Pauline Barrieu, Nadine Bellamy,
Bernard Sinclair-Desgagné

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Assessing Contaminated Land Cleanup Costs and Strategies*

Pauline Barrieu†
London School of Economics

Nadine Bellamy‡
Université d'Évry

Bernard Sinclair-Desgagné§
HEC Montréal

Abstract: The remediation of contaminated sites is often subject to substantial cost overruns. This persistent discrepancy between estimated and realized costs is chiefly responsible for misguided land use and wasteful delays in the reconversion of former industrial sites. In order to deal with incomplete information and uncertainty in this context, this paper draws on stochastic modelling and mathematical finance methods. We show that relatively simple and usable formulas can then be derived for better assessing cleanup strategies. These formulas apply to generic remediation technologies and scenarios. They are robust to misspecification of key parameters (like the effectiveness of a prescribed treatment). They also yield practical rules for decision making and budget provisioning.

Keywords: Contaminated sites, brownfields, remediation management, regulatory cost estimates, hitting-time models, real-options theory

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† Pauline Barrieu, Department of Statistics, London School of Economics, Houghton Street, London WC2A 2AE, United Kingdom; p.m.barrieu@lse.ac.uk

‡ Nadine Bellamy, Laboratoire de mathématiques et modélisation, Université d'Évry, 23 Boulevard de France, 91037 Évry Cedex, France; Nadine.Bellamy@maths.univ-evry.fr

§**Corresponding author:** Bernard Sinclair-Desgagné, Environmental Economics and Global Governance Chair, HEC Montréal, 3000 Côte-Sainte-Catherine, Montréal, Canada H3T 2A7; bsd@hec.ca .

> "The Pentagon was so sloppy when calculating the cost of removing perchlorate and other toxins from military sites that its estimate of $16 billion to $165 billion is almost worthless, congressional investigators said Tuesday." [1]

1. Introduction

Over the last decades, the number of potentially contaminated sites in most developed countries has grown to six or seven digits. In the European Union, more than 250,000 sites are deemed to be contaminated and requiring remediation (Swartjes 2011). In the United States, it is estimated that there are over 500,000 'brownfield' sites in urban, suburban and rural areas, and that their cleanup and redevelopment could cost more than $650 million (Bressler and Hannah 2000; Wernstedt et al. 2004).[2] While some of these sites might cover areas of several acres, most of them might only be 'micro-sites' of a few squared-yards. But their sheer number and growing opportunity cost, the threats they may pose to human health, wildlife, and land amenities, and the lasting stigma they can put on certain locations, activities, industries, firms or even individuals have now brought the matter on top of many policy makers' and corporate boards' agenda.

Dealing with contaminated sites raises a number of regulatory and business issues. Many of them have been addressed over the last 40 years, and the researchers' proposed remedies have then often been implemented into effective policies. Since insolvency and

[1] Lisa Friedman, "Report says cleanup cost miscaculated," *Los Angeles Daily News*, June 28, 2004.

[2] The U.S. Environmental Protection Agency (EPA) defines a brownfield as "an abandoned, idle or underused industrial and commercial facility where expansion and redevelopment is complicated by real or perceived environmental contamination." As pointed out by an anonymous referee, the large number of brownfield sites is mainly due to inadequate treatment of chemical liquids (such as oil) in the past (e.g., in gas, chemical industry, heavy industry, dry-cleaning, and filling stations works). The growing number of detected contaminated brownfield sites that was reported over the last three decades is to a very large extent a result of gradual improvement and enhancement of investigation activities.

limited liability often make it impossible to apply the polluter-pays principle, for instance, it has been found that assigning some responsibility to lenders and other deep-pocket parties up or down the value chain might contribute to set incentives right (see Alberini et al. 2005, and Hiriart and Martimort 2006, among others). Since the benefits of remediation can be unclear, several works have developed rigorous benefit-assessment techniques (see Swartjes 2011, Haninger et al. 2014, and the references therein). Since redevelopment projects are frequently deterred by the perspective of unexpected costs and liability expenses, two applied policy prescriptions are to preventively tax land users (which amounts to enforcing precautionary savings on the implicated parties), as the *Superfund* Program does in the United States, and to enhance risk sharing by the introduction of proper insurance, financial and organizational means (see Yin et al. 2011, Zhang et al. 2012, Medda et al. 2012, and Schroeder 2013, among many others). Coping with abandoned sites and the difficulty to trace back contamination to its originator(s), however, remains a major challenge for law makers. In the end, the effectiveness of any public policy, market remedy, insurance scheme or business plan inevitably depends on the reliability of cleanup cost estimates. This paper focuses on this question.

The evaluation of remediation investments is routinely done using the traditional net-present-value (NPV) method. A stylized example, taken from Zhang (2009, p. 1), illustrates the shortcomings of this approach.

> Two strategies are compared: strategy one is to implement P&T (pump and treat) for the entire decision time frame; strategy 2 is to implement PRB (permeable reactive barrier) for the entire decision time frame. The traditional method will value these two strategies based on their respective cash flows:

> \$10,000 per year for the former, \$30,000 on year 1 and \$1000 thereafter for the latter. Assuming a discount rate of 5% and a five-year horizon, the NPV of strategy 1 would be \$45,460, whereas the NPV of the PRB strategy would amount to only \$33,546. However, it is not taken into account that one technology (here: P&T) might be more flexible than the other one (here: PRB) if the conditions at the site develop differently than expected. What if the concentration of pollutants after two years of P&T is low enough to switch to a cheaper (\$5000 a year) alternative like monitored natural attenuation (MNA)? This would reduce the cost of strategy 1 to \$32,492. Or what if P&T turns out to meet the remediation target after only three years and can then be stopped? The NPV of strategy 1 over three years would then amount to \$28,594, which is much lower than the cost of strategy 2.

Basically, by ignoring uncertainty, learning and adaptation, the NPV approach overlooks the value of flexibility; it misses many contingency-based scenarios and might therefore lead to making costlier decisions. This important general fact was emphasized some time ago by Henry (1974) and Arrow and Fisher (1974). Herath and Park (2000; 2001) have conveyed it further to engineering economists, applying the more recent machinery of 'real options' put forward by Dixit and Pindyck (1994). The relevance of flexible strategies when dealing with contaminated sites was then successively pointed out by Bage et al. (2002, 2003), Wang and McTernan (2002), and Zhang (2009).

While the advantages of applying the logic of real options to site remediation projects are now well-understood, usable methods that would make valuing flexible strategies standard in practice are still lacking. To overcome this, and get operational means to correctly assess the costs of contingent cleanup strategies, this paper draws from the literatures on stochastic processes, mathematical finance and the theory of real options.

Specifically, our mathematical arguments borrow from the study of hitting times for a geometric Brownian motion (see, e.g., Jeanblanc et al. (2004), and the references therein).

The first part of this paper considers the estimation of remediation costs. This involves the computation of the probability of being compliant (i.e., reaching a contamination level that lies below the mandatory threshold) at various times including the required deadline as well as each intermediate time where monitoring is required. The second part next focuses on developing effective rules for decision making and budget provisioning. This requires the characterization of a *decisional* contamination threshold which allows to select a remediation approach (and a corresponding budget) that would keep the probability of not meeting the compliance deadline at an acceptable level.

These developments rely on somewhat involved mathematical tools. Yet, the obtained formulas and strategies stick to the most common (software-based) probabilistic depictions of contaminated sites and can be computed relatively easily (again using common softwares, like Excel). The formula's sensitivity to measurement errors or misspecification of key parameters (like the effectiveness of a chosen remediation technology) can also be readily grasped.

The rest of this text unfolds as follows. In the upcoming section, we introduce the notation and assumptions needed to represent typical remediation strategies. Section 3 next derives closed-form formulas for assessing the cost of such a strategy. These formulas' robustness to parametric variations is examined in Section 4. Section 5 proposes a simple systematic way to set a contingent (hence flexible) site remediation strategy based on our formulas. Section 6 addresses the important practical matter of budget provisioning. Some extensions are discussed in the concluding Section 7. All proofs are in the Appendix.

2. The basic model

Consider a piece of land that has been characterized as contaminated. In this case, some accountable entity - a private firm or a public body - has been identified. It must now deploy effective and affordable means in order to comply with regulation within a certain deadline. This situation can be modelled as follows.[3]

2.1 *The decisionmaker's information and regulatory constraints*

Thanks to the site characterization report, the decision-making entity holds a reasonably accurate picture of the land's contaminants, their respective properties, extent and concentration, and the hazards they may thus pose to human health and the environment (in view of the area's specific location, geology, weather exposure, and current or anticipated use).[4] This information supports a summary score y_0 - concretely, a 'risk index' or a soil 'quality rating' (Swartjes 2011, p. 33 and 49) - along with a forecast of this score's likely evolution as the contaminants get naturally dispersed or transform over time.

