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# Optimal Capital Controls and Real Exchange Rate

Policies:

## A Pecuniary Externality Perspective\*

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## Abstract

A new literature studies the use of capital controls to prevent financial crises. Within this new framework, we show that when exchange rate policy is costless, there is no need for capital controls. However, if exchange rate policy entails efficiency costs, capital controls become part of the optimal policy mix. When exchange rate policy is costly, the optimal mix combines prudential capital controls in tranquil times with policies that limit exchange rate depreciation in crisis times. The optimal mix yields more borrowing, fewer and less severe financial crises, and much higher welfare than with capital controls alone.

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# 1 Introduction

In response to the global financial crisis and its costly aftermath, a new policy paradigm emerged in which old fashioned government policies such as capital controls and other restrictions on credit flows became part of the standard crisis prevention policy toolkit (the so called macro-prudential policies). A few, large emerging market economies experimented with these tools. And even the traditionally conservative IMF changed its orthodox views on capital controls, advocating the use of such measures when other tools are not available or have run their course of action—see Blanchard and Ostry (2012) and IMF (2012).

The key rationale underpinning the use of capital controls is financial stability.<sup>1, 2</sup> The financial stability motive is the focus of the influential contributions of Korinek (2010) and Bianchi (2011).<sup>3</sup> Their analysis is based on variants of a common theoretical framework proposed by Mendoza (2002, 2010). In this framework, the scope for policy intervention arises because of a pecuniary externality stemming from the presence of a key relative price in the collateral constraint faced by private agents. In this environment, prudential interventions (i.e. before a financial crisis occurs) may be desirable because they can make agents internalize the consequences of this externality on their individual decisions. Capital controls in this setting can discourage financial excesses, reduce the amount that agents borrow, thereby lowering the probability of a financial crisis, and hence enhance welfare.

In this paper, we provide an integrated analysis of alternative policy tools that can be interpreted in terms of fiscal, monetary and macroprudential policies using the same model

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<sup>1</sup>Blanchard and Ostry (2012) make explicit reference to the pecuniary externality perspective when motivating the IMF’s view on the use of capital controls: *”If there are external effects from foreign borrowing (think of amplified crisis risks for the country, where the risks are not internalized by the borrower), then capital controls can act as Pigouvian taxes and constitute an optimal response at the country level, helping agents to internalize the external effects of their borrowing”.*

<sup>2</sup>Historically, as documented by Magud, Reinhart, and Rogoff (2011), capital controls have been adopted for fear of capital flows reversal, fear of excessive risk taking, and to contain asset price bubbles. Other traditional reasons include concerns for competitiveness and monetary policy independence—see more on these below.

<sup>3</sup>See also Lorenzoni (2008), Bianchi and Mendoza (2010), Jeanne and Korinek (2012, 2013) and Benigno et al. (2013).

economy studied by Bianchi (2011). A first main finding is that, when financial stability is the sole motive for policy intervention, the optimal policy design aims at supporting the value of the collateral and hence the agents' borrowing capacity during crises times. In this context, policies that support the real exchange rate (or more generally collateral price support policies) during a financial crisis dominates by a large margin prudential controls on capital flows from a welfare point of view. The dominance of price support policies relies, perhaps unrealistically, on the assumption that they are costless to use. Indeed, under the assumption that supporting the value of collateral during crisis times is costly, it becomes optimal to combine price support policies with macroprudential policies such as capital controls.

In our analysis, then, the rationale for macroprudential policies relies on the extent to which price support policies are cost-effective rather than the amount that agents borrow in the unregulated economy during tranquil times. This novel element emphasizes the interaction between ex-ante (normal times) and ex-post (crises times) policy interventions: when price support policy is fully effective in crises times (i.e. it is able to address the pecuniary externality distortion at no other cost) there is no scope for ex-ante policy intervention. However, if the policy is costly in crises times, it is optimal to adopt capital controls during normal times as a way to limit the occurrence of the crises, combined with price support policies in crises times to mitigate their severity. A second main finding is that the optimal combination of ex ante and ex post policy interventions achieves welfare gains of 1.10% of tradable consumption relative to the unregulated economy, which is much higher than the typical value found in the literature.

The vehicle to convey our messages is the the same model economy used by Bianchi (2011). This is a two-sector (tradables and nontradables) small open, endowment economy with an occasionally binding international borrowing constraint. Quantitatively, this model has been successful in reproducing the business cycle and the crisis dynamics properties of

a typical emerging market economy. In this class of models, a financial crisis event (also labelled a Sudden Stop in capital or credit flows) occurs when the constraint binds. In our model, the value of total current income generated both in the tradable and nontradeable sectors limits borrowing, denominated in units of tradable consumption. When the borrowing constraint binds, the decline in the relative price of nontradables generates a balance sheet effect and leads to a Fisherian debt-deflation spiral.

In this economy, there is a well defined scope for government intervention, but there are multiple instruments or tools with which policy could be conducted. The pecuniary externality arises from the fact that individual agents do not internalize the aggregate effect of their borrowing decisions on the relative price of nontradable goods, which is the price that enters the collateral constraint. There are three type of taxes that can be used to correct it: a tax/subsidy on foreign debt or a tax/subsidy on tradable consumption and a tax/subsidy on nontradable consumption. The tax on foreign debt is usually interpreted as a capital control, while taxes on either tradable or nontradable consumption can be interpreted as a real exchange rate interventions because they affect directly the relative price of nontradables.<sup>4</sup> Our policy analysis considers all three instruments and studies their relative effectiveness in welfare terms. Differently from the existing literature, this paper conducts the policy analysis following a Ramsey optimal taxation approach, assuming that the government budget is always balanced.

The paper first studies the Ramsey problem when capital controls are the only policy tool available, and the government budget constraint is balanced through lump-sum transfers/taxes. Consistent with Bianchi (2011) and Korinek (2010), in this case, the finding is that it is Ramsey optimal to limit the amount that agents borrow in normal times, while no action is needed during crises times. The reason why capital controls are not used by

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<sup>4</sup>The interpretation of the relative price of nontradables as the real exchange rate is standard in the literature. See for instance Bianchi (2011), Caballero and Lorenzoni (2014), Mendoza (2002), Korinek (2010), and Jeanne (2012). Alternatively, the consumption taxes (subsidies) could be interpreted more literally as domestic fiscal policy tools.

the Ramsey planner in crisis times is that, in this model, they cannot affect the allocation when a crisis occurs (i.e. when the borrowing constraint binds). Thus, in this setting, when capital controls are the only policy tool available, the best that the government can do is to reduce the probability that a crisis occurs by inducing the private sector to borrow less than in the decentralized equilibrium.

Next, the paper shows that a policy of supporting the real exchange rate during crisis times by relaxing the borrowing constraint when it binds, can achieve a much higher level of welfare. In fact, such a policy can undo the borrowing constraint completely and, as a result, support an equilibrium in which agents behave as if they were in an unconstrained allocation. Importantly, as we shall see, this policy is time-consistent. The policy can be implemented with a subsidy on non tradable consumption or a tax on tradable consumption. The result hinges on the ability of the Ramsey planner to manipulate the value of collateral with consumption taxes or subsidies that affect the relative price of nontradable goods without creating any other distortions. Indeed, since these consumption taxes or subsidies are rebated or financed through lump-sum transfers or taxes, they are costless and do not entail further distortions.

Finally the paper shows that, when lump-sum transfers/taxes are no longer available, price support policies during crisis times become costly, and capital controls in normal times complement exchange rate policy in crises times under the optimal policy mix. When ex-post policies are costless they can be used all the way to remove the borrowing constraint, and there is no need to engage in ex-ante policy interventions such as capital controls. But when the use of ex-post policies entails efficiency losses or costs (such as when there are other policy objectives to be traded off for financial stability), then ex-ante policy interventions like prudential capital controls are called for to maximize welfare. Notice that this rationale for the use of ex-ante policy interventions is not related to the amount that agents borrow in the competitive equilibrium of the economy without government intervention. Under the

optimal policy with both instruments, there is more borrowing, fewer and less severe crises, and as a result much higher welfare, with gains of 1.10% of tradable consumption relative to the unregulated economy as compared to only .41% with capital controls alone.

The paper relates to a few other recent contributions in the literature on pecuniary externalities which focused both on ex ante and ex post policies. Benigno et al. (2013) analyze the extent to which private agents overborrow or underborrow in a production version of our economy. They show that the allocation chosen by a social planner away from the crisis depends on the planner's ability to mitigate a crisis, should one occur. Benigno et al. (2013) do not analyze any implementation issues or optimal policy problems. Jeanne and Korinek (2013) study the time-consistent mix of ex-ante macroprudential regulation and ex-post bailout transfers in a three-period economy in which the relative price that enters the borrowing constraint is an asset price. The presence of the asset price in the policy problem opens the door to a time-consistency issue, which is their main focus and it is not present in our model. Jeanne and Korinek (2013) also study the role of ex-ante and ex-post policies in their model, but they restrict the set of policy tools along two dimensions: first they restrict the use of distortionary taxation to the contingency in which the constraint binds, while we allow the policy maker to choose freely which instrument to use both in normal and crisis times; and second, they do not consider all the possible tools in the context of their model.

Other new theoretical approaches rationalized the use of capital controls. One approach motivates the use of capital controls with the possibility of manipulating the intra or intertemporal terms of trade—conceptually analogous to the use of tariffs to manipulate the goods' terms of trade (Costinot, Lorenzoni and Werning (2014) and De Paoli and Lipinska (2013)). Schmitt-Grohe and Uribe (2016) examine the role of capital controls in an economy with downward nominal wage rigidity and a fixed exchange rate regime. They focus on competitiveness issues and are silent on the financial stability motive we focus on. Farhi and Werning (2012) study capital controls as a way to address the impossibility to simultaneously



have an open capital account, a fixed exchange regime, and an independent monetary policy (as known as the "impossible trilemma"). Likewise, Devereux and Yetman (2014) analyze capital controls as a way to restore monetary policy effectiveness when the nominal interest rate reaches the zero lower bound in a global liquidity trap context.

Other approaches have focused on the role of capital controls when there are multiple distortions or objectives. For instance, Brunnermeier and Sannikov (2015) show that restrictions to capital flows can be welfare improving in an economy with multiple goods, incomplete financial markets, and inefficient production, but do not discuss issues of optimal mix between ex-ante and ex-post interventions. Ottonello (2015) studies optimal exchange rate policy with downward nominal wage rigidity, flexible exchange rates, and a borrowing constraint like the one in our model. His analysis focuses on a restricted set of instruments similar to Jeanne and Korinek (2013) and discusses the trade-offs that exchange rate policy faces between competitiveness and financial stability considerations.

More broadly, our paper shares the emphasis on price support policies that limit the depreciation of the real exchange rate during crisis times with the work of Chang, Cespedes and Velasco (2012). They examine the role of other unconventional policy tools such as credit policies and direct interventions in the foreign exchange market, but they do not compute optimal policy and focus on the transmission mechanism of alternative policy tools.<sup>5</sup> In an different framework, Martin and Ventura (2014) also suggest policies that relax the collateral constraint by properly managing the size of "bubbles". More generally, our study of alternative policy tools is related to the work by Correia, Nicolini and Teles (2008) in which the role of price stickiness for the design of monetary policy depends on the existence of alternative fiscal policy tools. Finally, in terms of the solution techniques, we apply the same algorithm proposed by Benigno et al. (2012) to solve numerically for the Markov

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<sup>5</sup>In an optimizing neoclassical framework without credit frictions, Calvo, Reinhart and Vegh (1995) also analyze the role of real exchange rate targeting as a stabilization policy tool.

Perfect optimal policy problem in the context of a production version of our economy in which a time-consistency issue arises.

The rest of the paper is organized as follows. Section 2 describes the model environment, the scope for government intervention, and the alternative government instruments that we consider. Section 3 studies optimal capital control policy. Section 4 analyzes optimal real exchange rate policy. Section 5 considers the joint use of capital controls and real exchange rate policies when lump-sum transfers/taxes are not available, as well as some robustness exercises. Section 6 relates the main results of the paper to countries's experience with capital controls and price support policies over the past 20 years or so. Section 7 concludes. The numerical solution methods we use as well as other technical material including proofs and extensions are reported in an appendix for online publication, together with the computer code for the implementation of the analysis in the paper.

## 2 The model environment

This section describes our model economy and discusses its key assumptions. It then characterizes the competitive equilibrium of the model. Next it identifies the externality that gives rise to scope for government intervention. And finally, it discusses the alternative government policy instruments analyzed in the rest of the paper.

Consider a small open economy in which there is a continuum of households  $j \in [0, 1]$  that maximize the utility function

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_t^j)\}, \quad (1)$$

where  $C_t^j$  is the consumption basket for an individual  $j$  at time  $t$ , and  $\beta$  is the subjective discount factor.  $E_0$  denotes the conditional expectation at time 0. Assume that the period

utility function is isoelastic:

$$u(C_t^j) \equiv \frac{1}{1-\rho} (C_t^j)^{1-\rho}.$$

The consumption basket,  $C_t$ , is a CES aggregate of tradable and nontradable goods (omitting the subscript  $j$  to simplify notation):

$$C_t \equiv \left[ \omega^{\frac{1}{\kappa}} (C_t^T)^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{\frac{\kappa-1}{\kappa}} \right]^{\frac{\kappa}{\kappa-1}}. \quad (2)$$

The parameter  $\kappa$  is the elasticity of intratemporal substitution between consumption of tradable and nontradable goods, while  $\omega$  is the relative weight of the two goods in the utility function.

Normalizing the price of tradable goods to 1 and denoting the relative price of the nontradable goods with  $P^N$ , the aggregate price index is given by

$$P_t = \left[ \omega + (1-\omega) (P_t^N)^{1-\kappa} \right]^{\frac{1}{1-\kappa}}.$$

Note here that there is a one-to-one link between the aggregate price index  $P$  and the relative price  $P^N$ .

Households maximize utility subject to their budget constraint (expressed in units of tradable consumption) and a borrowing constraint. The asset menu includes only a one-period bond denominated in units of tradable consumption. Each household has two stochastic endowment streams of tradable and non-tradable output,  $\{Y_t^T\}$  and  $\{Y_t^N\}$ . For simplicity, assume that both  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support. Therefore the current state of the economy can be completely characterized by the triplet  $\{B_t, Y_t^T, Y_t^N\}$ . Thus, the budget constraint each household faces is

$$C_t^T + P_t^N C_t^N + B_{t+1} = Y_t^T + P_t^N Y_t^N + (1+r) B_t, \quad (3)$$

where  $B_{t+1}$  denotes the bond holding at the end of period  $t$ , and  $(1 + r)$  is the given world gross interest rate.

Access to international financial markets is not only incomplete but also imperfect in the sense that, by assumption, the amount that each individual can borrow is limited by a multiple of his current total income:

$$B_{t+1} \geq -\frac{1 - \phi}{\phi} [Y_t^T + P_t^N Y_t^N]. \quad (4)$$

One justification of (4) is in terms of liquidity constraints. By this interpretation, lenders require households to finance a fraction  $\phi$  of their current expenses—which include consumption, debt repayments and taxes—out of current income (see Mendoza (2002) for this interpretation):

$$\phi (Y_t^T + P_t^N Y_t^N) \geq C_t^T + P_t^N C_t^N - (1 + r)B_t. \quad (5)$$

In fact, by combining (5) with (3) we obtain (4). Another justification of (4) appeals to an environment in which the borrower engages in fraudulent activities during the period in which the debt is contracted and prevents creditors from seizing any future income—see Bianchi (2011) for this interpretation. Note here that (5) depends on pre-tax income rather than post-tax income. So, in an environment with default, the individual who defaults is left with her/his full tax-obligation.<sup>6</sup>

At the empirical level, as Mendoza (2002) and Bianchi (2011) emphasize, a specification in terms of current income is consistent with evidence on the determinants of access to credit markets, on lending criteria and guidelines used in mortgage and consumer financing (e.g., Jappelli 1990, Jappelli and Pagano, 1989). The assumption that nontradable goods can be pledged as collateral is consistent with the evidence reported by Tornell and Westermann (2005) on the use of international credit to finance booms in the nontradable sector.

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<sup>6</sup>See section 5 for a discussion of what happens when households can default on their tax obligation as well, and the borrowing constraint depends on post-tax income.

The key feature of (4) is that it captures currency mismatches in the balance sheet of the economy—see Krugman (1999). In fact borrowing is denominated in units of tradable consumption, while both the tradable and the nontradable endowment can be pledged as collateral. Indeed, currency mismatches have been one of the main vulnerability of emerging market economies in the numerous financial crises of the 1990s and the 2000s and continued to be a crucial policy challenge in the post global financial crisis period—see for instance Cespedes, Chang and Velasco (2004, 2012), Shin (2013), and Acharya et al. (2015) for a discussion.

From a model perspective, a financial crisis occurs when the constraint binds and the model dynamic changes nonlinearly; an event that is endogenous in the model. Yet the long-run business cycle properties of the economy are only marginally affected by the crises events (Mendoza, 2002, 2010). A unique feature of this model environment, therefore, is to nest endogenous financial crisis dynamics triggered by small exogenous disturbances within regular business cycles.

In our small open economy, the motive for borrowing arises from the assumption that  $\beta(1+r) < 1$  so that agents are impatient compared to foreign lenders. However, we also assume that there is a lower bound on debt strictly greater than the natural debt limit,  $\underline{B} > B^n$ , such that  $B_t \geq \underline{B}$ , for all  $t$ .<sup>7</sup> This lower bound guarantees that the competitive equilibrium of the economy without government intervention and without the international borrowing constraint (4) is well defined. In particular, it guarantees that there is an ergodic distribution of debt with finite support, and both tradable and nontradable consumption have a strictly positive lower bound, while the nontradable price also has finite support with strictly positive lower bound. In order to focus on non-trivial policies, the assumption that the competitive equilibrium allocation always violates the borrowing constraint (4) is also

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<sup>7</sup>If  $C^T$  and  $C^N$  are strong substitutes, this constraint may bind; since the evidence is that  $C^T$  and  $C^N$  are complements, this possibility is remote.

necessary, when  $B_t = \underline{B}$ , given  $Y_t^T$  and  $Y_t^N$ .<sup>8</sup>

Note finally that, as the online appendix discusses, our calibration, and in particular the assumption that tradable and nontradable goods are complement ( $\kappa < 1$ ), rules out the possibility of multiple equilibria.

## 2.1 Competitive equilibrium

The online appendix provides a full characterization of the competitive equilibrium of the economy with the borrowing constraint and no government intervention. In this equilibrium, households maximize (1) subject to (3) and (4) by choosing  $C_t^N$ ,  $C_t^T$  and  $B_{t+1}$ . The intratemporal allocation between tradeable and non tradeable goods is given by

$$\left( \frac{(1 - \omega) C_t^T}{\omega (C_t^N)} \right)^{\frac{1}{\kappa}} = P_t^N. \quad (6)$$

Goods market clearing for tradeable goods yields

$$C_t^T = Y_t^T - B_{t+1} + (1 + r) B_t, \quad (7)$$

while for non tradeable goods it is:

$$C_t^N = Y_t^N. \quad (8)$$

The quantitative properties of this equilibrium are well known (see Mendoza (2002) and Bianchi (2011)). Here, it is important to point only out that, as Bianchi (2011) illustrated, this very same model can account well for key business cycles statistics as well as the incidence and severity of financial crises in a typical emerging market economy like Argentina. Throughout the paper, therefore, whenever numerical methods are employed, for illustrative purposes, all parameter values of the model are set exactly like in Bianchi (2011)—and a

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<sup>8</sup>This restriction amounts to a lower bound on  $\phi$ .

summary table is reported in the online appendix.

