedules, and the incentive constraint (7) with unbundling becomes

$$\bar{t} \frac{\partial V(e;\theta)}{\partial e} = \frac{\partial C(e)}{\partial e}.$$

Since $\beta(\theta)\bar{t} + r(\theta) \geq .5 > \bar{t}$, effort is always lower under the constraint with unbundling.[22]

Corollary 3. *If toeholds are the only source of private benefits, unbundling lowers post-takeover incentives such that the bidder exerts effort as if she owned only a toehold.*

This highlights a tension between outside and inside governance: While unbundling resolves the frictions faced by a control-seeking "outsider," thereby facilitating a takeover, it lowers her incentives to improve firm value, should she become the controlling "insider." In effect, it lowers incentives from a level commensurate with a majority stake ($\beta\bar{t} + r \geq .5$) to a level commensurate with a minority stake ($\bar{t} < .5$) – which calls into question what the additional benefit of acquiring control through a takeover is relative to exercising governance through the kind of investor activism discussed in Section 4.4.

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[22]Unbundling clearly reduces equilibrium effort when the bidder's maximum private benefits do not exceed the total security benefits associated with majority ownership. This is naturally true for toeholds, since any admissible toehold is worth less than a majority stake in the same firm. In the case of diversion, by comparison, this is only true if no more than half of the total firm value can be diverted by a controlling shareholder.

6 Empirical implications

The choice of payment method as a means to overcome asymmetric information is a prominent theme in the literature on mergers and acquisitions. Unlike existing papers in this literature, we find that including equity is unlikely to provide signaling benefits. Because of the free-rider problem in our framework, it serves as a signaling device only if the bidder has pre-existing assets that appreciate as a result of the takeover, will merge these assets with the target, and crucially, possesses *no* private information about these assets that is independent of her information about the target. It is highly unlikely that this last condition holds in practice.

P1 Equity as a means of payment is unlikely to resolve asymmetric information problems in tender offers.

The other explanation put forward in the literature against the use of equity in tender offers focuses on the time delay imposed by the mandatory pre-registration of securities, to which tender offers seem to be more sensitive than negotiated control transfers (Martin, 1996).[23] Empirically, the means of payment indeed correlates strongly with the mode of acquisition. Eckbo (2009) finds that, among about 16,000 U.S. takeovers between 1980 and 2005, equity was used in roughly a quarter of the tender offers but more than two-thirds of the mergers.[24] While this is suggestive, it should be noted that the choice of acquisition mode is not exogenous, so that the correlation may be spurious.

Our analysis suggests that, instead of equity, derivatives which provide the seller with a call option-like claim help to overcome the asymmetric information problem. Earnouts, whereby the seller obtains additional future compensation if the business subsequently surpasses certain financial targets, are an example of such a contingent claim frequently used in the acquisition of private firms. There is also evidence of limited use of convertibles (Finnerty and Yan, 2009). As regards tender offers specifically, we are not aware of any systematic evidence on the use of such contingent claims, but we surmise that they may not be commonly used because of impracticality or post-takeover considerations.

First, the objective of many tender offers is to take the target firm private or to merge the target with other assets. In both cases it becomes difficult to define the underlying asset for derivatives, especially when the bidder has further private information about the other assets (see Section 3.4). Also, like shares, derivatives would require preregistration before the tender offer can be consummated. Second, unbundling control and ownership and relinquishing the "upside" may dilute incentives to improve firm value, possibly to an extent that questions the purpose of acquiring a majority stake (see Section 5).[25]

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[23]Other factors that influence the choice between cash and equity in takeovers – such as capital structure, financial slack, and taxes – are not specific to the mode of acquisition.

[24]The fact that equity is a prevalent means of payment in management-negotiated mergers is fully consistent with our result, as such negotiations are not subject to the free-rider problem. Also, while our result states that equity is not a viable signaling device in tender offers, it does not preclude the possibility that bidders might use equity for other reasons such as, e.g., liquidity constraints.

[25]To address these issues – measurement and post-takeover incentives – earnout structures often imposes *post-closing covenants* on the buyer, requiring, e.g., (i) that the buyer keep books and records for the seller to inspect, (ii) that the buyer maintain a minimum level of working capital in the business, (iii) that the buyer abstain from any change of control or sale transaction, or (iv) that the buyer make a commercially reasonable effort to reach the earn-out targets. The monitoring necessary to enforce these covenants will be performed by the seller of a private firm but not by dispersed shareholders following a tender offer – precisely due to the free-rider problem.

Neither merging, pre-registration, nor post-takeover incentives are an obstacle for investor activism. Activist investors trade shares and derivatives on secondary markets and they have engagement objectives that require a temporary intervention – such as sale of an asset, change in capital structure, or replacement of directors – with the intention of exiting a target once the stated objective is achieved. We would argue that it is precisely because activists do not subsequently become "insiders" and their involvement is *short-term*, which is often cited as a cause for concern, that they can take advantage of unbundling ownership and control as an effective and value-enhancing strategy.

P2 Empty voting increases firm value.

In a large data set of Schedule 13-D filings[26] by U.S. activist hedge funds, Brav et al. (2008) find that in 16.8 percent of the cases the fund reported derivative positions at the time of the filing, but most of these positions raised rather than offset the fund's economic interest in the target. Though as noted by the authors, there is a lack of reliable data since the disclosure of over-the-counter derivatives and short positions is not mandatory.

In any case, it would not be inconsistent with our arguments if activists held primarily long (derivative) positions in the target firm at the time of disclosure. In our analysis in Section 4.4, empty voting enables the activist to capture the full value improvement in her initial stake (α). Her profits are hence determined both (i) by her ability to cheaply accumulate a stake prior to disclosing her activist intent and (ii) by her ability to increase her influence as cheaply as possible after the disclosure. Our empty voting result pertains to (ii). As concerns (i), it may be argued that taking long derivative positions in the target prior to disclosure allows the funds to build up an initial stake cheaply. In fact, avoidance of disclosure is precisely the motivation in the main example Hu and Black (2006) provide for such a derivative position, which they refer to as *hidden (morphable) ownership*. By contrast, empty voting becomes relevant post-disclosure once the market knows of the activist's intentions (as in our analysis of Section 4.4), and in particular, only when the conflict between activist and incumbent management escalates into a shareholder vote. Because activists prefer to achieve their objective without resorting to (though threatening with) a proxy fight, it may not be surprising that they hold hidden ownership but not yet empty voting positions at the time of the 13-D filing.

Instead, one should expect empty voting positions to materialize ahead of impending shareholder votes, especially contested ones. Using large data samples from the U.S. and the U.K., Christofferson et al. (2007) document that share borrowing spikes on voting record dates, especially when the vote is close, and that such activity biases the voting outcome towards shareholder proposals (or against management proposals), which suggests that it enhances the influence of outsiders. Note that such "vote trading" is optimal for uninformed passive shareholders in that it allows them to lend control to activists while capturing the *full* economic gain from activism on their shares, i.e., to free-ride. Christofferson et al. (2007) further find, consistent with our

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[26]Schedule 13D filings are mandatory under Section 13(d) of the 1934 Exchange Act and must be filed with the U.S. Securities and Exchange Commission within 10 days of acquiring more than 5% of any class of securities of a publicly traded company by investors who have an interest in influencing the management of the company.

model of free-riding shareholders (exhibiting full "bargaining power" regarding security benefits but none regarding control benefits), that the price of the votes is virtually zero.

While we have treated takeovers and activism as separate examples of the outside governance problem, there is an interesting connection between the two in practice: Activists often put the target "into play" for acquirers. Studying 13-D filings from 1993 to 2006, Greenwood and Schor (2009) find that announcement and long-term abnormal returns are high only for targets ultimately acquired and that returns to activism decline in periods of low takeover activity, findings corroborated by Becht et al. (2014) in a data set of about 1,800 international cases. In light of our analysis, one may speculate whether activism to force a sale of the firm is a more lucrative intervention than a tender offer (with the same intended outcome) because it can use *temporary* "unbundling" to bring about a *permanent* control change without compromising post-acquisition incentives – a Trojan Horse unlocking the target for Barbarians at the Gate.[27]

Several developments over the last decades may have strengthened this advantage of activists – exercising temporary influence in excess of ownership – over acquirers who, by definition, seek permanent influence by way of majority ownership: The 1992 proxy reform removing restrictions on communication between shareholders[28], advances in communications technology, and innovations in derivatives markets have made it easier to mobilize votes without acquiring ownership for a (potential) proxy fight. In the same time period, overall takeover activity has been on the rise but the fraction of hostile takeovers has been in decline (Betton et al., 2008, Figure 9); hedge fund activism has increased tremendously and correlates with the number of interventions that lead to takeovers (Becht et al., 2014, Figure 6); and the use of unbundling in the nexus of corporate votes, activism, and control contests has become more frequent (Hu and Black, 2008, Table 1). The co-occurrence of these patterns accords well with the overall message of our theory.[29]

Going back to tender offers, our results imply that bidder gains and bid premia (target shareholder gains) should be positively related to the fraction of the takeover surplus that the bidder extracts as private benefits, provided that the exclusion mechanism qualifies as a viable signal. Of the exclusion mechanisms explored in this paper, only toeholds and takeover leverage seem to meet the necessary conditions of informativeness and commitment. The signaling potential of toeholds was first noted by Chowdhry and Jegadeesh (1994). Empirically, larger toeholds are indeed positively correlated with bidder gains, consistent with our prediction, but negatively correlated with bid premia (Eckbo, 2009).[30] Bid premia are, however, inherently difficult to measure since prior expectations of a takeover impound part of the premium into the pre-offer price, which is of particular concern when the bidder accumulates a toehold. Our prediction regarding takeover leverage

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[27]A recent example involves the pharmaceutical company Valeant and the hedge fund activist William Ackman. In tacit agreement with Valeant, Ackman accumulated a 9.7 percent stake in Allergan, a target Valeant is interested in, and then announced his intention of pressuring the incumbent management to agree to a merger with Valeant (De La Merced et al., 2014).

[28]See, e.g., Sharara and Hoke-Witherspoon (1993), Bradley et al. (2010), and Fos (2013).

[29]Collaborations like the one between Valeant and William Ackman (see fn. 23), whether explicit or implicit, may in part drive the shift from hostile takeovers to investor activism observed by recent news articles such as Davidoff (2013): "But unlike hostile takeovers, there is a real fear on Wall Street of the activists. For now, the question is whether activism will remain on the upswing and be the disciplining force that the hostile takeover occupied."

