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INFERENCE AND TESTING BREAKS IN LARGE DYNAMIC PANELS WITH STRONG CROSS SECTIONAL DEPENDENCE

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ABSTRACT. In this paper we provide a new Central Limit Theorem for estimators of the slope papers in large dynamic panel data models (where both n and T increase without bound) in the presence of, possibly, strong cross-sectional dependence. We proceed by providing two related tests for breaks/homogeneity in the time dimension. The first test is based on the *CUSUM* principle; the second test is based on a Hausman-Durbin-Wu approach. Some of the key features of the tests are that they have nontrivial power when the number of individuals, for which the slope parameters may differ, is a "negligible" fraction or when the break happens to be towards the end of the sample, and do not suffer from the incidental parameter problem. We provide a simple bootstrap algorithm to obtain (asymptotic) valid critical values for our statistics. An important feature of the bootstrap is that there is no need to know the underlying model of the cross-sectional dependence. A Monte-Carlo simulation analysis sheds some light on the small sample behaviour of the tests and their bootstrap analogues. We implement our test to some real economic data.

JEL classification: C12, C13, C23

Key words: Large dynamic panel data models. Cross-sectional strong-dependence. Central Limit Theorems. Homogeneity. Bootstrap algorithms.

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1. INTRODUCTION

Nowadays it is widely recognized that economic agents are interrelated due to common factors, contagion, spillovers and so on. This dependence has been systematically neglected until quite recently in econometrics, possibly, due to a lack of a clear framework to characterize such a dependence which is exacerbated by the fact that, contrary to time series data, there is an absence of a clear or natural ordering of the data. In response to this, in the last decade or so, a huge amount of work has been directed to the study of cross-sectional dependence and several approaches or models have been put forward.

One way to model the cross-sectional dependence among individuals is by using a common (unobserved) factor models as in Andrews (2005), Pesaran (2006) or Bai (2009). A second approach is based on the "distance" of individuals located on a regular pattern in the plane, or lattice. It recognizes that data may be collected on a regular lattice as a consequence of planned experiments or a result of a systematic sampling scheme. Applications which use this type of data cover various areas like environmental, urban, agricultural economics as well as economic geography among others. Early examples of this are the celebrated papers by Mercer and Hall (1911) on wheat crop yield data and Batchelor and Reed (1924) on fruit trees, that were further analyzed by Whittle (1954). Other examples are given in Cressie and Huang (1999) and Fernández-Casal et al. (2003). Examples of lattice models in environment economics include Mitchell et al. (2005), who study the effect of CO_2 on crops, and Genton and Koul (2008), who analyze the effect of pollutants transported by winds on the yield of barley in UK.

A third approach to explain or model cross-sectional dependence is through the introduction of measures related to economic and/or geographical distance. This approach was advocated by Conley (1999) and followed by Chen and Conley (2001). The benefit of this approach, similar to lattice models, is that the statistical behaviour is reminiscent of that in standard time series analysis. Another approach that has received a lot of attention is the so-called SAR model, where the dependence is modelled as a linear transformation of "n" (sample size) independent and identically distributed (*iid*) random variables. This approach, considered as a variant of the model considered in Whittle (1954), was advocated in the geographic-economic literature by Cliff and Ord (1973) and it has been extensively employed in the econometric literature, see for instance Lee (2004) and Kelejian and Prucha (2007) among many others. One of the main difference with lattice data is that, contrary to the latter approach, we cannot consider the data/individuals as being collected in a systematic fashion. It is precisely this difference which makes the estimation and study of its properties more difficult and challenging.

In this paper, we characterize the cross-sectional dependence of, say the sequence $\{u_i\}_{i\in\mathbb{N}}$, through a model of the form $u_i = \sum_{j=0}^{\infty} a_j(i)\,\varepsilon_j$, where $\{\varepsilon_j\}_{j\in\mathbb{N}}$ are *iid* random variables. This approach was also considered by Robinson (2011) and Lee and Robinson (2013) and it has a strong resemblance with the well known Wold decomposition for time series sequences. Our motivation for using this approach is that it enables us to generate more general dependence structures than the SAR models can generate, in particular it permits dependence structures with "*strong-dependence*" or "*long-memory*", see our Definition 1 below. With this view, the SAR model can be considered as a particular scenario to the approach followed in this paper as we explain further in Section 2.

Let us introduce what we understand by "*strong-dependence*".

Definition 1. *The generic sequences* $\{\nu_{it}\}_{t\in\mathbb{Z}}$, $i\in\mathbb{N}^+$ *are* "strong-dependence" *if the sequence*

$$\frac{1}{n}\sum_{i,j=1}^{n}|\varphi_\nu(i,j)|$$

is not bounded in n*, where we denote*

$$\varphi_\nu(i,j)=Cov(\nu_{it},\nu_{jt}). \tag{1.1}$$

Our Definition 1 draws a lot of similarities with one of the characterizations often employed to describe "*long-memory*" dependence for a time series sequence $\{q_t\}_{t\in\mathbb{Z}}$. That is, where $\{q_t\}_{t\in\mathbb{Z}}$ exhibits the property of "*long-memory*" if $\frac{1}{T}\sum_{t,s=1}^{T}|Cov(q_t,q_s)|$ is not bounded in T, the sample size. A similar definition for cross-sectional "*weak-dependence*" was used in Sarafidis and Wansbeek (2010). While Chudik, Pesaran and Tosetti (2011) also consider the presence of *strong-* and *weak-dependence* in large panels, they describe the dependence using a factor model, whereas ours is closer related to that given for time series sequences or SAR models. Finally observe that our definition of "*strong-dependence*" does not involve or require any ordering of the observations or the definition of some economic/geographical metric across observations.

This paper is therefore concerned with inference in (linear) dynamic panel data models exhibiting, possibly, strong cross-sectional dependence when both the number of cross-section units and time increase to infinity. Our dynamic panel data model is

$$y_{it}=\alpha_t+\eta_i+\sum_{\ell=1}^{k_1}\rho_{t\ell}y_{i(t-\ell)}+\theta_t'z_{it}+u_{it},\quad i=1,...,n,\qquad t=1,...,T, \tag{1.2}$$

where θ_t is a $k_2\times 1$ vector of unknown parameters, $\{z_{it}\}_{t\in\mathbb{Z}}$ is a vector of exogenous covariates and $\{u_{it}\}_{t\in\mathbb{Z}}$ is the sequence of error terms, $i\in\mathbb{N}^+$. As usual α_t and η_i represent respectively the time and individual fixed effects. We shall assume that the sequences $\{z_{it}\}_{t\in\mathbb{Z}}$, $i\in\mathbb{N}^+$, are mutually independent of the error term $\{u_{it}\}_{t\in\mathbb{Z}}$, $i\in\mathbb{N}^+$, although not necessarily independent from the fixed effects α_t or η_i. More specific conditions on the sequences $\{u_{it}\}_{t\in\mathbb{Z}}$ and exogenous variables $\{z_{it}\}_{t\in\mathbb{Z}}$, $i\in\mathbb{N}^+$, will be given in Conditions $C1$ and $C2$ respectively in Section 2 below.

One of our main interest in the paper is to incorporate this cross-sectional dependence structure to further enhance the already extensive literature on (dynamic) panel data models. With this view, the main objectives in this paper are twofold. The first goal is to discuss and examine the asymptotic properties, and provide a new *Central Limit Theorem*, of estimators of the slope parameters of (1.2) when the cross-sectional dependence of the error sequences $\{u_{it}\}_{t\in\mathbb{Z}}$ and covariates $\{z_{it}\}_{t\in\mathbb{Z}}$, $i\in\mathbb{N}^+$, are (possibly) "*strong-dependent*". In particular, we provide very mild and general conditions to guarantee that the estimators of the parameters of the model are asymptotically normal. Our Central Limit Theorem results extend substantially the work by Kapoora, Kelejian and Prucha (2007), Yu, DeJong and Lee (2008) or Lee and Yu (2010) among others, as we allow for more general cross-sectional dependence structures that permits "*strong-dependence*". However to do so, we need to extend a Central Limit Theorem provided in Phillips and Moon (1999) to allow both for time and cross-sectional dependence. In their work, the sequences of random variables, say $\{\psi_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}$, are assumed to be such that $\{\psi_{it}\}_{t\in\mathbb{Z}}$ and $\{\psi_{jt}\}_{t\in\mathbb{Z}}$ are independent, which is a condition ruled out in our scenario. Unlike Phillips and Moon (1999), see also Hahn and Kürsteiner (2002), we cannot view the sequences as being independent in one of its dimensions. In addition, as we allow for "*strong-dependence*", we cannot use results and arguments based on any type of "*strong-mixing*" arguments, so that results in Jenish and Prucha (2009, 2012) cannot be used in our framework either. On the other hand, similar to what happens with time series regression models, see Robinson and Hidalgo (1997), we do need to restrict the strength of the cross-sectional dependence to guarantee that our estimator of the slope parameters converge in distribution with the standard root-nT rate and, more importantly, that they are asymptotically normal, see also Hidalgo (2003). As the work by Robinson and Hidalgo (1997) suggests, we might, of course, relax the strength of dependence at the expense of further complication in the mathematical apparatus by using some type of "weighted" fixed effect estimator. See our discussion of the conditions in the next section for further details and insights.

Our second main goal in this paper is to examine tests for breaks or homogeneity of the slope parameters in the model (1.2). Although similar models as the one in (1.2) have been considered, their interest has focussed on detecting the presence of heterogeneity across the cross-section units, that is the interest is on whether the slope parameters are the same for all $i \geq 1$. See for instance Pesaran and Smith (1995) or Pesaran and Yamagata (2008) whose framework and ours mainly differ in that our conditions are somehow milder than theirs and we allow for very general type of cross-sectional dependence that may exhibit some type of "*strong-dependence*" behaviour. Specifically, denoting in what follows $\beta_t = \left(\{\rho_{t\ell}\}_{\ell=1}^{k_1}; \theta_t'\right)'$, we are interested in the null hypothesis

$$H_0 : \beta_t = \beta \quad \text{for all } [T\epsilon] \leq t \leq T - [T\epsilon], \tag{1.3}$$

where $0 \leq \varepsilon \leq \frac{1}{2}$, with the alternative hypothesis being the negation of the null.

Alternatively, drawing notation and arguments from the time series literature, since our panel model (1.2) can be written as

$$y_{it} = \eta_i + \alpha_t + \beta' x_{it} + \delta' x_{it} \mathcal{I}(t > t_0) + u_{it}, \quad i = 1, ..., n, \quad t = 1, ..., T,$$

where in what follows we shall abbreviate $\left(\{y_{i,t-\ell}\}_{\ell=1}^{k_1}; z_{it}'\right)'$ by x_{it}, we might write our hypothesis as

$$H_0 : \delta = 0 \quad \text{for all } [T\epsilon] \leq t_0 \leq T - [T\epsilon],$$

where $0 \leq \epsilon \leq 1/2$ against the alternative hypothesis

$$H_1 : \exists \ [T\epsilon] \leq t_0 \leq T - [T\epsilon], \qquad \delta \neq 0.$$

In this respect, we can view our work as an extension of the relatively scarce work of breaks in the context of multivariate equations. See nevertheless the work by Bai, Lumsdaine and Stock (1998) for multivariate models and Bai (2000) on VAR models; see also Qu and Perron (2007). While their framework is for a fixed, and thus finite, n, in this paper we are concerned with a setup which allows "n" to increase with no limit as well. So, we can regard our hypothesis testing as one for structural breaks when the number of sequences, say $i = 1, ..., n$, increases with no limit. Hence we are in a framework of testing for many, possibly thousands, hypotheses simultaneously, see for instance Fan, Hall and Yao (2007). The testing for breaks has also some resemblance to the problem of testing whether a function or curve is constant, with the function of interest being $\beta_t = \beta(t/T)$ and we want to test $H_0 : \beta(\tau) = \beta$ for all $\tau \in [0,1]$. See also the work by Juhl and Xiao (2013) for the latter interpretation of the test.

We now make some general comments about our hypothesis testing. Although we explicitly consider the scenario of abrupt "breaks" when testing our hypothesis in (1.3), our tests also have nontrivial power when the change is gradual, that is when under the (local) alternative the slope parameters β_t move to their new regime as a continuous function in t; see our discussion in Section 3.3 below. A second point to mention is that implicitly we are assuming that the break, if there were any, would be an interior point of a compact subset of $[0,1]$; the introduction of weight functions (or normalizations as in Andrews, 1993) discussed in Section 2 below, effectively guarantees the latter (see also our more explicit comments after Theorem 2 and Corollary 1 below). It might then be of interest to see what would happen with the behaviour of the test when we allow the break to happen towards the end of the sample, namely $T - m_0 \leq t_0$, where m_0 can be a finite positive constant. Recall that in typical situations, we take $\epsilon = .05$ or $.10$, so that we leave 10% or 20% of the data out. However this choice is no more than arbitrary and the power of the test may depend on its choice. The technical aspects of such a case are completely different as one can observe from recent work by Hidalgo and Seo (2013). In fact, for the latter scenario, it is apparent that one would need strong approximation results for an increasing dimensional vector of partial sums of random variables in our setting. Although some preliminary ideas and results might be drawn from the recent work in Chernozhukov et al. (2013), they are unfortunately not immediately useful for the purpose of testing for breaks towards the end of the sample and more importantly their work need to be extended when the assumption of independence is dropped. This situation is beyond the scope of this manuscript. Nevertheless, we do pay particular attention to the type of alternative models that our tests are able to detect and more specifically their behaviour under local alternatives. Scenarios that raise very naturally in our context: (**i**) the consequences when the time of the break is towards the end of the sample, that is the break time k_0 satisfies $k_0 > T - [h_T]$, where $[h_T]$ may satisfy $[h_T] = o(T)$; (**ii**) the consequences when the number of sequences/individuals for which a break exists is negligible when compared to the number of individuals in the sample; and (**iii**) the consequences when the breaks are at different times for different individuals or a combination of all of them. Of course one can imagine a combination of all three scenarios. We shall discuss some issues regarding the consistency of our tests in scenarios (**i**) and (**ii**).

