Qi Zhang

The Balassa–Samuelson relationship: services, manufacturing and product quality

Article (Accepted version)
Refereed

Original citation:
Zhang, Qi (2017) *The Balassa–Samuelson relationship: services, manufacturing and product quality.* Journal of International Economics, 106 . pp. 55-82. ISSN 0022-1996

DOI: 10.1016/j.jinteco.2016.10.004

The Balassa-Samuelson Relationship: A Micro-Analysis[☆]

Qi Zhang[*]

London School of Economics, 32 Lincoln's Inn Fields, London WC2A 2AE, UK

University of Oxford, Manor Road Building, Oxford OX1 3UQ, UK

Abstract

This paper shows that the specification of the Balassa-Samuelson relationship, i.e. the positive relationship between a country's per capita income and its national price level, at the level of the products used in constructing national price levels is different for 'services' and 'manufactures'. It further offers a new candidate explanation for the B-S relationship in manufactures, which appeals to mismeasured quality. This explanation yields a second, distinctive, testable prediction: controlling for per capita income, a non-monotonic relationship should exist between a country's income inequality and its national price level. I show that this second prediction is consistent with empirical evidence.

Keywords: price level; income distribution; inequality; quality; Balassa-Samuelson.

JEL classifications: C43; D31; E31; F31; L15.

[☆]I thank the World Bank for granting me access to the ICP 2005 benchmark dataset. Financial support from the Dorothy Hodgkin Postgraduate Awards at the London School of Economics are gratefully acknowledged.

[*]*Email address*: `q.zhang4@lse.ac.uk`

1. Introduction

The Balassa-Samuelson (B-S) relationship, first introduced in 1964[1], links the per capita income level of a country to its price level, as measured by a broad price index. Balassa and Samuelson showed that as we move towards richer countries, the measured price level becomes higher (a.k.a. the Penn effect). This represents an apparent violation of Purchasing Power Parity (PPP). Balassa and Samuelson proposed an explanation based on an appeal to the presence of a service component in the national price level. Although rich countries are relatively more productive in the traded goods sector, it is hard for them to establish technological superiority in the labor-intensive service sector. Therefore, the prices of services will reflect local wages, and will rise relative to the prices of traded goods as we move from poor countries to rich countries.[2]

The point of departure of the present paper lies in a purely empirical analysis of the standard dataset used in the literature. By plotting the B-S relationship for each of the individual products used in constructing the standard national price level, it is shown that the form of the relationship differs sharply between two groups of products. Goods in the first group are essentially 'pure services' such as hairdressing. Goods in the second group are essentially non-services, such as manufactures. It should be expected that goods in the first group, all of whose costs are 'local', should not exhibit PPP. By deleting this (large) group, I recalculate a new price level. As I am left primarily with manufactures, the B-S relationship is not expected to hold. Yet I show that the B-S relationship remains intact for the new price level, i.e. PPP is violated.

The central aim of this paper is to propose a new candidate explanation for the apparent violation of PPP exhibited in the B-S relationship. Since this explanation, unlike the traditional explanation, does not appeal to the presence of a 'local services' element in the national price level, it is natural to test the present model using both the standard price level, and the modified price level that excludes the 'services type' products. Thus, I am not excluding the part played by services industries in driving the B-S relationship. Rather, I am suggesting that it is not the whole story.

The candidate explanation for the apparent violation of PPP that I develop in what follows, is based on an appeal to 'mismeasured quality'. The idea that quality is not fully adjusted for in conventional price indices is a long standing theme in the industrial organization literature. The idea can be traced to the hedonic price literature (Griliches, 1961), and it has been re-explored in recent work by Pakes (2003, 2005).

[1] Balassa (1964) and Samuelson (1964).

[2] This classic explanation has been heavily criticized in the literature. For example, Rogoff (1996) showed that although there is empirical support for the model when comparisons are made between the set of 'all poor countries' and the set of 'all rich countries', this effect is not statistically significant within either the poor countries group or the rich countries group.

In the context of calculating national price levels, the ICP (International Comparison Program) has made process in controlling for quality since the introduction of structured product descriptions (SPDs) in the 2005 round. The SPDs are used to define products, the prices of which are collected in every country. The ideal is to have the descriptions as precisely defined as possible, so that the prices collected based on these are comparable across countries. However, the design of SPDs has to balance the trade-off between comparability and representativity, so SPDs cannot be designed precisely enough to pin down quality. For example, due to different expenditure patterns in different countries, maybe a precisely defined item within an expenditure category is a typical item in the consumption basket of country A and hence is representative of its expenditure pattern, but the item may not be representative in country B, even though its price might be observable in country B. In this case, instead of using the precisely defined description to match items between the two countries and missing the criteria of representativity, the ICP may use a less precisely defined product description so that both countries can have representative products to match the new description. As a result, the important characteristics that have caused quality to exist in the disaggregated trade prices as shown in the literature, such as producer characteristics (brand and country of origin), are generally missing in the SPDs. Therefore, SPDs still leave large room for cross-country quality variations.

In the present setting, failure to make a full adjustment for quality levels means that, since the average quality level of goods consumed is higher in richer countries, a spurious correlation will appear between a country's income level and its price level.

The model developed in this paper is a hedonic pricing model *à la* Rosen (1974). The key assumption is that income elasticity of quality is positive and is higher for nontraded goods. The motivation for this assumption is discussed below.

Within this model, we will examine the relation between a country's income level, and a price level that is not fully quality adjusted. Given that consumers in the countries with higher per capita income tend to spend more on the nontraded ('quality') goods, this will imply a higher measured price level in richer countries (the B-S or Penn effect). While this first result is intuitively obvious, the model also leads to a less obvious, and distinctive, prediction: controlling for per capita income, income inequality also has an effect on the national price level and the effect is decreasing in per capita income. The intuition behind the second prediction is that in the model of Rosen (1974), where consumers' demand for quality rises in income and the price schedule of quality depends on the distribution of income, the average price of quality in the market will be the average paid by consumers with different income levels and hence will depend on the distribution of income. In testing the empirical predictions of the theory in what follows, I first show that the second prediction is consistent with empirical evidence using a price level based on all expenditure categories. I then repeat the exercise using a modified price level which excludes the 'pure services' categories. It is shown that even when services are excluded, the B-S relationship survives, and the second distinctive prediction of the present theory is consistent with the evidence.

This paper contributes to the international pricing literature by providing an alternative candidate explanation for the national price level, which is based on mismeasured quality. In the existing literature, international price differentials, i.e. the violation of PPP, have been explained by transportation and distribution costs. Crucini and Yilmazkuday (2014) use the retail prices of products with the same characteristics to show that transportation and distribution costs account for over 50% of international price dispersion at the aggregate level. Variable mark-ups have also been used to explain why the prices of the same product vary across countries. In Simonovska (2015), the mark-up of the same apparel or footwear rises with a country's per capita income because per capita income increases the consumption of each positively-consumed variety and drives down the elasticity of substitution between these varieties. In Alessandria and Kaboski (2011), firms charge higher mark-ups for consumers in richer countries as these consumers value time at a premium and hence they are less willing to spend time on searching for low prices. Hummels and Lugovskyy (2009) model the positive relationship between mark-ups and per capita income in the ideal varieties framework, in which consumers with higher income are willing to spend more to get closer to their ideal variety. In addition, a series of papers in the literature have used quality to explain international price differentials. Goldberg and Verboven (2001) and Imbs et al. (2010) show quality accounts for a large share of the international dispersion in the prices of cars and TVs. Schott (2004) uses product level U.S. import data to show unit values are higher for exporters with relatively high productivity. This finding contradicts the standard trade theory and suggests the role of quality in explaining the unit values. Crozet, Head and Mayer (2012) introduce directly measured quality to the Melitz (2003) model of firm heterogeneity in the case of French Champagne industry and show that quality increases firm-level prices even controlling for productivity. Hallak (2006) shows that countries with high per capita income tend to buy goods with higher qualities, which imply higher prices. Hallak and Schott (2011), Khandelwal (2010) and Feenstra and Romalis (2014) disentangle quality from trade unit values and show the important role of quality in explaining the cross-country variations in unit values. In contrast, Sutton and Trefler (2016) identify the quality of a good using a product range, i.e. the range of incomes defined by the income levels of the poorest and richest exporters of the good, and show how quality affects the mark-up of the good in a general equilibrium framework. Choi, Hummels and Xiang (2009) use quality to explain the correlation between income distribution and the distribution of import prices. Bekkers, Francois and Manchin (2012) also study the relationship between import prices and two moments of income distribution: per capita income and income inequality, but they focus on the role of hierarchic demand[3] in driving the relationship. Similarly, this paper also tries to link income distribution with prices, but instead of the prices of imports, the emphasis is on the national price level. Fajgelbaum, Grossman and Helpman (2011) provide a demand-based explanation for the pattern of trade in vertically differentiated goods, while in this paper a demand-based explanation is given for the role of vertically differentiated goods in the national price level. Bergstrand (1991) distinguishes empirically the competing theories of the national price level, which include the supply-based explanations by Balassa (1964), Samuelson (1964) and Bhagwati (1984) and a demand-based explanation. And he finds empirical support for the demand-based explanation.[4]

The rest of this paper is structured as follows. Section 2 develops the basic empirical analysis leading to the splitting of the product set into 2 groups. Section 3 presents the model, in which the price schedule of vertically differentiated goods is endogenously determined by income distribution. The market equilibrium is used to examine how income distribution affects price levels and expenditure shares. Section 4 tests the model's predictions at the aggregate level and at an intermediate level. Section 5 concludes.

2. The B-S Relationship at the Product Level

The point of departure of the present paper lies in a purely empirical analysis of the standard dataset used in the literature. More specifically, the B-S relationship is explored at the disaggregate level. The disaggregate price levels used in this paper are from the International Comparison Program (ICP) 2005 benchmark dataset, which includes the price levels of 129 basic headings for 146 countries.[5] Basic headings are defined as the most disaggregate price level, at which there exists matching expenditure data from national accounts. These price levels of basic headings are just the disaggregate price levels underlying the national price level in the Penn World Table.

One of the standard specifications characterizing the B-S relationship is

$$p = a + bw, \tag{1}$$

[3]Consumers extend their set of goods consumed as income rises.

[4]Hassan (2015) shows that the pattern of structural change can lead to an U-shape relationship between per capita income and national price levels. He mainly uses the supply side to explain national price differentials resulting from structural change. My demand-side model and empirics complements the supply-side model and empirics by Hassan (2015). In particular, when I include covariates intended to capture Hassan's work, I find that my own key variables fall in magnitude by about one third but remain very statistically significant.

[5]All the results in this paper hold both when the 129 basic headings are used and when individual and collective services are excluded, as they are classified as comparison-resistant areas by the ICP due to their imperfect price concepts.

where p denotes the national price level and w denotes GDP per capita.[6] Here we begin from specification (1), but we re-interpret it so as to distinguish explicitly between local costs, represented here as labour costs[7], and costs associated with internationally tradable items. We note that PPP is expected to apply only to the second element.

Now this distinction between local and international costs has always been regarded as (at least a contributing factor in) driving the apparent violation of PPP. But here, we make this explicit, with a view to identifying and separating the industries, for which costs are mainly local labour costs.

The specification in (1) can be interpreted as follows: if the market is competitive, price is equal to marginal cost, which is a weighted average of local labour costs and international costs,

$$p = MC = \varphi w + (1 - \varphi)\iota, \tag{2}$$

where φ is the weight and ι is the world price of the internationally tradable items, and is constant across countries. Therefore, its reduced form yields the linear specification in (1). And linking (2) to (1) tells us that b is equal to φ and a is equal to $(1 - \varphi)\iota$. To identify and separate those products with local labour costs as their major costs, I run the regression implied by (1), on a product by product basis, and plot the estimates of a and b in Figure 1.[8]

The open diamonds represent services (designated the 'S-group') and the dark squares represent non-services (designated the 'M-group'). Please see Appendix B.2 for the product classification. On inspection, we have two observations. First, we notice that there is a sharp split between pure services and non-services. For services, we expect that φ is large so b is large and a is small. Indeed, b is high with a mean 0.615 and a standard deviation 0.215 while a is statistically not very different from 0 with a mean 0.382 and a standard deviation 0.235, suggesting that their price levels are mainly determined by local labour costs. For non-services, φ is smaller. So we expect that b is smaller and a is larger. Indeed, b is low with a mean 0.427 and a standard deviation 0.253 while a is significantly positive with a mean 0.815 and a standard deviation 0.302. This is consistent with the fact that their price levels rely less on local labour costs. In addition, a multivariate test of means can also be performed to test the hypothesis that both the means of as and the means of bs are equalized between the S-group and the M-group, allowing for heterogeneous covariance matrices across the two groups. The test result shows that the hypothesis is rejected at 1% level with the F-statistic being equal to 40.13. Second,

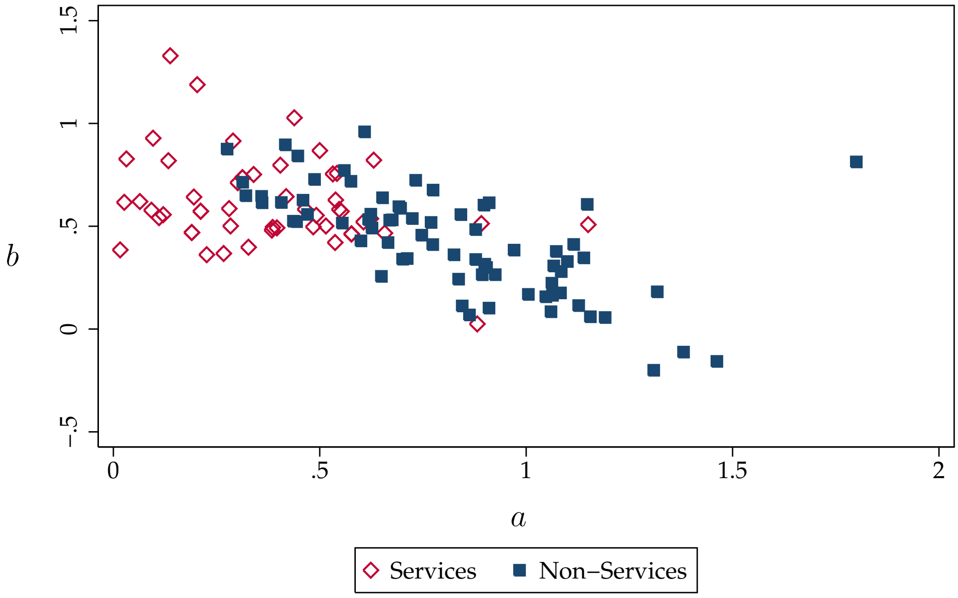
[6]This specification is used in Balassa (1964).

[7]Other local costs are rent, electricity etc., but these non-tradable items should, like labour, reflect local conditions.

[8]Appendix C.1 provides a modified version of Figure 1 showing the industry labels, in which we can see more clearly the role played by tradables in the service sectors.

there is a large amount of heterogeneity in the estimates of a and b within non-services. Some non-services do not exhibit the B-S relationship as their estimates of b are close to zero. While some other non-services exhibit a strong B-S relationship with a significantly positive estimate of b.

Figure 1: The B-S Estimation: Services vs. Non-Services



Notes: a and b are the intercept and slope from the B-S estimation. The open diamonds and the dark squares represent the estimates from services and non-services respectively. Data from Zimbabwe are not included in the estimation due to its volatile exchange rate in 2005. The basic heading of Passenger Transport by Air is excluded from the figure, as its B-S relationship is downward-sloping.

Now these observations can also be analysed using alternative specifications. A second and more popular formulation of the B-S relationship is

$$p = a + b \ln w, \tag{3}$$

where the log of GDP per capita is used on the r.h.s. In Appendix C.2, I plot the price level relative to the U.S. ($p/p_{U.S.}$) against $\ln(GDP\ per\ capita/US\ GDP\ per\ capita)$ separately for each of the ICP basic headings.[9] In each plot, one data point corresponds to one country. We expect the plots to have one of the following three patterns.

[9] Two basic headings, "Net Purchases Abroad" and "Balance of Exports and Imports", are excluded, as they are not supposed to be covered by the scope of this paper.

1. In the extreme where $\varphi = 1$ so that all costs are local, we expect $p = w$ and hence

$$p/p_{U.S.} = \exp(\ln(GDP\ per\ capita/US\ GDP\ per\ capita)).$$

That is, we expect an exponential relationship. Panel (a) of Figure 2 displays this relationship for the case of Hospital Services.

2. In the other extreme where $\varphi = 0$[10] so that all costs are international, we expect $p = a$. That is, the data should line up horizontally. Panel (b) of Figure 2 displays this relationship for Other Cereals.

3. Intermediate cases should display a weaker exponential relationship. Panel (c) of Figure 2 displays this relationship for Furniture.

The Figure 2 plots are representative. The more interested reader should look at all 127 plots in Appendix C.2 to convince himself or herself that these three types are very much in evidence and appear where expected, i.e. the exponential relationship occurs for most services and the horizontal relationship occurs for most highly-traded goods.

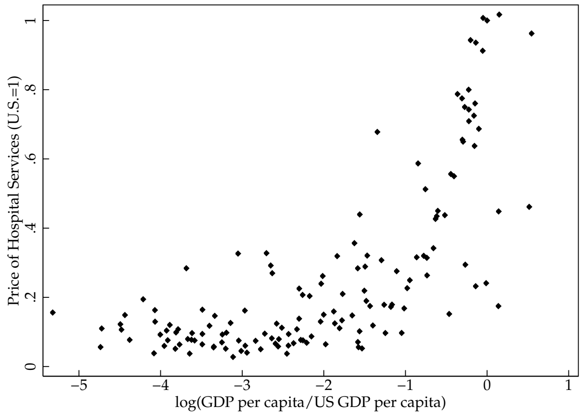
Now the central theme of this paper is that, even when we eliminate those products that are primarily 'services', whose costs are predominantly local labour costs, the B-S relationship remains clear, i.e. the violation of PPP is still evident. This can be seen in Figure 3, where the B-S relationships, in its basic form (1), are plotted using the national price level and the price level of non-services.[11] Both show a positive relationship. In addition, to check that the B-S relationship found for the price level of non-services is not due to the aggregation method used in aggregating the prices at the product level to the price level of non-services, I also pool the B-S scatter plots of all M-group products in Appendix C.2 together and fit the pooled scatter plot with a non-parametric regression curve without imposing any functional form. Figure 4 shows that the non-parametric fitted curve also displays a B-S relationship.[12] The above results motivate our search for an alternative contributing factor, that will hold even among industries that are pure 'manufacturing' industries (M-group products).

[10] Of course, φ can never be exactly 0 because there are always some labour costs.

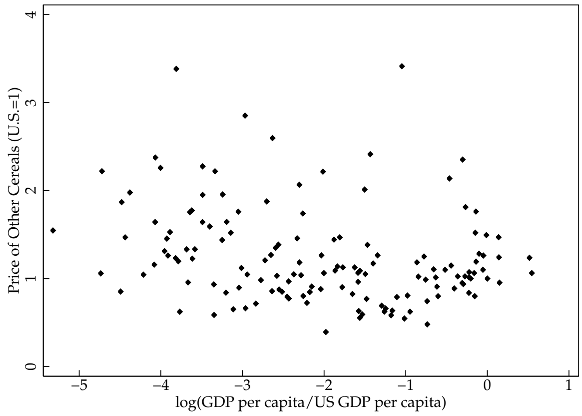
[11] The price levels of non-services are constructed using the standard Geary and Khamis (GK) method by the PWT and the ICP Benchmark Dataset 2005.

[12] A small number of observations above 3 are not plotted.