[30]Eckbo (2009) cites a possible alternative explanation for these empirical patterns, which is that larger toeholds by the initial bidder may deter potential rival bidders.

is novel.

P3 In bootstrap tender offer acquisitions, bidder returns and bid premia increase with takeover leverage.

There exist only few empirical studies on takeover financing. Schlingemann (2004) finds that debt raised during the year before a bid announcement is not significantly related to bidder gains. In contrast, Martynova and Renneboog (2009) document that the cumulative average abnormal returns for bidders in the 120 days around bid announcements are significantly larger for debt-financed acquisitions. Most relevant to our paper, Bharadwaj and Shivdasani (2003) study 115 tender offers and find a positive relation between bidder returns and the fraction of the acquisition price funded by bank loans. A direct test of our hypothesis would have to focus on the impact of leverage in bootstrap acquisitions within a sample of uncontested tender offers.[31]

7 Conclusion

Control contestability is an important aspect of corporate governance. It allows an outsider who can increase firm value, the sum of security benefits and control benefits, to buy into the firm to influence corporate decisions. In the absence of frictions, the current owners always agree to sell sufficiently many shares, and such transactions ensure that control is allocated to its most efficient user.

Public equity markets and dispersed ownership lead to frictions that impede such efficient control transactions. Individually, dispersed shareholders lack the incentives to acquire costly information or to engage in costly coordination. This free-rider behavior has ramifications for their collective behavior: They do not exercise their control rights and hence care only about security benefits. In addition, they do not sell their shares unless they capture the whole value improvement in security benefits that the outsider can generate. Their lack of information increases their reluctance to sell even further because they suspect the outsider to understate the value improvement in security benefits. These frictions are arguably most conspicuous in tender offers, but also afflict shareholder activism.

In a tender offer framework, we show how and to what extent the outsider can overcome these frictions. Because the outsider is forced to concede all gains in security benefits to free-riding shareholders, she cannot credibly reveal any information by voluntarily giving up such gains. Instead, the outsider must not only enjoy private benefits of control, but she must also be able to commit to forgo (part of) these benefits in a manner that is informative about the security benefits. We analyze various bidding strategies employed in practice and show that takeover leverage and toeholds, for example, meet these conditions whereas cash-equity offers and bid restrictions do not.

The reason the bidder has to relinquish control benefits, even though free-riding shareholders are willing to tender if paid only the security benefits, is that common shares bundle cash flow rights and voting rights.

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[31]Asymmetric information is crucial to our prediction that bid premia increase with takeover leverage. In Müller and Panunzi (2004)'s symmetric information setting, takeover leverage is negatively correlated with bid premia (though positively with bidder returns) given that its purpose is to lower the target shareholders' reservation price. As tentative evidence, they cite Maloney et al. (1993) and Lang et al. (1991) who find that bidder leverage is positively related to bidder returns but negatively to target shareholder returns, but note that both studies consider *preexisting* leverage, as opposed to debt raised as part of a bootstrap acquisition.

Unbundling these rights reduces the extent to which frictions in the trade of security benefits "spill over" to the transfer of control. Indeed, we show that it is optimal for the outsider to assume derivative positions that "bet" against the firm to drive a wedge between her equity voting power and her net economic interest. This strategy remains optimal when we adapt our framework to analyze activist investors, in which context the strategy has been labeled empty voting. This insight, interpreted more broadly, suggests that excess returns to activist investors derive from strategies that lever their influence above and beyond their equity positions. Such separation of ownership and control, typically considered a source of corporate governance problems, is an efficient response to the frictions that dispersed ownership causes for control contestability.

Decoupling control from ownership enables bidders (activist investors) to overcome the target (passive) shareholders' lack of coordination and information without relinquishing private benefits. Unbundling need therefore not necessarily undermine the outsiders' incentives to improve firm value once in control. Decreasing the equity stake required for control clearly weakens incentives, but preserving the private benefits of control produces a countervailing effect. That said, in settings where the negative effect dominates, a tension arises between control contestability and post-takeover incentives. For example, when the source of profit is the appreciation of equity owned prior to the bid, the optimal unbundling strategy reduces post-takeover incentives to a level commensurate with the initial minority stake, as opposed to a majority stake. In this case it is open to question what benefits a takeover affords compared to activism. We would argue that these insights are pertinent to current developments in the market for corporate control.

Proofs

Proof of Corollary 1

Corollary 1 follows from the equivalence of mixed offers and restricted cash-only offers which the subsequent lemma establishes. Consider a bid for r shares that offers a cash price C and t shares in the post-takeover firm.

Lemma 1. *Under full information, the restricted mixed offer (r,C,t) and the restricted cash-only offer (r^{co}, C^{co}) with $C^{co} = C$ and $r^{co} = r - t$ are payoff-equivalent.*

Proof. To succeed, the mixed offer must satisfy the free-rider condition $C + tX \geq rX$, or equivalently

$$C/r + (t/r)X \geq X. \tag{9}$$

Given the condition is satisfied, all shareholders tender, and the bidder's payoff is

$$\Phi(X) + r[X - (C/r + (t/r)X)]. \tag{10}$$

Rearranging the free-rider condition (9) to

$$C \geq (r - t)X$$

and the bidder's payoff (10) to

$$\Phi(X) + (r - t)X - C$$

shows that the restricted cash-*only* offer (r^{co}, C^{co}) with $C^{co} = C$ and $r^{co} = r - t$ is payoff-equivalent for any X. ∎

Proof of Proposition 2

Given that $\Phi'(\cdot) \geq 0$, we show that there exists a schedule $\{\alpha(\cdot), r(\cdot), P(\cdot)\}$ with $\alpha' > 0$, $r' \geq 0$ and $P' > 0$ that can be supported as a separating equilibrium.

Quasi-concavity. Suppose that the proposed schedule satisfies (3) for $\hat{X} = X$ for all $X \in \mathscr{X}$. This schedule then makes the objective function quasi-concave. Specifically, under this schedule,

$$\partial \Pi / \partial \hat{X} = \alpha'(\hat{X})\Phi(X) + r'(\hat{X})K(\hat{X};X) + r(\hat{X})K'(\hat{X};X) - r'(\hat{X})P(\hat{X}) - r(\hat{X})P'(\hat{X}) \tag{11}$$

is non-negative for $\hat{X} \leq X$ and non-positive for $X \geq \hat{X}$.

For $\hat{X} = X$, the first-order condition (3) becomes

$$r(\hat{X})P'(\hat{X}) + r'(\hat{X})P(\hat{X}) = \alpha'(\hat{X})\Phi(\hat{X}) + r'(\hat{X})K(\hat{X};\hat{X}) + r(\hat{X})K'(\hat{X};\hat{X}).$$

Substituting the right-hand side into (11) and using $K(\hat{X};X) = X + k[1 - \alpha(\hat{X})]\Phi(X)$ yields

$$\partial\Pi/\partial\hat{X} = \alpha'(\hat{X})\Phi(X) + r'(\hat{X})[X + k[1 - \alpha(\hat{X})]\Phi(X)] - r(\hat{X})k\alpha'(\hat{X})\Phi(X)$$
$$- \alpha'(\hat{X})\Phi(\hat{X}) - r'(\hat{X})[\hat{X} + k[1 - \alpha(\hat{X})]\Phi(\hat{X})] + r(\hat{X})k\alpha'(\hat{X})\Phi(\hat{X}). \quad (12)$$

By rearranging, we obtain

$$\partial\Pi/\partial\hat{X} = \alpha'(\hat{X})\left[\Phi(X) - \Phi(\hat{X})\right] - r(\hat{X})k\alpha'(\hat{X})\left[\Phi(X) - \Phi(\hat{X})\right]$$
$$+ r'(\hat{X})k[1 - \alpha(\hat{X})]\left[\Phi(X) - \Phi(\hat{X})\right] + r'(\hat{X})\left[X - \hat{X}\right]$$

$$\partial\Pi/\partial\hat{X} = \underbrace{\left[[1 - r(\hat{X})k]\alpha'(\hat{X}) + k[1 - \alpha(\hat{X})]r'(\hat{X})\right]}_{\Gamma}\left[\Phi(X) - \Phi(\hat{X})\right] + r'(\hat{X})\left[X - \hat{X}\right].$$

The assumption $\Phi'(\cdot) \geq 0$ implies that $\Phi(X) \geq \Phi(\hat{X})$ when $\hat{X} \leq X$ and that $\Phi(X) \leq \Phi(\hat{X})$ when $\hat{X} \geq X$. Since $\alpha' > 0$ and $r' \geq 0$ and therefore $\Gamma \geq 0$ for $k = 0$ as well as $k = 1$, it follows that

$$\partial\Pi/\partial\hat{X} \text{ is } \begin{cases} \text{non-negative} & \text{for } \hat{X} < X \\ 0 & \text{for } \hat{X} = X \\ \text{non-positive} & \text{for } \hat{X} > X \end{cases}.$$

Thus, the proposed schedule makes $\Pi(\hat{X};X)$ weakly quasi-concave over $\mathscr{X}$. This also holds for $r'(\hat{X}) = 0$, in which case all bidder types propose the same bid restriction.

Local optimality. Condition (3) is a functional equation for $\alpha(\cdot)$, $r(\cdot)$ and $P(\cdot)$ with two degrees of freedom. To derive an example of an incentive-compatible schedule, we set $r(\cdot) = 0.5$. Then, using $K(\hat{X};X) = X + k[1 - \alpha(\hat{X})]\Phi(X)$, condition (3) for $\hat{X} = X$ simplifies to

$$0.5P'(\hat{X}) = \alpha'(\hat{X})\Phi(X) - 0.5[k\alpha'(\hat{X})\Phi(\hat{X})]$$
$$\alpha'(\hat{X}) = \frac{P'(\hat{X})}{(2 - k)\Phi(\hat{X})}$$

Integrating on both sides over $[X,\overline{X}]$ yields

$$\int_X^{\overline{X}} \alpha'(u)du = \int_X^{\overline{X}} \frac{P'(u)}{(2 - k)\Phi(u)}du \quad \Leftrightarrow \quad \alpha(\overline{X}) - \alpha(X) = \int_X^{\overline{X}} \frac{P'(u)}{(2 - k)\Phi(u)}du.$$

As the highest-valued type does not have to relinquish any private benefits $[\alpha(\overline{X}) = 1]$,

$$\alpha(X) = 1 - \int_X^{\overline{X}} \frac{P'(u)}{(2 - k)\Phi(u)}du. \quad (13)$$

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Free-rider condition. Equation (13) has one degree of freedom, which we use to select a price function that satisfies the free-rider condition $P(X) \geq K(X;X) = X + k[1 - \alpha(X)]\Phi(X)$.