Finally the paper describes a bootstrap approach for our estimators and tests. The motivation for this comes from the fact that the Monte-Carlo simulation experiment suggests that critical values drawn from the asymptotic distribution do not provide a good approximation to the finite sample behaviour of the test. One main reason for this originates from our general/mild conditions on the cross-sectional dependence which may result in a poor "nonparametric" estimator of the covariance structure of our statistics. In such a situation bootstrap techniques may be employed in the hope to improve the finite sample behaviour. To that end, we shall describe and examine two very simple bootstrap algorithms which have the appealing feature that there is no need to provide any estimate of the covariance structure of the error term. As a consequence, the bootstrap algorithms avoid the rather unpleasant need of time series inspired bootstrap methods which depend on (or make use of) some type of some "ad hoc" distance among the errors (observations), and hence there is no need to choose any bandwidth parameter, as is the case with time series, to implement a valid bootstrap approach. One of our findings is that the size of our tests is not affected by the choice of ϵ (trimming).

The remainder of the paper is organized as follows. In the next section, we discuss the regularity conditions of our model and provide a Central Limit Theorem for the slope parameters of the model (1.2) given either heterogeneity or homogeneity of the slope parameters. Section 3 discusses our test procedures for the null hypothesis of homogeneity. A whole broad family of tests are provided that make use of a weighting function $w(\tau)$, where typical choices are $w(\tau) = 1$ and $w(\tau) = \tau^{1/2}(1-\tau)^{1/2}$. We discuss local alternatives and consistency of our tests, showing that our tests have nontrivial power for sequences converging to zero faster than elsewhere, see Pesaran and Yamagata (2008). Their tests therefore have zero asymptotic relative efficiency when compared to ours. Section 4 discusses a bootstrap approach to our tests in view of the fact that the asymptotic distribution sometimes might provide a poor approximation to the finite sample critical values. A second motivation for the use of the bootstrap is that in model (1.2), say, the covariance structure can be quite complicated, so that bootstrap algorithms may be the only suitable solution to even compute valid critical values for the test. Section 5 presents a Monte Carlo simulation experiment to shed some light on the finite sample performance of our tests and the behaviour of the bootstrap counterpart and Section 6 presents an empirical application where we test for structural breaks in a growth model. Section 7 gives a summary and describe possible extension of our results in several directions of interest. The proofs of our main results are provided in an Appendix, which for space considerations has been relegated to the authors' accompanying website http://personal.lse.ac.uk/schafgans/tba.

2. REGULARITY CONDITIONS AND ASYMPTOTIC PROPERTIES OF THE SLOPE PARAMETER ESTIMATORS

Before we discuss and describe the statistical properties for estimators of the parameters β_t in (1.2), we first introduce a set of regularity conditions on the model and discuss the statistical properties of the covariates and error term. We assume that, for all $t \geq 1$, all the roots of the polynomials $\left|1 - \sum_{\ell=1}^{k_1} \rho_{t\ell} L^{\ell}\right| = 0$ are outside the unit interval, so we are not considering panel data models with possible unit roots under either the null or the alternative hypothesis as in Phillips and Moon (1999) or Im, Pesaran and Shin (2003).

Our regularity conditions are given next.

C1: $\{u_{it} = \sigma_i v_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$, *are zero mean sequences of random variables, where* $0 < \sigma^{-1} < \sigma_i < \sigma < \infty$ *and the sequences* $\{v_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$, *satisfy*

(i) *$E(v_{it} \mid \mathcal{V}_{i,t-1}) = 0$; $E(v_{it}^2 \mid \mathcal{V}_{i,t-1}) = 1$ and finite fourth moments, with $\mathcal{V}_{i,t}$ denoting the $\sigma-$algebra generated by $\{v_{is},\, s \le t\}$.*

(ii) *For all $t \in \mathbb{Z}$,*

$$v_{it} = \sum_{\ell=1}^{\infty} a_\ell(i)\, \varepsilon_{\ell t}, \qquad \sum_{\ell=1}^{\infty} |a_\ell(i)|^2 < \infty,$$

where $\{\varepsilon_{\ell t}\}_{t\in\mathbb{Z}}$, $\ell \in \mathbb{N}^+$, are zero mean independent identically distributed (iid) random variables with finite fourth moments. The weights $\{a_\ell(i)\}_{i=1}^n$ satisfy

$$\sup_{\ell\ge1} \sum_{i=1}^{n} |a_\ell(i)|^2 < \infty. \tag{2.1}$$

C2: *$\{z_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$, are sequences of random variables such that:*

(i)
$$z_{it} = \mu_t + \sum_{k=0}^{\infty} c_k(i)\, \chi_{i,t-k}, \qquad \sum_{k=0}^{\infty} c_k k^{1/2} < \infty,$$

where, denoting by $\|B\|$ the norm of the matrix B, $c_k = \max_{i\ge1} \|c_k(i)\|$ and $E(\chi_{it} \mid \Upsilon_{i,t-1}) = 0$; $Cov(\chi_{it} \mid \Upsilon_{i,t-1}) = \Sigma_\chi$ and $E\|\chi_{it}\|^4 < \infty$, with $\Upsilon_{i,t}$ denoting the $\sigma-$algebra generated by $\{\chi_{is},\, s \le t\}$.

(ii) *The sequences of random variables $\{\chi_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$, are such that*

$$\chi_{it} = \sum_{\ell=1}^{\infty} b_\ell(i)\, \eta_{\ell t}, \qquad \sum_{\ell=1}^{\infty} \|b_\ell(i)\|^2 < \infty,$$

where $\{\eta_{\ell t}\}_{t\in\mathbb{Z}}$, $\ell \in \mathbb{N}^+$, are zero mean iid random variables with finite fourth moments.

(iii) *Denoting $\Sigma_{x,i} = Cov(x_{it}; x_{it})$, we have that*

$$0 < \Sigma_x = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \Sigma_{x,i}. \tag{2.2}$$

C3: *For all $i \in \mathbb{N}^+$, the sequences $\{u_{it}\}_{t\in\mathbb{Z}}$ and $\{z_{it}\}_{t\in\mathbb{Z}}$ are mutually independent and*

$$0 < \max_{1\le i\le n} \sum_{j=1}^{n} \|\varphi_u(i,j)\, \varphi_z(i,j)\| < \infty, \tag{2.3}$$

where for any $i, j \ge 1$, as defined in (1.1),

$$\varphi_u(i,j) = \mathrm{Cov}(u_{it}; u_{jt}), \quad \varphi_z(i,j) = \mathrm{Cov}(z_{it}; z_{jt}).$$

C4: *$T, n \to \infty$ such that $n/T^2 \to 0$.*

We now comment on our conditions. Conditions $C1$ and $C2$ indicate that we do not allow for temporal dependence on the errors $\{u_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$. Of course, it is possible to relax the latter condition, allowing u_{it} to follow a model similar to that for z_{it} as given in $C2$, in which case we might name (1.2) a "*stochastic difference equation panel model*". The only major difference that we might encounter is that in the latter scenario the estimation procedure would involve instrumental variables with $\{z_{i,t-\ell}\}_{\ell=1}^{k_1}$ as natural instruments for $\{y_{i,t-\ell}\}_{\ell=1}^{k_1}$. However, this is beyond the scope of the present manuscript as it will only add some extra lengthy technicalities and/or considerations which are well known when $n = 1$.

While both cross-sectional and temporal dependence are allowed to be present at the same time on $\{z_{it}\}_{t\in\mathbb{Z}}$, as it would then be the case for $\{y_{it}\}_{t\in\mathbb{Z}}$, we have assumed otherwise a separable covariance dependence structure as it is known in the argot of the spatio-temporal literature. See for instance Cressie and Huang (1999) or Gneiting (2002). Indeed a simple algebra yields that

$$Cov\left(z_{it}; z_{js}\right) = \gamma_{z,i}\left(|t-s|\right)\varphi_z\left(i,j\right), \tag{2.4}$$

where $\gamma_{z,i}\left(\ell\right) =: \sum_{k=0}^{\infty} c_k\left(i\right) c_{k+|\ell|}\left(i\right)$ and $\varphi_z\left(i,j\right) =: \varphi_\chi\left(i,j\right)$. This type of dependence is often assumed in empirical work due to its practicality and also in view of the difficulty to write down explicit models when the covariance structure of the data is not separable. Nevertheless, it should be noted that the separability condition can be tested, see for instance Matsuda and Yajima (2004). Of course, we can modify this condition allowing the sequences $\{z_{it}\}_{t\in\mathbb{Z}}$, $i\in\mathbb{N}^+$, to satisfy some type of mixing condition such as L^4-Near Epoch dependence with size greater than or equal to 2, see Davidson (1994). The latter type of dependence might be useful from a theoretical/technical point of view if we allow, say that the errors exhibits some form of nonlinear type of dependence and/or we allow them to suffer from heteroscedasticity of the type $\sigma^2\left(z_{it}\right)$. Another model where the latter type of dependence proves to be very convenient from a technical point of view is when we have a nonlinear dynamic panel models, say

$$y_{it} = \eta_i + \alpha_t + g\left(y_{i,t-1}; \rho_t\right) + \theta_t' z_{it} + u_{it}, \quad i = 1,...,n, \quad t = 1,...,T,$$

similar to the nonparametric model examined in Hjellvik, Chen and Tjøstheim (2004). Since the conclusions of our results should follow with L^4-Near Epoch dependence as it has been shown in an ample number of situations, we have decided to keep $C1$ and $C2$ are they stand to facilitate the proof of the CLT of our estimators which is non standard and requires modifications of existing results due to our mild conditions. On the other hand, our condition that $\sum_{k=0}^{\infty} c_k k^{1/2} < \infty$ rules out temporal "*strong-dependence*" for the regressors z_{it}, and hence on y_{it}. There is no doubt that we can relax this assumption to allow for "*strong-dependence*" among the regressors z_{it} as well as the errors u_{it}, at the expense of complicating our technical appendix quite considerably. However, as there are multiple examples where the results follow whether the data is "*weak-dependence*" or "*strong-dependence*" we have decided to keep our condition $C2$ for simplicity. Regardless on whether we allow the latter relaxation on the Conditions $C1$ and $C2$, the conditions are quite mild and, as stated, our proofs already are quite technical. Also, notice that $C2$ (**ii**) implies that

$$\sup_{\ell\geq 1}\sum_{i=1}^{n}\left\|b_\ell\left(i\right)\right\|^2 < \infty.$$

It is worth noticing that we are not assuming that the temporal dynamic behaviour of the sequences $\{z_{it}\}_{t\in\mathbb{Z}}$, $i\in\mathbb{N}^+$, is common among the cross sectional units, so that we allow for some form of heterogeneity in the second moments of the data. That is,

$$\begin{aligned} Cov\left(z_{it}; z_{it}\right) &= \left(\sum_{k=0}^{\infty} c_k\left(i\right)\right)\Sigma_\chi\left(\sum_{k=0}^{\infty} c_k\left(i\right)\right)' \\ &= \Sigma_{z,i}, \end{aligned}$$

which is constant in "i" if $c_k(i) = c_k$ for all $k \geq 0$. This is in line with the assumption in Pesaran and Smith (1995). In addition, we allow for some trending behaviour which is in tune with Kim and Sun (2013). However, when $\mathcal{T}(r)$ or $\mathcal{T}^{\Delta}(r)$ given below in (3.1) and (3.3) respectively are evaluated at $r = T$, then there is no difference whether $Ez_{it} = \mu_t$ or $Ez_{it} = \mu$, say. Our conditions relax the moment conditions needed elsewhere, for instance those in Pesaran and Yamagata (2008), who assume finite moments of order greater than 4.

We next turn our focus on the discussion of the cross-sectional dependence induced by our Conditions $C1$ and $C2$. As elsewhere, see Lee and Robinson (2013), we allow for cross-sectional dependence to be driven by the models outline in parts (**ii**) of Conditions $C1$ and $C2$. In this sense our conditions relax considerably models employed elsewhere, for instance, our conditions allow the usual SAR (or more generally $SARMA$) models. Indeed, by definition of the SAR model, we have

$$\begin{aligned} u &= (I - \omega W)^{-1} \varepsilon \\ &= (I + \Xi)\varepsilon, \quad \Xi = \left(\psi_j(i)\right)_{i,j=1}^{n}, \end{aligned}$$

so that $u_i = \sum_{j=0}^{n} \psi_j(i)\varepsilon_j$, which implies that the SAR model can be regarded as a particular model of that allowed in $C1$ or $C2$. In addition, it is worth noting that in $C1$ the sequence $\sum_{i=1}^{n} |a_\ell(i)|$ is permitted to grow with n, which is not the case with the SAR model. So, in this case our conditions are weaker than those typically assumed when cross-sectional dependence is allowed. Of course we can allow the weights $a_\ell(i)$ to depend also on the sample size "n" as it is often done in SAR models with weight matrices W rowed normalized, however, the latter does not add anything different. With $\sigma_i < \sigma < \infty$, moreover, we observe that $\left(\sum_{\ell=1}^{\infty}\sum_{i=1}^{n} |a_\ell(i)|^2\right)^{-1} \to_{n\nearrow\infty} 0$. While an alternative approach to model, possibly "*long-memory*", cross-sectional dependence is through the presence of common (unobserved) factors, as in Pesaran (2006) and Bai (2009), we have decided to follow the model assumed in $C1$ due to its similarities with time series models and the fact that it can be considered as a natural generalization of the empirically popular SAR models. Finally, we can mention that $C2$ (**iii**) implies that we can allow for some form of multicollinearity among the regressors z_{it}, but only for a fraction of individuals, as (2.2) indicates that all we need is that on "average" there is no multicollinearity.