Taking stock of measurement errors and the effect of random natural events (such

[3] Two warnings are in order here. First, our model intends to be general, but most of the procedures and technologies we mention below for concreteness apply mainly to *soil* remediation rather than groundwater treatment (the two categories of intervention). Second, this section begins on what is of course a highly stylized account of a real situation. 'Site characterization' is itself a complex process which calls upon many types of scientific expertise (geochemistry, geohydrology, toxicology, biology, etc.) and involves sophisticated multivariate and geostatistical tools (such as Kriging interpolation). As for regulatory constraints, they are often the result of demanding tradeoffs and negotiations between stakeholders (land owners, local residents, national and local public authorities, urban developers, scientists and engineers, and even international institutions), which seek to strike a fine balance between economic development and concerns about ecosystems and human health. For a more complete picture, see the introduction and contributed chapters in Swartjes (2011).

[4] There are thousands of listed contaminants. The most frequently encountered ones include metals and metalloids (cadmium, lead, copper, zinc, arsenic, ...), non-metal inorganic substances (cyanides, ...), aromatic hydrocarbons (benzene, toluene, ...), organic pollutants (polychlorinated biphenyls or PCBs, chloroethylenes, ...), and petroleum hydrocarbons. For a standard classification of contaminants and their respective health impacts, see Swartjes (2011).

as weather conditions, rainfall, etc.), any score estimation would be non-deterministic; concretely, a provided rating (now or in the future) may then be granted a margin which follows a Normal distribution (Swartjes 2011, p. 37). For our modelling purposes, this suggests that the evolution of the site's score can be captured by the stochastic process

$$Y_t = y_0 e^{\left(\mu_0 - \frac{\sigma^2}{2}\right)t + \sigma W_t}, \tag{1}$$

or equivalently

$$\begin{aligned} dY_t &= Y_t \left(\mu_0 dt + \sigma dW_t\right) \\ Y_0 &= y_0 \ , \end{aligned} \tag{2}$$

where the index $t \in [0, \infty)$ stands for time. This process, a geometric Brownian motion, embodies (i) the site's estimated capacity to naturally regenerate, through the average rate of pollution decay $\mu_0 < 0$, and (ii) the uncertainty surrounding the value of a stated score at any time t, through the random term σdW_t where σ is a positive real number and $(W_t; t \geq 0)$ is a standard Brownian motion defined on a reference probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The probability measure $\mathbb{P}$ formally represents the decision-maker's beliefs based on the available information; hence, letting $\mathbb{E}(\cdot)$ denote mathematical expectations under $\mathbb{P}$, we have that $\mathbb{E}\left(\frac{Y_t}{y_0}\right) = e^{\mu_0 t}$, which corresponds to the characterization report's prediction concerning the likely average evolution of the site's contamination score.

All things considered (site characterization, environmental law, the stakeholders' expressed preferences, etc.), the regulator deems any score above a certain threshold L to be unacceptable. However, the site characterization report indicates that $y_0 > L$. After negotiations, the responsible entity is given a time span T to work on the site so the measured contamination rating at the end of the period, Y_T, would be such that $Y_T \leq L$.

Failure to comply will result in the entity having to pay a penalty ζ, which we assume to be proportional to the difference between L and Y_T, i.e.

$$\zeta = k_{\text{fine}} \left(Y_T - L\right)^+ \quad , \tag{3}$$

where $k_{\text{fine}} > 0$ may not only account for the tax inflicted by the regulator but also the reputational stigma from violating the law, damaging nature, hurting the local residents' property value, and endangering people's health. For safety reasons, the regulator also requires the entity to perform a number N of monitorings at some pre-specified times

$$t_1 = \frac{1}{N}T,\ t_2 = \frac{2}{N}T,\ ..., t_k = \frac{k}{N}T,\ ...,\ t_N = T\ . \tag{4}$$

Each monitoring will cost a constant pre-agreed amount.[5] Of course, as soon as the mandated contamination level L is reached, remediation efforts will stop. From now on, we let T_L^Y denote the first time the threshold L is hit, i.e. $T_L^Y = \inf\{t\ /\ Y_t \leq L\}$.

2.2 *Remediation technologies*

The decision-making entity is now contemplating a set of technologies that could deal with the problem.[6] Whether a remediation technology is feasible or not, to begin with, depends on the actual contaminants (a given treatment may be suitable for organic materials but not for metals), the area's relevant features (e.g., peculiar soil characteristics such as texture, permeability, moisture content, etc.), the regulatory constraints, the technology's side effects, and certain regulatory standards ('landfarming', for instance, which involves thin spreading of excavated dirty soil, may be forbidden under some jurisdictions).

[5] Site monitoring is often outsourced to a specialized firm subject to a fixed-price contract.

[6] Kahn et al. (2004) provides an exhaustive overview of site remediation technologies, their respective properties and some real-life applications, together with a valuable list of references. The technology descriptions that follow draw without restraints from this article.

Aside from feasibility, remediation technologies are also usually classified into two distinct groups. The first subset includes what are called '*ex situ*' technologies. These approaches proceed by first excavating the contaminated soil and ship it elsewhere for handling. Possible modes of treatment then include:

(d) *Biopiles*, which consist in stacking the contaminated soil and then boost natural depollution processes using aeration or other devices;

(e) *Incineration*, where the contaminated soil is heated in order to release petroleum waste and/or destroy organic contaminants.

Ex situ technologies are radical and effective. Accordingly, and to simplify matters, let's assume that using one of these means will immediately bring the decision-making entity into compliance. The downside, however, is that ex situ technologies are largely irreversible and generally expensive.

The second group comprises what are called '*in situ*' technologies. Contrary to the above, an in situ approach requires no removal of soil material. Common examples are:

(a) *Bioventing*, which consists in injecting air into the contaminated media at a rate designed to maximize biodegradation;

(b) *Soil flushing*, in which contaminated soil is flooded with a solution that carries contaminants to a spot where they can be removed;

(c) *Natural attenuation*, which relies on natural processes to degrade contaminants, reduce their concentration, and/or bind them to the soil matrix so that their spreading is retarded and/or reduced.[7]

[7]Note that the precise definition of natural attenuation may vary from country to country; in some

In situ technologies are typically cheaper and more flexible.[8] They can usually be combined with or followed by other modes of treatment. But they require a longer time horizon on average, and their outcome is uncertain. This suggests that, under an in situ approach, the site contamination score will evolve according to the stochastic process

$$
\begin{aligned}
& dY_t = Y_t\left(\mu dt + \sigma dW_t\right) \\
& Y_0 = y_0 \ ,
\end{aligned}
\tag{5}
$$

where pollution decreases at a faster (but finite) mean rate $\mu \leq \mu_0$.[9]

Once the feasible remediation technologies and their respective properties are established, the decisionmaker then needs to assess and compare the cost of different cleanup strategies. This issue is taken up in the next section.

3. Some cost assessment formulas

Two generic remediation strategies will now be considered: uninterrupted in situ remediation, and interim in situ intervention followed by an ex situ approach. We shall compute the expected cost of each strategy, assuming a constant discount rate r.[10] Other decision criteria, dealing specifically with risk aversion, are indicated in the Conclusion.

3.1 *Relying on an in situ technology*

Suppose the decisionmaking entity chooses to proceed using only an in situ approach. According to the experts, the selected technology has a mean effectiveness rate given by μ.

cases, contaminants dilution is also included. We thank an anonymous referee for this remark.

[8] There are exceptions of course, such as 'vitrification' - an in situ technique which aims to melt soil or other earthen materials at extremely high temperatures (1600-2000 ^{o}C), thereby immobilizing most inorganic and destroying organic pollutants. But this confirms rather than invalidates our main point.

[9] Equality holds here for natural attenuation.

[10] This is without loss of generality. Allowing for a stochastic discount rate in the present framework is equivalent to changing the reference probability measure (El Karoui et al. 1998).