For $k = 0$, one such price schedule is $P(X) = X$. Using this schedule in (13) yields $\alpha(X) = 1 - \int_X^{\overline{X}} [2\Phi(u)]^{-1} du$. Clearly, these two schedules satisfy the free-rider condition and the incentive compatibility constraint.

For $k = 1$, consider the price schedule $P(X) = \frac{X+z}{\overline{X}+z}\overline{X}$. Using this schedule in (13), and then substituting the resulting expression for $\alpha(X)$ into the free-rider condition yields

$$\frac{X+z}{\overline{X}+z}\overline{X} \geq X + \left[\frac{\overline{X}}{\overline{X}+z}\int_X^{\overline{X}} \frac{1}{\Phi(u)}du\right]\Phi(X).$$

Collecting the terms with X on the left-hand side yields

$$\left(\frac{\overline{X}}{\overline{X}+z} - 1\right)X \geq \left[\frac{\overline{X}}{\overline{X}+z}\int_X^{\overline{X}} \frac{1}{\Phi(u)}du\right]\Phi(X) - \frac{\overline{X}}{\overline{X}/z+1}.$$

As $z \to \infty$, the left-hand side converges to $-X$ whereas the right-hand side converges to $-\overline{X}$. Consequently, there exists some $\bar{z}$ such that, for all $z > \bar{z}$, both the free-rider condition and the incentive compatibility constraints are satisfied.

Cut-off type. The condition (3) puts a constraint on how equilibrium profits vary across types in equilibrium. By the envelope theorem, and using $K(X;X) = X + k[1 - \alpha(X)]\Phi(X)$, we have that equilibrium profits must be increasing at the rate

$$\partial\Pi(X;X)/\partial X = [\alpha(X) + k[1 - \alpha(X)]r(X)]\Phi'(X) + r(X)$$

for *any* schedule that satisfies (3).

Given an equilibrium exists, the cut-off type X_S^c is given by

$$\int_{X_S^c}^{\overline{X}} \left\{[\alpha(u) + k[1 - \alpha(u)]r(u)]\Phi'(u) + r(u)\right\} du = \Phi(\overline{X}). \tag{14}$$

Under the proposed equilibrium schedule, bidder types below X_S^c incur a loss under the proposed schedule. Hence, they prefer not making a bid over making the bid prescribed by the proposed schedule. The option of not making a bid does not undermine the non-mimicking constraints. Under the proposed schedule, the bidder prefers a loss-making offer to offers made by higher-valued types. A fortiori, she also prefers a zero-profit offer over the latter.

Out-of-equilibrium beliefs. The proposed schedule can be supported as a separating equilibrium under the out-of-equilibrium beliefs that any deviation comes from the highest bidder type, $\overline{X}$. Under these beliefs, the target shareholders do not tender their shares in response to a deviation bid $(\tilde{r}, \tilde{\alpha}, \tilde{P})$ unless $\tilde{P} \geq \overline{X}$. Any

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such bid, however, is weakly dominated by $(0.5, 1, \overline{X})$, which is the equilibrium bid of the highest bidder type. Since $(0.5, 1, \overline{X})$ is mimicking-proof, any successful deviation bid is—by implication—unattractive under the proposed out-of-equilibrium beliefs.

Bid restriction. The above separating schedule with $r(\cdot) = 0.5$ implies that the bid restriction is a redundant signal. By contrast, the private benefit retention rate α and the price P are indispensable as signals. First, if α is invariant across types, Proposition 1 applies. Second, a uniform price in a separating equilibrium must satisfy $P = \overline{X}$. But then all bidder types $X < \overline{X}$ prefer the offer $(0.5, 1, \overline{X})$, which always succeeds irrespective of shareholder beliefs, to any other offer with $P = \overline{X}$. Hence, they would pool. ∎

Diversion and controlling stake

Here, we assume that the bidder's cannot contractually commit not to extract the maximum private benefits ex post, but that her post-takeover controlling stake affects either her ability or her willingness to extract private benefits.

Case 1 (Entrenchment). Suppose the bidder's *ability* to extract private benefits increases with the size of her controlling stake such that the maximum fraction $\bar{\phi}$ of total value extracted as private benefits *increases* in r. The bidder's payoff function is then

$$\bar{\phi}(r)V + r[(1 - \bar{\phi}(r))V - P]$$

where $\bar{\phi}'(r) > 0$.

By defining $\Phi(V) \equiv \bar{\phi}(1)V$, $\alpha(r) \equiv \bar{\phi}(r)V / \Phi(V)$, and $X \equiv V - \Phi(V)$, the objective function can be written

$$\alpha(r)\Phi(V) + r\{X + [1 - \alpha(r)]\Phi(V) - P\}$$

and the free-rider condition as $P \geq X + [1 - \alpha(r)]\Phi(V)$. This tender offer game is thus isomorphic to our generic framework with $k = 1$ and the additional constraint $\alpha(r) = r$. There exist $\bar{\phi}(\cdot)$ such that a separating equilibrium can be constructed under this additional constraint. One example is $\alpha = r$, which implies $\bar{\phi}(r) = r\bar{\phi}(1)$ where $\bar{\phi}(1)$ can be any constant in $(0, 1]$. We show in the context of the merger game discussed in Section 3.1.4 that this additional constraint, $\alpha = r$, can be satisfied.

Case 2 (Alignment). Suppose private benefit extraction is costly such that the bidder's *incentives* to extract private benefits decreases with the size of her controlling stake (Burkart, Gromb and Panunzi, 1998). Specifically, suppose the bidder incurs a cost of $c(\phi)$ when she extracts a fraction ϕ of the total value as private benefits, where $c'(\cdot) > 0$ and $c''(\cdot) > 0$. We proceed by backward induction. Once in control with an ownership stake $r \geq 0.5$, the bidder chooses ϕ to maximize $\phi V + r(1 - \phi)V - c(\phi)$, which is concave in ϕ due to $c''(\cdot) > 0$. The optimal extraction rate $\phi^* = \phi^*(r, V)$ solves the first-order condition $c'(\phi^*) = (1 - r)V$.

At the time of the tender offer, the bidder hence solves the problem

$$\max_{r,P} \phi^*(r,V)V + r[(1-\phi^*(r,V))V - P] - c(\phi^*(r,V)),$$

which can be written as

$$\max_{\hat{V}} \phi^*(r(\hat{V}),V)V + r(\hat{V})\{[1-\phi^*(r(\hat{V}),V)]V - P(\hat{V})\} - c(\phi^*(r(\hat{V}),V)).$$

The first-order derivative of this objective function is

$$\frac{\partial \phi^*(r(\hat{V}),V)}{\partial r}r'(\hat{V})V + r'(\hat{V})V - \phi^*(r(\hat{V}),V)r'(\hat{V})V - r(\hat{V})\frac{\partial \phi^*(r(\hat{V}),V)}{\partial r}r'(\hat{V})V$$

$$- r'(\hat{V})P(\hat{V}) - r(\hat{V})P'(\hat{V}) - [1-r(\hat{V})]\frac{\partial \phi^*(r(\hat{V}),V)}{\partial r}r'(\hat{V})V \quad (15)$$

where we use the first-order condition $c'(\phi^*(r(\hat{V}),V)) = [1-r(\hat{V})]V$ from the bidder's private benefit extraction problem. This can be simplified to

$$[1-\phi^*(r(\hat{V}),V)]r'(\hat{V})V - r'(\hat{V})P(\hat{V}) - r(\hat{V})P'(\hat{V}).$$

We now show that any $r(\cdot)$-schedule that satisfies the first-order condition renders the bidder's optimization quasi-convex, and hence violates the sufficient condition for a maximum.

First-order condition. In a separating equilibrium, (15) equals zero for $\hat{V} = V$ for all V, which can then be written

$$0 = r'(\hat{V})\left\{P(\hat{V}) - [1-\phi^*(r(\hat{V}),\hat{V})]\hat{V}\right\} + r(\hat{V})P'(\hat{V}). \quad (16)$$

In a separating equilibrium, lower-valued types reveal their type so as to pay a lower price; that is, $P'(\hat{V}) > 0$. Further, by the free-rider condition, $P(\hat{V}) - [1-\phi^*(r(\hat{V}),\hat{V})]\hat{V} \geq 0$. Thus, a necessary condition for (16) to be satisfied is that $r'(\hat{V}) < 0$.

Quasi-convexity. Substituting for $r'(\hat{V})P(\hat{V}) + r(\hat{V})P'(\hat{V})$ in (15) by using (16) gives the first-order derivative

$$\left[1-\phi^*(r(\hat{V}),V)\right]r'(\hat{V})\left(V - \hat{V}\right) \quad (17)$$

For $r'(\hat{V}) < 0$, this derivative is non-positive for all $\hat{V} < V$ and non-negative for all $\hat{V} > V$. This means that, for any $r(\cdot)$ such that (16) can be satisfied, the bidder's objective function is quasi-*convex*, and the solution to (16) identifies a global *minimum*. Thus, there exists no separating offer schedule in this case. We conclude with two remarks. First, Burkart, Gromb and Panunzi (1998) choose a cost function of the form $c(\phi) = \ell(\phi)V$, which the above analysis encompasses. Second, unlike in the other tender offer games studied in this paper, here the offer schedule affects the total takeover surplus (not just the division of surplus). This makes it more attractive for high types to profit from mimicking low types as opposed to extracting (more)

private benefits, which undermines separation.

Mergers and probabilistic outcomes

As argued in the text, we can map the merger model into the generic framework with $k = 0$ and the additional constraint $r = \gamma\alpha$. For the sake of completeness, we show that a separating equilibrium can be constructed under this additional constraint.