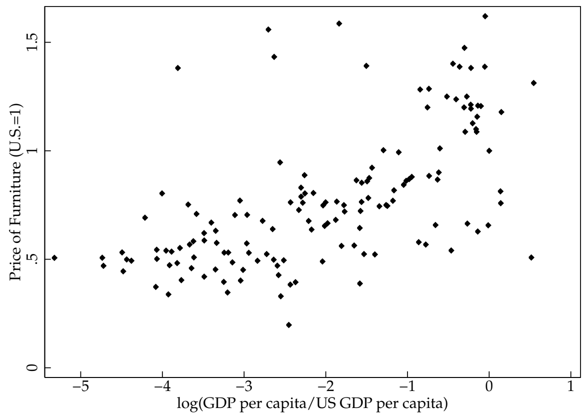
Figure 2: Three Archetypal Forms of Price-Wealth Relationships



(a) Hospital Services

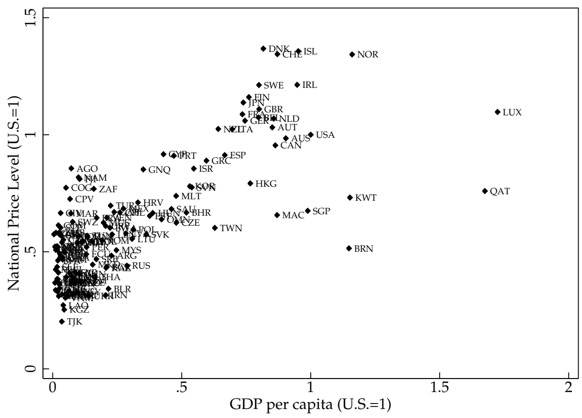


(b) Other Cereals

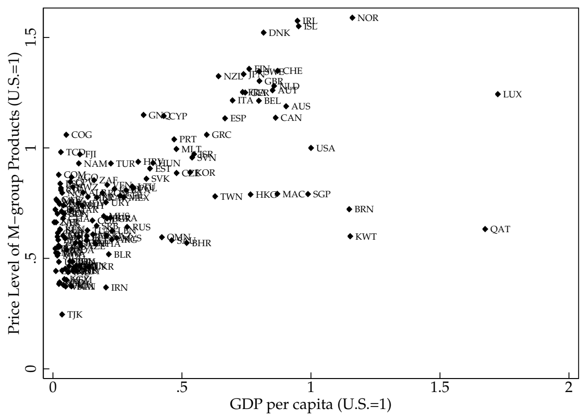


(c) Furniture

Figure 3: The B-S Relationship for Alternative Price Indexes (Based on Equation (1))



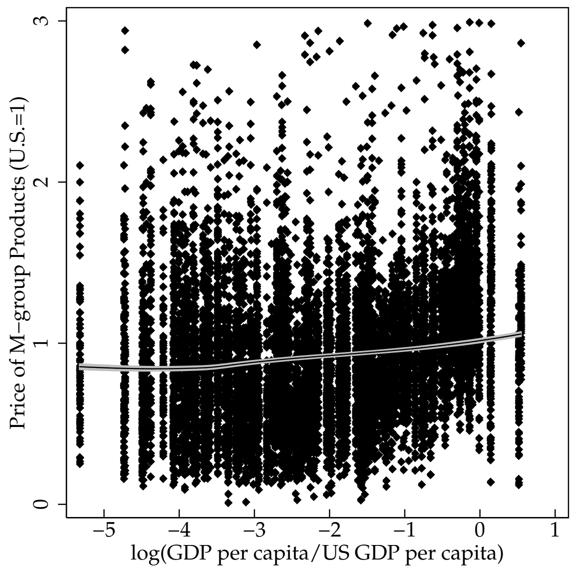
(a) The B-S Relationship Using the National Price Level



(b) The B-S Relationship Using the Modified Price Level that Excludes 'Services Type' Products

In the next section, I develop an alternative candidate explanation for the B-S re-

Figure 4: The Pooled Scatter Plots of all M-group Products



Notes: The non-parametric curve with 95% confidence intervals is obtained using local-mean smoothing with a bandwidth of 1.5.

lationship, appropriate to the M-group products, based on an appeal to mismeasured quality.

3. A 'Mismeasured Quality' Interpretation

This section begins with the basic intuition behind the distinctive prediction of the model, i.e. the non-monotonic relationship between income inequality and the national price level. Then it describes the consumer's problem, in which an empirically motivated key assumption is reflected, the producer's problem and the market equilibrium. Finally, an analytical solution of the equilibrium is used to derive propositions regarding how income distribution affects expenditure shares and price levels at disaggregate and aggregate levels.

3.1. The Basic Intuition

The model developed in this paper is a hedonic pricing model *à la* Rosen (1974), in which consumers and firms choose their optimal positions along an equilibrium price schedule $p(z)$, where z is a vector of characteristics of the good in question. As firms take the equilibrium price schedule as given in their profit maximization problem, the price of any quality good is equal to its marginal cost. Hence the mark-up is fixed to be 1 and does not play any role in this paper.

The focus of the analysis lies in establishing a relationship between a country's level of income, and – more importantly – the form of income distribution in the country, and the pattern of demand for both 'quality' goods and 'homogeneous' goods.

The novel prediction of the model is that controlling for per capita income, income inequality also affects the national price level. The basic intuition of this is: a higher income inequality implies that a consumer has to pay more to switch from a low quality good to a high quality good as the prices of high quality goods are raised by the newly created rich people. As a result, consumers will lower the share of income spent on quality goods and increase the share on homogeneous goods. A simple illustration of this mechanism is: suppose a country's per capita income is μ^* and its income distribution is made up of three income groups with equal population. The top income group consumes a top quality good, the middle income group consumes a middle quality good and the bottom income group consumes a bottom quality good. Given the same Cobb-Douglas utility function, everyone spends a same fraction θ of his/her income on quality goods. As quality cannot be fully controlled for, the implied price level of quality goods will depend on the average expenditure on it, which is equal to $\mu^*\theta$. Now consider a mean-preserving spread of income distribution, which means that there are now less people in the middle income group and more people in the top and bottom groups. This income redistribution will increase the quantity demanded of the top and bottom quality goods. As the marginal cost function of the quality good is increasing in the number of units produced, the increased demand will increase the prices of the top and bottom quality goods, resulting in a more convex price function. With a more convex price function of quality goods and the unit elasticity of substitution in the Cobb-Douglas utility function, all consumers will spend a lower fraction θ' $(< \theta)$ of income on quality goods. As a result, its measured price level will be lower, as the average expenditure on quality goods, is now reduced to $\mu^*\theta'$. This is the mechanism through which income inequality affects the national price level in the present model.

3.2. The Model I: The Consumer's Problem

On the consumer's side, in every country there is a unit mass of consumers indexed by income level c. The income distribution is assumed (conventionally) to follow the Pareto distribution characterized by two parameters k_c and c_m, where c_m is the lower bound of income level c and k_c is the shape parameter. Hence the probability density function of income is

$$f(c) = k_c c_m^{k_c} c^{-(k_c+1)}, k_c > 1, c \in [c_m, \infty).$$

Quality enters the utility function in a way similar to Flam and Helpman (1987), who introduce quality to their model of North-South trade. That is, consumer preferences are represented by a utility function $v(x, z)$, where x is the quantity of a homogeneous good, denoted by X, and z is the quality of a vertically differentiated good denoted by Z. In this paper, I further assume that the homogeneous good is tradable and the vertically differentiated good is nontradable and hence locally priced. Therefore, a consumer with income c chooses the quantity of the homogeneous good and the quality level of the vertically differentiated good to solve the following utility maximization problem:

$$\max_{x,z} v(x,z)$$
$$s.t.\ c = x + p(z). \tag{4}$$

where $p(z)$ is the nonlinear hedonic price function of the vertically differentiated good, which is set locally, and the price of the homogeneous good is one.

The utility function assumed above reflects two key assumptions in the consumer's problem. The first is that income elasticity of quality varies across goods and is high for many goods. To reflect this, I include both homogeneous goods (with zero income elasticity of quality) and vertically differentiated goods (with positive income elasticity of quality) in the model. I motivate this assumption by reference to Bils and Klenow (2001), which provide empirical support for this assumption. They estimate income elasticities of quality and income elasticities of quantity for 66 durable goods and find that income elasticity of quality is positive and non-negligible.[13] For many goods in their sample, income elasticity of quality is higher relative to that of quantity. For example, for goods such as window coverings, curtains and drapes, the quality consumed rises sharply relative to the quantity as household income rises. In contrast, for goods such as clothing and footwear, the quantity consumed tends to increase by more than the quality as household income rises. The second assumption is that income elasticity of quality is higher for nontraded good. To reflect this, the homogeneous good is assumed to be tradable and the vertically differentiated good is assumed to be nontradable in the model. This assumption can be supported by a significant negative relationship between tradability and income elasticity of quality for the goods studied in Bils and Klenow (2001). The details are shown in Appendix C.4. In addition to tradable homogeneous goods and non-traded vertically differentiated goods, I abstract from other types of goods for the following reasons. First, I let traded goods be homogeneous for analytical simplicity. Second, homogeneous nontradables, which are the focus of the B-S model, are not included in the present model. Furthermore, the significant negative relationship between tradability and income elasticity of quality also suggests that we can abstract from other goods and focus on the traded homogeneous goods and the nontraded vertically differentiated goods

[13] Appendix C.3 provides additional empirical evidence on the relative importance of income elasticities of quality and quantity for food, houses, clothes and vehicles.

in the model without loss of generality.

In what follows I assume the utility function $v(x, z)$ is a standard Cobb-Douglas utility function, i.e. $v(x, z) = x^\alpha z^\beta$, $\alpha + \beta = 1$. Given this, the Lagrangian for the problem in (4) is given by

$$L = x^\alpha z^\beta + \lambda[c - x - p(z)].$$

First-order necessary conditions imply that

$$\frac{v_z}{v_x} = p'(z) \text{ and hence } \frac{\beta}{\alpha}\frac{x}{z} = p'(z).$$

Combining it with the budget constraint $x = c - p(z)$, we have

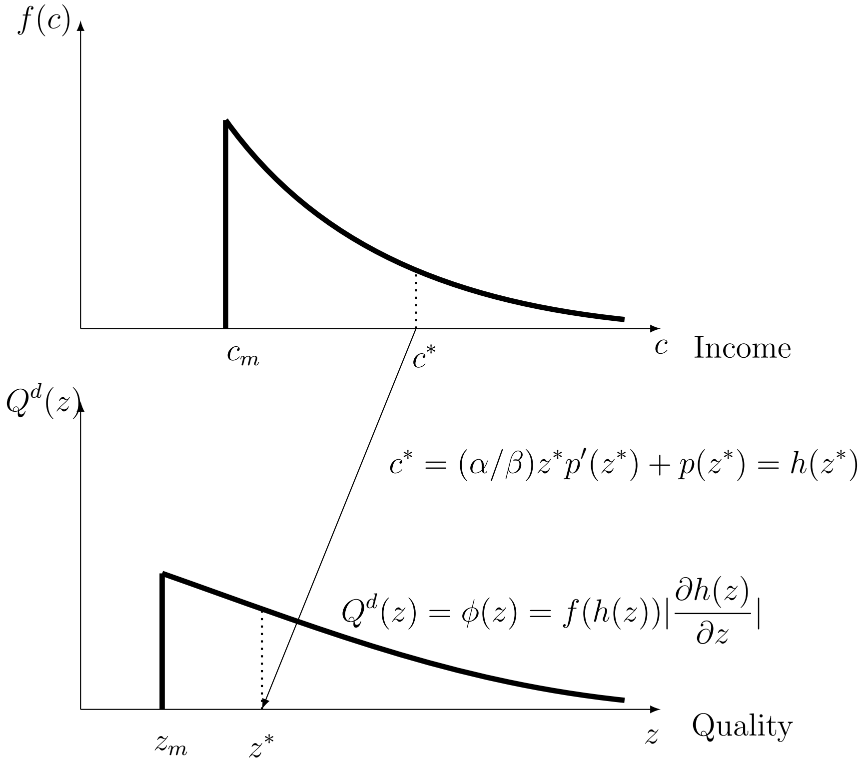
$$c = (\alpha/\beta) z p'(z) + p(z), \tag{5}$$

which implies that if a consumer chooses a vertically differentiated good with quality z, then his/her income is equal to $(\alpha/\beta) z p'(z) + p(z)$. We denote this consumer's income conditional on choosing quality z by $h(z)$. Given the pdf of income $f(c)$ and the relationship between income and quality $c = h(z)$ or $z = h^{-1}(c)$, we can write down the pdf of z, which is denoted as $\phi(z)$, as follows:

$$\phi(z) = f(h(z)) \left| \frac{\partial h(z)}{\partial z} \right|, \tag{6}$$

where $|\cdot|$ denotes the absolute value. The mapping from the pdf of c to the pdf of z can be illustrated in Figure 5.

Figure 5: Mapping From Income Distribution to Distribution of Quality Demanded



Notes: Given the exogenous income distribution $f(c)$ and the relationship between income and quality derived from the utility maximization problem $c = h(z)$, the distribution of quality demanded $Q^d(z)$ can be obtained. $c^* = (\alpha/\beta)z^*p'(z^*) + p(z^*)$ is the relationship between an individual's income c^* and the quality he/she chooses to consume z^*.

Substituting for $h(z)$ and $f(\cdot)$ in (6) yields

$$\begin{aligned}\phi(z) &= f(\frac{\alpha}{\beta}zp'(z) + p(z))\left|\frac{\partial[\frac{\alpha}{\beta}zp'(z) + p(z)]}{\partial z}\right| \\ &= k_c c_m^{k_c}[\frac{\alpha}{\beta}zp'(z) + p(z)]^{-(k_c+1)}\left|\frac{\alpha}{\beta}[p'(z) + zp''(z)] + p'(z)\right|.\end{aligned}$$

If we denote the quantity demanded for the good with quality z by $Q^d(z)$, then the market demand in a small interval dz near quality z is given by the product of the pdf of quality around z, $\phi(z)$, and the length of the interval:

$$Q^d(z)dz = k_c c_m^{k_c}[\frac{\alpha}{\beta}zp'(z) + p(z)]^{-(k_c+1)}\left|\frac{\alpha}{\beta}[p'(z) + zp''(z)] + p'(z)\right| dz. \tag{7}$$

3.3. The Model II: The Producer's Problem

Although different countries have different income distributions, the supply side is the same for all countries. In every country, there is a unit mass of firms producing vertically differentiated goods indexed by product quality z. The distribution of the firms is assumed to be the Pareto distribution characterized by two parameters k_z and z_m, where z_m is the lower bound of product quality z and k_z is the shape parameter.[14] Therefore, the pdf of z takes the following form:

$$g(z) = k_z z_m^{k_z} z^{-(k_z+1)},\ k_z > 1,\ z \in [z_{m,}\infty). \tag{8}$$

As in Rosen (1974), producers in all countries are assumed to have the same cost function $\Delta(M, z) = A_z M^\tau z^\gamma$, $\tau > 1, \gamma > 1$, where A_z is the productivity parameter and M denotes the number of units of the product with quality z that the firm produces. It is assumed that $\tau > 1$ and $\gamma > 1$, which ensure that total cost is a convex function in M and z.

The producers are price takers. Furthermore, it is assumed that the producers can vary M but not z. (i.e. a producer's quality is a given parameter in the short run). Therefore, the producer's problem is to maximize profit by choosing its output level M:

$$\max_M Mp(z) - \Delta(M, z).$$

The first-order conditions imply that

$$p(z) = \frac{\partial \Delta}{\partial M} = A_z \tau M^{\tau-1} z^\gamma.$$

Thus,

$$M(z) = (\frac{p(z)}{A_z \tau z^\gamma})^{1/(\tau-1)}. \tag{9}$$

If we denote the firms' output of z as $Q^s(z)$, then the market supply in a small interval dz near quality z is given by the product of the pdf of the firms around z, the quantity

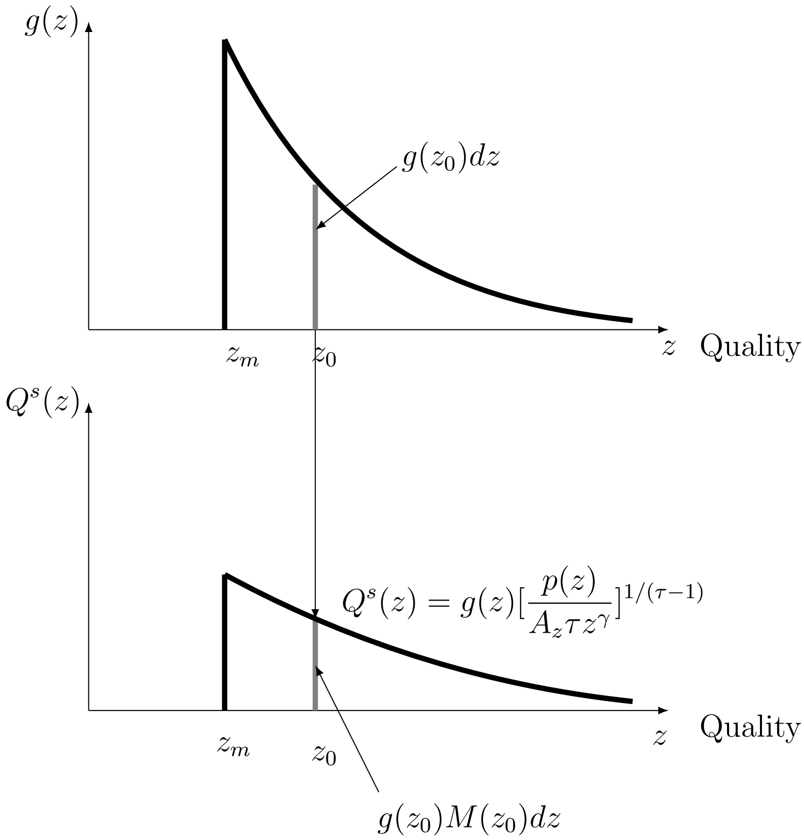
[14] The reasons for using the Pareto distribution, a power law probability distribution, are not only that a closed form solution can be obtained but also that this assumption is consistent with empirical evidence. Gaffeo, Gallegati and Palestrini (2003) analyze the average size distribution of a pool of the G7 group firms over the period 1987-2000. They find that the empirical distributions are all consistent with the power law. Luttmer (2007) also shows that the firm size distribution follows the Pareto distribution over much of its range using the U.S. data in 2002. In the model of this paper, the quality of a firm is a power transformation of the size of the firm, so it is reasonable to assume quality also follows the Pareto distribution as the Pareto distribution is close under power transformation.

supplied by each firm producing quality z and the length of the interval:

$$\begin{aligned} Q^s(z)dz &= g(z)M(z)dz \\ &= k_z z_m^{k_z} z^{-(k_z+1)} (\frac{p(z)}{A_z \tau z^\gamma})^{1/(\tau-1)} dz. \end{aligned} \tag{10}$$

Figure 6 shows the mapping from the distribution of firms by quality to the distribution of quality supplied.

Figure 6: Mapping From Firm Distribution by Quality to Distribution of Quality Supplied



Notes: Given the initial distribution of firms $g(z)$ in (8) and the output of each firm $M(z)$ in (9), the distribution of quality supplied $Q^s(z)$ can be derived, as illustrated, and takes the form of (10). $g(z_0)dz$ is the number of firms that produce z_0 and $g(z_0)M(z_0)dz$ is total quantity of z_0 supplied.

3.4. Market Equilibrium

An equilibrium is defined as a triple $\{z(c), M(z), p(z)\}$, where $z(c)$ is the policy function for consumers, $M(z)$ is the policy function for producers, and $p(z)$ is the price schedule, such that:

1. $z(c)$ solves the consumer's utility maximization problem taking $p(z)$ as given.
2. $M(z)$ solves the producer's profit maximization problem taking $p(z)$ as given.

3. Market clearing: demand is equal to supply for Z goods, i.e., $Q^s(z) = Q^d(z)$ for all z.

3.5. An Analytical Solution

In equilibrium, we must have the market clearing condition $Q^s(z)dz = Q^d(z)dz$. It can be used to solve for a solution[15] to the equilibrium price function, in which the parameters of the price function can be expressed as functions of the income distribution parameters k_c and c_m. This is summarized in Lemma 1.