We next discuss our Condition $C3$. The first important point to remark is that expression (2.3) does not imply that

$$g_u(n) = \frac{1}{n}\sum_{i,j=1}^{n} |\varphi_u(i,j)| \quad \text{nor} \quad g_z(n) = \frac{1}{n}\sum_{i,j=1}^{n} \|\varphi_z(i,j)\|$$

are bounded with n, i.e. that $g_u(n) + g_z(n) < C$, although it does imply that

$$0 < \lim_{n\to\infty} \left\| \frac{1}{n}\sum_{i,j=1}^{n} \varphi_u(i,j)\varphi_z(i,j) \right\| < \infty. \tag{2.5}$$

In fact, $g_u(n)$ and/or $g_z(n)$ can be such that they diverge to infinity with n, in which case $\{z_{it}\}_{t\in\mathbb{Z}}$ and $\{u_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$ are "*strong-dependent*" sequences. On the other hand, their combined cross-sectional dependence, that is the dependence of the sequence $\{w_{it} = (z_{it} - E(z_{it}))u_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$, satisfies

$$g_w(n) = \frac{1}{n}\sum_{i,j=1}^{n} \|\varphi_w(i,j)\| < C,$$

so that $\{w_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$ is "*weakly-dependent*". We do point out that due to the dynamic aspect of our panel data model (2.3), (2.5) does impose some restriction on the rate of divergence of $g_u(n)$ and $g_z(n)$. To see this, suppose for the sake of argument that $\varphi_u(i,j) = \varphi_u(|i-j|)$. In the introduction various examples were given where $\varphi_u(i,j) = \varphi_u(|i-j|)$, i.e., when lattice type of data is available, so that we can "locate" our individuals in some form of equally space distance or when the dependence is related to some "economic/geographical" distance as in Conley (1999). Given $\varphi_u(|i-j|) \simeq |i-j|^{2d_u-1}$ and $\varphi_z(|i-j|) \simeq |i-j|^{2d_z-1}$ with $0 < d_u < 1/4$ and $0 < d_z < 1/4$ (so that both u_{it} and z_{it} are "*strong-dependent*"), $d_u + d_z < 1/2$ in (2.3) which ensures w_{it} is "weakly dependent". However, it could also fit the framework of Jenish and Prucha (2012), who regard observations as lying on an irregularly spaced pattern. It is worth emphasizing that our assumptions do not imply any type of strong-mixing condition as in Jenish and Prucha (2012) as that would require that at least $g_u(n) + g_z(n) < C$ and typically involves the notion of falling off of dependence as $|i-j|$ increases, which is not very relevant to all spatial situations of interest, see Lee and Robinson (2013). In fact, drawing similarities with time series literature, using Ibragimov and Rozanov (1978, *Ch.* 4), it suggests that our condition rules out any form of *weak-dependence*, such as strong-mixing, in $\{w_{it}\}_{t\in\mathbb{Z}}$, $i \in N^+$. In addition, and keeping in mind our previous comments on the behaviour of $\varphi_u(i,j)$, (2.3) yields that

$$0 < \lim_{n\to\infty} \frac{1}{n}\sum_{i,j=1}^{n} \varphi_u^2(i,j) < \infty,$$

so that $u_{it}^2 - E\left(u_{it}^2\right)$ behaves as if it were a "*weakly-dependent*" sequence. Finally (2.3) also implies that

$$\max_{1\le i\le n}\left|\sum_{j=1}^{n}\varphi_u(i,j)\right|\left\|\sum_{j=1}^{n}\varphi_z(i,j)\right\| = O\left(n^{1-\zeta}\right) \tag{2.6}$$

for some $\zeta > 0$.

Condition (2.5) bears similarities to a condition found in classical time series regression models with possible "*strong-dependence*". There the condition is that

$$\int_{-\pi}^{\pi} f_{u_i}(\lambda) f_{z_i}(\vartheta - \lambda)\, d\lambda = f_i(\vartheta) \quad \vartheta \in (-\pi, \pi]$$

is a continuous function at $\vartheta = 0$, where $f_{u_i}(\lambda)$ and $f_{z_i}(\lambda)$ denote respectively the spectral density functions of $\{u_{it}\}_{t\in\mathbb{Z}}$ and the regressors $\{z_{it}\}_{t\in\mathbb{Z}}$, see for instance Robinson and Hidalgo (1997) and Hidalgo (2003). We then view (2.3), or (2.5), as the counterpart of the last displayed expression in regression models with cross-sectional dependence.

Finally, Condition $C4$ is identical to that of Pesaran and Yamagata (2008). We could relax this assumption allowing n to grow to infinity at least as fast as $T^{-1} = O\left(\log^{-1} n\right)$, but that would be at the expense of efficiency requiring the use of instrumental variables; see above comments for Condition $C1$.

Before presenting our first main result, let us introduce some notation. In what follows, we denote the "*average*" long-run variance as

$$V_1 = \lim_{n\to\infty} \frac{1}{n} \sum_{i,j=1}^{n} \{\varphi_u(i,j)\,\varphi_x(i,j)\}. \tag{2.7}$$

For generic sequences $\{\varsigma_{it}\}_{t=1}^{T}$, $i = 1, ..., n$, we write

$$\begin{aligned} &\widetilde{\varsigma}_{it} = \varsigma_{it} - \overline{\varsigma}_{\cdot t} - \overline{\varsigma}_{i\cdot} + \overline{\varsigma}_{\cdot\cdot} \\ &\text{with } \overline{\varsigma}_{\cdot t} = \frac{1}{n}\sum_{i=1}^{n}\varsigma_{it}; \quad \overline{\varsigma}_{i\cdot} = \frac{1}{T}\sum_{i=1}^{T}\varsigma_{it}; \quad \overline{\varsigma}_{\cdot\cdot} = \frac{1}{T}\sum_{t=1}^{T}\varsigma_{\cdot t}. \end{aligned} \tag{2.8}$$

The transformation in (2.8) allows us to remove the individual and time effects η_i and α_t from the model (1.2). While under the alternative, this transformation yields

$$\widetilde{y}_{it} = \beta_t'\widetilde{x}_{it} + \frac{1}{T}\sum_{s=1}^{T}(\beta_t - \beta_s)'(x_{is} - \overline{x}_{\cdot s}) + \widetilde{u}_{it}, \quad i = 1, ..., n \text{ and } t = 1, ..., T,$$

under the null we have

$$\widetilde{y}_{it} = \beta'\widetilde{x}_{it} + \widetilde{u}_{it}, \quad i = 1, ..., n \text{ and } t = 1, ..., T.$$

In the absence of individual fixed effects, the "standard" transformed regressors $x_{it}^{\dagger} = x_{it} - \overline{x}_{\cdot t}$ would appear under both the null and the alternative hypothesis. Also, in view of $C1$ and $C2$, it is obvious that we can take $Ex_{it} = 0$ as $\widetilde{x}_{it}$ is invariant to additive constants to x_{it}.

Let $\widehat{\beta}_{FE}$ be the fixed effect estimator of the slope parameters, i.e.

$$\widehat{\beta}_{FE} = \left(\sum_{i=1}^{n}\sum_{t=1}^{T}\widetilde{x}_{it}\widetilde{x}_{it}'\right)^{-1}\left(\sum_{i=1}^{n}\sum_{t=1}^{T}\widetilde{x}_{it}\widetilde{y}_{it}\right), \tag{2.9}$$

and, for all $t \geq 1$, consider

$$\widehat{\beta}_t = \left(\sum_{i=1}^{n}\widetilde{x}_{it}\widetilde{x}_{it}'\right)^{-1}\left(\sum_{i=1}^{n}\widetilde{x}_{it}\widetilde{y}_{it}\right). \tag{2.10}$$

Finally with $\Sigma_x > 0$ as in $C2$, define

$$V_2 = \Sigma_x^{-1} V_1 \Sigma_x^{-1}.$$

We now give our main result of this section.

Theorem 1. *Under Conditions $C1 - C4$ and $\beta_t = \beta$, we have that*

(a) $(Tn)^{1/2}\left(\widehat{\beta}_{FE} - \left(\beta - \frac{b}{T}\right)\right) \xrightarrow{d} \mathcal{N}(0, V_2)$

(b) $n^{1/2}\left(\widehat{\beta}_{t_1} - \beta, ..., \widehat{\beta}_{t_\ell} - \beta\right)' \xrightarrow{d} \mathcal{N}(0, I_\ell \otimes V_2)$, *for any finite $\ell \geq 1$.*

Proof. The proof of this result or any other will be given in the Appendix . □

Remark 1. (**i**) *If* $\lim n/T > 0$ *we have the well-known asymptotic bias due to the incidental parameter problem in the linear dynamic panel model (Hahn and Kürsteiner, 2002). In the absence of additional* z *regressors, the asymptotic bias,* b/T*, equals* $(1+\beta)(\frac{1}{n}\sum_{i=1}^{n}\varphi(i,i) - \frac{1}{n^2}\sum_{i,j=1}^{n}\varphi_u(i,j))/T$ *when* $n/T^3 \to 0$*. This asymptotic bias reduces to that of Hahn and Kürsteiner, in the absence of cross-sectional dependence. As* $n/T^2 \to 0$ *we can ignore asymptotic bias in (b).*
(**ii**) *The estimators* $\widehat{\beta}_t$ *and* $\widehat{\beta}_s$ *are asymptotically independent if* $s \neq t$*. This is the case because* $Cov(u_{it}, u_{js}) = 0$ *for all* $s \neq t$ *by* $C1$*.*
(**iii**) *Under the alternative hypothesis, i.e.* $\beta_t \neq \beta$*, we have that Theorem 1 still holds true but with some minor changes. Indeed, when* $\beta_t \neq \beta$*, we can easily extend our arguments to show that*

$$\textbf{(a)} \quad (Tn)^{1/2}\left(\widehat{\beta}_{FE} - \left(\frac{1}{T}\sum_{t=1}^{T}\beta_t - \frac{b}{T}\right)\right) \xrightarrow{d} \mathcal{N}(0, V_2 + W)$$

$$\textbf{(b)} \quad n^{1/2}\left(\widehat{\beta}_{t_1} - \beta_{t_1}, ..., \widehat{\beta}_{t_\ell} - \beta_{t_\ell}\right)' \xrightarrow{d} \mathcal{N}(0, I_\ell \otimes V_2)\,, \text{ for any finite } \ell \geq 1,$$

where

$$W = \Sigma_x^{-1} \lim_{n,T\to\infty} \frac{1}{nT} Var\left(\sum_{i=1}^{n}\sum_{t=1}^{T} x_{it}x_{it}'\left[\beta_t - \frac{1}{T}\sum_{s=1}^{T}\beta_s\right]\right)\Sigma_x^{-1}.$$

So, only the fixed-effect estimator results of Theorem 1 are affected. The asymptotic bias, when the coefficient on $y_{i.t-1}$ *changes, remains* $O(T^{-1})$*.*

Recalling our definition of V_2, Theorem 1 indicates that to provide inferences about the slope parameters, we need a consistent estimator of the "*average*" long-run variance V_1 in (2.7). In our particular setup, we propose the following very simple estimator

$$\widehat{V}_1 = \frac{1}{T}\sum_{t=1}^{T}\left\{\left(\frac{1}{n^{1/2}}\sum_{i=1}^{n}\widetilde{x}_{it}\widehat{u}_{it}\right)\left(\frac{1}{n^{1/2}}\sum_{i=1}^{n}\widetilde{x}_{it}\widehat{u}_{it}\right)'\right\}, \tag{2.11}$$

where $\widehat{u}_{it} = \widetilde{y}_{it} - \widehat{\beta}_{FE}'\widetilde{x}_{it}$, $i = 1, ..., n$ and $t = 1, ..., T$. $\widehat{V}_1$ is a time-cluster estimator of the variance, see Driscoll and Kraay (1989) or Bester, Conley and Hansen (2011). It is worth remarking that in (2.11) we cannot employ $\widehat{u}_{it} = \widetilde{y}_{it} - \widehat{\beta}_t'\widetilde{x}_{it}$, as $\sum_{i=1}^{n}\widetilde{x}_{it}\widehat{u}_{it} = 0$ by definition. One important feature of the above estimator is that, contrary to the HAC estimators of Kelejian and Prucha (2007), or Kim and Sun (2013) there is no need to introduce any artificial "metric" among the cross-sectional observations.[1] It is not clear that this would be convenient, as changing the "metric" may yield a different estimate of V_1 and thereby induce potentially different outcomes in our inferences.

Proposition 1. *Under the same conditions of Theorem 1, we have*

$$\widehat{V}_1 - V_1 = o_p(1)\,.$$

Remark 2. *For our cluster estimator, relaxation of Condition* $C4$ *necessitates a modification to ensure its consistency. The first modification would be the use of instrumental variables for computation of the residuals, where lags of* z_{it} *are used as instruments for* y_{it-1} *(see also our comment*

[1] Vogelsang (2012) considers various cluster estimators of the variance for the static linear panel model in the presence of time-dependence. He does not explicitly discuss HAC corrections to account for the cross-sectional dependence either, only for the time dependence.

on Condition $C1$). Alternatively, we could correct our residuals directly for the asymptotic bias by estimating it. This would not necessarily enhance our estimator for the "average" long-run variance as estimation of it can be very inaccurate in view of the amount of cross-sectional dependence. We have decided not to pursue these routes as in many settings, as in ours, this condition appears sensible.

We now make some comments on Proposition 1. When $\beta_t \neq \beta$, the results in Proposition 1 does not hold true. The reason being that in this case $\widehat{\beta}_{FE}$ would only be a consistent estimator of $\lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}\beta_t$ as the remark that follows Theorem 1 indicates. There is a second way to obtain a consistent estimator of V_1 via bootstrap methods. Recall that this approach was one of the main motivations for the bootstrap in the original paper by Efron (1979) as a method to estimate the asymptotic covariance of estimators when they are not easy to compute or to provide an explicit formula. We will delay discussing this approach to Section 4 below.

3. TESTS FOR BREAKS

We now introduce two related tests for breaks of the slope parameters in our model (1.2). Our first approach to test H_0 in (1.3), a *CUSUM* type test, is based on the behaviour of

$$\mathcal{T}(r) = \frac{1}{(nT)^{1/2}} \sum_{t=1}^{r}\sum_{i=1}^{n} \widetilde{x}_{it}\left(\widetilde{y}_{it} - \widehat{\beta}'_{FE}\widetilde{x}_{it}\right), \quad r = 1, ..., T-1. \tag{3.1}$$

The intuition for $\mathcal{T}(r)$ is that under the null hypothesis, we expect that $\widetilde{x}_{it}\left(\widetilde{y}_{it} - \widehat{\beta}'_{FE}\widetilde{x}_{it}\right) \simeq \widetilde{x}_{it}u_{it}$ which has a mean equal to zero, while under the alternative hypothesis we have that $\widetilde{x}_{it}\left(\widetilde{y}_{it} - \widehat{\beta}'_{FE}\widetilde{x}_{it}\right)$ will develop a term of the type

$$\widetilde{x}_{it}\widetilde{x}'_{it}\left(\beta_t - \widehat{\beta}_{FE}\right) \simeq \widetilde{x}_{it}\widetilde{x}'_{it}\left(\beta_t - \frac{1}{T}\sum_{s=1}^{T}\beta_s\right),$$

see Remark 1. Under the alternative therefore, $\mathcal{T}(r)$ would be governed by the non-zero function

$$h(r) = \left\{\frac{1}{n}\sum_{i=1}^{n} E\left(\mathring{x}_{it}\mathring{x}'_{it}\right)\right\}\frac{n^{1/2}}{T^{1/2}}\sum_{t=1}^{r}\left(\beta_t - \frac{1}{T}\sum_{s=1}^{T}\beta_s\right),$$

where for generic sequences $\{\varsigma_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$, we denote

$$\{\mathring{\varsigma}_{it}\}_{t\in\mathbb{Z}} = \{\varsigma_{it} - E(\varsigma_{it})\}_{t\in\mathbb{Z}}, \quad i \in \mathbb{N}^+.$$

The preceding arguments suggest that one possible method to test the null hypothesis in (1.3) might be based on continuous functionals of $\mathcal{T}(r)$.