The first-order condition (3) with $k = 0$ and $r = \gamma\alpha$ is

$$\gamma\alpha(\hat{X})P'(\hat{X}) + \gamma\alpha'(\hat{X})P(\hat{X}) = \alpha'(\hat{X})\Phi(X) + \gamma\alpha'(\hat{X})X$$

$$P'(\hat{X}) = \frac{\alpha'(\hat{X})}{\alpha(\hat{X})}\left[\frac{1}{\gamma}\Phi(X) + X - P(\hat{X})\right].$$

In a separating equilibrium, this condition holds for $\hat{X} = X$. Further choosing the price schedule $P = X$ to satisfy the free-rider condition and inserting it into the first-order condition yields a differential equation for $\alpha(\cdot)$:

$$\frac{\gamma}{\Phi(X)} = \frac{\alpha'(X)}{\alpha(X)}.$$

Integrating on both sides yields

$$\int_X^{\overline{X}} \frac{\gamma}{\Phi(u)}du = \ln\alpha(\overline{X}) - \ln\alpha(X)$$

$$\alpha(X) = \exp\left[-\int_X^{\overline{X}} \frac{\gamma}{\Phi(u)}du\right]$$

which is increasing in X.

Proof of Proposition 3

By definition, every $X \in \mathcal{X}_{(r_P,\alpha_P,P_P)}$ satisfies the participation constraint (4). Define the posterior belief function $g(r_P,\alpha_P,P_P) \equiv E[K(\alpha_P,X)|X \in \mathcal{X}_{(r_P,\alpha_P,P_P)}]$. Note that $g(r_P,\alpha_P,0) = E[K(\alpha_P,X)] > 0$, which means that the free-rider condition is violated for $P_P = 0$. In contrast, $g(r_P,\alpha_P,K(\alpha_P,\overline{X})) = E[K(\alpha_P,X)|K(\alpha_P,X) \geq K(\alpha_P,\overline{X}) - \alpha_P\Phi(X)/r_P)] \leq K(\alpha_P,\overline{X})$, which means that the free-rider condition is satisfied for $P_P = K(\alpha_P,\overline{X})$. Hence, for any r_P and α_P, there exists a set of prices $\mathcal{P}(r_P,\alpha_P) \subset (0,K(\alpha_P,\overline{X})]$ which satisfy the free-rider condition.

In addition to satisfying the free-rider condition, a pooling equilibrium offer (r_P,α_P,P_P) must yield at least as high (positive) profits as any alternative bid (r',α',P'). Any bid (r',α',P') with $P' < K(\alpha',\overline{X})$ can be made to fail by choosing shareholders' off-equilibrium beliefs $E[K(\alpha',X)|(r',\alpha',P')] = K(\alpha',\overline{X})$, and can hence be ruled out as a profitable deviation. All bids (r',α',P') with $P' \geq K(\alpha',\overline{X})$ always succeed,

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irrespective of shareholders' off-equilibrium beliefs. Among these offers, the least costly, and hence most profitable, one is $(0.5, 1, K(1, \overline{X}))$. Hence, only offers that belong to some $\mathscr{P}(r_P, \alpha_P)$ and also satisfy the condition

$$\max\{\alpha_P \Phi(X) + r_P[K(\alpha_P, X) - P_P], 0\} \geq$$
$$\Phi(X) + 0.5[K(1, X) - K(1, \overline{X})] = \Phi(X) + 0.5(X - \overline{X}) \quad (18)$$

for all $X \in \mathscr{X}$ can be supported as Perfect Bayesian Equilibria. Such offers do exist: Consider some price $\hat{P} \in \mathscr{P}(0.5, 1)$. We know that $\hat{P} \leq K(1, \overline{X}) = \overline{X}$. Thus, using $(r_P, \alpha_P, P_P) = (0.5, 1, \hat{P})$ in (18) yields

$$\max\{\Phi(X) + 0.5(X - \hat{P}), 0\} \geq \Phi(X) + 0.5(X - \overline{X}),$$

which holds. ∎

Partially separating equilibria with multiple pooling prices

Consider the discontinuous private benefit function

$$\Phi(X) = \begin{cases} \underline{\Phi} & \text{for } X \in [0, A) \\ \overline{\Phi} & \text{for } X \in [A, \overline{X}] \end{cases}$$

with $\overline{\Phi} > \underline{\Phi} > 0$. We want to construct an equilibrium in which all types in $[X_1, A) \subseteq [0, A)$ offer a uniform contract $(\underline{r}, \underline{\alpha}, \underline{P})$, and all types in $[A, \overline{X}]$ offer a uniform contract $(\overline{r}, \overline{\alpha}, \overline{P})$. The two pooling offers $(\underline{r}, \underline{\alpha}, \underline{P})$ and $(\overline{r}, \overline{\alpha}, \overline{P})$ must satisfy the following incentive compatibility, participation, and free-rider constraints:

incentive compatibility **constraints**. In $[0, A)$, higher types have a stronger incentive to deviate to $(\overline{r}, \overline{\alpha}, \overline{P})$. Similarly, of all types in $[A, \overline{X}]$, type A has the strongest incentive to deviate to $(\underline{r}, \underline{\alpha}, \underline{P})$. Hence, if $(\underline{r}, \underline{\alpha}, \underline{P})$ and $(\overline{r}, \overline{\alpha}, \overline{P})$ satisfy the constraints

$$\underline{\alpha}\underline{\Phi} + \underline{r}(A - \underline{P}) \geq \overline{\alpha}\underline{\Phi} + \overline{r}(A - \overline{P})$$
$$\underline{\alpha}\overline{\Phi} + \underline{r}(A - \underline{P}) \leq \overline{\alpha}\overline{\Phi} + \overline{r}(A - \overline{P}) \quad ,$$

or equivalently,

$$\underline{r}(A - \underline{P}) - \overline{r}(A - \overline{P}) \geq (\overline{\alpha} - \underline{\alpha})\underline{\Phi}$$
$$\underline{r}(A - \underline{P}) - \overline{r}(A - \overline{P}) \leq (\overline{\alpha} - \underline{\alpha})\overline{\Phi} \quad , \quad \text{(IC)}$$

they satisfy the non-mimicking constraints of all types in $[0, A)$ and $[A, \overline{X}]$.

Participation constraints. Of all types in $[X_1, A)$, type 0 gets the smallest payoff from $(\underline{r}, \underline{\alpha}, \underline{P})$. Similarly, of all types in the $[A, \overline{X}]$, type A gets the smallest payoff from $(\overline{r}, \overline{\alpha}, \overline{P})$. Hence, if $(\underline{r}, \underline{\alpha}, \underline{P})$ and $(\overline{r}, \overline{\alpha}, \overline{P})$,

respectively, satisfy the participation constraints

$$\underline{\alpha}\Phi + \underline{r}(X_1 - \underline{P}) = 0$$
$$\overline{\alpha}\overline{\Phi} + \overline{r}(A - \overline{P}) \geq 0 \quad , \tag{PC}$$

they satisfy the participation constraints of all types in $[X_1, A)$ and $[A, \overline{X}]$. Note that A's participation constraint is implied by her incentive compatibility constraint. Note also that, if X_1's participation constraint binds, all types below X_1 prefer not to make a bid.

Free-rider conditions. Assuming $k = 0$ for this example, the free-rider conditions are

$$\underline{P} \geq E[X \mid X \in [X_1, A)]$$
$$\overline{P} \geq E[X \mid X \in [A, \overline{X}]] \quad . \tag{FR}$$

Equilibrium values. Together, (IC), (PC), and (FR) comprise a system of five equations with six unknowns. It is straightforward to construct an example: First, choose admissible values for $(\underline{r}, \underline{\alpha}, \underline{P})$ and $(\overline{r}, \overline{\alpha}, \overline{P})$ with $\underline{P} \in [0, A)$ and $\overline{P} \in [A, \overline{X}]$. to satisfy (IC), which is possible given that $\overline{\Phi} > \underline{\Phi} > 0$. Next, choose a type X_1 to satisfy (PC). Finally, choose a distribution of types within each interval such that (FR) is satisfied.

By means of illustration, here is a numeric example that satisfies all constraints: $\overline{X} = 9$, $A = 5$, $\overline{\Phi} = 6$, $\underline{\Phi} = 2$, $(\underline{r}, \underline{\alpha}, \underline{P}) = (0.5, 0.5, 3)$, $(\overline{r}, \overline{\alpha}, \overline{P}) = (1, 1, 7)$, $X_1 = 1$, and types are uniformly distributed on $[1, 5)$ and on $[5, 9]$.

Comparison to equilibrium with single pooling offer. Whether the above equilibrium is more or less efficient than equilibria with a single pooling offer depends on the type distribution G. Define the conditional distributions $\underline{G}(x) = \Pr[X \leq x \mid X \in [X_1, A)]$ and $\overline{G}(x) = \Pr[X \leq x \mid X \in [A, \overline{X}]]$. By means of illustration, consider shifts in probability mass between $[A, \overline{X}]$ and $[0, A)$ that affect the overall distribution G but do not change the conditional distributions $\underline{G}$ and $\overline{G}$. Such a shift changes $E(X)$, but neither $E[X \mid X \in [X_1, A)]$ nor $E[X \mid X \in [A, \overline{X}]]$. If so, the above equilibrium continues to be supported by the same contracts $(\underline{r}, \underline{\alpha}, \underline{P})$ and $(\overline{r}, \overline{\alpha}, \overline{P})$, since they still satisfy (IC), (PC), and (FR). But as $E(X)$ changes, equilibria with a single pooling price can become more or less efficient. In our numeric example, consider changes in the overall distribution that vary $E(X)$ but leave the distributions within $[1, 5)$ and $[5, 9]$ uniform. For any such shift, the above equilibrium continues to exist. In an equilibrium with the single pooling offer $(r_P, \alpha_P, P_P) = (0.5, 1, E(X))$, the cut-off type is determined by the participation constraint

$$\alpha_P \underline{\Phi} + r_P [X_P^c - E(X)] \geq 0,$$

which yields $X_P^c = \max\{0, 4 - E(X)\}$. Depending on $E(X)$, this equilibrium can be less or more efficient than the equilibrium with two pooling offers, since X_P^c can be larger or smaller than $X_1 = 1$.