Lemma 1. *The price function*

$$p(z) = b(k_c, c_m)z^{d(k_c)}, \;\; z \in [\bar{z}_m, \infty)$$

where

$$b(k_c, c_m) = \left\{ \frac{k_c c_m^{k_c} \left[\frac{\alpha}{\beta} \frac{k_z+\gamma/(\tau-1)}{k_c+1/(\tau-1)} + 1 \right]^{-k_c} \frac{k_z+\gamma/(\tau-1)}{k_c+1/(\tau-1)}}{k_z z_m^{k_z} (\frac{1}{A_z \tau})^{1/(\tau-1)}} \right\}^{(\tau-1)/[k_c(\tau-1)+1]}, \tag{11}$$

$$d(k_c) = \frac{k_z + \gamma/(\tau-1)}{k_c + 1/(\tau-1)}, \;\; d'(k_c) < 0 \tag{12}$$

and $\bar{z}_m$ is such that

$$c_m = \frac{\alpha}{\beta}\bar{z}_m p'(\bar{z}_m) + p(\bar{z}_m).$$

satisfies the market clearing condition $Q^s(z) = Q^d(z)$.

Proof: See Appendix A.1.

3.6. Income Distribution and Expenditure Shares

It can be shown that the expenditure share on Z goods, i.e. the price paid for Z good divided by income $p(z)/c$, crucially depends on the convexity of the price function d as

$$\frac{p(z)}{c} = \frac{p(z)}{\frac{\alpha}{\beta}zp'(z) + p(z)} = \frac{bz^d}{\frac{\alpha}{\beta}bdz^d + bz^d} = \frac{1}{\frac{\alpha}{\beta}d + 1}. \tag{13}$$

The first equality follows from the first order condition (5) and the second equality follows from the form of price function in Lemma 1. Furthermore, as the convexity d crucially depends on income distribution parameter k_c from (12), we can now derive the relationship between income distribution and expenditure shares. As the Gini coefficient and per capita income of the Pareto distribution can be expressed as analytical functions of the

[15] I adopt the standard method of undetermined coefficients to find a particular solution of the form $p(z) = bz^d$, with $b, d > 0$, which is then substituted into the market clearing condition to solve for the values of the parameters b and d (Zwillinger (1997), P415).

distribution parameters, viz. $Gini = 1/(2k_c - 1)$ and $\mu = c_m k_c/(k_c - 1)$, we can also think of the Pareto distribution as governed by two independent parameters $Gini$ and μ. Therefore, from (12) and (13) we can derive how the Gini coefficient and per capita income affect the convexity of the price function d and hence the expenditure shares. The results are summarized in Proposition 1.

Proposition 1. *(Income Distribution and Expenditure Shares) Income inequality has a positive impact on the expenditure share of X goods and a negative impact on the expenditure share of Z goods. Per capita income has no impact on expenditure shares.*

Proof: See Appendix A.2.

The intuition behind the first prediction of Proposition 1 is that an increase in the Gini coefficient implies an increase in d and hence a more convex price function. As a result, the marginal utility derived from Z goods becomes more expensive. Due to the high substitutability between X goods and Z goods, consumers will spend a smaller fraction of their expenditure on Z goods and more on X goods. This mechanism of how income inequality affects expenditure shares is crucial in determining how income inequality influences the price levels, which will be provided in the next subsection. The second prediction is a direct implication of the Cobb-Douglas utility function. The Cobb-Douglas utility function is used for analytical simplicity, as it can yield a closed form solution. However, with more general utility functions, such as the CES utility function, the results in this paper will still hold as long as the elasticity of substitution between the two goods is high enough.

3.7. Income Distribution and Price Levels

We now explore the implications of the model for disaggregate and aggregate price levels and how the implications are affected by the extent to which quality is adjusted. The two polar cases are:

1. Perfect quality measurement.

2. Quality is not fully adjusted using hedonics such as in Pakes (2003).

First, in the case of perfect quality measurement, quality can be fully observed, so in terms of the setting of the model we will be able to use the hedonic regression to estimate the parameters in the price function $p(z)$. Then the price schedule $p(z)$ from each country can be used to compare the price level of Z goods across countries. One way of constructing the price level of Z goods while minimizing the impact of quality in it is to use $p_j(z_0) = b_j z_0^{d_j}$ as the price level of Z goods in country j, where b_j and d_j are the estimated parameters in country j and z_0 is a constant level of quality across countries. As a result, the price levels of Z goods in country i and country j are given by $p_i(z_0) = b_i z_0^{d_i}$ and $p_j(z_0) = b_j z_0^{d_j}$. The difference between the two price levels are only due to the different bs and ds but not due to different qualities. Although in this case quality can be fully controlled for, (11) implies that there still exists the B-S relationship. To see this, suppose we keep a country's Gini coefficient constant and increase its per capita income, this implies a constant k_c but a higher c_m in the Pareto distribution. From (12) and (11), d will stay constant but b will go up, resulting in an upward shift of the price schedule $p(z)$. Hence, for any quality goods, the price will be higher than before. This is because a higher level of per capita income will increase the demand for the higher quality Z goods and the resulting higher output will increase their prices as the marginal cost is increasing in output. The magnitude of this B-S relationship can be measured by the elasticity of the price level $p(z_0)$ with respect to per capita income μ, which is equal to $k_c(\tau - 1)/[k_c(\tau - 1) + 1]$ from (11).

However, the above procedures face a prominent practical issue. The issue is that quality cannot be perfectly controlled for Z goods. Pakes (2003) shows how to use hedonics to adjust quality biases in the price indexes of quality goods due to the introduction of new goods in the case of PC, as the traditional price indexes used by the Bureau of Labor Statistics (BLS) do not make any adjustment for the differences in the utility per dollar between the new good and the good(s) it replaces. However, the adjustment procedures require a complete dataset on the characteristics of the goods, which is impossible in the case of cross-country price comparison. Without the level of quality being observed and the price schedule $p(z)$ being estimated, the common practice of constructing the price index of Z goods is to use some partial quality adjustment methods such as matching precisely defined items and calculate the average of the observed prices of all Z goods. As a result, the price index of Z goods will generally depend on the average expenditure on Z goods and these two will become equal when there is no quality adjustment at all. The consequence of the practice is that the resulting price level is less informative than in the case of perfect quality measurement as a higher price level could be either due to higher values of b and d in the price function or simply due to the fact that the prices of higher quality goods have been observed. This will lead to a higher B-S effect, i.e. a higher elasticity of the price level with respect to per capita income.

To see this, suppose there is no quality adjustment and we simply use the average expenditure on Z goods, which is equal to[16]

$$\overline{p} = \mu \frac{\beta}{\alpha \frac{k_z + \gamma/(\tau-1)}{(1/Gini+1)/2+1/(\tau-1)} + \beta}, \tag{14}$$

as its price level, then the elasticity of the price level with respect to per capita income μ is equal to 1, compared with a elasticity of $k_c(\tau - 1)/[k_c(\tau - 1) + 1]$ in the previous case of full quality adjustment. Motivated by the different elasticities in the two polar

[16] Please see Appendix A.3 for the derivation.

cases, I assume the general form of the relationship between the measured price level with partial quality adjustment $\overline{p}^*$ and the average expenditure $\overline{p}$ is $\overline{p}^* = \overline{p}^\eta$, where $k_c(\tau-1)/[k_c(\tau-1)+1] < \eta < 1$. Here, η is used to govern the extent to which quality is adjusted. $\eta - k_c(\tau-1)/[k_c(\tau-1)+1]$ is the part of the elasticity of the price level with respect to per capita income that is due to imperfect quality adjustment. Given this assumption, we can derive how income distribution affects the measured price levels with partial quality adjustment at the disaggregate and aggregate levels and the role played by η in it.

First, the effects of per capita income and income inequality on the price level of Z goods are derived. The results are summarized in Proposition 2.

Proposition 2. *(Income Distribution and the Disaggregate Price Level) Per capita income has a positive impact on the price level of Z goods ($\partial\overline{p}^*/\partial\mu > 0$), whereas income inequality has a negative impact ($\partial\overline{p}^*/\partial Gini < 0$). Moreover, the effect of income inequality on the price level of Z goods is decreasing in per capita income ($\partial^2\overline{p}^*/\partial Gini\partial\mu < 0$).*

Proof: See Appendix A.4.

Proposition 2 first shows that the B-S effect exists for the price level of Z goods. Moreover, from the proof, it is easy to see that the effect would disappear if η could go to zero. The dependence of the B-S effect on η implies that this effect can be attributed to two sources. One source is imperfect quality adjustment, which is governed by $\eta - k_c(\tau-1)/[k_c(\tau-1)+1]$. The other source is increasing marginal cost, which is governed by $k_c(\tau-1)/[k_c(\tau-1)+1]$. The magnitude of $k_c(\tau-1)/[k_c(\tau-1)+1]$ crucially depends on the cost function parameters τ and could be arbitrarily small as τ approaches 1 from above, while the magnitude of imperfect quality adjustment is exogenous and is likely to be large given the difficulty of controlling for quality in reality. Therefore, imperfect quality adjustment plays an important role in the B-S relationship of Z goods.

Proposition 2 also shows that keeping per capita income constant, income inequality can also affect the price level of Z goods. This is because the price level of Z goods depends on the average expenditure on Z goods, which is the product of the expenditure share and per capita income. As income inequality has a negative impact on the expenditure share of Z goods as shown in Proposition 1, income inequality can affect the price level of Z goods through the expenditure share and the effect is decreasing in per capita income. Moreover, from the proof the impact of income inequality increases as η goes up, suggesting the impact of income inequality decreases with the degree of quality adjustment. As the price level of X goods is constant across countries, the above effects of income distribution on the price level of Z goods will play important roles in the relationship between income distribution and the national price level.

To investigate the implications of income distribution for the national price level, we need to construct an aggregate price index for both X goods and Z goods. Here for analytical simplicity, I use the bilateral Fisher index, which is the geometric mean of the Laspeyres index and the Paasche index, as a proxy for the national price level. This is because the methods of computing the national price level in practice, such as the Eltero-Koves-Szulc (EKS) and Geary and Khamis (GK) methods, are not analytically tractable, but Deaton and Heston (2010) has shown that the national price level in the Penn World Table can be well approximated by the geometric mean of the Laspeyres index and the Paasche index. The Laspeyres and Paasche indices are both weighted averages of the price ratios of disaggregate price levels relative to the base country, the U.S. In the Laspeyres index, the weights are given by the expenditure shares of the base country, while in the Paasche index the weights are given by the expenditure shares of the country in question. In the model of this paper, there are two types of goods: X goods and Z goods, so the Paasche index P_P, the Laspeyres index P_L and the bilateral Fisher index P_F are defined as follows:

$$P_P \equiv \left(\frac{1}{1} share_x + \frac{\overline{p}_0^*}{\overline{p}^*} share_z\right)^{-1} = \left(\frac{\alpha d}{\alpha d + \beta} + \frac{\overline{p}_0^*}{\overline{p}^*}\frac{\beta}{\alpha d + \beta}\right)^{-1},$$

$$P_L \equiv \frac{1}{1} share_{x,0} + \frac{\overline{p}^*}{\overline{p}_0^*} share_{z,0} = \frac{\alpha d_0}{\alpha d_0 + \beta} + \frac{\overline{p}^*}{\overline{p}_0^*}\frac{\beta}{\alpha d_0 + \beta},$$

$$P_F \equiv \sqrt{P_P P_L},$$

where 0 is used in the subscript to denote variables from the base country, i.e. U.S. $\overline{p}_0^*$ and $\overline{p}^*$ are the measured price levels of Z goods in the base country and the country in question. Given the definition of the national price level, in Proposition 3 the results regarding how income distribution affects the national price level are shown. More specifically, to make the results comparable with the empirical tests in Section 4, the results regarding how the log of per capita income and the Gini coefficient affect the log of the national price level are shown.

Proposition 3. *(Income Distribution and the National Price Level) The impact of per capita income on the national price level is positive: the elasticity of the national price level with respect to per capita income is positive ($e_{P_F,\mu} \equiv (\partial P_F/\partial \mu)\mu/P_F > 0$). The impact of income inequality on the national price level is decreasing in per capita income: the semi-elasticity of the national price level with respect to income inequality is decreasing in per capita income ($\partial e_{P_F,Gini}/\partial \mu < 0$, where $e_{P_F,Gini} \equiv (\partial P_F/\partial Gini)/P_F$).*

Proof: See Appendix A.5.

Empirically, this proposition suggests that in a cross-sectional setting, one should regress the national price level on per capita income and the interaction of per capita income and the Gini coefficient, where we expect the coefficient on per capita income to be positive and the coefficient on the interaction term to be negative.

Given the constant price level of X goods, we can use the intuition derived from Proposition 2 regarding the effects of income distribution on the price level of Z goods to understand the effects of income distribution on the national price level. First, given the positive impact of per capita income on the price level of Z goods, per capita income will have a positive impact on the national price level, which is a weighted average of the price levels of X goods and Z goods. However, the elasticity of the national price level with respect to per capita income is less than that of the price level of Z goods as the elasticity of the price level of X goods with respect to per capita income is 0.

Second, although income inequality has a negative impact on the price level and the expenditure share of Z goods, it is not necessary that income inequality has a negative impact on the national price level. This is because the impact of income inequality on the national price level also depends on the relative magnitudes of the price ratio of X goods and Z goods relative to the base country as shown in the definition of the national price level.

Finally, as income inequality has a negative impact on the expenditure share of Z goods and the price level of Z goods and the national price level both depend on the average expenditure on Z goods, which is the product of the expenditure share on Z goods and per capita income, the impact of income inequality on the price level of Z goods and the national price level will be decreasing in per capita income.

To summarize, the model developed in this section predicts the B-S relationship, i.e. the positive relationship between per capita income and the national price level, and a non-monotonic relationship between income inequality and the national price level. And from the proof the magnitudes of all these impacts depends positively on η, which is negatively related to the extent to which quality is adjusted. Therefore the model provides a mismeasured quality explanation of the relationship between the national price level and income distribution.

Compared with this explanation, the other explanations of the B-S effect cited in the introduction generally have a demand side based on the representative agent framework, i.e. all the consumers in a country are represented by a single consumer. As a result, income distribution is degenerate and no relationship between income inequality and prices can be implied from these models. However, there are three exceptions, in which a non-degenerate income distribution is specified. They are Bekkers, Francois and Manchin (2012), Choi, Hummels and Xiang (2009) and Fajgelbaum, Grossman and Helpman (2011). Due to the presence of income distribution in these models, they can generate implications for the relationship between income distribution and prices. In Bekkers, Francois and Manchin (2012), they use trade data and find a positive relationship between per capita income of importing countries and import prices and a negative relationship between their income inequalities and import prices. They propose three models to explain these empirical findings, including a quality expansion model, an ideal variety model and a hierarchic demand model. All three models can generate a positive relationship between per capita income and import prices. But only the hierarchic demand model can generate a negative relationship between income inequality and import prices, while the other two models generate the opposite. As their paper is mainly about import prices, and my paper is mainly about national price levels, there is no direct comparison between the two papers. However, Proposition 2 of my paper does predict three relationships between income distribution and the price of Z goods, i.e. the vertically differentiated goods in my model, which are a positive relationship between per capita income and the price of Z goods, a negative relationship between income inequality and the price of Z goods and the impact of income inequality on the price of Z goods is decreasing in per capita income. If we view import prices in their paper as prices of vertically differentiated goods in my model, then their preferred explanation, i.e. the hierarchic demand model can generate the first two relationships in Proposition 2, i.e. the positive relationship between per capita income and import prices, and the negative relationship between income inequality and import prices but not the third relationship, i.e. the impact of income inequality on the price of Z goods is decreasing in per capita income. The third relationship is crucial in generating the nonlinear relationship between income inequality and national price levels in my paper, which is shown to be consistent with the data. Choi, Hummels and Xiang (2009) theoretically derive the general relationship between income distribution and distribution of import prices. However, no conclusive relationship between income inequality and price levels can be implied from their results, as if we want to do so, we have to make assumptions about the functional form of income distribution and how to aggregate prices into price levels. As a result, the relationship between income inequality and price levels will depend on the assumption made and will be in general ambiguous. Fajgelbaum, Grossman and Helpman (2011) develop a framework to study patterns of trade in horizontally and vertically differentiated products. In their model, a non-degenerate income distribution is specified and goods with higher quality imply higher prices. However, their model's prediction regarding how income inequality affects the average quality demanded in the economy is ambiguous. As a result, no conclusive relationship between income inequality and prices can be predicted by their model. Therefore, existing explanations of the B-S effect cannot be easily extended or modified to account for the empirical relationship between income distribution and the national price level found in my paper, but my paper provides a relatively simple model to account for that.

4. Empirical Evidence

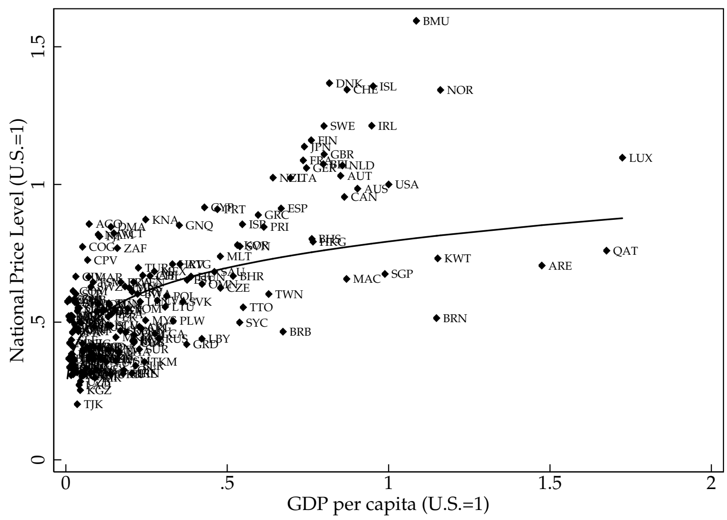
This section tests the model's predictions at two levels: the aggregate level and an intermediate level. The aggregate price levels are from the Penn World Table (PWT), which are commonly used to compare price levels across countries or compute real GDPs in constant internationally comparable dollars. To construct these price levels, the International Comparison Program (ICP) at the World Bank collects the prices of a detailed list of goods and services in selected countries and years. The countries from which the ICP has collected data on prices in a given year are called benchmark countries and the given year is called a benchmark year. The collected detailed prices are first aggregated into the price levels of basic headings. Basic headings are the most disaggregate level at which there exist matching expenditure data from national accounts. The PWT then aggregates the price levels at the basic heading level in the benchmark dataset into a national price level using expenditure data from national accounts as weights. For non-benchmark countries and years, the price levels are inferred from fitted values of price regression run on benchmark dataset. In this paper, I concentrate on the benchmark year 2005, for which national price levels are available in the PWT and disaggregate price levels are available in the ICP Benchmark Dataset 2005.

4.1. Testing the Model: the Standard Index

In addition to the B-S relationship, i.e. a positive relationship between per capita income and the national price level, the model developed in Section 3 also predicts that income inequality can affect the national price level and the impact is decreasing in per capita income. In this subsection, these model's predictions are tested using data at the aggregate level.

To show that not only per capita income but also income inequality is important in determining the national price level, I extend the regression in Rogoff (1996) by introducing the Gini coefficient as an additional explanatory variable to investigate if it can help to explain national price differentials.

Figure 7: Price Level versus GDP per capita in 2005 (U.S.=1)



Notes: The fitted line is obtained by regressing the logarithm of National Price Level on the logarithm of GDP per capita.

Table 1: Income Distribution and the National Price Level

	Relative Price Level $\log(P_j/P_{U.S.})$			
Independent Variable	(1)	(2)	(3)	(4)
$\log(Y_j/Y_{U.S.})$	0.186***	0.719***	0.919***	0.709***
	(0.0171)	(0.0854)	(0.117)	(0.0857)
Gini		-2.623***	-2.651***	-2.594***
		(0.510)	(0.606)	(0.510)
Gini$\times\log(Y_j/Y_{U.S.})$		-1.359***	-1.649***	-1.335***
		(0.218)	(0.307)	(0.218)
VAT			-0.00108	
			(0.00405)	
Population				-0.000182
				(0.000157)
Constant	-0.232***	0.763***	0.905***	0.759***
	(0.0409)	(0.192)	(0.241)	(0.192)
Observations	189	153	90	153
R^2	0.388	0.541	0.717	0.545

Notes: Standard errors are in parentheses. ***, ** and * indicate significantly different from zero at the 1%, 5% and 10% level respectively.