Our second approach is based on the observation that we can regard H_0 as testing whether the slope parameters β_t are the same across time, where for a given time period t, we estimate β_t as in (2.10). This test recognizes that under H_0, we can use the *mean group (MG) estimator*

$$\widehat{\beta}_{MG} = \frac{1}{T}\sum_{s=1}^{T}\widehat{\beta}_s, \tag{3.2}$$

see Pesaran, Shin and Smith (1999), as an estimator for the common slope parameters β. While under the null, for every t, $\widehat{\beta}_t - \widehat{\beta}_{MG}$ converges to zero in probability, under the alternative hypothesis $\widehat{\beta}_t - \widehat{\beta}_{MG}$ will develop a mean different than zero. Our Hausman-Durbin-Wu's type of statistic is then based on continuous functionals of

$$\mathcal{T}^{\Delta}(r) = \frac{n^{1/2}}{T^{1/2}} \sum_{t=1}^{r} \left(\widehat{\beta}_t - \widehat{\beta}_{MG} \right). \tag{3.3}$$

It is worth noticing that tests based on $\mathcal{T}(r)$ and $\mathcal{T}^{\Delta}(r)$ are related. Indeed, using the definition of $\widehat{\beta}_{FE}$, given in (2.9), we easily obtain

$$\begin{aligned} \mathcal{T}(r) &\equiv \frac{1}{(nT)^{1/2}} \sum_{t=1}^{r} \sum_{i=1}^{n} \widetilde{x}_{it} \left(\widetilde{y}_{it} - \widehat{\beta}'_{FE} \widetilde{x}_{it} \right) \\ &\simeq \frac{n^{1/2}}{T^{1/2}} \left(\sum_{t=1}^{r} \left(\frac{1}{n} \sum_{i=1}^{n} \widetilde{x}_{it} \widetilde{y}_{it} \right) - \frac{r}{T} \sum_{s=1}^{T} \left(\frac{1}{n} \sum_{i=1}^{n} \widetilde{x}_{is} \widetilde{y}_{is} \right) \right) \\ &= \Sigma_x \mathcal{T}^{\Delta}(r) (1 + o_p(1)), \end{aligned} \tag{3.4}$$

so that $\mathcal{T}^{\Delta}(r)$ is a "weighted" version of $\mathcal{T}(r)$ for any r. We point out that our tests have similarities with the Δ test in Pesaran and Yamagata (2008), see also Swamy (1970). However, as we will notice in Section 3.3 below, tests based on (3.1) or (3.3) can detect local alternatives which the Δ test cannot.

Let $\mathcal{B}(\tau)$ denote the standard Brownian motion in $[0,1]$ and $\mathcal{BB}(\tau) = \mathcal{B}(\tau) - \tau \mathcal{B}(1)$ the standard Brownian bridge. As the next theorem shows, our tests do not suffer from the incidental parameter problem.

Theorem 2. *Assuming $C1 - C4$, under H_0, we have that as $n, T \to \infty$,*

$$(\mathbf{a}) \quad \frac{1}{(nT)^{1/2}} \sum_{t=1}^{[T\tau]} \sum_{i=1}^{n} \widetilde{x}_{it} \left(\widetilde{y}_{it} - \widehat{\beta}'_{FE} \widetilde{x}_{it} \right) \overset{d}{\Longrightarrow} V_1^{1/2} \mathcal{BB}(\tau)$$

$$(\mathbf{b}) \quad \frac{n^{1/2}}{T^{1/2}} \sum_{t=1}^{[T\tau]} \left(\widehat{\beta}_t - \widehat{\beta}_{MG} \right) \overset{d}{\Longrightarrow} V_2^{1/2} \mathcal{BB}(\tau).$$

Remark 3. *Due to the differencing (see also (3.4)), the asymptotic bias cancels out as (asymptotically) it is independent of t and/or i.*

For any continuous mapping function $\varphi(\cdot)$, our tests are given by

$$\mathcal{T} = \varphi \left(\frac{\mathcal{T}'(r) \widehat{V}_1^{-1} \mathcal{T}(r)}{w^2(r/T)} \right) \quad \text{and} \quad \mathcal{T}^{\Delta} = \varphi \left(\frac{\mathcal{T}^{\Delta\prime}(r) \widehat{V}_2^{-1} \mathcal{T}^{\Delta}(r)}{w^2(r/T)} \right), \tag{3.5}$$

where $w(\tau)$, $\tau \in [0,1]$, is a weighting function that (**i**) is non-decreasing in a neighbourhood of 0, (**ii**) is non-increasing in a neighbourhood of 1, (**iii**) is positive on $(\eta, 1-\eta)$ and (**iv**) satisfies

$$\int_0^1 \frac{1}{\tau(1-\tau)} \exp\left(-c \frac{w^2(\tau)}{\tau(1-\tau)} \right) d\tau < \infty. \tag{3.6}$$

A standard weighting $w(\tau)$ function which satisfies these conditions is $w(\tau) = 1$. The common choice $w(\tau) = \tau^{1/2}(1-\tau)^{1/2}$, implicitly used in Andrews (1993) and many subsequent authors, on the other hand, fails to satisfy this condition (3.6). While the latter weight function provides a natural standardization of our test, as it represents the standard deviation of a standard Brownian Bridge, it does have the drawback of requiring trimming for values of τ close to 0 and 1. In fact, any weighting function that does not satisfy (3.6) is subject to the use of some trimming for values to close to 0 or to 1, which is a well known result, see for instance Shorack and Wellner (2009, *p*.462).

We then have the following result.

Corollary 1. *Assuming $C1-C4$, under H_0 and $w(\tau)$ satisfying* (3.6) *as $n,T\to\infty$, we have that*

$$
\begin{aligned}
&(\mathbf{a})\quad \mathcal{T} \overset{d}{\Longrightarrow} \varphi\left(\frac{(\mathcal{BB}(\tau))'(\mathcal{BB}(\tau))}{w^2(\tau)}\right)\\
&(\mathbf{b})\quad \mathcal{T}^{\Delta} \overset{d}{\Longrightarrow} \varphi\left(\frac{(\mathcal{BB}(\tau))'(\mathcal{BB}(\tau))}{w^2(\tau)}\right).
\end{aligned}
$$

Proof. The proof of this corollary follows easily by Proposition 1 and Theorem 2. Indeed Proposition 1 indicates that

$$
\begin{aligned}
\frac{\widehat{V}_1^{-1/2}}{(nT)^{1/2}}\sum_{t=1}^{[T\tau]}\sum_{i=1}^{n}\widetilde{x}_{it}\left(\widetilde{y}_{it}-\widehat{\beta}_{FE}'\widetilde{x}_{it}\right) &= \frac{V_1^{-1/2}}{(nT)^{1/2}}\sum_{t=1}^{[T\tau]}\sum_{i=1}^{n}\mathring{x}_{it}\left(\mathring{y}_{it}-\widehat{\beta}_{FE}'\mathring{x}_{it}\right)(1+o_p(1))\\
\widehat{V}_1^{-1/2}\frac{n^{1/2}}{T^{1/2}}\sum_{t=1}^{[T\tau]}\left(\widehat{\beta}_t-\widehat{\beta}_{MG}\right) &= V_1^{-1/2}\frac{n^{1/2}}{T^{1/2}}\sum_{t=1}^{[T\tau]}\left(\widehat{\beta}_t-\widehat{\beta}_{MG}\right)(1+o_p(1)).
\end{aligned}
$$

From here, Theorem 2 and the continuous mapping theorem yield the conclusion of the corollary. □

Corollary 1 indicates that when $w(\tau)=1$, we have

$$
\begin{aligned}
\max_{0<r<T}\left|\mathcal{T}(r)'\,\widehat{V}_1^{-1}\mathcal{T}(r)\right| &\overset{d}{\Longrightarrow} \max_{0<\tau<1}\left|(\mathcal{BB}(\tau))'(\mathcal{BB}(\tau))\right|\\
\max_{0<r<T}\left|\mathcal{T}^{\Delta}(r)'\,\widehat{V}_2^{-1}\mathcal{T}^{\Delta}(r)\right| &\overset{d}{\Longrightarrow} \max_{0<\tau<1}\left|(\mathcal{BB}(\tau))'(\mathcal{BB}(\tau))\right|,
\end{aligned}
$$

which correspond to a Kolmogorov-Smirnov's type of statistic. However when $w^2(\tau)=\tau(1-\tau)$, which corresponds to the weight function implicit in Andrews (1993), (3.6) is not satisfied so that as in Andrews (1993) we trim for values close to the boundary, that is we consider

$$
\begin{aligned}
\max_{[T\epsilon]<r<T-[T\epsilon]}\left|\frac{\mathcal{T}(r)'\,\widehat{V}_1^{-1}\mathcal{T}(r)}{w^2(r/T)}\right| &\overset{d}{\Longrightarrow} \max_{\epsilon<\tau<1-\epsilon}\left|\frac{(\mathcal{BB}(\tau))'(\mathcal{BB}(\tau))}{w^2(\tau)}\right|\\
\max_{[T\epsilon]<r<T-[T\epsilon]}\left|\frac{\mathcal{T}^{\Delta}(r)'\,\widehat{V}_2^{-1}\mathcal{T}^{\Delta}(r)}{w^2(r/T)}\right| &\overset{d}{\Longrightarrow} \max_{\epsilon<\tau<1-\epsilon}\left|\frac{(\mathcal{BB}(\tau))'(\mathcal{BB}(\tau))}{w^2(\tau)}\right|,
\end{aligned}
$$

for some $0<\epsilon<\frac{1}{2}$.

Of course, we can use other weighting functions $w(\tau)$ to target particular alternatives in a similar way as directional tests do in goodness-of-fit tests, see also Andrews and Ploberger (1994). We have not pursued this somewhat standard extension.

Neither have we pursued the scenario put forward in the introduction of $\epsilon \to 0$, as in Hidalgo and Seo (2013). They, basically, examine the consequences when no trimming is used when $w(\tau)$ fails the condition given in (3.6). Bear in mind, the purpose of trimming and the introduction of a weight function satisfying (3.6) is somehow to make $\max_{r<[T\epsilon]}$ or $\max_{T-[T\epsilon]<r}$ asymptotically negligible, as the asymptotic distribution becomes a Gumbel distribution when the latter is not true, see also Horváth (1993).

3.1. LOCAL ALTERNATIVES AND CONSISTENCY OF THE TESTS.

We now discuss the local alternatives for which the tests described in the previous two sections have nontrivial power and from there easily conclude their consistency. To that end, we begin by considering the local alternatives

$$H_a: \quad \beta_t = \beta + \delta_{nT}\mathcal{I}(t > t_0), \tag{3.7}$$

where $t_0 = [T\tau_0]$ for some $\tau_0 \in (\epsilon, 1-\epsilon)$ with $\epsilon > 0$, and δ_{nT} is a deterministic sequence depending on n and/or T. To shorten the discussion we will only explicitly handle the behaviour under H_a in (3.7) and discuss the consistency of tests based on $\mathcal{T}(r)$ and $\mathcal{T}^{\Delta}(r)$ given in (3.1) and (3.3), respectively.

For this purpose, introduce the "shift" function

$$\Xi(\tau) = (\tau - \tau_0)\mathcal{I}(\tau > \tau_0) - \tau(1-\tau_0). \tag{3.8}$$

We then establish the following result.

Proposition 2. *Assuming $C1-C4$, under H_a with $\delta_{nT} = \delta/(nT)^{1/2}$, $|\delta| > 0$, we have that as $n, T \to \infty$,*

$$\textbf{(a)} \quad \frac{1}{(nT)^{1/2}}\sum_{t=1}^{[T\tau]}\sum_{i=1}^{n}\widetilde{x}_{it}\left(\widetilde{y}_{it} - \widehat{\beta}'_{FE}\widetilde{x}_{it}\right) \overset{d}{\Longrightarrow} V_1^{1/2}\mathcal{BB}(\tau) + \delta\Sigma_x\Xi(\tau)$$

$$\textbf{(b)} \quad \frac{n^{1/2}}{T^{1/2}}\sum_{t=1}^{[T\tau]}\left(\widehat{\beta}_t - \widehat{\beta}_{MG}\right) \overset{d}{\Longrightarrow} V_2^{1/2}\mathcal{BB}(\tau) + \delta\Xi(\tau).$$

Proposition 2 indicates that the tests have no trivial power if the alternative hypothesis converges to the null at the rate $(nT)^{1/2}$. On the other hand, when $\delta_{nT}^{-1} = o\left((nT)^{1/2}\right)$, the statistic diverges to infinity, that is the test will reject with probability 1 as the sample size increases. Finally, when $\delta_{nT} = o\left((nT)^{-1/2}\right)$, the asymptotic distribution is identical to that obtained under the null hypothesis. This clearly improves on the local alternatives given in Pesaran and Yamagata (2008), who only were able to detect local alternatives $\delta_{nT} = O\left(n^{-1/4}T^{-1/2}\right)$. In this way, their test has zero asymptotic relative efficiency compared to ours.

While the "shift" function is asymptotically equivalent whether we include individual fixed effects or not, we do point out that for small samples the terms $\frac{1}{T}\sum_{s=1}^{T}(\beta_t - \beta_s)'(x_{is} - \overline{x}_{\cdot s})$, which vanish asymptotically but not with T small, may affect the finite sample power properties of our tests in a nonlinear way.

The consistency of the test is given in the following corollary.