Proof of Proposition 5

In any pooling equilibrium, there exists a subset $\mathscr{X}_{(r_P,\alpha_P,P_P)}$ of types that are active and make the same offer (r_P,α_P,P_P), which satisfies the free-rider condition $P_P \geq \mathrm{E}[K(\alpha_P,X)|X \in \mathscr{X}_{(r_P,\alpha_P,P_P)}]$. Denote the lowest type in that subset by $\underline{X}^\alpha \equiv \min \mathscr{X}_{(r_P,\alpha_P,P_P)}$. Clearly, $P_P > K(\alpha_P,\underline{X}) \geq \underline{X}$.

Consider the offer (r_P,α^d,P^d), with $\alpha^d = \alpha_P - \delta$. A given type X prefers this offer over the pooling offer if and only if

$$\alpha^d \Phi(X) + r_P\left[K(\alpha^d,X) - P^d\right] > \alpha_P \Phi(X) + r_P\left[K(\alpha_P,X) - P_P\right]$$

$$r_P P_P - r_P P^d > \left(\alpha_P - \alpha^d\right)\Phi(X) + r_P\left[K(\alpha_P,X) - K(\alpha^d,X)\right].$$

For $k = 0$,

$$r_P P_P - r_P P^d > \delta\Phi(X).$$

For $k = 1$,

$$r_P P_P - r_P P^d > (1 - r_P)\,\delta\Phi(X).$$

The right-hand side of the inequality is increasing in X for $\Phi'(\cdot) > 0$. Thus, if the inequality binds for some type X', then it holds for all and only types (weakly) lower than X'. Hence, we can choose P^d such that the inequality holds for all and only types (weakly) lower than $\underline{X}^\alpha$. For very small δ, this requires a very small decrease in P^d such that $P^d \geq K(\alpha^d,\underline{X}^\alpha)$. Under the intuitive criterion, the target shareholders assign the deviation offer to types $X \leq \underline{X}^\alpha$. Given $P^d \geq K(\alpha^d,\underline{X}^\alpha)$, the shareholders therefore never reject the deviation offer. This argument is applicable so long as $\alpha_P > 0$. Thus, any equilibrium with two types making the same offer does not survive the intuitive criterion. That is, the only pooling equilibrium that survives is the degenerate outcome in which only the highest bidder type makes a bid with $\alpha_P = 0$ and $P_P = K(0,\overline{X})$.

If $\Phi'(X) = 0$ over some interval of X, lower types cannot use deviations to a lower α to separate themselves. An alternative is to deviate to a lower bid restriction r or to an offer that has a smaller success probability q. While the former option is exhausted at $r_P = 0.5$, reducing the success probability is always available except in the degenerate case of $q_P = 0$. Consider a probabilistic pooling equilibrium (r_P,q_P,P_P). Denote the lowest type that makes this offer by $\underline{X}^q \equiv \min \mathscr{X}_{(r_P,q_P,P_P)} < P_P$. A deviation offer (r_P,q^d,P^d) with $q^d = q_P - \delta$ is preferred by type X if and only if

$$q^d\left[\Phi(X) + r_P(X - P^d)\right] > q_P\left[\Phi(X) + r_P(X - P_P)\right]$$

$$q_P r_P P_P - q^d r_P P^d > \left(q_P - q^d\right)\left[\Phi(X) + r_P X\right].$$

The right-hand side of the inequality is increasing in X. Thus, if the inequality binds for some type X', then it holds for all and only types (weakly) lower than X'. Hence, we can choose P^d such that the inequality

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holds for all and only types (weakly) lower than $\underline{X}^q$. For very small δ, this requires a very small decrease in P^d such that $P^d \geq \underline{X}^q$. Under the intuitive criterion, the target shareholders assign the deviation offer to types $X \leq \underline{X}^q$, and since $P^d \geq \underline{X}^q$, the shareholders accept the deviation offer. ∎

Proof of Proposition 6

For expositional convenience, we characterize the tender offer terms by the triple (r, α, M) where $M \equiv rP$. Consider the type $(\overline{X}, \underline{\Phi})$ and an arbitrary type $(X, \Phi) \neq (\overline{X}, \underline{\Phi})$. In any fully revealing equilibrium, type $(\overline{X}, \underline{\Phi})$ cannot be held to a profit lower than $\underline{\Phi}$ because she can always succeed with the bid $(r, 1, r\overline{X})$. At the same time, she cannot earn more than $\underline{\Phi}$ because of the free-rider condition. In order for type $(\overline{X}, \underline{\Phi})$ not to mimic type (X, Φ), the latter type must make an offer (r, α, M) which satisfies $\underline{\Phi} \geq r\overline{X} + \alpha\underline{\Phi} - M$, or equivalently

$$M \geq \underline{M} \equiv r\overline{X} - (1 - \alpha)\underline{\Phi}. \tag{19}$$

In addition, a truthful offer by (X, Φ) must also yield a higher profit than the "out-of-equilibrium" offer $(0.5, 1, 0.5\overline{X})$ which succeeds irrespective of target shareholder beliefs. That is, her offer (r, α, M) must satisfy $rX + \alpha\Phi - M \geq 0.5(X - \overline{X}) + \Phi$, or equivalently

$$M \leq \overline{M} \equiv (r - 0.5)X + 0.5\overline{X} - (1 - \alpha)\Phi. \tag{20}$$

The constraints (19) and (20) can be simultaneously satisfied if $\overline{M} \geq \underline{M}$ holds. Straightforward manipulations yield $(r - 0.5)(X - \overline{X}) \geq (1 - \alpha)(\Phi - \underline{\Phi})$. The left-hand side is non-positive, whereas the right-hand side is non-negative. In fact, this condition is violated unless all types with $X < \overline{X}$ offer $r = 0.5$ and $\alpha = 1$, in which case the condition holds with equality. Furthermore, since types that choose the same r and α cannot be separated on the basis of the cash price, satisfying the condition requires that all types with $X < \overline{X}$ make the same offer $(0.5, 1, M)$. ∎

Proof of Corollary 2

We reenter the proof of Proposition 2 right after the part that establishes quasi-concavity.

Local optimality. Instead of setting $r(\cdot) = 0.5$, we consider an arbitrary but constant bid restriction $r(\cdot) = r$. Using $K(\hat{X}; X) = X + k[1 - \alpha(\hat{X})]\Phi(X)$, the first-order condition (3) for $\hat{X} = X$ simplifies to

$$
\begin{aligned}
rP'(\hat{X}) &= \alpha'(\hat{X})\Phi(X) - r[k\alpha'(\hat{X})\Phi(\hat{X})] \\
\alpha'(\hat{X}) &= \frac{P'(\hat{X})}{(1/r - k)\Phi(\hat{X})}
\end{aligned}
$$

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Integrating on both sides over $[X, \overline{X}]$ yields

$$\int_X^{\overline{X}} \alpha'(u) du = \int_X^{\overline{X}} \frac{P'(u)}{(1/r - k)\Phi(u)} du \quad \Leftrightarrow \quad \alpha(\overline{X}) - \alpha(X) = \int_X^{\overline{X}} \frac{P'(u)}{(1/r - k)\Phi(u)} du.$$

As the highest-valued type does not have to relinquish any private benefits $[\alpha(\overline{X}) = 1]$,

$$\alpha(X) = 1 - \int_X^{\overline{X}} \frac{P'(u)}{(1/r - k)\Phi(u)} du. \tag{21}$$

Taking the derivative with respect to r yields

$$\frac{\partial \alpha(X)}{\partial r} = -\frac{1}{(1 - rk)^2} \int_X^{\overline{X}} \frac{P'(u)}{\Phi(u)} du < 0$$

for all X since (as can be easily shown) $P'(\cdot) \geq 0$. That is, the fraction of private benefits the bidder must relinquish in a separating equilibrium is strictly higher for every type, the larger the fraction of shares the bidder must buy.

It is straightforward to show that the next part in the proof of Proposition 2 on the free-rider condition applies, with the appropriate minor modifications, analogously to the case of $r(\cdot) = r$. For brevity, we skip to the part that analyzes the cut-off type.

Cut-off type. By the envelope theorem, and using $K(X; X) = X + k[1 - \alpha(X)]\Phi(X)$, we have that equilibrium profits must be increasing at the rate

$$\partial \Pi(X; X)/\partial X = \begin{cases} [\alpha(X) + r]\Phi'(X) + r & \text{for } k = 0 \\ [\alpha(X) - r\alpha(X) + r]\Phi'(X) + r & \text{for } k = 1 \end{cases}$$

for any schedule that satisfies (3).

For $k = 0$, we immediately see that the slope $\partial \Pi(X; X)/\partial X$ is increasing in r for all X. With the highest type's profit fixed at $\Phi(\overline{X})$, this implies that the cut-off type is higher when r is larger.

For $k = 1$, substituting in (21), we get

$$\begin{aligned} \frac{\partial^2 \Pi(X; X)}{\partial X \partial r} &= \frac{\partial}{\partial r} \left\{ [\alpha(X) - r\alpha(X) + r]\Phi'(X) + r \right\} \\ &= \frac{\partial}{\partial r} \left\{ \left[1 - r \int_X^{\overline{X}} \frac{P'(u)}{\Phi(u)} du \right] \Phi'(X) + r \right\} \\ &= 1 - \Phi'(X) \int_X^{\overline{X}} \frac{P'(u)}{\Phi(u)} du. \end{aligned} \tag{22}$$

To make the same argument as for $k = 0$, we want to show that (or when) this is positive. Compare (22) to

(21) for $k = 1$,

$$\alpha(X) = 1 - \frac{r}{1-r} \int_X^{\overline{X}} \frac{P'(u)}{\Phi(u)} du,$$

which is positive for all types that make a bid in equilibrium. This implies that (22) is positive for all X if

$$\Phi'(X) < \frac{r}{1-r}.$$

For $r \in [0.5, 1]$, $\frac{r}{1-r} \in [1, \infty]$, in which case the last inequality is implied by $\Phi'(X) < 1$.

Stochastic security benefits and signaling with derivatives

Let $X \in [0, +\infty)$ be a random variable. Suppose that there are n bidder types $\theta \in \Theta \equiv \{1, 2, \ldots, n\}$, each knowing the probability density function $f_\theta(X)$ of her post-takeover cash flows. In addition, assume that the family of densities $\{f_\theta(X)\}_{\theta \in \Theta}$ satisfies the strong monotone likelihood ratio property (SMLRP). That is, for all $\theta' > \theta$, $f_{\theta'}(X)/f_\theta(X)$ is strictly increasing.

