The coevolution of segregation, polarised beliefs and discrimination: the case of private vs. state education

The Coevolution of Segregation, Polarised Beliefs and Discrimination: The Case of Private vs. State Education

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Abstract: In this paper we analyze the coevolution of segregation into private and state schools, beliefs about the educational merits of different schools, and labour market discrimination. In a dynamic model, we characterize a necessary and sufficient condition on initial levels of segregation and beliefs under which full polarisation of beliefs and long run labour market discrimination are sustainable. The model suggests a new perspective on the long term effects of different policy interventions, such as integration, school vouchers and policies that are directly targeted towards influencing beliefs.

1 Introduction

In a 2013 YouGov survey, when asked: "Do state secondary schools give talented children a good education and allow them to achieve their full potential?", 58% of state school parents in the UK replied yes compared to only 48% of private school parents. Such beliefs also exhibit big differences across different political affiliations, socioeconomic status, and geographical locations. As in the UK, a US 2014 Gallop poll shows a 10% difference in the share of state school parents and private school parents who believe that state schools provide sufficiently good education.[2] A strong relation between polarised beliefs and individuals' background comes up in surveys on other issues. Golub and Jackson (2012), who study the effect of segregation on belief polarisation, cite polling data that shows that "In October 2004, 47% of Republican poll respondents believed that Iraq had weapons of mass destruction just before the 2003 invasion of that country, as opposed to only 9% of Democrats...This kind of disagreement occurs on many other important factual questions—for instance, whether temperatures on Earth are increasing over time."

The fact that segregation across networks -be it of schools, neighborhoods, political associations, friendships etc.- may affect and polarise beliefs, has received recent attention in the literature. Dustmann and Preston (2001) analyze how segregation in neighborhoods

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[2]See http://pdkintl.org/programs-resources/poll/ for the Annual PDK/Gallup Poll of the Public's Attitudes Toward the Public Schools.

affects attitudes towards minorities. Algann *et al* (2015) show that students' political opinions converge among friends, and Boisjoly *et al* (2006) and La Ferrara *et al* (2014) show that living in mixed-race housing lowers students' prejudiced beliefs. Kaufmann and Harris (2015) study the rise in segregation along racial lines in the UK and find significant effects of segregation on attitudes about the benefits of immigration.[3]

In explaining the relation between segregation and beliefs, the literature has mainly focused on one direction of causality, namely, the affect of segregation on beliefs and attitudes. However, while segregation can affect beliefs, it is important to take into account that segregation is often endogenous and may in itself depend on these beliefs. In particular, beliefs play an important role in many economic decisions that affect segregation in society. When people choose where to live, beliefs about crime rates and demographic compositions of different neighborhoods are important.[4] Indeed school choices -our main application in this paper- tend to be persistent and affected by parental background and beliefs. Evans and Tilley (2011) find that 43 per cent of the privately educated in the UK who have children have sent them to private schools, nearly five times the rate for parents who went to state schools.

To better understand segregation and its effect on beliefs, one has therefore to analyze the *coevolution* of segregation and beliefs. In this paper we analyze the coevolution of segregation in schools (private versus state), beliefs about the productivity of different school graduates, and the effect of this on labour market outcomes.[5] Schools are important in shaping beliefs as they involve intense socialization, where beliefs are formed and moulded by peers and by teachers. Moreover, in many countries, communities are segregated by the school choices of parents.[6]

[3]There is mounting evidence that segregation in western countries is on the rise. Reardon and Bischoff (2011) show the increased segregation in the US from 1970-2009. Chetty et al (2014) provide a snapshot of segregation in the US and its effect on social mobility. See also the Pew Research centre report on http://www.pewsocialtrends.org/2012/08/01/the-rise-of-residential-segregation-by-income/. Gentzkow and Shapiro (2011) compute an isolation index and show how face-to-face segregation -in voluntary associations, work, neighborhoods, family, trusted friends and political discussants- is much more significant as compared to the segregation implied by online and offline news consumption.

[4]Dustmann and Preston (2001) show that earlier studies that have only looked at one direction of causality, i.e., how segregation and social exclusion affect beliefs and attitudes towards minorities, have biased results due to neglecting location choices which depend on these beliefs.

[5]Private schools are termed independent or public in the UK, whereas state schools are termed public schools in the US. To avoid confusion we then use the terms private and state schools.

[6]One example is the "white flight" to the suburbs in the US which is considered to be motivated by the possibility of racial and socioeconomic segregation in schools. See Bradford and Kelejian (1973). More recent evidence on sorting by income driven by school choice is provided in Epple and Sieg (1999) and Calabrese, Epple, Romer and Sieg (2006). For efficiency implications of this, see Calabrese, Epple and Romano (2012).

Importantly, the different attitudes and beliefs about school graduates have real behavioral implications in labour markets, through occupational choice and employment decisions. For example, in the UK, private school male graduates are up to 10 per cent more likely to land top jobs than state school graduates with the same grades from the same university.[7] Indeed, 50% of private school students believe that people that attend their school will be 'very successful' in the world of work compared to 9% of state school pupils.[8] Similarly, in the US, private schools lead the tables in terms of placements into top universities, even though students from private schools or selective state schools perform no better than those in standard state schools in achievements tests.[9]

We analyse a model of non-overlapping generations with infinite periods and three stages in each period. In the *peer influence* stage, segregation affects beliefs: In this stage, individuals' beliefs about schools are shaped by their parents' beliefs and by their school peers. In the *labour market* stage, discrimination may arise based on such beliefs: Employers decide whether to hire an employee, given the school he graduated from and their own beliefs about the schools' effect on productivities. Labour market experience also entail learning about true productivities. In the *school choice* stage, beliefs and labour market discrimination affect segregation choices: In this stage, parents choose which school -state or private- to send their offspring to. The private school admits a fixed share of the population with the highest willingness to pay for the school.

Our analysis relies on two important "behavioral" assumptions. First, we assume that pupils in schools have selection bias.[10] In particular, we assume that individuals exchange information only with those in the same school and neglect to take into account that the selection into the school depends on beliefs.

Note that selection bias is not sufficient to create polarised beliefs as this depends on how people select into schools. Our second assumption will imply homophily, which together with selection bias will create the "echo chamber" effect. Specifically, we assume that parents decide on the school for their child by using "imperfect empathy", as in Bisin and Verdier (2001).[11] Parents base their decisions on their expectation about labour

[7] These facts are taken from a recent study by the Social Mobility and Poverty Commission in the UK, which tracked 20,000 students. There is also evidence that state school graduates outperform compared to the expectations about them, which is consistent with initial discrimination due to wrong beliefs and subsequent adjustment of beliefs due to learning. See a 2014 report by HEFCE.

[8] https://yougov.co.uk/news/2016/02/02/state-school-children-believe-privately-educated-p/

[9] See the 2007 study of the Centre of Education Policy, using NELS data from 1988-2000, which takes family background into account. See also Abdulkadiroğlu et al (2014) and Dobbie and Fryer (2014) who show that peers with high achievements or a school racial mix have no effect on pupils' attainment.

[10] For a recent experimental paper showing how people's choices are affected by selection bias see Enke (2016).

[11] The imperfect empathy assumption is also related to papers looking at choices of beliefs in social

market discrimination, but also on their expectations about how the school will affect their child's future beliefs and behaviour. Parents understand that the environment will influence their offspring's beliefs but think that the optimal course of behaviour is the one that follows their own beliefs (hence empathy is imperfect). This creates homophily, i.e., parents would rather their children segregate with like-minded others so that their child's belief does not stray too much from their own.[12] This endogenous homophily, along with selection bias, will imply that beliefs can become polarised.

We are interested in the following questions: Are segregation and polarised beliefs sustainable in the long run? That is, if some in society overestimate the productivity differences between state and private schools, do they indeed segregate so that beliefs in society become heterogenous and polarised in the long term? Can such echo chamber effects overcome true learning in the labour market? What factors might unravel segregation and discrimination in society?

We find a simple necessary and sufficient condition that characterises when segregation, polarised beliefs and discrimination persist in the long run. When the condition is satisfied, in all equilibria, there are polarised beliefs about the productivity of graduates from the different schools (over and above actual productivity differences). Parents who send their children to a private school believe that the difference between the schools is higher than what it really is. Parents who send their children to a state school realize that there is discrimination, believe it is not justified, and are priced out of the private school. Finally, those who went to the private (state) school will also send their children to the private (state) school. Thus, the "old boys" network is endogenously formed.[13]

The condition identifies environments in which the echo chamber effect outweighs true learning.[14] First, history matters; to create long run segregation and polarised beliefs, those in the private school have to start from a relatively low opinion about state school graduates. Second, the higher is the intensity of socialization in schools the easier it is to create segregation and polarisation. Finally, polarised beliefs are easier to sustain the

interactions. Benabou and Tirole (2006) describe a model in which individuals (or parents) censor information so as to distort their future selves' (or children's) beliefs. Levy and Razin (2012) model the choice of whether to be religious or secular as a choice that affects future beliefs. Benabou (2013) shows that the choices of individuals to ignore information depend in equilibrium on others' choices and on the complementarities of their actions with others.

[12] For other approaches to homophily see Baccara and Yariv (forthcoming) and Peski (2008).

[13] Network effects have also been mentioned as a possible reason for discrimination. Our analysis endogenises such networks. See Granovetter (1974), Marmaros and Sacerdote (2002), and Simon and Warner (1992).

[14] Wrong beliefs arise in our model not because individuals stop "experimenting", as in Piketty (1995) or Fudenberg and Levine (1993), but because their peers' beliefs are pessimistic enough and thus the echo chamber effect outweighs any positive learning.

less individuals learn about others from their labour market experience.

Importantly, the condition we identify is also necessary for segregation, implying that the cycle of segregation and polarised beliefs can be broken down. This arises in our model when those who segregate in the private school have sufficiently mixed beliefs so that belief polarisation cannot arise.

Our key contribution in this paper is to analyze how segregation and beliefs, along with discrimination, evolve endogenously.[15] A recent literature in economics highlights the role played by segregation in shaping beliefs in society, possibly through an echo chamber effect. For example, Golub and Jackson (2012) study how segregation affects information diffusion in social networks when individuals use the DeGroot heuristic to update their beliefs.[16] They show that homophily slows down the convergence to a consensus in society. Implicit in their analysis are selection bias and a correlation neglect bias that arises from repeated communication.[17]

Our model highlights the fact that what is important for polarised beliefs across society is that those with similar beliefs will segregate. In our model endogenous segregation can result in homophily, and thus, given selection bias, induces polarised beliefs which then further segregation. Related to this is a paper by Sethi and Yildiz (2016) who show how beliefs can polarise when individuals choose which experts to listen to. In their model one chooses a "familiar" expert from whom they can easily learn, whereas in our model an individual chooses their network of individuals with similar beliefs so that their offspring's belief does not stray away too much from their own. We also show how such polarisation in beliefs can have a direct and dynamically persistent effect on economic outcomes such as wages and employment.[18]

Our second contribution is to provide a model of socialisation, or peer influence, which relies on individuals learning from one another. The notion that physical proximity affects

[15] Benabou (1993) and Epple and Romano (2008) focus on the relation between segregation in education and complementarities: Benabou (1993) shows how segregation inefficiently arises due to complementarities in education, and Epple and Romano (2008) use complementarities in education and income to derive segregation between state and private schools. We differ from this literature by focusing on beliefs as the key element that fuels -and is fuelled by- segregation.