This strategy entails three types of expenses: (i) a fixed cost of implementation, $C^I(\mu)$, to be paid initially, which might be close to 0 for natural attenuation and increase as the efficiency parameter μ grows in absolute value, (ii) the operational and monitoring cost $C_1^{O\&M}(\mu)$ based on a constant total expected cost c incurred meanwhile the treatment is applied (this cost comprises operational expenses related to, say, electricity to operate pumps, chemicals to mix flushing solutions, etc., as well as the cost of monitoring the site at the end of the period), and (iii) the penalty ζ defined by Equation (3). The present value of penalties and total operational and monitoring expenditures are random numbers given respectively by

$$\begin{aligned} C_1^{O\&M}(\mu) &= c\sum_{k=1}^{N} e^{-rt_k}\mathbf{1}_{t_k<T_L^Y} \text{ , and} && (6)\\ C_1^{P}(\mu) &= e^{-rT}k_{\text{fine}}\left(Y_T-L\right)\times\mathbf{1}_{T<T_L^Y} \quad . && (7)\end{aligned}$$

Let $C_1(\mu)$ be the total cost associated with this strategy, i.e.

$$C_1(\mu) = C^I(\mu) + C_1^{O\&M}(\mu) + C_1^P(\mu) \quad . \tag{8}$$

It is possible to obtain a closed form expression for this cost's expected value. This expression will use the following notation:

$$\begin{aligned}\mathbf{F}_\mu^-(a,b,t) &\doteqdot \Phi\left(d_2(a,b,t)\right) - \left(\tfrac{L}{y_0}\right)^{\frac{2\left(\mu-\frac{1}{2}b^2\right)}{a^2}} \times \Phi\left(d_1(a,b,t)\right)\\ \mathbf{F}_\mu^+(a,b,t) &\doteqdot \Phi\left(d_2(a,b,t)\right) - \left(\tfrac{L}{y_0}\right)^{\frac{2\left(\mu+\frac{1}{2}b^2\right)}{a^2}} \times \Phi\left(d_1(a,b,t)\right)\end{aligned}$$

where Φ refers to the standard Normal cumulative distribution function, and $d_1(a,b,t) \doteqdot \frac{\frac{1}{a}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{a}-\frac{b}{2}\right)t}{\sqrt{t}}$, $d_2(a,b,t) \doteqdot \frac{-\frac{1}{a}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{a}-\frac{b}{2}\right)t}{\sqrt{t}}$.

PROPOSITION 1: The expected present value of the total cost associated with using an in situ technology with mean effectiveness rate μ is given by:

$$\begin{aligned}\mathbb{E}\left(C_1\left(\mu\right)\right) &= C^I\left(\mu\right) + c\sum_{k=1}^{N} e^{-r\frac{k}{N}T}\mathbf{F}_\mu^-\left(\sigma,\sigma,\tfrac{k}{N}T\right) \\ &+k_{\text{fine}}e^{-rT}\left(y_0 e^{\mu T}\mathbf{F}_\mu^+\left(\sigma,-\sigma,T\right) - L\mathbf{F}_\mu^-\left(\sigma,\sigma,T\right)\right)\end{aligned}$$

This formula will prove useful for decision-making and budget provisioning, as Sections 5 and 6 below will show. It might look rather cumbersome, but it involves only standard normal distributions and can thus be easily encoded. Quite intuitively, the value of $\mathbb{E}\left(C_1\left(\mu\right)\right)$ decreases with the required contamination threshold L and goes up with the operational costs c and the punishment k_{fine}.

3.2 *Combining in situ and ex situ approaches*

Suppose now that the decision-maker commits to treating the contaminated soil ex situ after having applied an in situ technology with effectiveness parameter μ for n periods. This strategy avoids paying the penalty ζ. Its total cost $C_2\left(n,\mu\right)$ consists of (i) the initial cost $C^I\left(\mu\right)$ for implementing the in situ technology, (ii) the operational and monitoring cost c incurred over the period $t_n < t_N$, and (iii) the implementation cost $C_2^V\left(n,\mu\right)$ of the ex situ technology, that is:

$$C_2\left(n,\mu\right) = C^I\left(\mu\right) + C_2^{O\&M}\left(n,\mu\right) + C_2^V\left(n,\mu\right)\ , \tag{9}$$

where $C_2^{O\&M}\left(n,\mu\right)$ is the total operational and monitoring cost given by

$$C_2^{O\&M}\left(n,\mu\right) = c\sum_{k=1}^{n} e^{-rt_k} \times \mathbf{1}_{t_k<T_L^Y}\ . \tag{10}$$

Let's assume that the cost of an ex situ approach is proportional to the contamination score at time t_n; this cost's present value is then

$$C_2^V(n,\mu) = e^{-rt_n} k_{\text{ex situ}} Y_{t_n} \times \mathbf{1}_{t_n < T_L^Y} \tag{11}$$

where $k_{\text{ex situ}}$ is a positive constant. A closed form expression for the expected total cost in this case is again at hand.

PROPOSITION 2: The expected present value of the total cost associated with a strategy involving an ex situ approach that will take place after having used an in situ technology of effectiveness parameter μ up to time t_n is given by:

$$\mathbb{E}(C_2(n,\mu)) = C^I(\mu) + c\sum_{k=1}^{n} e^{-\frac{rk}{N}T}\mathbf{F}_\mu^-\left(\sigma,\sigma,\frac{k}{N}T\right) + k_{\text{ex situ}} y_0 e^{-(r-\mu)t_n}\mathbf{F}_\mu^+(\sigma,-\sigma,t_n) \tag{12}$$

The expexted cost $\mathbb{E}(C_2(n,\mu))$ increases with c and $k_{\text{ex situ}}$, and decreases with the mandatory contamination standard L.

Examples of $\mathbb{E}(C_1(\mu))$ and $\mathbb{E}(C_2(n,\mu))$ for different values of n and μ and selected but fixed parameters c, r, and σ are depicted in Figure 1. Both these expected costs are convex and decreasing in $\mid \mu \mid$ (i.e. as the situ technology becomes more effective). The latter also goes down when switching to an ex situ treatment is delayed (i.e. as n increases). When the absolute value of μ is low, combining in situ and ex situ approaches is less expensive than relying on in situ treatment only; the reverse holds when the absolute value of μ is high. From now on, we shall assume this situation to hold.[11]

Insert Figure 1 about here.

Of course, a remediation strategy could be more complex than the one described here,

[11] This will always be the case if operational and monitoring costs are not too large.

involving, say, two in situ technologies (e.g., natural attenuation, then soil flushing) followed by the excavation and incineration of the remaining contaminated soil. Provided each remediation technology fits the assumptions made above, the approach which underlies our propositions can handle these cases as well and generate similar formulas. Indeed, as the Appendix shows, our assumptions allow to invoke a key lemma, shown in Douady (1998) and Jeanblanc et al. (2004), which expresses the probability the random time threshold T_L^Y is greater than some given time t as the weighted difference of two Normal cumulative distributions; computing expected costs formulas similar to the ones in Propositions 1 and 2 is then straightforward.

Thanks to Propositions 1 and 2, the formulas' sensitivity with respect to one essential parameter - the predicted effectiveness rate μ of an in situ treatment - can readily be assessed, as we will now see.

4. Robustness

Suppose now that, instead of ascribing a point estimate to the effectiveness rate of a given in situ technology, the decision-maker deems this rate μ to lie between some reasonable bounds $\underline{\mu}$ and $\overline{\mu}$. This belief may be based on laboratory tests, as well as the information conveyed by the site characterization report. The next proposition then sets an interval on the random total costs.

Proposition 3: Assume there exist two negative constants $\underline{\mu}$ and $\overline{\mu}$ such that $-\infty < \underline{\mu} \leq \mu \leq \overline{\mu} < 0$. Then we have (i) $C_1\left(\underline{\mu}\right) \leq C_1\left(\mu\right) \leq C_1\left(\overline{\mu}\right)$ for a pure in situ intervention, and (ii) $C_2\left(n, \underline{\mu}\right) \leq C_2\left(n, \mu\right) \leq C_2\left(n, \overline{\mu}\right)$ under a mixed in situ/ex situ approach.

This result is rather strong, since it is written in terms of the random total costs themselves, not in terms of their expected value. One important benefit is to allow better forecasts of future expenses, hence better provisioning of financial, organizational and technological resources. Provisioning is further addressed in Section 6.

The decision-making entity might also want to assess the cost difference from using two distinct in situ techniques. Our next statement provides a bound on this difference (this time, however, in expected values).