To construct a separating equilibrium, we allow the bidder to pay cash and issue bonds and a cash-settled "knock-in" call option. This is a latent call option with an exercise price of S that becomes activated only once the security benefits X exceeds some "trigger" level $T > S$. To simplify the exposition, we further assume that $\Phi = 0$.

Proposition 10. *In the tender offer game with stochastic post-takeover security benefits, a separating equilibrium exists if SMLRP holds. All bidder types make a bid and purchase the target shares for a combination of cash, bonds and knock-in call options.*

The equilibrium offer in Proposition 10 provides target shareholders with both "upside participation" and "downside protection". The bidder primarily wants to signal a low value. To this end, she issues knock-in options that transfer some high cash flow realizations to the target shareholders. However, the use of knock-in options makes the bidder prone to being mimicked by (even) lower-valued types. To remove doubts about the value of the offered options, she must include bonds to separate herself from lower-valued types. Thus, the bidder's need to separate herself from lower-valued types through offering downside protection *derives endogenously* from the bidder's *primary* intention to distinguish herself from higher-valued types by offering upside participation.

Proof. We first establish the following two auxiliary results.

Lemma 2. *For all $\theta' > \theta$, there exists a unique $X_\theta(\theta') \in (0, \infty)$ s.t.*

$$f_\theta(X) \begin{cases} > f_{\theta'}(X) & \text{for all } X < X_\theta(\theta') \\ < f_{\theta'}(X) & \text{for all } X > X_\theta(\theta') \end{cases}.$$

Proof. By SMLRP, for all $\theta' > \theta$, there is a unique $X_\theta(\theta') \in (0, \infty)$ s.t.

$$f_{\theta'}(X)/f_\theta(X) \begin{cases} < 1 & \text{for } X < X_\theta(\theta') \\ = 1 & \text{for } X \in \mathscr{X}(\theta, \theta') \\ > 1 & \text{for } X > X_\theta(\theta') \end{cases}.$$

Otherwise, if $f_{\theta'}(X)/f_\theta(X)$ is either always larger or always smaller than 1, it cannot be that $F_\theta(\infty) = F_{\theta'}(\infty)$. This implies the result. $\square$

Lemma 3. *For all* $\theta'' > \theta' > \theta$, $X_{\theta'}(\theta'') \geq X_\theta(\theta')$.

Proof. Suppose to the contrary that

$$X_{\theta'}(\theta'') < X_\theta(\theta'). \tag{$\bigstar$}$$

By Lemma 2, it then follows that

(a) For $X \in (0, X_{\theta'}(\theta''))$: $\quad \frac{f_{\theta''}(X)}{f_{\theta'}(X)} < 1$ and $\frac{f_{\theta'}(X)}{f_\theta(X)} < 1 \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} < 1$

(b) For $X = X_{\theta'}(\theta'')$: $\quad \frac{f_{\theta''}(X)}{f_{\theta'}(X)} = 1$ and $\frac{f_{\theta'}(X)}{f_\theta(X)} < 1 \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} < 1$

(c) For $X \in (X_{\theta'}(\theta''), X_\theta(\theta'))$: $\quad \frac{f_{\theta''}(X)}{f_{\theta'}(X)} > 1$ and $\frac{f_{\theta'}(X)}{f_\theta(X)} < 1 \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} \gtrless 1$

(d) For $X = X_\theta(\theta')$: $\quad \frac{f_{\theta''}(X)}{f_{\theta'}(X)} > 1$ and $\frac{f_{\theta'}(X)}{f_\theta(X)} = 1 \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} > 1$

(e) For $X \in (X_\theta(\theta'), \infty)$: $\quad \frac{f_{\theta''}(X)}{f_{\theta'}(X)} > 1$ and $\frac{f_{\theta'}(X)}{f_\theta(X)} > 1 \Rightarrow \frac{f_{\theta''}(X)}{f_\theta(X)} > 1$

Observe that (i) $f_{\theta''}(X) = f_{\theta'}(X)$ for $X = X_{\theta'}(\theta'')$ and (ii) $f_{\theta'}(X) = f_\theta(X)$ for $X = X_\theta(\theta')$. SMLRP implies that $f_{\theta''}(X)/f_\theta(X) \leq 1$ in case (c), and hence that (iii) $f_{\theta''}(X) = f_\theta(X)$ for $X = X_\theta(\theta')$. Points (ii) and (iii) together imply that (iv) $f_{\theta''}(X) = f_{\theta'}(X)$ for $X = X_\theta(\theta')$. Given that $f_{\theta''}(X) > f_{\theta'}(X)$ in case (c), points (iv) and (i) can only be reconciled with SMLRP if $X_{\theta'}(\theta'') = X_\theta(\theta')$. However, this contradicts inequality ($\bigstar$). $\square$

Main proof. The proof proceeds in two steps. In the first step, we compare adjacent types and analyze local incentive compatibility. In the second step, we show that an offer which is locally mimicking-proof is also globally mimicking-proof.

Local incentive compatibility. Consider a type θ who, for each target share, offers a cash price P_θ, a debt claim with face value D, and a (cash-settled) knock-in call option with exercise price S_θ and trigger level T_θ.

Absent private benefits, a fully efficient equilibrium requires that the bidder's cash price is weakly lower than the expected value of the cash flow rights that she acquires. At the same time, the free-rider condition requires that the cash price is weakly higher than the expected value of the transferred cash flow rights. Both

constraints can only be satisfied simultaneously if they are both binding:

$$P_\theta = \int_D^{T_\theta} (X - D) f_\theta(X) dX + \int_{T_\theta}^\infty (S_\theta - D)^+ f_\theta(X) dX.$$

Consequently, every truthful offer must yield zero bidder profits.

(i) The next higher type $\theta + 1$ does not mimic θ iff

$$-P_\theta + \int_D^{T_\theta} (X - D) f_{\theta+1}(X) dX + \int_{T_\theta}^\infty (S_\theta - D)^+ f_{\theta+1}(X) dX \leq 0.$$

Substituting for P_θ, the inequality can be written as

$$(S_\theta - D)^+ \int_{T_\theta}^\infty [f_{\theta+1}(X) - f_\theta(X)] dX \leq \int_D^{T_\theta} [f_\theta(X) - f_{\theta+1}(X)] (X - D) dX. \tag{23}$$

Set $T_\theta = X_\theta(\theta + 1)$. By Lemma 2, both integrals are then *strictly* positive for any $D < T_\theta = X_\theta(\theta + 1)$, in which case there exists a $S_\theta > 0$ such that (23) is satisfied.

(ii) Analogously, the next lower type $\theta - 1$ does not mimic θ iff

$$(S_\theta - D)^+ \int_{T_\theta}^\infty [f_{\theta-1}(X) - f_\theta(X)] dX \leq \int_D^{T_\theta} [f_\theta(X) - f_{\theta-1}(X)] (X - D) dX. \tag{24}$$

Set $D = X_{\theta-1}(\theta)$. By Lemma 2, the right-hand side is then *strictly* positive. By Lemma 3, $T_\theta = X_\theta(\theta + 1) \geq X_{\theta-1}(\theta)$ so that the left-hand side integral is *strictly* negative. So, (24) holds.

Global incentive compatibility. We now consider in turn types higher than $\theta + 1$ and types lower than $\theta - 1$.

(i) Given $T_\theta = X_\theta(\theta + 1)$ and $D = X_{\theta-1}(\theta)$, consider now the incentive compatibility constraint of an arbitrary type $\theta^+ > \theta + 1$ vis-à-vis type θ:

$$[S_\theta - X_{\theta-1}(\theta)]^+ \int_{X_\theta(\theta+1)}^\infty [f_{\theta^+}(X) - f_\theta(X)] dX \leq$$

$$\int_{X_{\theta-1}(\theta)}^{X_\theta(\theta+1)} [f_\theta(X) - f_{\theta^+}(X)] [X - X_{\theta-1}(\theta)] dX.$$

Defining $\eta(X) \equiv f_{\theta+1}(X) - f_{\theta^+}(X)$, write the inequality as

$$[S_\theta - X_{\theta-1}(\theta)]^+ \int_{X_\theta(\theta+1)}^\infty [f_{\theta+1}(X) - f_\theta(X) - \eta(X)] dX \leq$$

$$\int_{X_{\theta-1}(\theta)}^{X_\theta(\theta+1)} [f_\theta(X) - f_{\theta+1}(X) + \eta(X)] [X - X_{\theta-1}(\theta)] dX. \tag{25}$$

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By Lemma 3, $X_{\theta+1}(\theta^+) \geq X_\theta(\theta+1)$ so that $\eta(X) > 0$ for all $X < X_\theta(\theta+1)$. This implies that the right-hand side of (25) is larger than the right-hand side of (23), and hence *strictly* positive. Turning to the left-hand side, because

$$-\int_{X_\theta(\theta+1)}^\infty \eta(X)dX = \int_0^{X_\theta(\theta+1)} \eta(X)dX - \int_0^\infty \eta(X)dX = \int_0^{X_\theta(\theta+1)} \eta(X)dX > 0,$$

the integral on the left-hand side of (25) is larger than the integral on the left-hand side of (23), and hence *strictly* positive. We conclude that—for $T_\theta = X_\theta(\theta+1)$ and $D = X_{\theta-1}(\theta)$—there exists a *strictly* positive price, $S_\theta > 0$, such that no type $\theta^+ > \theta$ mimics type θ.