[16] See also Dandekar et al (2013).

[17] See also De Marzo, Vayanos and Zwiebel (2003) and Glaeser and Sunstein (2009) for theoretical models of echo chamber effects stemming from correlation neglect. See Enke and Zimmerman (2015) for an experimental analysis of correlation neglect. In our model there is no correlation neglect. The naive social learning literature has also focused on the effects of non-Bayesian learning on whether societies converge to have the right or wrong beliefs. See Eyster and Rabin (2010) and Gagnon-Bartsch and Rabin (2015).

[18] Polarisation of beliefs has also been considered in the political economy literature, namely its effect on political outcomes. See Barber and McCarty (2013) and Levy and Razin (2015).

beliefs and social attitudes has been traditionally studied in sociology, psychology and criminology and has been coined the Contact Hypothesis by Allport (1954). In this spirit, our model also provides a rationale for school integration, over and above the one explored in the economics literature (which focuses mainly on academic achievement through peer effects):[19] In our model successful integration will change wrong stereotypes and potentially unravel the cycle of wrong beliefs, stereotypes and discrimination in the long-run.

The literature has provided many explanations for labour market discrimination. Statistical discrimination models follow either Arrow (1973), who focuses on self-fulfilling beliefs, or Phelps (1972) who focuses on noisy information.[20] In all these models individuals' beliefs are correct at least on average. Our approach is different: We show how, even in the absence of differences, individuals may strongly believe that such differences exist. We provide a specific mechanism -endogenous segregation and the echo chamber effect- to illustrate how wrong beliefs overcome true information in the long run. Also in this literature, Austen-Smith and Fryer (2005) and Chaudhuri and Sethi (2008) show how investment in skills changes as a result of peer effects in segregated neighborhoods.[21] Our analysis is complementary: We endogenise segregation, and allow *beliefs* to be what segregation exacerbates.[22]

We also add to the discrimination literature by shedding some light on common practices in school and university admissions. Our condition identifies environments in which, when private schools admit pupils according to their willingness to pay, they will not necessarily survive in the long-run. Alternative selection criteria, based on beliefs, values or culture, will allow such schools to increase their long term survival. This accords with a common practice of many private schools and universities in the UK and in the US which hold interviews, often with parents as well, or have a legacy criterion for admission.

Note also that in our model discrimination is according to an acquired trait, such as the

[19] See for example Sacerdote (2011)

[20] For a survey on models of statistical discrimination see Fang and Moro (2010). Coate and Loury (1993) and Moro and Norman (2004) are recent examples that follow Arrow (1973) to show how discrimination or segregation arise due to self fulfilling asymmetric equilibria. Fryer and Jackson (2008) is a recent example of a categorization model which can rationalize the use of stereotypes. Black (1995), and Lang, Manove and Dickens (2005) analyze labour market discrimination with wrong beliefs, which can result from noisier information. Peski and Szentes (2013) analyze "spontaneous" discrimination which arises from repeated game considerations, without beliefs on productivity.

[21] Calvo-Armengol and Jackson (2004) show how information about job opportunities flows in some networks but not in others, resulting in discrimination. This is also related to the literature on identity, as in Akerlof and Kranton (2000).

[22] Also related to self-fulfilling beliefs is the work on "social assets" by Mailath and Postlewaite (2006). In their work as in ours private education becomes valuable when others value it.

school one had attended. The analysis can be extended to also include inherited traits such as ethnicity, race or gender. We discuss these issues in Section 5.

2 The model

We consider a non-overlapping generations model with a continuum of dynasties, each indexed by $i \in [0,1]$. Each dynasty consists of one individual at any period $t \in \{1,...\}$, which at the end of the period is replaced by one offspring.

There are two kinds of schools, state and private. The focus of our analysis will be on how (endogenous) segregation in different schools can foster polarised (and hence sometimes wrong) beliefs about the productivity of the graduates of the different schools. For simplicity, we assume that whatever the school an individual goes to, his actual productivity is the same and equal to one. We extend the analysis to different productivities later on. For the purpose of our analysis it is not important whether productivity arises through innate ability, standard peer effects, or through quality of teaching.[23]

We consider a simple form of beliefs. Assume that all dynasties believe that the productivity of private school graduates is 1. They also believe that the productivity of a state school graduate is either 0 or 1. Specifically, at any date $t \geq 1$, each dynasty holds a belief that the productivity of graduates of state schools is 1 with probability $q_i^t \in (0,1]$ and 0 otherwise, with some $F^t(.)$ describing the distribution of $q_i^t \in (0,1]$ in the population. The beliefs of different dynasties will change within periods and across time as we describe below, and we will analyse where beliefs and behaviour will converge to.

At any period t, all individuals go through the following phases:

Stage 1: *Peer influence in schools.* Individuals start school with the belief q_i^t inherited from their parents. They communicate with peers and update their beliefs.

Stage 2: *Labour market.* School graduates become employees or employers and are randomly matched. Employers observe the schooling history of employees and make employment decisions given their beliefs. Employees and employers receive wage/profits respectively. Labour market experience is potentially informative about the productivity of state school graduates.

Stage 3: *Parental school choice.* Following labour market experience, individuals have one offspring each, to which they transmit their beliefs. They then choose a school for their offspring, private or state, to maximize their perceived offspring's payoff in the labour market.

[23] There is a large literature on sorting in schools according to parental income and innate ability, which can potentially affect productivity. See for example Epple and Romano (1998). MacLeod and Urquiola (2009) shows that even if private schools select a higher ability students, they then might invest lower effort so that the value added of the private school is low and overall productivity will not be much higher.

Note that individuals receive utility in the employment phase, in the form of wages or profits, and when considering their offspring's wages and profits in the next period. Attending school does not in itself entail any utility.

We now describe the specific stages of the model.

2.1 Peer influence in schools

Our key assumption is that there is peer influence in schools and that pupils have "selection bias" when updating their beliefs based on what they learn in school.[24] To model peer influence, we consider a simple communication model in which individuals -in the same school- transmit their beliefs q_i^t truthfully to each other.[25] Specifically, let $f_J^t(.)$ denote the distribution over the inherited beliefs of pupils in school $J \in \{s, p\}$ at period t. We assume that individual i in school J is randomly matched and exchanges beliefs with n others in the same school. Sobel (2014) and Levy and Razin (2016) show that if individuals believe that they had all started with a common uniform prior (on $\{0, 1\}$), and that their posterior beliefs q_i^t were formed by receiving private conditionally independent signals, then these posterior beliefs are sufficient statistics for rational Bayesian updating. We can therefore describe the evolution of beliefs of an individual i in school J at time t, from q_i^t, before he interacts with others, to $q_{i,g}^t$, after communicating with n peers and graduating, according to the following process (for the proof that Bayesian updating results in the formulation below, see Sobel 2014 and Levy and Razin 2016):

$$q_{i,g}^t \equiv \chi(\mathbf{q}^t, q_i^t) = \frac{q_i^t \prod_{k=1,..,n} q_k^t}{q_i^t \prod_{k=1,..,n} q_k^t + (1 - q_i^t) \prod_{k=1,..,n} (1 - q_k^t)}, \tag{1}$$

where $\mathbf{q}^t$ is a vector of beliefs $(q_1^t, ..q_n^t)$ of length n, each belief drawn from $f_J^t(.)$.

Note that individuals have a selection bias: They only learn from those that they interact with while not taking into account that those who chose to attend the other school may have different beliefs. This will play an important role if the school choice

[24] Peer influence occurs among individuals in the same school only. As in the network literature which assumes that individuals communicate with those they are connected to, in our model as well information exchange arises only among those that interact with one another. An important difference however is that in our model the "network", i.e., the identity of the individuals in the different schools, will be endogenously determined by the parents, given their beliefs.

[25] While individuals exchange beliefs over the productivity of state school graduates, this can be more generally thought of as exchanging views that "our school is better than other schools", or that "attending a private school is the only way to succeed in life". We take the view therefore that exchanges between individuals consist of transmission of beliefs, rather than signals, and, as in the social learning literature, that such transmission is sincere.

decision leads to segregation of beliefs. Apart from selection bias, the belief updating process follows from Bayesian updating assuming conditional independence in the *initial* beliefs. That is, there is no correlation neglect in the model.[26] Denote by $f^t_{J,g}(.)$ the distribution of beliefs of graduates from school J.

It is easy to see from (1) that the belief updating function satisfies the following properties. First, confident individuals are persuasive. For example, if some individual has extreme beliefs at one or zero, then he fully convinces all others. Second, beliefs upon graduation are monotone; they increase in the parent's beliefs and in peers' beliefs. Finally, belief updating can also exhibit *polarisation*: For a set of beliefs where all are higher (lower) than a half, then updated beliefs would be higher (lower) than the maximum (minimum) belief in the set. We will show later on that dynasties with pessimistic beliefs will tend to segregate together; the feature of polarisation will then facilitate the echo chamber effect as an agglomeration of pessimistic beliefs will overcome true learning in the labour market.

2.2 The labour market: employment and learning

In this model we show that polarisation and wrong beliefs can have real economic consequences, and specifically, we focus on labour market discrimination. As in Black (1995), discrimination due to wrong beliefs will typically be characterised by losses for employers and employees alike. State school employees will be discriminated against, whereas employers with wrong beliefs will incur loses due to suboptimal behaviour, e.g., they forgo opportunities to employ. We focus on the simplest labour market interaction that delivers these two features, but our results hold for a more general environment.

Specifically, let school graduates at time t make up the two sides of the labour market. Any individual, disregarding her education path, becomes an employer with probability γ and an employee with probability $1-\gamma$ (the analysis can be extended to asymmetric probabilities). Employers and employees are matched randomly. To fix ideas, let $\gamma < \frac{1}{2}$, so an employer is matched for sure, while an employee is matched with probability $\frac{\gamma}{1-\gamma} < 1$ (this is not important and can be reversed).

We assume that an employee always prefers to work, while the employer decides whether to employ the individual he is matched with. The gain for the employer is the productivity of the employee, and the cost is a wage w (paid to the employee) drawn independently for each match according to a uniform distribution on $[0,1]$.[27] The randomness of the

[26] Note that given the continuum of dynasties, two dynasties will meet with probability 0. Thus, conditional on initial beliefs being derived from independent sources, all future exchanged beliefs would still be conditionally independent.

[27] The uniformity of the distribution is not essential for the results but simplifies exposition. The distribution needs to be full support.

wage is a simple way to model employers with wrong beliefs taking suboptimal actions. In particular, an employer who is matched with a private school graduate, about which all have correct beliefs, will always employ as $w \leq 1$. On the other hand, an employer from dynasty i will employ a state school graduate employee at time t only if $w \leq q_{i,g}^t$. If an employer decides not to employ, no profits or wages can be earned for that period. Thus, whenever $q_{i,g}^t < 1$, the behaviour of the employer is suboptimal as the true productivity of the worker is 1. In the Appendix we extend the analysis to an alternative labour market model in which wages are determined by Nash bargaining.