PROPOSITION 4: Let μ^* and μ^+, $\mu^* > \mu^+$, represent two in situ technologies' respective mean effectiveness rate. Then:

i) If only in situ technologies are involved, for some constant δ_N we have that

$$0 \leq \mathbb{E}\left[C_1\left(\mu^*\right) - C_1\left(\mu^+\right)\right] \quad \leq \left[C^I\left(\mu^*\right) - C^I\left(\mu^+\right)\right] + \delta_N\left(\mu^* - \mu^+\right) + e^{-rT} k_{\text{fine}} e^{\left(\mu^* - \mu^+\right)T}$$

ii) And if an ex situ approach is deployed at time t_n, for some constant δ_n we have

$$0 \leq \mathbb{E}\left[C_2\left(n, \mu^*\right) - C_2\left(n, \mu^+\right)\right] \quad \leq \left[C^I\left(\mu^*\right) - C^I\left(\mu^+\right)\right] + \delta_n\left(\mu^* - \mu^+\right) + e^{-rt_n} k_{\text{ex situ}} e^{\left(\mu^* - \mu^+\right)t_n}$$

The constants δ_N and δ_n summarize the bounds on the expectations, which are given by some finite deterministic series (see the Appendix).

The accountable entity could use this result on at least two sorts of occasions. First, for a number of reasons (e.g., social acceptability, local employment, technology transfers, etc.), implementing the most effective or the cheapest in situ technology could still be questioned; while it may then be difficult to assign monetary value to these additional considerations, proposition 4 gives an order of magnitude for how great this value should be in order to compensate for the extra spending. Second, suppose that two subcontractors are bidding for implementing the in situ treatment, each one proposing a distinct approach; the latter inequalities offer a way to judge whether the gap between bids is plausible (a legitimate concern if the winning firm is going to operate under a cost-sharing contract).

The above statements were obtained under predetermined cleanup strategies. Thanks to these results, however, the next section will now address the design of such a strategy.

5. Contingent strategies

Suppose the site's depollution has already begun using a given in situ technology with effectiveness rate μ. We shall now consider a rule that the decisionmaking entity could use at any time t in order to decide whether or not to switch to an ex situ approach.

Since operational and monitoring expenses are often significant, and postponing the eradication of contamination entails the provisioning of valuable resources, the accountable entity may not want to pursue in situ treatment when the odds of succeeding are low. This view can be expressed by holding a 'tolerance level' α, $0 < \alpha < 1$, (which might also embed the entity's tolerance of risk, since the outcome from continuing the in situ treatment is always uncertain) and a rule stipulating that

An ex situ approach is adopted whenever
the site's score exceeds some dynamic trigger b_t^α *defined as*

$$b_t^\alpha = \max(L, \beta_t^\alpha) \ \text{ where } \beta_t^\alpha = \inf\left\{b_t \ / \ \mathbb{P}\left(Y_T > L \ / \ Y_t = b_t\right) \geq \alpha\right\} \ . \tag{13}$$

The rule thus asks to keep up with in situ treatment as long as meeting the regulator's demands at or before time T remains within acceptable sight in probabilistic terms, or as long as the site's current risk index Y_t makes reaching the legal threshold L on time sufficiently likely (likelier than α).

The following technical assumption will prove useful in characterizing b_t^α.

Assumption: Let Φ denote the standard Normal cumulative distribution function.
Then $\Phi\left(\frac{\left(\mu-\frac{\sigma^2}{2}\right)\sqrt{T}}{\sigma}\right) \leq \alpha \leq 0.5$.

The second inequality, $\alpha \leq 0.5$, seems reasonable, for a properly accountable entity would hardly tolerate a large probability of being found noncompliant. As to the first inequality, notice that μ is negative, which implies that $\Phi\left(\frac{\left(\mu-\frac{\sigma^2}{2}\right)\sqrt{T}}{\sigma}\right) < 0.5$. Further justification will come after the next proposition.

Proposition 5: For $0 < \alpha < 1$ and time $t < T$, the trigger b_t^α defined by (14) can be written as

$$b_t^\alpha = \max[L,\ L \exp\left(\sigma\sqrt{T-t}\Phi^{-1}(\alpha) - \left(\mu - \frac{\sigma^2}{2}\right)(T-t)\right)] \tag{14}$$

As it can be seen more easily now, assuming that $\Phi\left(\frac{\left(\mu-\frac{\sigma^2}{2}\right)\sqrt{T}}{\sigma}\right) \leq \alpha$ ensures that the coefficient β_0^α in (13) is such that $\beta_0^\alpha \geq L$. This condition seems natural: if it does not hold, then $b_0^\alpha = L$ so an ex situ treatment should have been adopted right away.

Taking stock of proposition 5, one can now consider the behavior of the trigger b_t^α as time elapses. The next figure illustrates the pattern for different values of α.[12] As

[12] We focus here on α, as the decision-making entity should determine its tolerance level *after* the

expected, when α gets smaller, so the accountable entity's tolerance of failure decreases, the subjective lower bound b_t^α for site contamination scores that would mandate a radical intervention goes down.[13] Other observable features are formally stated in our last proposition.

Insert Figure 2 about here.

PROPOSITION 6: The decision trigger b_t^α has the following properties:

i) $b_T^\alpha = L$;

ii) it is decreasing in the current time t ;

iii) it is a convex function of the current time t.

These attributes make the above decision rule rather sensible. Under this rule, compliance is ultimately achieved (property i), and the pressure to switch to a fully dependable ex situ treatment grows more and more as the deadline T looms (properties ii and iii).

As we shall now see, taking on such a trigger strategy additionally allows to handle budget provisioning - an important practical matter - in a systematic fashion.

6. Budget provisioning

Before engaging in any remediation project, the responsible entity would normally have to put aside some capital and build provisions. Suppose that she plans to use the remediation strategy defined in the preceding section. Suitable provisions could then be established as follows.

remediation technologies have been selected (so these technologies can be considered as given). Besides, considering various values for the parameters μ and σ would yield the same qualitative pattern for b_t^α.

[13] The reader might observe that the values $\mu = -.02$ and $\sigma = .03$ used here differ from the values $-0.9 \leq \mu \leq -0.1$ and $\sigma = 0.2$ employed in Figure 1. The only reason is that it makes each graph as easy to read as possible.

Let $\widehat{\tau}$ denote the monitoring time at which the in situ treatment is terminated and replaced by an ex situ approach, i.e.

$$\widehat{\tau} = \inf \{ t \ / \ t \in \{0, t_1, t_2, ..., t_N\} \quad \text{and} \quad Y_t \geq b_t^{\alpha} \} \ .$$

If the probability of ever switching to an ex situ technology before time T is lower than some predetermined level β, i.e.

$$\max_{n \in \{0,1,...,N-1\}} \{ \mathbb{P} (\widehat{\tau} = t_n) \} < \beta \ ,$$

the decisionmaking entity should make provisions according to the expected cost $E(C_1(\mu))$ of using a pure in-situ technology. Otherwise, she should set set aside an amount $\min_n E(C_2(n, \mu))$ corresponding to the cheapest mixed re*mediation strategy.*

Thanks to the above results, computing the needed probabilities $\mathbb{P}(\widehat{\tau} = t_n)$ turns out to be relatively tractable. Indeed, for $n = 0$, either $Y_0 < b_0^{\alpha}$ and then $\mathbb{P}(\widehat{\tau} = 0) = 0$, or $Y_0 \geq b_0^{\alpha}$ and then $\mathbb{P}(\widehat{\tau} = 0) = 1$.

For $n = 1$, we have $\mathbb{P}(\widehat{\tau} = t_1) = \mathbb{P}\left(Y_0 < b_0^{\alpha} \cap Y_{t_1} \geq b_{t_1}^{\alpha}\right)$. When $Y_0 < b_0^{\alpha}$, this expression becomes

$$\mathbb{P}(\widehat{\tau} = t_1) = \mathbb{P}\left(Y_{t_1} \geq b_{t_1}^{\alpha}\right) = \Phi\left(-d\left(t_1, b_{t_1}^{\alpha}\right)\right)$$

with

$$d\left(t_1, b_{t_1}^{\alpha}\right) = \frac{\ln \frac{b_{t_1}^{\alpha}}{y_0} - \left(\mu - \frac{\sigma^2}{2}\right) t_1}{\sigma \sqrt{t_1}}$$

For $n = 2$, the probability $\mathbb{P}(\widehat{\tau} = t_n)$ can be written as

$$\mathbb{P}(\widehat{\tau} = t_2) = \mathbb{P}\left(Y_0 < b_0^{\alpha} \cap Y_{t_1} < b_{t_1}^{\alpha} \cap Y_{t_2} \geq b_{t_2}^{\alpha}\right)$$

