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Buying high and selling low: Stock repurchases and persistent asymmetric information[1]

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Abstract

Share prices generally fall when a firm announces a seasoned equity offering (SEO). A standard explanation is that an SEO communicates negative information to investors. We show that if repeated capital market transactions are possible, this same asymmetry of information between firms and investors implies that some firms also repurchase shares in equilibrium. A subset of these firms directly profit from repurchases, while other firms repurchase in order to improve the terms of a subsequent SEO. The possibility of repurchases reduces both SEOs and investment. Overall, our analysis highlights the importance of analyzing SEOs and repurchases in a unified framework.

Share prices generally fall in response to a firm's announcement of a seasoned equity offering (SEO).[1] The standard explanation for this empirical regularity is that a firm has information that investors lack, and a SEO reveals to investors that the firm's information is negative (see, in particular, Myers and Majluf (1984)). In equilibrium firms with negative information issue equity, and accept the negative share price response because the SEO provides funding for a profitable investment. In contrast, firms with positive information prefer to pass up the investment rather than issue equity at a low price.

But if a firm really does have superior information relative to its investors, a firm's decision problem is more complicated than simply deciding whether or not to undertake an SEO. In particular, a firm with positive information potentially has the incentive to repurchase shares, both to (A) directly profit from the repurchase transaction, and also to (B) communicate positive information to investors, and thereby improve the terms of a subsequent SEO. However, most existing SEO models are static,[2] which eliminates both incentives. For motive (A), this follows from the no-trade theorem (Milgrom and Stokey 1982): investors can infer that a repurchase offer comes from a currently undervalued firm, and hence prefer to retain their shares. Motive (B) is inherently dynamic, in that requires at least two rounds of equity transactions to implement. Yet motive (A) figures prominently in managerial survey responses, while Billett and Xue (2007) provide empirical evidence for (B).[3]

In this paper we analyze a dynamic version of the standard SEO model. Our main result is that repurchases motived by both (A) and (B) indeed arise in equilibrium. Moreover, these

[1]See Asquith and Mullins (1986), along with a large subsequent literature, summarized in Eckbo et al (2007).

[2]In exceptions such as Lucas and McDonald (1990, 1998) and Hennessy, Livdan and Miranda (2010), a firm's informational advantage only lasts one period. In contrast, in our paper the information asymmetry is persistent. In papers such as Constantinides and Grundy (1989), which we discuss in detail below, firms engage in two rounds of transactions, but the second transaction is a deterministic function of the first.

[3]Brav et al (2005) survey managers. Consistent with motive (A), a very large fraction of managers agree (Table 6) that the "Market price of our stock (if our stock is a good investment, relative to its true value)" is an important factor. The article also reports survey evidence consistent with motive (B): a very large fraction of managers agree (Table 3) that "Repurchase decisions convey information about our company to investors."

repurchases interact with SEOs. In particular, we show that the possibility of repurchases reduces the likelihood that a firm undertakes an SEO. The equilibrium outcome of the static game, in which bad firms issue and good firms do nothing, is fragile, in the sense that in the dynamic setting this outcome fails a standard refinement ("Never dissuaded once convinced," NDOC, discussed in detail below). In contrast, equilibria featuring repurchases satisfy NDOC.

Our results suggest that the standard explanation of the negative price-reaction to SEO announcements is incomplete, in the sense that the equilibrium that underpins the explanation must also feature repurchases, a point that is absent in the standard explanation. Nonetheless, our analysis still delivers the negative price-reaction to SEO announcements (this is the "selling low" of the title), because in equilibrium an SEO is a negative signal relative to *both* the alternatives of repurchasing and doing nothing. The primary empirical implications of our analysis are that (I) some firms repurchase, and then subsequently engage in an SEO, (II) in such cases, the dollar value of the SEO exceeds the dollar value of the repurchase, (III) the cumulative share price response to a repurchase announcement and then subsequent SEO announcement is more favorable than the cumulative response to an SEO announcement made without a prior repurchase. All three predictions are consistent with the empirical findings of Billett and Xue (2007). Additional empirical implications are that (IV) some firms repurchase in order to profit from the repurchase transaction, and (V) repurchasing firms have good investment opportunities and are credit constrained. As noted, (IV) is consistent with survey responses, while as discussed in subsection **??**, (V) has some support in existing empirical studies.