### 2.1.1 Unconstrained Equilibrium

As shown later, two of the government policy instruments that we consider, when used optimally, can completely remove the effects of the constraint (4) and achieve an allocation that is identical to the competitive equilibrium of the model without the borrowing constraint (4). In what follows this allocation is referred to as the "unconstrained equilibrium" (UE) and it is characterized in the online appendix.<sup>9</sup>

In the deterministic steady state of the model, since agents are impatient, the allocation will tend to converge towards the natural debt limit.<sup>10</sup> In our stochastic economy, agents engage in precautionary saving so that the probability of hitting the natural debt limit is zero.

Also note that the unconstrained equilibrium characterizes an allocation in which financial markets are incomplete so that there are inefficient variations in consumption due to the lack of state contingent debt. For completeness, the online appendix describes the first best allocation in which agents in the small open economy have access to state contingent securities and compare it with the unconstrained allocation.

## 2.2 Pecuniary externality

In order to understand the rationale for policy intervention in our model, we first follow the recent literature—e.g., Lorenzoni (2008), Korinek (2010) and Bianchi (2011)—and focus on a benevolent social planner problem with restricted planning abilities. The rest of the paper then focuses on Ramsey optimal policy. In particular, start by assuming that the social planner can directly choose the level of debt subject to the credit constraint while allowing

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<sup>9</sup>The existence of a lower bound on debt which is strictly greater than the natural debt limit guarantees that the competitive allocation without borrowing constraint has an ergodic distribution of debt with finite support under the assumption that  $\beta(1+r) < 1$ .

<sup>10</sup>In our model, this level equals (minus) the annuity value of the lowest tradable endowment value.

goods markets to clear competitively. Unlike the representative agent in the competitive equilibrium of the model, the social planner internalizes the effects of his/her borrowing decisions on the equilibrium relative price of nontradables. This is relevant in our set up because, when the constraint binds, the agents' borrowing capacity depends on the value of the collateral, which in turn is determined endogenously by the equilibrium relative price of nontradables.

### 2.2.1 Social planning problem

Specifically, the social planner maximizes (1) subject to the same borrowing constraint (4) that private agents face and the market clearing conditions for tradables and nontradables goods (7) and (8). In specifying this problem, the equilibrium price of nontradables is determined competitively according to the pricing rule (6). This condition also serves as a constraint on the planning problem to eliminate  $P_t^N$  from the borrowing constraint.<sup>11</sup>

Figure 1 illustrates graphically the consequences of the presence of the borrowing constraint by comparing the policy functions of the endogenous variables  $(C_t^T, B_{t+1}, P_t^N)$  for a negative one-standard deviation shock.<sup>12</sup> Consider the three allocations we defined above: the competitive equilibrium with borrowing constraint (CE), the social planner problem (SP) and the unconstrained equilibrium (UE).<sup>13</sup>

Figure 1 shows the difference between the policy functions of the constrained (CE and SP) on the one hand and the unconstrained (UE) equilibrium allocations on the other. In particular, the UE allocation features a much higher level of tradable consumption and

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<sup>11</sup>This formulation is usually referred to as "constrained-efficient" planning problem in the literature. A second possibility, sometimes referred to as the "conditionally-efficient" problem, is to determine this relative price by imposing as a constraint on the problem the competitive equilibrium *policy function* (in our case  $P_t^N = f^{CE}(B_t, Y_t^N, Y_t^T)$ ). In our endowment economy, however, these two definitions give exactly the same result and do not affect the normative analysis. See Kehoe and Levine (1993) and Lorenzoni (2008) for more details and a discussion.

<sup>12</sup>A policy function is the non-linear equilibrium relation between the endogenous variables of the model and its exogenous and endogenous states (in our case, the triplet  $\{B_t, Y_t^N, Y_t^T\}$ ).

<sup>13</sup>Complete solutions for these allocations have to be computed numerically with the the global solution methods that the online appendix describes.



debt, as well as a higher relative price of nontradable goods, compared to the CE and SP allocations. In the absence of the borrowing constraint, agents can borrow freely from international capital markets to smooth consumption for any given stock of existing debt. In contrast, the CE and SP allocations are relatively close: they diverge only slightly in the region in which the constraint is not binding, but it is expected to bind in the future; otherwise they coincide exactly, including particularly in the region in which the constraint binds.

In the CE and the SP allocations, in the region in which the constraint binds (i.e., when there is a financial crisis in our model), both consumption of tradables and the relative price of nontradables fall sharply.<sup>14</sup> This decline is the consequence of the so-called "Fisherian deflation" or fire sale mechanism emphasized in the financial crisis literature. When borrowing is constrained, consumption is lower relative to the desired amount in the unconstrained equilibrium. Lower tradable consumption is accompanied by a decline in relative price of nontradables, which in turn reduces the value of collateral, tightening borrowing capacity and reducing tradable consumption further, a feedback loop that results in an even lower relative price of nontradables and tradable consumption.

As emphasized by Lorenzoni (2008), Korinek (2010) and Bianchi (2011), when the constraint does not bind (i.e. in normal times), but it is expected to bind in the future with some positive probability, agents in the competitive equilibrium consume more than in the social planner allocation. As the appendix shows, this difference arises because individual agents don't take into account the additional benefit of reducing consumption today, which in turn represents the marginal benefit of consuming more when the constraint binds in the future.

Note however that, in this endowment economy, for a given state  $\{B_t, Y_t^N, Y_t^T\}$  in which the constraint binds, the CE and the SP allocations coincide. For a given amount of existing

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<sup>14</sup>In the figure, the binding region starts in correspondence to the kink in the policy functions.

debt, tradable consumption will be the same in the two allocations since it is constrained by the borrowing limit. The equilibrium relative price of nontradables is also equalized, since the consumption of nontradables is pinned down by its endowment.<sup>15</sup>

### 2.3 Alternative policy instruments

While there is a well defined scope for government intervention, in this economy, there is a variety of instruments or tools with which policy could be conducted. In fact, there are at least three types of taxes that can be used: a tax/subsidy on debt, a tax/subsidy on tradable consumption and a tax/subsidy on nontradable consumption. Our policy analysis considers all of them, studying their relative effectiveness in welfare terms if used one at the time as well as their joint use.<sup>16</sup>

The policy analysis is conducted as a Ramsey optimal taxation exercise, assuming that the government budget is always balanced. Thus, for given policy instrument(s), the Ramsey planner maximizes the representative household’s utility function, subject to the resources constraints and the first order conditions of its maximization problem.

**Tax on debt** The first policy tool that we examine is a tax  $\tau_t^B$  ( $< 0$ ) or a subsidy ( $> 0$ ) on one-period debt issued at time  $t$ ,  $B_{t+1}$ . This instrument is usually referred to as a capital control.<sup>17</sup> With lump-sum transfers/taxation, the government budget constraint is:

$$T_t = \tau_t^B B_{t+1}, \tag{9}$$

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<sup>15</sup>Recall that the relative price of nontradables is proportional to the ratio of tradable over nontradable consumption.

<sup>16</sup>Note that any given allocation could possibly be implemented by different tax instruments. This indeterminacy in the tax instruments depends on the fact that there are two decisions margins but we are considering three possible policy tools—see also Costinot et al. (2014) on this.

<sup>17</sup>One of the best known cases of a use of such a tool is the Brazilian IOF tax. See Pereira and Harris (2012) for a detailed account of this actively researched country case.

where  $T_t$  denotes the lump sum transfer or tax. In this case, the household's budget constraint in the competitive equilibrium of the model becomes

$$C_t^T + P_t^N C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1}(1 + \tau_t^B) + (1 + r) B_t, \quad (10)$$

while the liquidity constraint becomes

$$\phi(Y_t^T + P_t^N Y_t^N) \geq C_t^T + P_t^N C_t^N - (1 + r) B_t + T_t. \quad (11)$$

Combining these three constraints, gives rise to the same international borrowing constraint as before, so that access to international financial market continues to be constrained by (4).

**Taxes on consumption** The other two policy tools are consumption taxes on non-tradable and on tradable goods. Both policy tools influence directly equation (6) and affect the relative price of nontradable goods,  $P_t^N$ , which in the context of our economy is a proxy for the real exchange rate—see for example Mendoza (2002), Caballero and Lorenzoni (2014), Korinek (2010), Bianchi (2011), Jeanne (2012), and Schmitt-Grohe and Uribe (2015) for the same interpretation. For this reason, in what follows, these two tools are referred to as “real exchange rate policy” or “exchange rate policy” for brevity. Alternatively these taxes can be also interpreted as fiscal devaluation/revaluation when monetary policy tools are not available (i.e. in a fixed exchange rate regime or in a currency union).

With a tax on nontradable consumption,  $(1 + \tau_t^N)$ , the household's budget constraint becomes

$$C_t^T + P_t^N(1 + \tau_t^N)C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1} + (1 + r) B_t, \quad (12)$$

where  $\tau_t^N > (<) 0$  is now a tax (or a subsidy) on nontradable consumption and  $T_t > (<) 0$  is a government lump-sum transfer (or tax). As in the case of capital controls, the government

budget balances period by period:

$$T_t = \tau_t^N P_t^N C_t^N. \quad (13)$$

In this case, the liquidity constraint becomes

$$\phi(Y_t^T + P_t^N Y_t^N) \geq C_t^T + (1 + \tau_t^N) P_t^N C_t^N - (1 + r)B_t + T_t, \quad (14)$$

which combined with the individual and the government budget constraints above determines the same international borrowing constraint as before (4).

With a tax on tradable consumption,  $(1 + \tau_t^T)$ , the household now faces the following budget constraint:

$$(1 + \tau_t^T)C_t^T + P_t^T C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1} + (1 + r)B_t. \quad (15)$$

The government budget constraint continues to balance period by period:

$$T_t = \tau_t^T C_t^T, \quad (16)$$

and the borrowing constraint remains as in (4).

### 3 Optimal capital controls

This section studies the optimal Ramsey problem when the policy tool is  $\tau_t^B$ . The Ramsey problem for  $\tau_t^B$  is to choose a competitive equilibrium that maximizes (1). More formally:

**Definition 1** *For a given  $\{B_0\}$  and assuming that  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support, the Ramsey problem for  $\tau_t^B$  is to choose a competitive*

equilibrium that maximizes

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_t)\},$$

subject to the resource constraints (7) and (8), the government budget constraint (9), the borrowing constraint

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N], \quad (17)$$

and the first order conditions of the household,

$$u'(C_t) C_{CT} (1 + \tau^B) = \lambda_t + \beta (1 + r) E_t [u'(C_{t+1}) C_{CT}], \quad (18)$$

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N. \quad (19)$$

The following proposition that qualifies the main result of Bianchi (2011) can now be stated:

**Proposition 1** *In an economy defined by (1), (4), (12) and (9) with a tax on  $\tau_t^B$  as the government instrument, the Ramsey optimal policy with  $\tau^B$  as instrument replicates the social planner allocation (SP). Moreover the optimal policy is time-consistent.*

PROOF: see online appendix.

A few remarks are in order here. As discussed by Bianchi (2011) and noted in the previous section, when the constraint binds (i.e.  $\lambda_t^{SP} > 0$ ), the social planner allocation coincides with the competitive equilibrium allocation, and therefore it is optimal to set  $\tau_t^B = 0$ . When the constraint does not bind, but can bind with positive probability in the next period (i.e.  $\lambda_t^{SP} = 0$ , but  $E_t[\lambda_{t+1}^{SP} \Sigma_{t+1}^{SP}] > 0$  in equation (44) in the online appendix), the optimal state contingent  $\tau_t^B$  is a tax on borrowing ( $\tau_t^B < 0$ ). Thus, it is optimal to engage in a policy intervention even when the constraint does not bind but might bind in the future. In this sense the optimal policy is “prudential” or “precautionary” in nature. Intuitively, since  $\tau_t^B$

is impotent during the crisis, the best thing that policy can do, conditional on having only the tax on debt as instrument, is to reduce the probability that a crisis occurs by limiting the amount that agents borrow in equilibrium (i.e. by taxing  $B_{t+1}$ ). Note also that, in the region in which the constraint binds ( $\lambda_t > 0$ ), any value of  $\tau_t^B$  can implement the social planner allocation.<sup>18</sup>

Figure 2 plots the policy function of  $\tau_t^B$ , for a negative one-standard deviation shock, that solves the optimal policy problem above and replicates the SP allocation, as well as the welfare gains for  $\tau_t^B$  as a function of current bond holdings. Figure 3 reports the ergodic distributions of debt in the CE and the SP allocations. Table 1 reports the ergodic mean of debt as a share of (annual) income in units of tradable consumption, the unconditional probability of a financial crisis in the model, as well as the average welfare gain associated with this policy instrument relative to the CE.<sup>19</sup>

When the economy approaches the binding region, the tax rate goes to zero; before the crisis hits, the higher is the probability that the constraint binds, the higher is the tax on borrowing. Looking at the welfare gains it is evident that they also peak when the constraint binds, but revert to zero slower than the tax rate. The welfare gains of optimal capital controls persist past the level of debt at which the constraint binds because entering a crisis with less debt makes the crisis relatively less costly (see Figure 8 below and its discussion on this latter point). As Figure 3 and Table 1 illustrate, the policy intervention reduces the debt/income ratio and the likelihood of a financial crisis. This implies that the economy, on average, will borrow less under the optimal capital control policy than in the competitive equilibrium, but will experience fewer and less severe financial crises (Table 1 and Figure 8).

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<sup>18</sup>This is true as long as  $\tau^B$  is less than  $\bar{\tau}^B$ , i.e. the maximum value of the tax rate consistent with the constraint being binding.

<sup>19</sup>See the appendix on the the solution method for the SP allocation and the computation of the welfare gains.

## 4 Optimal exchange rate policy

Let us now focus on use of consumption taxes financed with lump-sum taxation discussed above. Specifically, consider first the nontradable consumption tax, as the online appendix shows that the tax on tradable consumption can achieve the same results when used optimally. We refer to this policy tool as "costless exchange rate policy" because it can be implemented without incurring in any efficiency cost associated with its financing. Section 5 discusses the case of distortionary financing.

### 4.1 Nontradable tax

Like before, define first the Ramsey problem when  $\tau_t^N$  is the policy instrument.

**Definition 2** *For a given  $\{B_0\}$  and assuming that  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support, the Ramsey problem for  $\tau_t^N$  as instrument is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_j)\},$$

*subject to the resource constraints (7) and (8), the government budget constraint (13), the borrowing constraint*

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N]. \quad (20)$$

*and the first order conditions of the household*

$$u'(C_t)C_{CT} = \lambda_t + \beta(1+r) E_t [u'(C_{t+1})C_{CT}], \quad (21)$$

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N (1 + \tau_t^N). \quad (22)$$

It is important to point out that the non-tradable consumption tax directly affects the

relative price of nontradables (i.e. the real exchange rate). Note also that, in normal times and in the unconstrained equilibrium, the determination of the consumption of tradable and non-tradable goods is independent from  $P_t^N$ . Therefore,  $\tau_t^N$  is neutral when the constraint does not bind. In fact, the Euler equation and the goods market equilibrium conditions are all that is needed to determine consumption of tradables and nontradables when the constraint does not bind. In contrast, when the constraint binds,  $\tau_t^N$  is no longer neutral because changes in  $P_t^N$  affect the value of the collateral, and hence the consumption of tradable goods.

The next proposition says that, when used optimally, this policy instrument can achieve the unconstrained allocation (i.e., it assures that the borrowing constraint is never strictly binding in the equilibrium of our economy so that  $\lambda_t = 0$  for all  $t$ ). To characterize the solution of this Ramsey problem, one can follow the same two steps of the previous proposition. Thus, first characterize a policy rule for  $\tau_t^N$  that decentralizes the unconstrained competitive equilibrium. Then show that this equilibrium is the one that solves the Ramsey problem above.

**Proposition 2** *In an economy defined by (1), (4), (12) and (13) with a tax on non-tradable consumption  $\tau_t^N$  as the government instrument, there exists a policy for  $\tau_t^N$  that decentralizes the unconstrained allocation. This policy is Ramsey optimal and time-consistent.*

PROOF: see online appendix.

Several remarks are also in order here. The proposition above implies that exchange rate policy always dominates capital control policy in welfare terms. Under this policy, it is possible to undo the constraint completely and replicate the unconstrained equilibrium. In contrast, capital controls can only limit (by reducing the probability of hitting the constraint) the distortionary effects of the pecuniary externality associated with the constraint, but not the constraint itself. As we can see from Table 1, these welfare differences can be quantitatively very large.



How does exchange rate policy work? The intuition for the result is that (22) directly links the tax rate to the relative price of nontradables. When the borrowing constraint does not bind, the policy tool is neutral in the sense that it affects  $P_t^N$ , but not the consumption allocation. In contrast, when the constraint binds, the tax is no longer neutral and can be used to affect the value of collateral in the borrowing constraint, and hence also tradable consumption. By subsidizing the consumption of nontradable goods, the policy increases its relative price. Crucially, when the constraint binds, a higher relative price increases the value of collateral and avoids the debt-deflation mechanism that would otherwise ensue.<sup>20</sup>

In equilibrium, agents anticipate that this policy will undo the constraint when it binds and will behave as if the constraint does not exist (i.e. like in the unconstrained allocation). As we can see from Figure 1, for a given endowment of nontradable goods, the unconstrained allocation (UE) entails a much higher price of nontradables and consumption of tradable goods during tranquil times than in the two constrained allocations (the CE and SP). Eventually (i.e. in finite time) our economy will hit the borrowing constraint because agents are relatively impatient. When that happens, under the optimal policy,  $\tau^N$  will be set so that the multiplier on the constraint is zero (i.e. the constraint is just binding).

Notice here that the policy function for  $\tau^N$  is time-consistent, and hence promising to eliminate the borrowing constraint by supporting the relative price of nontradable whenever the constraint binds is fully credible in equilibrium.

This optimal policy can also be implemented with a fixed tax rate. Since any policy schedule  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  can achieve the unconstrained allocation, for any  $\hat{\tau}_t^N$  that undoes the borrowing constraint, there also exists a fixed subsidy  $\tau_{fix}^N \in (-1, \hat{\tau}_t^N]$  that replicates such allocation. This consideration is important because it simplifies the practical implementation of the policy. Indeed, for a given exogenous state  $\{Y_t^T, Y_t^N\}$  with finite support, it is possible

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<sup>20</sup>For this reason, in broader terms, we can interpret this policy also as a collateral price support policy, aimed at putting a floor under asset price valuations during a financial crisis. In the specific case of our model, it takes the form of a fiscal intervention aimed at supporting the relative price of non tradeable that enters the borrowing constraint.

to determine the corresponding fixed subsidy for which the constraint does not bind. The fixed level of  $\tau_{fix}^N$  will be such that  $\tau_{fix}^N \in (-1, \hat{\tau}_{t,\min}^N]$ , where  $\hat{\tau}_{t,\min}^N$  is the lowest value of the subsidy given the finite support of the exogenous states.

Welfare gains from this policy tool are two orders of magnitude higher than the gains from implementing the SP allocations in Table 1. Figure 4 plots the implied optimal  $\tau^N$  as a function of current debt position and the associated welfare gains for a negative one-standard deviation shock. The implied subsidy and the welfare gains associated with it increase with the level of debt. As Figure 4 illustrates, this optimal policy subsidizes nontradable consumption, limiting the downward pressure on the relative price of non tradable goods. As a result, agents can borrow and consume much more in both good and bad times. In this case, however, the probability of a crisis is zero, despite the fact that borrowing and consumption are much higher than in the CE or the SP (Table 1).

Note here that, for our calibration (which is the same as in Bianchi, 2011), agents are very impatient and the incentive to borrow dominates the precautionary motive that tends to contain their borrowing. The relative strength of this “impatience” effect implies that even when the initial net foreign assets position is positive, agents will borrow up to the borrowing limit, so that a tax subsidy on nontradable consumption is needed to relax the credit constraint. As the current debt position worsens, the state contingent tax subsidy becomes bigger, tending towards the lower bound of -1.