If $Y_0 < b_0^{\alpha}$, then

$$\mathbb{P}(\widehat{\tau} = t_2) = \mathbb{P}\left(Y_{t_2} \geq b_{t_2}^{\alpha} | Y_{t_1} < b_{t_1}^{\alpha}\right) \mathbb{P}\left(Y_{t_1} \geq b_{t_1}^{\alpha}\right) = \mathbb{P}\left(Y_{t_2} \geq b_{t_2}^{\alpha} | Y_{t_1} < b_{t_1}^{\alpha}\right) \Phi\left(d\left(t_1, b_{t_1}^{\alpha}\right)\right)$$

and

$$\begin{aligned}
&\mathbb{P}\left(Y_{t_2} \geq b^{\alpha}_{t_2} | Y_{t_1} < b^{\alpha}_{t_1}\right) \\
= &\int_0^{b^{\alpha}_{t_1}} \mathbb{P}\left(Y_{t_2} \geq b^{\alpha}_{t_2} | Y_{t_1} < b^{\alpha}_{t_1}\right) f\left(d\left(t_{1,}y\right)\right) dy \\
= &\int_0^{b^{\alpha}_{t_1}} \frac{1}{\sqrt{2\pi}} \mathbb{P}\left(y \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\left(t_2 - t_1\right) + \sigma\left(W_{t_2} - W_{t_1}\right)\right) \geq b^{\alpha}_{t_2}\right) \exp\left(-\frac{d\left(t_{1,}y\right)^2}{2}\right) dy \\
= &\int_0^{b^{\alpha}_{t_1}} \frac{1}{\sqrt{2\pi}} \Phi\left(d\left(t_1, t_2, y\right)\right) \exp\left(-\frac{d\left(t_{1,}y\right)^2}{2}\right) dy
\end{aligned}$$

where f is the density function of the standard Normal distribution, and

$$d\left(t_1, t_2, y\right) = \frac{\ln \frac{y}{L} + \left(\mu - \frac{\sigma^2}{2}\right)\left(T + t_1\right) - \Phi^{-1}(\alpha)\sigma\sqrt{T - t_2}}{\sigma\sqrt{t_2 - t_1}} \qquad \text{(replacing } b^{\alpha}_{t_2} \text{ by its explicit value)}$$

$$d\left(t_{1,}y\right) = \frac{\ln \frac{y}{y_0} - \left(\mu - \frac{\sigma^2}{2}\right)t_1}{\sigma\sqrt{t_1}}$$

Hence,

$$\mathbb{P}\left(\widehat{\tau} = t_2\right) = \int_0^{b^{\alpha}_{t_1}} \frac{1}{\sqrt{2\pi}} \Phi\left(d\left(t_1, t_2, y\right)\right) \exp\left(-\frac{d\left(t_{1,}y\right)^2}{2}\right) dy \times \Phi\left(d\left(t_1, b^{\alpha}_{t_1}\right)\right)$$

The general case $n \geq 3$ is handled in the Appendix.

7. Concluding remarks

Around the world, policymakers, regulators, land owners, local communities, industrial firms, insurance companies, etc. have to deal with the remediation of contaminated sites. One important hurdle is that cleaning up a contaminated site is often subject to significant cost uncertainties. This paper's motivation was to cope with this issue. Drawing on studies of random hitting times carried out in mathematical finance and real-options theory, we computed closed-form tractable expressions for assessing the expected cost of cleanup strategies, and derived simple rules for selecting remediation technologies and provisioning financial resources.

Several extensions of this study appear to be natural at this stage.

First, while the rules we examined look rather reasonable and applicable, these rules were not shown to be optimal. Seeking the best decision in the present framework would constitute a nontrivial and certainly worthwhile exercise in stochastic optimization. The obtained solution would then have to be compared with the above rules, both analytically and/or through simulations.

Secondly, although the decisionmaker's risk aversion could somewhat be captured by the 'tolerance' thresholds introduced in Sections 5 and 6, more precise ways to represent risk aversion can be called for. One should then turn to other decision criteria, such as a mean-variance criterion or some of the convex risk measures developed in mathematical finance (see, e.g., Artzner et al. 1999).

Third, as the ultimate test for the current propositions and their possible extensions lies in their concrete application, one would want to see the approach outlined in this paper implemented in algorithms and softwares. A related complementary challenge would also be to understand better how our cleanup cost assessment method might affect remediation management projects overall, through changing stakeholders' perceptions, say, or impacting negotiations with the regulator.

APPENDIX

To start with, let us state a general result that is essential for computing the formulas in Propositions 1 and 2.

LEMMA: Let $(X_t)_{t\geq 0}$ and $\left(m_t^X\right)_{t\geq 0}$ be the processes defined respectively by $X_t = \nu t + W_t$ and $m_t^X = \inf\{X_s, 0 \leq s \leq t\}$. For $\xi \leq 0$, we have

$$\mathbb{P}\left(m_t^X \geq \xi\right) = \Phi\left(\frac{-\xi + \nu t}{\sqrt{t}}\right) - e^{2\nu\xi}\Phi\left(\frac{\xi + \nu t}{\sqrt{t}}\right)$$

where Φ is the standard Normal cumulative distribution function: $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$.

Proofs can be found in Douady (1998) or Jeanblanc et al. (2004).

PROOF OF PROPOSITION 1:

Since

$$\begin{aligned}\mathbb{P}\left(t_k < T_L^Y\right) &= \mathbb{P}\left[\inf\{Y_t, 0 \leq t \leq t_k\} > L\right] \\ &= \mathbb{P}\left[\inf\left\{\left(\tfrac{\mu}{\sigma} - \tfrac{\sigma}{2}\right)t + W_t, 0 \leq t \leq t_k\right\} > \tfrac{1}{\sigma}\ln\left(\tfrac{L}{y_0}\right)\right] \quad ,\end{aligned}$$

and $L < y_0$ by assumption, the Lemma entails that:

$$\begin{aligned}\mathbb{P}\left(t_k < T_L^Y\right) \quad = \quad & \Phi\left(\frac{-\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right) + \left(\frac{\mu}{\sigma} - \frac{\sigma}{2}\right)\frac{k}{N}T}{\sqrt{\frac{k}{N}T}}\right) \\ & - \left(\frac{L}{y_0}\right)^{\frac{2\left(\mu - \frac{1}{2}\sigma^2\right)}{\sigma^2}} \times \Phi\left(\frac{\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right) + \left(\frac{\mu}{\sigma} - \frac{\sigma}{2}\right)\frac{k}{N}T}{\sqrt{\frac{k}{N}T}}\right)\end{aligned}$$

Hence,

$$\mathbb{E}\left(C_1^{O\&M}(\mu)\right) = c\sum_{k=1}^{N} e^{-r\frac{k}{N}T}\left(\begin{array}{c}\Phi\left(\frac{-\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{\sigma}-\frac{\sigma}{2}\right)\frac{k}{N}T}{\sqrt{\frac{k}{N}T}}\right) \\ -\left(\frac{L}{y_0}\right)^{\frac{2\left(\mu-\frac{1}{2}\sigma^2\right)}{\sigma^2}} \times \Phi\left(\frac{\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{\sigma}-\frac{\sigma}{2}\right)\frac{k}{N}T}{\sqrt{\frac{k}{N}T}}\right)\end{array}\right)$$

Moreover,

$$
\begin{aligned}
\mathbb{P}^{\sigma}\left(T<T_L^Y\right) &= \mathbb{P}^{\sigma}\left[\inf\left\{\left(\tfrac{\mu}{\sigma}-\tfrac{\sigma}{2}\right)t+W_t, 0\leq t\leq T\right\}>\tfrac{1}{\sigma}\ln\left(\tfrac{L}{y_0}\right)\right] \\
&= \mathbb{P}^{\sigma}\left[\inf\left\{\left(\tfrac{\mu}{\sigma}+\tfrac{\sigma}{2}\right)t+W_t^{\sigma}, 0\leq t\leq T\right\}>\tfrac{1}{\sigma}\ln\left(\tfrac{L}{y_0}\right)\right] ,
\end{aligned}
$$

where the probability measure $\mathbb{P}^{\sigma}$ is equivalent to $\mathbb{P}$ and is defined by the Radon-Nikodym derivative

$$
\left.\frac{d\mathbb{P}^{\sigma}}{d\mathbb{P}}\right|_{\mathcal{F}_T} = e^{\sigma W_T-\frac{1}{2}\sigma^2 T}
$$

and where the process $W_t^{\sigma}=W_t-\sigma t$ is a $\mathbb{P}^{\sigma}$-Brownian motion. Thanks again to the Lemma, we obtain

$$
\begin{aligned}
\mathbb{P}^{-\sigma}\left(T<T_L^Y\right) &= \Phi\left(\frac{-\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{\sigma}+\frac{\sigma}{2}\right)T}{\sqrt{T}}\right) \\
&\quad -\left(\frac{L}{y_0}\right)^{\frac{2\left(\mu+\frac{1}{2}\sigma^2\right)}{\sigma^2}}\times\Phi\left(\frac{\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{\sigma}+\frac{\sigma}{2}\right)T}{\sqrt{T}}\right)
\end{aligned}
$$