(ii) Given $T_\theta = X_\theta(\theta+1)$ and $D = X_{\theta-1}(\theta)$, consider now the incentive compatibility constraint of an arbitrary type $\theta^- < \theta - 1$ vis-à-vis type θ:

$$[S_\theta - X_{\theta-1}(\theta)]^+ \int_{X_\theta(\theta+1)}^\infty [f_{\theta^-}(X) - f_\theta(X)] dX \leq$$

$$\int_{X_{\theta-1}(\theta)}^{X_\theta(\theta+1)} [f_\theta(X) - f_{\theta^-}(X)] [X - X_{\theta-1}(\theta)] dX.$$

Defining $\zeta(X) \equiv f_{\theta-1}(X) - f_{\theta^-}(X)$, write the inequality as

$$[S_\theta - X_{\theta-1}(\theta)]^+ \int_{X_\theta(\theta+1)}^\infty [f_{\theta-1}(X) - f_\theta(X) - \zeta(X)] dX \leq$$

$$\int_{X_{\theta-1}(\theta)}^{X_\theta(\theta+1)} [f_\theta(X) - f_{\theta-1}(X) + \zeta(X)] [X - X_{\theta-1}(\theta)] dX. \quad (26)$$

By Lemma 3, $X_{\theta-1}(\theta) \geq X_{\theta^-}(\theta-1)$ so that $\zeta(X) > 0$ for all $X > X_{\theta-1}(\theta)$. This implies that the right-hand side of (26) is larger than the right-hand side of (24), and hence *strictly* positive. Turning to the left-hand side, again by Lemma 3, $X_\theta(\theta+1) \geq X_{\theta-1}(\theta) \geq X_{\theta^-}(\theta-1)$ so that $\zeta(X) > 0$ for all $X > X_\theta(\theta+1)$. This implies that the left-hand side integral of (26) is smaller than the left-hand side integral of (24), and hence *strictly* negative. So, (26) holds. We conclude that—for $T_\theta = X_\theta(\theta+1)$, $D = X_{\theta-1}(\theta)$, and $S_\theta > 0$—no type $\theta^- < \theta$ mimics type θ. ∎

Proof of Proposition 8

We first derive the optimal bid in a modified setting in which the bidder can commit to an effort level prior to being in control, and then show that the same outcome obtains in a setting without such commitment.

Setting with commitment. Suppose the bidder can choose both $\mathscr{C}$ and e to maximize her payoff

$$\Pi(\mathscr{C}, e) = \beta B(e; \theta) - C(e) + rK(e; \theta) - rP - r\mathscr{S}(e, \theta)$$

subject to non-mimicking constraints and the free-rider condition. For any chosen e, r, and β, her payoff is maximized by P and $\mathscr{S}$ that minimize the overall payment and thus make the free-rider condition bind. A cash price $P = K(e;\theta)$ and a call option with strike price $S = P$ achieve this. Because this offer is also mimicking-proof (Proposition 7), it is accepted by shareholders. This reduces the bidder's payoff to her *net private benefits* $\hat{\Phi}(e;\theta) = \beta B(e;\theta) - C(e)$. The first-order condition

$$\beta \frac{\partial B(e;\theta)}{\partial e} = \frac{\partial C(e)}{\partial e}$$

pins e down as a function of β and θ: $e(\beta,\theta)$. By the envelope theorem,

$$\frac{\partial \hat{\Phi}(\beta, e(\beta,\theta))}{\partial \beta} = B(e(\beta,\theta);\theta) > 0.$$

So, the bidder optimally chooses $\beta^* = 1$ and $e^* = e(1,\theta)$, which is the solution to (7).

Setting without commitment. The original problem differs only in that it has the additional constraint (8). The above outcome hence remains optimal if it is feasible. This is indeed the case because the contract $\mathscr{C}^* = (1, r, K(e^*;\theta), \mathscr{S}^*)$ where $\mathscr{S}^*$ is a call option with strike price $S^* = K(e^*;\theta)$ induces $e^* = e(1,0)$ also in the absence of commitment.

Consider the incentive constraint (8) under $\mathscr{C}^*$. For $e \leq e^*$, the call option is worthless, and the first-order derivative of the objective function under $\mathscr{C}^*$ with respect to e is

$$\frac{\partial B(e;\theta)}{\partial e} - \frac{\partial C(e)}{\partial e} + r \frac{\partial K(e;\theta)}{\partial e}. \tag{27}$$

Given e^* satisfies (7), the difference between the first two terms is positive for $e \leq e^*$, and (27) is hence strictly positive (i.e., the bidder wants to strictly increase e) as long as $e \leq e^*$. For $e \geq e^*$, the call options are worth $r\mathscr{S}(e;\theta) = r[K(e;\theta) - K(e^*;\theta)]$, making (8) identical to (7), yielding e^* as the solution. ∎

Proof of Proposition 9

We first establish sufficient conditions under which a separating equilibrium exists. Afterwards, we discuss the countervailing effects that unbundling has on post-takeover effort. Last, we provide a toehold example.

Preliminaries. In the contracting environment of Section 3 ($\mathscr{S} = 0$), bid restrictions are redundant for signaling. Hence, we can fix $r = \bar{r}$. Denote the bidder's payoff under optimal effort, or value function, as

$$W(\beta, P) \equiv \max_e \beta B(e;\theta) - C(e) + \bar{r}K(e;\theta) - \bar{r}P = \underbrace{\beta B(e(\beta,\theta);\theta) - C(e(\beta,\theta))}_{\mathscr{B}(\beta;\theta)} + \underbrace{\bar{r}K(e(\beta,\theta);\theta)}_{\mathscr{K}(\beta;\theta)} - \bar{r}P$$

where $\mathscr{B}(\beta;\theta)$ and $\mathscr{K}(\beta;\theta)$ are net private benefits and security benefits under optimal effort as a function of β and θ. The optimal effort $e(\beta,\theta)$ is implicitly defined by the first-order condition (8). Since B, X, and

C are twice continuously differentiable, $\mathscr{B}(\beta;\theta)$, and $\mathscr{K}(\beta;\theta)$ are twice continuously differentiable.

We first establish an auxiliary result regarding $\mathscr{K}(\beta;\theta)$.

Lemma 4. *Security benefits under optimal effort $\mathscr{K}(\beta;\theta)$ are increasing in θ.*

Proof. First, we show that

- $B_e, X_e \geq 0$ with at least one strict inequality

- $B_{e\theta}, X_{e\theta} \geq 0$

- $B_{ee}, X_{ee} \leq 0$.

Since, by assumption, $\frac{\partial B}{\partial X} \geq 0$, $\frac{\partial V}{\partial B}$ and $\frac{\partial V}{\partial X}$ must have the same sign. Further, given $X + kB = V$ and hence $\frac{\partial X}{\partial V} + k\frac{\partial B}{\partial V} = 1$, $\frac{\partial B}{\partial V}$ and $\frac{\partial X}{\partial V}$ must both be positive, and at least one strictly so. The same is then true for $B_e = \frac{\partial B}{\partial V}V_e$ and $X_e = \frac{\partial X}{\partial V}V_e$ since $V_e > 0$. Similarly, since $V_{e\theta} \geq 0$, $V_{ee} \leq 0$, $V_\theta > 0$, we have $B_{e\theta} = \frac{\partial B}{\partial V}V_{e\theta} \geq 0$, $X_{e\theta} = \frac{\partial X}{\partial V}V_{e\theta} \geq 0$, $B_{ee} = \frac{\partial B}{\partial V}V_{ee} \leq 0$, $X_{ee} = \frac{\partial X}{\partial V}V_{ee} \leq 0$, $B_\theta = \frac{\partial B}{\partial V}V_\theta > 0$, and $X_\theta = \frac{\partial X}{\partial V}V_\theta > 0$.

Second, writing out K, the bidder's objective function is

$$\beta B(e;\theta) - C(e) + \bar{r}\left[X(e;\theta) + (1-\beta)kB(e;\theta)\right] - \bar{r}P,$$

and the first-order condition with respect to effort is

$$\left[\beta + r(1-\beta)k\right]B_e + rX_e - C_e = 0.$$

The solution e^*, by the implicit function theorem, satisfies the partial derivative

$$\frac{\partial e}{\partial \theta} = -\frac{\left[\beta + r(1-\beta)k\right]B_{e\theta} + rX_{e\theta}}{\left[\beta + r(1-\beta)k\right]B_{ee} + rX_{ee} - C_{ee}} \geq 0.$$

The inequality follows from the first step and $C_{ee} > 0$.

Third, from the definition $\mathscr{K}(\beta;\theta) = X(e(\beta,\theta);\theta) + (1-\beta)kB(e(\beta,\theta);\theta)$, it follows that

$$\frac{\partial \mathscr{K}(\beta;\theta)}{\partial \theta} = X_\theta + X_e\frac{\partial e}{\partial \theta} + (1-\beta)k\left[B_\theta + B_e\frac{\partial e}{\partial \theta}\right]$$

As shown in the first step, all terms on the right-hand side are positive, and B_θ and X_θ are strictly so. □

We now return attention to the contract choice. At the time of the tender offer, the bidder maximizes the value function W with respect to β and P subject to the free-rider condition (and shareholder beliefs). To construct a separating equilibrium, reformulate the bidder's problem as a direct mechanism:

$$\max_{\hat{\theta}} \mathscr{B}(\beta(\hat{\theta});\theta) + \bar{r}\mathscr{K}(\beta(\hat{\theta});\theta) - \bar{r}P(\hat{\theta})$$

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subject to the free-rider condition

$$P(\hat{\theta}) \geq \mathscr{K}(\beta(\hat{\theta}); \hat{\theta}),$$

where $\hat{\theta}\,(\theta)$ is the bidder's reported (true) type. Assuming a price schedule $P(\cdot)$ under which the free-rider condition is binding, the problem can be written as

$$\max_{\hat{\theta}} \mathscr{B}(\beta(\hat{\theta}); \theta) + \bar{r}\mathscr{K}(\beta(\hat{\theta}); \theta) - \bar{r}\mathscr{K}(\beta(\hat{\theta}); \hat{\theta})$$

Note that the last two terms do not cancel.