To quantify what a more realistic policy can achieve in welfare terms, consider the case of a fix, 10 percent nontradable subsidy. Such intervention yields an average relative price of on nontradables that is approximately 10 percent less depreciated than in the competitive equilibrium, with an average welfare gain of 0.4 percent of permanent consumption. This is about the same as the gain attained with the optimal capital control policy, which nonetheless is a state contingent tax schedule (Table 1).

## 4.2 Tradable tax

This section highlights the differences between the working of the tax on nontradable and the one on tradable consumption, as the appendix shows that they achieve the same allocation and welfare gains.

The tax on tradable consumption  $\tau_t^T$  affects not only the intratemporal relative price, but also the intertemporal allocation of resources through the Euler equation. Despite this difference, it is possible to find a policy for  $\tau_t^T$  that replicates the unconstrained allocation, like in the case of the nontradable consumption tax  $\tau_t^N$ . The difference between the two policies is that the subsidy on nontradable consumption requires financing through lump sum taxes, while the revenues from the tax on tradables will be rebated as lump sum transfers to private agents. From a practical standpoint, this is important though, as fiscal space is typically limited in the midst of a financial crisis.

Indeed, both policy tools ( $\tau_t^T$  and  $\tau_t^N$ ) could be interpreted more strictly in terms of fiscal policy actions or more broadly as policy aimed at targeting the real exchange rate. In a way, the latter interpretation is related to the recent literature that proposes to manipulate the real exchange rate through the use of fiscal tools—see for instance Lipinska and Von Tadden (2009), Franco (2011) and Fahri, Gopinath and Itskhoki (2013). The difference is that here we want to limit the depreciation of the real exchange rate for financial stability purposes, while in the literature on fiscal devaluation the idea is to engineer a devaluation to gain competitiveness.

## 5 Optimal capital controls and exchange rate policy with distortionary financing

The previous section showed that exchange rate policy dominates capital control policy in welfare terms when it can be implemented in a costless manner—i.e., by using lump-sum

transfers to finance the nontradable subsidy, and hence without introducing any additional distortion. The key point is that, in a debt-deflation environment, optimal policy aims at relaxing the collateral constraint. In particular, optimal policy aim at supporting the relative price that influences the borrowing constraint and, in principle, can undo the effects of a binding constraint completely. The result however hinges on the ability of the Ramsey planner to manipulate the price of collateral without costs, because our instruments operate in the context of a balanced budget in which lump sum transfers or taxes are available.

This section departs from this key assumption and considers an environment in which lump-sum transfers/taxes are not available, the government budget balances with distortionary financing, and it is costly to manipulate the price of the collateral with consumption taxes. As Section 6 below discusses further, the distortionary financing cost introduced can be interpreted more broadly as representing another friction in the economy, a second objective of exchange rate policy, or any situation in which managing the real exchange rate during a financial crisis is difficult. For this reason we label these set of interventions "costly exchange rate policy".

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Given the structure of our endowment economy, there are two possibilities for the government budget constraint. In the first one, the set of taxes is arbitrarily restricted to  $\tau_t^B$  and  $\tau_t^N$ . In the second one, all all the tax instruments discussed thus far,  $\tau_t^B$ ,  $\tau_t^N$ , and  $\tau_t^T$  can be used. So let's look at each of them in turn.

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<sup>21</sup>In a model with heterogenous agents, lump-sum instruments will have distributional implications. Here we abstract from these issues, but still consider an environment in which lump sum instruments are not available.

## 5.1 Two policy instruments

In the first case, the government budget constraint becomes:

$$\tau_t^B B_{t+1} = \tau_t^N P_t^N C_t^N, \quad (23)$$

with the tax on borrowing financing the subsidy on nontradable goods or, alternatively, the subsidy rebating the revenue from taxing debt. The following definition states the corresponding Ramsey problem.

**Definition 3** *For a given  $\{B_0\}$  and assuming that  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support, the Ramsey problem for  $\tau_t^N$  and  $\tau_t^B$  when (23) holds is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_t^j)\},$$

*subject to (7) and (8) and (20), and the first order conditions of the households*

$$u'(C_t) C_{CT} (1 + \tau_t^B) = \lambda_t + \beta (1 + r) E_t [u'(C_{t+1}) C_{CT}], \quad (24)$$

$$\frac{(1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N (1 + \tau_t^N). \quad (25)$$

There is no analytical solution for this problem, so one has to rely on numerical methods. To do so, note first that, given our chosen instruments (i.e.  $\tau_t^N$  and  $\tau_t^B$ ), the problem is time consistent.<sup>22</sup> Next, We use the computational algorithm that exploits the Markov-Perfect nature of the equilibrium, proposed by Benigno et al. (2012) and summarized in

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<sup>22</sup>To see this, note that the problem above can be reduced to a static one by considering the restricted case in which the Ramsey planner maximizes agents' utility subject to (7), (8), (20) and (25). One can then solve for the allocations, the multiplier on the credit constraint and the relative price. Next, use (23) and (24) to retrieve the path of taxes. The online appendix also provides an alternative proof based on the equivalence between the commitment and the time-consistent solution of the problem above.

the appendix. Note that, for comparison purposes, the economy continues to be calibrated exactly as in Bianchi (2011). The rest of this section reports and discusses only the actual solution.

Figure 5 plots the policy function under the optimal policy for  $\tau_t^N$  and  $\tau_t^B$  and the associated welfare gains in terms of tradeable consumption as a function of current bond holdings for a negative one-standard deviation shock. Figure 6 describes the policy function for  $B_{t+1}$ ,  $C_t^T$  and  $P_t^N$  under the optimal policy (OP, dashed line) and the competitive allocation (CE, solid line). Figure 7 reports the ergodic distribution of debt. In order to assess the severity of the crisis, Figure 8 also reports the ergodic distribution of total consumption growth in unit of tradable consumption during crisis times (i.e., the change in consumption from  $t - 1$  to  $t$ , given that the economy is in a financial crisis in period  $t$ ). For this purpose, a crisis is identified, as in Bianchi (2011), by a constraint that binds strictly and a debt reduction larger than one-standard deviation. In these plots, the constraint binds at a level of debt of about -0.95, where the policy rules display a kink.

As Figure 5 shows, when exchange rate policy is costly, there is scope for both ex-post and ex-ante interventions. During crises times, like before, the optimal policy requires subsidies to non-tradable consumption to limit the depreciation of the relative price of nontradable goods. This is now financed by a tax on the amount that agents borrow. During normal times, the optimal policy requires capital controls whose revenues are rebated in the form of subsidies to nontradable consumption.

The optimal policy mix depends crucially on the interaction between ex-ante and ex-post interventions. In the context of our simple economy, this interaction is affected by the way the policy interventions are financed. When financing of ex-post intervention is not costly (i.e. there are lump-sum taxes) policies aimed at supporting the market price of collateral are fully effective and can achieve the unconstrained allocation. In contrast, when financing of ex-post intervention is distortive, preventing excessive depreciation of the real exchange

rate becomes costly, and the optimal policy weights the marginal benefit of relaxing the borrowing constraint with the distortion introduced by capital controls. Indeed, when the constraint binds, the tax on debt affects  $C_{t+1}$  through (24). Since the ex-post policy becomes costly, it is no longer fully effective in addressing the pecuniary externality, and it becomes optimal to intervene during normal times to reduce the probability of meeting the borrowing constraint. Consistent with this, a comparison of Figure 2 and Figure 5 shows that the optimal tax rate on debt, in the region where the constraint is not binding, is now much smaller than the case in which capital control is the only government instrument.

There are three other features of the optimal policy that are noteworthy. First, when the constraint is not binding, while the tax on the amount that agents borrow affects their borrowing decision, the subsidy to nontradable consumption is neutral and is equivalent to a lump-sum transfer. On the other hand when the constraint binds, both instruments affect the real allocation.

Second, in this more general set up, like in the case of costless exchange rate policy, there continues to be more borrowing and consumption than in the competitive equilibrium despite the fact that the economy experiences fewer and less severe crises (see Table 1 and Figure 8). The Ramsey planner achieves this by choosing a different allocation of consumption, with relatively more consumption of tradable goods compared to the competitive equilibrium allocation. As a consequence, the welfare gains of this optimal policy mix are more than twice as large as those in which only capital controls are used, and continue to be larger the more indebted is the economy (see Figure 5 and Table 1). This is consistent with the old adage that where borrowing is allocated is at least as important as how much borrowing takes place.

Third and finally, agents borrow more than in the competitive equilibrium allocation during normal times even though optimal policy requires a tax on the amount agents borrow (Figure 6 and 7). Intuitively, on the one hand, agents want to borrow less because their

borrowing is taxed; on the other hand, they are willing to borrow more since crises events are mitigated (only in part in this case) by policy intervention (see Figure 8 and Table 1, respectively). Indeed, the real exchange rate depreciates less during crises times compared to the competitive equilibrium allocation and allows agents to consume more (Figure 6). In this setting, therefore, the rationale for capital controls is not related to the amount that agents borrow in the unregulated economy, the so called "overborrowing" on which the existing literature focused on, but rather to the relative (in)effectiveness of the ex post intervention.

## 5.2 Three policy instruments

Consider now the second possibility in which all available distortionary taxes can be combined to balance the budget:

$$\tau_t^B B_{t+1} = \tau_t^N P_t^N C_t^N + \tau_t^T C_t^T. \quad (26)$$

In this situation, it is possible to show analytically that there is a combination of the three tools that can achieve the unconstrained allocation even if there are no lump-sum transfers/taxes. In the appendix, we prove that we can always combine the triplet of policy tools  $(\tau_t^N, \tau_t^T, \tau_t^B)$  to undo the international borrowing constraint.

The policy implication of this last exercise echoes what was emphasized earlier: the set of instruments and their relative effectiveness is crucial for the optimal policy design. The third instrument addresses the distortion introduced by the second one in crisis times. Intuitively, it is possible to use the tax on tradable goods to undo the efficiency losses caused by the use of tax on borrowing when policy aims at supporting the real exchange rate. This is consistent with the notion that with enough instruments we can always undo a friction—cfr. Correia, Nicolini, and Teles (2008). In our specific context, this implies that the challenge for the policy maker is to identify the particular combination of instruments that are most effective in addressing the pecuniary externality and its interaction with the others relevant frictions in the economy.



### 5.3 Extensions

Our analysis shows that, in economies with occasionally binding collateral constraints, it is optimal to design policies aimed at relaxing the constraint when it binds, and the desirability of policies aimed at preventing crisis depends on the cost-effectiveness of such crisis resolution interventions. Here we want to discuss some extensions of the basic framework and show that the main messages of our optimal policy analysis are robust in this model environment.

**Imperfect exchange rate intervention** Consider first a more realistic case of costly (or less than fully effective) ex-post intervention, which we label "imperfect exchange rate intervention," for example, because it is not be feasible to perfectly control the real exchange rate during the financial crisis. In practice, this might be due to imperfect credibility or because of limited availability of foreign exchange reserves needed to support the nominal exchange rate, as we discuss in section 6 below.

To model this idea (fully developed in the appendix), assume that when it is not feasible to implement the desired level of the subsidy  $\tau^N$ ,  $\tau^N$  must be set to zero. The appendix shows that under this policy it is not always possible to relax the collateral constraint, so that it becomes optimal to intervene ex-ante, from a Ramsey planner perspective, like in the case of distortionary financing discussed above. Intuitively, when exchange rate intervention is imperfect, crises become more costly events, and it is desirable to tax borrowing in normal times to limit the probability that a crisis occurs.

**Borrowing constraint with post-tax income** A second extension is a situation in which the borrowing constraint depends on post-tax income as follows:

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N - T_t]. \quad (27)$$

The appendix shows that, under this assumption, changes in  $\tau^N$  financed through  $T_t$ , will not relax the borrowing constraint<sup>23</sup>. Despite this, it is possible to show that the general principle of optimal policy design in this class of models that we stressed above continues to hold, but may require the use of different policy tools. In fact, in this specific case, the policy maker would need to use the tax on borrowing,  $\tau^B$ , along with lump-sum transfers/taxes to relax the constraint. When (27) holds and binds, the Ramsey planner can increase the value of the collateral by transferring resources to the household via  $T_t$ , at the a cost of higher  $\tau^B$ . Indeed, the appendix shows that this case is isomorphic to the one in which the borrowing constraint depends on pre-tax income, and the available policy tools are  $\tau^N$  and  $\tau^B$ . The only difference is that the borrowing constraint is now relaxed through  $T_t$  rather than by engineering an increase  $P_t^N$ .

**Production** Lastly, note that in a production economy there would be multiple margins on which the pecuniary externality can distort decisions—see for instance the economy analyzed by Benigno et al. (2012 and 2013). In this case, Benigno et al. (2012) show that exchange rate policy alone via  $\tau^N$  cannot remove the borrowing constraint or even achieve the SP allocation of that economy. For example, the optimal policy for  $\tau^N$  alone, in that more general setting, is a tax in normal times and a subsidy in crisis times. The policy, therefore, has both a crisis prevention element aimed at containing the frequency of financial crises, and a crisis resolution element aimed at mitigating their effect by relaxing the constraint when they do occur. To achieve the SP allocation of that economy or to remove the borrowing constraint altogether, however, would require the use of multiple tools. Indeed, in that model environment, collateral price support policies induce distortions in the allocation of labor between tradeable and non-tradeable production that requires the use of an offsetting policy tool to fully restore efficiency or contain the cost of trying to remove the constraint.

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<sup>23</sup>This can be seen by combining (27) with the government budget constraint  $T_t = \tau_t^N P_t^N C_t^N$ .

## 6 Discussion

Our model is useful to discuss exchange rate policy in the real world, and the implied "optimal" policies are consistent with the experience of emerging market economies over the past 20 years or so.

A first implication of our analysis is on the role of collateral price support policies when they can be implemented in a costless manner. In the context of the model, these policies take the form of a fiscal subsidy to the consumption of nontradable goods, financed in a lump sum manner. This contains the fall of the relative price of nontradable (or the depreciation of the real exchange rate in our model) that typically occurs during a sudden stop of capital inflows. If such a policy is feasible, our analysis shows that, not only it contains the crisis when one occurs, but it also eliminates the scope for any prudential measure such as capital controls. This is because the intervention can remove the borrowing constraint altogether, which is the only source of inefficiency in our model economy. Of course, in reality, there are other distortions, possibly leading to different conclusions as it is discussed below.

Which policies in the real world can support the real exchange rate and how costly are they to implement? In practice, the real exchange rate is typically supported by defending the nominal exchange rate by selling previously accumulated foreign exchange reserves. And while accumulating or borrowing foreign exchange reserves is costly, drawing them down at any particular point in time is costless.

For example, during the global financial crisis, Brazil and Mexico faced a sudden stop in private capital inflows following the Lehmann's collapse in September 2008. The Brazilian Real depreciated by more than 20 percent in a month against the US dollar, and the central bank intervened heavily to defend it as predicted by our model. As Mesquita and Toros (2010) emphasize, the main motivation for this intervention was the vulnerability of the non-financial corporate sector to the depreciation of the Real because of their exposure in the derivative market to US dollar swaps (proxied in our model with borrowing in units of

tradable consumption). A similar experience was shared by Mexico when large corporate entities were also exposed to foreign currency derivatives at the time of Lehmann's collapse. In their account of the Mexican experience, Chang, Cespedes and Velasco (2012) emphasize how the response of the policy authorities consisted in foreign exchange market intervention with the objective to limit the depreciation of the Mexican Peso in the face of currency mismatches in the corporate sector balance sheet.

More broadly, in the context of the recent US and European financial crises, the prescription of our model can be interpreted as interventions that avoid the collapse of asset prices when a crisis occurs. In this sense, our results not only rationalizes the need to set a floor under the exchange rate as in the emerging market crises of the 1990s and the 2000s, but also the non-conventional policies of purchasing risky assets to contain "fire sales" and the asset deflation spirals that characterized the United States and European crises.<sup>24</sup>

Official reserves however are always limited, and this limited availability exposes countries to costly speculative attacks. Many emerging market countries learned this lesson the hard way in the 1990s and the 2000s as speculative attacks on limited pools of foreign exchange reserves broke many pegs: Mexico in 1994, Thailand in 1997, South Korea and Indonesia in 1998, Russia in 1998, Brazil in 1999, Argentina in 2001, Uruguay in 2002, etc. Once out of reserves, these countries had to borrow foreign currency from the IMF under tight macroeconomic adjustment programs to contain the initial devaluations. Indeed, as predicted by our model, supporting the exchange rate was a crucial component of all adjustment programs supported by the IMF in Indonesia, South Korea, and Brazil during the period 1997-1999 (IMF Independent Evaluation Office, 2003). These adjustment programs turned out to be economically and politically very costly. As a result, after these crises, countries started to accumulate very large pools of official reserves to deploy in support of the exchange

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<sup>24</sup>It is possible to show that the small open economy studied here is isomorphic to an environment in which domestic banks intermediate foreign saving and households borrow using a domestic asset as collateral. In that context, optimal ex post policy, when warranted, supports domestic asset prices. See Bianchi (2011) and Cesa-Bianchi and Rebucci (2016) for more discussion of that set up.

rate in the case of sudden halt in capital inflows as they did during the global financial crisis.<sup>25</sup> But even when accomplished gradually rather than borrowing from the IMF, reserve accumulation is costly. Countries must save in a precautionary manner in a riskless asset while continuing to borrow in risky instruments as long as they are in a net debtor position. So there is a carry cost, or premium for holding reserves.

Perhaps even more importantly, exchange rate policy also has other objectives than that of maintaining financial stability by mitigating the effects of currency mismatches. Exchange rate policy is typically tasked to also address competitiveness issues and to contribute to macroeconomic stability by helping to manage inflation. In our model, financial stability is the only policy objective, as there are no other frictions justifying intervening in the economy for macroeconomic stabilization or competitiveness reasons.

If we were to introduce other frictions and hence policy objectives, a trade off would emerge similar to the one we studied in the previous section by assuming distortionary financing of exchange rate policy or imperfect exchange rate interventions. One example is a government that targets the real exchange rate by manipulating the nominal parity in the presence of both a borrowing constraint like ours and a nominal rigidity. In this case, the advantage of keeping the exchange rate relatively appreciated is to support the agents' borrowing capacity. The disadvantage would be to cause unemployment in response to shocks. Our distortionary cost captures the essence of costly ex post interventions in the presence of other distortions or government objectives.<sup>26</sup>

Indeed, the second main policy implication of our analysis is that, if financial crises cannot be contained or mitigated without incurring significant costs, or there are additional

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<sup>25</sup>Emerging markets official reserves (excluding gold) increased from about one trillion US dollar in 2000 to over 6 trillions in 2012 according to IMF IFS data (or about a third of world GDP valued at current US dollars). While this spectacular accumulation of reserve assets cannot be explained entirely by prudential or precautionary motives, most empirical studies concur that precautionary saving is the most important determinant of this process.

<sup>26</sup>Ottonello (2015) studies optimal exchange rate policy in such a set up and illustrates the trade off between increasing agents' borrowing capacity (dubbed credit access policy) and unemployment.

distortions to consider associated with their resolution, a policy of crisis prevention becomes part of the optimal policy mix, such as for instance using capital controls in a countercyclical manner. However, when both ex ante and ex post interventions are used jointly under the optimal policy mix, the level of the tax on borrowing is much lower than when the capital control is used as the only instrument (cfr. Figure 2 and 5).

This latter result is consistent with available empirical evidence on the use of capital controls. As Fernandez, Rebucci, and Uribe (2015) pointed out, if countries were to use capital controls in a prudential manner as implied by Proposition 1, we should observe active use of countercyclical capital controls. However, when Fernandez et al (2015) looked at a large number of countries over the period 1995-2011, they found that capital controls are virtually flat during episodes of boom and bust in output or the current account.