The intuition for our result that an equilibrium must feature repurchases is as follows. On the one hand, if investor beliefs following a repurchase offer are that a firm is bad, then good firms certainly profit from repurchases, since the price they pay is low. On the other hand, if investor beliefs following a repurchase offer are good, then any firm interested in undertaking an SEO can benefit by first repurchasing, so as to improve investor beliefs. So in either case, repurchases arise in equilibrium.

Before proceeding, it is worth noting that in practice firms may repurchase their own stock either via tender offers or open market repurchase programs. Our formal model corresponds both to open market repurchase programs in which firms actually act as announced, and to Dutch-auction tender offers in which the price paid to investors depends on the level of investor interest. In the US, open market repurchases are the dominant form of repurchase, and most of the empirical evidence that we cite deals with this case.

A perhaps surprising feature of open market repurchases in the US is that firms are not legally bound to follow their announced programs. Nonetheless, Stephens and Weisbach (1998) find that "74 to 82 percent of the shares targeted at the time of the original announcement are subsequently repurchased," while Oded (2009) documents a mean completion rate of 92%. Moreover, many event studies document a significant abnormal return of 2-4% in share prices in response to an announcement of share repurchases (e.g., Ikenberry, Lakonishok and Vermaelen (1995), Peyer and Vermaelen (2009)), suggesting that announcements are not pure cheap-talk.[4]

There is a large existing literature on the information content of firms' capital structure decisions. The idea that firms repurchase their stock to signal they are good can be traced back to the old intuition that *retaining* equity is a useful signal (Leland and Pyle (1977)). Similarly, Example 1 of Brennan and Kraus (1987) has a good firm simultaneously repurchasing debt and issuing equity. The debt repurchase allows the firm to signal that it is good.

A large literature studies signaling in static payout models. Although many of these models are written in terms of dividend payouts rather than repurchases, the economic effects would apply similarly to repurchases, and so we discuss both together.

[4]Related to this discussion, in an early study of repurchases, Barclay and Smith (1988) find evidence that the announcement of a repurchase program is followed by an increased bid-ask spread, which they interpret as an increase in adverse selection, which they in turn interpret as investors being unsure about whether or not they are trading against the firm. However, in general subsequent research has not supported this original finding (see the discussion in Grullon and Ikenberry (2000)).

In one branch of this literature (e.g., Bhattacharya (1979), Vermaelen (1984), Miller and Rock (1985)), good firms pay out cash to show that they have high cash flows. Bad firms do not mimic because they have low cash flows, and so paying out cash necessitates either costly external financing or distorts investment.[5] An important assumption in this branch of the literature is that a firm's objective (exogenously) includes the interim share price, because, for example, a subset of shareholders need to sell at an interim date.

A second branch of the literature (e.g., John and Williams (1985), Ambarish, John and Williams (1987), Williams (1988)) directly assumes that pay outs are costly: in particular, dividends are tax-inefficient.[6] Firms then issue equity to finance an investment. Good firms pay out, while bad firms do not. Because of this separation, good firms are able raise the funds they need for investment in a less dilutive way. Bad firms do not mimic good firms because they would pay the same cost (inefficient cash pay outs), but benefit less because dilution is less costly to them than it is to good firms.

In both these sets of papers, the economic function of pay outs is that they destroy value. In our analysis, repurchases destroy value for bad firms, but in contrast to pay outs in the above papers, they create value for good firms. Consequently, our model ties together the use of repurchases as a signal by some firms (as a prelude to share issuance and investment) with the ability of other firms to profitably engage in repurchases. In this sense, repurchases in our model differ both from money burning (Daniel and Titman (1995)) and wasteful advertisement (Milgrom and Roberts (1986)), but instead constitute a wealth transfer between different firms. Moreover, a novel prediction of our model is that both good and bad firms repurchase, but intermediate firms may not repurchase.

Constantinides and Grundy (1989) and Chowdhry and Nanda (1994) both highlight a

[5] Bhattacharya (1979) and Miller and Rock (1985) are written in terms of dividends, while Vermaelen (1984) is written in terms of repurchases. Ofer and Thakor (1987) describe a model with similar ingredients in which firms can both issue dividends and repurchase shares, where both actions are costly, and characterize which form of payout firms prefer. We consider the robustness of our analysis to multi-dimensional signaling in Section **??**.