To stack the odds against wrong beliefs, we assume that the labour market experience entails learning: Following the employment phase, all those who were either employed by or employed a state school graduate, receive a signal that increases their beliefs. To simplify the information structure, assume that individuals believe that the signal indicates that the productivity of the employee/employer is 1 with accuracy $\tau \in (0.5, 1)$. Post employment, individuals then update beliefs to $q_{i,e}^t$, which, by Bayes rule, is given by,

$$q_{i,e}^t = \frac{q_{i,g}^t \tau}{q_{i,g}^t \tau + (1 - q_{i,g}^t)(1 - \tau)} \text{ if } i \text{ was in an active match with a state school graduate,} \tag{2}$$

and $q_{i,e}^t = q_{i,g}^t$ otherwise. Denote by $f_e^t(.)$ the distribution of beliefs in the population as a whole following the employment activity at time t.

2.3 Parental school choice

Following employment, individuals become parents, transmit their beliefs to their children, i.e., $q_i^{t+1} = q_{i,e}^t$, and decide which school to send them to. Let Δ_i^t be the willingness to pay of a parent from dynasty i for the private school. In other words, Δ_i^t is the perceived future payoff of the child in the labour market in period $t+1$, conditional on attending the private school, relative to that payoff conditional on attending the state school. The parent will calculate this expected labour market payoff from each school given her forecast of how the offspring's beliefs will change in each school, and her forecast of the behaviour of employers in period $t+1$. Importantly, the parent would compute Δ_i^t given what she believes to be the "true" expected state school productivity, $q_{i,e}^t$. Note that a parent here only cares about her direct offspring.

The willingness to pay Δ_i^t (which we will derive formally in the next Section) will consist of two components. First, in the model, state school graduate employees may be discriminated against. If parents foresee this correctly, they will -disregarding their own beliefs- prefer to send their child to a private school. Second, parents may understand that their child's belief will change which will affect his behaviour as an employer. Parents will therefore prefer to send their children to the school that will induce the *best beliefs*

from their own point of view because they are concerned about their offspring potentially making a hiring error in the labour market. This assumption, of "imperfect empathy", implies that they would prefer their offspring's beliefs to stay as close as possible to their own. Such induced homophily would facilitate belief polarisation.

Note that even though parents realize that their offspring will be influenced by others' beliefs, the parents themselves do not update their beliefs anymore. Imperfect empathy implies "old age rigidity" of beliefs. This phenomenon is recently documented in the literature (see Ortoleva and Snowberg 2015). In our model, young individuals are influenced by the environment they grow up in, but when they grow old and after they have accumulated job market experience, parents believe that their assessment is the correct one. They are aware however that the environment they choose to raise their children in (that is, the school they send them to), will affect their children's beliefs and hence their future behaviour.

We model "school choice" in the following simple way: We assume that at any time t, the private school admits a share ρ of individuals with the largest Δ_i^t, conditional on $\Delta_i^t \geq 0$ (and the largest share feasible subject to $\Delta_i^t \geq 0$ if this constraint is satisfied for a share lower than ρ). Thus, the school admits all those with the largest willingness to pay, subject to the school's capacity constraint.

Note that we abstract away from any income differences that might prevent some from attending the private school. Income differences will only exacerbate segregation (see the discussion in Section 4.1). For now, dynasties only differ in their beliefs, which will determine their willingness, but not their ability, to pay for the private school.

2.4 Equilibrium definition

An equilibrium in the infinite game given some initial state in Period 1 (an allocation of a measure ρ of dynasties to the private school) is a dynamic process of school peer influence, labour market behaviour, and parental school choice, in which, at any period t, the following is satisfied:

(i) *Optimal employers' behaviour*: At any period $t = 1, 2, ..,$ employers employ state school graduates only if $w \leq q_{i,g}^t$ and always employ private school graduates.

(ii) *Belief updating:* For each dynasty i, and period $t = 1, 2, .., ,$ q_i^t is updated to $q_{i,g}^t$ according to (1) given the distribution of beliefs $f_p^t()$ and $f_s^t()$ in period $t = 1, 2...,$ and $q_{i,g}^t$ is updated following employment to $q_{i,e}^t$ according to (2).

(iii) *Correct parental beliefs and optimal school choice:* Parents at period $t = 1, 2, ...$ compute Δ_i^t using imperfect empathy and their own beliefs $q_{i,e}^t$, given a correct expectations of how their offspring's beliefs will change in each school, and the correct expectation

of the equilibrium behaviour of employers in period $t+1$.[28] The measure ρ of dynasties with the highest Δ_i^t are admitted to the private school.

Proposition 0: *An equilibrium exists.*[29]

3 Segregation, polarisation and discrimination

Where will society converge to in the long run? To answer this question we need to put some structure on the initial distribution of beliefs and allocation to schools. We consider the following initial state: In Period 1 there is a proportion ρ in the private school with beliefs $q_i^1 = q < 1$ and a proportion $1-\rho$ in the state school with beliefs $q_i^1 = 1$. Given this simple initial state, we will now characterise all long-run equilibria. We then show how the result extends to other initial states, see the remark below.

We start with a benchmark in which there is no peer influence in schooling. This case can be modeled by assuming that peers do no affect each others' beliefs, so that $n = 0$. The next result characterises the long term distribution of beliefs in any sequence of equilibria.[30]

Proposition 1: *Let $n = 0$. In any sequence of equilibria all dynasties converge to have beliefs at $q_i^\infty = 1$.*

When there is no interaction in school, a dynastic belief can only change in the labour market, and this is due to true learning. As any dynasty with wrong beliefs will always learn with a strictly positive probability, all dynasties will converges to have beliefs which put probability one on the truth. Therefore, to sustain long run polarisation of beliefs, we must have peer influence in schools.

Assume now that $n > 0$. Our main result characterizes a simple necessary and sufficient condition on q : When it is low enough society will converge to polarised beliefs, and if it is high enough all will converge to have the same -true- belief.

[28] That is, parents know the present distribution of beliefs $f_e^t(.)$, the equilibrium school choices of other parents (and hence the correct distribution of beliefs in school J in period $t+1$, $f_J^{t+1}(.)$), and have the correct forecast of the distribution of beliefs following graduation from school J, $f_{J,g}^{t+1}(.)$.

[29] All proofs are in the Appendix.

[30] This result holds for any initial state.

Proposition 2: *Let* $n > 0$ *and let*

$$q^*(\tau, n) \equiv \frac{(1-\tau)^{\frac{1}{n}}}{\tau^{\frac{1}{n}} + (1-\tau)^{\frac{1}{n}}}.$$

Then: (i) ***Polarisation****: Whenever* $q \leq q^*(\tau, n)$*, in all equilibria, a constant set of dynasties of measure* ρ *attends the private school, discriminates against state school graduates, and has beliefs that converge to a degenerate distribution on 0. (ii)* ***No Polarisation:*** *Whenever* $q > q^*(\tau, n)$*, in all equilibria, all individuals converge to have the same, correct, beliefs. (iii)* ***Comparative Statics:*** *The cutoff* $q^*(\tau, n)$ *decreases in* τ*, the intensity of learning in the labour market, and increases in* n*, the intensity of peer influence.*

Other initial states: (i) The result remains exactly the same if in the initial state the dynasties in the private school do not have the same beliefs but some distribution of beliefs over $(0, q]$. (ii) The result remains exactly the same if the initial state is generalized to allow initial wrong beliefs in the state school as well. We show this in Proposition 2A in the Appendix.

A sufficient level of initial segregation can therefore lead to long-run segregation, polarised beliefs, and labour market discrimination. Even if dynasties are exposed to many positive signals, or get relatively accurate information in the labour market (a higher τ), they are also exposed to many negative beliefs of peers about the productivity of state school graduates. The tendency of individuals to exchange information with like-minded individuals (which arises in our model due to imperfect empathy) leads to the echo chamber effect which overcomes new and true information that is learned in the labour market.

When $q \leq q^*(\tau, n)$, in all equilibria the process of convergence to extreme beliefs is characterized by a constant set of private school dynasties that send their offspring to the private school, have beliefs that converge to the lowest value, and employ private school graduates with a higher probability than state school graduates.

How does endogenous segregation persist to polarise beliefs sufficiently in the long run? And when does it break? For segregation to arise, beliefs in the private school have to be sufficiently low throughout time. In other words, dynasties with sufficiently low beliefs need to have the highest willingness to pay for the private school (compared with others in the population). On the other hand, the process of segregation and belief polarisation would break down if the distribution of beliefs in both schools are sufficiently mixed, in other words, when it is not necessarily the dynasties with the lowest beliefs that choose to attend the private school. To see how the simple condition above determines in which environments these cases arise, we need to understand the trade-offs facing the parents when they consider their school choice.

Consider Δ_i^t, the relative labour market benefit from sending a child to the private school vis a vis the state school. The parent in dynasty i will compute her offspring's utility given her own beliefs, $q_{i,e}^t$, and her forecast of how the offspring's beliefs will change in each school, depending on her forecast of the behaviour of employers in period $t+1$.

Suppose that at period t the beliefs of those in the state school are still all at one, as in the initial state. In equilibrium, a parent realises that at period $t+1$, all employers hire a private school graduate employee, and that a state school employer hires all. There are therefore two events in which school choice is relevant. First, with probability $(1-\gamma)\frac{\gamma}{1-\gamma}\rho = \gamma\rho$ the offspring becomes an employee and is matched with an employer who is a private-school graduate. In this case, if she herself went to the private school she is employed for sure (and receives in expectations a wage equal to $\frac{1}{2}$), but if she went to the state school she will only be employed with some probability (and thus receive a lower wage on average). Specifically, she is employed by an individual from dynasty l only if $w \leq q_{l,g}^{t+1}$. Thus, given the (correct) equilibrium future distribution $f_{p,g}^{t+1}(.)$ of private-school graduates' employers from dynasties l and their beliefs $q_{l,g}^{t+1}$, the relative benefit from attending a private school in this case is:

$$\frac{1}{2} - \bar{w}^{t+1} \tag{3}$$
$$\text{where } \bar{w}^{t+1} \equiv \int_0^1 \left[\int_0^{q_{l,g}^{t+1}} w dw\right] f_{p,g}^{t+1}(q_{l,g}^{t+1})\, dq_{l,g}^{t+1} \leq \frac{1}{2}$$

This term $\frac{1}{2} - \bar{w}^{t+1}$ is non-negative and would induce *all* parents to prefer to send their child to the private school. As long as there are heterogenous beliefs in the population, state school graduates employees are discriminated against and it is better to attend a private school.