First, to show that the problem with the Balassa-Samuelson relationship pointed out in Rogoff (1996) still persists in 2005, Figure 3 in Rogoff (1996) is replicated in Figure 7 using the 2005 data in PWT 7.1. I exclude Zimbabwe from the figure due to its very volatile exchange rate in 2005. As shown in Figure 3 of Rogoff (1996), Figure 7 still shows that the B-S relationship performs well for the whole sample but does not perform well either within poor countries or within rich countries, suggesting the need to find other important explanatory variables of the national price level.

Guided by the importance of income inequality in the model, I replicate the regressions in Rogoff (1996) but adding the Gini coefficient as an extra explanatory variable, as shown in Table 1. $P_j/P_{U.S.}$ is the price level of country j relative to the United States, and $Y_j/Y_{U.S.}$ is country j's relative per capita income. Results from Regressions (1) to (4) are consistent with the B-S relationship that countries with higher per capita income tend to have higher price levels as the estimated coefficients of relative income are all significantly positive.

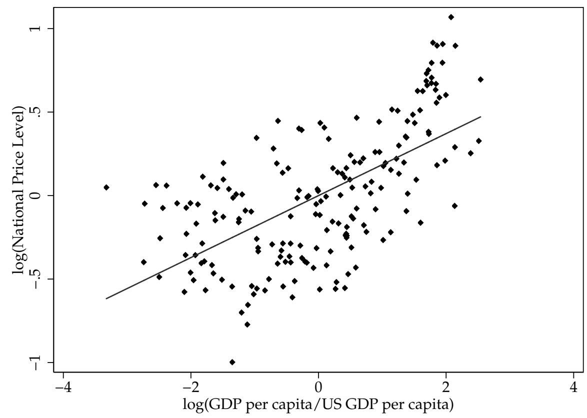
In Regression (2), we turn to the key prediction of the paper. When the Gini coefficient and the interaction of the Gini coefficient and the relative income are included in Regression (2), the coefficients of the Gini coefficient and the interaction term are significantly negative. This implies a significant correlation between income inequality and the national price level, and this correlation is decreasing in per capita income. Notice that the coefficient on income has risen substantially. However, when we evaluate the overall effect of income at the mean value of the Gini coefficient (0.408), we find that the overall impact of income is 0.165 ($= 0.719 - 1.359 \times 0.408$), which is consistent with the result in Regression (1). Therefore, Regression (2) is a better specification as it can capture both the B-S relationship and the nonmonotonic relationship between income inequality and the national price level. This explains why R^2 increases from 0.388 in Regression (1) to 0.541 in Regression (2).[17]

To check that these regression results are not driven by outliers, the partial regression plots for Regression (2) are also shown below. Panel (a) of Figure 8 is the bivariate plot of $\log(P_j/P_{U.S.})$ against $\log(Y_j/Y_{U.S.})$. Panel (b), (c) and (d) show the partial regression plots (controlling for other regressors) of $\log(P_j/P_{U.S.})$ against $\log(Y_j/Y_{U.S.})$, $Gini$ and $Gini \times \log(Y_j/Y_{U.S.})$ respectively. We can see that after controlling for other regressors, the slope of the B-S relationship in the partial regression plot (Panel (b)) is higher than in the bivariate plot (Panel(a)). And the partial regression plots in Panel (c) and (d) show significant negative relationships, as estimated in Regression (2). From these plots, it is

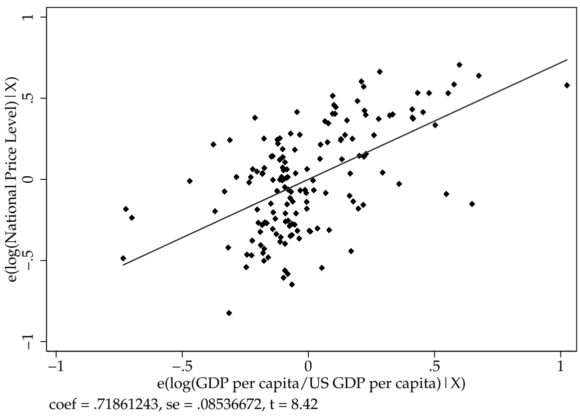
[17] Notice that the sample size changes from (1) to (2) due to the fact that the Gini coefficients are not available for some countries. It can be shown that restricting the sample in (1) to the sample in (2) won't change the estimation results much. Therefore, any inference drawn from (1) and (2) are not caused by the different sample sizes in the two regressions.

clear that the estimated relationship in Regression (2) are not driven by a small number of influential observations.

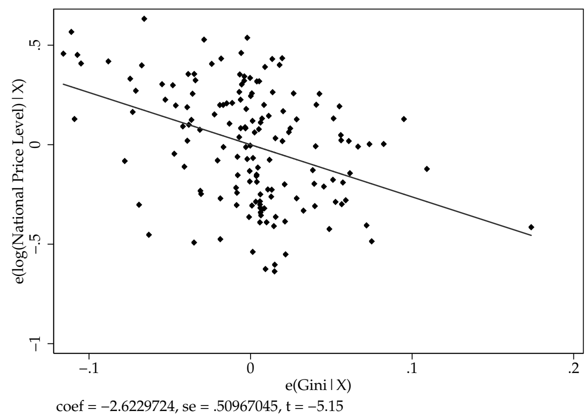
Figure 8: The B-S Relationship and Partial Regression Plots of Regression (2)



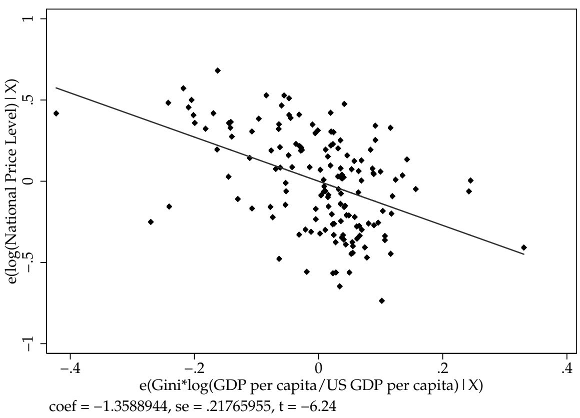
(a) Bivariate B-S Relationship



(b) $\log(Y_j/Y_{U.S.})$



(c) *Gini*



(d) $Gini \times \log(Y_j/Y_{U.S.})$

Therefore, the results in Regression (2) are consistent with what the model predicts in Proposition 3: per capita income has a positive impact on the national price level, income inequality also has a significant impact on the national price level and the impact is decreasing in per capita income. As robustness checks, Regressions (3) and (4) control for VAT[18] and population, with the latter being a proxy for market size to eliminate the potential impact of quality on the price level that is due to market size. The estimation results show that the inclusion of these two control variables does not change the estimation result in (2). Moreover, both control variables are not significant at the 10%

[18]VAT stands for Value Added Tax. If a country has no VAT system, then the country's sales tax or goods and services tax (GST) will be used instead. These taxes are included in the final prices paid by consumers, hence these tax rates are included in the regression in order to control for their impact on the national price level.

level.

Therefore, the results in Table 1 confirm the model's predictions and show that income inequality has a non-monotonic impact on the national price level. Moreover, the extra amount of explanatory power it provides is significant.

4.2. Testing the Model: the 'M-goods' Index

As the Balassa-Samuelson explanation of the national price level relies on the service component in the national price level, in addition to the above test using the national price level, I also want to test my model using a modified price index excluding all the services. So I replicate the estimations in Table 1, but do so for the price level of M-group (non-service) products and the national price level, which are both constructed using the standard method used by the PWT, i.e. the Geary-Khamis (GK) method[19], and the price levels at the basic heading level from the ICP Benchmark Dataset 2005.[20]

All the estimation results are shown in Table 2.[21] $P_{j,M}/P_{U.S.,M}$ is the price level of M-group products of country j relative to the United States. In the first two regressions, the relationships between the constructed national price level and income distribution are consistent with the empirical results in Section 4.1. The slope coefficient in Regression (1), i.e. the elasticity of the national price level with respect to per capita income is significantly positive. In Regression (2), when the Gini coefficient and the interaction of the Gini coefficient and per capita income are included, the coefficients of the Gini coefficient and the interaction term are significantly negative. Regression (2) also raises R^2 from 0.527 in Regression (1) to 0.622.

In Regression (3) and (4), the same regressions are run for the price level of M-group products. On inspection, we can find that all the above qualitative results in Regression (1) and (2) in terms of the significance and the sign of estimated coefficients and the improvement in R^2 also hold for the case of M-group products. In Regression (3), per capita income has a significantly positive impact on the price level. Regression (4) includes the Gini coefficient and the interaction term as additional regressors and all estimated coefficients are significant. The explanatory power of the two additional regressors also increases R^2.

[19]Using the EKS method generates very similar results.

[20]The national price level is constructed by aggregating the price levels of all basic headings. The price level of M-group products is constructed by aggregating the price levels of all non-service basic headings. The classification of basic headings is based on OECD (2006).

[21]The different sample size in Table 1 and 2 are due to the ability to construct the M-group price index and the aggregate price index for Table 2. In Table 1, the national price levels in 2005 are from PWT 7.1, which covers 190 countries in 2005. Therefore, in column 1 of Table 1, the number of observations is 189 as Zimbabwe is excluded due to its very volatile exchange rate in 2005. While in Table 2, in order to construct the price index for the M-group products, I have to use the price levels of basic headings from the ICP Benchmark Dataset 2005. However, this dataset covers less countries than PWT 7.1 does and it only covers 146 benchmark countries.

Table 2: Income Distribution and the National Price Level: M-group products

	Aggregate: $\log(P_j/P_{U.S.})$		M-group: $\log(P_{j,M}/P_{U.S.,M})$	
	(1)	(2)	(3)	(4)
$\log(Y_j/Y_{U.S.})$	0.269***	0.900***	0.154***	0.757***
	(0.0213)	(0.115)	(0.0175)	(0.0875)
Gini Index		-2.981***		-2.961***
		(0.715)		(0.544)
Gini Index$\times \log(Y_j/Y_{U.S.})$		-1.619***		-1.546***
		(0.298)		(0.226)
Constant	-0.180***	0.922***	-0.0168	1.083***
	(0.0525)	(0.261)	(0.0432)	(0.198)
Observations	145	132	145	132
R^2	0.527	0.622	0.352	0.551

Notes: Standard errors are in parentheses. ***, ** and * indicate significantly different from zero at the 1%, 5% and 10% level respectively. The price level of the M-group products and the aggregate price level are aggregated using the GK method.

However, the improvement in R^2 is much greater than the improvement from Regression (1) to Regression (2). In Regression (3), given the small slope coefficient, R^2 is only 0.352, but including the Gini index and the interaction term increases R^2 to 0.551 in Regression (4). This implies that, compared with per capita income, income inequality provides much more explanatory power for the price level of M-group products than for that of services.

These empirical results can actually shed some light on the explanatory power of the B-S model. The B-S explanation relies on the service component in the national price level. However, it is shown that using a modified price index excluding all services basic headings, the B-S relationship and the second distinctive prediction of the present theory survive. Therefore, the present theory provides a full explanation for the fact that both per capita income and income inequality matter for the price level of non-services, which cannot be addressed by the B-S model.

5. Conclusion

This paper provides a new candidate explanation for the B-S relationship, which is based on an appeal to mismeasured quality. The key assumption is that income elasticity of quality is non-negligible and tends to be higher for nontraded goods. The model implies the B-S relationship, and a new distinctive prediction: keeping per capita income constant, income inequality has a significant impact on the national price level and the impact is decreasing in per capita income. I show that this second prediction is consistent with empirical evidence. Furthermore, as the B-S model relies on the services component in the national price level, I also repeat the empirical test using a modified index excluding the 'pure services' categories. It is shown that even when services are excluded, the B-S relationship, and the second distinctive prediction of the present theory survive.

In addition, the model's predictions can also be supported by over-time evidence. When the panel estimation of the B-S effect in Rodrik (2008) is extended to take into account the impact of income inequality in various specifications, the estimation results are consistent with the relationships between income distribution and the national price levels predicted in this paper. The details are in an online appendix.

Appendix A

A.1. Proof of Lemma 1

Equating (7) and (10) yields

$$k_z z_m^{k_z} z^{-(k_z+1)}[\frac{p(z)}{A_z \tau z^{\gamma}}]^{1/(\tau-1)} dz$$
$$= k_c c_m^{k_c}[\frac{\alpha}{\beta} z p'(z) + p(z)]^{-(k_c+1)} \left| \frac{\alpha}{\beta}[p'(z) + z p''(z)] + p'(z) \right| dz$$

$$k_z z_m^{k_z}(\frac{1}{A_z \tau})^{1/(\tau-1)} z^{-(k_z+1)-\gamma/(\tau-1)} p(z)^{1/(\tau-1)}$$
$$= k_c c_m^{k_c}[\frac{\alpha}{\beta} z p'(z) + p(z)]^{-(k_c+1)} \left| \frac{\alpha}{\beta}[p'(z) + z p''(z)] + p'(z) \right|. \tag{A.1}$$

This is a second-order nonlinear non-autonomous differential equation defining $p(z)$, $z \in [\overline{z}_{m,}\infty)$. The boundary condition is

$$c_m = \frac{\alpha}{\beta} \overline{z}_m p'(\overline{z}_m) + p(\overline{z}_m).$$

Here $\overline{z}_m$ is the lowest quality that is viable in the equilibrium, which is determined by the lowest income c_m and the equilibrium price function $p(z)$.

I adopt the standard method of undetermined coefficients to find a particular solution of the form $p(z) = bz^d$, with $b,\ d > 0$, which is then substituted into the market clearing condition to solve for the values of the parameters b and d (Zwillinger (1997), P415).

The first and second derivatives of the postulated price function form are

$$p'(z) = bdz^{d-1} \text{ and } p''(z) = bd(d-1)z^{d-2}. \tag{A.2}$$

Substituting (A.2) into (A.1) yields

$$\begin{aligned} & k_z z_m^{k_z} (\frac{b}{A_z \tau})^{1/(\tau-1)} z^{-(k_z+1)+(d-\gamma)/(\tau-1)} \\ = & k_c c_m^{k_c} [b(\frac{\alpha}{\beta}d+1)]^{-(k_c+1)} (\frac{\alpha}{\beta}d^2+d) b z^{-d(k_c+1)+d-1}. \end{aligned} \tag{A.3}$$

Since (A.3) holds for any value of z and both l.h.s. and r.h.s. are power functions of z, it must be true that the two parameters of the power functions on both sides are equal:

$$k_z z_m^{k_z} (\frac{b}{A_z \tau})^{1/(\tau-1)} = k_c c_m^{k_c} [b(\frac{\alpha}{\beta}d+1)]^{-(k_c+1)} (\frac{\alpha}{\beta}d^2+d) b \tag{A.4}$$

and

$$-(k_z+1) + (d-\gamma)/(\tau-1) = -d(k_c+1) + d - 1. \tag{A.5}$$

From (A.5),

$$d(k_c) = \frac{k_z + \gamma/(\tau-1)}{k_c + 1/(\tau-1)}. \tag{A.6}$$

and it is easy to show that $d'(k_c) < 0$.

Substituting (A.6) into (A.4) can solve for the other parameter b in the price function:

$$b(k_c, c_m) = \left\{ \frac{k_c c_m^{k_c} \left[\frac{\alpha}{\beta} \frac{k_z+\gamma/(\tau-1)}{k_c+1/(\tau-1)} + 1 \right]^{-k_c} \frac{k_z+\gamma/(\tau-1)}{k_c+1/(\tau-1)}}{k_z z_m^{k_z} (\frac{1}{A_z \tau})^{1/(\tau-1)}} \right\}^{(\tau-1)/[k_c(\tau-1)+1]}. \tag{A.7}$$

Therefore, a solution to the market equilibrium is

$$\begin{aligned} p(z) &= b(k_c, c_m) z^{d(k_c)} \\ &= \left\{ \frac{k_c c_m^{k_c} \left[\frac{\alpha}{\beta} \frac{k_z+\gamma/(\tau-1)}{k_c+1/(\tau-1)} + 1 \right]^{-k_c} \frac{k_z+\gamma/(\tau-1)}{k_c+1/(\tau-1)}}{k_z z_m^{k_z} (\frac{1}{A_z \tau})^{1/(\tau-1)}} \right\}^{(\tau-1)/[k_c(\tau-1)+1]} z^{\frac{k_z+\gamma/(\tau-1)}{k_c+1/(\tau-1)}}, \ z \in [\overline{z}_m, \infty), \end{aligned}$$

where $\overline{z}_m$ satisfies

$$c_m = \frac{\alpha}{\beta} \overline{z}_m p'(\overline{z}_m) + p(\overline{z}_m).$$

Q.E.D.

A.2. Proof of Proposition 1

Given the expenditure share of Z goods in (13), the impact of income inequality, i.e. the impact of the Gini coefficient, is equal to

$$\begin{aligned} & \frac{\partial[p(z)/c]}{\partial Gini} \\ = & \frac{\partial[\beta/(\alpha d+\beta)]}{\partial Gini} \\ = & \frac{\partial[\beta/(\alpha d+\beta)]}{\partial d}\frac{\partial d}{\partial Gini}. \end{aligned}$$

Since $d = [k_z(\tau-1)+\gamma]/[k_c(\tau-1)+1]$ and $Gini = 1/(2k_c-1)$, it can be shown that $\partial d/\partial Gini > 0$. Moreover, $\partial[\beta/(\alpha d+\beta)]/\partial d < 0$, so $\partial[p(z)/c]/\partial Gini < 0$, i.e. income inequality has a negative impact on the expenditure share of Z goods.

As $Gini = 1/(2k_c-1)$ and $\mu = c_m k_c/(k_c-1)$, changing per capita income while keeping the Gini coefficient constant has no impact on k_c or d and hence expenditure shares. Q.E.D.

A.3. Derivation of the Average Expenditure on Z goods $\bar{p}$

Given the market equilibrium price schedule $p(z)$, we can calculate the average expenditure on Z goods $\bar{p}$, which is the total expenditure on Z goods divided by the total number of units as follows:

$$\begin{aligned} \bar{p} &= \frac{\int_{\bar{z}_m}^{\infty} p(z)Q^s(z)dz}{\int_{\bar{z}_m}^{\infty} Q^s(z)dz} \\ &= \frac{\int_{\bar{z}_m}^{\infty} bz^d k_z z_m^{k_z}(\frac{b}{A_z\tau})^{1/(\tau-1)} z^{-(k_z+1)+(d-\gamma)/(\tau-1)}dz}{\int_{\bar{z}_m}^{\infty} k_z z_m^{k_z}(\frac{b}{A_z\tau})^{1/(\tau-1)} z^{-(k_z+1)+(d-\gamma)/(\tau-1)}dz} \\ &= b\frac{\int_{\bar{z}_m}^{\infty} z^{-(k_z+1)+(d-\gamma)/(\tau-1)+d}dz}{\int_{\bar{z}_m}^{\infty} z^{-(k_z+1)+(d-\gamma)/(\tau-1)}dz} \\ &= b\left[\frac{z^{-(k_z+1)+(d-\gamma)/(\tau-1)+d+1}}{-(k_z+1)+(d-\gamma)/(\tau-1)+d+1}\,|_{\bar{z}_m}^{\infty}\right] / \left[\frac{z^{-(k_z+1)+(d-\gamma)/(\tau-1)+1}}{-(k_z+1)+(d-\gamma)/(\tau-1)+1}\,|_{\bar{z}_m}^{\infty}\right]. \end{aligned}$$

Since from (12), $-(k_z+1)+(d-\gamma)/(\tau-1)+d+1 = (1-k_c)d < 0$,

$$\bar{p} = \frac{-(k_z+1)+(d-\gamma)/(\tau-1)+1}{-(k_z+1)+(d-\gamma)/(\tau-1)+d+1} b\bar{z}_m{}^d. \tag{A.8}$$

As $c_m = (\alpha/\beta)\bar{z}_m p'(\bar{z}_m) + p(\bar{z}_m)$ from Lemma 1 and $p(z) = bz^d$,

$$b\bar{z}_m^d = [\beta/(\alpha d+\beta)]c_m. \tag{A.9}$$

Therefore, (A.8) and (A.9) imply

$$\overline{p} = \frac{-(k_z+1)+(d-\gamma)/(\tau-1)+1}{-(k_z+1)+(d-\gamma)/(\tau-1)+d+1}\frac{\beta}{\alpha d+\beta}c_m. \tag{A.10}$$

Substituting (12) into (A.10) yields

$$\overline{p} = \frac{k_c}{k_c-1}c_m\frac{\beta}{\alpha\frac{k_z+\gamma/(\tau-1)}{k_c+1/(\tau-1)}+\beta}.$$

As the Gini coefficient and the mean of the Pareto income distribution are equal to $1/(2k_c-1)$ and $k_c c_m/(k_c-1)$, we can express the average expenditure in terms of the mean and the Gini coefficient:

$$\overline{p} = \mu\frac{\beta}{\alpha\frac{k_z+\gamma/(\tau-1)}{(1/Gini+1)/2+1/(\tau-1)}+\beta}. \tag{A.11}$$

which is just equal to the product of per capita income μ and the expenditure share on Z goods given (12), (13) and $Gini = 1/(2k_c-1)$.