[6] Ambarish, John and Williams (1987) and Williams (1988) both allow for repurchases. In both papers, and in contrast to our paper, there is a single date at which both repurchases and issues occur, and so there is no possibility of a firm repurchasing at one date to improve the terms of issue at a subsequent date.

central characteristic of repurchases, namely that they are more expensive when investor beliefs about the firm are more positive. Importantly, this is the opposite of the case for share issues. In Constantinides and Grundy (1989), this feature of repurchases allows a fully-separating efficient equilibrium to exist under some circumstances: firms commit to spend excess cash from a fundraising stage on repurchases, and this means that "the more management inflates the perceived value of the claim [issued at the fundraising stage], the more capital it raises and the more shares it must repurchase from the outsiders at the inflated price." In Chowdhry and Nanda (1994), this same force means that low value firms do not repurchase, because (by a parameter assumption) investor beliefs are always such that the price is too high. As such, in Chowdhry and Nanda's model firms distribute cash via repurchases only when investor information is negative relative to the firm's information, and via dividends otherwise. Our paper complements the analysis of these authors by showing that, perhaps surprisingly, the positive relation between repurchase prices and investor beliefs means that some equilibrium repurchases must occur.[7] The reason is that good firms will repurchase unless investor beliefs associate repurchase offers with very good firms; but even in this case, repurchases remain attractive, since by repurchasing today a firm changes investor beliefs and hence improves the terms of a subsequent share issue (see subsection **??**).

Oded (2005) demonstrates how a good firm can signal its type via a repurchase program that gives it the option but not the obligation to repurchase at a future date. A firm repurchases shares from current investors who are hit by a liquidity shock: hence the no-trade theorem does not apply, and a repurchase program gives the firm a valuable repurchase option, which by itself increases the current share price. However, the firm's current investors understand they will suffer by trading against the firm, and this generates countervailing downwards pressure on the current share price. But the repurchase option is more valuable to good firms, because by an important assumption they are riskier as well as higher in value

[7] Note that the timing of equilibrium repurchases is very different in our model from that of Constantinides and Grundy (1989). In our model, a firm first repurchases, and then subsequently may issue equity to fund an investment. In Constantinides and Grundy, a firm instead first issues a security to raise funding, with a commitment to disburse excess cash via repurchases.

than bad firms. Hence there is an equilibrium in which good firms announce a repurchase program but bad firms do not mimic, because they would pay the same cost (downwards pressure on today's share price) but gain a less valuable option. In contrast to our paper, Oded does not consider repurchases as a prelude to equity issuance.

The above discussion focuses on papers that study repurchases in the context of asymmetric information between a firm and its investors. Naturally, there are other factors that affect the incidence of repurchases—perhaps most notably, the differential tax treatment of repurchases and dividend payments. Another example is Huang and Thakor's (2013) suggestion that repurchases are useful because they reduce disagreement among a firm's shareholders. Grullon and Ikenberry (2000) offer a good survey of the literature on repurchasing.

In many microstructure models (e.g., Kyle (1985)) an informed trader buys and sells shares in many periods. Closest to our paper are the small number of papers in which the informed trader trades in opposite directions in different periods, e.g., buys in early periods then sells in later periods (see Kyle and Viswanathan (2008), and references therein). However, the economic motivation for this dynamic strategy is very different from in our paper: in these prior papers, the informed trader buys early in order to extract more money from the noise/liquidity traders he sells to later. In contrast, in our paper the informed trader (the firm) always *sells* at a fair price. The firms that make trading profits in our setting are very good firms who buy undervalued shares in early rounds. The source of these profits is the losses experienced by worse firms who also buy in early rounds.

A relatively small literature studies dynamic models of trade under asymmetric information without noise/liquidity traders. Noldeke and van Damme (1990) and Swinkels (1999) study a labor market model where education acts as a signal. Fuchs and Skrzypacz (2013) study trade of a single indivisible asset that is more highly valued by buyers than the seller. They focus on whether more trading opportunities increase or reduce welfare. Kremer and Skrzypacz (2007) and Daley and Green (2012) study a similar model in which information arrives over time. In contrast to these papers, in our model both sales and repurchases are possible; trade is in divisible shares; and the gains from trade arise from the possibility of financing a profitable investment. Perhaps closest to the current paper are Morellec and Schurhoff (2011) and Strebulaev, Zhu and Zryumov (2014). Both papers study dynamic models in which a firm with long-lived private information chooses a date to raise outside financing and invest. In both papers, issue and investment are tied together (by assumption), and the combination of repurchases with subsequent equity issue—which is our main focus—is not examined. Instead, the main results of both papers concern the timing of investment. A contemporaneous paper by Ordonez, Perez-Reyna and Yogo (2013) studies a dynamic model of debt issuance. In a model with moral hazard in place of adverse selection, DeMarzo and Urosevic (2006) study the dynamics of a large shareholder selling off his stake in a firm.