In summary, we conclude from this review of country experiences over the past 20 years or so that, while the mechanics of optimal policies implied by our model are different than those typically implemented in the around the world, the general principle followed by these policies is very much consistent with them. The policies implied by our analysis are also consistent with the available empirical evidence on the use of capital controls, in stark contrast to those implied by the existing literature.

## 7 Conclusion

In response to the recent global financial crisis, a new policy paradigm emerged in which old fashioned forms of government interventions such as capital controls and other quantitative restrictions on credit flows—the so called macro-prudential policies—have become part of the standard policy toolkit. Arguably, macro-prudential policies are desirable because they can help prevent financial crises that otherwise would be too costly to endure or contain with only ex post interventions.

In this paper we study the optimal policy mix of ex post, crisis management policy tools

and ex ante, crisis prevention policy tools. We first show that when the Ramsey planner can choose among different policy tools, ex post collateral price support policies dominate prudential policy measures in welfare terms by a very large margin. This dominance is conditional on the extent to which price support policies do not entail efficiency losses. Indeed, when collateral price support policies can be used without costs, there is no need for macro prudential policies. In contrast, when crisis management policies are not fully effective because they are costly to implement, ex-ante policies such as capital controls can be rationalized as a complement to collateral price support policies that limit the occurrence of crises. The joint use of ex ante and ex post policies achieves a welfare gain of more than 1 percent of permanent consumption in our model; a gain that is twice as large as the welfare gain of using only capital controls.

Our analysis is conducted in the context of a relatively simple quantitative model, but in reality the trade-offs that policymakers face are richer than the ones implied by our framework. For instance, there are benefits from a more depreciated exchange rate in terms of the classical expenditure switching effect of the exchange rate that are not incorporated into our analysis. To an extent, our model can be interpreted as one in which balance-sheet considerations dominate other exchange rate policy motives, but we acknowledge that a richer model would be needed to quantify these issues. We regard the study of optimal monetary and macro-prudential policy in a quantitative model in which pecuniary externalities interact with nominal rigidities as an area of fruitful future research.

## References

- [1] Acharya Viral, Stephen Cecchetti, José De Gregorio, Sebnem Kalemli-Ozcan, Philip R. Lane, and Ugo Panizza (2015), "Emerging economy corporate debt: The threat to financial stability", <http://www.voxeu.org/article/emerging-economy-corporate-debt-threat-financial-stability>, VoxEu..
- [2] Blanchard, O. and J. Ostry (2012), "The multilateral approach to capital controls", <http://www.voxeu.org/article/multilateral-approach-capital-controls>, VoxEu.
- [3] Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E.R. Young (2013), "Financial Crises and Macro-Prudential Policies," *Journal of International Economics*, Vol. 89(2), pp. 453-470.
- [4] Benigno, G., H. Chen, C. Otrok, A. Rebucci, and E.R. Young (2012), "Optimal Policy for Macro-Financial Stability, " CEPR Discussion Paper No. 9223.
- [5] Bianchi, J. (2011), "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review* 101(7), pp. 3400-3426.
- [6] Bianchi, J. and E.G. Mendoza (2010), "Overborrowing, Financial Crises and 'Macro-Prudential' Taxes," NBER Working Paper No. 16091.
- [7] Brunnermeier, M. and Sannikov, Y. (2014) "A Macroeconomic Model with a Financial Sector", *American Economic Review*, Vol. 104, No. 2 pp. 379-42.
- [8] Brunnermeier, M. and Sannikov, Y. (2015) "International Credit Flows and Pecuniary Externality", *American Economic Journal: Macroeconomics* 7.1 (2015): 297-338.
- [9] Calvo, Guillermo, Carmen Reinhart and Carlos Vegh (1995), "Targeting the Real Exchange Rate: Theory and Evidence", *Journal of Development Economics*, Vol 47, 97-133.



- [10] Caballero Ricardo and Guido Lorenzoni (2014), "Persistent Appreciations and Overshooting: A Normative Analysis", *IMF Economic Review*, Vol. 62, No. 1, pp. 1-47.
- [11] Céspedes Luis Felipe, Roberto Chang, and Andrés Velasco (2004), "Balance Sheets and Exchange Rate Policy," *American Economic Review*, Vol. 94, No. 4, pp. 1183-1193.
- [12] Céspedes F. L., Chang, R., and A. Velasco (2012) "Is Inflation Target Still on Target?", NBER Working Paper No. 18570.
- [13] Céspedes F. L., Chang, R., and A. Velasco (2013) "Financial Intermediation, Exchange Rates, and Unconventional Policy in an Open Economy", NBER Working Paper No. 18431.
- [14] Correia, I., Juan P. Nicolini, Pedro Teles (2008) "Optimal Fiscal and Monetary Policy: Equivalence Results", *Journal of Political Economy* 116, 1, pp.141-170.
- [15] Costinot Arnaud , Guido Lorenzoni, Ivan Werning (2014) "A Theory of Capital Controls as Dynamic Terms-of-Trade Manipulation", February, *Journal of Political Economy*, 122, 1.
- [16] De Paoli Bianca and Anna Lipińska (2013), "Capital Controls: A Normative Analysis," Federal Reserve Bank of New York Staff Reports No. 600, February 2013.
- [17] Devereux, Michael B. and James Yetman (2014), "Capital Controls, Global Liquidity Traps and the International Policy Trilemma", *Scandinavian Journal of Economics*, Vol. 116, No. 1, pp. 158-189.
- [18] Farhi Emmanuel and Ivan Werning (2012), "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates" NBER Working Paper No. 18199.
- [19] Farhi Emmanuel, Gita Gopinath and Oleg Itkhoki (2013), "Fiscal Devaluation", *Review of Economic Studies*, April 2014, Volume 81 (2).

- [20] Franco Francesco (2011): “Improving competitiveness through fiscal devaluation, the case of Portugal,” unpublished manuscript, Universidade Nova de Lisboa.
- [21] Fernandez Andres, Alessandro Rebucci, Martin Uribe (2015), ”Are Capital Controls Countercyclical?”, *Journal of Monetary Economics*, Vol. 76, pp. 1-14.
- [22] IMF Independent Evaluation Office (2003), ” *The IMF and Recent Capital Account Crises: Indonesia, Korea, and Brazil*”, Evaluation Report, Independent Evaluation Office, IMF, Washington DC, 2003.
- [23] IMF (2012) “The liberalization and management of capital flows: an institutional view”, IMF staff paper, November 2012 (<http://www.imf.org/external/np/pp/eng/2012/111412.pdf>).
- [24] Jappelli, Tullio (1990), ”Who is credit constrained in the US economy?” *The Quarterly Journal of Economics*, pp. 219–234.
- [25] Jappelli, Tullio, and Marco Pagano (1989), ”Consumption and capital market imperfections: An international comparison”, *American Economic Review*, 79(5): 1088-1105.
- [26] Jeanne, O. (2012) “Capital Flow Management”, *AER Papers and Proceedings*, Volume 102, No. 3.
- [27] Jeanne, O. and A. Korinek (2012), “Managing Credit Booms and Busts: A Pigouvian Taxation Approach,” unpublished manuscript, Johns Hopkins University.
- [28] Jeanne, O. and A.Korinek (2013), “Macroprudential Regulation Versus Mopping Up After the Crash”, NBER wp no 18675.
- [29] Kehoe, T.J. and D. Levine (1993), “Debt-Constrained Asset Markets,” *Review of Economic Studies* 60(4), pp. 865-888.

- [30] Korinek, A. (2010), Regulating capital flows to emerging markets: An externality view, manuscript, Johns Hopkins University.
- [31] Krugman, P. (1999), “Balance Sheets, the Transfer Problem, and Financial Crises,” *International Tax and Public Finance* 6(4), pp. 459-472.
- [32] Lipińska, Ania, and L. von Thadden (2009): “Monetary and Fiscal Policy Aspects of Indirect Tax Changes in a Monetary Union,” ECB Working Papers No. 1097.
- [33] Lorenzoni, G. (2008), “Inefficient Credit Booms,” *Review of Economic Studies* 75(3), pp. 809-833.
- [34] Magud, N.E., C.M. Reinhart, and K.S. Rogoff (2011), “Capital Controls: Myth and Reality – A Portfolio Balance Approach,” NBER Working Paper 16085.
- [35] Martin, Alberto and Jaume Ventura, 2014. “Managing Credit Bubbles” . NBER Working Papers 19960, National Bureau of Economic Research.
- [36] Mendoza, E.G. (2002), “Credit, Prices, and Crashes: Business Cycles with a Sudden Stop,” in Edwards, S. and J.A. Frankel (eds.), *Preventing Currency Crises in Emerging Markets*, University of Chicago Press and NBER.
- [37] Mendoza, E.G. (2010), “Sudden Stops, Financial Crises and Leverage: A Fisherian Deflation of Tobin’s Q,” *American Economic Review* 100(5), pp. 1941-1966.
- [38] Mesquita, M and M Toros (2010): “Brazil and the 2008 panic”, BIS Papers, no 54, pp 113–20.
- [39] Ottonello P., ”Optimal exchange-rate policy under collateral constraints and wage rigidity.” Manuscript, Columbia University, 2015.
- [40] Pereira da Silva L. A. and R. E. Harris” (2012), ”Sailing through the Global Financial Storm: Brazil’s Recent Experience with Monetary and Macroprudential Policies to Lean

Against the Financial Cycle and Deal with Systemic Risks”, Central Bank of Brazil WP No. 290, August.

[41] Schmitt-Grohe, S. and M. Uribe (2015) “Prudential Policy for Peggers”, *Journal of Political Economy*, forthcoming.

[42] Tornell, Aaron, and Frank Westermann (2005), ”Boom-bust cycles and financial liberalization”, *MIT Press Books*.

## A Appendix (for online publication)

### A.1 Competitive equilibria

In the competitive equilibrium of the economy with the borrowing constraint and without government intervention (which we also call the "constrained allocation" for brevity), households maximize (1) subject to (3) and (4) by choosing  $C_t^N$ ,  $C_t^T$  and  $B_{t+1}$ . The Lagrangian for this problem is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \begin{array}{l} \frac{1}{1-\rho} C_t^{1-\rho} + \lambda_t \left( B_{t+1} + \frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N] \right) + \\ \mu_t (Y_t^T + P_t^N Y_t^N - B_{t+1} + (1+r) B_t - C_t^T - P_t^N C_t^N), \end{array} \right]$$

with  $\lambda_t$  and  $\mu_t$  denoting the multipliers on the borrowing constraint and the budget constraint, respectively. The first order conditions of this problem are:

$$C_T : u'(C_t) C_{C^T} = \mu_t, \quad (28)$$

$$C_N : u'(C_t) C_{C^N} = \mu_t P_t^N, \quad (29)$$

$$B_{t+1} : \mu_t = \lambda_t + \beta (1+r) E_t [\mu_{t+1}]. \quad (30)$$

$$\lambda_t \left\{ B_{t+1} + \frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N] \right\} = 0 \quad (31)$$

Combining (28) and (29) we have:

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N. \quad (32)$$

This constrained allocation can now be characterized completely by the first order conditions (30), (31) and (32) and the goods market equilibrium conditions:

$$C_t^T = Y_t^T - B_{t+1} + (1 + r) B_t, \quad (33)$$

and

$$C_t^N = Y_t^N. \quad (34)$$

### A.1.1 Parameter values

The parameter values of the model are set exactly as in Bianchi (2011):

<b>Structural parameters</b>	<b>Values</b>
Elasticity of substitution between tradable and non-tradable goods	$\kappa = .83$
Intertemporal substitution and risk aversion	$\rho = 2$
Credit constraint parameter	$\phi = 0.75758$ 1/
Relative weight of tradable and non-tradable goods	$\omega = 0.31$
Discount factor	$\beta = 0.91$

<b>Exogenous variables</b>	<b>Values</b>
World real interest rate	$r = 0.04$
Steady state endowments	$Y^N = Y^T = 1$

<b>Endowment process</b>	
Autocorrelation Matrix	$\begin{bmatrix} 0.901 & 0.495 \\ -0.453 & 0.225 \end{bmatrix}$
Variance-Covariance Matrix	$\begin{bmatrix} 0.00219 & 0.00162 \\ 0.00162 & 0.00167 \end{bmatrix}$

<b>Average values in the ergodic distribution</b>	<b>Values</b>
Net foreign assets	$B = -0.91$

1/ This value of  $\phi$  implies a value for  $\kappa = .32$  in Bianchi's (2011) notation.

### A.1.2 Ruling out multiple equilibria

The borrowing constraint can induce multiple equilibria due to the possibility of a self-fulfilling decline in the relative price of nontradables that can reduce the value of the collateral, and the consumption of tradable goods, in a manner compatible with the initial decline in the relative price of nontradables. More formally, by combining the borrowing constraint (4), the budget constraint (3) and the pricing equation we obtain:

$$C_t^T = B_t(1+r) + \left(1 + \frac{1-\phi}{\phi}\right) \left[ Y_t^T + \frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} \right] \equiv f(C_t^T).$$

When the elasticity of intratemporal substitution is less than 1 (goods are complements), a sufficient condition for unicity is that the derivative of the RHS of this expression with respect to  $C^T$  ( $f'(C^T)$ ), evaluated at the intersection point with the LHS is greater than 1. Indeed, when  $B_t(1+r) + \left(1 + \frac{1-\phi}{\phi}\right) Y_t^T < 0$  and  $\kappa < 1$ , we have that  $\lim_{C^T \rightarrow 0} f'(C^T) = 0$  and  $\lim_{C^T \rightarrow \infty} f'(C^T) = \infty$ . These conditions, which are satisfied in our calibration, combined with the assumption that  $f'(C^T) > 1$  evaluated at the intersection point, guarantee that the equilibrium is unique—see also the discussion of Jeanne and Korinek (2012).

Another issue that might arise in our model, given our specification of the borrowing constraint, is the possibility that, when the amount that the planner borrows increases, then the relative price of nontradable rises the value of the collateral by more than the increase in  $B_{t+1}$ , thus leading to a relaxation of the borrowing constraint. Our calibration also rules out the possibility of such a perverse dynamic.



### A.1.3 Unconstrained allocation

In terms of equilibrium conditions, the allocation without the borrowing constraint, which we call the "unconstrained allocation", is fully characterized by the following equations:

$$u'(C_t^{UE})C_{C^T}^{UE} = \mu_t^{UE}, \quad (35)$$

$$u'(C_t^{UE})C_{C^N}^{UE} = \mu_t^{UE} (P_t^N)^{UE}, \quad (36)$$

$$\mu_t^{UE} = \beta (1 + r) E_t [\mu_{t+1}^{UE}], \quad (37)$$

along with the goods market equilibrium conditions (7) and (8).<sup>27</sup>

### A.1.4 First best allocation

With perfect access to complete asset markets, an allocation that we call the "first best", households maximize utility subject to their budget constraint state-by-state and period-by-period, expressed in units of tradable consumption.

Let's assume agents can trade a set of state-contingent securities at time 0, which pay one unit of tradables in a particular state at time  $t$ . Let  $z^t$  describe the history of shocks at time  $t$ , and let  $\pi_0(z^t)$  denotes the time-zero probability of a particular history, with  $Q_0(z^t)$  denoting the time-zero price of a security paying one unit of tradables at time  $t$  in history  $z^t$ . We denote with  $\pi_t(z_{t+1}|z^t)$  the probability of being in state  $z_{t+1}$  in time  $t + 1$  conditional on a given history at time  $t$ . Then the period budget constraint of an individual agent is:

$$\sum_{z_{t+1}} Q_t(z_{t+1}|z^t) B_{t+1}^j(z_{t+1}) + C_t^T + P_t^N C_t^N = Y_t^T(z^t) + P_t^N Y_t^N(z^t) + B_t^j.$$

Note here that state-contingent securities can only pay off in terms tradable goods.<sup>28</sup> We

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<sup>27</sup>Since we assume that there is a lower bound on debt strictly greater than the natural debt limit,  $\underline{B} > B^n$ , the ergodic distribution of external debt is strictly above the natural debt limit.

<sup>28</sup>This is a natural restriction, as agents in different countries can only promise to transfer tradable

also continue to assume that in our economy there is a lower bound on debt that is strictly greater than the natural debt limit,  $\underline{B} > B^n$ . Since nontradable consumption is equal to its endowment, which is positive in every state of nature, this lower bound guarantees that tradable consumption has a strictly positive lower bound. Thus, the following constraint needs to be satisfied:

$$B_{t+1}(z^{t+1}) \geq \underline{B}(z^{t+1})$$

or

$$C_t^T > 0.$$

Optimal behavior in this economy can be characterized in terms of the following first order conditions:

$$C_T : (C_t)^{\frac{1}{\kappa}-\rho} \omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}} = \mu_t(z^t), \quad (38)$$

$$C_N : (C_t)^{\frac{1}{\kappa}-\rho} (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}} = \mu_t(z^t) P_t^N, \quad (39)$$

$$B_{t+1}(z^{t+1}) : Q_t(z_{t+1}|z^t) \mu_t(z^t) = \beta \pi(z_{t+1}|z^t) \mu_{t+1}(z^{t+1}). \quad (40)$$

Combining (38) and (39) we have:

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N. \quad (41)$$

We can then combine (38) and (39) with the definition of the price index to get:

$$P_t = \left[ \omega \left( \frac{(C_t)^{\frac{1}{\kappa}-\rho} \omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}}{\mu_t(z^t)} \right)^{1-\kappa} + (1-\omega) \left( \frac{(C_t)^{\frac{1}{\kappa}-\rho} (1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\mu_t(z^t)} \right)^{1-\kappa} \right]^{\frac{1}{1-\kappa}}.$$

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goods to each other in different states because, by their nature, non-tradable goods cannot be transferred internationally. In equilibrium non-tradable consumption must still equal non-tradable output in each state, for each country.

From this expression we obtain

$$(\mu_t P_t)^{1-\kappa} = (C_t)^{\frac{(1-\kappa)(1-\rho\kappa)}{\kappa}} (C_t)^{\frac{\kappa-1}{\kappa}},$$

which becomes

$$\mu_t P_t = C_t^{-\rho}$$

and holds for every state of nature. Since the price of a state contingent asset is common across countries and it is the same as the one that the small open economy faces, we have that

$$Q_t(z_{t+1}|z^t)\mu_t^*(z^t) = \beta\pi(z_{t+1}|z^t)\mu_{t+1}^*(z^{t+1}),$$

where  $\mu^*$  denote the rest of the world marginal utility of tradeable consumption. It is now easy to obtain

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\rho} \frac{P_t}{P_{t+1}} = \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\rho} \frac{P_t^*}{P_{t+1}^*}.$$

Iterating on this condition and defining  $\delta_0 \equiv \left(\frac{C_0}{C_0^*}\right)^{-\rho} \frac{P_0^*}{P_0}$ , we finally obtain the familiar, complete market risk sharing condition

$$\left(\frac{C_t^*}{C_t}\right)^{-\rho} = \delta_0 \frac{P_t^*}{P_t},$$

which links the ratio of the national price levels, or the real exchange rate, to the consumption differential with the rest of the world.

Figure 1A plots the lifetime utility or the first best and unconstrained allocations and shows that the two differs significantly only for high level of initial debt.

## A.2 Social planner allocation

A benevolent social planner maximizes (1) subject to the same borrowing constraint (4) that private agents face and the market clearing conditions for tradables and nontradables goods (7) and (8).