A.4. Proof of Proposition 2

Given (14), $\tau > 1$ and $\overline{p}^* = \overline{p}^\eta$, we have

$$\frac{\partial\overline{p}^*}{\partial\mu} = \eta\overline{p}^{\eta-1}\frac{\partial\overline{p}}{\partial\mu} = \eta\overline{p}^{\eta-1}\frac{\beta}{\alpha\frac{k_z+\gamma/(\tau-1)}{(1/Gini+1)/2+1/(\tau-1)}+\beta} > 0,$$

and

$$\begin{aligned}
&\frac{\partial\overline{p}^*}{\partial Gini}\\
= \; &\eta\overline{p}^{\eta-1}\frac{\partial\overline{p}}{\partial Gini}\\
= \; &\eta\overline{p}^{\eta-1}\mu\frac{-\alpha\beta}{\left[\alpha\frac{k_z+\gamma/(\tau-1)}{(1/Gini+1)/2+1/(\tau-1)}+\beta\right]^2}\\
&\times\frac{k_z+\gamma/(\tau-1)}{[(1/Gini+1)/2+1/(\tau-1)]^2}\frac{1}{2Gini^2} < 0.
\end{aligned} \tag{A.12}$$

Thus, the impact of per capita income on the price level of Z goods is positive and the impact of income inequality is negative.

Differentiating (A.12) with respect to μ yields

$$
\begin{aligned}
&\frac{\partial^2 \overline{p}^*}{\partial Gini \partial \mu} \\
= \; & \eta[(\eta-1)\overline{p}^{\eta-2}\frac{\partial \overline{p}}{\partial \mu}\mu + \overline{p}^{\eta-1}] \frac{-\alpha\beta}{\left[\alpha \frac{k_z+\gamma/(\tau-1)}{(1/Gini+1)/2+1/(\tau-1)} + \beta\right]^2} \\
& \times \frac{k_z+\gamma/(\tau-1)}{[(1/Gini+1)/2+1/(\tau-1)]^2}\frac{1}{2Gini^2} \\
= \; & \eta^2\overline{p}^{\eta-1}\frac{-\alpha\beta}{\left[\alpha \frac{k_z+\gamma/(\tau-1)}{(1/Gini+1)/2+1/(\tau-1)} + \beta\right]^2} \frac{k_z+\gamma/(\tau-1)}{[(1/Gini+1)/2+1/(\tau-1)]^2}\frac{1}{2Gini^2} < 0.
\end{aligned}
$$

Therefore, the impact of income inequality on the price level of Z goods is decreasing in per capita income. Q.E.D.

A.5. Proof of Proposition 3

Given the definition of the Paasche index,

$$
\begin{aligned}
P_P &= \left(1 \cdot \frac{\alpha d}{\alpha d+\beta} + \frac{\overline{p}_0^*}{\overline{p}^*}\frac{\beta}{\alpha d+\beta}\right)^{-1} \\
&= \left(\frac{\alpha d}{\alpha d+\beta} + \frac{(\mu_0 \frac{\beta}{\alpha d_0+\beta})^\eta}{(\mu \frac{\beta}{\alpha d+\beta})^\eta}\frac{\beta}{\alpha d+\beta}\right)^{-1} \\
&= \left(\frac{\alpha d}{\alpha d+\beta} + \mathcal{R}\frac{\beta}{\alpha d+\beta}\right)^{-1}, \qquad (A.13)
\end{aligned}
$$

where $\mathcal{R} \equiv \overline{p}_0^*/\overline{p}^* = [\mu_0(\alpha d+\beta)]^\eta/[\mu(\alpha d_0+\beta)]^\eta$.

Therefore,

$$\frac{\partial \mathcal{R}}{\partial \mu} = -\eta\mathcal{R}\mu^{-1} < 0 \qquad (A.14)$$

and

$$\frac{\partial \mathcal{R}}{\partial d} = \eta\mathcal{R}\frac{\alpha}{\alpha d+\beta} > 0. \qquad (A.15)$$

The impact of per capita income on the Paasche index can be obtained using (A.13) and (A.14):

$$
\begin{aligned}
\frac{\partial P_P}{\partial \mu} &= -\left(\frac{\alpha d}{\alpha d+\beta} + \mathcal{R}\frac{\beta}{\alpha d+\beta}\right)^{-2}\frac{\beta}{\alpha d+\beta}\frac{\partial \mathcal{R}}{\partial \mu} \\
&= \left(\frac{\alpha d}{\alpha d+\beta} + \mathcal{R}\frac{\beta}{\alpha d+\beta}\right)^{-2}\eta\mathcal{R}\mu^{-1}\frac{\beta}{\alpha d+\beta} > 0. \qquad (A.16)
\end{aligned}
$$

Hence, the impact of per capita income on the Paasche index is positive.

Similarly, the impact of income inequality on the Paasche index can be obtained by differentiating (A.13) with respect to *Gini*:

$$
\begin{aligned}
\frac{\partial P_P}{\partial Gini} &= -\left(\frac{\alpha d}{\alpha d+\beta}+\mathcal{R}\frac{\beta}{\alpha d+\beta}\right)^{-2}\left[\frac{\alpha(\alpha d+\beta)-\alpha\alpha d}{(\alpha d+\beta)^2}+\frac{\partial \mathcal{R}}{\partial d}\frac{\beta}{\alpha d+\beta}+\mathcal{R}\frac{-\alpha\beta}{(\alpha d+\beta)^2}\right]\frac{\partial d}{\partial Gini} \\
&= \frac{(1-\eta)\mathcal{R}-1}{(\alpha d+\mathcal{R}\beta)^2}\alpha\beta\frac{\partial d}{\partial Gini}. \qquad (A.17)
\end{aligned}
$$

To see how the impact of income inequality on the Paasche index changes with per capita income, we can calculate the following second-order partial derivative using (A.17):

$$
\begin{aligned}
\frac{\partial^2 P_P}{\partial Gini\partial\mu} &= \frac{(1-\eta)(\alpha d+\mathcal{R}\beta)-2\beta[(1-\eta)\mathcal{R}-1]}{(\alpha d+\mathcal{R}\beta)^3}\frac{\partial \mathcal{R}}{\partial\mu}\alpha\beta\frac{\partial d}{\partial Gini} \\
&= \frac{(1-\eta)\alpha d-(1-\eta)\mathcal{R}\beta+2\beta}{(\alpha d+\mathcal{R}\beta)^3}\frac{\partial \mathcal{R}}{\partial\mu}\alpha\beta\frac{\partial d}{\partial Gini} \qquad (A.18)
\end{aligned}
$$

Now using (A.16), (A.17) and (A.18), we can derive how per capita income affects the semi-elasticity of the Paasche index with respect to income inequality:

$$
\begin{aligned}
&\frac{\partial e_{P_P,Gini}}{\partial\mu} \\
&= \frac{\partial(\frac{\partial P_P}{\partial Gini}\frac{1}{P_p})}{\partial\mu} \\
&= \frac{\partial^2 P_P}{\partial Gini\partial\mu}\frac{1}{P_p}+\frac{\partial P_P}{\partial Gini}\frac{1}{-P_P^2}\frac{\partial P_P}{\partial\mu} \\
&= \frac{(1-\eta)\alpha d+\beta}{\alpha d+\beta}\frac{1}{(\alpha d+\mathcal{R}\beta)^2}\alpha\beta\frac{\partial d}{\partial Gini}\mathcal{R}(-\eta)\mu^{-1}, \qquad (A.19)
\end{aligned}
$$

which implies that $\partial e_{P_P,Gini}/\partial\mu < 0$.

Similarly, given the definition of the Laspeyres index,

$$
\begin{aligned}
P_L &= (1-\frac{\beta}{\alpha d_0+\beta})+\frac{\overline{p}^*}{\overline{p}_0^*}\frac{\beta}{\alpha d_0+\beta} \\
&= (1-\frac{\beta}{\alpha d_0+\beta})+\frac{[\mu(\alpha d_0+\beta)]^\eta}{[\mu_0(\alpha d+\beta)]^\eta}\frac{\beta}{\alpha d_0+\beta} \\
&= (1-\frac{\beta}{\alpha d_0+\beta})+\mathcal{R}^{-1}\frac{\beta}{\alpha d_0+\beta}. \qquad (A.20)
\end{aligned}
$$

The impact of per capita income on the Laspeyres index can be obtained using (A.20) and (A.14):

$$
\begin{aligned}
\frac{\partial P_L}{\partial\mu} &= -\mathcal{R}^{-2}\frac{\beta}{\alpha d_0+\beta}\mathcal{R}(-\eta)\mu^{-1} \\
&= \mathcal{R}^{-1}\frac{\beta}{\alpha d_0+\beta}\eta\mu^{-1} > 0. \qquad (A.21)
\end{aligned}
$$

In addition, the impact of income inequality on the Laspeyres index can be obtained using (A.20):

$$\begin{aligned}\frac{\partial P_L}{\partial Gini} &= -\mathcal{R}^{-2}\frac{\beta}{\alpha d_0+\beta}\frac{\partial \mathcal{R}}{\partial Gini}\\ &= -\mathcal{R}^{-1}\frac{\beta}{\alpha d_0+\beta}\eta\frac{\alpha}{\alpha d+\beta}\frac{\partial d}{\partial Gini}.\end{aligned} \tag{A.22}$$

Furthermore, differentiating (A.21) with respect to $Gini$ yields

$$\frac{\partial^2 P_L}{\partial\mu\partial Gini} = -\mathcal{R}^{-2}\frac{\beta}{\alpha d_0+\beta}\eta\mu^{-1}\frac{\partial \mathcal{R}}{\partial d}\frac{\partial d}{\partial Gini}. \tag{A.23}$$

Now, using (A.21), (A.22) and (A.23), we can derive how per capita income affects the semi-elasticity of the Laspeyres index with respect to income inequality:

$$\begin{aligned}\frac{\partial e_{P_L,Gini}}{\partial\mu} &= \frac{\partial[\frac{\partial P_L}{\partial Gini}\frac{1}{P_L}]}{\partial\mu}\\ &= \frac{\partial^2 P_L}{\partial Gini\partial\mu}\frac{1}{P_L}+\frac{\partial P_L}{\partial Gini}\frac{1}{-P_L^2}\frac{\partial P_L}{\partial\mu}\\ &= \mathcal{R}^{-2}\frac{\beta}{\alpha d_0+\beta}\eta^2\frac{\alpha}{\alpha d+\beta}\frac{\partial d}{\partial Gini}\frac{1}{P_L}[-\mathcal{R}+\frac{1}{P_L}\frac{\beta}{\alpha d_0+\beta}]\mu^{-1}.\end{aligned} \tag{A.24}$$

From the definition of P_L we have

$$P_L\frac{\alpha d_0+\beta}{\beta} = \frac{\alpha d_0+\beta}{\beta}-1+\mathcal{R}^{-1}.$$

Thus,

$$\frac{1}{P_L}\frac{\beta}{\alpha d_0+\beta} = \frac{1}{(\alpha d_0+\beta)/\beta-1+\mathcal{R}^{-1}} = \frac{1}{\alpha d_0/\beta+\mathcal{R}^{-1}}. \tag{A.25}$$

Substituting (A.25) into (A.24) yields

$$\begin{aligned}\frac{\partial e_{P_L,Gini}}{\partial\mu} &= \mathcal{R}^{-2}\frac{\beta}{\alpha d_0+\beta}\eta^2\frac{\alpha}{\alpha d+\beta}\frac{\partial d}{\partial Gini}\frac{1}{P_L}[-\mathcal{R}+\frac{1}{\alpha d_0/\beta+\mathcal{R}^{-1}}]\mu^{-1}\\ &= \mathcal{R}^{-2}\frac{\beta}{\alpha d_0+\beta}\eta^2\frac{\alpha}{\alpha d+\beta}\frac{\partial d}{\partial Gini}\frac{1}{P_L}[\frac{-\mathcal{R}\alpha d_0/\beta-1+1}{\alpha d_0/\beta+\mathcal{R}^{-1}}]\mu^{-1}\\ &= \mathcal{R}^{-2}\frac{\beta}{\alpha d_0+\beta}\eta^2\frac{\alpha}{\alpha d+\beta}\frac{\partial d}{\partial Gini}\frac{1}{P_L}[\frac{-\mathcal{R}\alpha d_0/\beta}{\alpha d_0/\beta+\mathcal{R}^{-1}}]\mu^{-1}.\end{aligned} \tag{A.26}$$

Hence, $\partial e_{P_L,Gini}/\partial\mu < 0$.

Next, we will use the above results to show the signs of $e_{P_{F,\mu}}$ and $\partial e_{P_{F,Gini}}/\partial\mu$. According to the definition of $e_{P_F,\mu}$ and P_F,

$$\begin{aligned} e_{p_F,\mu} &= \frac{\partial P_F}{\partial \mu}\frac{\mu}{P_F} \\ &= \frac{\partial (P_P P_L)^{\frac{1}{2}}}{\partial \mu}\frac{\mu}{P_F} \\ &= \frac{1}{2}(P_P P_L)^{-\frac{1}{2}}(\frac{\partial P_P}{\partial \mu}P_L + \frac{\partial P_L}{\partial \mu}P_P)\frac{\mu}{P_F}\ , \end{aligned}$$

which is positive as $\partial P_P/\partial \mu > 0$ and $\partial P_L/\partial \mu > 0$.

Finally,

$$P_F = (P_P P_L)^{\frac{1}{2}}$$

implies

$$\log P_F = \frac{1}{2}\log P_P + \frac{1}{2}\log P_L.$$

Differentiating both side with respect to *Gini* yields

$$\frac{1}{P_F}\frac{\partial P_F}{\partial Gini} = \frac{1}{2}\frac{1}{P_P}\frac{\partial P_P}{\partial Gini} + \frac{1}{2}\frac{1}{P_L}\frac{\partial P_L}{\partial Gini},$$

which implies

$$e_{P_F,Gini} = \frac{1}{2}e_{P_P,Gini} + \frac{1}{2}e_{P_L,Gini},$$

according to the definitions of $e_{P_F,Gini}$, $e_{P_P,Gini}$ and $e_{P_L,Gini}$, i.e. the semi-elasticities of P_F, P_P and P_L with respect to *Gini*. Therefore,

$$\frac{\partial e_{P_F,Gini}}{\partial \mu} = \frac{1}{2}\frac{\partial e_{P_P,Gini}}{\partial \mu} + \frac{1}{2}\frac{\partial e_{P_L,Gini}}{\partial \mu},$$

which is negative as $\partial e_{P_P,Gini}/\partial \mu < 0$ and $\partial e_{P_L,Gini}/\partial \mu < 0$ from (A.19) and (A.26). Q.E.D.

Appendix B

B.1. Data Sources

Data on disaggregate price parities and expenditure at the basic heading level come from the ICP Benchmark 2005. Data on national price level, real per capita income and population are taken from the Penn World Table 7.1 (PWT 7.1). All the conclusions in this papers also hold for PWT 8.0 and PWT 8.1.

Data on the Gini coefficient are taken from the World Bank: World Development Indicators. For those countries with the Gini coefficient missing in 2005, a linear interpolation is used, if there are at least one observation before 2005 and one observation after 2005 for a country. If the available observations for a country are all before 2005 or all after 2005, then the nearest neighbour interpolation is used. VAT data are from KPMG "Corporate and Indirect Tax Survey 2011".

Data on housing quantities and value come from 2007 Survey of Consumer Finances (SCF) Chartbook. Data on vehicle quantity and expenditure are taken from 2005 Consumer Expenditure Survey (CEX) Tables. Expenditure and quantity for different types of clothes are extracted from 2005 CEX Interview Survey microdata. Data on OECD consumption expenditure in 2005 are downloaded from OECD iLibrary.

The relative importance of income elasticity of quality for 66 durable goods is based on Table 4 of Bils and Klenow (2001). The tradability for each good is constructed as the ratio of imports to gross domestic demand, using the data from US Benchmark Input-Output Data in 2002.

B.2. Product Classification

According to OECD (2006), all basic headings are classified into ND (non-durable), SD (semi-durable), D (durable), S (service), IS (individual service), CS (collective service) and IG (investment goods) by nature of its output. In this paper, the basic headings of services, including S (service), IS (individual Service) and CS (collective service) are defined as the S-group products, while the rest of the basic headings are defined as the M-group products.

Appendix C

C.1. A Modified Version of Figure 1 Showing the Industry Labels

Figure 1 Modified: The B-S Estimation: Services vs. Non-Services

Notes: a and b are the intercept and slope from the B-S estimation. The open diamonds and the dark squares represent the estimates from services and non-services respectively. Data from Zimbabwe are not included in the estimation due to its volatile exchange rate in 2005. The basic heading of Passenger Transport by Air is excluded from the figure, as its B-S relationship is downward-sloping.

Please see above a modified version of Figure 1 showing the industry labels for all the data points. The correspondence table between the industry labels and the full industry names is shown below. We can see that the service sectors that depend little on tradeables, such as housing and hairdress, have large positive estimates of b, while the service sectors depend more on tradeables, such as telephone service, have small estimates of b. So this modified figure provides more supporting evidence for using the estimates of a and b to separate the industries, for which costs are mainly local labour costs, from the industries, for which costs are mainly the international costs of tradeables.

Table: The Correspondence Table between Industry Labels and Industry's Full Names

Basic Heading Code	Label	Full Name
1101111	rice	Rice
1101112	cereal_ot	Other cereals and flour
1101113	bread	Bread
1101114	bakery_ot	Other bakery products
1101115	pasta	Pasta products
1101121	beef	Beef and veal
1101122	pork	Pork
1101123	lamb	Lamb, mutton and goat
1101124	poultry	Poultry
1101125	meat_ot	Other meats and preparations
1101131	seafood_fresh	Fresh or frozen fish and seafood
1101132	seafood_preserved	Preserved fish and seafood
1101141	milk_fresh	Fresh milk
1101142	milk_preserved	Preserved milk and milk products
1101143	cheese	Cheese
1101144	egg	Eggs and egg-based products
1101151	butter	Butter and margarine
1101153	fat_ot	Other edible oils and fats
1101161	fruit_fresh	Fresh or chilled fruit
1101162	fruit_preserved	Frozen, preserved or processed fruits
1101171	vegetable_fresh	Fresh or chilled vegetables
1101172	potato_fresh	Fresh or chilled potatoes
1101173	vegetable_preserved	Frozen or preserved vegetables
1101181	sugar	Sugar
1101182	jam	Jams, marmalades and honey
1101183	confectionery	Confectionery, chocolate and ice cream
110119	food_product	Food products n.e.c.