1 Example

Firms have cash 1, and the opportunity to invest 11.5 at date 2 in a project that subsequently yields 11.9. Hence firms need to raise additional funds of 10.5 in order to invest. Firms can either repurchase (buy) or issue (sell) shares at each of dates 1 and 2. All uncertainty is resolved at date 3, and firms act to maximize their date 3 share price. The initial number of shares is normalized to 1. Firm assets-in-place a are uniformly distributed over $[6, 12]$.

If date 1 transactions are exogenously ruled out, and transactions are only possible at date 2, this setting is simply a version of Myers and Majluf with a continuum of firm types. We first describe an equilibrium of this benchmark.[8] Firms $a \leq 7.42$ raise funds 10.5 by issuing $\frac{10.5}{8.11}$ shares at a price $P^{MM} = 8.11$, and then invest. Firms $a > 7.42$ do nothing. To see that the price P^{MM} is fair, note the expected value of a conditional on $a \leq 7.42$ is 6.71

[8] The equilibrium described entails firms either raising just enough outside financing to fund the investment, or else doing nothing. Other equilibria exist in which issuing firms raise strictly more funds than required. However, all equilibria of the benchmark are characterized by a cutoff firm type such that firms below this cutoff issue and invest, while firms above this cutoff do nothing; see Proposition ?? below.

and that P^{MM} solves

$$P^{MM} = \frac{11.9 + 6.71}{1 + \frac{10.5}{P^{MM}}}.$$

Given the issue price P^{MM}, the date 3 share price of firm $a = 7.42$ is 8.42 if it does nothing, and is $\frac{11.9+7.42}{1+\frac{10.5}{8.11}} = 8.42$ if it issues and invests. Hence firms $a < 7.42$ strictly prefer to issue and invest, while firms $a > 7.42$ find issue too dilutive, and strictly prefer to do nothing.

Our paper's focus is the case in which transactions are possible at both dates 1 and 2. In this case, the following is a Perfect Bayesian equilibrium (PBE), illustrated in Figure 1:

- At date 1, firms with assets-in-place in either $[6.75, 7.07]$ or $[10.55, 12]$ spend cash 1 to repurchase $\frac{1}{11.55}$ shares at a price $P_1 = 11.55$. The remaining firms do nothing.
- At date 2, firms with assets-in-place below 6.75 raise funds 10.5 by issuing $\frac{10.5}{7.77}$ shares at a price $P_2^D = 7.77$, and invest. Firms with assets-in place in $[6.75, 7.07]$ raise funds 11.5 by issuing $\frac{11.5}{8.00}$ shares at a price $P_2^{RI} = 8.00$, and invest. The remaining firms do nothing.

We verify this is an equilibrium. First, conditional on firms behaving this way, the repurchase and issue prices are fair, as follows. The date 2 issue-after-repurchase price $P_2^{RI} = 8.00$ is fair, since it solves

$$P_2^{RI} = \frac{E\left[a|a \in [6.75, 7.07]\right] + 11.9}{1 - \frac{1}{11.55} + \frac{11.5}{P_2^{RI}}}.$$

The date 2 direct issue price $P_2^D = 7.77$ is fair, since it solves

$$P_2^D = \frac{E\left[a|a \in [6, 6.75]\right] + 11.9}{1 + \frac{10.5}{P_2^D}}.$$

The date 1 repurchase price is fair, since conditional on date 1 repurchase there is a probability $\frac{7.07-6.75}{12-10.55+7.07-6.75} = 0.18$ that the date 2 price is $P_2^{RI} = 8.00$ and a probability $\frac{12-10.55}{12-10.55+7.07-6.75} = 0.82$ that it is $\frac{E[a|a \in [10.55,12]]}{1-\frac{1}{11.55}} = 12.34$, and so the expected date 2 price is 11.55.