In specifying this problem, the equilibrium price of nontradables is determined competitively according to the pricing rule (6) that serves also as a constraint to the planning problem. By substituting the relative price of nontradables,  $P_t^N$  in the borrowing constraint (4) with the competitive pricing rule (6) we can write the Lagrangian of the planning problem as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \begin{aligned} & \frac{1}{1-\rho} (C_t)^{1-\rho} + \mu_{1,t}^{SP} (Y_t^T - B_{t+1} + (1+r) B_t - C_t^T) + \\ & + \mu_{2,t}^{SP} (Y_t^N - C_t^N) + \lambda_t^{SP} \left( B_{t+1} + \frac{1-\phi}{\phi} \left[ Y_t^T + \left( \frac{(1-\omega)(C_t^T)}{\omega Y_t^N} \right)^{\frac{1}{\kappa}} Y_t^N \right] \right) \end{aligned} \right],$$

where  $\mu_{1,t}^{SP}$ ,  $\mu_{2,t}^{SP}$  and  $\lambda_t^{SP}$  denote the multipliers and the superscript  $SP$  distinguishes them from those in the constrained and unconstrained allocations. The planner must choose the optimal path for  $C_t^T$ ,  $C_t^N$  and  $B_{t+1}$ , and the first order conditions for its problem are:

$$C_T : u'(C_t^{SP}) C_{C^T}^{SP} + \lambda_t^{SP} \Sigma_t^{SP} = \mu_{1,t}^{SP}, \quad (42)$$

$$C_N : u'(C_t^{SP}) C_{C^N}^{SP} = \mu_{2,t}^{SP}, \quad (43)$$

$$B_{t+1} : \mu_{1,t}^{SP} = \lambda_t^{SP} + \beta (1+r) E_t [\mu_{1,t+1}^{SP}]. \quad (44)$$

$$\lambda_t^{SP} \left\{ B_{t+1}^{SP} + \frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N] \right\} = 0 \quad (45)$$

where  $\Sigma_t^{SP} \equiv \frac{1-\phi}{\phi} \frac{\partial P_t^N}{\partial C_t^T} Y_t^N = \frac{1-\phi}{\phi} \frac{1}{\kappa} \frac{(1-\omega)}{\omega} \left( \frac{(1-\omega)(C_t^T)}{\omega} \right)^{\frac{1}{\kappa}-1} (Y_t^N)^{\frac{\kappa-1}{\kappa}}$ .

The key difference between the planning allocation and the competitive one (with the borrowing constraint) follows from examining equations (42) and (28). The planner inter-

nalizes the consequences of her/his decisions on  $P_t^N$ . When the constraint binds ( $\lambda_t^{SP} > 0$ ), there is an additional benefit in consuming an extra unit tradable consumption, represented by the term  $\lambda_t^{SP}\Sigma_t$ . This term captures the increase in the price of non-tradable goods associated with the marginal increase in tradable consumption. As we discuss in the paper, this difference between the two margins has intertemporal implications and affects agents behavior also when the constraint does not bind.

### A.3 Optimal policy with a tax on debt

**Proof of Proposition 1** *In an economy defined by (1), (4), (12) and (9) with a tax on  $\tau_t^B$  as the government instrument, the Ramsey optimal policy with  $\tau^B$  as instrument replicates the social planner allocation (SP). Moreover the optimal policy is time-consistent.*

PROOF: Let's consider first a less restricted version of the Ramsey problem in which the planner maximizes (1) subject to (7) and (8), (17) and (19). This problem corresponds to the social planner one (SP) defined above. The solution of the Ramsey problem for  $\tau^B$  cannot achieve a higher welfare than the social planner allocation because the Ramsey problem is more restricted than the social planner problem—by equation (18).

We conjecture that the two allocation coincide. To verify this, note that the Euler equation for the social planner problem is

$$u'(C_t^{SP})C_{C_t^T}^{SP} + \lambda_t^{SP}\Sigma_t^{SP} = \lambda_t^{SP} + \beta(1+r)E_t[u'(C_{t+1}^{SP})C_{C_{t+1}^T}^{SP} + \lambda_{t+1}^{SP}\Sigma_{t+1}^{SP}]. \quad (46)$$

It is easy to see that, if the Ramsey planner chooses  $(1 + \tau_t^B)$  in equation (18) so that

$$\tau_t^B = (u'(C_t^{SP})C_{C_t^T}^{SP})^{-1}(\lambda_t^{SP}\Sigma_t^{SP} - \beta(1+r)E_t[\lambda_{t+1}^{SP}\Sigma_{t+1}^{SP}]), \quad (47)$$

the Euler equations (46) and (30) become identical. It follows that the solution of the Ramsey problem for  $\tau^B$  and the social planner problem above coincide, and the expression (47) is

the Ramsey optimal policy for  $\tau_t^B$ .

Moreover, since the optimal policy for  $\tau^B$  decentralizes the social planner problem, which is a recursive problem that can be represented by value function iteration and only depends on the current state ( $\{B_t, Y_t^N, Y_t^T\}$ ), the equilibrium is subgame perfect and the policy rule (47) is time-consistent.

QED.

## A.4 Optimal policy with a tax on nontradable consumption

**Proof of Proposition 2** *In an economy defined by (1), (4), (12) and (13) with a tax on non-tradable consumption  $\tau_t^N$  as the government instrument, there exists a policy for  $\tau_t^N$  that decentralizes the unconstrained allocation. This policy is Ramsey optimal and time-consistent.*

PROOF: For a given state  $\{Y_t^N, Y_t^T, B_t\}$ , let  $B_{t+1}^{UE}$  be the next-period debt and  $P_t^{N,UE}$  the current period relative price of non tradable goods in the economy defined by (1) and (3) but without the credit constraint (4)—i.e., in the unconstrained economy satisfying (35)-(37).

Next, let  $\hat{P}_t^N$  be the minimum price such that the credit constraint would be met if it were present. Thus:

$$\hat{P}_t^N = \max \left\{ 0, -\frac{B_{t+1}^{UE} + \frac{1-\phi}{\phi} Y_t^T}{\frac{1-\phi}{\phi} Y_t^N} \right\}.$$

If we set  $\tau^N$  such that  $\hat{P}_t^N(1 + \tau_t^N) \leq P_t^{N,UE}$ , then the credit constraint does not bind. In other words, let  $\hat{\tau}_t^N = P_t^{N,UE}/\hat{P}_t^N - 1$ , then any  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  would eliminate the credit constraint ( $\lambda_t = 0 \forall t$ ) if it were present, and the competitive equilibrium of the economy would coincide with the unconstrained allocation, which eventually converges to the lower debt bound,  $\underline{B}$ .

Now, in the economy with the credit constraint, the Ramsey planner maximizes (1) subject to (7) and (8), (22), (20), and (21). In this problem, any policy schedule such that  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  can achieve an allocation that satisfies the first order conditions (35)-(37) of

the unconstrained allocation. Since  $\tau_t^N$  can affect the allocation only when the constraint binds, but it is neutral when the constraint does not bind, the Ramsey planner can achieve at best the unconstrained allocation. Thus, the tax policy  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  is the optimal solution of the Ramsey problem. Moreover, any  $\tau_t^N \in (-1, \hat{\tau}_t^N]$  is completely determined by the current state  $\{B_t, Y_t^T, Y_t^N\}$  and therefore it is time-consistent.

QED.

## A.5 Optimal policy with a tax on tradable consumption

The Ramsey problem when  $\tau_t^T$  is the policy instrument is as follows.

**Definition 4** *For a given  $\{B_0\}$  and assuming that  $\{Y_t^T\}$  and  $\{Y_t^N\}$  are Markov processes with finite, strictly positive support, the Ramsey problem for  $\tau_t^T$  is to choose a competitive equilibrium that maximizes*

$$U^j \equiv E_0 \sum_{t=0}^{\infty} \{\beta^t u(C_t)\},$$

*subject to the resource constraints*

$$C_t^T = Y_t^T - B_{t+1} + (1+r)B_t, \quad (48)$$

$$C_t^N = Y_t^N, \quad (49)$$

*the borrowing constraint*

$$B_{t+1} \geq -\frac{1-\phi}{\phi} [Y_t^T + P_t^N Y_t^N]. \quad (50)$$

*the government budget constraint (16) and the first order conditions of the household*

$$\frac{u'(C_t)C_{C_t^T}}{1+\tau_t^T} = \lambda_t + \beta(1+r)E_t \left[ \frac{u'(C_{t+1})C_{C_{t+1}^T}}{1+\tau_{t+1}^T} \right]. \quad (51)$$

with

$$\frac{(1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = \frac{P_t^N}{1 + \tau_t^T}. \quad (52)$$

The tax on tradable consumption affects not only the intratemporal relative price in (52), but also the intertemporal allocation of resources in (51). Despite this difference, the next proposition shows that it is possible to find a policy for  $\tau_t^T$  that replicates the unconstrained allocation like in the case of the nontradable consumption tax  $\tau_t^N$ .

**Proposition 3** *In an economy defined by (1), (3), (15) and (16) with a tax on tradable consumption  $\tau_t^T$  as the government instrument, there exists a policy for  $\tau_t^T$  that decentralizes the unconstrained allocation and it is time-consistent.*

PROOF: Let the optimal non-tradable consumption tax be  $\hat{\tau}_t^N$ . In the Ramsey problem for  $\tau_t^T$ , if we set  $\frac{1}{1+\tau_t^T} = 1 + \tau_t^N$  we can achieve the unconstrained allocation, and  $\lambda_t \equiv 0 \forall t$ . However, since  $\tau_t^T$  affects also the intertemporal allocation of resources (51) we need to show that there is a constant  $\tau_t^T$  such that the intertemporal margin is not affected.

To do so, we first note that, by imposing  $\lambda_t \equiv 0$  and setting  $\tau_t^T$  so that

$$\frac{1}{1 + \tau_t^T} = \frac{\beta(1 + r)E_t \left[ \frac{u'(C_{t+1}^{UN})C_{t+1}^{UN}}{1 + \tau_{t+1}^T} \right]}{E_t[u'(C_{t+1}^{UN})C_{t+1}^{UN}]}, \quad (53)$$

the Euler equations of the Ramsey problem and the unconstrained equilibrium coincide. It follows that the tax rate  $\tau_t^T$  that satisfies (53) must be constant (otherwise the intertemporal margin would be distorted).

By inspection of the unconstrained allocation, the non-tradable price has a strictly positive lower limit. Therefore there exists  $\underline{\tau}^T$  (this is, the lower level of the tax on tradables compatible with the strictly positive lower limit on the relative price of nontradables) such that the borrowing constraint (4) is always satisfied for any  $\tau^T \geq \underline{\tau}^T$ . Thus, any constant tax policy of the form  $\tau_t^T \equiv \tau^T \geq \underline{\tau}^T$  is an optimal policy such that the competitive equilibrium



replicates the unconstrained equilibrium. As  $\underline{\tau}^T$  is completely determined by the current state  $\{B_t, Y_t^T, Y_t^N\}$  it is time-consistent.

QED.

## A.6 Optimal policy with distortionary financing

### A.6.1 Time consistency of optimal policy with two instruments

We now prove formally that the Ramsey problem with two distortionary policy instruments is time-consistent.

Ramsey optimal policy with two distortionary policy instruments solves the following problem:

$$(\{B_{t+1}^R\}, \{(\tau_t^N)^R\}, \{(\tau_t^B)^R\}) \doteq \arg \max_{(\{B_{t+1}\}, \{\tau_t^N\}, \{\tau_t^B\})} \sum_{t=0}^{\infty} \beta^t U(C_t^T, C_t^N), \quad (54)$$

subject to conditions (7), (8), (20), (24) and (25) for all  $t = 0, 1, \dots$ .

The time consistent optimal policy solves the following recursive problem

$$(B_{t+1}^C, (\tau_t^N)^C, (\tau_t^B)^C) \doteq \arg \max_{(B_{t+1}, \tau_t^N, \tau_t^B)} U(C_t^T, C_t^N) + \beta V^C(B_{t+1}) \quad (55)$$

subject to conditions (7), (8), (20), (24) and (25) at time  $t$ . Here  $V^C(\cdot)$  is the household value function under the time consistent optimal policy, i.e.

$$V^C = \sum_{t=0}^{\infty} \beta^t U(C_t^{TC}, C_t^{NC})$$

where  $\{C_t^{TC}\}$  and  $\{C_t^{NC}\}$  are sequences of tradeable and nontradable consumptions based on the time consistent optimal policy. Note that the state of economy at time  $t$  is  $B_t$ , the current level of debt. Hence, the value function depends solely on  $B_t$ . We want to establish that, in our economy, the Ramsey optimal policy is time consistent, i.e.  $B_{t+1}^R = B_{t+1}^C, \tau_t^{NR} =$

$$\tau_t^{NC}, \tau_t^{BR} = \tau_t^{BC}.$$

To prove this, we shall take the following steps. First, we show that this is the case in a three-period version of these two problems. Second, we look at a four-period case and show that this can be reduced to the 3-period case. Next we show that we can always reduce an  $n$ -period case to a  $n - 1$ -period one for any  $n > 4$ . This establishes, by induction, that in any finite-period version of our model economy the two policy regimes coincide. Finally, under the auxiliary assumption that the period utility function and the marginal utility of consumption are bounded in the feasible set, we prove that Ramsey optimal policy in the finite-period model converges to Ramsey optimal policy in a infinite-horizon version of our economy.

**Three-period model** We start by examining the 3-period version of the original Ramsey optimal policy problem:

$$\begin{aligned} & (\{B_1^{R'}, B_2^{R'}\}, \{\tau_0^{NR'}, \tau_1^{NR'}\}, \{\tau_0^{BR'}, \tau_1^{BR'}\}) \doteq \\ & \arg \max_{(\{B_1, B_2\}, \{\tau_0^N, \tau_1^N\}, \{\tau_0^B, \tau_1^B\})} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^C(B_2), \end{aligned} \quad (56)$$

subject to (7), (8), (20), (24) and (25) for all  $t = 0, 1, \dots$ .

It is easy to see that the only potential source of difference between the two policy regimes comes from the Euler equation (24). In fact, when we optimize at time  $t = 1$  in the time-consistent regime, we do not take into account that the choice of  $B_2$  affects the Euler equation at time  $t = 0$ ,

$$U_{C_0^T}(1 + \tau_0^B) = \lambda_0 + \beta(1 + r)U_{C_1^T},$$

since from (7)  $B_2$  affects  $U_{C_1^T}$ .

However this can result in differences between the two policy regimes only if  $B_2$  affects  $U(C_0^T)$  and  $U(C_1^T, C_1^N) + \beta V^C(B_2)$  in opposite ways. Specifically, in order for the following

two problems

$$\begin{aligned} & \max_{(\{B_1, B_2\}, \{\tau_0^N, \tau_1^N\}, \{\tau_0^B, \tau_1^B\})} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^C(B_2) \\ &= \max_{(B_1, \tau_0^N, \tau_0^B)} U(C_0^T, C_0^N) + \beta \left( \max_{(B_2, \tau_1^N, \tau_1^B)} U(C_1^T, C_1^N) + \beta^2 V^C(B_2) \right), \end{aligned} \quad (57)$$

to coincide, it is sufficient that the following derivatives have the same sign:

$$\frac{\partial U_0(B_2)}{\partial B_2} \quad \text{and} \quad \frac{\partial U_1(B_1, B_2)}{\partial B_2}, \quad (58)$$

where

$$U_0(B_2) \doteq \max_{(B_1, \tau_0^N, \tau_0^B)} U(C_0^T, C_0^N),$$

subject to (7), (8), (20), (24) and (25) at time  $t = 0$ , and

$$U_1(B_1, B_2) \doteq \max_{(\tau_1^N, \tau_1^B)} U(C_1^T, C_1^N) + \beta V^C(B_2)$$

subject to (7), (8), (20), (24) and (25) at time  $t = 1$ .

If this restriction holds, the maximization of  $U_1(B_1, B_2)$  with respect to  $B_2$ , yields the same optimal value of  $B_2$  that maximizes  $U_0(B_2)$ . Therefore the maximization can be done in a step-wise way (which gives the time consistent optimal policy) for the Ramsey program on the left hand side of the equality (57).

Thus, in order to show that in our economy Ramsey optimal policy is time consistent we need to establish (58). To do this, we are going to show that both  $U_0(B_2)$  and  $U_1(B_1, B_2)$  are decreasing functions of  $B_2$ , given  $B_1$ . In fact, it is straightforward to see that the function  $U_0(B_2)$  is a decreasing function of  $B_2$ , since if the household knows that in period 2 she can borrow more, she is able to consume more in period 1, and through the Euler equation (24), she can also consume more in period 0.

Next we want to show that  $U_1(B_1, B_2)$  is also a decreasing function of  $B_2$ , for given  $B_1$ . Let

$B_2^*$  be the borrowing level in the competitive equilibrium without the borrowing constraint or any tax intervention. So  $U_1(B_1, B_2)$  must achieves its maximum at  $B_2^*$ . Therefore  $U_1(B_1, B_2)$  decreases for any  $B_2 \geq B_2^*$ . We shall show that  $B_2^C \geq B_2^*$  in the optimal plan that maximize  $U_1(B_1, B_2)$  subject to (7), (8), (20), (24) and (25) for  $t = 1$ .

We know from our optimal policy analysis on the individual tax instruments, that the optimal policy is such that  $\tau_1^{NC} \leq 0$  and  $\tau_1^{BC} \leq 0$ . If the borrowing constraint is not binding, we have from the Euler equation (24) that

$$U_{C_1^T}(1 + \tau_1^B) = \beta(1 + r)U_{C_2^T}.$$

And if  $\tau_1^B < 0$  and the  $B_2 < B_2^*$ , we would have

$$U_{C_1^T}(1 + \tau_1^B) < U_{C_1^{T*}} = \beta(1 + r)U_{C_2^{T*}} < \beta(1 + r)U_{C_2^T},$$

which is a contradiction.<sup>29</sup> Therefore we conclude that if the borrowing constraint is not binding,  $B_2^C \geq B_2^*$ .

If the constraint is binding, from the Euler equation (24) we have that  $\lambda > 0$ . Suppose that  $B_2' \geq B_2^*$  is optimal in the economy without the borrowing constraint. We want to show that the optimal policy in the economy with the borrowing constraint has  $B_2 \geq B_2'$ . Suppose this is not the case. Then we would have

$$U_{C_1^T}(1 + \tau_1^B) - \lambda < U_{C_1^{T'}}(1 + \tau_1^B) = \beta(1 + r)U_{C_2^{T'}} < U_{C_2^T},$$

which again contradicts the Euler equation (24).<sup>30</sup> Therefore we must have that  $B_2^C \geq B_2^{C'} \geq B_2^*$ . Combining the previous two arguments, it follows that  $U_1(B_1, B_2)$  is also a decreasing

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<sup>29</sup>The first inequality comes from  $C_1^T > C_1^{T*}$  and the fact that  $U_{C_1^T}$  is a decreasing function of  $C_1^T$ . The second inequality comes from  $C_2^T < C_2^{T*}$  and the same logic.

<sup>30</sup>The inequalities follow from the same reasonings in the case of a nonbinding borrowing constraint.

function of  $B$  and hence has the same sign of  $U_0(B_2)$ , which proves that 58 holds.

**Finite-period model** Let us now look first at the case of a four-period model. We will show that this case can be reduced to the 3-period model above. In a four-period version of our model, the Ramsey program solves the following problem

$$\max_{(\{B_i\}_{i=1}^3, \{\tau_i^N\}_{i=0}^2, \{\tau_i^B\}_{i=0}^2)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 U(C_2^T, C_2^N) + \beta^3 V^C(B_3),$$

subject to (7), (8), (20), (24) and (25) for  $t = 1, \dots, 3$ .

Now note first that

$$U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N)$$

is decreasing in  $B_3$  by the same reasoning as in the 3-period model, and that

$$U(C_2^T, C_2^N) + \beta V^C(B_3)$$

is also decreasing in  $B_3$ , since  $B_3 \geq B_3^*$  where  $B_3^*$  is the competitive equilibrium borrowing level without the borrowing constraint or tax interventions. Therefore we have that

$$\begin{aligned} & \max_{(\{B_i\}_{i=1}^3, \{\tau_i^N\}_{i=0}^2, \{\tau_i^B\}_{i=0}^2)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 U(C_2^T, C_2^N) + \beta^3 V^C(B_3) \\ &= \max_{(\{B_i\}_{i=1}^2, \{\tau_i^N\}_{i=0}^1, \{\tau_i^B\}_{i=0}^1)} U(C_0^T, C_0^N) + \beta U(C_1^T, C_1^N) + \beta^2 V^C(B_2), \end{aligned}$$

where

$$V^C(B_2) = \max_{B_3, \tau_2^N, \tau_2^B} U(C_2^T, C_2^N) + \beta V^C(B_3)$$

subject to (7), (8), (20), (24) and (25) for  $t = 3$ . Thus, we reduced a four-period model into a three-period model. It follows that the Ramsey optimal policy is time consistent in a four-period version of our model.