The second case in which a benefit or a loss from attending a private school arises, is when the offspring becomes an employer and is matched with an employee who is a state school graduate. This happens with probability $\gamma(1-\rho)$. If she is a private school graduate, she would employ the state school graduate if $w \leq q_{i,g}^{t+1} = \chi(\mathbf{q}^{t+1}, q_{i,e}^t)$. Upon employing, she would pay some $w \leq q_{i,g}^{t+1}$, and gain -from the point of view of her parent- an expected payoff of $q_{i,e}^t - w$. Taking expectations over all possible peer influence vectors, $\mathbf{q}^{t+1}$, going to the private school in this event would generate:

$$\pi_i^t \equiv \int_{\mathbf{q}^{t+1}} \left[\int_0^{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)} (q_{i,e}^t - w) dw\right] f_{p,n}^{t+1}(\mathbf{q}^{t+1})\, d\mathbf{q}^{t+1},$$

which depends on the school choices of other parents or in other words the expected equilibrium distribution over vectors of length n of peer beliefs in the private school, $f_{p,n}^{t+1}(\mathbf{q}^{t+1})$. If on the other hand the individual is a state school graduate, she would

employ the matched state school graduate for sure, gaining -in the eyes of her parent- an expected net payoff of $q^t_{i,e} - \frac{1}{2}$. Thus, the relative gain/loss from attending a private school in this event is:

$$\hat{\Delta}^t_i \equiv \pi^t_i - (q^t_{i,e} - \frac{1}{2}). \tag{4}$$

This term $\hat{\Delta}^t_i$ embodies the imperfect empathy assumption, and can be negative or positive. If the beliefs of the parent are low, the induced behaviour of the offspring if she goes to the state school is far worse than that induced by the private school, yielding $\hat{\Delta}^t_i > 0$. However, if the beliefs of the parent are relatively high, sending the child to a private school might represent a loss as peer influence might substantially decrease the offspring's beliefs. When $q^t_{i,e} = 1$ then $\hat{\Delta}^t_i = 0$, as in this case $\chi(\mathbf{q}^{t+1}, q^t_{i,e}) = 1$ for all $\mathbf{q}^{t+1}$.

Putting (3) and (4) together, we have that given a parental belief $q^t_{i,e}$, the benefit from a private school vis a vis a state school is:

$$\Delta^t_i = \gamma\rho(\frac{1}{2} - \bar{w}^{t+1}) + \gamma(1-\rho)\hat{\Delta}^t_i \tag{5}$$

The value of $q^*(\tau, n)$ insures that dynasties in the private school, following peer influence and labour market experience, *will end up with lower beliefs than they had started with.* As this process continues, their beliefs will converge to zero. At the same time this also implies that these dynasties want to send their children to private schools more than state school parents do. Specifically, we show that when the beliefs of all dynasties i in the private school are low enough, then $\hat{\Delta}^t_i > 0$, implying that there will be no "contamination" or replacement by individuals with beliefs at 1.

Our result also shows when endogenous segregation break down. Specifically, we find that if the condition is not satisfied, then those who segregate in the private school have sufficiently mixed beliefs so that belief polarisation cannot arise. To see how this comes about, suppose that initial beliefs are above $q^*(\tau, n)$. This implies that while the belief of some may decrease (if for example they do not hire a state school graduate), there would be a share of individuals for whom beliefs, following peer influence and labour market experience, will increase. But under a contemplated segregation, beliefs of many in the private school must converge to zero; this implies that for the dynasties whose beliefs keep increasing with time, peer influence in the private school would, at some point, induce a large swing in beliefs (and thus suboptimal future labour market behaviour). In other words, we would have that for some private school dynasties, $\hat{\Delta}^t_i < 0$. But for state school parents $\hat{\Delta}^t_i = 0$, as their offspring is not influenced in the private school. Therefore, state school parents will have higher willingness to pay for the private school and will be able to send their offspring there. A process of contamination of beliefs will start and segregation will eventually break down.[31]

[31] The fact that the school has a capacity constraint and always admits a fixed share ρ is not important

In reality, segregation would not be that easy to break. It might be that new entrants to the private school from state school dynasties would not immediately interact with others and fully convince them, as the literature on "within school segregation" discusses (see Echenique *et al* 2006). Even informal interactions may depend on background, beliefs, or income. This implies that segregation is underestimated in the model or that its breakdown would be a slow process. We discuss in Section 4.1 other ways which make polarisation and segregation harder to break.

Unequal productivities: We now consider a simple extension of the model that illustrates that what is important in our analysis is not the fact that productivities in the two schools are equal, but that individuals might overestimate the differences.

Suppose that the true productivity of state school graduates is some $\theta < 1$ (while that of private school graduates is 1, known to all). Thus beliefs at period t are that the productivity of state school graduates is θ with some probability $q_i^t > 0$, and zero otherwise. We again start with the initial condition that in Period 1, a share ρ is in the private school and believe that state school productivity is θ with probability $q_i^1 = q < 1$ and a share $1 - \rho$ is in the state school and has beliefs $q_i^1 = 1$. We can then extend our proof of Proposition 2 in a straightforward way to show more generally:

Proposition 3: *Suppose that the true productivity of state school graduates is $\theta < 1$. Then Proposition 2 holds.*

Note that true beliefs in this case would imply that individuals would prefer to hire private school graduates. However, if we start from sufficient segregation in beliefs across schools, then in the long run wrong beliefs will persist and polarise, and we would have discrimination over and above what it should be. This again would be a result of endogenous segregation of specific dynasties with sufficiently low beliefs.

4 Discussion

We first discuss factors that increase the instances of polarisation, segregation and discrimination. We then discuss policies that can reduce the occurrence of such phenomena.

4.1 The persistence of polarisation and segregation

Segregation and long run polarised beliefs may be underestimated in our model. We now discuss some factors which, in reality, may increase its occurrence.

for this result. More generally, if the private school admits those with the highest willingness to pay (as all pricing mechanisms would), those with the "wrong" beliefs for the school will eventually enter.

Interviews and the legacy criterion: Proposition 2 had illustrated that segregation will eventually break down if the initial state does not consist of sufficiently many individuals with low beliefs in the private school. The reason for this is that the willingness to pay to the private school of some with intermediate beliefs will at some point become lower than those with very high beliefs, implying a contamination of the pool of beliefs in the private school. This would hold with more general pricing mechanisms that rely on willingness to pay.

One strategy of private schools to avert this is to base admittance on the actual beliefs held by the dynasty member. Accepting those with low enough beliefs may not maximize profits, but will certainly maximize the chances of survival of the private school. Our model can then shed light on why many private schools and top universities use interviews or legacy criteria for admissions. In the prism of our model, interviews allow schools to screen students according to views and family values, making sure that only those with the "right" beliefs or values are admitted.[32] A legacy criteria allows universities to maintain a significant cohort in the school coming from the same families, which allows the school to better control and protect specific values and belief systems.[33]

Income effects: We have abstracted away from income effects. It is clear that an unequal distribution of income or wealth would only exacerbate segregation. The private school is the costly school. Even if a parent from a state school dynasty has a higher willingness to pay for the private school compared with a parent from a private school dynasty, she may not have the means to do it. This implies that it may not be as easy to break segregation as we had described above

Another element in our model which is affected by income is learning on the job market. If getting a placement in a company demands a period of unpaid internship, then those with low income -which by definition are more likely to be in state education- are less likely to be able to afford it. This means that there are less opportunities to learn about their type.

4.2 How to stop segregation and polarisation?

In our model, segregation and polarised beliefs induce inefficiencies: Discrimination is distortionary as state schools graduates miss employment opportunities, while private

[32]Indeed, web sites that provide advice for private schools' prospective students in the UK explain that it is important to mention in an interview that previous generations in the family have attended the school. Interview advice for the parents themselves includes "expect to talk about yourself..remember to discuss long term values for your child".

[33]The Ivy League institutions are estimated to admit 10% to 30% of each entering class using this factor. The former president of Harvard University, Larry Summers, has stated: "Legacy admissions are integral to the kind of community that any private educational institution is."

school employers miss opportunities to employ.

As discussed above, income effects, school ethos, or other network effects can all play a role to increase segregation. What can decrease its incidence? We discuss some possibilities below.

The long-term effects of integration: The literature on integration in schools has mainly focused on academic achievements of participants as the potential benefit of integration (see Sacerdote 2011 and Angrist and Lang 2004). Our analysis suggests that integration might have long-term effects: In our model, integration in schools would affect the beliefs of individuals through peer influence and so their behaviour in the labour market later on in their lives. A recent empirical literature shows how beliefs indeed evolve as a result of integration (see Boisjoly *et al* 2006, Kaufman and Harris 2015, La Ferrara *et al* 2014 and Algann *et al* 2015).

As with the potential effect of integration on academic achievement, the actual level of interaction among the different students is important. If for example pupils from state school dynasties are integrated into a private school but interact only with one another, the beliefs of the private school dynasties will not be affected. Thus, integration has to be happening in practice for such policies to have an effect.

Labour market and anti-discrimination policies: Proposition 2 shows that better learning in the labour market (captured by the parameter τ) will increase the instances in which we have convergence to long run wrong beliefs. This can arise for example when we have a longer probation period for employees before they can be let off, so that employers can learn better about state school graduates.

Our framework also allows us to assess common anti-discrimination policy interventions. Consider a simple policy which induces private school employers to hire more graduates from state schools. One way to model the consequences of such a policy is to assume that a private school employer hires a state school graduate if $w < q^t_{i,g} + \lambda$, for some $\lambda > 0$. This can be thought of as a subsidy given to an employer who hires a state school graduate or a punishment for the employer who does not abide by anti-discrimination laws. In any case, the instances of hiring state school graduates by employers would increase.

While this policy will surely decrease discrimination, it will not have a direct effect on the incidence of segregation and polarisation. Specifically, the necessary and sufficient condition for segregation we had derived above will not be affected by λ. The condition is computed at the worst case scenario for segregation, in which a state school graduate is hired in any case. Whenever the condition is satisfied, then in all equilibria beliefs in the private school will converge to zero and state school graduates will be employed by private employers iff $w < \lambda$. As long as $\lambda < 1$, some discrimination will persist. Thus,

anti-discrimination policies -as long as they are not absolute and fully enforced- would be more effective coupled with policies that also affect beliefs directly.

Vouchers, subsidies and scholarships: The US educational voucher system has been heavily scrutinized; vouchers give individuals the money to attend a private or a charter school. The key argument for vouchers is that school choice improves competition and as a result can have a positive effect on public schools as well. The key argument against vouchers is that of cream skimming, as the best students would leave the public system.[34] Our analysis can provide a complementary way to look at the effect of vouchers or subsides.

While most US states have provided vouchers to low-income families or in specific school districts, a recent Nevada program is the most far-reaching, incentivizing parents to abandon public schools with annual $5,100 education savings accounts (ESAs) for private schooling - regardless of household wealth or income level. Note however that if such vouchers are given to all, it is still the case that individuals would differ in their willingness to pay for the private school. The private school through its pricing will naturally choose those with the highest willingness to pay. This system will therefore have a minimal effect in our model. In the equilibria with segregation, private schools could charge a sum on top of $5,100, which would reflect the willingness to pay of those with low beliefs.[35]

Consider then the more traditional voucher system according to which subsidies are given only to those with low-income, who typically attend state schools. Consider an equilibrium with segregation in which at some period t, all dynasties with beliefs below q^t attend the private school. If the voucher or subsidy given to state school dynasties is sufficient to overcome the difference in the willingness to pay between a state school dynasty and the highest type q^t in the private school, the segregation equilibrium may break down. But as we show below, long-term segregation equilibria may still arise. The reason is that those with lower beliefs always have a higher willingness to pay than those with higher beliefs, and that this wedge increases when private schools become more exclusive.