Second, firms respond optimally to the stated repurchase and issue prices. If a firm repurchases then issues, it has $1 - \frac{1}{11.55} + \frac{11.5}{8.00} = 2.35$ shares outstanding at date 3. If a firm issues directly, it has $1 + \frac{10.5}{7.77} = 2.35$ shares outstanding at date 3. Hence the date 3 share price of a firm with assets-in-place a under both these alternatives is

$$\frac{11.9 + a}{2.35},$$

while the date 3 share price from repurchasing at date 1 and then doing nothing is

$$\frac{a}{1 - \frac{1}{11.55}} = 1.09a$$

and the date 3 share price from doing nothing at both dates is simply

$$1 + a.$$

Out of these three alternatives, firms with assets-in-place below 7.07 obtain the highest payoff from either repurchasing and then investing, or directly issuing and investing; they are indifferent between the two options. Firms with assets-in-place between 7.07 and 10.55 obtain the highest payoff from doing nothing. Finally, firms with assets-in-place above 10.55 obtain the highest payoff from repurchasing at date 1 and then doing nothing.[9]

Discussion:

Firms with assets-in-place $a > 10.55$ repurchase shares for strictly less than their true value, $a+1$, and so make strictly positive profits. The reason investors accept the lower price is that these firms pool with worse firms (namely, firms with a between 6.75 and 7.07). But this raises the question of why these worse firms are prepared to repurchase. They do so in order to improve the terms at which they can subsequently issue. If instead they attempt

[9]We have established that firms act optimally when their choice set is limited to the four equilibrium strategies. This still leaves open the possibility that a firm could profitably deviate to some other strategy. Off-equilibrium beliefs that deter such deviations are specified in the proof of Proposition **??**.

to issue equity directly, they obtain a worse price: specifically, they issue shares at a price 7.77 rather than 8.00. As in the title of our paper, these firms buy high and sell low.

The intermediate interval of firms with between 7.07 and 10.55 find issue too dilutive, as in Myers and Majluf, and also find repurchase too expensive.

Firms with $a > 10.55$ strictly profit from their repurchase transactions, even though these transactions fail to create any value. The ultimate source of these profits is that the investing firms with $a \leq 7.07$ end up paying a premium to raise capital. By this, we mean that if firms $a \leq 7.07$ could all credibly pool and issue directly, the issue price P would satisfy $P = \frac{11.9+\frac{1}{2}(6+7.07)}{1+\frac{10.5}{P}}$, i.e., $P = 7.94$, and so the payoff of each firm $a < 7.07$ would be $\frac{11.9+a}{1+\frac{10.5}{7.94}} = \frac{11.9+a}{2.32}$, which is higher than they get in the above equilibrium.

A related observation is that the equilibrium of the Myers and Majluf setting, where repurchase is impossible, entails investment by firms with assets-in-place between 6 and a cutoff level strictly above 7.07. The reason more investment occurs in this case is that low-a firms are able to issue without subsidizing the profitable repurchases of high-a firms.

This raises the question of whether low-a firms are able to avoid the cross-subsidy even when repurchases are possible. The answer is that they cannot, at least not in any equilibrium satisfying NDOC (see Proposition **??**). The reason is that, in any equilibrium, high-a firms make strictly positive profits. This property arises because firms with a close to the maximum value of 12 make strictly positive profits unless investors require a price close to 13 to sell their stock to a repurchasing firm, or in other words, investors interpret a repurchase offer as coming from a firm with a high a. But equilibrium zero-profits for high-a firms are inconsistent with such beliefs, because high-a firms could then profitably deviate to repurchase at date 1, triggering very positive beliefs, and then issue at a very favorable price at date 2 (since the beliefs inherited from date 1 are very positive).

2 Model and preliminary results

Our model is essentially the same as Myers and Majluf (1984). The only substantive difference is that whereas Myers and Majluf consider a firm's interactions with the equity market at just one date, we consider two possible dates. As we will show, this additional feature generates equilibrium share repurchases.

There are four dates, $t = 0, 1, 2, 3$; an all-equity firm, overseen by a manager; and at each of dates 1 and 2, a large number of risk-neutral investors who trade the firm's stock. We normalize the date-0 number of shares to 1.

At date 0, the manager of the firm privately learns the value of the firm's existing assets ("assets-in-place"). Write a for the expected value of these existing assets, where $a \in [\underline{a}, \overline{a}]$, and the distribution of a is given by measure μ. We assume the distribution of a has full support on $[\underline{a}, \overline{a}]$, and admits a bounded and continuous density function, denoted by f. In addition to assets a, the firm has cash (or other marketable securities) with a value S.

At the end of date 2, the firm has an opportunity to undertake a new project. (In Section **??**, we extend the model to allow for a choice of investment timing, with the firm able to invest at either date 1 or date 2.) The project requires an initial investment I and generates an expected cash flow $I + b$. For simplicity, we assume that b is common knowledge; in other words, we focus on a version of the Myers and Majluf environment in which asymmetric information is about assets-in-place, not investment opportunities. Throughout, we assume $I > S$, so that the firm needs to raise external financing to finance the investment I.