110121	coffee	Coffee, tea and cocoa
110122	soft_drink	Mineral waters, soft drinks, fruit and vegetable juices
1102111	spirits	Spirits
1102121	wine	Wine
1102131	beer	Beer
110220	tobacco	Tobacco
1103111	clothing_material	Clothing materials and accessories
1103121	garment	Garments
1103141	clothing_repair	Cleaning and repair of clothing
1103211	footwear	Footwear
1103221	footwear_repair	Repair and hire of footwear
110410	housing_rental	Actual and imputed rentals for housing
110430	housing_repair	Maintenance and repair of the dwelling
110440	housing_water	Water supply and miscellaneous services relating to the dwelling
110442	housing_ot	Miscellaneous services relating to the dwelling
110451	electricity	Electricity
110452	gas	Gas
110453	fuel_ot	Other fuels
110511	furniture	Furniture and furnishings
110512	carpet	Carpets and other floor coverings
110513	furniture_repair	Repair of furniture, furnishings and floor coverings
110520	household_textile	Household textiles
110531	appliances_major	Major household appliances whether electric or not
110532	appliances_small	Small electric household appliances
110533	applicances_repair	Repair of household appliances
110540	utensil	Glassware, tableware and household utensils
110551	tool_major	Major tools and equipment
110552	tool_small	Small tools and miscellaneous accessories
110561	nondurable_housegood	Non-durable household goods

1105621	domestic_service	Domestic services
1105622	houseservice	Household services
110611	pharmaceutical	Pharmaceutical products
110612	medical_ot	Other medical products
110613	therapeutical	Therapeutical appliances and equipment
110621	medical_service	Medical Services
110622	dental	Dental services
110623	paramedical	Paramedical services
110630	hospital	Hospital services
110711	car	Motor cars
110712	motor_cycle	Motor cycles
110713	bicycle	Bicycles
110722	transport_fuel	Fuels and lubricants
		for personal transport equipment
110723	transport_repair	Maintenance and repair
		of personal transport equipment
110724	transport_ot	Other services in respect
		of personal transport equipment
110731	transport_railway	Passenger transport by railway
110732	transport_road	Passenger transport by road
110733	transport_air	Passenger transport by air
110734	transport_sea	Passenger transport by sea and inland waterway
110735	transport_combined	Combined passenger transport
110736	transport_otservice	Other purchased transport services
110810	postal	Postal services
110820	telephone_equipment	Telephone and telefax equipment
110830	telephone_service	Telephone and telefax services
110911	audio_visual_equipment	Audio-visual, photographic
		and information processing equipment
110914	recording	Recording media
110915	audio_visual_repair	Repair of audio-visual, photographic

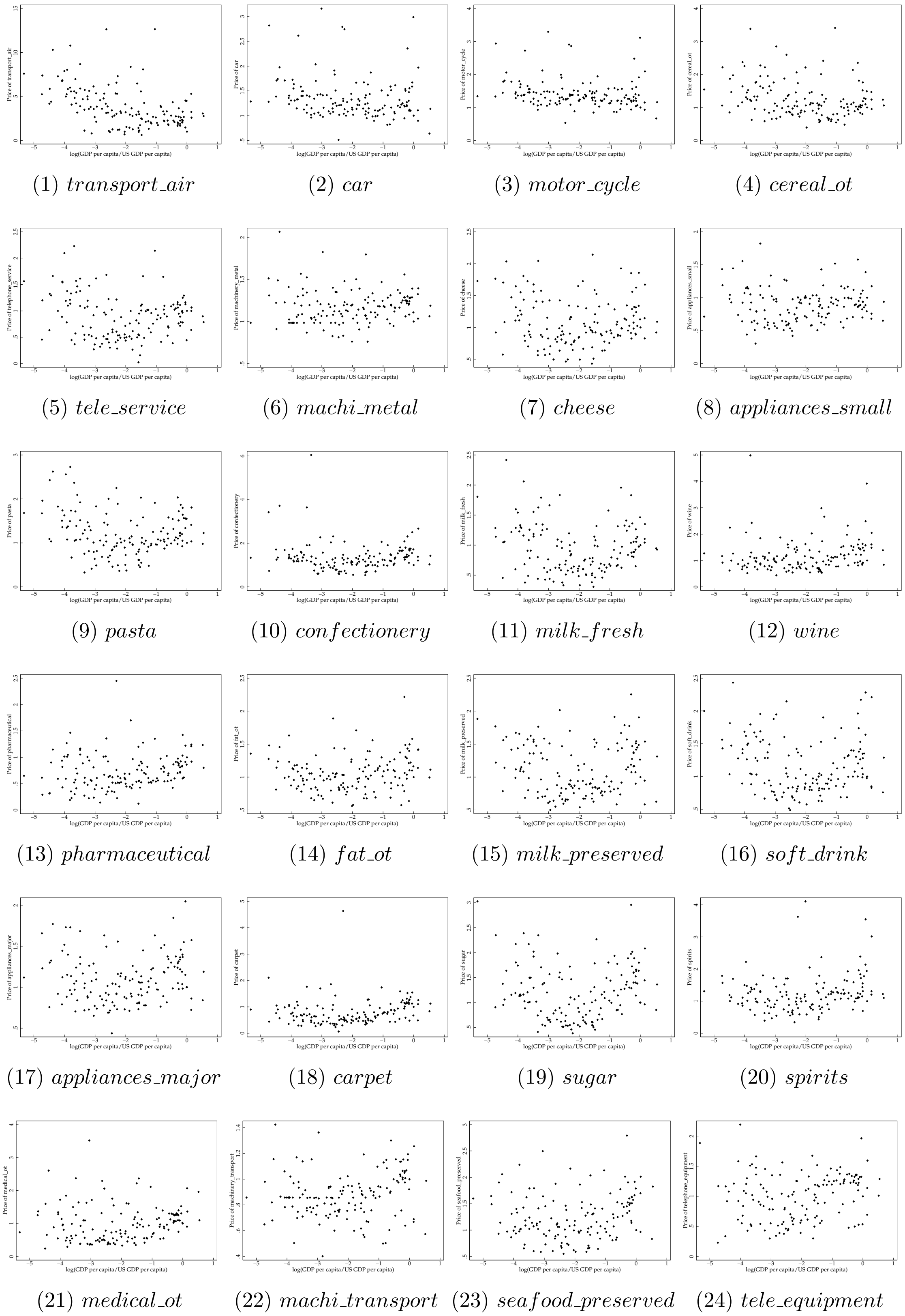
		and information processing equipment
110921	recreation_major	Major durables for outdoor and indoor recreation
110931	recreation_ot	Other recreational items and equipment
110933	garden	Gardens and pets
110935	pet	Veterinary and other services for pets
110941	sport	Recreational and sporting services
110942	cultural	Cultural services
110943	chance_game	Games of chance
110950	papers_book	Newspapers, books and stationery
110960	holiday	Package holidays
111000	education	Education
111110	catering	Catering services
111120	accommodation	Accommodation services
111211	hairdress	Hairdressing salons and
		personal grooming establishments
111212	personal_care	Appliances, articles and products for personal care
111220	prostitution	Prostitution
111231	jewellery	Jewellery, clocks and watches
111232	personaleffect_ot	Other personal effects
111240	social_protection	Social protection
111250	insurance	Insurance
111261	financialservice	FISIM
111262	financialservice_ot	Other financial services n.e.c
111270	service_ot	Other services n.e.c.
111300	netpurchase_abroad	Net purchases abroad
130221	govhealth_compensation	Compensation of employees(Government health)
130222	govhealth_intermediate	Intermediate consumption(Government health)
130223	govhealth_surplus	Gross operating surplus(Government health)
130224	govhealth_nettax	Net taxes on production(Government health)
130225	govhealth_sale	Receipts from sales(Government health)
130421	govedu_compensation	Compensation of employees(Government education)

130422	govedu_intermediate	Intermediate consumption(Government education)
130423	govedu_surplus	Gross operating surplus(Government education)
130424	govedu_nettax	Net taxes on production(Government education)
130425	govedu_sale	Receipts from sales(Government education)
140111	govcollective_compensation	Compensation of employees(Government collective)
140112	govcollective_intermediate	Intermediate consumption(Government collective)
140113	govcollective_surplus	Gross operating surplus(Government collective)
140114	govcollective_nettax	Net taxes on production(Government collective)
140115	govcollective_sale	Receipts from sales(Government collective)
150110	machinery_metal	Metal products and equipment
150120	machinery_transport	Transport equipment
150210	construction_residential	Residential buildings
150220	construction_nonresidential	Non-residential buildings
150230	construction_civilengineer	Civil engineering works
150300	construction_ot	Other products

C.2. Characterizing Disaggregate B-S Relationships

To get an idea of how the B-S effect varies across products and countries, we plot the scatter diagrams of the price level against GDP per capita (in logarithm) relative to the US for each basic heading in Figure C.1. On inspection, we can identify a clear and striking empirical pattern: for some products, the B-S relationship is weak and disperse. For example, in the case of cars (2) and small appliances (8), the B-S effect is insignificant and GDP per capita has no predictive power for the price levels. While for other products, the B-S relationship is highly nonlinear: within low- and middle- income countries, the relationship between per capita income and the price level is weakly positive, while within high income countries, there is a sudden increase in the slope of the positive relationship. For example, in the case of compensation of employees in government-produced health services (121), the B-S relationship is not stable across countries. The elasticity of the price level with respect to per capita income is large within rich countries, while the elasticity is not significant among low- and middle- income countries.

Figure C.1: Disaggregate Price Level of Basic Headings and GDP Per capita



(1) *transport_air* (2) *car* (3) *motor_cycle* (4) *cereal_ot*

(5) *tele_service* (6) *machi_metal* (7) *cheese* (8) *appliances_small*

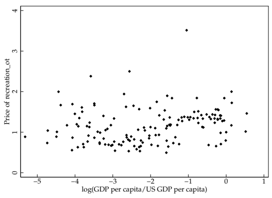
(9) *pasta* (10) *confectionery* (11) *milk_fresh* (12) *wine*

(13) *pharmaceutical* (14) *fat_ot* (15) *milk_preserved* (16) *soft_drink*

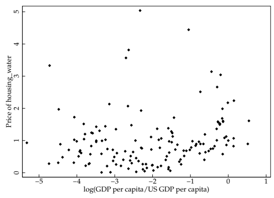
(17) *appliances_major* (18) *carpet* (19) *sugar* (20) *spirits*

(21) *medical_ot* (22) *machi_transport* (23) *seafood_preserved* (24) *tele_equipment*

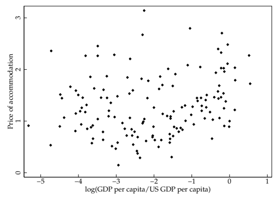
Figure C.1 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita



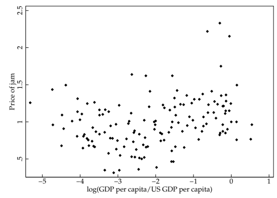
(25) *recreation_ot*



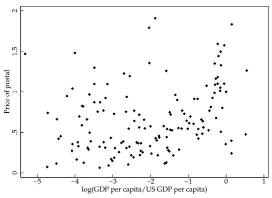
(26) *housing_water*



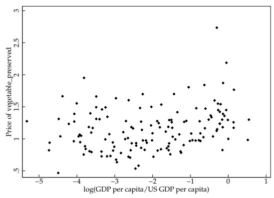
(27) *accommodation*



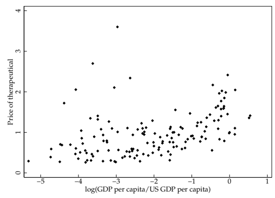
(28) *jam*



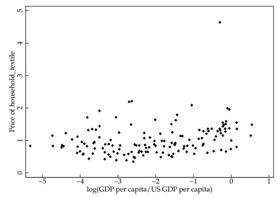
(29) *postal*



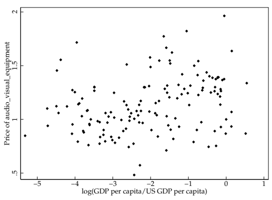
(30) *veg_preserved*



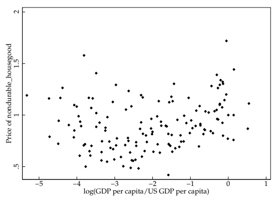
(31) *therapeutical*



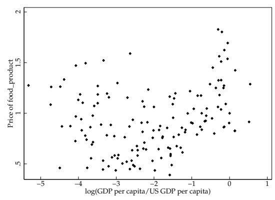
(32) *household_textile*



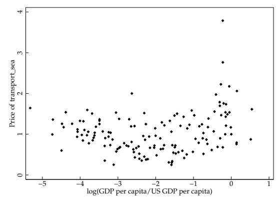
(33) *audio_visual_eqp*



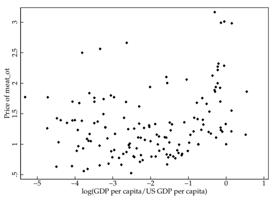
(34) *nondurable_hsgd*



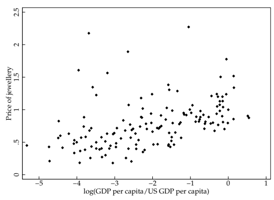
(35) *food_product*



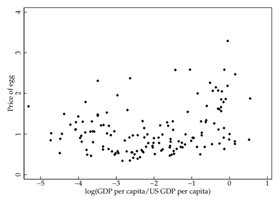
(36) *transport_sea*



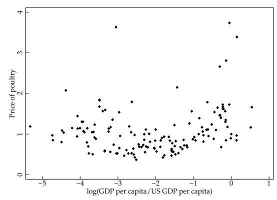
(37) *meat_ot*



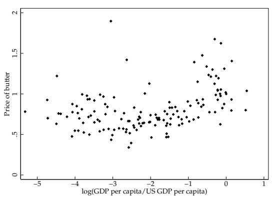
(38) *jewellery*



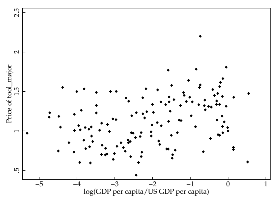
(39) *egg*



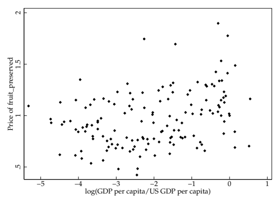
(40) *poultry*



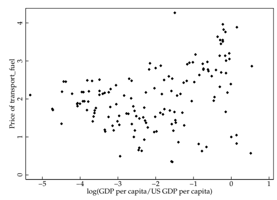
(41) *butter*



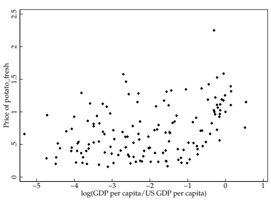
(42) *tool_major*



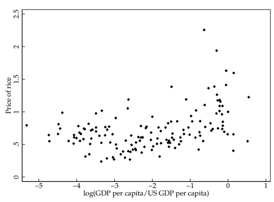
(43) *fruit_preserved*



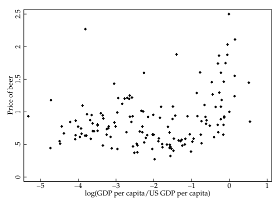
(44) *transport_fuel*



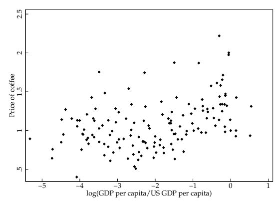
(45) *potato_fresh*



(46) *rice*

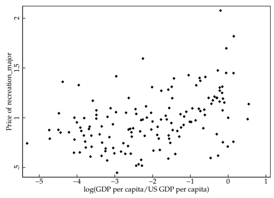


(47) *beer*

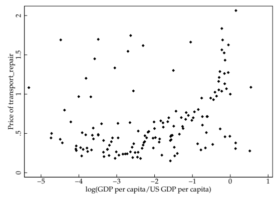


(48) *coffee*

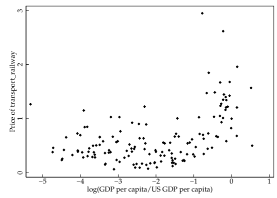
Figure C.1 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita



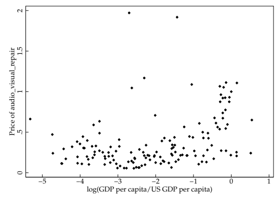
(49) *recreation_major*



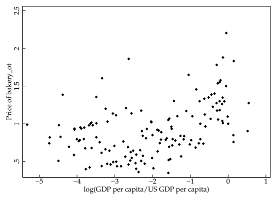
(50) *transport_repair*



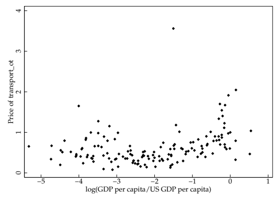
(51) *transport_railway*



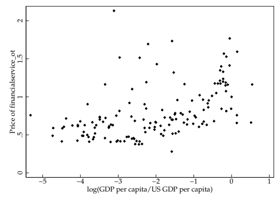
(52) *audio_visual_rep*



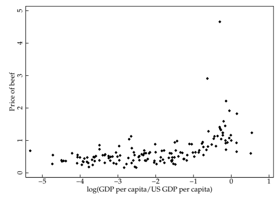
(53) *bakery_ot*



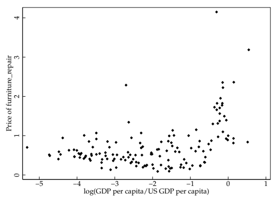
(54) *transport_ot*



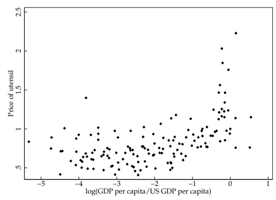
(55) *financialser_ot*



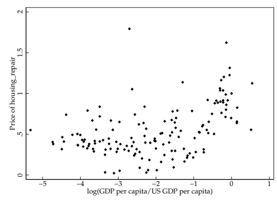
(56) *beef*



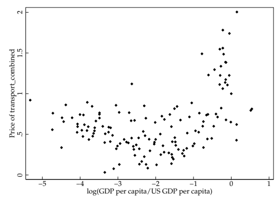
(57) *furniture_repair*



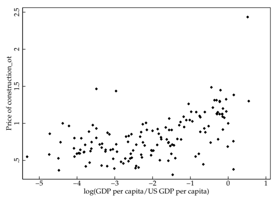
(58) *utensil*



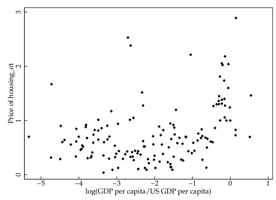
(59) *housing_repair*



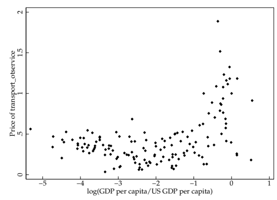
(60) *transport_comb*



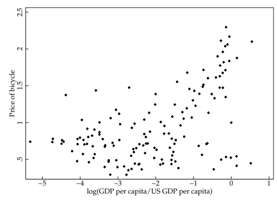
(61) *const_ot*



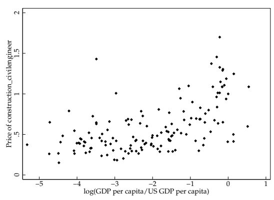
(62) *housing_ot*



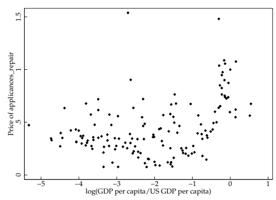
(63) *transport_otser*



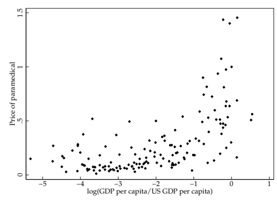
(64) *bicycle*



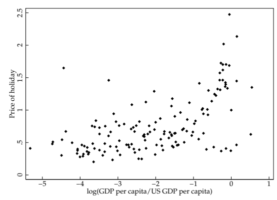
(65) *const_civileng*



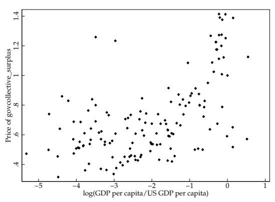
(66) *applicances_rep*



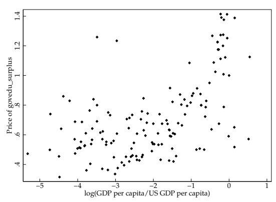
(67) *paramedical*



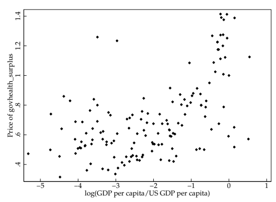
(68) *holiday*



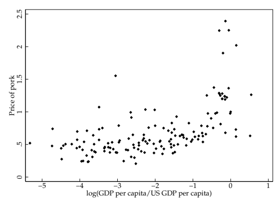
(69) *govcol_surplus*



(70) *govedu_surplus*

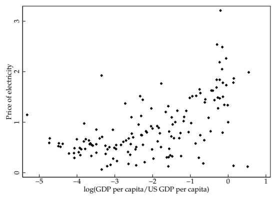


(71) *govhealth_surplus*

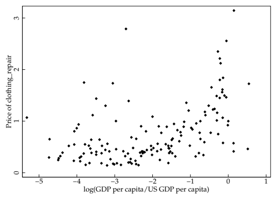


(72) *pork*

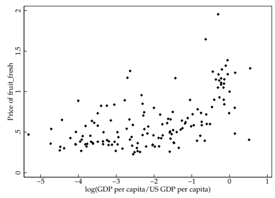
Figure C.1 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita



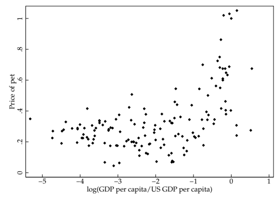
(73) *electricity*



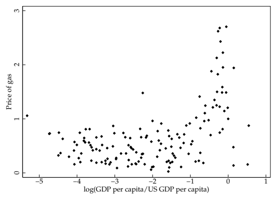
(74) *clothing_repair*



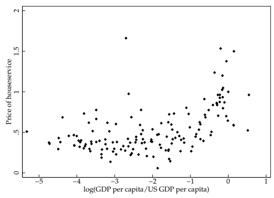
(75) *fruit_fresh*



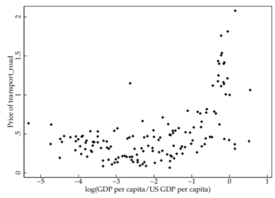
(76) *pet*



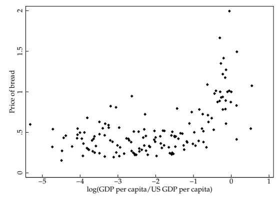
(77) *gas*



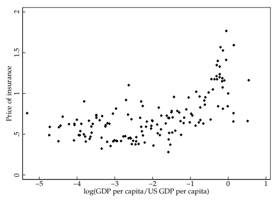
(78) *houseservice*



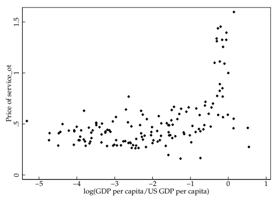
(79) *transport_road*



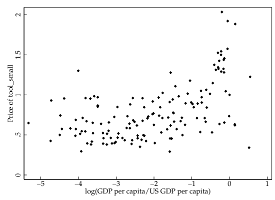
(80) *bread*



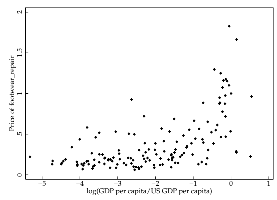
(81) *insurance*



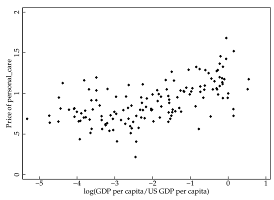
(82) *service_ot*



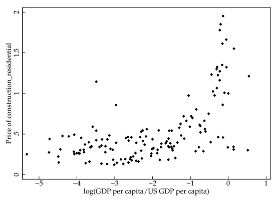
(83) *tool_small*



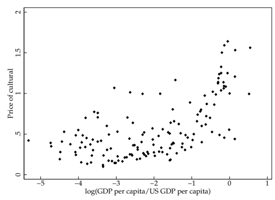
(84) *footwear_repair*



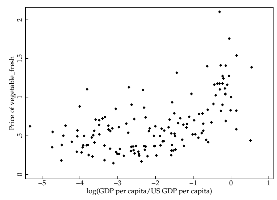
(85) *personal_care*



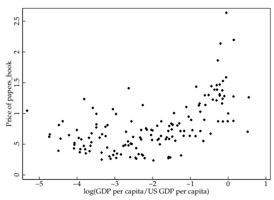
(86) *const_residential*



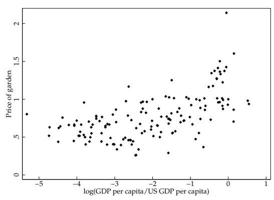
(87) *cultural*



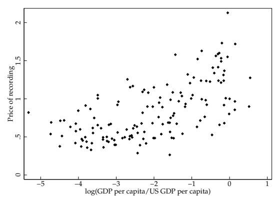
(88) *vegetable_fresh*



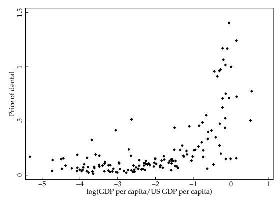
(89) *papers_book*



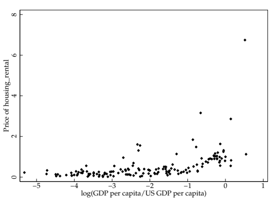
(90) *garden*



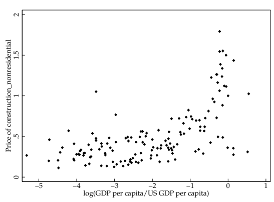
(91) *recording*



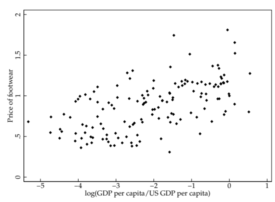
(92) *dental*



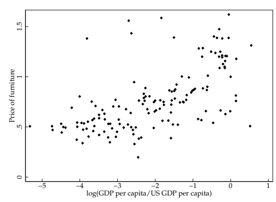
(93) *housing_rental*



(94) *const_nonresid*

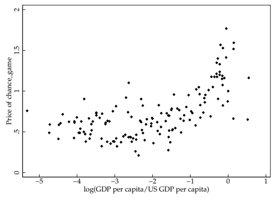


(95) *footwear*

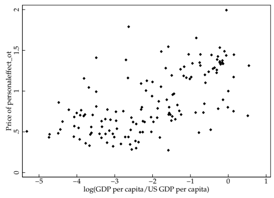


(96) *furniture*

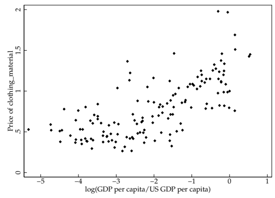
Figure C.1 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita



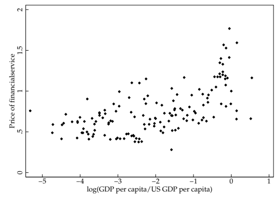
(97) *chance_game*



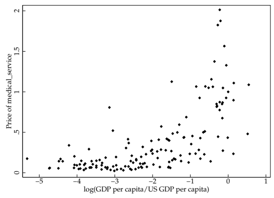
(98) *personaleffect_ot*



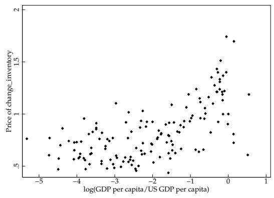
(99) *clothing_material*



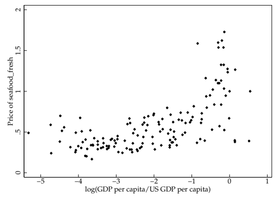
(100) *financialservice*



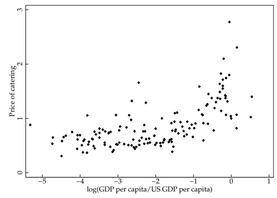
(101) *medical_service*



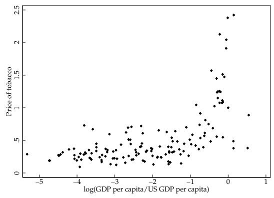
(102) *change_invent*



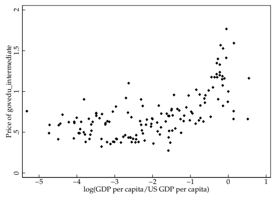
(103) *seafood_fresh*



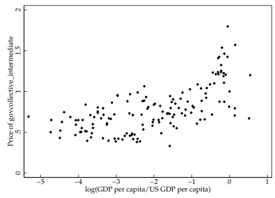
(104) *catering*



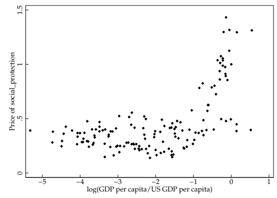
(105) *tobacco*



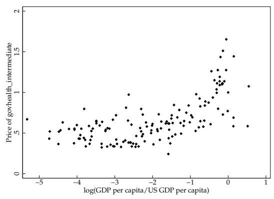
(106) *govedu_interme*



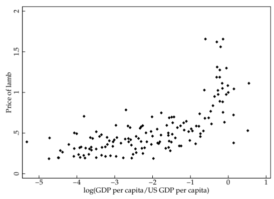
(107) *govcol_interme*



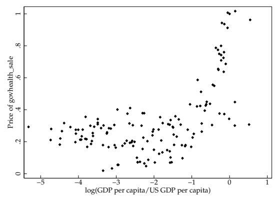
(108) *social_protection*



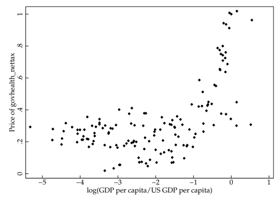
(109) *govhealth_inter*



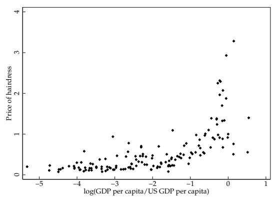
(110) *lamb*



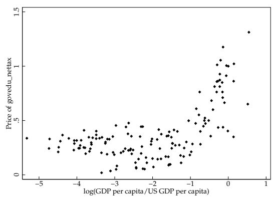
(111) *govhealth_sale*



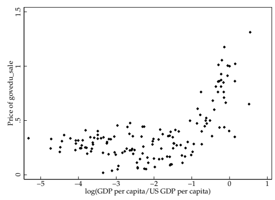
(112) *govhealth_nettax*



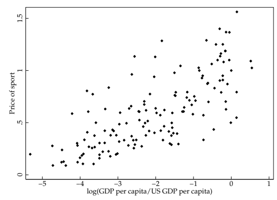
(113) *hairdress*



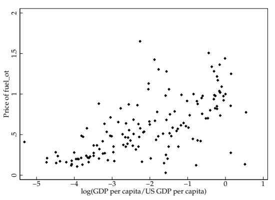
(114) *govedu_nettax*



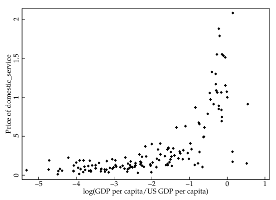
(115) *govedu_sale*



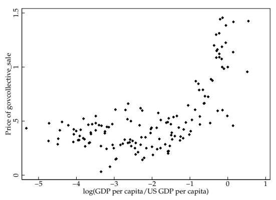
(116) *sport*



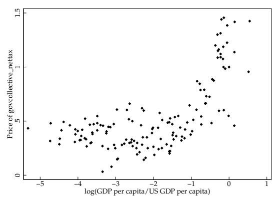
(117) *fuel_ot*



(118) *domestic_service*

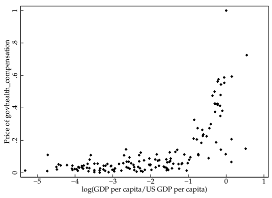


(119) *govcol_sale*

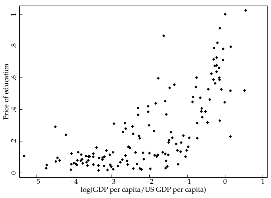


(120) *govcol_nettax*

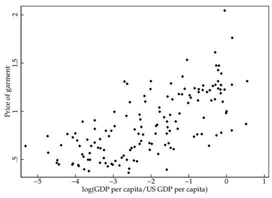
Figure C.1 Continued: Disaggregate Price Level of Basic Headings and GDP Per capita



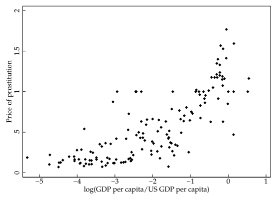
(121) *govhealth_comps*



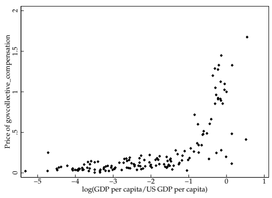
(122) *education*



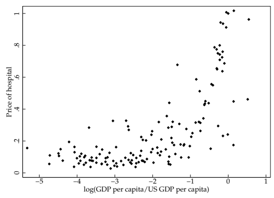
(123) *garment*



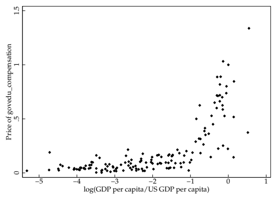
(124) *prostitution*



(125) *govcol_compensa*



(126) *hospital*



(127) *govedu_comps*

Notes: Zimbabwe is excluded from the figures due to its volatile exchange rate in 2005.

C.3. Income Elasticities of Quality and Quantity

In the traditional literature, if the price function is linear and the unit price is constant, prices usually do not play a role and consumption (physical quantity) is equivalent to consumption expenditure. Hence consumption quantity and consumption expenditure are used interchangeably.

However, as expenditure is in general the product of quantity and unit price, which depends on the quality of the good, income elasticity of expenditure can be decomposed into income elasticity of quantity and income elasticity of quality (unit price) as follows:

$$\begin{aligned} e_{expenditure} &= \frac{dlog(consumption\ expenditure)}{dlog(income)} \\ &= \frac{dlog(quantity \times unit\ price)}{dlog(income)} \\ &= \frac{d[log(quantity) + log(unit\ price)]}{dlog(income)} \\ &= \frac{dlog(quantity)}{dlog(income)} + \frac{dlog(unit\ price)}{dlog(income)} \\ &= e_{quantity} + e_{quality}. \end{aligned}$$

As consumption goods differ in their income elasticities of quantity and quality as shown in Bils and Klenow (2001), here I divide all consumption goods into two groups according to the relative magnitudes of these two elasticities. The first type of goods, which is denoted as X goods, has a very low income elasticity of quality but a high income elasticity of quantity. The other type of goods, which is denoted as Z goods, has a very high income elasticity of quality but a low income elasticity of quantity. Expenditure and quantity data from the U.S. are used to identify which type a particular consumption category belongs to. We focus on four categories of consumption goods, namely food, housing, clothes and vehicles. It is shown below that the income elasticities of quantity of food and housing are close to zero and the income elasticity of quality of clothes is close to zero, while both elasticities are nonzero for vehicles.

Table C.1 shows that these four categories plus hotels and restaurants account for on average more than 60% of the total expenditure within OECD countries in 2005, and the expenditure on those goods with low income elasticity of quantity, such as food and housing, constitutes on average around 70% of the total expenditure on these five categories. Food at restaurant is a substitute of food at home and staying at a hotel is a substitute of staying at home. Therefore, hotels and restaurants are also classified as Z goods, just like food and housing. Hence it is important to incorporate Z goods into the model due to its significant expenditure share.

Table C.1: Expenditure Shares of Food, Housing, Apparel, Transportation and Restaurants and Hotels within OECD countries in 2005

	Share of the Five Categories	Share of Z goods	Share of Z goods within the Five Categories
Max	0.751	0.508	0.805
Min	0.466	0.314	0.630
Average	0.610	0.426	0.700
Standard Deviation	0.051	0.041	0.041

Food

As has been documented in the literature, calorie intake does not vary with permanent income across households. Specifically, Aguiar and Hurst (2005) find that employed household heads with a higher level of income consume similar amounts of calories as employed household heads with a lower level of income. However, conditional on log calories, they find that the income elasticities of vitamin A and vitamin C are over 0.30 and the income elasticities of vitamin E and calcium are 0.17 and 0.08, respectively. In addition, the income elasticity of cholesterol is negative.

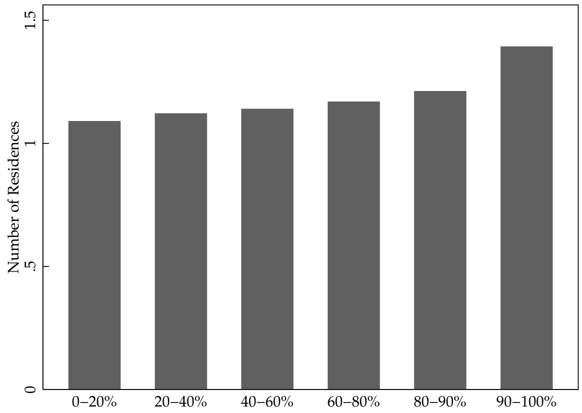
The results suggest that the income elasticity of quantity for food is very close to zero. Households with higher income do not consume a larger quantity of food than households with lower income. Instead, they consume higher quality foods, such as those rich in vitamin and calcium. On the other hand, low income households consume cheaper calories by having more fat and cholesterol in their food.

Housing

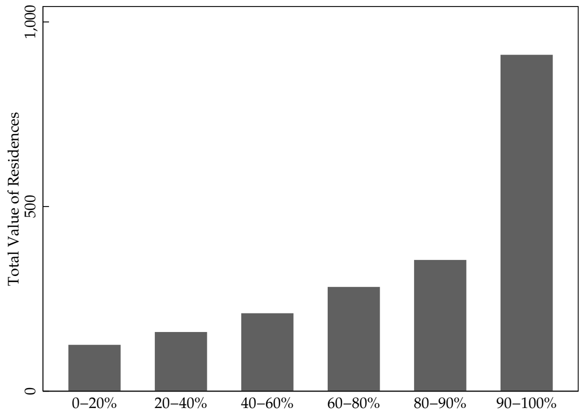
As for housing consumption, the number of houses counts as quantity whereas other characteristics of a house, such as square meters, all count as quality. Micro-evidence has shown that the price function of housing is nonlinear. For example, Anderson (1985) estimates the hedonic price function of housing, i.e. regressing housing prices on characteristics of houses, which include structural characteristics of the house, improvements to the house, physical characteristics of the lot, neighbourhood characteristics, etc. He shows that the price function is convex. Even if housing quality is defined by square meters, the price function is still estimated as a convex function. For example, Coulson (1992) estimates a nonparametric response of housing price to floorspace size. The marginal price is estimated to be increasing, which implies a convex price function. Mason and Quigley (1996) estimate the hedonic price indices for downtown Los Angeles and they find the price function is convex in size (1000 sq ft). Also, Bao and Wan (2004) find that the sale price per square foot is increasing in gross area controlling for other characteristics using Hong Kong data.

I use data from the Survey of Consumer Finances (SCF) to compute the number of residences, total value of residences and value per residence by percentile of income. Figure C.2 plots the results for 2004.