Note that in our model the highest expected utility an individual can gain in the labour market is $\frac{1}{2}$, either from being an employer who pays the expected wage for the most productive employee, or from being an employee who gains the highest expected wage. We therefore consider vouchers with value $V \in [0, \frac{1}{2})$.

[34] See Hoxby (1994).

[35] Indeed More than 80% of Nevada private schools cost an average of $4,800 more than the voucher amount each year, according to a survey by Educate Nevada Now.

Proposition 4: *For any voucher value $V < \frac{1}{2}$, there exist a cutoff $q_V^*(\tau, n)$, with $q_V^*(\tau, n)$ decreasing in V, $q_0^*(\tau, n) = q^*(\tau, n)$ as defined in Proposition 2 and $q_{V \to \frac{1}{2}}^*(\tau, n) \to 0$, such that long-term segregation arises in all equilibria when all dynasties with $q \leq q_V^*(\tau, n)$ attend the private school.*

In the Appendix we show that in any equilibrium with segregation, the willingness to pay for dynasties in the private school increases (compared to state school dynasties) when the school is more exclusive (so that the cutoff is lower). Thus segregation can still arise, but the set of long term equilibria with segregation will be smaller. This can be determined endogenously by the private schools, as following the introduction of vouchers, they can respond by making entry more exclusive (by using interviews, testing for ability etc.). The incentive of schools to do so stems from the fact that they can only make profits if there is polarisation of beliefs in society.

The winners and losers from segregation: One feature of the basic model is that when school fees are taken into account, then in the long term, private school dynasties have a lower per period utility than state school dynasties. While private school graduates enjoy a better employment market, they also pay private school fees which extract their willingness to pay. This willingness to pay however is in part based on wrong beliefs which is not recuperated in the market later on.

This may be an artefact of the simple model we are using, as we had stacked the model against segregation and its benefits. As discussed above, fees may be lower and still prevent state school dynasties from entering if for example admission policies are partly based on interviews. This would imply that the willingness to pay of private school parents is not extracted. In addition, the constraint in our model that state school dynasties need to be deterred from entry with a low willingness to pay may be exaggerated: they may have high willingness to pay but no means to do so, due to income effects discussed above. In a more general model, wrong beliefs can also be two-sided, that is, state school dynasties may also overestimate the benefits of state schools. Thus state school dynasties can lose as employers as well.

Even so, it might be true that private education is indeed not beneficial in reality. Recent data in the UK supports the observation that private schools are not beneficial on average, when parental background and school fees are taken into consideration.[36]

[36] A report by the Social Market Foundation calculated that the higher educational achievement and university degrees accrued by private school pupils translated to £193,000 in higher earnings between the ages of 26 and 42. After adjusting for family background and social circumstances, the private school pay advantage was £57,000. However, ten years' worth of average private school fees in 1980 would be similar in scale -around £56,000 in today's prices. See http://www.theguardian.com/education/2014/jul/03/subsidy-independent-school-fees.

5 Concluding remarks

In this paper we analyse a model of the coevolution of segregation and polarisation of beliefs. Our model is one of segregation along acquired traits which fits many applications including primary and secondary schooling (private versus state, secular versus religious). Higher education is another good example. The forces that shape segregation and beliefs in our model could be behind what some have dubbed "the higher education bubble".[37]

Segregation often occurs along inherited traits, such as race, ethnicity or gender. An interesting line of research would be to add inherited traits into our analysis. In a world in which it is illegal to discriminate along such traits, schooling and location choices might be used as proxies to allow *de facto* segregation and discrimination to persist.

In practice, the ADA and Title VII of the Civil Rights Act have been used to challenge alleged "educational discrimination" practices. A famous example is Griggs v. Duke Power Company from 1971, in which the employer had adopted a high-school diploma requirement for all positions in four of its five departments without "meaningful study" of its "relationship to job-performance ability", based on the untested belief that doing so would "improve the overall quality of the work force." The Court decided that the requirement was unlawful because it had a disparate impact on African Americans, who had high school diploma rates far lower than Whites in the relevant geographical area, and because the requirement was not job related for the positions in question and consistent with business necessity. The Court stated:[38] "The evidence...shows that employees who have not completed high school...have continued to perform satisfactorily and make progress in departments for which the high school...criterion [is] now used...The facts of this case demonstrate...the infirmity of using diplomas or degrees as fixed measures of capability. History is filled with examples of men and women who rendered highly effective performance without the conventional badges of accomplishment in terms of certificates, diplomas, or degrees. Diplomas and tests are useful servants, but Congress has mandated the commonsense proposition that they are not to become masters of reality." The courts and the EEOC have applied the holding in Griggs consistently, and Congress confirmed it when it amended Title VII in the Civil Rights Act of 1991.

6 Appendix

6.1 Appendix A: Proofs

Proof of Proposition 0: At any period t, construct the self correspondence $\Gamma : [0,1] \to 2^{[0,1]}$ in the following way. For any set of beliefs S of measure ρ that is composed of

[37] http://www.economist.com/blogs/schumpeter/2011/04/higher_education

[38] Griggs, 401 U.S. at 431-33.

intervals in $[0,1]$, the correspondence assigns a measure ρ of dynasties that have the highest Δ_i^t, given that the set S attends the private school.[39] To complete the description of the correspondence, assign any individual that is indifferent between attending the state and private school to the private school. For any S, $\Gamma(S)$ is never empty. Thus, this correspondence is closed. Moreover, the set $[0,1]$ is compact. By Ok (2004), the set $[0,1]$ has the fixed set property so that every closed self correspondence has a fixed set. We can therefore always construct a sequence of fixed sets for $t = 1, 2, ...$■

Proof of Proposition 1: At period 1 all dynasties in the private school have beliefs at $q_i^1 = q$ where $q > 0$. Note that as $n = 0$ for all such dynasties, $q_i^t \geqslant q$. Any dynasty at period t has a strictly positive probability, bounded below by $q\gamma(1-\rho)$, of employing a state school graduate and therefore updating their belief to $\frac{q_i^t\tau}{q_i^t\tau+(1-q_i^t)(1-\tau)} > q_i^t$. Therefore, all dynasties will converge to a belief q_i^∞ such that $q_i^\infty = \frac{q_i^\infty\tau}{q_i^\infty\tau+(1-q_i^\infty)(1-\tau)} \Leftrightarrow q_i^\infty = 1$.■

Proof of Proposition 2: The proof below uses notation that is introduced in Section 3 in the discussion that follows the statement of Proposition 2.

Preliminaries: For short, let $q^*(\tau, n) \equiv q^*$. Note that: (i) the condition in the Proposition implies that $q^* < 0.5$; (ii) When the individual with beliefs q^* interacts in the worst case scenario with n other individuals with the same beliefs, his beliefs upon graduation become $q_g^* \equiv \frac{q^{*n+1}}{q^{*n+1}+(1-q^*)^{n+1}} < q^*$ (as $q^* < 0.5$). If such an individual later on employs a state school graduate, his beliefs post-employment become $q_e^* = \frac{q_g^*\tau}{q_g^*\tau+(1-q_g^*)(1-\tau)}$. If $\frac{q_g^*\tau}{q_g^*\tau+(1-q_g^*)(1-\tau)} = q^*$, then the dynasty's beliefs at the end of the period are still at $q^* < \frac{1}{2}$. The condition in the Proposition insures that $\frac{q_g^*\tau}{q_g^*\tau+(1-q_g^*)(1-\tau)} = q^*$. It is easy to check that the mapping of q to $\frac{q_g\tau}{q_g\tau+(1-q_g)(1-\tau)}$ has a unique fixed point q^* on (0,1), and moreover that for all $q < q^*$, $\frac{q_g\tau}{q_g\tau+(1-q_g)(1-\tau)} < q$, and for all $q > q^*$, $\frac{q_g\tau}{q_g\tau+(1-q_g)(1-\tau)} > q$. Note that q^* represents the worst case scenario, so that dynasties that will not experiment, will have lower beliefs than q^*.

With some abuse of notation, let Δ_1^t denote the relative benefit from the private school of individuals with beliefs at 1.

Sufficiency: We know from the above that beliefs in the private school always remain below $q^* < \frac{1}{2}$. We will now show that for all $q_{i,e}^t < \frac{1}{2}$, given that beliefs in the private school are interior, then $\hat{\Delta}_i^t > \hat{\Delta}_1^t = 0$, and thus only the original set of dynasties will continue to attend the private school, and individuals from the state school will not "contaminate"

[39] Note that at period t the domain of the set of beliefs is not necessarily connected but it is closed and bounded.

beliefs in the private school. We therefore need to show that:

$$\begin{aligned}&\hat{\Delta}_i^t\\&=\pi_i^t-(q_{i,e}^t-\frac{1}{2})\\&=\int_{\mathbf{q}^{t+1}}(\int_0^{\chi(\mathbf{q}^{t+1},q_{i,e}^t)}(q_{i,e}^t-w)dw)dF_{p,n}^{t+1}(\mathbf{q}^{t+1})-(q_{i,e}^t-\frac{1}{2})>0,\end{aligned}$$

where $F_{p,n}^{t+1}(\mathbf{q}^{t+1})$ is the distribution over vectors $\mathbf{q}^{t+1}$ of length n of beliefs of children in the private school as inherited from their parents (recall that $q_i^{t+1}=q_{i,e}^t$).

Note however that $q_{i,e}^t<\frac{1}{2}$, and that $\int_0^{\chi(\mathbf{q}^{t+1},q_{i,e}^t)}(q_{i,e}^t-w)dw=\chi(\mathbf{q}^{t+1},q_{i,e}^t)q_{i,e}^t-\chi(\mathbf{q}^{t+1},q_{i,e}^t)^2/2>0$ as such function is strictly concave, is maximised at $q_{i,e}^t$, and $\chi(\mathbf{q}^{t+1},q_{i,e}^t)\leq q_{i,e}^t$ as beliefs in the private school are lower than a half. This implies that $\hat{\Delta}_i^t>0$.[40]

Finally note that the beliefs in the private school would converge to the singleton 0 as they are always below a half. This concludes the sufficiency part as the measure ρ of private school dynasties dynasties will have a strictly high willingness to pay for the private school compared with state school dynasties.

Necessity: Suppose that the condition in the proposition is violated so that $[\frac{q}{1-q}]^{n-1}>\frac{1-\tau}{\tau}$. This implies that $q>q^*$ and that at any point in time, all those in $(q^*,q]$, if they draw only peers from this set, will have new beliefs that are higher then what they started with. In particular, one can find sequences of dynasties whose beliefs will converge to one. We will show that this implies that we can find a positive measure of such dynasties, for whom eventually $\hat{\Delta}_i^t<\hat{\Delta}_1^t=0$.

Note that given the binary state of nature, dynasties must converge to have beliefs on zero, one or a half (the latter can arise as if all have beliefs at a half, then the updated beliefs will be half as well). We next show that long term segregation implies that a strictly positive measure of dynasties will converge to beliefs of zero.