We make the following extremely mild assumption on the relation between b and the density function f of asset values:

Assumption 1 *The density f equals $1/b$ at at most countably many values of a.*

The assumption is satisfied generically. It is used only in the proof of equilibrium existence (Propositions **??** and **??**).

At each of dates $t = 1, 2$, the firm can issue new equity and/or repurchase existing equity.

Equity issues and repurchases take place as follows. The manager offers to buy or sell a fixed dollar amount s_t of shares, where $s_t > 0$ corresponds to share repurchases and $s_t < 0$ corresponds to share issues. Investors respond by offering a quantity of shares in exchange. In other words, if $s_t > 0$ each investor offers a number of shares he will surrender in exchange for s_t; and if $s_t < 0$, each investor offers a number of shares he will accept in return for paying the firm $-s_t$.

At $t = 1$, there is a small probability $\alpha \geq 0$ that the firm is exogenously unable to execute a capital market transaction. Concretely, the need to make decisions about other matters may exhaust a firm's decision-making capacity; or random factors may result in a firm's board failing to reach the consensus needed to authorize capital market transactions.[10] The possibility of $\alpha > 0$ is required only for Part (II) of Proposition **??**, which establishes the existence of an equilibrium satisfying NDOC. Even here, the result holds for arbitrarily small values of $\alpha > 0$. We term a firm that is exogenously unable to access capital markets at date 1 as *inactive*, and any other firm as *active* (regardless of whether the firm actually takes a date 1 action).[11]

(Note that both a and $I + b$ are expected values, so our model allows for very volatile cash flows. In particular, we assume that there is enough cash flow volatility that it is impossible for firms to issue risk free debt. In general, the choice between risky debt and equity under asymmetric information is non-obvious; see Fulghieri, Garcia and Hackbarth (2014) for a recent characterization. In Section **??** we discuss the robustness of our analysis to allowing for other securities.)

At date 3, the true value of the firm is realized, including the investment return, and the firm is liquidated.

Write P_3 for the date-3 liquidation share price, and write P_1 and P_2 for the transaction

[10] Board approval is required for both SEOs and repurchase programs.

[11] One could also analyze a version of the model with a symmetric assumption at date 2, i.e., at each of dates 1 and 2, there is a probability α that a firm is exogenously unable to execute a capital market transaction. Adding this possibility at date 2 has no impact on our results, but adds unnecessary complexity to the analysis.

price of the shares at dates $t = 1, 2$. Because the number of investors trading at each of dates 1 and 2 is large, competition among investors implies that the date t share price is

$$P_t = E\left[P_3 | \text{date } t \text{ information, including firm offer } s_t\right]. \tag{1}$$

The manager's objective is to maximize the date 3 share price, namely

$$P_3 = \frac{S - s_1 - s_2 + a + b\mathbf{1}_{\text{investment}}}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}}, \tag{2}$$

where $\mathbf{1}_{\text{investment}}$ is the indicator function associated with whether the firm undertakes the new project, and the denominator reflects the number of shares outstanding at date 3. Note that in the case that only share issues are possible, the manager's objective function coincides with the one specified in Myers and Majluf (1984), which is to maximize the utility of existing ("passive") shareholders. In our setting, where repurchases are possible, the manager's objective function can be interpreted as maximizing the value of passive shareholders, who neither sell nor purchase the firm's stock at dates 1 and 2. Alternatively, the manager's objective can be motivated by assuming that the manager himself has an equity stake in the firm, and is restricted from trading the firm's shares on his own account.[12]

For use throughout, observe that (**??**) and (**??**), together with the fact that the firm invests whenever it has sufficient funds, imply that the date 2 price conditional on s_1 and s_2

[12]Note that if the manager also put weight on a high date 1 share price this would further increase the manager's incentives to repurchase equity. On the other hand, it is important for our analysis that the manager does not fully internalize the welfare of date 0 shareholders who sell at date 1: in particular, our analysis requires that if a manager is able to repurchase shares at less than their true value, then he does so. As discussed in the text, one justification is that the manager seeks to maximize the value of his own equity stake. A second justification is that when a firm repurchases its own stock, it may not be its existing shareholders who sell shares to the firm; instead, the firm's repurchase offer may be filled by short-sellers of the firm's stock. Attaching zero welfare weight to short-sellers is analogous to the Myers and Majluf assumption of attaching zero welfare weight to new purchasers of the firm's shares.