Formula (9) now follows from the fact that

$$
\mathbb{E}\left[Y_T\times\mathbf{1}_{T<T_L^Y}\right]=y_0e^{\mu T}\mathbb{P}^{\sigma}\left(T<T_L^Y\right)
$$

and

$$
\begin{aligned}
\mathbb{E}\left(C_1^P(\mu)\right) &= \mathbb{E}\left[k_{\text{fine}}e^{-rT}\left(Y_T-L\right)\times\mathbf{1}_{T<T_L^Y}\right] \\
&= k_{\text{fine}}e^{-rT}\mathbb{E}\left[Y_T\times\mathbf{1}_{T<T_L^Y}\right]-k_{\text{fine}}Le^{-rT}\mathbb{P}\left(T<T_L^Y\right) \qquad \blacksquare
\end{aligned}
$$

PROOF OF PROPOSITION 2:

Using the same line of arguments as in the previous derivation, the expected present value of turning to incineration at time at time t_n can be expressed as

$$\begin{aligned}
\mathbb{E}\left(C_2^V(n,\mu)\right) &= k_{\text{ex situ}}\mathbb{E}\left[e^{-rt_n}.Y_{t_n}\times \mathbf{1}_{t_n<T_L^Y}\right] = k_{\text{ex situ}}y_0e^{-(r-\mu)t_n}\mathbb{P}^\sigma\left(t_n<T_L^Y\right)\\
&= k_{\text{ex situ}}e^{-(r-\mu)t_n}\left[\begin{array}{c}\Phi\left(\frac{-\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{\sigma}+\frac{\sigma}{2}\right)t_n}{\sqrt{t_n}}\right)\\ -\left(\frac{L}{y_0}\right)^{\frac{2\left(\mu+\frac{1}{2}\sigma^2\right)}{\sigma^2}}\times\Phi\left(\frac{\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{\sigma}+\frac{\sigma}{2}\right)t_n}{\sqrt{t_n}}\right)\end{array}\right]
\end{aligned}$$

And the expected operational and monitoring costs are similarly given by

$$\begin{aligned}
\mathbb{E}\left(C_2^{O\&M}(n,\mu)\right) &= c\sum_{k=1}^{n}e^{-rt_k}\mathbb{E}\left(\mathbf{1}_{t_k<T_L^Y}\right)\\
&= c\sum_{k=1}^{n}e^{-\frac{rk}{N}T}\left[\begin{array}{c}\Phi\left(\frac{-\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{\sigma}-\frac{\sigma}{2}\right)\frac{k}{N}T}{\sqrt{\frac{k}{N}T}}\right)\\ -\left(\frac{L}{y_0}\right)^{\frac{2\left(\mu-\frac{1}{2}\sigma^2\right)}{\sigma^2}}\times\Phi\left(\frac{\frac{1}{\sigma}\ln\left(\frac{L}{y_0}\right)+\left(\frac{\mu}{\sigma}-\frac{\sigma}{2}\right)\frac{k}{N}T}{\sqrt{\frac{k}{N}T}}\right)\end{array}\right] \qquad \blacksquare
\end{aligned}$$

Proof of Proposition 3:

i) Notice that the function[14] T_L^Y is non-decreasing in μ and so are the functions $1_{T<T_L^Y}$ and $1_{t_k<T_L^Y}$. As

$$C_1(\mu) = C^I(\mu) + c\sum_{k=1}^{N}e^{-rt_k}\mathbf{1}_{t_k<T_L^Y} + e^{-rT}k_{\text{fine}}(Y_T-L)\times\mathbf{1}_{T<T_L^Y} \quad ,$$

the function $C_1(\mu)$ is also non-decreasing in μ. Hence the result.

ii) The functions T_L^Y and $C^I(\mu)$ are non-decreasing in μ. Therefore, the desired result is obtained from the equality

$$C_2(n,\mu) = C^I(\mu) + c\sum_{k=1}^{n}e^{-rt_k}\times\mathbf{1}_{t_k<T_L^Y} + e^{-rt_n}k_{\text{ex situ}}Y_{t_n}\times\mathbf{1}_{t_n<T_L^Y} \qquad \blacksquare$$

[14] The dependency in μ of this function and the subsequent functions is not explicitely indicated, but comes from the process Y.

Proof of Proposition 4:

For $i = 1, 2$, we denote by Y^i the solution of the stochastic differential equation $dY_t^i = Y_t^i \left(\mu^i dt + \sigma dW_t\right)$, $Y_0^i = y_0$.

i) Technology in situ

The difference $C_1\left(\mu^1\right) - C_1\left(\mu^2\right)$ consists of the following three terms :

$$\begin{aligned} C_1\left(\mu^1\right) - C_1\left(\mu^2\right) &= C^I\left(\mu^1\right) - C^I\left(\mu^2\right) \\ &+ c\sum_{k=1}^N e^{-rt_k}\left[\mathbf{1}_{t_k<T_L^{Y^1}} - \mathbf{1}_{t_k<T_L^{Y^2}}\right] \\ &+ e^{-rT}.k_{\text{fine}}\left[\left(Y_T^1 - L\right)\times\mathbf{1}_{T<T_L^{Y^1}} - \left(Y_T^2 - L\right)\times\mathbf{1}_{T<T_L^{Y^2}}\right] \end{aligned}$$

Let us first consider the second term $c\sum_{k=1}^N e^{-rt_k}\left[\mathbf{1}_{t_k<T_L^{Y^1}} - \mathbf{1}_{t_k<T_L^{Y^2}}\right]$

From $\mu^1 > \mu^2$ we get

$$0 \le c\sum_{k=1}^N e^{-rt_k}\left[\mathbf{1}_{t_k<T_L^{Y^1}} - \mathbf{1}_{t_k<T_L^{Y^2}}\right] \le c\sum_{k=1}^N e^{-rt_k}\mathbf{1}_{T_L^{Y^2}<t_k<T_L^{Y^1}}$$

Hence

$$\mathbb{E}\left(c\sum_{k=1}^N e^{-rt_k}\left[\mathbf{1}_{t_k<T_L^{Y^1}} - \mathbf{1}_{t_k<T_L^{Y^2}}\right]\right) \le c\sum_{k=1}^N e^{-rt_k}\mathbb{P}\left(T_L^{Y^2} < t_k < T_L^{Y^1}\right)$$

We have

$$\begin{aligned} \mathbb{P}\left(T_L^{Y^2} < t_k < T_L^{Y^1}\right) &= \mathbb{P}\left(\left(\mu^1 - \tfrac{1}{2}\sigma^2\right)t_k > \ln\left(\tfrac{L}{y_0}\right) - \sigma W_{t_k} > \left(\mu^2 - \tfrac{1}{2}\sigma^2\right)t_k\right) \\ &= \mathbb{P}\left(\frac{\ln\left(\frac{L}{y_0}\right) - \left(\mu^1 - \frac{1}{2}\sigma^2\right)t_k}{\sigma\sqrt{t_k}} < \frac{W_{t_k}}{\sqrt{t_k}} < \frac{\ln\left(\frac{L}{y_0}\right) - \left(\mu^2 - \frac{1}{2}\sigma^2\right)t_k}{\sigma\sqrt{t_k}}\right) \\ &= \frac{1}{\sqrt{2\pi}} \times \int_{d(\mu^1,t_k)}^{d(\mu^2,t_k)} e^{-\frac{x^2}{2}}dx \end{aligned}$$

with $d\left(\mu, t\right) = \dfrac{\ln\left(\frac{L}{y_0}\right) - \left(\mu - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$. From this we deduce

$$\begin{aligned}
\mathbb{E}\left(c\sum_{k=1}^{N}e^{-rt_k}\left[\mathbf{1}_{t_k<T_L^{Y^1}}-\mathbf{1}_{t_k<T_L^{Y^2}}\right]\right) &\leq \frac{c}{\sqrt{2\pi}}\sum_{k=1}^{N}e^{-rt_k}\int\limits_{d(\mu^1,t_k)}^{d(\mu^2,t_k)}e^{-\frac{x^2}{2}}dx \\
&\leq \frac{c}{\sqrt{2\pi}}\sum_{k=1}^{N}e^{-rt_k}\left(d(\mu^2,t_k)-d(\mu^1,t_k)\right) \\
&\leq \frac{c\left(\mu^1-\mu^2\right)}{\sigma\sqrt{2\pi}}\sum_{k=1}^{N}e^{-rt_k}\sqrt{t_k}
\end{aligned}$$