First note that if long term segregation exists, it cannot be that almost all dynasties converge to have beliefs at one. If this is the case then the benefit from sending one's child to the private school, for any parent, will converge to that of state school parents.

The next step is to note that if the measure of dynasties whose beliefs converge to zero goes to zero, then it must be that the measure of dynasties whose beliefs converge to one, goes to one. To see this note that if there was in the limit a strictly positive measure of

[40] We have used the uniform distribution of wages in computing $\int_0^{\chi(\mathbf{q}^{t+1},q_{i,e}^t)}(q_{i,e}^t-w)dw$. In fact any distribution that first order stochastically dominates the uniform will satisfy the condition. For other distributions, one would potentially need to add another condition to insure that no "contamination" arises but there would always be a low enough initial q^* that would satisfy such condition. The necessary part does not depend on the uniform distribution at all.

dynasties with beliefs converging to a half, this would unravel as these dynasties would learn from work experience with strictly positive probability, and their beliefs would drift towards one.

We will now look at the strictly positive measure of dynasties initially at $(q^*, q]$ whose beliefs are increasing and show that as a measure $\frac{\alpha}{2}$ of other dynasties have beliefs below ε, then their loss from peer influence in the private school is large.

To do so, we focus on such dynasties initially in $(q^*, q]$ whose beliefs increase sufficiently slow so that others can converge to have beliefs below ε. Specifically, note that by choosing which individuals in the private school members of a dynasty interact with each period, and whether they learn in the labour market, we can construct a feasible sequence of beliefs for a dynasty $\{q_i^t\}_{t=1}^{\infty}$ that lingers as long as we wish below a half, i.e. for any T we can find such a sequence such that $q_i^t \to 1$ but such that $q_i^t < 0.5$ for any $t < T$. Note further that by continuity of $\chi(\mathbf{q}^{t+1}, q_{i,e}^t)$ and full support we can also construct a whole set of such sequences all going to one in a similar rate.

Note now that for dynasty i at time t with beliefs $q_{i,e}^t$ we have:

$$\begin{aligned}
&\hat{\Delta}_i^t - \hat{\Delta}_1^t \\
&= \int_{\mathbf{q}^{t+1}} \left(\int_0^{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)} (q_{i,e}^t - 1) dw \right) - \int_{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)}^1 (1-w) dw \, dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) + (1 - q_{i,e}^t) \\
&< - \int_{\mathbf{q}^{t+1}} \left(\int_{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)}^1 (1-w) dw \right) dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) + (1 - q_{i,e}^t)
\end{aligned}$$

Now choose ε and T_ε such that a measure $\frac{\alpha}{2}$ of dynasties have beliefs below ε. Note that if i interacts with n of these dynasties, the measure of all such interactions is $(\frac{\alpha}{2})^n$. Choose a sequence of a strictly positive measure of dynasties whose beliefs converge to one slow enough so that at T_ε their distance from one is $\eta > 0$ such that $\chi(\mathbf{q}^{t+1}, q_{i,e}^t)$ is at most 2ε. Then we will have:

$$\hat{\Delta}_i^t - \hat{\Delta}_1^t < -(\frac{\alpha}{2})^n \int_{2\varepsilon}^1 (1-w) dw + \eta < 0$$

whenever ε and η are low enough (note that α does not depend on ε). Thus in the next period a strictly positive measure of state school parents will send their kids to the private school.

Once a strictly positive measure of dynasties with beliefs at 1 enter the school, this will be the case for all future periods (as from that point onwards, the measure of dynasties with beliefs less than 1 is smaller than ρ). They will then "infect" whoever they meet and so all dynasties will converge to beliefs of 1 which is a contradiction to long term segregation.■

Proof of Proposition 2A: We now prove the results stated following Proposition 2 in the text, extending the initial condition to two cases. First, note that as case (i) states, if initial beliefs are in $(0, q^*]$, then nothing in the proof above changes. We now consider case (ii), where the initial beliefs in the state school are in $[\bar{q}, 1]$ and the initial beliefs in the private school are in $(0, q^*]$.

Let now Δ_1^t denote the relative benefit from the private school to a dynasty with beliefs at 1. Note first that whenever q^* is sufficiently low and $\bar{q}$ is sufficiently high, then $\hat{\Delta}_l^t - \hat{\Delta}_1^t \leq 0$ for any dynasty l with $q_{l,e}^t > \bar{q}$.

To see why note that in the state school, the beliefs of a dynasty always increase, as all other beliefs are greater than a half. This implies that the lowest bound on utility is attained when $\chi(\mathbf{q}^{t+1}, q_{l,e}^t) = 1$. For $q_{l,e}^t = 1 - \eta$, this utility for the event of becoming an employer and hiring a state school graduate amounts to $1 - \eta - \frac{3}{4}$. On the other hand, the best utility for this type in the private school would be when the interaction is with the highest type, q^*. If the resulting beliefs, satisfy $\frac{q^{*n}(1-\eta)}{q^{*n}(1-\eta)+(1-q^*)^n\eta} < 1 - 2\eta$, then the utility from this best interaction is lower implying that $\hat{\Delta}_l^t < \hat{\Delta}_1^t = 0$. This arises when $\frac{q^{*n}}{(1-q^*)^n} < \frac{1-2\eta}{2(1-\eta)}$. For a small enough η, this holds for all small enough $q^* < \frac{1}{2}$, which is satisfied by our condition. Thus $\hat{\Delta}_l^t - \hat{\Delta}_1^t \leq 0$.

This implies that for long term segregation, we need $\hat{\Delta}_l^t > \hat{\Delta}_1^t = 0$, for all $q_{l,e}^t \leq q^*$, while we know that segregation breaks down if $\hat{\Delta}_l^t < \hat{\Delta}_1^t = 0$ for some $q_{l,e}^t > q^*$ which had attended the private school at period t.

Consider the sufficiency then. If in equilibrium the same original dynasties in the private school remain in the private school, beliefs only decrease and converge to zero. To see that these dynasties remain in the school we need to show for any dynasty i in the private school, that $\hat{\Delta}_i^t > \hat{\Delta}_1^t = 0$ at any t, conditional on the set of dynasties in the private school remaining below q^*. Taking into consideration the distribution over vectors of beliefs in the state school, $F_{s,n}^{t+1}(\mathbf{q}^{t+1})$, the condition then becomes:

$$\int_{\mathbf{q}^{t+1}} \left(\int_0^{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)} (q_{i,e}^t - w) dw \right) dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) - \int_{\mathbf{q}^{t+1}} \left(\int_0^{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)} (q_{i,e}^t - w) dw \right) dF_{s,n}^{t+1}(\mathbf{q}^{t+1}) > 0$$

Note that as above, the left element (the gain from the private school) is bounded from below by 0. On the other hand, for $\bar{q}$ sufficiently high, the best case scenario is for beliefs to end up at $\frac{q_{i,e}^t \bar{q}^n}{q_{i,e}^t \bar{q}^n + (1-q_{i,e}^t)(1-\bar{q})^n} \to 1$ yielding a limit of a negative utility of $q_{i,e}^t - 0.5$. Thus the condition above is satisfied.

Now consider the necessity. When segregation arises, as in the proof of Proposition 2, we can repeat the same proof as above, as for segregation to hold in the long term it is a necessary condition that $\hat{\Delta}_i^t - \hat{\Delta}_1^t \geq 0$. The only issue may arise if the dynasty whose beliefs increase and we follow is being replaced by a state school dynasty with some $q < 1$; we then continue to follow this new private school dynasty with these beliefs.■

Proof of Proposition 3: There are now three events in which the education path is relevant on the equilibrium path. First, with probability $(1-\gamma)\frac{\gamma}{1-\gamma}\rho = \gamma\rho$ the individual becomes an employee and is matched with an employer who is a private-school graduate. In this case, if she herself went to a private school she is employed for sure (and receives an expected wage of a half), but if she went to a state school she will only be employed with some probability. Specifically, she is employed by an individual from dynasty l only if $w < \theta q_{l,g}^{t+1}$. Thus, given some conjectured $f_g^{t+1}(.)$, the future distribution of employers' beliefs, the relative benefit for attending a private school in this case is:

$$\frac{1}{2} - \bar{w}^{t+1} \geq 0 \tag{6}$$
$$\text{where } \bar{w}^{t+1} \equiv \int_0^1 (\int_0^{\theta q_{l,g}^{t+1}} w dw) f_{p,g}^{t+1}(q_{l,g}^{t+1}) dq_{l,g}^{t+1}$$

Second, with probability $(1-\gamma)\frac{\gamma}{1-\gamma}(1-\rho) = \gamma(1-\rho)$, a graduate would meet a state school employer who would always employ a private school graduate but would only employ the state school graduate if $w < \theta$, i.e., the difference is:

$$\frac{1}{2} - \int_0^{\theta} w dw = \frac{1}{2} - \frac{\theta^2}{2}.$$

The third case in which a benefit or a loss from attending a private school arises, is when an individual becomes an employer and is matched with an employee who is a state school graduate (which happens with probability $\gamma(1-\rho)$). If she is a state school graduate herself, she would employ him if $w < \theta$, gaining -in the eyes of her parent- expected productivity of $\theta q_{i,e}^t$ while paying in expectations $\frac{\theta}{2}$. On the other hand, if she is a private school graduate, she would employ only if $w < \theta q_{i,g}^{t+1}$. Upon employing, she would pay some $w < \theta q_{i,g}^{t+1}$, and gain -from the point of view of her parent- productivity of $\theta q_{i,e}^t$. Thus we have that in this event the gain/loss from attending a private school, is:

$$\hat{\Delta}_i^t \equiv \pi_i^t - \int_0^{\theta} (\theta q_{i,e}^t - w) dw = \pi_i^t - \theta(\theta q_{i,e}^t - \frac{\theta}{2}) \tag{7}$$

where π_i^t is the expected gain, in the eyes of the parent, from the offspring being a private school graduate employer:

$$\pi_i^t \equiv \int_{\mathbf{q}^{t+1}} [\int_0^{\theta\chi(\mathbf{q}^{t+1}, q_{i,e}^t)} (\theta q_{i,e}^t - w) dw] f_{p,n}^{t+1}(\mathbf{q}^{t+1}) d\mathbf{q}^{t+1}$$

We therefore have that for a parent with belief $q_{i,e}^t$, the benefit from a private school vis a vis state school is:

$$\Delta_i^t = \gamma\rho(\frac{1}{2} - \bar{w}^{t+1}) + \gamma(1-\rho)(\frac{1}{2} - \frac{\theta^2}{2}) + (1-\gamma)(1-\rho)\hat{\Delta}_i^t. \tag{8}$$

Sufficiency: We know from the above that beliefs in the private school always remain below $q^* < \frac{1}{2}$. We will now show that for all $q < \frac{1}{2}$, given that beliefs in the private school are interior, then $\Delta_i^t > \Delta_1^t \Leftrightarrow \hat{\Delta}_i^t > \hat{\Delta}_1^t$. This implies that only the original dynasties remain in the school. We therefore need to show that:

$$\int_{\mathbf{q}^{t+1}} [\int_0^{\theta\chi(\mathbf{q}^{t+1},q_{i,e}^t)} (\theta q_{i,e}^t - w)dw] f_{p,n}^{t+1}(\mathbf{q}^{t+1})d\mathbf{q}^{t+1} - (\theta^2 q_{i,e}^t - \frac{\theta^2}{2}) > 0 = \hat{\Delta}_1^t.$$

A sufficient condition for this is that for any $\mathbf{q^{t+1}}$:

$$\int_0^{\theta\chi(\mathbf{q^{t+1}},q_{i,e}^t)} \theta q_{i,e}^t dw - \int_0^{\theta\chi(\mathbf{q^{t+1}},q_{i,e}^t)} w dw - \int_0^{\theta} \theta q_{i,e}^t dw + \int_0^{\theta} w dw > 0 \Leftrightarrow \tag{9}$$

$$\int_{\theta\chi(\mathbf{q^{t+1}},q_{i,e}^t)}^{\theta} (w - \theta q_{i,e}^t)dw > 0 \Leftrightarrow$$

where the last inequality holds whenever $q_{i,e}^t < \frac{1}{2}$ and $\chi(\mathbf{q^{t+1}},q_{i,e}^t) < 1$. This concludes the sufficiency part as the measure ρ of private school dynasties dynasties will have a strictly high willingness to pay for the private school compared with state school dynasties. Finally note that the beliefs in the private school would converge to the singleton 0 as they are always below a half.