A separate but related issue is whether a firm's shareholders could improve firm value by contracting with the firm's manager, as suggested by Dybvig and Zender (1991). For example, as Corollary **??** below demonstrates, before the value of assets a is realized, a firm's shareholders would ideally like to prohibit a firm from repurchasing its shares. However, and parallel to Persons' (1994) response to Dybvig and Zender, such a commitment is vulnerable to renegotiation because once a is realized high-a firms can generate value for their shareholders by repurchasing.

is

$$P_2(s_1, s_2) = \frac{S - s_1 + E[a|s_1, s_2] + b\mathbf{1}_{S-s_1-s_2 \geq I}}{1 - \frac{s_1}{P_1}}. \tag{3}$$

Iterating, (**??**) and (**??**), together with the law of iterated expectations, imply that the date 1 price conditional on s_1 and the unconditional date 0 price are respectively

$$\begin{aligned} P_1(s_1) &= S + E\left[a + b\mathbf{1}_{S-s_1-s_2 \geq I} | s_1\right] && (4) \\ P_0 = E[P_3] &= S + E[a] + b \times [\text{fraction of firms that invest}]. && (5) \end{aligned}$$

From (**??**) and (**??**), the payoff of firm a from (s_1, s_2) is

$$\begin{aligned} &\frac{S-s_1-s_2+a+b\mathbf{1}_{S-s_1-s_2 \geq I}}{\left(1 - \frac{s_1}{P_1}\right)\left(1 - \frac{s_2}{S-s_1+E[a|s_1,s_2]+b\mathbf{1}_{S-s_1-s_2 \geq I}}\right)} \\ &= \frac{S-s_1-s_2+a+b\mathbf{1}_{S-s_1-s_2 \geq I}}{\left(1 - \frac{s_1}{S+E\left[a+b\mathbf{1}_{S-s_1-s_2 \geq I}|s_1\right]}\right)\left(1 - \frac{s_2}{S-s_1+E[a|s_1,s_2]+b\mathbf{1}_{S-s_1-s_2 \geq I}}\right)} \end{aligned}. \tag{6}$$

We characterize the perfect Bayesian equilibria (PBE) of this game. We restrict attention to pure strategy equilibria in which all investors hold the same beliefs off-equilibrium. We focus on equilibria in which all firms play a best response (as opposed to equilibria in which almost all firms play a best response).[13]

Finally, we state here a simple monotonicity result, which we use repeatedly:

Lemma 1 *If a' and a'' are either both active firms, or both inactive firms, and conduct capital transactions (s_1', s_2') and (s_1'', s_2'') respectively, with $S - s_1' - s_2' > S - s_1'' - s_2''$, then $a' < a''$.*

In other words, better firms raise fewer funds across dates 1 and 2. An immediate corollary of Lemma **??** is:

[13]Given a perfect Bayesian equilibrium in which almost all firms play a best response, one can easily construct an equilibrium in which all firms play a best response by switching the actions of the measure zero set of firms who originally did not play a best response. Because only a measure zero set of firms are switched, the original set of beliefs remain valid.

Corollary 1 *In any equilibrium, there exist cutoffs* $a^*, a^{i*} \in [\underline{a}, \overline{a}]$ *such that all active (respectively, inactive) firms* $a < a^*$ *(respectively,* $a < a^{i*}$*) invest and all active firms* $a > a^*$ *(respectively, inactive firms* $a > a^{i*}$*) do not invest.*

3 One-period benchmark

Before proceeding to our main analysis, we characterize the equilibrium of the benchmark model in which firms can only issue or repurchase shares at date 2, with the date 1 issue/repurchase decision s_1 exogenously set to 0. The main conclusion of this section is that the Myers and Majluf conclusion holds: only the lowest-a firms issue and invest, and repurchases play no meaningful role. In other words, the addition of the possibility of repurchases to the Myers and Majluf environment is, by itself, inconsequential. Instead, our results further below are driven by the possibility of firms engaging in capital transactions at multiple dates.