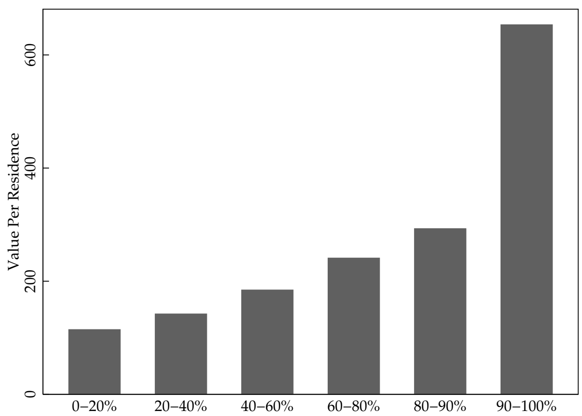
Figure C.2: Housing Quantity and Quality by Percentile of Income in 2004



(a) Number of Residences



(b) Total Value of Residences (thousands of dollars)



(c) Value Per Residence (thousands of dollars)

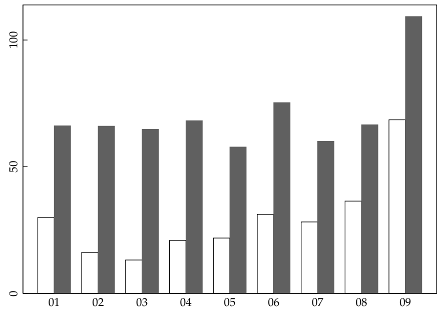
Notes: The families without a residence are excluded from the figures. Value per residence is obtained by dividing total value of residences by the average number of residences.

By inspection, one can notice that as we move from low percentiles of income to high percentiles of income, the number of residences only changes modestly and almost all the variations in total value of residence is due to the variations in value per residence. This implies a very low income elasticity of quantity.

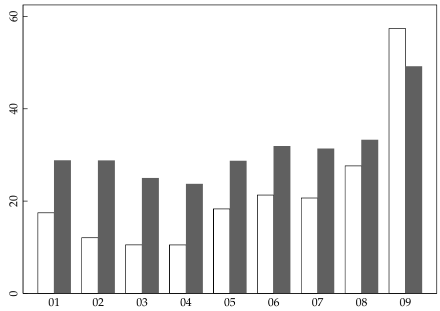
Clothes

The detailed expenditure and quantity data on clothes are extracted from the raw data files of 2005 Consumer Expenditure Survey (CEX). The total expenditure on a certain type of clothes is divided by the number of clothes to get the unit price. Figure C.3 plots the total expenditure and the unit price for different types of clothes across nine income classes. The white bars denote the total expenditure and the dark bars denote the unit price. It is shown that for all types of clothes, unit prices are almost the same for all income classes, which suggests a very low income elasticity of quality.

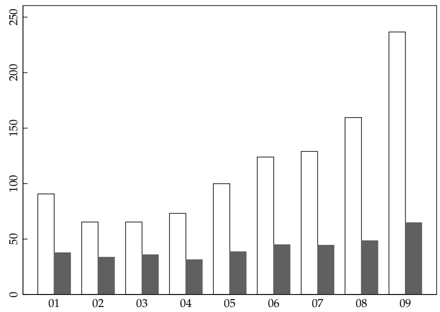
Figure C.3: Total Expenditure and Unit Price for Different Types of Clothes in Dollars by Income Class in 2005



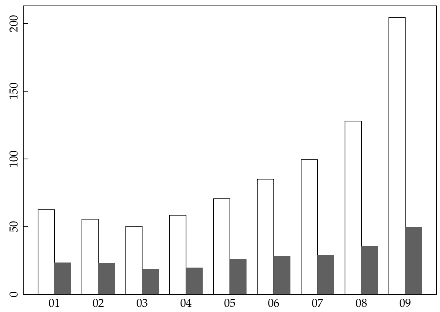
(a) Coats



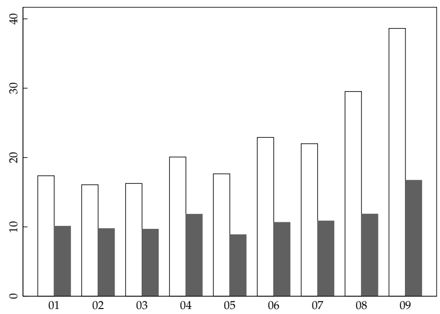
(b) Sweaters



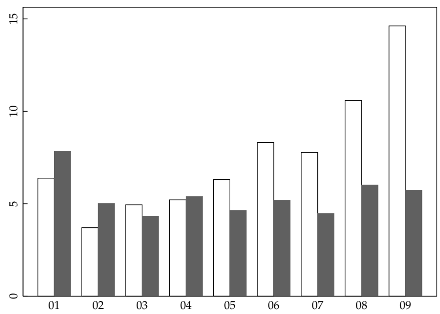
(c) Pants



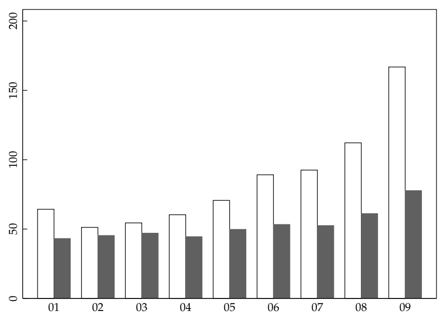
(d) Shirts



(e) Undergarments



(f) Hosiery



(g) Footwear

Notes: White bars denote total expenditure and dark bars denote unit price. The unit price is defined as the ratio of total expenditure to the average number of units. The nine income classes are: less than $5,000, $5,000 to $9,999, $10,000 to $14,999, $15,000 to $19,999, $20,000 to $29,999, $30,000 to $39,999, $40,000 to $49,999, $50,000 to $69,999, $70,000 and over.

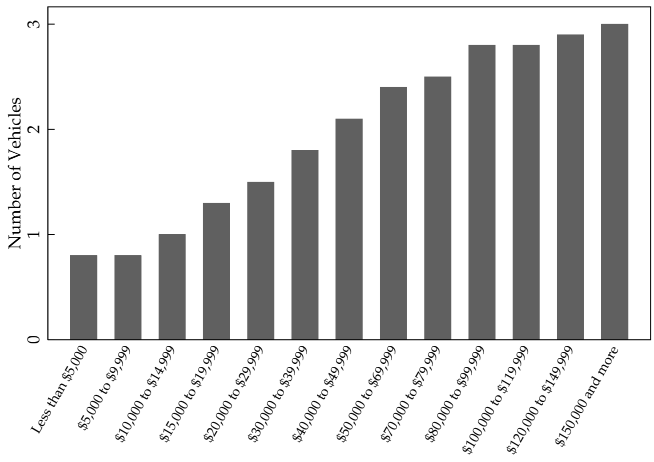
Vehicles

Data on quantity and expenditure of vehicles are also taken from the CEX. Figure C.4 plots the number of vehicles, total value of vehicle purchases and unit value per vehicle across income classes. It shows that neither income elasticity of quantity nor income elasticity of quality is close to zero. As we move from lower income groups to higher income groups, both quantity and quality increase significantly.

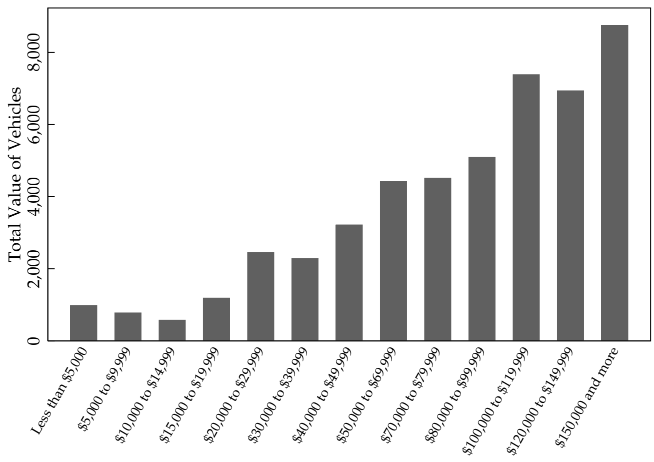
In addition to the four consumption categories noted above, Bils and Klenow (2001) also document the relative importance of income elasticities of quality and quantity for 66 durable goods in the CEX. Their results are consistent with the above evidence. For example, the income elasticity of quality of clothes is very low and the income elasticities of quantity and quality of vehicles are of the same magnitude.

In summary, the evidence presented shows that the income elasticities of quantity of food and housing are close to zero, the income elasticity of quality of clothes is close to zero and both are nonzero for vehicles. Some may argue that the observed elasticities are equilibrium outcomes, which are endogenous, so the observed patterns cannot be taken as primitives in the model. However, the observed elasticities, such as the zero income elasticity of quantity of food and housing, are not due to equilibrium outcomes, but due to the nature of the goods. For example, daily calorie intake has to be within a certain range regardless of income and for convenience, a household usually has one primary residence.

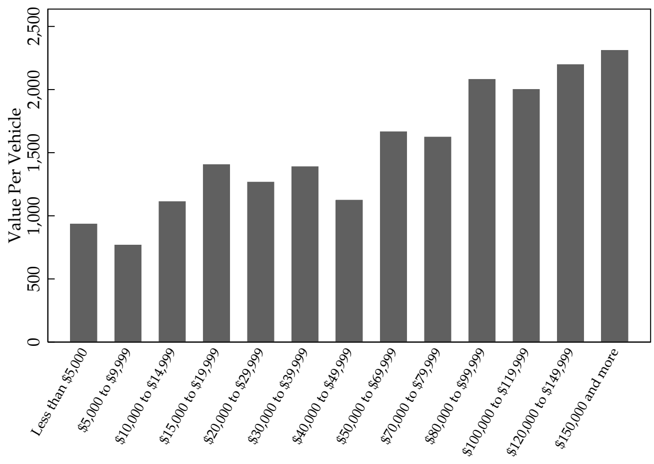
Figure C.4: Vehicle Quantity and Quality by Percentile of Income in 2005



(a) Number of Vehicles



(b) Total Value of Vehicles (dollars)



(c) Value Per Vehicle (dollars)

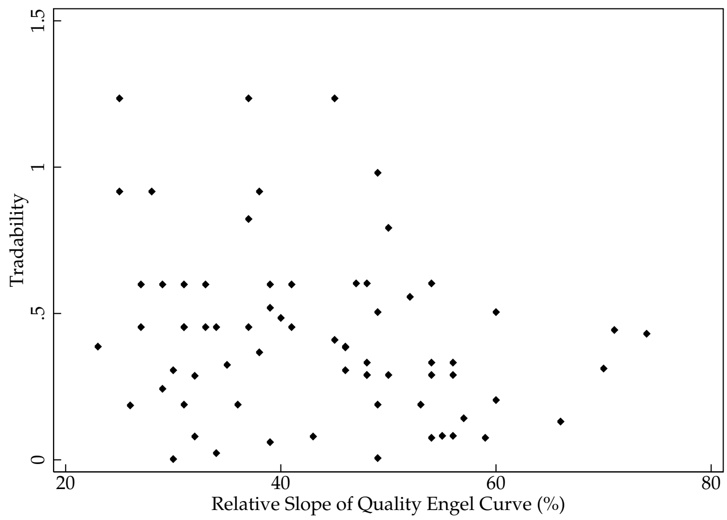
Notes: Value per vehicle is obtained by dividing total value of vehicles by the average number of vehicles.

C.4. Tradability and Income Elasticities of Quality and Quantity

In addition to the fact that consumption goods differ in income elasticities of quality and quantity, empirical evidence also suggests that goods with relatively high income elasticity of quality tend to be nontraded goods, while goods with relatively high income elasticity of quantity tend to be traded goods. This is the empirical motivation for why this paper just focuses on two types of goods in the model without loss of generality: traded homogeneous goods and nontraded vertically differentiated goods.

To show the correlation between tradability and income elasticities of quality and quantity, this subsection focuses on the 66 durable goods studied in Bils and Klenow (2001). For these goods, they have estimated the relative importance of income elasticities of quality and quantity, as measured by the relative slope of the quality Engel curve, which is the slope of quality Engel curve relative to the sum of the slopes of quality and quantity Engel curves. To measure the tradability of these goods, I first match the goods in Bils and Klenow (2001) with the commodities in the 2002 Benchmark Input-Output Tables of the U.S. Then for each commodity in the Input-Output Tables, I define a tradability measure using the ratio of imports to gross domestic demand, i.e. the sum of consumption, investment and government demand for the commodity. This tradability measure will be used as a proxy for the tradability of the corresponding good in Bils and Klenow (2001). The motivation for the measure is that it can show us the extent to which the domestic demand of these goods must rely on imports. A higher ratio, i.e. a higher degree of dependence on imports, implies that the good is more likely to be priced internationally rather than locally and hence it is more like a tradable good. Figure C.5 plots the tradability measure against the relative slope of the quality Engel curve and shows that there is a negative relationship between the two, which can be confirmed by their correlation coefficient -0.244 with a p-value of 0.043. In addition, their spearman's rank correlation is -0.239 with a p-value of 0.048. The significant negative relationship between tradability and income elasticity of quality suggests that we may abstract from other goods and focus on the traded homogeneous goods and the nontraded vertically differentiated goods in the model in order to make a sharp contrast of the roles played by these two types of goods.

Figure C.5: Tradability vs. Income Elasticity of Quality



Notes: The data points in the figure represent the 66 durable goods studied in Bils and Klenow (2001). The relative slopes of quality Engel curve are based on their estimates. The tradability is defined to be the ratio of imports to gross domestic demand.

Acknowledgments

I thank Alan M. Taylor, the editor, and two anonymous referees for very helpful comments that improved the paper. With particular thanks to John Sutton for his invaluable advice and continuous encouragement. I have benefited greatly from advice by Albert Marcet, Alex Michaelides, Philipp Schmidt-Dengler and Daniel Trefler. I also thank Gianluca Benigno, Francesco Caselli, Raj Chetty, Bernardo Guimaraes, Ethan Ilzetzki, Philipp Kircher, Bruce Lyons, Rachel Ngai, Martin Pesendorfer, Jörn-Steffen Pischke, Christopher Pissarides, Danny Quah, Mark Schankerman, Kevin Sheedy, Silvana Tenreyro, Tommaso Valletti, Jaume Ventura and other seminar participants for their helpful comments.

References

Aguiar, Mark and Erik Hurst, "Consumption versus Expenditure," *Journal of Political Economy*, 2005, *113* (5), 919–948.

Alessandria, George and Joseph P. Kaboski, "Pricing-to-Market and the Failure of Absolute PPP," *American Economic Journal: Macroeconomics*, 2011, *3* (1), 91–127.

Anderson, John E., "On Testing the Convexity of Hedonic Price Functions," *Journal of Urban Economics*, 1985, *18* (3), 334–337.

Balassa, Bela, "The Purchasing-Power Parity Doctrine: A Reappraisal," *Journal of Political Economy*, dec 1964, *72* (6), 584–596.

Bao, Helen X. H. and Alan T. K. Wan, "On the Use of Spline Smoothing in Estimating Hedonic Housing Price Models: Empirical Evidence Using Hong Kong Data," *Real Estate Economics*, 2004, *32* (3), 487–507.

Bekkers, Eddy, Joseph Francois, and Miriam Manchin, "Import prices, income, and inequality," *European Economic Review*, 2012, *56* (4), 848–869.

Bergstrand, Jeffrey H., "Structural Determinants of Real Exchange Rates and National Price Levels: Some Empirical Evidence," *American Economic Review*, 1991, *81* (1), 325–334.

Bhagwati, Jagdish N., "Why Are Services Cheaper in the Poor Countries?," *Economic Journal*, 1984, *94* (374), 279–286.

Bils, Mark and Peter J. Klenow, "Quantifying Quality Growth," *American Economic Review*, 2001, *91* (4), 1006–1030.

Choi, Yo Chul, David Hummels, and Chong Xiang, "Explaining Import Quality: The Role of the Income Distribution," *Journal of International Economics*, 2009, *77* (2), 265–275.

Coulson, N. Edward, "Semiparametric Estimates of the Marginal Price of Floorspace," *Journal of Real Estate Finance and Economics*, 1992, *5* (1), 73–83.

Crozet, Matthieu, Keith Head, and Thierry Mayer, "Quality Sorting and Trade: Firm-level Evidence for French Wine," *Review of Economic Studies*, nov 2012, *79* (2), 609–644.

Crucini, Mario J. and Hakan Yilmazkuday, "Understanding Long-Run Price Dispersion," *Journal of Monetary Economics*, sep 2014, *66*, 226–240.

Deaton, Angus and Alan Heston, "Understanding PPPs and PPP-Based National Accounts," *American Economic Journal: Macroeconomics*, 2010, *2* (4), 1–35.

Fajgelbaum, Pablo, Gene M. Grossman, and Elhanan Helpman, "Income Distribution, Product Quality, and International Trade," *Journal of Political Economy*, 2011, *119* (4), 721–765.

Feenstra, Robert C. and John Romalis, "International Prices and Endogenous Quality," *Quarterly Journal of Economics*, 2014.

Flam, Harry and Elhanan Helpman, "Vertical Product Differentiation and North-South Trade," *American Economic Review*, 1987, *77* (5), 810–822.

Gaffeo, Edoardo, Mauro Gallegati, and Antonio Palestrini, "On the size distribution of firms: additional evidence from the G7 countries," *Physica A: Statistical Mechanics and its Applications*, 2003, *324* (1-2), 117–123.

Goldberg, Pinelopi Koujianou and Frank Verboven, "The Evolution of Price Dispersion in the European Car Market," *Review of Economic Studies*, 2001, *68* (4), 811–848.

Griliches, Zvi, "Hedonic Price Indexes for Automobiles: An Econometric Analysis of Quality Change," in "The Price Statistics of the Federal Government," National Bureau of Economic Research, 1961, pp. 137–196.

Hallak, Juan Carlos, "Product Quality and the Direction of Trade," *Journal of International Economics*, 2006, *68* (1), 238–265.

_ **and Peter K. Schott**, "Estimating Cross-country Differences in Product Quality," *Quarterly Journal of Economics*, 2011, *126* (1), 417–74.

Hassan, Fadi, "The Price of Development: The Penn-Balassa-Samuelson Effect Revisited," *Working Paper*, 2015.

Hummels, David and Volodymyr Lugovskyy, "International Pricing in a Generalized Model of Ideal Variety," *Journal of Money, Credit, and Banking*, 2009, *41*, 3–33.

Imbs, Jean M., Haroon Mumtaz, Morten O. Ravn, and Helene Rey, "One TV, One Price?," *Scandinavian Journal of Economics*, 2010, *112* (4), 753–781.

Khandelwal, Amit, "The Long and Short (of) Quality Ladders," *Review of Economic Studies*, 2010, *77* (4), 1450–76.

Luttmer, Erzo G. J., "Selection, Growth, and the Size Distribution of Firms," *Quarterly Journal of Economics*, 2007, *122* (3), 1103–1144.

Mason, Carl and John M. Quigley, "Non-parametric Hedonic Housing Prices," *Housing Studies*, 1996, *11* (3), 373–385.

Melitz, Marc J., "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71* (6), 1695–1725.

OECD, *Eurostat-OECD Methodological Manual on Purchasing Power Parities* 2006.

Pakes, Ariel, "A Reconsideration of Hedonic Price Indexes with an Application to PC's," *American Economic Review*, 2003, *93* (5), 1578–1596.

_ , "Hedonics and the Consumer Price Index," *Annales d'Economie et de Statistique*, 2005, (79-80), 729–748.

Rodrik, Dani, "The real exchange rate and economic growth," *Brookings Papers on Economic Activity*, 2008.

Rogoff, Kenneth, "The Purchasing Power Parity Puzzle," *Journal of Economic Literature*, 1996, *34* (2), 647–668.

Rosen, Sherwin, "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, 1974, *82* (1), 34–55.

Samuelson, Paul A, "Theoretical Notes on Trade Problems," *Review of Economics and Statistics*, 1964, *46* (2), 145–154.

Schott, Peter K., "Across-Product versus Within-Product Specialization in International Trade," *Quarterly Journal of Economics*, 2004, *119* (2), 647–678.

Simonovska, Ina, "Income Differences and Prices of Tradables: Insights from an Online Retailer," *The Review of Economic Studies*, oct 2015, *82* (4), 1612–1656.

Sutton, John and Daniel Trefler, "Capabilities, Wealth and Trade," *Journal of Political Economy*, 2016, *124* (3).

Zwillinger, Daniel, *Handbook of Differential Equations*, Academic Press, 1997.