Now, since the sum $\sum_{k=1}^{N}e^{-rt_k}\sqrt{t_k}$ does not depend on μ^i, $i=1,2$, we can assert that there exists a constant δ_N that only depends on N and such that

$$\mathbb{E}\left(c\sum_{k=1}^{N}e^{-rt_k}\left[\mathbf{1}_{t_k<T_L^{Y^1}}-\mathbf{1}_{t_k<T_L^{Y^2}}\right]\right)\leq\delta_N\left(\mu^1-\mu^2\right)$$

We now consider the third term $+e^{-rT}.k_{\text{fine}}\left[(Y_T^1-L)\times\mathbf{1}_{T<T_L^{Y^1}}-(Y_T^2-L)\times\mathbf{1}_{T<T_L^{Y^2}}\right]$

From $\mu^1>\mu^2$ we deduce

$$0\leq(Y_T^1-L)\times\mathbf{1}_{T<T_L^{Y^1}}-(Y_T^2-L)\times\mathbf{1}_{T<T_L^{Y^2}}\leq Y_T^1-Y_T^2$$

and therefore

$$\mathbb{E}\left(e^{-rT}.k_{\text{fine}}\left[\left(Y_T^1-L\right)\times\mathbf{1}_{T<T_L^{Y^1}}-\left(Y_T^2-L\right)\times\mathbf{1}_{T<T_L^{Y^2}}\right]\right)\leq e^{-rT}.k_{\text{fine}}.e^{\left(\mu^1-\mu^2\right)T}$$

ii) Technology ex situ

The difference $C_2\left(\mu^1\right)-C_2\left(\mu^2\right)$ consists of three terms as follows :

$$C^I\left(\mu\right)+c\sum_{k=1}^{n}e^{-rt_k}\times\mathbf{1}_{t_k<T_L^Y}+e^{-rt_n}.k_{\text{ex situ}}Y_{t_n}\times\mathbf{1}_{t_n<T_L^Y}$$

$$\begin{aligned}
C_2\left(\mu^1\right)-C_2\left(\mu^2\right) &= C^I\left(\mu^1\right)-C^I\left(\mu^2\right) \\
&+c\sum_{k=1}^{n}e^{-rt_k}\left[\mathbf{1}_{t_k<T_L^{Y^1}}-\mathbf{1}_{t_k<T_L^{Y^2}}\right] \\
&+e^{-rt_n}.k_{\text{ex situ}}\left[Y_{t_n}^1\times\mathbf{1}_{t_n<T_L^{Y^1}}-Y_{t_n}^1\times\mathbf{1}_{t_n<T_L^{Y^1}}\right]
\end{aligned}$$

and the proof is similar to the previous one. ■

Proof of Proposition 5:

We focus on the characterization of

$$\beta_t^{\alpha} = \inf \{ b_t \;/\; \mathbb{P}(Y_T > L \;/\; Y_t = b_t) \geq \alpha \}$$

The inequality $\mathbb{P}(Y_T > L \;/\; Y_t = b_t) \geq \alpha$ can be written on the form $\mathbb{P}\left(Y_T^{t,b_t} > L\right) \geq \alpha$ where

$$Y_T^{t,b_t} \doteq b_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)(T-t) + \sigma(W_T - W_t)\right)$$

Thus

$$\mathbb{P}\left(Y_T^{t,b_t} > L\right) = \mathbb{P}\left(U > \frac{\ln \frac{L}{b_t} - \left(\mu - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) \qquad \text{where } U \sim N(0,1)$$

and therefore

$$\begin{aligned} \mathbb{P}\left(Y_T^{t,b_t} > L\right) = \mathbb{P}(Y_T > L \;/\; Y_t \geq b_t) \geq \alpha \quad &\Longleftrightarrow \Phi\left(\frac{\ln \frac{b_t}{L} + \left(\mu - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) \geq \alpha \\ &\Longleftrightarrow \frac{\ln \frac{b_t}{L} + \left(\mu - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \geq \Phi^{-1}(\alpha) \end{aligned}$$

We finally get

$$\frac{\ln \frac{b_t^{\alpha}}{L} + \left(\mu - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \geq \Phi^{-1}(\alpha)$$

and the desired result. ■

Proof of Proposition 6:

i) This condition follows immediately from Equation (14).

ii) Let us first recall that

$$b_t^{\alpha} = \begin{cases} \beta_t^{\alpha} & \text{if } \beta_t^{\alpha} > L \\ L & \text{otherwise} \end{cases}$$

where $\beta_t^{\alpha} = L \exp\left(a\sqrt{T-t} + b(T-t)\right)$. Let $I(\alpha)$ be defined as

$$I(\alpha) = \{ t \;/\; b_t^{\alpha} = \beta_t^{\alpha} \}$$

Obviously, $I(\alpha) = \left\{ t \ / \ a\sqrt{T-t} + b(T-t) > 0 \right\}$ and from this, we get that the map $t \in I(\alpha) \to \beta_t^\alpha$ is decreasing with respect to the current time t. Therefore, the same property of decreasing monotonicity holds for b_t^α.

iii) The sign of the second derivative $\frac{\partial^2 \beta_t^\alpha}{\partial t^2}$ is that of $\left(\frac{a}{2\sqrt{T-t}} + b \right)^2 - \frac{a}{4(T-t)\sqrt{T-t}}$. Now for t in $I(\alpha)$ we have

$$\begin{aligned}
\left(\frac{a}{2\sqrt{T-t}} + b \right)^2 - \frac{a}{4(T-t)\sqrt{T-t}} &> \left(\frac{a}{2\sqrt{T-t}} - \frac{a}{\sqrt{T-t}} \right)^2 - \frac{a}{4(T-t)\sqrt{T-t}} \\
&= \frac{a^2}{4(T-t)} - \frac{a}{4(T-t)\sqrt{T-t}} = \frac{a\left(a\sqrt{T-t} - 1\right)}{4(T-t)\sqrt{T-t}} > 0
\end{aligned}$$

Hence, $t \in I(\alpha) \to \beta_t^\alpha$ is convex and therefore, b_t^α is also convex. ■

Computing $\mathbb{P}(\widehat{\tau} = t_n)$:

For $n \geq 3$, one can use the following general argument

$$\begin{aligned}
& \mathbb{P}(\widehat{\tau} = t_n) \\
= {} & \mathbb{P}\left(Y_0 < b_0^\alpha \cap Y_{t_1} < b_{t_1}^\alpha \cap Y_{t_2} < b_{t_2}^\alpha \cap ... \cap Y_{t_{n-1}} < b_{t_{n-1}}^\alpha \cap Y_{t_n} \geq b_{t_n}^\alpha \right) \\
= {} & \mathbb{P}\left(Y_{t_n} \geq b_{t_n}^\alpha \mid \bigcap_{i=0}^{n-1} Y_{t_i} < b_{t_i}^\alpha \right) \mathbb{P}\left(\bigcap_{i=0}^{n-1} Y_{t_i} < b_{t_i}^\alpha \right) \\
= {} & \mathbb{P}\left(Y_{t_n} \geq b_{t_n}^\alpha \,| Y_{t_{n-1}} < b_{t_{n-1}}^\alpha \right) \mathbb{P}\left(\bigcap_{i=0}^{n-1} Y_{t_i} < b_{t_i}^\alpha \right)
\end{aligned}$$

Provided that $Y_0 < b_0^\alpha$, the latter writes

$$\begin{aligned}
\mathbb{P}(\widehat{\tau} = t_n) &= \mathbb{P}\left(Y_{t_n} \geq b_{t_n}^\alpha \mid \bigcap_{i=1}^{n-1} Y_{t_i} < b_{t_i}^\alpha \right) \mathbb{P}\left(\bigcap_{i=1}^{n-1} Y_{t_i} < b_{t_i}^\alpha \right) \\
&= \mathbb{P}\left(Y_{t_n} \geq b_{t_n}^\alpha \,| Y_{t_{n-1}} < b_{t_{n-1}}^\alpha \right) \mathbb{P}\left(\bigcap_{i=1}^{n-1} Y_{t_i} < b_{t_i}^\alpha \right)
\end{aligned}$$