Necessity: We can repeat the same proof as in Proposition 2 to show that whenever long term segregation arises and the condition is violated, then we can look at the strictly positive measure of dynasties at $(q^*, q]$ whose beliefs are increasing and show that as a measure $\frac{\alpha}{2}$ of other dynasties have beliefs below ε, then their loss from peer influence in the private school is large. To do so, we again focus on such dynasties in $(q^*, q]$ whose beliefs increase sufficiently slow so that others can converge to have beliefs below ε. Note now that,

$$\begin{aligned}
&\hat{\Delta}_i^t - \hat{\Delta}_1^t \\
&= \int_{\mathbf{q}^{t+1}} (\int_0^{\theta\chi(\mathbf{q}^{t+1},q_{i,e}^t)} (\theta q_{i,e}^t - \theta)dw) - \int_{\theta\chi(\mathbf{q}^{t+1},q_{i,e}^t)}^{\theta} (\theta - w)dw) dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) + \theta^2(1 - q_{i,e}^t) \\
&< -\int_{\mathbf{q}^{t+1}} (\int_{\theta\chi(\mathbf{q}^{t+1},q_{i,e}^t)}^{\theta} (\theta - w)dw) dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) + \theta^2(1 - q_{i,e}^t)
\end{aligned}$$

Now choose ε and T_ε such that a measure $\frac{\alpha}{2}$ of dynasties have beliefs below ε. Note that if i interacts with n of these dynasties, the measure of all such interactions is $(\frac{\alpha}{2})^n$. Choose a sequence of a strictly positive measure of dynasties whose beliefs converge to one slow enough so that at T_ε their distance from one is $\eta > 0$ such that $\chi(\mathbf{q}^{t+1}, q_{i,e}^t)$ is at most 2ε. Then we will have:

$$\hat{\Delta}_i^t - \hat{\Delta}_1^t < -(\frac{\alpha}{2})^n (\int_{\theta 2\varepsilon}^{\theta} (\theta - w)dw) + \theta^2\eta < 0$$

whenever ε and η are low enough (note that α does not depend on ε). Thus in the next period a strictly positive measure of state school parents will send their kids to the private school. We can then follow the remainder of the proof of Proposition 2.■

Proof of Proposition 4: We will show that $\hat{\Delta}_i^t$ for $q_i = q^*(\tau, n)$, that is, the type at the cutoff, is monotonically decreasing in $q^*(\tau, n)$. With some abuse of notation, denote this by $\hat{\Delta}_{q^*}^t$. Thus for more exclusive private schools in terms of their beliefs, $\hat{\Delta}_{q^*}^t - \hat{\Delta}_1^t = \hat{\Delta}_{q^*}^t$ will be higher.

$$\begin{aligned}
&\hat{\Delta}_{q^*}^t \\
&= \pi_{q^*}^t - (q_{q^*,e}^t - \frac{1}{2}) \\
&= \int_{\mathbf{q}^{t+1}} (\int_0^{\chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)} (q_{q^*,e}^t - w) dw) dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) - (q_{q^*,e}^t - \frac{1}{2}).
\end{aligned}$$

Taking a derivative w.r.t q^*, we get

$$\frac{\partial q_{q^*,e}^t}{\partial q^*} (\int_{\mathbf{q}^{t+1}} (\frac{\partial \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)}{\partial q_{q^*,e}^t} (q_{q^*,e}^t - \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)) + \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)) dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) - 1)$$

Note that $\frac{\partial \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)}{\partial q_{q^*,e}^t}(q_{q^*,e}^t - \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)) + \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t) < 1$ and thus the above is negative as required. To see why, note that the above is $\frac{\partial \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)}{\partial q_{q^*,e}^t} q_{q^*,e}^t + (1 - \frac{\partial \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)}{\partial q_{q^*,e}^t}) \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)$. As $\frac{\partial \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)}{\partial q_{q^*,e}^t} < 1$, we have a convex combination of two beliefs that are both lower than a half. To see why $\frac{\partial \chi(\mathbf{q}^{t+1}, q_{q^*,e}^t)}{\partial q_{q^*,e}^t} < 1$, let x be the geometric average of the vector of beliefs $\mathbf{q}^{t+1}$, so that $x = (\prod_{i=1}^{n} q_i)^{\frac{1}{n}}$. Thus we can write $\chi(\mathbf{q}^{t+1}, q_{q^*,e}^t) = \frac{x^n q_{q^*,e}^t}{x^n q_{q^*,e}^t + (1 - q_{q^*,e}^t)(1-x)^n}$. The derivative of this w.r.t. $q_{q^*,e}^t$ is $\frac{x^n (1-x)^n}{\left(q_{q^*,e}^t x^n - q_{q^*,e}^t (1-x)^n + (1-x)^n\right)^2}$. We need to show that $x^{\frac{n}{2}} (1-x)^{\frac{n}{2}} < q_{q^*,e}^t x^n - q_{q^*,e}^t (1-x)^n + (1-x)^n$, or that $1 < q_{q^*,e}^t \frac{x}{1-x}^{n/2} + \frac{1-x}{x}^{n/2} (1 - q_{q^*,e}^t)$. Note however that $0.5 \frac{x}{1-x} + \frac{1-x}{x}(0.5) > 1$, and as $q_{q^*,e}^t, x < 0.5$, for all higher n, we have the required result.

Finally note that the highest willingness to pay is $\hat{\Delta}_{q^*}^t|_{q^* \to 0} \to \frac{1}{2}$.■

6.2 Appendix B: Endogenous wages

In this section of the Appendix we offer an alternative formulation of the labour market in which wages are determined by Nash bargaining. Suppose that in any period, for any employer, a position is made vacant. The new position requires a cost to train any new hire. The cost c is drawn unifromly from $[0, 1]$. If hired for the position, an employee with

productivity $\theta \in \{0,1\}$ will yield a net rent for the employer of $\theta - c$. If the employee is not hired, both the employer and the employee get zero in that period.

Given a belief $q_{i,g}^t$ of the employer, if the employee is hired, the total rents in the eyes of the employer are $q_{i,g}^t - c$. For simplicity of exposition we assume that the employee and the employer equally share these perceived rents generated by the match. Any (fixed) interior allocation is fine for our analysis.[41] Thus, a private school graduate will be employed by all, whereas a state school graduate employee will be employed by an employer from dynasty i at period t if and only if $c \leq q_{i,g}^t$.

With this formulation we can now prove Proposition 2:

Sufficiency: We know from the above that beliefs in the private school always remain below $q^* < \frac{1}{2}$. We will now show that for all $q < \frac{1}{2}$, given that beliefs in the private school are interior, then $\hat{\Delta}_i^t > \hat{\Delta}_1^t$. This implies that only the original dynasties remain in the school. We therefore need to show that:

$$\begin{aligned}
&\hat{\Delta}_i^t \\
&= \pi_i^t - (q_{i,e}^t - \frac{3}{4}) \\
&= \int_{\mathbf{q}^{t+1}} (\int_0^{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)} (q_{i,e}^t - c - \frac{\chi(\mathbf{q}^{t+1}, q_{i,e}^t) - c}{2}) dc) dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) - (q_{i,e}^t - \frac{3}{4}) > 0 = \hat{\Delta}_1^t.
\end{aligned}$$

A sufficient condition for this is that for any $\mathbf{q^{t+1}}$:

$$\int_0^{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)} (q_{i,e}^t - c - \frac{\chi(\mathbf{q}^{t+1}, q_{i,e}^t) - c}{2}) dc) dF_{p,n}^{t+1}(\mathbf{q}^{t+1}) - (q_{i,e}^t - \frac{3}{4}) > 0,$$

and this holds as $q_{i,e}^t < \frac{1}{2}$. Finally note that the beliefs in the private school would converge to the singleton 0 as they are always below a half. This concludes the sufficiency part as the measure ρ of private school dynasties dynasties will have a strictly high willingness to pay for the private school compared with state school dynasties.

Necessity: We can repeat exactly the same proof as in Proposition 2, to show that eventually we would have a dynasty in the private school for whom $\Delta_i^t < \Delta_s^t$. Specifically, to show this, the condition would become:

[41] When the employer makes a take-it-or-leave-it offer he will offer a wage of zero to the employee leaving the whole surplus $q_{i,g}^t - c$ to himself. When the employee makes a take-it-or-leave-it offer he will offer a wage of $q_{i,g}^t - c$ leaving the employer with zero rents. When the bargaining power of the two sides is more even, they will share the surplus $q_{i,g}^t - c$ more evenly.

$$
\begin{aligned}
&\hat{\Delta}_i^t - \hat{\Delta}_1^t \\
&= (1 - q_{i,e}^t) + \int_{\mathbf{q}^{t+1}} \Big(\int_0^{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)} (q_{i,e}^t - 1 + \frac{1 - \chi(\mathbf{q}^{t+1}, q_{i,e}^t)}{2}) dc \Big) \\
&\quad - \int_{\mathbf{q}^{t+1}} \int_{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)}^1 (1 - c - \frac{1-c}{2}) dc \Big] f_{p,n}^{t+1}(\mathbf{q}^{t+1}) d\mathbf{q}^{t+1} \\
&< \chi(\mathbf{q}^{t+1}, q_{i,e}^t)(\frac{1 - \chi(\mathbf{q}^{t+1}, q_{i,e}^t)}{2}) - \int_{\mathbf{q}^{t+1}} \int_{\chi(\mathbf{q}^{t+1}, q_{i,e}^t)}^1 (\frac{1-c}{2}) dc \Big] \; f_{p,n}^{t+1}(\mathbf{q}^{t+1}) \; d\mathbf{q}^{t+1} + (1 - q_{i,e}^t)
\end{aligned}
$$

Now choose ε and T_ε such that a measure $\frac{\alpha}{2}$ of dynasties have beliefs below ε. Note that if i interacts with n of these dynasties, the measure of all such interactions is $(\frac{\alpha}{2})^n$. Choose a sequence of a strictly positive measure of dynasties whose beliefs converge to one slow enough so that at T_ε their distance from one is $\eta > 0$ such that $\chi(\mathbf{q}^{t+1}, q_{i,e}^t)$ is at most 2ε. Then we will have:

$$
\hat{\Delta}_i^t - \hat{\Delta}_1^t < 2\varepsilon - (\frac{\alpha}{2})^n \int_{2\varepsilon}^1 (\frac{1-c}{2}) dc + \eta < 0
$$

whenever ε and η are low enough (note that α does not depend on ε). Thus in the next period a strictly positive measure of state school parents will send their kids to the private school. This would imply a breakup of segregation.■

References

[1] Abdulkadiroğlu, A., J. Angrist, and P. Pathak (2014), The Elite Illusion: Achievement Effects at Boston and New York Exam Schools. Econometrica, 82: 137–196.