By using the same method, we can always reduce an  $n$ -period model into  $n - 1$  period model for any  $n > 4$ . By induction, therefore, we showed that the Ramsey optimal policy for any finite-period version of our economy is time consistent.

**Infinite-horizon model** If we can establish the convergence of the Ramsey optimal policy problem for a finite-period version of our model to an infinite-period version, we will have established that Ramsey optimal policy is time consistent for (54). To do so, we need an additional assumption, i.e. that both  $U(\cdot)$  and  $U_{C^T}(\cdot)$  are bounded in the feasible set.

Define now the following mapping  $T : \mathcal{F}_b \rightarrow \mathcal{F}_b$ , where  $\mathcal{F}_b$  is the set of bounded continuous function defined on  $[\underline{B}, 0] \times \mathbb{R}^+$ ,

$$T(V)(B, \mu) \doteq \min_{\gamma \geq 0} \max_{B', \tau^B} U(C^T, C^N) - \mu(1 + r)U_{C^T} + \gamma(U_{C^T} - \lambda) + \beta V(B', \gamma),$$

subject to (7), (8), (20), (24) and (25).

From the assumption that both  $U(\cdot)$  and  $U_{C^T}(\cdot)$  are bounded, it follows that  $\lambda$  is bounded. Following Marcet and Marimon (2011), we also conclude that  $T$  is a contraction mapping.

In addition, we note that  $T^n(V)(B, 0)$  is the welfare function of a Ramsey optimal plan for a  $n$ -period economy with  $V(\cdot)$  as the final period utility. Therefore from a standard contraction mapping argument we have that

$$V^*(\cdot) \doteq \lim_{n \rightarrow \infty} T^n(V)(\cdot)$$

is well defined and is uniformly converging.  $V^*(\cdot)$  will be the fixed point of the contraction mapping and is the welfare function of the infinite-period economy under the Ramsey optimal policy.

By the uniform convergence of the welfare function, the finite-period Ramsey optimal policy converges to the infinite-period Ramsey optimal policy. Therefore we established that

the Ramsey optimal policy for (54) is time consistent.

QED

### A.6.2 Three distortionary policy instruments

We now focus on the case in which all three distortionary policy tools are available to the policy maker (see (26)).

Suppose that the triplet of policy tools  $(\tau_t^N, \tau_t^T, \tau_t^B)$  can completely remove the borrowing constraint (4). The Euler equation for this economy would be:

$$\frac{1 + \tau_t^B}{1 + \tau_t^T} \mu_t = \beta(1 + r) E_t \frac{\mu_{t+1}}{1 + \tau_{t+1}^T}. \quad (59)$$

Remember now that the Euler equation for the unconstrained economy is:

$$\mu_t^{UE} = \beta(1 + r) E_t [\mu_{t+1}^{UE}]. \quad (60)$$

By comparing (59) and (60), we can see that in order to replicate the unconstrained equilibrium the triplet of policy tools  $(\tau_t^N, \tau_t^T, \tau_t^B)$  must satisfy:

$$\frac{1 + \tau_t^B}{1 + \tau_t^T} = \frac{E_t \frac{\mu_{t+1}}{1 + \tau_{t+1}^T}}{E_t [\mu_{t+1}^{UE}]}. \quad (61)$$

In addition, from the government budget constraint, we need to have

$$\tau_t^T (C_t^T)^{UE} + \tau_t^N (C_t^N)^{UE} + \tau_t^B B_{t+1}^{UE} = 0. \quad (62)$$

And from the borrowing constraint, we must have that

$$B_{t+1}^{UE} \geq -\frac{1 - \phi}{\phi} \left[ Y_t^T + (P_t^N)^{UE} Y_t^N \frac{1 + \tau_t^T}{1 + \tau_t^N} \right]. \quad (63)$$

To find the tax policy  $\{\tau_t^B, \tau_t^T, \tau_t^N\}$  that solves (61) to (63) we proceed recursively as follows. Denote the stochastic steady state level of debt by  $\underline{B}$ , and by  $B_0$  the level of debt in the unconstrained equilibrium at which the constraint would become binding exactly in the constrained economy. Now define  $B_t = B^{UE}(B_{t-1})$ , where  $B^{UE}(\cdot)$  is the policy function in the unconstrained equilibrium. From this policy function, we can obtain  $B_0 > B_1 > \dots > B_t > B_{t+1} > \dots > \underline{B}$ , so that  $\{B_k\}$  is a debt trajectory in the unconstrained solution starting from  $B_0$ .

Starting from  $k = 0$ , for any  $B \in (B_1, B_0]$ , we can compute

$$\frac{1 + \tau^T(B)}{1 + \tau^N(B)} = - \frac{B^{UE}(B) + \frac{1-\phi}{\phi} Y_t^T}{\frac{1-\phi}{\phi} (P_t^N)^{UE}(B) Y_t^N}$$

from (63). Let's set  $\tau_0^T(B) \equiv 0$  in that interval and use the expression above to obtain  $\tau_0^N(B)$ . The value of  $\tau_0^B(B)$  in the  $(B_1, B_0]$  interval can then be determined by the government budget constraint (62).

Next, consider  $k = 1$  and the associated interval  $(B_2, B_1]$ . Since we have already determined the value of  $\tau_0^T(B)$  and  $\tau_0^B(B)$  for  $B \in (B_1, B_0]$ , by using (61), we can obtain the value of  $\tau_1^T(B)$ . Again by assuming the borrowing constraint (61) is binding exactly, we can determine the value of  $\tau_1^N(B)$ . Last, by using the government budget constraint (62) we can determine the value of  $\tau_1^B(B)$  and update to  $k = 2$ .

By iterating recursively, we can always find the tax policy that replicates the unconstrained solution in an economy with the borrowing constraint.

QED

## A.7 Extensions

In this section we provide details of the extensions discussed in the paper.



### A.7.1 Imperfect exchange rate intervention

Let us consider a situation in which both the capital control tax  $\tau^B$  and the non-tradable consumption tax  $\tau^N$  are available to the government, and the budget is balanced with a lump-sum tax  $T$ . However, the non-tradable consumption tax  $\tau^N$  can be used only some of the times, depending on the realization of an exogenous random variable  $\varepsilon_t \in \{0, 1\}$  that is known to the government at the beginning of each period  $t$ . This random loss of access to  $\tau^N$  could capture imperfect credibility of exchange rate policy or limited availability of revenue to implement the subsidy. With such an imperfect form of exchange rate intervention, the household budget constraint becomes

$$C_t^T + (1 + \varepsilon_t \tau_t^N) P_t^N C_t^N = Y_t^T + P_t^N Y_t^N + T_t - B_{t+1}(1 + \tau_t^B) + (1 + r)B_t;$$

the government budget now is

$$\varepsilon_t \tau_t^N P_t^N C_t^N + \tau_t^B B_{t+1} = T_t;$$

and the non-tradable pricing equation now is

$$\frac{(1 - \omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = (1 + \varepsilon_t \tau_t^N) P_t^N.$$

In order to study this case, we proceed in two steps. We first design the optimal policy for  $\tau_t^N$  conditional on the realization of  $\varepsilon_t$  and a given capital control tax  $\tau^B$ . We then optimize over  $\tau^B$  taking into account that the optimal policy for  $\tau_t^N$  might not be always available.

Similarly to what we found in proposition 2 in the paper, optimal policy for  $\tau_t^N$  can undo the collateral constraint. Indeed, it is straightforward to see that regardless of the policy for

$\tau^B$ , the time-consistent optimal policy for  $\tau^N$  is

$$\tau_t^N \in \left\{ \begin{array}{ll} \left( -1, \hat{\tau}_t^N \doteq \frac{P_t^{N,TC(\tau^B)}}{\hat{P}_t^N} - 1 \right], & \varepsilon_t = 1, \\ \text{any,} & \varepsilon_t = 0. \end{array} \right\}$$

Here we have denoted with  $P_t^{N,TC(\tau^B)}$  the non-tradable price under any time-consistent policy for  $\tau^B$ . Also

$$\hat{P}_t^N = \max \left\{ 0, -\frac{B_{t+1}^{TC(\tau^B)} + \frac{1-\phi}{\phi} Y_t^T}{\frac{1-\phi}{\phi} Y_t^N} \right\},$$

where  $B_{t+1}^{TC(\tau^B)}$  denotes borrowing under the same time-consistent policy of  $\tau^B$ . Essentially, under this policy for  $\tau^N$ , the government can remove the borrowing constraint.

We then proceed with the second step and study the optimal policy for  $\tau^B$  taking into account that the optimal policy for  $\tau_t^N$  might not be always available. To do so, we transform the optimization problem into an equivalent one in which there is only  $\tau^B$ , the budget balancing lump-sum tax  $T$ , and a modified version of the credit constraint:

$$(1 - \varepsilon_t) \left( B_{t+1} + \frac{1-\phi}{\phi} (Y_t^T + P_t^N Y_t^N) \right) \geq 0.$$

This version of the borrowing constraint takes into account that, when exchange rate policy is available, i.e.  $\varepsilon_t = 1$ , the constraint can be removed. A line of argument similar to the one used in the proof of Proposition 1 in the paper can show that there exists the following time-consistent optimal policy for  $\tau_t^B$ :

$$\tau_t^B = (u'(C_t^{SP(\varepsilon)}) C_{C_t^T}^{SP(\varepsilon)})^{-1} ((1 - \varepsilon_t) \lambda_t^{SP(\varepsilon)} \Sigma_t^{SP(\varepsilon)} - \beta(1+r) E_t[(1 - \varepsilon_{t+1}) \lambda_{t+1}^{SP(\varepsilon)} \Sigma_{t+1}^{SP(\varepsilon)}]),$$

and it coincides with the Ramsey optimal policy. Here we denoted  $SP(\varepsilon)$  as a social planner problem in which the credit constraint binds at time  $t$  only when  $\varepsilon_t = 0$ . And this optimal policy  $\tau_t^B$  achieves the allocation corresponding to  $SP(\varepsilon)$ . Thus, this shows that, with

imperfect credibility, prudential capital controls are part of the optimal policy design, like in the case in which we have distortionary financing analyzed in the paper. Notice however that, here, the optimal  $\tau_t^B$  depends on the likelihood that  $\varepsilon_t = 0$ : the higher the probability that exchange rate policy is not available, the higher the level of  $\tau_t^B$ .

### A.7.2 Borrowing constraints with pre-tax income

We first show that when the constraint is expressed in terms of post-tax income, and the policy tools available to the Ramsey planner is  $\tau^N$  with  $\tau_t^N P_t^N C_t^N = T_t$ , the tax/subsidy on non-traded goods is ineffective. We have that

$$B_{t+1} \geq -\frac{1-\phi}{\phi} \left[ Y_t^T + \left( \tilde{P}_t^N \right) Y_t^N - T_t \right]$$

with

$$\tilde{P}_t^N = \frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = P_t^N (1 + \tau_t^N)$$

so that combining the previous two equations with the government budget constraint we obtain

$$\begin{aligned} B_{t+1} &\geq -\frac{1-\phi}{\phi} \left[ Y_t^T + \left( \tilde{P}_t^N \right) Y_t^N - T_t \right] = \\ &= -\frac{1-\phi}{\phi} \left[ Y_t^T + P_t^N (1 + \tau_t^N) Y_t^N - \tau_t^N P_t^N C_t^N \right] = \\ &= -\frac{1-\phi}{\phi} \left[ Y_t^T + P_t^N Y_t^N \right] \end{aligned}$$

This implies that, when the collateral depends on post tax income then any changes in  $\tau^N$  is neutralized by corresponding changes in the lump sum transfer/tax and it is not effective in affecting the collateral constraint.

Here we show that, when the borrowing constraint is expressed in terms of post-tax income, and the policy tools available to the Ramsey planner is  $\tau^B$  with  $\tau_t^B B_{t+1} = T_t$ ,

the allocation under optimal policy for this instrument is identical to the one in which the borrowing constraint is defined in terms of pre-tax income and the instrument are  $\tau_t^N, \tau_t^B$  with  $\tau_t^N P_t^N C_t^N = \tau_t^B B_{t+1}$ . Rewrite the borrowing constraint as

$$\begin{aligned} B_{t+1} &\geq -\frac{1-\phi}{\phi} \left[ Y_t^T + \left( \tilde{P}_t^N \right) Y_t^N - T_t \right] = \\ &= -\frac{1-\phi}{\phi} \left[ Y_t^T + \left( \tilde{P}_t^N \right) Y_t^N - \tau_t^B B_{t+1} \right] \end{aligned} \quad (64)$$

where

$$\tilde{P}_t^N = \frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}}$$

Now, if the borrowing constraint depends on pre-tax income and we have  $\tau_t^N P_t^N C_t^N = \tau_t^B B_{t+1}$ , we can write the borrowing constraint as

$$\begin{aligned} B_{t+1} &\geq -\frac{1-\phi}{\phi} \left[ Y_t^T + P_t^N Y_t^N \right] = \\ &= -\frac{1-\phi}{\phi} \left[ Y_t^T + \frac{\tilde{P}_t^N Y_t^N}{(1+\tau_t^N)} \right] \\ &= -\frac{1-\phi}{\phi} \left[ Y_t^T + \left( \tilde{P}_t^N \right) Y_t^N - \frac{\tau_t^N \tilde{P}_t^N Y_t^N}{(1+\tau_t^N)} \right] \end{aligned} \quad (65)$$

where

$$\frac{(1-\omega)^{\frac{1}{\kappa}} (C_t^N)^{-\frac{1}{\kappa}}}{\omega^{\frac{1}{\kappa}} (C_t^T)^{-\frac{1}{\kappa}}} = \tilde{P}_t^N = P_t^N (1+\tau_t^N)$$

Comparing (64) and (65) we can see that the two coincide since

$$\frac{\tau_t^N \tilde{P}_t^N Y_t^N}{(1+\tau_t^N)} = \tau_t^N P_t^N C_t^N = \tau_t^B B_{t+1}.$$

## A.8 Numerical solution methods

Here we describe how we compute the different equilibria numerically. We start by rewriting the competitive equilibrium of the model with the borrowing constraint. We can summarize this equilibrium with the following set of nonlinear functional equations:

$$\begin{aligned}
\mu(B, Y^T, Y^N) &= \beta(1+r)E[\mu(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})] \\
&\quad + \max\{\lambda(B, Y^T, Y^N), 0\}^2 \\
\mu(B, Y^T, Y^N) &= \left(C^{\frac{\kappa-1}{\kappa}}\right)^{(1-\rho)\frac{\kappa}{1-\kappa}-1} \omega^{\frac{1}{\kappa}} C^T(B, Y^T, Y^N)^{-\frac{1}{\kappa}} \\
C^T(B, Y^T, Y^N) &= (1+r)B + Y^T - B'(B, Y^T, Y^N) \\
P^N(B, Y^T, Y^N) &= \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(\frac{Y^N}{C^T(B, Y^T, Y^N)}\right)^{-\frac{1}{\kappa}} \\
\max\{-\lambda(B, Y^T, Y^N), 0\}^2 &= B'(B, Y^T, Y^N) + \frac{1-\phi}{\phi}Y^T + P^N(B, Y^T, Y^N)Y^N,
\end{aligned}$$

where

$$C^{\frac{\kappa-1}{\kappa}} \equiv \left[\omega^{\frac{1}{\kappa}}(C^T(B, Y^T, Y^N))^{\frac{\kappa-1}{\kappa}} + (1-\omega)^{\frac{1}{\kappa}}(Y_t^N)^{\frac{\kappa-1}{\kappa}}\right].$$

We then convert the complementary slackness conditions for the borrowing constraint in these equations into a nonlinear equation, following Garcia and Zangwill (1981).

### A.8.1 The constrained and unconstrained competitive equilibrium

Given an initial guess for the marginal utility of tradable consumption tomorrow  $\mu^0(B', Y^{T'}, Y^{N'})$ , the set of nonlinear functional equations above can be solved at each point in the state space  $(B, Y^T, Y^N)$  to obtain an updated function  $\mu^1(B, Y^T, Y^N)$ . This process is then iterated to convergence. We use a cubic spline to approximate the  $\mu^0(B', Y^{T'}, Y^{N'})$  function at values of  $B'$  that are not on the grid for  $B$ . We obtain the lifetime utility in the competitive

equilibrium using the following Bellman equation:

$$V^{CE}(B, Y^T, Y^N) = \frac{1}{1-\rho} C^{1-\rho} + \beta E [V^{CE}(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})].$$

The allocation corresponding to the unconstrained competitive equilibrium is computed in a similar fashion, except that the complementary slackness condition is omitted.

### A.8.2 The social planning problem

The solution of the social planning problem solves the following standard dynamic programming problem:

$$V^{SP}(B, Y^T, Y^N) = \max_{C^T, B', P^N} \left\{ \frac{1}{1-\rho} C^{1-\rho} + \beta E [V^{SP}(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})] \right\}$$

subject to

$$\begin{aligned} C^T + B' &\leq (1+r)B + Y^T \\ B' &\geq -\frac{1-\phi}{\phi} (Y^T + P^N Y^N) \\ P^N &= \left( \frac{1-\omega}{\omega} \right)^{\frac{1}{\kappa}} \left( \frac{Y^N}{C^T} \right)^{-\frac{1}{\kappa}}. \end{aligned}$$

Again, we approximate the value function with a cubic spline and solve the constrained optimization problem using feasible sequential quadratic programming with analytical derivatives.

### A.8.3 Markov-Perfect optimal policy

To compute the Ramsey optimal control program with two instruments we exploit time-consistent nature of the problem and use the method proposed by Benigno et al. (2012). That method is related to Klein, Krusell, and Rios-Rull (2009): the main difference being that the

algorithm that we use does not require that the policy functions are differentiable (which in general would not hold in our environment due to the occasionally-binding constraint) but only that they are continuous.

The optimal policy problem for  $\tau_N$  and  $\tau_B$  is also solved iteratively. The current government solves the following problem

$$V^{OP}(B, Y^T, Y^N) = \max_{\tau_N, \tau_B, C^T, P^N, B', \mu, \lambda} \left\{ \frac{1}{1-\rho} C^{1-\rho} + \beta E [V^{OP}(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})] \right\}$$

subject to

$$\begin{aligned} (1 + \tau_B) \mu &= \beta(1+r) E [\mu(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'})] + \max\{\lambda, 0\}^2 \\ \mu &= \left(C^{\frac{\kappa-1}{\kappa}}\right)^{(1-\rho)\frac{\kappa-1}{\kappa}-1} \omega^{\frac{1}{\kappa}} (C^T)^{-\frac{1}{\kappa}} \\ C^T &= (1+r)B + Y^T - B' \\ P^N &= \left(\frac{1-\omega}{\omega}\right)^{\frac{1}{\kappa}} \left(\frac{Y^N}{C^T}\right)^{-\frac{1}{\kappa}} \frac{1}{1+\tau_N} \\ \max\{-\lambda, 0\}^2 &= B' + \frac{1-\phi}{\phi} (Y^T + P^N Y^N) \\ 0 &= \tau_N P^N Y^N + \tau_B B'(B, Y^T, Y^N). \end{aligned}$$

We then guess both the continuation value function and the future marginal utility function, solve the optimization problem using feasible sequential quadratic programming with analytical derivatives, and then update both functions to convergence.<sup>31</sup> Both functions are approximated with cubic splines.