Local optimality. Full revelation requires a schedule $\beta(\cdot)$ such that the first-order condition

$$\frac{\partial \mathscr{B}(\beta(\hat{\theta}); \theta)}{\partial \beta}\beta'(\hat{\theta}) + \bar{r}\frac{\partial \mathscr{K}(\beta(\hat{\theta}); \theta)}{\partial \beta}\beta'(\hat{\theta}) - \bar{r}\frac{\partial \mathscr{K}(\beta(\hat{\theta}); \hat{\theta})}{\partial \beta}\beta'(\hat{\theta}) - \bar{r}\frac{\partial \mathscr{K}(\beta(\hat{\theta}; \hat{\theta})}{\partial \hat{\theta}} = 0$$

is satisfied at $\hat{\theta} = \theta$ for every θ, which is the case when

$$\frac{\partial \mathscr{B}(\beta(\theta); \theta)}{\partial \beta}\beta'(\theta) = \bar{r}\frac{\partial \mathscr{K}(\beta(\theta); \theta)}{\partial \theta}. \tag{28}$$

Since $\frac{\partial \mathscr{K}(\beta; \theta)}{\partial \theta} > 0$ (Lemma 4), this can only hold if the derivatives on the left-hand side have the same sign. The solution $\beta(\cdot)$ must also satisfy monotonicity and the boundary condition $\beta(\overline{\theta}) = 1$. Hence, $\beta'(\theta) > 0$ and $\frac{\partial \mathscr{B}(\beta(\theta); \theta)}{\partial \beta} > 0$ for all θ. That is, under the separating schedule, higher types retain a larger fraction of their gross private benefits, and every type must on the margin gain if it could retain more of its gross private benefits. Moreover, higher types exert more effort not only because they are more productive (see the second step in the proof of Lemma 4) but also because $r(\theta) = \bar{r}$ and $\beta'(\theta) > 0$. As a result, higher types generate greater baseline security benefits X.[32]

Rewrite the differential equation (28) as

$$\beta' = \bar{r}\frac{\frac{\partial \mathscr{K}(\beta, \theta)}{\partial \theta}}{\frac{\partial \mathscr{B}(\beta, \theta)}{\partial \beta}} \equiv F(\beta, \theta).$$

With $\mathscr{B}(\beta; \theta)$ and $\mathscr{K}(\beta; \theta)$ twice continuously differentiable, and hence $F(\beta, \theta)$ and $F_\beta(\beta, \theta)$ continuous in $(0, 1] \times \Theta$, there exists a unique solution pinned down by the boundary condition $\beta(\overline{\theta}) = 1$ (cf. Sydsaeter et al., 2008, p. 217).[33]

---

[32]Note that the first-order condition does not require baseline net private benefits $\mathscr{B}(\beta; \theta)$ to be increasing in θ for a given β. This is useful because such a condition is not necessarily easy to satisfy, since the basic "overworking" problem created by the interaction between moral hazard and the free-rider problem can be worse for higher types. However, note that at the same time this does not preclude that $\mathscr{B}(\beta; \theta)$ is increasing under the *equilibrium* schedule.

[33]The restriction to $\beta \in (0, 1]$ is inconsequential as $\Phi(\beta, \theta) = 0$ implies $\beta > 0$ for all θ. That is, any bid that confers non-negative profits on the bidder must have $\beta > 0$.

Quasi-concavity. The first-order derivative of the bidder's objective under the direct mechanism is

$$\frac{\partial W}{\partial \hat{\theta}} = \frac{\partial \mathscr{B}(\beta(\hat{\theta});\theta)}{\partial \beta}\beta'(\hat{\theta}) + \bar{r}\frac{\partial \mathscr{K}(\beta(\hat{\theta});\theta)}{\partial \beta}\beta'(\hat{\theta}) - \bar{r}\frac{\partial \mathscr{K}(\beta(\hat{\theta});\hat{\theta})}{\partial \beta}\beta'(\hat{\theta}) - \bar{r}\frac{\partial \mathscr{K}(\beta(\hat{\theta});\hat{\theta})}{\partial \hat{\theta}}.$$

Using (28) to substitute for the last term and rearranging yields

$$\frac{\partial W}{\partial \hat{\theta}} = \beta'(\hat{\theta})\left[\frac{\partial \mathscr{B}(\beta(\hat{\theta});\theta)}{\partial \beta} - \frac{\partial \mathscr{B}(\beta(\hat{\theta});\hat{\theta})}{\partial \beta}\right] + \bar{r}\beta'(\hat{\theta})\left[\frac{\partial \mathscr{K}(\beta(\hat{\theta});\theta)}{\partial \beta} - \frac{\partial \mathscr{K}(\beta(\hat{\theta});\hat{\theta})}{\partial \beta}\right]. \qquad (29)$$

If the private benefits retention rate and productivity are "complements" in inducing the bidder to generate net private benefits and security benefits, i.e., if

$$\frac{\partial^2 \mathscr{B}(\beta;\theta)}{\partial \beta \partial \theta}, \frac{\partial^2 \mathscr{K}(\beta;\theta)}{\partial \beta \partial \theta} \geq 0,$$

then the bracketed terms in (29) are non-negative for $\hat{\theta} < \theta$ and non-positive for $\hat{\theta} > \theta$, thus rendering the bidder's objective function (weakly) quasi-concave in $\hat{\theta}$. Note that these conditions pertain to values created under optimal effort, and (at least) not (directly) to the relationship between effort and productivity in the production technology.

Equilibrium multiplicity. Provided the conditions for quasi-concavity are met, separating equilibria can be supported for multiple $\bar{r}$. Also, there may exist separating equilibria with non-uniform bid restrictions.

Toehold example(s). In the case of toeholds, we have $k = 0$

$$
\begin{aligned}
B(e;\theta) &= \bar{t}V(e;\theta) \\
X(e;\theta) &= V(e;\theta) = K(e;\theta)
\end{aligned}
$$

where $V(e;\theta)$ denotes gross firm value. The incentive constraint (8) without unbundling becomes

$$\beta\bar{t}\frac{\partial V(e;\theta)}{\partial e} + r\frac{\partial V(e;\theta)}{\partial e} = \frac{\partial C(e)}{\partial e},$$

and the incentive constraint (7) with optimal unbundling becomes

$$\bar{t}\frac{\partial V(e;\theta)}{\partial e} = \frac{\partial C(e)}{\partial e}.$$

A comparison between these two constraints immediately reveals that effort must be lower in the second one because $\beta\bar{t} + \bar{r} \geq .5 > \bar{t}$. This is true independent of (further assumptions on) the production technology and regardless of whether the equilibrium is pooling or separating. For examples in which separating equilibria exist, we must put more structure on the production technology.

Example 1: Marginal return to effort increasing in bidder type. Assume

$$
\begin{aligned}
V(e;\theta) &= \theta e + \underline{V}\\
C(e) &= \frac{c}{2}e^2.
\end{aligned}
$$

Fixing $r = \bar{r}$, the bidder maximizes $\beta\bar{t}(\theta e + \underline{V}) - \frac{c}{2}e^2 + \bar{r}(\theta e + \underline{V}) - rP$. Her optimal effort is given by

$$
e(\beta,\theta) = (\beta\bar{t} + \bar{r})\frac{\theta}{c}
$$

so that

$$
\begin{aligned}
\mathscr{B}(\beta;\theta) &= \beta\bar{t}V(e(\beta,\theta);\theta) - C(e(\beta,\theta)) = (\beta\bar{t} - \bar{r})(\beta\bar{t} + \bar{r})\frac{\theta^2}{2c} + \beta\bar{t}\underline{V}\\
\mathscr{K}(\beta;\theta) &= V(e(\beta,\theta);\theta) = (\beta\bar{t} + \bar{r})\frac{\theta^2}{c} + \underline{V}.
\end{aligned}
$$

We assume that $\underline{V}$ is large enough that $\mathscr{B}(1;\overline{\theta}) > 0$ for some $\bar{r} \geq .5$, i.e.,

$$
\underline{V} > -(\bar{t} - \bar{r})(\bar{t} + \bar{r})\frac{\overline{\theta}^2}{2\bar{t}c}.
$$

This guarantees that (at least) the highest type can make a profitable bid under symmetric information, and hence also under a separating equilibrium schedule. The differential equation for local optimality becomes

$$
\beta' = \bar{r}\frac{2(\beta\bar{t} + \bar{r})\frac{\theta}{c}}{\beta\bar{t}\left(2\bar{t}\frac{\theta^2}{2c} + \underline{V}\right)} \equiv F(\beta,\theta),
$$

where $F(\beta,\theta)$ and $F_\beta(\beta,\theta)$ are continuous for $(\beta,\theta) \in (0,1] \times [\underline{\theta},\overline{\theta}]$, and its solution identifies a separating equilibrium schedule since the conditions for quasi-concavity are satisfied,

$$
\begin{aligned}
\frac{\partial^2\mathscr{B}(\beta;\theta)}{\partial\beta\partial\theta} &= 2\beta\bar{t}^2\frac{\theta}{c} \geq 0\\
\frac{\partial^2\mathscr{K}(\beta;\theta)}{\partial\beta\partial\theta} &= 2\bar{t}\frac{\theta}{c} \geq 0,
\end{aligned}
$$

with the cut-off type pinned down by the requisite zero-profit condition.

Example 2: Marginal return to effort constant across bidder types. Assume

$$
\begin{aligned}
V(e;\theta) &= \theta + e\\
C(e) &= \frac{c}{2}e^2.
\end{aligned}
$$

Fixing $r = \bar{r}$, the bidder maximizes $\beta \bar{t}(\theta + e) - \frac{c}{2}e^2 + \bar{r}(\theta + e) - rP$. Her optimal effort is given by

$$e(\beta, \theta) = \frac{\beta \bar{t} + \bar{r}}{c}$$

(which in this example is independent of θ). Thus,

$$\begin{aligned}
\mathscr{B}(\beta; \theta) &= \beta \bar{t} V(e(\beta, \theta); \theta) - C(e(\beta, \theta)) = \beta \bar{t} \theta - \frac{\bar{r}^2 - \beta^2 \bar{t}^2}{2c} \\
\mathscr{K}(\beta; \theta) &= V(e(\beta, \theta); \theta) = \theta + \frac{\beta \bar{t} + \bar{r}}{c}.
\end{aligned}$$

We assume that $\mathscr{B}(1; \overline{\theta}) > 0$ for some $\bar{r} \geq .5$, i.e.,

$$\overline{\theta} > \frac{\bar{r}^2 - \bar{t}^2}{2\bar{t}c}.$$

This guarantees the existence of types that make a positive profit under symmetric information, and hence also under the separating schedule. The differential equation for local optimality becomes

$$\beta' = \frac{\bar{r}}{\bar{t}\theta + \frac{\beta \bar{t}^2}{c}} \equiv F(\beta, \theta),$$

where $F(\beta, \theta)$ and $F_\beta(\beta, \theta)$ are continuous for $(\beta, \theta) \in (0, 1] \times [\underline{\theta}, \overline{\theta}]$, and its solution identifies a separating equilibrium schedule since the conditions for quasi-concavity are satisfied,

$$\begin{aligned}
\frac{\partial^2 \mathscr{B}(\beta; \theta)}{\partial \beta \partial \theta} &= \bar{t} \geq 0 \\
\frac{\partial^2 \mathscr{K}(\beta; \theta)}{\partial \beta \partial \theta} &= 0 \geq 0,
\end{aligned}$$

with the cut-off type pinned down by the requisite zero-profit condition. ∎

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