Corollary 2. *Assuming $C1-C4$, under H_a with $\delta_{nT}=\delta$ for all n and T, we have that*

$$(\mathbf{a})\quad \Pr\left\{\varphi\left(\frac{\mathcal{T}(r)'\,\widehat{V}_1^{-1}\mathcal{T}(r)}{w^2(r/T)}\right)>a\right\}\to 1$$

$$(\mathbf{b})\quad \Pr\left\{\varphi\left(\frac{\mathcal{T}^{\Delta}(r)'\,\widehat{V}_2^{-1}\mathcal{T}^{\Delta}(r)}{w^2(r/T)}\right)>a\right\}\to 1$$

for any $a>0$ and continuous $w(\tau)$.

Proof. The proof is standard from Proposition 2, so it is omitted. □

Remark 4. (**i**) *It is important to mention that we have not assumed that $w(\tau)$ satisfies* (3.6) *on purpose. The reason is that under the alternative hypothesis we have assumed $\tau_0\in(\epsilon,1-\epsilon)$ for some $\epsilon>0$. Of course, if $w(\tau)$ would satisfy* (3.6)*, we then could take $\epsilon=0$. However, we do not want to lengthen the paper with this unnecessary and rather trivial discussion.*

(**ii**) *Our main conclusion in this section does not depend on the fact that the break or heterogeneity of the slope parameters is abrupt in nature. Indeed, suppose that we replace H_a in* (3.7) *by the following alternative hypothesis*

$$H_a:\quad \beta_t=\beta+\frac{1}{(nT)^{1/2}}\left\{\sum_{\ell=1}^{L}\delta_\ell\mathcal{I}(t>t_\ell)+\delta\left(\frac{t}{T}\right)\right\},$$

where $\delta(\tau)$ is a continuous (smooth) function in $\tau\in(0,1)$ while $|\delta_\ell|>0$, $\ell=1,..,L$ permits discrete jumps. The only difference lies in the form of the shift function $\Xi(\tau)$ appearing in (3.8)*. Indeed, with the (local) alternatives given in the last displayed expression, the shift function $\Xi(\tau)$ becomes*

$$\Xi(\tau)=\sum_{\ell=0}^{L}\delta_\ell\mathcal{I}(\tau>\tau_\ell)-\tau\sum_{\ell=1}^{L}\delta_\ell(1-\tau_\ell)+\int_0^\tau\delta(v)\,dv-\tau\int_0^1\delta(v)\,dv.$$

It is clear that $\Xi(\tau)$ is different from zero in a set $\Lambda\subset[0,1]$ with positive Lebesgue measure. Indeed, suppose for simplicity that $\delta_\ell=0$ for all $\ell\geq 0$, then

$$\Xi(\tau)=\int_0^\tau\delta(v)\,dv-\tau\int_0^1\delta(v)\,dv.$$

In that case $\Xi(\tau)=0$ for all $\tau\in(0,1)$ if and only if $\delta(\tau)$ is a constant function which is ruled out as it would imply that $H_a\subset H_0$. To see this, we notice that $\Xi(\tau)=\int_0^\tau\left\{\delta(v)-\overline{\delta}\right\}dv$, where $\overline{\delta}=\int_0^1\delta(v)\,dv$. But $\Xi(\tau)=0$ for all $\tau\in(0,1)$ if and only if $\delta(v)=\overline{\delta}$ for all $v\in(0,1)$.

We finish the section commenting on the power of the tests in the situations mentioned in the introduction, namely (**i**) when the time of the break is towards the end of the sample and (**ii**) when the number of individuals for which a break exists is negligible compared to n. We ignore the presence of individual fixed effect here for simplicity.

We first consider (**i**). Assume that $\beta_t = \beta$ if $t \leq T - T_0$ ($T_0 =: h_T$ with $[h_T] = o(T)$) and $\beta_t = \beta + \delta$ otherwise. Consider the decomposition

$$\begin{aligned} &\frac{1}{T}\sum_{t=1}^{\widetilde{T}}\left(\widehat{\beta}_t - \widehat{\beta}_{MG}\right) \qquad (3.9) \\ = \ &\frac{1}{T}\sum_{t=1}^{\widetilde{T}}\left(\beta_t - \frac{1}{T}\sum_{s=1}^{T}\beta_s\right) + \frac{1}{T}\sum_{t=1}^{\widetilde{T}}\left\{\left(\widehat{\beta}_t - \beta_t\right) - \left(\widehat{\beta}_{MG} - \frac{1}{T}\sum_{s=1}^{T}\beta_s\right)\right\}. \end{aligned}$$

The second term on the right of (3.9) is $O\left((nT)^{-1/2}\right)$, whereas the first term equals

$$\begin{cases} -\delta\frac{\widetilde{T}}{T}\frac{T_0}{T} & \text{if } \widetilde{T} < T - T_0 \\ -\delta\left(\frac{T-T_0}{T}\right)\left(\frac{T-\widetilde{T}}{T}\right) & \text{if } \widetilde{T} \geq T - T_0. \end{cases}$$

So, we have that

$$\frac{n^{1/2}}{T^{1/2}}\sum_{t=1}^{[T\tau]}\left(\widehat{\beta}_t - \widehat{\beta}_{MG}\right) = O_p(1) - \begin{cases} \frac{n^{1/2}T_0}{T^{1/2}}\delta\tau & \text{if } [T\tau] < T - T_0 \\ \frac{n^{1/2}(T-T_0)}{T^{1/2}}\delta(1-\tau) & \text{if } [T\tau] \geq T - T_0, \end{cases}$$

implying that tests based on $\mathcal{T}^{\Delta}(r)$ will diverge and hence be consistent if $C^{-1} < \frac{n^{1/2}T_0}{T^{1/2}}$ for some positive finite constant C. The same conclusions are drawn regarding tests based on $T(r)$. Regarding the relative growth of n and T, you can see that if $T_0 > C^{-1}T^{1/2}$ the condition of consistency is automatically satisfied. When T_0 is a constant then we need that T does not grow faster than n to infinity. On the other hand, when $T_0 \in \left(\left[CT^{1/4}\right], \left[CT^{1/2}\right]\right)$, we have that $\frac{n^{1/2}T_0}{T^{1/2}} > C^{-1}\frac{n^{1/2}}{T^{1/4}}$ which diverges to infinity because $T/n^2 = o(1)$. We also point out here that when $n = 1$, the condition for consistency, i.e., that T_0 does not grow slower than $T^{1/2}$, corresponds to the result obtained for the LM_τ test in Hidalgo and Seo (2013).

Next we consider the situation (**ii**). Suppose for sake of argument that the break occurs at $\tau_0 = 1/2$, and that it only occurs for the first $\iota(n)$ individuals with the condition that $\iota(n) = o(n)$. Again we examine the behaviour of $\mathcal{T}^{\Delta}(r)$. After standard algebra, we have that

$$\widehat{\beta}_t = O_p(1) + \begin{cases} \beta & \text{if } t < T/2 \\ \beta + \delta\frac{\iota(n)}{n} & \text{if } t \geq T/2. \end{cases}$$

So, we obtain that

$$\frac{n^{1/2}}{T^{1/2}}\sum_{t=1}^{[T\tau]}\left(\widehat{\beta}_t - \widehat{\beta}_{MG}\right) = O_p(1) - \frac{1}{2}\begin{cases} \delta\frac{\iota(n)[T\tau]}{n^{1/2}T^{1/2}} & \text{if } [T\tau] < \frac{1}{2}T \\ \delta\frac{\iota(n)(T-[T\tau])}{n^{1/2}T^{1/2}} & \text{if } [T\tau] \geq \frac{1}{2}T, \end{cases}$$

which implies that test based on $\mathcal{T}^{\Delta}(r)$ will diverge and hence be consistent if $C^{-1} < T^{1/2}\iota(n)/n^{1/2}$.

4. **BOOTSTRAP ALGORITHM**

One of our motivations for introducing a bootstrap algorithm for our tests (and estimators) is that our tests suffer small sample biases which in some cases, as supported by our Monte Carlo experiments, can be quite substantial. Among other reasons, these biases may be due to the fact that the asymptotic distribution yields a poor approximation in finite samples given our estimator of the long run variance V_1. In such situations the bootstrap approach can, as is well known, provide a tool to improve its finite sample behaviour. A quick glance at our conditions in Section 2, may suggest that a bootstrap mechanism may not be easy to implement (let alone to establish its validity) since one of the basic requirements for its validity is that the bootstrap algorithm should preserve the covariance structure. Drawing analogies with the time series literature, one may be tempted to use the block bootstrap principle. However, since there is no obvious ordering of the data in the cross-sectional dimension, it is not clear that a block bootstrap would work in our context or what its sensitivity would be to a particular chosen ordering of the data (over and above the problem of how to choose the block size). Instead, we propose here a valid bootstrap algorithm with the interesting feature that it is computationally simple, mainly due to the observation that there is no need to estimate, either by parametric or nonparametric methods, the cross-sectional dependence of the error term. Moreover the bootstrap has the additional attractive feature that we do not need to choose any tuning parameter for its implementation, as would be the case with a moving block bootstrap type of bootstrap.

More specifically, we provide two bootstrap algorithms. The first bootstrap procedure is described in the following *4 STEPS.*

***STEP 1*:** We compute the residuals $\{\widehat{u}_{it}\}_{t=1}^{T}$, $i = 1, ...n$, as

$$\widehat{u}_{it} = \widetilde{y}_{it} - \sum_{\ell=1}^{k_1} \widehat{\rho}_{t\ell} \widetilde{y}_{i(t-\ell)} - \widehat{\theta}_t^{\prime} \widetilde{z}_{it}, \quad i = 1, ..., n; \quad t = 1, ..., T$$

and obtain the centered residuals as

$$\check{u}_{it} = \widehat{u}_{it} - \frac{1}{T} \sum_{t=1}^{T} \widehat{u}_{it}. \tag{4.1}$$

Remark 5. *The motivation to employ* (4.1) *to center the residuals will become apparent when looking at the next* STEP 2*.*

***STEP 2*:** Denoting $\check{U}_t = \{\check{u}_{it}\}_{i=1}^{n}$, we do standard random resampling from the empirical distribution of $\{\check{U}_t\}_{t=1}^{T}$. The bootstrap sample is denoted by $\{U_t^*\}_{t=1}^{T}$.

***STEP 3*:** Generate the bootstrap dynamic panel data model as

$$\widetilde{y}_{it}^* = \sum_{\ell=1}^{k_1} \widehat{\rho}_{MG,\ell} \widetilde{y}_{i(t-\ell)} + \widehat{\theta}_{MG}^{\prime} \widetilde{z}_{it} + u_{it}^*, \quad i = 1, ..., n, \quad t = 1, ..., T, \tag{4.2}$$

where $\widehat{\rho}_{MG,\ell}$, $\ell = 1, ..., k_1$, and $\widehat{\theta}_{MG}$ are the *MG estimators* in (3.2).

***STEP 4*:** Compute the test statistics using model (4.2) as if it were the true panel regression model. That is, for $r = 1, ..., T-1$,

$$\begin{aligned} \mathcal{T}^*(r) &= \frac{1}{(nT)^{1/2}} \sum_{t=1}^{r} \sum_{i=1}^{n} \widetilde{x}_{it} \left(\widetilde{y}_{it}^* - \widehat{\beta}_{FE}^{\prime *} \widetilde{x}_{it} \right) \\ \mathcal{T}^{\Delta *}(r) &= \left(\frac{n}{T} \right)^{1/2} \sum_{t=1}^{r} \left(\widehat{\beta}_t^* - \widehat{\beta}_{MG}^* \right). \end{aligned}$$

In the latter step, $\widehat{\beta}^*_{FE}$ denotes the fixed effect estimator of the slope parameters β, i.e.

$$\widehat{\beta}^*_{FE} = \left(\sum_{i=1}^{n}\sum_{t=1}^{T} \widetilde{x}_{it}\widetilde{x}'_{it}\right)^{-1} \left(\sum_{i=1}^{n}\sum_{t=1}^{T} \widetilde{x}_{it}\widetilde{y}^*_{it}\right),$$

and $\widehat{\beta}^*_t = \left(\sum_{i=1}^{n} \widetilde{x}_{it}\widetilde{x}'_{it}\right)^{-1} \left(\sum_{i=1}^{n} \widetilde{x}_{it}\widetilde{y}^*_{it}\right)$ with $\widehat{\beta}^*_{MG}$ denoting the *MG bootstrap estimator*

$$\widehat{\beta}^*_{MG} = \frac{1}{T}\sum_{s=1}^{T} \widehat{\beta}^*_s.$$

Before establishing the validity of the bootstrap tests $\mathcal{T}^*$ and $\mathcal{T}^{\Delta *}$ (defined below), we establish the following results:

Theorem 3. *Assuming Conditions $C1 - C4$, we have that (in probability)*

$$(\mathbf{a}) \quad (Tn)^{1/2}\left(\widehat{\beta}^*_{FE} - \left(\widehat{\beta}_{MG} - \frac{\hat{b}}{T}\right)\right) \xrightarrow{d^*} \mathcal{N}(0, V_2)$$

$$(\mathbf{b}) \quad n^{1/2}\left(\widehat{\beta}^*_{t_1} - \widehat{\beta}_{MG}, ..., \widehat{\beta}^*_{t_\ell} - \widehat{\beta}_{MG}\right)' \xrightarrow{d^*} \mathcal{N}(0, I_\ell \otimes V_2) \quad \textit{for any finite } \ell \geq 1.$$

Remark 6. *As in Theorem 1, if $\lim n/T > 0$ we get an asymptotic bias term in (a) due to the incidental parameter problem in the linear dynamic panel model. In the absence of additional z regressors, $\hat{b}/T$ equals $(1+\hat{\beta}_{MG})(\frac{1}{Tn}\sum_{i=1}^{n}\sum_{t=1}^{T} \hat{u}^2_{it} - \frac{1}{Tn^2}\sum_{i,j=1}^{n}\sum_{t=1}^{T} \hat{u}_{it}\hat{u}_{jt})/T$ when $n/T^3 \to 0$.*

Recalling that $V_2 = \Sigma_x^{-1} V_1 \Sigma_x^{-1}$, a consistent bootstrap estimator of the "*average*" long-run variance V_1, is given by

$$\widehat{V}^*_1 = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{n^{1/2}}\sum_{i=1}^{n} \widetilde{x}_{it}\widehat{u}^*_{it}\right)\left(\frac{1}{n^{1/2}}\sum_{i=1}^{n} \widetilde{x}_{it}\widehat{u}^*_{it}\right)',$$

and $\widehat{u}^*_{it} = \widetilde{y}^*_{it} - \widehat{\beta}^{*\prime}_{FE}\widetilde{x}_{it}$, $i = 1, ..., n$ and $t = 1, ..., T$, as the next proposition establishes.