We set a large number of grid points in the  $B$  dimension (1550), with most of them concentrated at the lower end of the debt range where the constraint may bind. The joint process for  $(Y^T, Y^N)$  is approximated as a Markov chain with 49 states (7 in each dimension) using the method of Gospodinov and Lkhagvasuren (2013). Invariant distributions were

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<sup>31</sup>We use analytical derivatives, particularly for the continuation value function, as numerical derivatives produce solutions that are "choppy" for the tax variables (but not the other endogenous variables).

produced using the nonstochastic method from Young (2010), except for the frequency of crises which are estimated using a simulated sample of 10,000,000 observations.

#### A.8.4 Welfare calculations

To compute the welfare equivalents, we solve the following functional equation:

$$\tilde{V}^{CE}(B, Y^T, Y^N; \chi) = \frac{1}{1-\rho} C^{1-\rho} + \beta E \left[ \tilde{V}^{CE}(B'(B, Y^T, Y^N), Y^{T'}, Y^{N'}; \chi) \right];$$

where  $\chi$  is a proportional increment to tradable consumption, and the decision rules are those from the competitive equilibrium. We use 200 grid points for  $\chi$ , evenly-spaced. We then solve the following nonlinear equations for  $\chi(B, Y^T, Y^N)$ :

$$V^{SP}(B, Y^T, Y^N) = \tilde{V}^{CE}(B, Y^T, Y^N; \chi),$$

to obtain the welfare gain from moving to the SP allocation;

$$V^{OP}(B, Y^T, Y^N) = \tilde{V}^{CE}(B, Y^T, Y^N; \chi),$$

to obtain the welfare gain from moving to the OP allocation; and

$$V^{UA}(B, Y^T, Y^N) = \tilde{V}^{CE}(B, Y^T, Y^N; \chi)$$

to obtain the welfare gain from moving to the unconstrained allocation. These equations are solved using the Brent's method, with linear interpolation between grid points for  $\chi$ .



## References

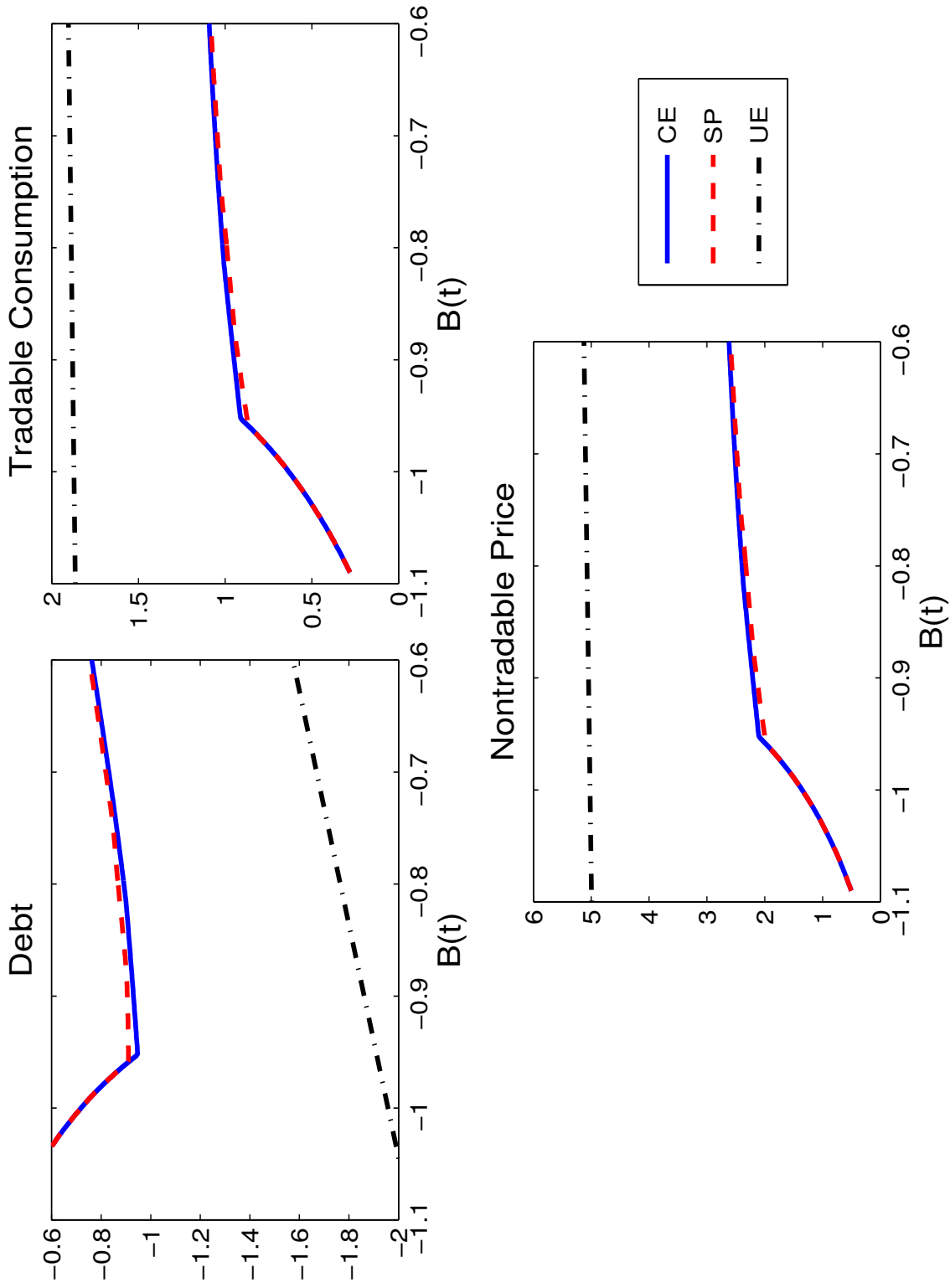
- [1] Chari, V.V. ,Patrick Kehoe and Ellen McGrattan (2002) “Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?”, *Review of Economic Studies*, v.69, pp 533-563.
- [2] Gospodinov, Nikolay and Lkhagvasuren, Damba, (2013): ”A moment-matching method for approximating vector autoregressive processes by finite-state Markov chains,” Federal Reserve Bank of Atlanta Working Paper 2013-05.
- [3] Klein, Paul, Per Krusell, and Jose-Victor Rios-Rull 2008. “Time-consistent public policy.” *Review of Economic Studies* 75(3), 789-808.
- [4] Young, Eric R. (2010), ”Solving the Incomplete Markets Model with Aggregate Uncertainty Using the Krusell-Smith Algorithm and Non-Stochastic Simulations,” *Journal of Economic Dynamics and Control* 34(1), pp. 36-41.

Table 1: Ergodic Averages

	Debt to Income	Prob. of Crisis	Welfare Gain
CE	-29.2%	6.7%	NA
SP	-28.4%	1.2%	0.41%
UE	NA	0.0%	33.8%
OP	-30.5%	4.9%	1.10%

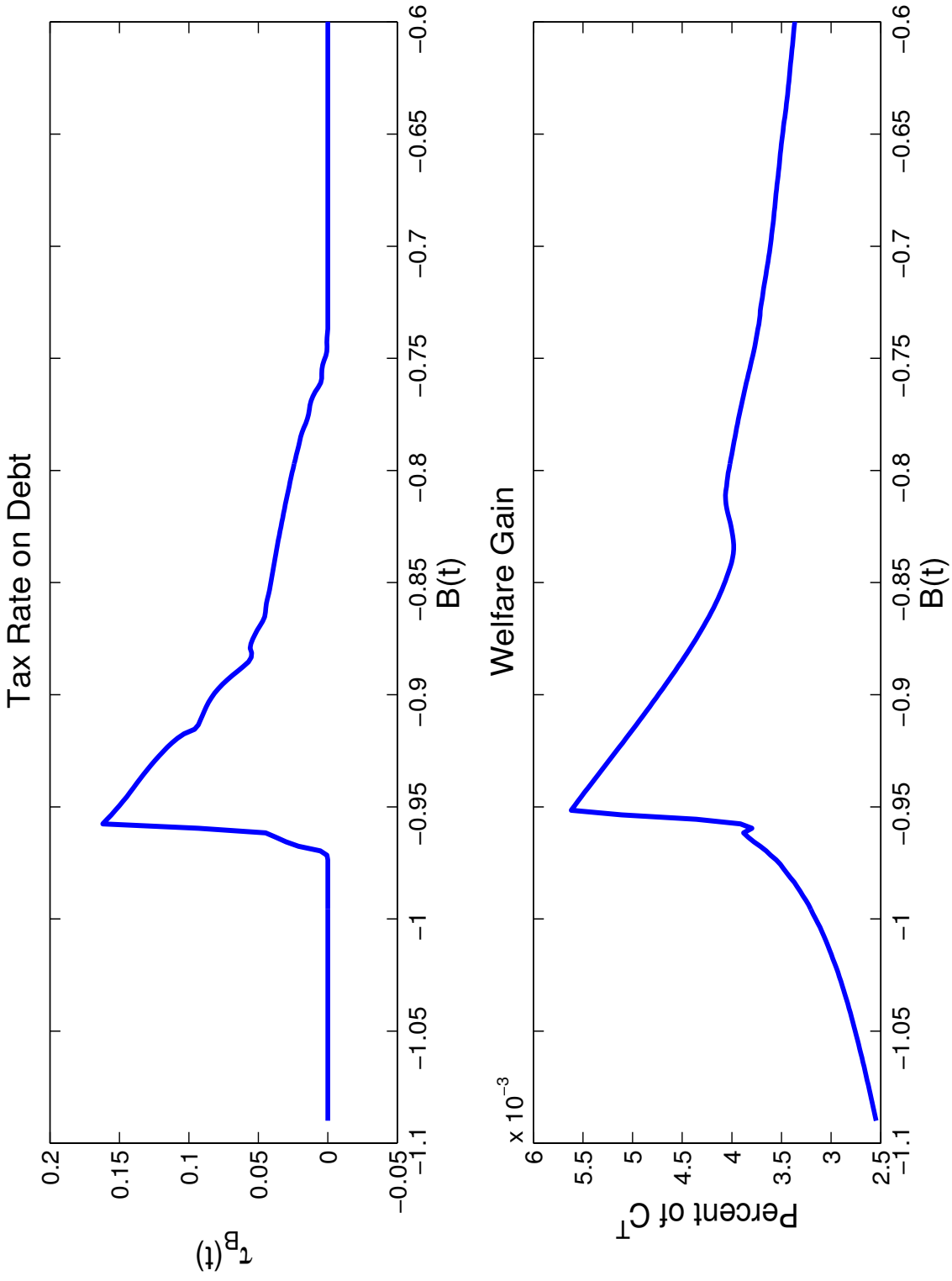
Notes: CE denotes the competitive equilibrium allocation; SP the social planner allocation; UE the unconstrained equilibrium; OP the optimal policy equilibrium with both debt tax and nontradable consumption tax. The table reports ergodic means (in percent). Welfare gains are relative to the CE and are measured in unit of tradable consumption.

Figure 1: Alternative Allocations: Decision Rules



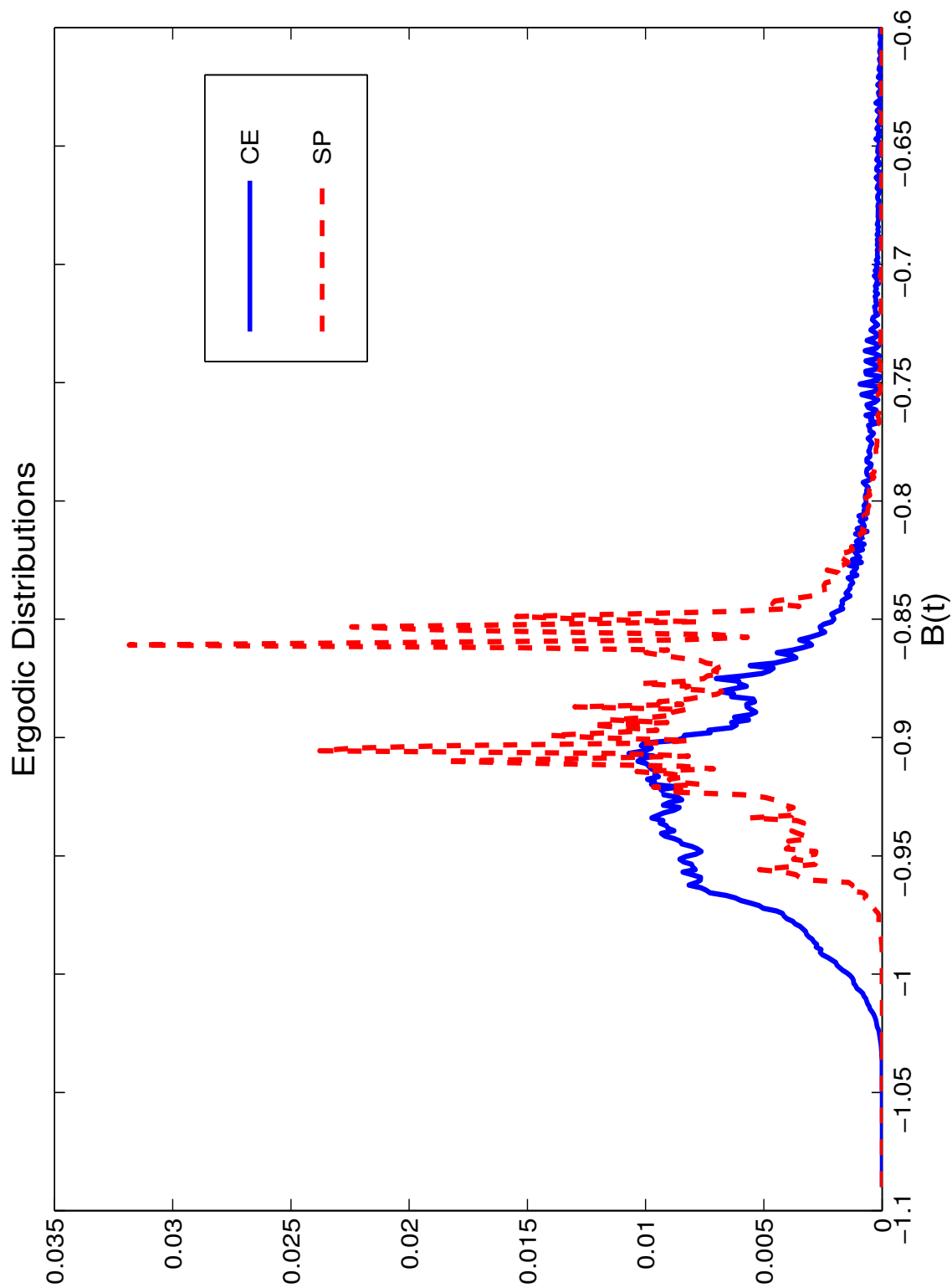
Notes: CE denotes the competitive equilibrium allocation; SP the social planner allocation; UE the unconstrained equilibrium. The figure plots the equilibrium decision rules or policy functions of the endogenous variables plotted conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 2: Optimal Capital Control Policy



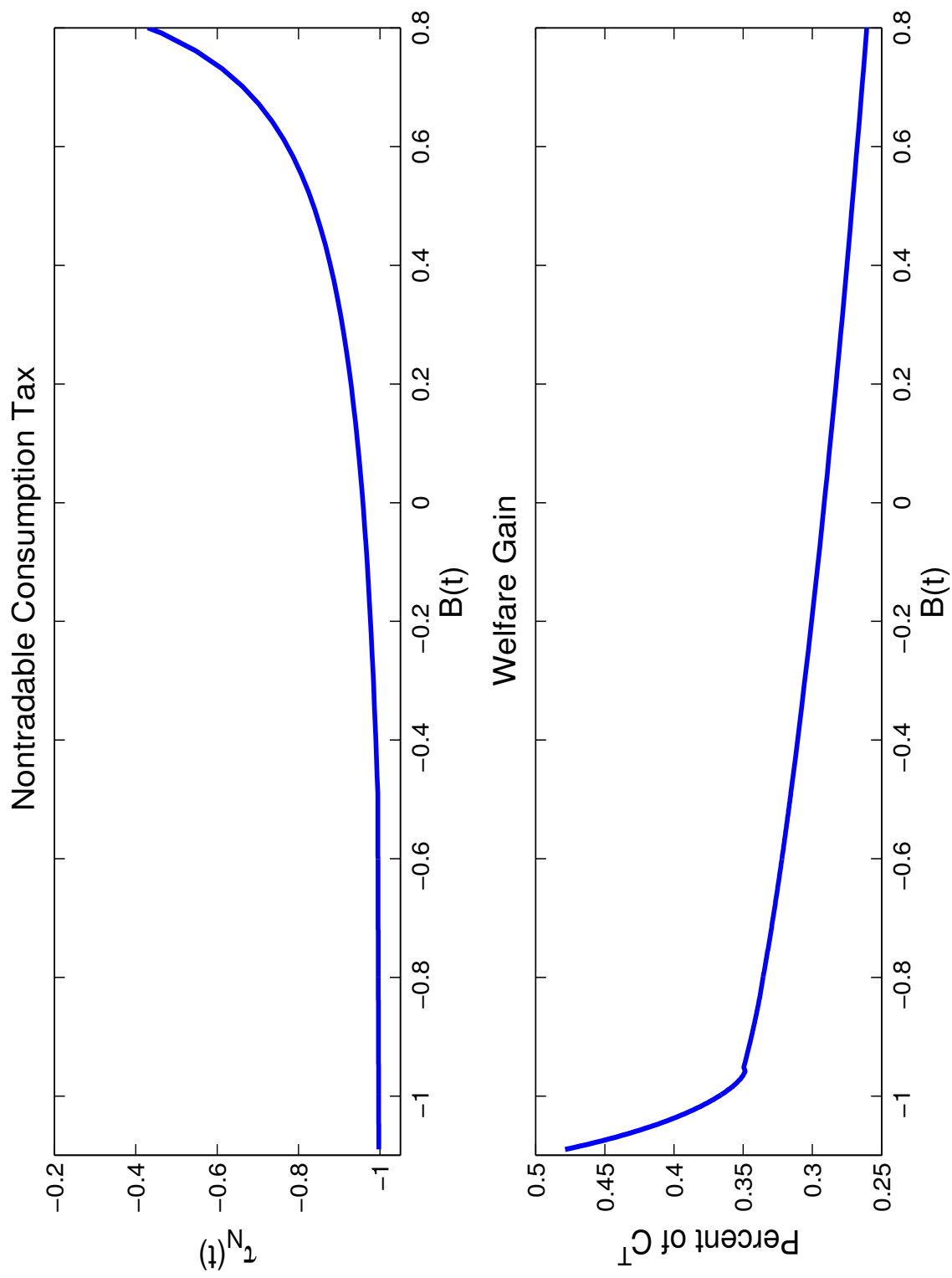
Notes: The figure plots the optimal debt tax rate and the associated welfare gain relative to the competitive equilibrium conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 3: Optimal Capital Control Policy