[2] Akerlof, R. and R. Kranton (2000), Economics and Identity, The Quarterly Journal of Economics, 115, pp. 715-753.

[3] Algann, Y., N. Dalvit, and Q-A. Do (2015), How Social Networks Shape our Beliefs: A Natural Experiment Among Future French Politicians, mimeo.

[4] Allport G. W. (1954), The nature of prejudice. Cambridge, MA: Perseus Books.

[5] Angrist, J.,and K. Lang (2004), Does School Integration Generate Peer Effects? Evidence from Boston's Metco Program, American Economic Review, 94(5), 1613-1634.

[6] Arrow, K. J. (1973), The Theory of Discrimination, in Discrimination in Labor Markets, ed.by O. Ashenfelter, and A. Rees, pp. 3-33. Princeton University Press.

[7] Austen-Smith D. and R. Fryer (2005). An Economic Analysis of 'Acting White'. Quarterly Journal of Economics.

[8] Baccara, M. and L. Yariv (forthcoming), "Choosing Peers: Homophily and Polarization in Groups", Journal of Economic Theory.

[9] Barber, M. and N. McCarty (2013), The Causes and Consequences of polarisation, in *Negotiating Agreement in Politics,* eds. Cathie Jo Martin and Jane Mansbridge. APSA task force report.

[10] Benabou, R. (1993), Workings of a City: Location, Education, and Production, Quarterly Journal of Economics, 108, 619-652.

[11] Benabou, R. (2013), Groupthink: Collective Delusions in Organizations and Markets, Review of Economic Studies 80, 429-462.

[12] Benabou, R and J. Tirole (2006), Belief in a Just World and Redistributive Politics, Quarterly Journal of Economics, 121(2), pp. 699-746.

[13] Bisin, A. and T. Verdier (2001), The economics of cultural transmission and the dynamics of preferences, Journal of economic theory 97(2), 298-319.

[14] Black, D. (1995), Discrimination in an Equilibrium Search Model, Journal of Labor Economics, Vol. 13, No. 2, pp. 309-334.

[15] Boisjoly, J., G.J. Duncan, M. Kremer, D. M. Levy, and J. Eccles. (2006). Empathy or Antipathy? The Impact of Diversity. American Economic Review, 96(5): 1890-1905.

[16] Bradford, D. H. Kelejian (1973), An Econometric Model of the Flight to the Suburbs, Journal of Political Economy, Vol. 81, No. 3, pp. 566-589.

[17] Calabrese, S., D. Epple, and R. Romano (2012), Inefficiencies from Metropolitan Political and Fiscal Decentralization: Failures of Tiebout Competition, Review of Economic Studies 79(3), 1081-1111.

[18] Calabrese, S., D. Epple, T. Romer, and H. Sieg (2006), Local public good provision: Voting, peer effects, and mobility. Journal of Public Economics, 90, 959-981.

[19] Calvó-Armengol A, Jackson M. (2004), The effects of social networks on employment and inequality. American Economic Review 94:426–54.

[20] Chaudhuri, S. and R. Sethi (2008), Statistical Discrimination with Peer Effects: Can Integration Eliminate Negative Stereotypes?, Review of Economic Studies, Volume 75, Issue 2, pp. 579-596.

[21] Chetty, R., Hendren, N., Kline, P., & Saez, E. (2014). Where is the land of opportunity? The geography of intergenerational mobility in the United States (No. w19843). National Bureau of Economic Research.

[22] Coate, S. and G. Loury (1993), Will affirmative action policies eliminate stereotypes?, American Economic Review 83(5), 1220-40.

[23] Dandekar, P., Goelb, A. and Leec, D. T., (2013), "Biased assimilation, homophily, and the dynamics of polarization", *Proceedings of the National Academy of Sciences in the US*, Vol. 110 no. 15.

[24] De Marzo P. M., Vayanos D, Zwiebel J. (2003). Persuasion bias, social ináuence and unidimensional opinions. Q. J. Econ.

[25] Dobbie W. and R. Fryer (2014), The Impact of Attending a School with High-Achieving Peers: Evidence from New York City Exam Schools, American Economic Journal: Applied Economics 6(3):58-75.

[26] Dustmann, C. and I., Preston (2001), Attitudes to Ethnic Minorities, Ethnic Context and Location Decisions, The Economic Journal. Volume 111, Issue 470, pages 353–373, March.

[27] Echenique F., R. Fryer, and A. Kaufman (2006), Is School Segregation Good or Bad?. American Economic Review.

[28] Enke, B. (2016), "What You See Is All There Is", Manuscript, University of Bonn.

[29] Enke, B., and F. Zimmerman (2015), "Correlation Neglect in Belief Formation", mimeo.

[30] Epple, D., and R. Romano (2008), Educational Vouchers And Cream Skimming, International Economic Review, vol. 49(4), pages 1395-1435.

[31] Epple, D., and R. Romano (1998), Competition between Private and Public Schools, Vouchers, and Peer-Group Effects, American Economic Review 88: 33-62.

[32] Epple, D. and H. Sieg (1999), Estimating Equilibrium Models of Local Jurisdictions. Journal of Political Economy, 107 (4), 645-681.

[33] Evans and Tilley (2011), Private schools and public divisions: the influence of fee-paying education on social attitudes, in British Scoial Attitudes No. 28. London: National Centre for Social Research.

[34] Eyster, E. and M. Rabin (2010), Naïve herding in rich-information settings. American Economic Journal: Microeconomics, 2 (4). pp. 221-243.

[35] Fang, H., and A. Moro (2010), Theories of Statistical Discrimination and Affirmative Action: A Survey, in Handbook of Social Economics, ed. by J. Benhabib, A. Bisin, and M. Jackson.

[36] Fryer, R. and M. Jackson (2008), A Categorical Model of Cognition and Biased Decision Making, Contributions in Theoretical Economics 8(1).

[37] Fudenberg, D. and D. Levine (1993), Self Confirming Equilibria, Econometrica 61(3), 523-545.

[38] Gagnon-Bartsch, T. and M. Rabin (2015), Naive Social Learning, Mislearning, and Unlearning, mimeo.

[39] Gentzkow, M. and J. Shapiro (2011), Ideological Segregation Online and Offline, Quarterly Journal of Economics, 126 (4).

[40] Glaeser, E. and C. R. Sunstein (2009), Extremism and social learning, Journal of Legal Analysis, Volume 1, Number 1.

[41] Golub, B. and M. Jackson (2012), "How Homophily Affects the Speed of Learning and Best-Response Dynamics", Quarterly Journal of Economics, pp. 1287–1338.

[42] Granovetter, M. (1974), Getting a Job, Chicago University Press.

[43] Hoxby, C. (1994), Do Private Schools Provide Competition for Public Schools? www.nber.org/papers/w4978.

[44] Kaufmann, E. and G. Harris (2015), 'White Flight' or Positive Contact? Local Diversity and Attitudes to Immigration in Britain, Comparative Political Studies, 1–28.

[45] La Ferrara, E., J. Burns and L. Corno (2014), Interaction, prejudice and performance: evidence from randomly assigned peers in South Africa. Presentation in the CEPR public economics conference, Warwick University, December 2014.

[46] Lang, K., M. Manove and W. Dickens (2005), Racial Discrimination in Labor Markets with Posted Wage Offers, American Economic Review, 95(4), 1327-1340.

[47] Levy, G. and. R. Razin (2016), Correlation Capacity, Ambiguity and Group Shifts, mimeo.

[48] Levy, G. and R. Razin (2015), Correlation Neglect, Voting Behaviour and Information Aggregation, American Economic Review, 105 (4), 1634-1645.

[49] Levy, G. and R. Razin (2012), Religious Beliefs, Religious Participation and Cooperation, American Economic Journal: Microeconomics, 4(3), pp. 121-151.

[50] Mailath, G. and A. Postlewaite (2006), Social Assets, International Economic Review 47(4), pp. 1057-1091.

[51] Marmaros, D.,and B. Sacerdote (2002), Peer and Social Networks in Job Search, European Economic Review, 46(4-5), 870-879.

[52] MacLeod, B. and M. Urquiola (2015), Reputation and School Competition, American Economic Review, 105(11): 3471–3488.

[53] Moro, A., P. Norman (2004), A General Equilibrium Model of Statistical Discrimination, Journal of Economic Theory, 114, 1-30.

[54] Ok, E. (2004), Fixed Set Theory for Closed Correspondences with Applications to Self-Similarity and Games, Nonlinear Analysis 56, pp. 309-330.

[55] Ortoleva, P., and E. Snowberg (2015), "Overconfidence in Political Behavior." American Economic Review, 105(2): 504-35.

[56] Peski, M. and B. Szentes (2013), Spontaneous Discrimination, American Economic Review, vol. 6, 2412-2436.

[57] Peski, M. (2008), Complementarities, Group Formation and Preferences for Similarity, mimeo.

[58] Phelps, E. (1972), The Statistical Theory of Racism and Sexism, American Economic Review 62: 659-661.

[59] Piketty, T. (1995). Social Mobility and Redistributive Politics, Quarterly Journal of Economics 110: 551-83.

[60] Reardon, S.F., & Bischoff, K. (2011). More Unequal and More Separate: Growth in the Residential Segregation of Families by Income, 1970-2009. Report prepared for the Russell Sage US2010 project. www.s4.brown.edu/us2010/Data/Report/report111111.pdf.

[61] Sacerdote, B. (2011), Peer Effects in Education: How Might They Work, How Big Are They and How Much Do We Know Thus Far?, in Handbook of the Economics of Education Volume 3, ed. by E. Hanushek, S. Machin,and L. Woessmann, pp. 249-277. Elsevier.

[62] Sethi, R. and M. Yildiz (2016), "Communication with Unknown Perspectives", mimeo.

[63] Simon, C. and J. T. Warner (1992), Matchmaker, Matchmaker: The Effect of Old Boy Networks on Job Match Quality, Earnings, and Tenure, Journal of Labor Economics, Vol. 10, No. 3, pp. 306-330.

[64] Sobel, J. (2014), On the Relationship between Individual and Group Decisions, Theoretical Economics 9, 163–185.