Proposition 3. *Assuming $C1 - C4$, we have that*

$$\widehat{V}^*_1 - V_1 = o_{p^*}(1).$$

Remark 7. *The same remark as given after Proposition 1 applies here.*

We now give the validity of our bootstrap test.

Theorem 4. *Assuming $C1 - C4$ and $w(\tau)$ satisfying (3.6), we have that as $n, T \to \infty$, in probability*

$$(\mathbf{a}) \quad \mathcal{T}^* = \varphi\left(\frac{\mathcal{T}^*(r)'\left(\widehat{V}^*_1\right)^{-1}\mathcal{T}^*(r)}{w^2(r/T)}\right) \overset{d^*}{\Longrightarrow} \varphi\left(\frac{(\mathcal{BB}(\tau))'(\mathcal{BB}(\tau))}{w^2(\tau)}\right)$$

$$(\mathbf{b}) \quad \mathcal{T}^{\Delta *} = \varphi\left(\frac{\mathcal{T}^{\Delta *}(r)'\left(\widehat{V}^*_2\right)^{-1}\mathcal{T}^{\Delta *}(r)}{w^2(r/T)}\right) \overset{d^*}{\Longrightarrow} \varphi\left(\frac{(\mathcal{BB}(\tau))'(\mathcal{BB}(\tau))}{w^2(\tau)}\right),$$

where $\varphi(\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous functional.

While the first bootstrap algorithm is given under $C1$ with $E\left[v_{it}^2 \mid \mathcal{V}_{i,t-1}\right] = \sigma^2$, the second allows $E\left[v_{it}^2 \mid \mathcal{V}_{i,t-1}\right] = \sigma_t^2$, $i \in \mathbb{N}^+$. While a rigorous proof of the validity of the next bootstrap algorithm in the presence of conditional heteroscedasticity is beyond the scope of this paper, its validity under $C1$ can be proven quite similarly and has therefore been left out. The second bootstrap algorithm is described in the next *4 STEPS*.

STEP **$1'$:** We compute the residuals as

$$\widehat{u}_{it} = \widetilde{y}_{it} - \sum_{\ell=1}^{k_1} \widehat{\rho}_{\ell,FE} \widetilde{y}_{i,t-\ell} - \widehat{\theta}_{FE}' \widetilde{z}_{it}, \quad i = 1, ..., n, \quad t = 1, ..., T.$$

Let the centered residuals be $\check{u}_{it} = \widehat{u}_{it} - \frac{1}{T} \sum_{t=1}^{T} \widehat{u}_{it}$.

STEP **$2'$:** Generate a random sample $\{\xi_t\}_{t=1}^T$ with zero mean and unit variance and obtain the bootstrap error terms as

$$\{u_{it}^*\} = \{\check{u}_{it} \xi_t\}, \quad i = 1, ..., n, \quad t = 1, ..., T.$$

Remark 8. *It is important to emphasize that while one might be tempted to obtain the residuals under the alternative hypothesis (as we did in the previous bootstrap), this would not be possible here. The reason for this is that it would translate into a bootstrap statistic that would be identically zero. Indeed, it is not difficult to see that its behaviour is governed by that of*

$$\sum_{i=1}^{n} u_{it}^* \widetilde{x}_{it} = \xi_t \sum_{i=1}^{n} \check{u}_{it} \widetilde{x}_{it} = 0$$

by orthogonality between residuals and regressors.

STEP **$3'$:** Generate the bootstrap panel data model as

$$\widetilde{y}_{it}^* = \sum_{\ell=1}^{k_1} \widehat{\rho}_{\ell,FE} \widetilde{y}_{i,t-\ell} + \widehat{\theta}_{FE}' \widetilde{z}_{it} + u_{it}^*, \quad i = 1, ..., n, \quad t = 1, ..., T. \tag{4.3}$$

STEP **$4'$:** Compute the bootstrap analogues of our statistics $\mathcal{T}(r)$ and $\mathcal{T}^{\Delta}(r)$ with (4.3) as our dynamic panel regression model.

Remark 9. (**i**) *The second bootstrap approach is similar to that in Chan and Ogden* (2009) *and can be regarded as a wild-type bootstrap with increasing dimensional vectors. In this sense, we can view the bootstrap as a generalization or extension of bootstrapping $VAR(P)$ models, say, when the dimension of the (time series) sequence n grows with no limit. Notice that in the case of finite n, a standard approach to bootstrap VAR models is to obtain the bootstrap errors as $\{e_t \xi_t\}_{t=1}^T$, where ξ_t is a scalar sequence and e_t denote residuals.*
(**ii**) *We have assumed that the sequence $\{\xi_t\}_{t=1}^T$ has mean zero and unit variance. In the standard wild bootstrap algorithm, it is often suggested that the random variables ξ_t should also have unit skewness. As our purpose is to illustrate and describe a valid bootstrap in our scenario, we have ignored this.*

One major and important difference between the two bootstrap algorithms is that in the latter algorithm we cannot use the residuals obtained under the alternative hypothesis, that is $\widehat{u}_{it} = \widetilde{y}_{it} - \widehat{\beta}_t' \widetilde{x}_{it}$, which is why we use $\widehat{\beta}_{FE}$ given in (2.9) there instead. It is well known that the use of residuals obtained under the null in the bootstrap, although needed to establish its validity, may suffer from inferior power properties than similar bootstraps where the residuals are computed under the alternative hypothesis. Indeed this is corroborated in our simulation results and reinforces the observation that for bootstrapped tests to have good power properties the residuals should be computed under the alternative hypothesis when possible. The heuristic explanation for this comes from the observation that residuals that are computed under the null hypothesis will not "estimate" the true error term when the alternative hypothesis is true.

In both bootstrap algorithms, specifically as it relates to *STEP* 3 and *STEP* 3′, we have kept $y_{i,t-\ell}$ as an explanatory covariate instead of $y^*_{i,t-\ell}$ as is typically done in time series data, see e.g. Neumann and Kreiss (1998).

We conclude this section by providing a bootstrap estimator for V_2, and hence $V_1 = \Sigma_x V_2 \Sigma_x$, for use in our tests given in (3.5). To that end, suppose that we compute $\widehat{\beta}^*_{FE}$, as in *STEP* 4, for B bootstrap samples *STEPS 2* and *3*, that is

$$\widehat{\beta}^*_{FE}(b) = \left(\sum_{i=1}^{n} \sum_{t=1}^{T} \widetilde{x}_{it} \widetilde{x}_{it}' \right)^{-1} \left(\sum_{i=1}^{n} \sum_{t=1}^{T} \widetilde{x}_{it} \widetilde{y}^*_{it}(b) \right), \quad b = 1, ..., B,$$

where

$$\widetilde{y}^*_{it}(b) = \sum_{\ell=1}^{k_1} \widehat{\rho}_{MG,\ell} \widetilde{y}_{i(t-\ell)} + \widehat{\theta}'_{MG} \widetilde{z}_{it} + u^*_{it}(b), \quad i = 1, ..., n, \quad t = 1, ..., T,$$

and $\{U^*_t(b)\}_{t=1}^T$ with $U^*_t(b) = \{u^*_{it}(b)\}_{i=1}^n$. The estimate for V_2 we may use in our tests (3.5) then is given by

$$\widehat{V}^*_2 = \frac{1}{B} \sum_{b=1}^{B} \left(\widehat{\beta}^*_{FE}(b) - \frac{1}{B} \sum_{v=1}^{B} \widehat{\beta}^*_{FE}(v) \right)^2$$

which would replace $\widehat{V}_2$ when making inferences.

5. **FINITE SAMPLE BEHAVIOUR.**

In this section we present a Monte-Carlo experiment that illustrates the performance of our tests in finite samples. We consider the typical weighting functions $w(\tau) = 1$ and $w(\tau) = \tau^{-1/2}(1-\tau)^{-1/2}$ and we compare the bootstrap algorithms used to obtain valid critical values, revealing that both typically outperform the use of asymptotic critical values.

The data generating processes we consider are

$$\begin{aligned} DGP1 &: \quad y_{it} = \alpha_t + \eta_i + \rho y_{i,t-1} + \theta z_{it} + \delta_\theta z_{it} 1(t > t_0) + u_{it} \\ DGP2 &: \quad y_{it} = \alpha_t + \eta_i + \rho y_{i,t-1} + \delta_\rho y_{i,t-1} 1(t > t_0) + \theta z_{it} + u_{it} \end{aligned}$$

for $i = 1, ..., n$ and $t = 1, ..., T$. We allow for breaks in the slope of the strictly exogenous variable z_{it} (δ_θ) and the lagged dependent variable y_{it-1} (δ_ρ) and consider different scenarios for the time of the break (t_0). The time fixed effects α_t and individual fixed effects η_i are drawn independently

($\alpha_t \sim IIDN(1,1)$ and $\eta_i \sim IIDN(1,1)$) and are held fixed across replications. The regressor, z_{it}, is a strictly exogenous regressor generated as

$$z_{it} = \alpha_t + v_{it} \text{ with } v_{it} = \rho_{z_i} v_{i,t-1} + \sqrt{1-\rho_{z_i}^2}\vartheta_{it}$$

and either (i) $\rho_{z_i} = 0$ (no temporal dependence), (ii) $\rho_{z_i} = 0.5$ or 0.9 (individual-homogenous autoregressive time dependence), or (iii) $\rho_{z_i} \sim IIDU[0.05, 0.95]$ (individual-heterogeneous autoregressive time dependence). Several cross-sectional dependence scenarios are considered for z_{it} (ϑ_{it}) : no spatial dependence, weak spatial dependence and strong spatial dependence. In the absence of cross-sectional dependence, ϑ_{it} (and therefore z_{it}) is $IIDN(0, \sigma_{z_i}^2)$ for $i = 1, .., n$ with $\sigma_{z_i}^2 = 1$. We consider two weak spatial dependence formulations. First we follow Lee and Robinson (2013). Here random locations for individual units are drawn along a line, denoted $s = (s_1, ...s_n)'$ with $s_i \sim IIDU[0, n]$. Keeping these locations fixed across replications, ϑ_{it} are generated independently as scalar normal variables with mean zero and covariances $cov(\vartheta_{it}, \vartheta_{jt}) = \sigma_{z_i}\sigma_{z_j}(0.5)^{|s_i - s_j|}$, ensuring z_{it} exhibits an exponential decay in dependence with distance across individuals. Second, we consider a polynomial decay of dependence in z_{it} with distance across individuals. Using the linear time dependence representation, $\vartheta_{it} = \sigma_i \left(\sum_{\ell=1}^{\infty} c_\ell\left(i\right) e_{\ell t}\right)$, we chose $c_\ell(i) = |s_\ell - s_i|^{-10}$ where s_i and s_ℓ are random locations (drawn independently from $IIDU[0, n]$) and $e_{\ell t} \sim IIDN(0, 1)$; σ_i is such that $Var(\vartheta_{it}) = \sigma_{z_i}^2$. For the strong spatial dependence setting, we use $c_\ell(i) = |s_\ell - s_i|^{-0.9}$ instead.[2]

While not allowing for any temporal dependence in u_{it}, we consider the same scenarios for the cross-sectional dependence for the error term where, in the absence of cross-sectional dependence, we assume $u_{it} \sim IIDN(0, \sigma_{u_i}^2)$ for $i = 1, ..., n$ with $\sigma_{u_i}^2 = 1$. The earlier discussion of the cross-sectional dependence scenarios for ϑ_{it} then, suitably modified, holds for u_{it}.

In the tables below, we report empirical size and power of our tests at the nominal 5% level for various pairs of n and T using 10,000 simulations. The columns labelled $\mathcal{T}_\varepsilon$ relate to the $CUSUM$ based test, while $\mathcal{T}_\varepsilon^\Delta$ relate to the associated Hausman-Durbin-Wu type, or slope based, test. When $\varepsilon = 0$, they present the untrimmed version of the tests with $w(\tau) = 1$; for the trimmed versions of the test ($\varepsilon > 0$) we apply $w\left(\tau\right) = \tau^{-1/2}\left(1-\tau\right)^{-1/2}$. Under the null $H_0 : \delta = 0$ with $\delta = (\delta_\rho | \delta_\theta)'$ both DGPs are identical. We let $\rho = 0.5$ and $\theta = 1$.

In this paper, we only report the simulation results for the base case in which our strictly exogenous regressor z_{it} does not exhibit any temporal dependence. This allows us to focus on the impact the cross-sectional dependence of z_{it} and u_{it} have on our tests.[3] The empirical size of our tests for the joint null $H_0 : \delta = 0$ against $H_0 : \delta \neq 0$ in either DGP is provided in Table 1.

Insert Table 1 around here

[2] In the polynomial case, we use $\max(1, |s_\ell - s_i|)$ as our measure of distance; not imposing such a censoring would remove all dependence in settings where for some (ℓ, i) s_ℓ and s_i lie very close together.

[3] Simulations that allow for heterogeneity across individuals (i.e., non-constant $\sigma_{z_i}^2$ and $\sigma_{u_i}^2$) or temporal dependence of z_{it} are available on our supporting website http://personal.lse.ac.uk/schafgans/tba. We also include simulations that suggest our tests are robust to the presence of fixed individual heterogeneity in z_{it}.