Moreover,

$$\begin{aligned}
&\mathbb{P}\left(\bigcap_{i=1}^{n-1} Y_{t_i} < b^{\alpha}_{t_i}\right) \\
&= \mathbb{P}\left(Y_{t_{n-1}} < b^{\alpha}_{t_{n-1}} \mid \bigcap_{i=1}^{n-2} Y_{t_i} < b^{\alpha}_{t_i}\right) \mathbb{P}\left(\bigcap_{i=1}^{n-2} Y_{t_i} < b^{\alpha}_{t_i}\right) \\
&= \mathbb{P}\left(Y_{t_{n-1}} < b^{\alpha}_{t_{n-1}} \mid Y_{t_{n-2}} < b^{\alpha}_{t_{n-2}}\right) \mathbb{P}\left(\bigcap_{i=1}^{n-2} Y_{t_i} < b^{\alpha}_{t_i}\right)
\end{aligned}$$

Therefore,

$$\mathbb{P}\left(\widehat{\tau} = t_n\right) = \mathbb{P}\left(Y_{t_n} \geq b^{\alpha}_{t_n} \, |Y_{t_{n-1}} < b^{\alpha}_{t_{n-1}}\right) \prod_{i=2}^{n-1} \mathbb{P}\left(Y_{t_i} < b^{\alpha}_{t_i} \mid Y_{t_{i-1}} < b^{\alpha}_{t_{i-1}}\right)$$

We need then to compute the various conditional expectations

$$\begin{aligned}
&\mathbb{P}\left(Y_{t_n} \geq b^{\alpha}_{t_n} \, |Y_{t_{n-1}} < b^{\alpha}_{t_{n-1}}\right) \\
&= \int_0^{b^{\alpha}_{t_{n-1}}} \mathbb{P}\left(Y_{t_n} \geq b^{\alpha}_{t_n} \, |Y_{t_{n-1}} = y\right) f\left(d\left(t_{n-1,}y\right)\right) dy \\
&= \int_0^{b^{\alpha}_{t_{n-1}}} \frac{1}{\sqrt{2\pi}} \mathbb{P}\left(y \exp\left(\left(\mu - \frac{\sigma^2}{2}\right)\left(t_n - t_{n-1}\right) + \sigma\left(W_{t_n} - W_{t_{n-1}}\right)\right) \geq b^{\alpha}_{t_n}\right) \exp\left(-\frac{d\left(t_{n-1,}y\right)^2}{2}\right) dy \\
&= \int_0^{b^{\alpha}_{t_{n-1}}} \frac{1}{\sqrt{2\pi}} \Phi\left(d\left(t_{n-1}, t_n, y\right)\right) \exp\left(-\frac{d\left(t_{n-1,}y\right)^2}{2}\right) dy
\end{aligned}$$

where, as previously introduced, f is the standard Normal density function, Φ is the standard Normal cumulative distribution function and[15]

$$\begin{aligned}
d\left(t_{n-1}, t_n, y\right) &= \frac{\ln \frac{y}{L} + \left(\mu - \frac{\sigma^2}{2}\right)\left(T + t_{n-1}\right) - \Phi^{-1}(\alpha)\sigma\sqrt{T - t_n}}{\sigma\sqrt{t_n - t_{n-1}}} \\
d\left(t_{n-1,}y\right) &= \frac{\ln \frac{y}{y_0} - \left(\mu - \frac{\sigma^2}{2}\right)t_{n-1}}{\sigma\sqrt{t_{n-1}}}
\end{aligned}$$

[15] In the first expression, we replace $b^{\alpha}_{t_n}$ by its explicit value.

Moreover for any $i \geq 2$ we have:

$$\mathbb{P}\left(Y_{t_i} < b^{\alpha}_{t_i} \mid Y_{t_{i-1}} < b^{\alpha}_{t_{i-1}}\right) = \int_0^{b^{\alpha}_{t_{i-1}}} \mathbb{P}\left(Y_{t_i} < b^{\alpha}_{t_i} \,|Y_{t_{i-1}} = y\right) f\left(d\left(t_{i-1,}y\right)\right) dy$$

$$= \int_0^{b^{\alpha}_{t_{i-1}}} \frac{1}{\sqrt{2\pi}} \Phi\left(-d\left(t_{i-1}, t_i, y\right)\right) \exp\left(-\frac{d\left(t_{i-1,}y\right)^2}{2}\right) dy \qquad \blacksquare$$

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Figure 1. Comparison of EC1 and EC2
$\sigma = 0.2$

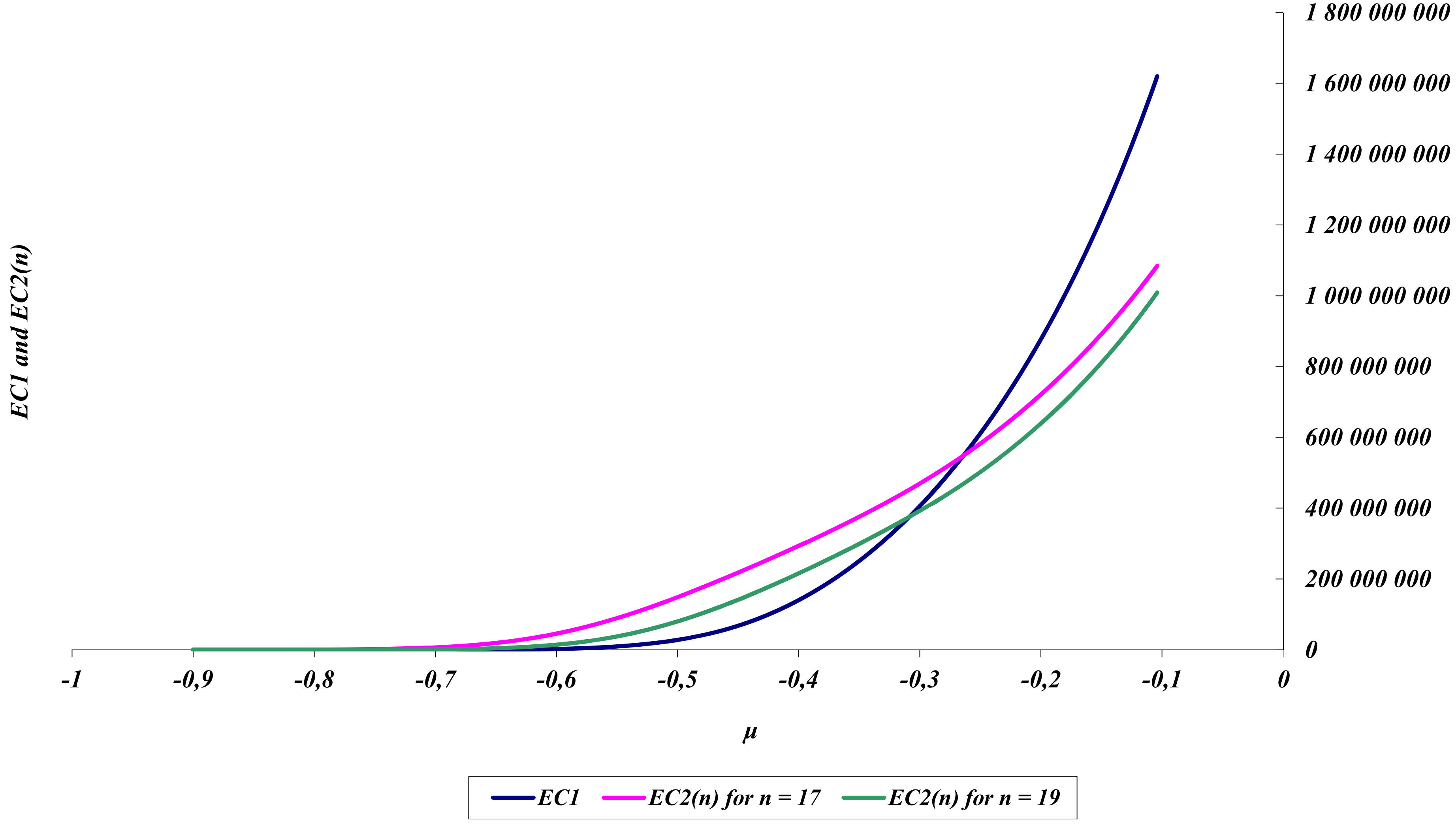

Figure 2. Variation of $\beta(t)/L$

σ = 3 %and μ = - 2 %