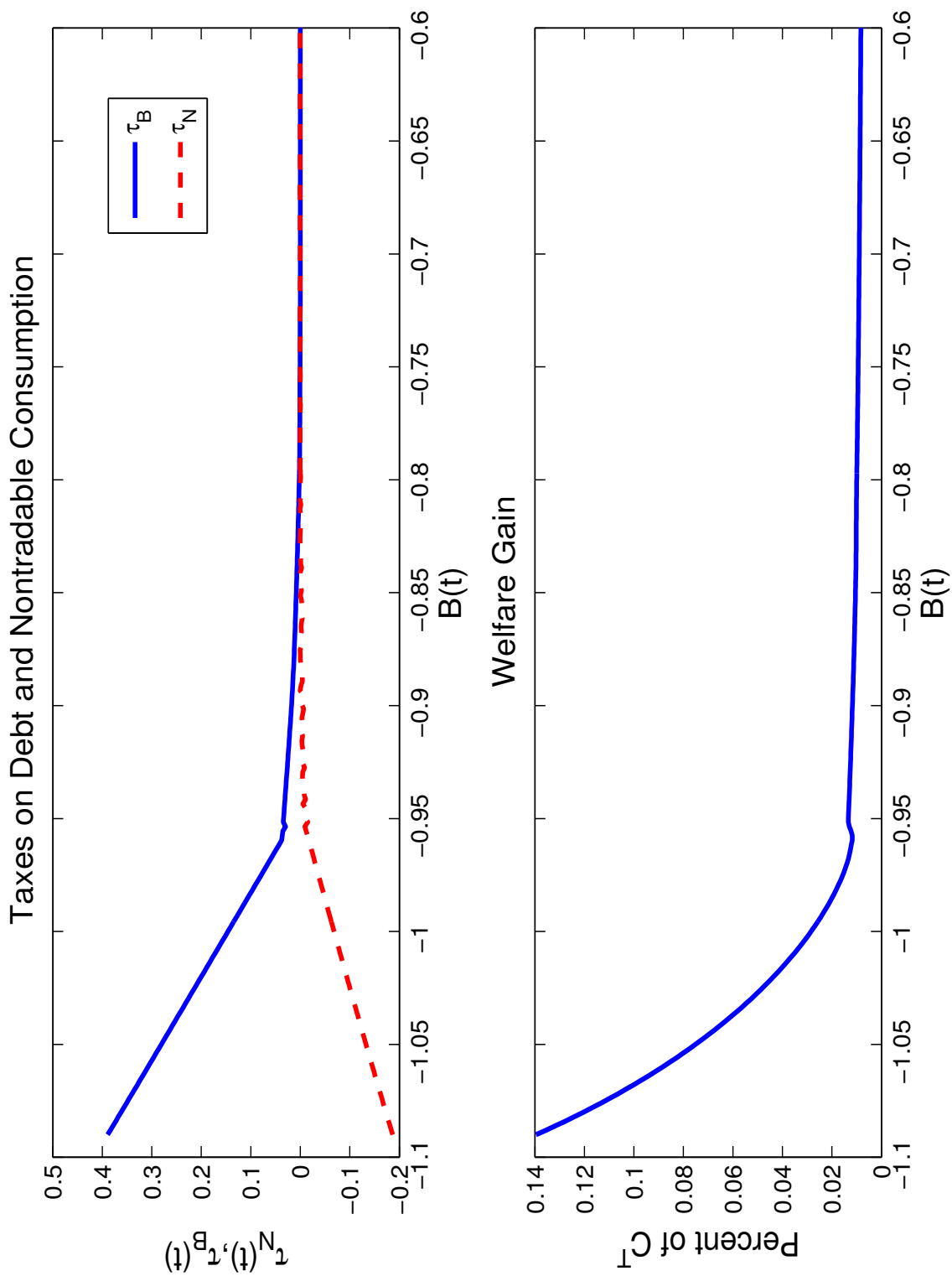
Notes: The figure plots the ergodic distribution of debt in units of tradable consumption in the competitive equilibrium (CE) and the social planner allocations (SP). Borrowing decreases from left to right on the x-axis.

Figure 4: Optimal Exchange Rate Policy



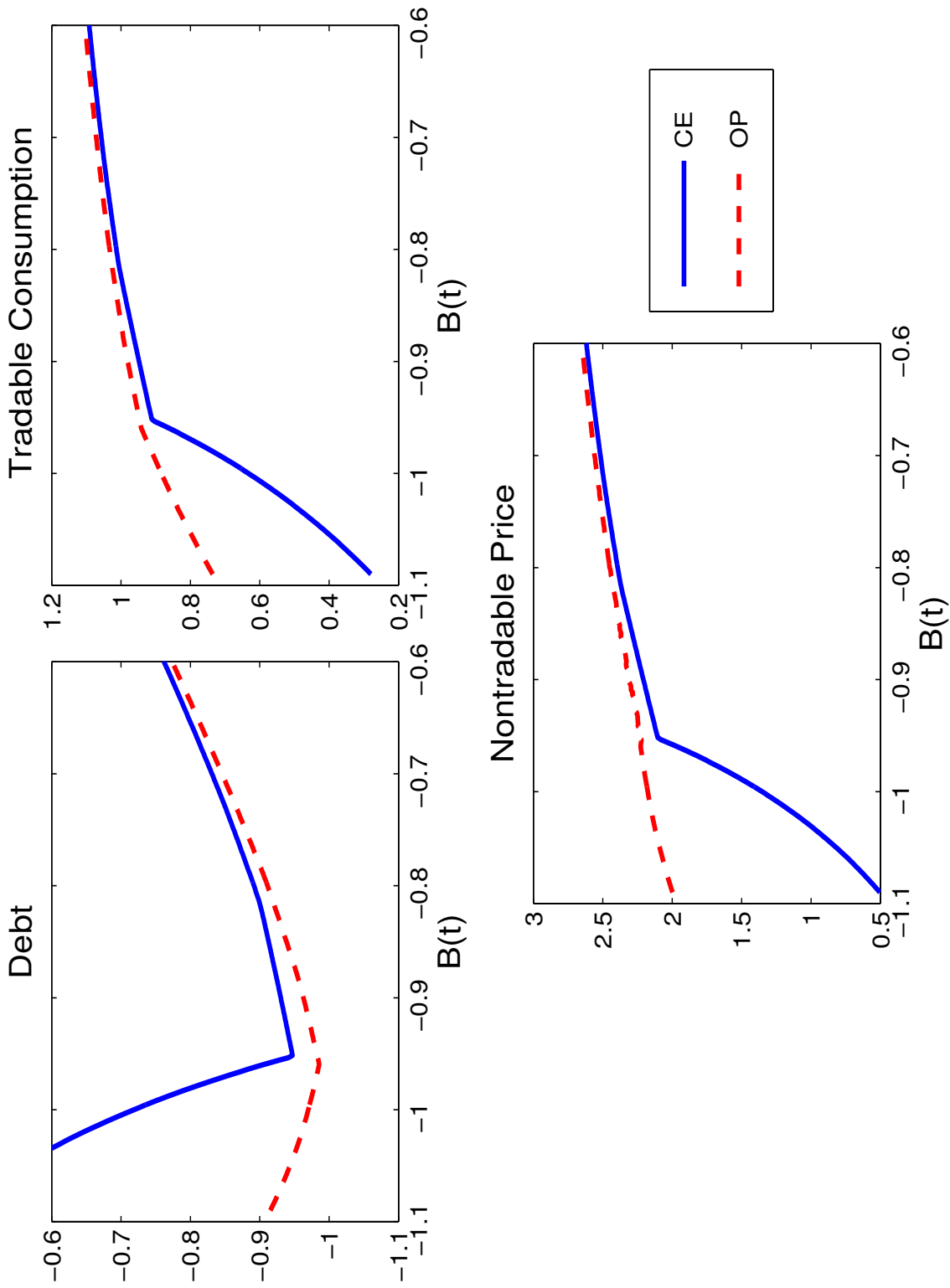
Notes: The figure plots the optimal nontradable consumption tax rate and the associated welfare gain relative to the competitive equilibrium conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

Figure 5: Optimal Capital Control and Exchange Rate Policy



Notes: The figure plots the optimal debt and nontradable consumption tax rates and the associated welfare gain relative to the competitive equilibrium, conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

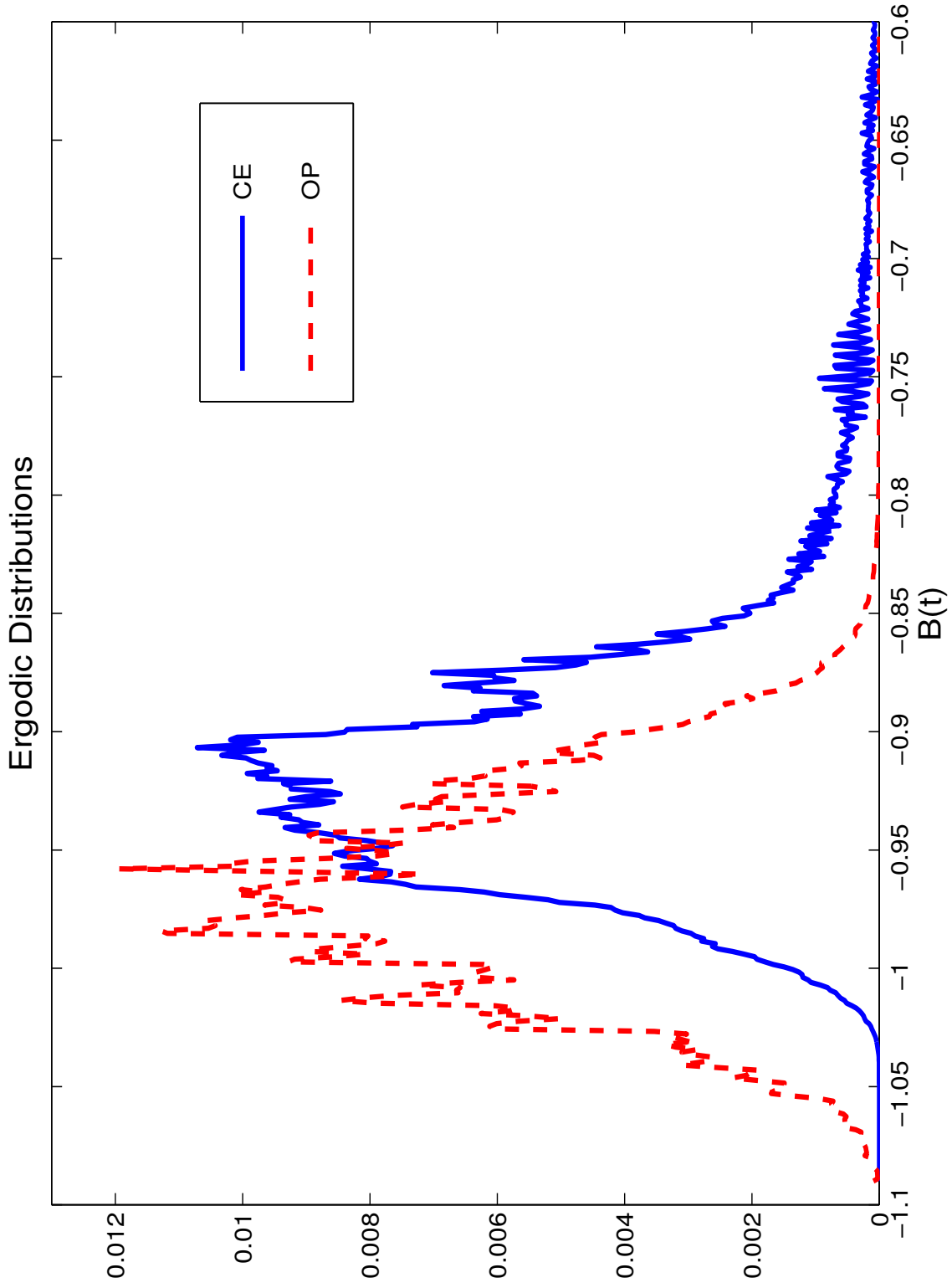
Figure 6: Optimal Capital Control and Exchange Rate Policy: Decision Rules



Notes: CE denotes the competitive equilibrium allocation; OP the optimal policy equilibrium with both debt tax and nontradable consumption tax. The figure plots the equilibrium decision rules of the endogenous variables plotted conditional on one-standard deviation shocks. Borrowing decreases from left to right on the x-axis.

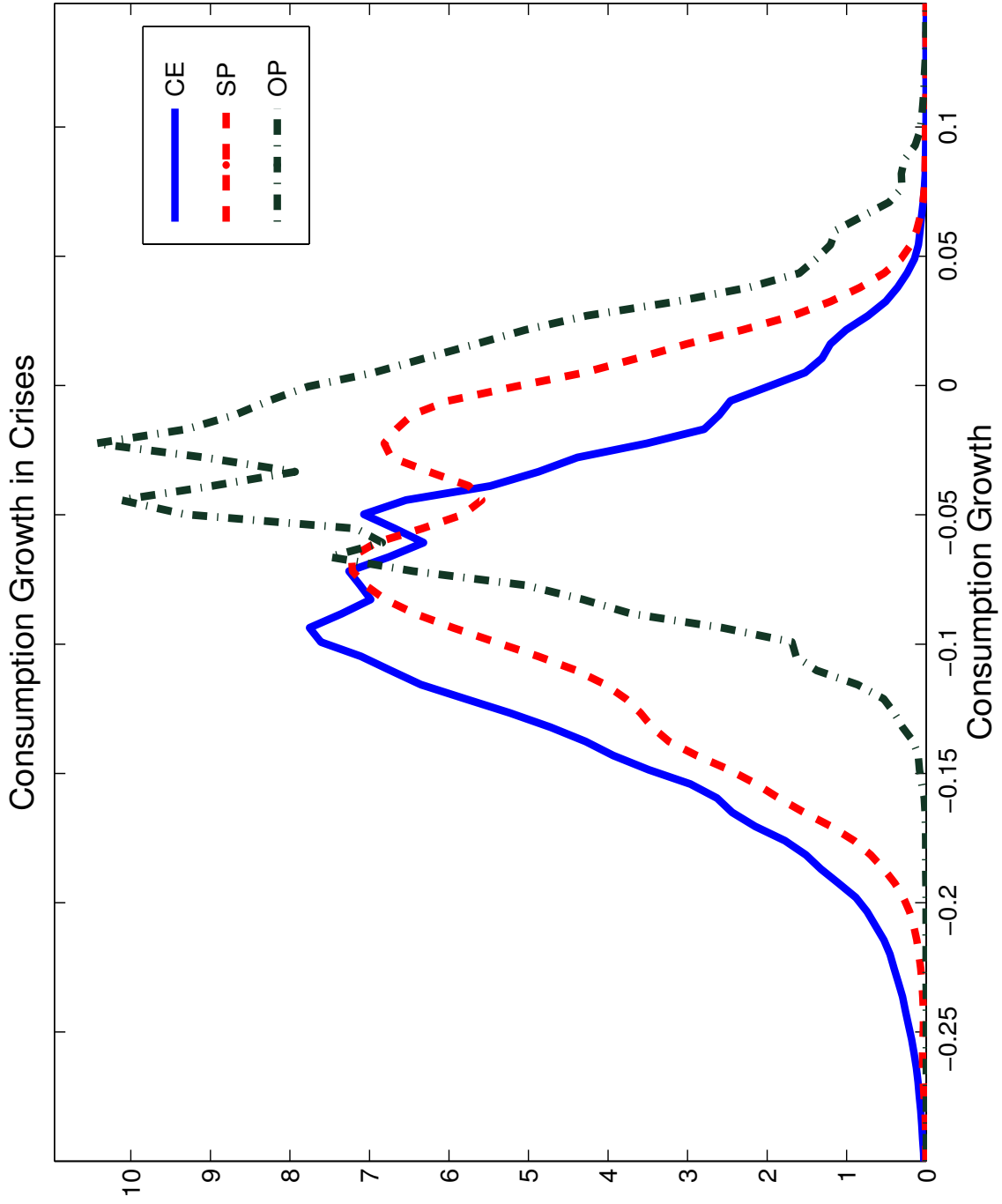


Figure 7: Optimal Capital Control and Exchange Rate Policy



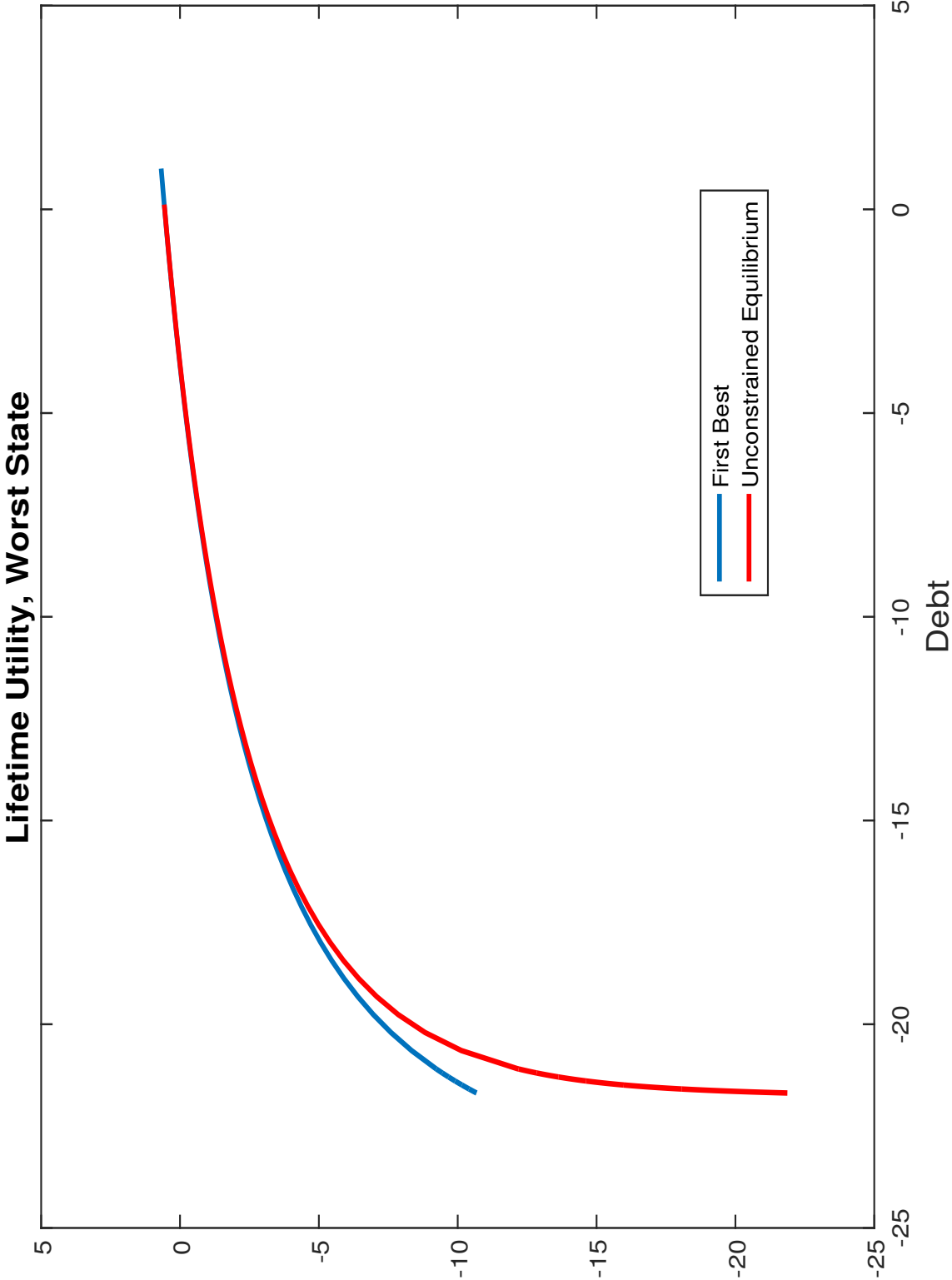
Notes: The figure plots the ergodic distribution of debt in units of tradable consumption in the competitive equilibrium (CE) and the optimal policy equilibrium with both debt tax and nontradable consumption tax (OP). Borrowing decreases from left to right on the x-axis.

Figure 8: Comparing Policy Regimes



Notes: CE denotes the competitive equilibrium allocation; SP the social planner allocation; OP the optimal policy equilibrium with both debt tax and nontradable consumption tax. The figure plots the ergodic distribution of consumption growth in the period after the constraint was binding. Borrowing decreases from left to right on the x-axis.

Figure 9: Appendix Figure 1.A



Notes: The figure compares the utility of the representative agent in the case of worst shock in the complete market and the unconstrained allocations.