The exact asymptotic critical values from Estrella (2003) with $p = 2$ are used to obtain the empirical size of the trimmed version of the test. They suggest that in finite samples, the *CUSUM* based test is undersized for all cross-sectional dependence scenarios; the slope based test on the other hand appears oversized when n is quite small $(n = 25)$, especially in the presence of stronger cross-sectional dependence. The empirical sizes based on the two bootstrap algorithms are given for both the trimmed and untrimmed versions of the test. In general, the empirical size of our tests based on the bootstrap algorithm are much closer to the nominal size, with the Efron bootstrap yielding in most scenarios an empirical size closest to the nominal size. For example, with small sample sizes ($n = T = 25$) the empirical size of the untrimmed *CUSUM* test $\mathcal{T}_0$ based on the Efron bootstrap equals 0.047 in the absence of spatial dependence, 0.048 and 0.051 in the presence of respectively exponential and polynomial weak spatial dependence, versus 0.043 in the presence of strong spatial dependence. The performance of the $\mathcal{T}_\varepsilon^\Delta$ test vis-a-vis the $\mathcal{T}_\varepsilon$ test suggests a worsening of the coefficient based test with the level of spatial dependence. For small sample sizes ($n = T = 25$) the empirical size of the untrimmed coefficient test, $\mathcal{T}_0^\Delta$, based on the Efron bootstrap equals 0.044 in the absence of spatial dependence, 0.041 and 0.038 in the presence of respectively exponential and polynomial weak spatial dependence, versus 0.013 in the presence of strong spatial dependence. For the $\mathcal{T}_\varepsilon^\Delta$ test to remain properly sized, the cross sectional sample needs to be larger when the level of spatial dependence increases. The simulations do reveal fluctuation in the empirical sizes associated with the level of trimming of our test. Increasing the trimming generally improves the size of the tests with $w(\tau) = \tau^{-1/2}(1-\tau)^{-1/2}$ but this obviously limits the possibility of detecting a break closer to the end of the sample due to this trimming. In view of this, the good performance of the untrimmed tests with $w(\tau) = 1$ is useful. The results for the empirical size are comparable to those obtained in the absence of individual fixed effects (see Hidalgo and Schafgans, 2015).

We present the empirical size of the slope-based test for the associated individual hypotheses $H_0 : \delta_\theta = 0$ (DGP1) and $H_0 : \delta_\rho = 0$ (DGP2) on our accompanying website. Exact asymptotic critical values for the untrimmed individual tests are based on asymptotic critical values from $\sup_\tau |BB(\tau)|$ (with $\sup_\tau \sqrt{(H\mathcal{T}^\Delta(r))' \left(H\widehat{V}_2 H'\right)^{-1} H\mathcal{T}^\Delta(r)} \overset{d}{\Longrightarrow} \sup_\tau |BB(\tau)|$ with $H = (1:0)$ and $(0:1)$, respectively); for $\varepsilon > 0$ we use Estrella (2003) with $p = 1$. The empirical sizes of the individual coefficient tests are comparable for δ_θ and δ_ρ and both are of the same order of magnitude as the joint test size. The rejection rates for the untrimmed tests based on the asymptotic values of the supremum of the Browning bridge are generally larger than the rejection rates for the trimmed tests relying on Estrella's exact asymptotic critical values. This is also the case for the rejection rates associated with the Wild bootstrap algorithm.

Table 2 presents the power of our tests, when the break is either in the middle, $t_0 = [0.5T]$, or in the second half of the sample, $\tau_0 = [0.8T]$, for DGP1 (where we only have a break in the slope of the strictly exogenous variable z_{it}) with $\delta_\theta = 0.5$ and $\delta_\rho = 0$. The table provides the power of the joint hypothesis for the *CUSUM* test, $\mathcal{T}_\varepsilon$, and the slope-based test, $\mathcal{T}_\varepsilon^\Delta$. The power (size) of the associated individual slope-based tests are available from our accompanying website. In Table 2, we observe that even with small sample sizes our tests have high power in detecting a break in

θ.

Insert Table 2 around here

As expected, the power is lower when the break lies closer to the end of the sample. Using the Efron bootstrap algorithm the power is 0.984 for $\mathcal{T}_{0.10}$ and 0.854 for $\mathcal{T}^{\Delta}_{0.10}$ in the absence of spatial dependence when the break lies in the middle of the sample, against 0.864 for $\mathcal{T}_{0.10}$ and 0.679 for $\mathcal{T}^{\Delta}_{0.10}$ when the break lies towards the end of the sample. Moreover, the power decreases with the cross-sectional dependence. In general, the power of the $\mathcal{T}_{\varepsilon}$ test is higher than the $\mathcal{T}^{\Delta}_{\varepsilon}$ test using the bootstrap based critical values. Nevertheless, when focussing on the power associated with the individual coefficient test ($H_0 : \delta_\theta = 0$), the $\mathcal{T}^{\Delta}_{\varepsilon}$ test performs comparable to the $\mathcal{T}_{\varepsilon}$ test in detecting the break in θ. Clearly the power of an individual coefficient based test for a single break is higher than the power of a joint coefficient based test. When both n and T equal 100, the power of the tests (joint $\mathcal{T}_{\varepsilon}$ and $\mathcal{T}^{\Delta}_{\varepsilon}$ and individual for $\mathcal{T}^{\Delta}_{\varepsilon}$) equals 1 for all but the strong spatial dependence setting in which case it is close to one. Finally, the empirical power of our tests based on the Efron bootstrap typically exceeds the Wild bootstrap based ones as expected. The power of observing a break in the slope of z_{it} is similar whether we include the individual fixed effect or not (see Hidalgo and Schafgans, 2015).

Table 3, by symmetry, presents the power of our tests, when the break is either in the middle, $t_0 = [0.5T]$, or in the second half of the sample, $\tau_0 = [0.8T]$, for DGP2 with $\delta_\rho = 0.1$ and $\delta_\theta = 0$. The table again provides the power of the joint hypothesis for the *CUSUM* test, $\mathcal{T}_{\varepsilon}$, and the slope-based test, $\mathcal{T}^{\Delta}_{\varepsilon}$, while the power (size) of the associated individual slope-based test are available from our accompanying website. In Table 3, we observe that for both tests, the power of detecting our break in the autoregressive coefficient is smaller than the power of the break we considered in the slope of the strictly exogenous regressor.

Insert Table 3 around here

For example, in the presence of exponential weak dependence and small samples ($n = T = 25$), $\mathcal{T}_0$ reveals a 0.129 power of detecting our break in ρ based on Efron bootstrap against a 0.770 power of detecting our break in θ. Obviously a larger break in ρ would be easier to detect. The power of the $\mathcal{T}_{\varepsilon}$ test is again generally higher than $\mathcal{T}^{\Delta}_{\varepsilon}$ test using the bootstrap based critical values, a difference which is reduced when focussing on the power associated with the single coefficient test ($H_0 : \delta_\rho = 0$). For $n = T = 100$, we observe that the loss in power of detecting a break in the autocorrelation coefficient increases with the amount of spatial dependence. For instance, using $\mathcal{T}^{\Delta}_0$ the power of detection based on the Efron bootstrap drops from 1.00 in absence of spatial dependence, to 0.945 and 0.925 in the weak spatial dependence setting to 0.401 in the strong spatial dependence setting. For the T^{Δ}_{ε} test, the power of detecting a break in the autocorrelation coefficient is, as expected, smaller when the break is later in the sample. While this holds for the untrimmed *CUSUM* base test $\mathcal{T}_0$ as well, there appear some nonlinearities for the trimmed *CUSUM* based test that are absent when there are no individual fixed effects (see Hidalgo and Schafgans, 2015).

As our simulations on our accompanying website reveal our tests seem robust to the presence of fixed individual heterogeneity in z_{it}, and the introduction of heterogeneity has little impact on the size of our tests while typically enhancing the power of our test in the presence of spatial dependence. The introduction of (individual-heterogeneous) autoregressive time dependence of the regressor z_{it} also has little impact on the size of our tests. The introduction of time dependence in z_{it} is typically accompanied by a reduction in the power of our tests to detect a break in θ in small samples, which is slightly more pronounced for the coefficient based test, $\mathcal{T}_{\varepsilon}^{\Delta}$, and is strongest in the absence of spatial dependence. The deterioration of the power of detecting a break in θ associated with the presence of (individual-heterogeneous) autoregressive time dependence is less strong in the absence of individual fixed effects (see also Hidalgo and Schafgans, 2015). Increasing the time dependence in z_{it} generally reduces the power to detect such a break in the presence of weak and strong dependence; only when both individual and time fixed effects are included do we observe a nonmonotonic relationship when there is strong dependence. In absence of cross sectional dependence, the power of our tests to detect a break in the autocorrelation coefficient also deteriorates in the presence of time dependence in z_{it}. In the presence of spatial dependence, on the other hand, the power of our tests to detect such a break typically increases with the level of time dependence in the regressor when $n = T = 100$, although this is not always the case when the sample is small. These results are obtained in the absence of individual effects as well.

6. **AN APPLICATION TO ECONOMIC GROWTH DATA**

In this empirical illustration we apply our test for structural break to a growth regression equation. Since spatial correlations are all-pervasive in international trade, it is important in growth regression analysis to account for the presence of cross-sectional dependence. While, for example, Yu and Lee (2012) and Parent and LeSage (2012) attempt to model regional spillovers using a spatial autoregressive setup (both specify a row-normalized contiguity weighting matrix associated with US states sharing common borders), we allow our error terms to exhibit a more general, and potentially strong, cross-country correlation structure which does not require us to specify the exact dependence structure.

Specifically, we consider the dynamic panel model

$$g_{Y,it} = \sum_{\ell=1}^{k_1} \rho_{\ell,t} g_{Y,it-\ell} + \beta_{L,t} g_{L,it} + \beta_{K,t} g_{K,it} + \beta_{H,t} g_{H,it} + \alpha_i + \gamma_t + \varepsilon_{it},$$
$$i = 1, .., n, \; t = 1, ..., T$$

where g_Y denotes the growth in GDP, g_L the growth in labour, and g_K and g_H the growth in physical and human capital, respectively. To account for business cycle fluctuations, we allow for temporal dependence by relating growth in GDP to past growth in GDP. The country fixed effects account for differences in technology or taste across countries, alleviating the endogeneity issues inherent in cross-sectional growth regressions, while the time fixed effect accounts for macroeconomic shocks. Unlike Su and Chen (2013), who allow for interactive fixed effects, we consider the usual additive fixed effects structure in accordance with our theoretical setup.

We use annual data from the Penn World Table 8.1 (see also Feenstra et al., 2015) and consider a country sample (NONOIL) similar to that used by Islam (1995) and Mankiw, Romer and Weil (1992); the sample excludes countries for which oil production is the dominant industry. We use the $RGDP^{NA}$ series to evaluate the growth in GDP of economies over time (measured in constant national prices, obtained from national accounts data for each country). For the factors of production, we use the series: EMP to measure growth in labour (employment), RK^{NA} (measured in constant national prices) to measure growth in physical capital and HC (a measure based on average years of schooling from Barro and Lee, 2012) to measure growth in human capital. The data spans the period 1961-2011. Data availability on employment, in particular, reduced our country sample relative to that of Islam (1995).[4] We use employment as it is the more appropriate variable to use to measure the growth in labour. In this we follow, e.g., Zhang, Su and Phillips (2012), who test for common trends in panel data with fixed effects using OECD growth data.

In Table 4 our test results are presented, where we consider two values for the number of lagged dependent variables, k_1, namely $k_1 = 1, 2$. The table presents the location of our break (date) and the associated p-values of our tests as indicated by our asymptotic critical values or obtained by our proposed bootstrap methods. We provide both trimmed ($\varepsilon > 0$) and untrimmed ($\varepsilon = 0$) variants of our tests $\mathcal{T}_\varepsilon$ and $\mathcal{T}_\varepsilon^\Delta$ and for the Hausman type test, $\mathcal{T}_\varepsilon^\Delta$, we consider both the joint test for homogeneity of (β, ρ) with $\beta = (\beta_L, \beta_K, \beta_H)$ and the 'individual' tests on β and ρ separately.

For the Hausman based tests (joint or individual) there is strong evidence for a structural break following the conclusion of the Uruguay Round in 1995/6. It is widely regarded as the most profound institutional reform of the word trading system since the GATT's establishment, which tackled trade barriers covering trade in all goods, not just manufactured products but included agricultural and textile products as well. It saw the phase-out of the multi-fibre arrangement governing trade in textiles and imposed rules and disciplines on agricultural subsidies and the GATT rules were extended to cover trade in services and intellectual property rights, see also Bowen et al. (1998). While the untrimmed $CUSUM$ based test with $k_1 = 2$ supports this finding (with p-values of 0.005) the trimmed $CUSUM$ based test detects the break earlier in the mid to late 60s around the less influential GATT Kennedy trade Round. When we restrict the sample to the period 1970-2011, both the $CUSUM$ based test (trimmed and untrimmed) and the Hausman based test (untrimmed) find evidence of a break in 1995 (though not significant using the trimmed version of the test). Evidence of a break around the dot com bubble, 2001, is found as well, e.g., in the trimmed Hausman based test for ρ and the joint trimmed Hausman test when $k_1 = 1$. When

[4] In recognition of the limited availability of the working age population (used by Mankiw, Romer and Weil, 1992) and employment, Islam decided to use total population instead. Our NONOIL sample includes: Australia, Austria, Argentina, Bangladesh, Belgium, Bolivia, Brazil, Cameroon, Canada,Chile, Colombia, Congo, Costa Rica, Côte d'Ivoire, Denmark, Dominican Republic, Ecuador, Egypt, Finland, France, Germany, Ghana, Greece, Guatemala, Hong Kong, India, Indonesia, Ireland, Israël, Italy, Jamaica, Japan, Jordan, Kenya, Malawi, Malaysia, Mali, Mexico, Morocco, Mozambique, Netherlands, New Zealand, Niger, Norway, Pakistan, Peru, Phillippines, Portugal, Republic of Korea, Senegal, Singapore, South Africa, Spain, Sri Lanka, Syrian Arab Republic, Sweden, Switzerland, Thailand, Tanzania, Trinidad and Tobago, Tunisia, Turkey, UK, Uganda, Uruguay, USA, Venezuela, Zambia, Zimbabwe.

comparing the p-values for our tests, we observe that we typically have a higher power to detect a break in $(\beta_L, \beta_K, \beta_H)$ when using the Efron bootstrap algorithm instead of the Wild bootstrap as expected. This also corresponds to our observations from the Monte Carlo simulations in the presence of strong (weak) spatial dependence. For the $\mathcal{T}_{\varepsilon}^{\Delta}$-test there appears a stronger evidence of a structural break using the asymptotic critical values than indicated when using the bootstrap. In our Monte Carlo simulation we also found the power for this test, in the presence of strong spatial dependence, to be lower using the bootstrap than suggested by the asymptotic distribution. Given the sample size for this empirical example, we therefore need to be somewhat careful with the use of the asymptotic critical values.

Overall, the ability for our tests to detect meaningful structural breaks in the presence of cross-sectional dependence seems to be confirmed by these results.

Insert Table 4 around here

7. CONCLUSIONS AND EXTENSIONS

The paper has examined several issues related to inference in large dynamic panel data models. Specifically, we have developed a Central Limit Theorem for the estimators of the slope parameters when the errors and the covariates might exhibit "strong" cross-sectional dependence. To that end, we have modified existing results given in Phillips and Moon (1999) to allow for dependence in both time and cross-section dimensions. From here, we have described and examined two different, but similar, tests for the null hypothesis of homogeneity of the slope parameters of the model. Unlike the asymptotic for our the slope parameters, our tests do not suffer from the incidental parameter problem associated with the linear dynamic panel model. Because the small sample properties of the test were not very satisfactory, we have described two bootstrap algorithms with the attractive feature that their implementation does not require any previous knowledge of the cross-sectional dependence or selection of any tuning/bandwidth parameters (as is normally the case when using moving block bootstraps with time series data).

A possible limitation of the conditions we imposed is that it rules out temporal dependence for the errors. That is, we might want to change $C1$ to

$C1'$: $\{u_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$, are linear sequences of zero mean random variables given by

$$u_{it} = \sum_{\ell=0}^{\infty} a_\ell(i)\,\varepsilon_{i,t-\ell}; \qquad \sum_{\ell=0}^{\infty} |a_\ell|\,\ell^{1/2} < \infty,$$

where $a_\ell = \sup_{i\in\mathbb{N}} |a_\ell(i)|$, and $\{\varepsilon_{it}\}_{t\in\mathbb{Z}}$, $i \geq 1$, are sequences of independent distributed random variables satisfying $\sup_{i\in\mathbb{N}} E\left(\varepsilon_{it}^4\right) = \sup_{i\in\mathbb{N}} \mu_i < \infty$ and

$$\lim_{T\nearrow\infty} \sup_{i\in\mathbb{N}^+} \sum_{t_1,t_2,t_3=1}^{T} |Cum\,(u_{it_1}; u_{it_2}; u_{it_3}; u_{i0})| < \infty.$$

When we change $C1$ to $C1'$, inspections of our proofs suggests the main qualitative results of the paper would follow, all we need to do would be to employ *instrumental variable* methods or some type of Hatanaka's "efficient" estimator to estimate the parameters of the model. We have not followed this route for notational simplicity as our basic conclusions should not be affected, except that the proofs would become lengthier. The change, though, would necessitate a modification of our bootstrap algorithm to accommodate the temporal dependence of the errors $\{u_{it}\}_{t\in\mathbb{Z}}$, $i \in \mathbb{N}^+$, and the estimator of the long run variance V_1. Details of this are beyond the scope of this paper and we hope to address these issues in a different paper.

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TABLE 1. Size of the slope homogeneity test

	No spatial dependence						Weak Spatial dependence (exponential)					
Test	Asymptotic		Wild Bootstrap		Efron Bootstrap		Asymptotic		Wild Bootstrap		Efron Bootstrap	
(n, T)	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$
$(25, 25)$												
$\varepsilon = 0.00$			.055	.034	.047	.044			.057	.029	.048	.041
$\varepsilon = 0.05$	.010	.032	.039	.019	.067	.045	.009	.044	.039	.013	.054	.035
$\varepsilon = 0.10$	.008	.036	.050	.025	.058	.044	.007	.050	.049	.017	.049	.036
$(100, 25)$												
$\varepsilon = 0.00$			.040	.030	.053	.047			.049	.036	.054	.044
$\varepsilon = 0.05$	.011	.006	.042	.019	.068	.038	.008	.011	.040	.021	.056	.038
$\varepsilon = 0.10$	.009	.009	.046	.026	.060	.040	.009	.014	.047	.028	.056	.040
$(25, 100)$												
$\varepsilon = 0.00$			.058	.050	.051	.051			.062	.044	.049	.040
$\varepsilon = 0.05$	.032	.058	.043	.033	.055	.048	.033	.057	.039	.020	.058	.039
$\varepsilon = 0.10$	.028	.057	.050	.040	.051	.048	.031	.060	.051	.029	.052	.037
$(100, 100)$												
$\varepsilon = 0.00$			.057	.055	.048	.051			.051	.053	.045	.048
$\varepsilon = 0.05$	.029	.030	.036	.032	.051	.046	.025	.027	.030	.028	.047	.044
$\varepsilon = 0.10$	.029	.032	.047	.046	.049	.052	.026	.032	.045	.040	.047	.044

	Weak Spatial dependence (polynomial)						Strong Spatial dependence					
Test	Asymptotic		Wild Bootstrap		Efron Bootstrap		Asymptotic		Wild Bootstrap		Efron Bootstrap	
(n, T)	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$
$(25, 25)$												
$\varepsilon = 0.00$			.060	.024	.051	.038			.056	.016	.043	.013
$\varepsilon = 0.05$	.008	.065	.040	.013	.061	.037	.015	.160	.046	.012	.057	.012
$\varepsilon = 0.10$	.009	.072	.049	.016	.057	.039	.010	.164	.051	.014	.051	.012
$(100, 25)$												
$\varepsilon = 0.00$			.047	.035	.050	.043			.053	.028	.043	.036
$\varepsilon = 0.05$	.007	.011	.036	.021	.056	.040	.007	.034	.034	.016	.050	.033
$\varepsilon = 0.10$	.008	.015	.044	.026	.053	.041	.006	.039	.044	.021	.049	.034
$(25, 100)$												
$\varepsilon = 0.00$			.060	.040	.047	.038			.062	.027	.051	.009
$\varepsilon = 0.05$	.031	.081	.041	.022	.053	.040	.040	.126	.040	.015	.057	.009
$\varepsilon = 0.10$	.028	.081	.055	.029	.054	.043	.032	.123	.053	.018	.057	.010
$(100, 100)$												
$\varepsilon = 0.00$			.063	.056	.052	.055			.055	.043	.050	.034
$\varepsilon = 0.05$	.025	.034	.038	.033	.054	.047	.032	.034	.039	.023	.053	.031
$\varepsilon = 0.10$	.028	.037	.053	.043	.055	.050	.027	.040	.047	.035	.052	.036

Table 2. Power of the slope homogeneity test, DGP1

	No spatial dependence						Weak Spatial dependence (exponential)					
Test	Asymptotic		Wild Bootstrap		Efron Bootstrap		Asymptotic		Wild Bootstrap		Efron Bootstrap	
(n,T)	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$
$t_0 = [0.5T]$												
$(25, 25)$												
$\varepsilon = 0.00$			.996	.942	.996	.942			.867	.547	.849	.556
$\varepsilon = 0.05$	.803	.789	.958	.759	.976	.834	.280	.407	.601	.260	.645	.360
$\varepsilon = 0.10$	.862	.837	.978	.817	.984	.854	.348	.465	.708	.319	.702	.382
$(100, 100)$												
$\varepsilon = 0.00$			1.00	1.00	1.00	1.00			1.00	1.00	1.00	1.00
$\varepsilon = 0.05$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\varepsilon = 0.10$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$t_0 = [0.8T]$												
$(25, 25)$												
$\varepsilon = 0.00$			.676	.343	.690	.400			.360	.128	.337	.158
$\varepsilon = 0.05$	.515	.536	.568	.332	.838	.658	.157	.251	.270	.102	.410	.242
$\varepsilon = 0.10$	.593	.600	.710	.430	.864	.679	.194	.289	.363	.136	.447	.252
$(100, 100)$												
$\varepsilon = 0.00$			1.00	1.00	1.00	1.00			1.00	1.00	1.00	1.00
$\varepsilon = 0.05$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\varepsilon = 0.10$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

	Weak Spatial dependence (polynomial)						Strong Spatial dependence					
Test	Asymptotic		Wild Bootstrap		Efron Bootstrap		Asymptotic		Wild Bootstrap		Efron Bootstrap	
(n,T)	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$
$t_0 = [0.5T]$												
$(25, 25)$												
$\varepsilon = 0.00$			.807	.409	.795	.447			.417	.076	.368	.039
$\varepsilon = 0.05$	.220	.394	.502	.160	.601	.252	.043	.285	.155	.029	.195	.019
$\varepsilon = 0.10$	.285	.447	.621	.205	.647	.284	.056	.315	.234	.036	.242	.021
$(100, 100)$												
$\varepsilon = 0.00$			1.00	1.00	1.00	1.00			1.00	1.00	1.00	1.00
$\varepsilon = 0.05$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$\varepsilon = 0.10$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
$t_0 = [0.8T]$												
$(25, 25)$												
$\varepsilon = 0.00$			.305	.107	.294	.127			.155	.029	.128	.016
$\varepsilon = 0.05$	.128	.245	.231	.072	.379	.167	.048	.221	.096	.021	.147	.018
$\varepsilon = 0.10$	.158	.283	.337	.100	.414	.182	.050	.240	.146	.025	.168	.018
$(100, 100)$												
$\varepsilon = 0.00$			1.00	1.00	1.00	1.00			.994	.987	.994	.978
$\varepsilon = 0.05$	1.00	1.00	1.00	1.00	1.00	1.00	0.995	.995	.982	.985	.996	.994
$\varepsilon = 0.10$	1.00	1.00	1.00	1.00	1.00	1.00	0.996	.996	.994	.993	.998	.996

Table 3. Power of the slope homogeneity test, DGP2

	No spatial dependence						Weak Spatial dependence (exponential)					
Test	Asymptotic		Wild Bootstrap		Efron Bootstrap		Asymptotic		Wild Bootstrap		Efron Bootstrap	
(n,T)	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$
$t_0 = [0.5T]$												
$(25, 25)$												
$\varepsilon = 0.00$			.079	.058	.095	.103			.067	.040	.071	.065
$\varepsilon = 0.05$	.020	.048	.065	.029	.109	.091	.018	.060	.063	.020	.084	.059
$\varepsilon = 0.10$	.021	.057	.087	.042	.113	.096	.016	.068	.072	.026	.083	.063
$(100, 100)$												
$\varepsilon = 0.00$			.999	.999	.999	.999			.891	.895	.906	.913
$\varepsilon = 0.05$	.992	.995	.993	.995	.994	.997	.732	.764	.755	.777	.800	.830
$\varepsilon = 0.10$	.994	.996	.996	.996	.998	.998	.768	.795	.816	.815	.835	.854
$t_0 = [0.8T]$												
$(25, 25)$												
$\varepsilon = 0.00$			.094	.046	.081	.060			.074	.036	.068	.050
$\varepsilon = 0.05$	.045	.055	.120	.035	.174	.092	.030	.064	.089	.022	.117	.059
$\varepsilon = 0.10$	.048	.063	.159	.052	.180	.093	.030	.072	.107	.032	.114	.062
$(100, 100)$												
$\varepsilon = 0.00$			1.00	1.00	1.00	1.00			.981	.961	.977	.945
$\varepsilon = 0.05$	1.00	1.00	1.00	1.00	1.00	1.00	.988	.977	.979	.979	.992	.987
$\varepsilon = 0.10$	1.00	1.00	1.00	1.00	1.00	1.00	.990	0.981	.991	.988	.994	.988

	Weak Spatial dependence (polynomial)						Strong Spatial dependence					
Test	Asymptotic		Wild Bootstrap		Efron Bootstrap		Asymptotic		Wild Bootstrap		Efron Bootstrap	
(n,T)	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$	$\mathcal{T}_\varepsilon$	$\mathcal{T}_\varepsilon^\Delta$
$t_0 = [0.5T]$												
$(25, 25)$												
$\varepsilon = 0.00$			.071	.033	.076	.064			.060	.023	.062	.019
$\varepsilon = 0.05$	.016	.077	.062	.016	.088	.053	.016	.144	.049	.013	.057	.016
$\varepsilon = 0.10$	.016	.087	.076	.022	.088	.058	.013	.156	.060	.015	.061	.016
$(100, 100)$												
$\varepsilon = 0.00$			.866	.852	.879	.869			.372	.329	.379	.327
$\varepsilon = 0.05$	.678	.716	.726	.720	.769	.784	.192	.210	.206	.195	.258	.221
$\varepsilon = 0.10$	.715	.751	.788	.773	.802	.808	.211	.237	.279	.235	.294	.240
$t_0 = [0.8T]$												
$(25, 25)$												
$\varepsilon = 0.00$			.079	.031	.071	.049			.070	.021	.056	.016
$\varepsilon = 0.05$	.025	.085	.084	.018	.117	.056	.028	.189	.076	.015	.090	.017
$\varepsilon = 0.10$	.026	.095	.106	.024	.119	.058	.024	.202	.088	.017	.095	.017
$(100, 100)$												
$\varepsilon = 0.00$			.970	.937	.966	.918			.614	.444	.583	.373
$\varepsilon = 0.05$	.975	.959	.966	.960	.983	.973	.636	.549	.612	.521	.686	.564
$\varepsilon = 0.10$	.977	.966	.982	.972	.988	.976	.658	.595	.703	.597	.722	.588

Table 4. Empirical application: growth model

	NONOIL N=69,T=52 (1960–2011)							
	$k_1 = 1$				$k_1 = 2$			
		p-values				p-values		
	date	Asy	Wild	Efron	date	Asy	Wild	Efron
$\mathcal{T}_\varepsilon$ -test								
$\varepsilon = 0.00$	1975		.000	.002	1995		.000	.005
$\varepsilon = 0.05$	1965	$< .05$	.034	.000	1965	$< .05$	.045	.001
$\varepsilon = 0.10$	1970	$< .05$	.004	.000	1970	$< .05$	.005	.000
$\mathcal{T}_\varepsilon^\Delta$ -test (β, ρ)								
$\varepsilon = 0.00$	1995		.019	.008	1995		.012	.014
$\varepsilon = 0.05$	2001	$< .01$	.067	.047	1996	$< .01$	.103	.115
$\varepsilon = 0.10$	2001	$< .01$	.058	.032	1996	$< .01$	.068	.079
$\mathcal{T}_\varepsilon^\Delta$ -test (β)								
$\varepsilon = 0.00$	1995		.011	.011	1995		.005	.002
$\varepsilon = 0.05$	1995	$< .01$	.054	.057	1995	$< .01$	.033	.027
$\varepsilon = 0.10$	1995	$< .01$	.045	.032	1995	$< .01$	.021	.011
$\mathcal{T}_\varepsilon^\Delta$ -test (ρ)								
$\varepsilon = 0.00$	1996	$< .01$	.020	.008	1996		.006	.013
$\varepsilon = 0.05$	2001	$< .01$	.058	.023	2001	$< .01$	.058	.051
$\varepsilon = 0.10$	2001	$< .01$	.054	.021	2001	$< .01$	.037	.040