The key reason that firms do not take advantage of repurchases in a one-period model is the no-trade theorem (Milgrom and Stokey (1982)). Even though firms enjoy an informational advantage relative to investors, they are unable to profit from this advantage.[14]

Proposition 1 *In the one-period benchmark, the set of firms that repurchase is of measure 0.*

Proposition **??** establishes that, in the one-period benchmark, a firm's ability to repurchase its own stock plays no meaningful role. Accordingly, the equilibria of the one-period benchmark coincide with those of the standard Myers and Majluf (1984) setting, as formally established by the next result:

[14] Bond and Eraslan (2010) study trade between differentially-informed parties in a common-values setting. The no-trade theorem does not apply because the eventual owner of the asset takes a decision that affects the asset's final cash flow. Trade affects the information available to the party making the decision. A similar force allows repurchases to occur in the full model, analyzed below: trade of shares at date 1 affects a firm's ability to raise finance at date 2.

Proposition 2 *In any equilibrium, there exists $a^* \in (\underline{a}, \bar{a}]$ such that almost all firms below a^* issue the same amount s^* and invest, while almost all other firms receive the same payoff as doing nothing (i.e., $P_3 = S + a$).*

Proposition **??** characterizes properties an equilibrium must possess. However, it does not actually establish the existence of an equilibrium. But, this is easily done. In particular, fix any s^* such that $S - s^* \geq I$, and define a^* as the solution to

$$a^* = \max\left\{a \in [\underline{a}, \bar{a}] : \frac{S - s^* + a^* + b}{1 - \frac{s^*}{S + E[a|a \in [\underline{a}, a^*] + b]}} \geq S + a^*\right\}. \tag{7}$$

There is an equilibrium in which all firms with assets below a^* issue and raise an amount $-s^*$, while firms with assets above a^* do nothing. Off-equilibrium-path beliefs are such that any other offer to issue (i.e., $s < 0$ and $s \neq s^*$) is interpreted as coming from the worst type $\underline{a}$, and any offer to repurchase (i.e., $s > 0$) is interpreted as coming from the best type $\bar{a}$.

Observe that if

$$\frac{I + \bar{a} + b}{1 + \frac{I - S}{S + E[a] + b}} \geq S + \bar{a},$$

this benchmark model has an equilibrium in which all firms invest. In this case, asymmetric information about a does not cause any social loss. In order to focus on the economically interesting case in which asymmetric information distorts investment, for the remainder of the paper we impose the following assumption:

Assumption 2 $\frac{1}{1-\alpha} \frac{I + \bar{a} + b}{1 + \frac{I - S}{S + E[a] + b}} < S + \bar{a}$.[15]

For use below, note that Assumption **??** implies

$$\bar{a} > E[a] + b > \underline{a} + b. \tag{8}$$

[15]The factor $\frac{1}{1-\alpha}$ is included on the LHS to ensure that, in the full model, there is no equilibrium in which all active firms invest, even if they receive a subsidy a cross-subsidy from firms that are exogenously inactive at $t = 1$. If there are no inactive firms ($\alpha = 0$), Assumption **??** is simply $\frac{I + \bar{a} + b}{1 + \frac{I - S}{S + E[a] + b}} < S + \bar{a}$.

A final point to note about the one-period benchmark is that it possesses multiple equilibria. One source of multiplicity stems from the net issue size, $-s^*$. This can be seen from equation (**??**), which determines the marginal investing firm a^*: different choices of s^* such that $S - s^* \geq I$ lead to different solutions a^*. A second potential source of multiplicity is that, even fixing s^*, the inequality in (**??**) may hold with equality at multiple different values of a. Economically, there may be an equilibrium in which the marginal investing firm has a high a, leading to a favorable issue price, which enables high-a firms to invest without suffering too much dilution; and another equilibrium in which the marginal firm has a low a, leading to an unfavorable issue price, which implies that only low-a firms are prepared to suffer the dilution cost associated with investment.

The first source of multiplicity can be eliminated by appealing to equilibrium refinements. In particular, the only choice of s^* that survives D1 is $s^* = S - I$.[16] The second source of multiplicity can be eliminated by imposing further conditions on the distribution of a. In particular, if the following condition holds, then for any s^* such that $S - s^* \geq I$ there is a unique equilibrium of the one-period benchmark associated with this s^*.

Condition 1 $\frac{E[\tilde{a}|\tilde{a} \leq a]+b-a}{S+E[\tilde{a}|\tilde{a} \leq a]+b}$ *is strictly decreasing in* a.

Condition **??** holds, for example, whenever a is distributed uniformly. Condition **??** is used only in Proposition **??**.

Lemma 2 *If Condition* **??** *holds, then for any* s^* *satisfying* $S - s^* \geq I$*, there is a unique equilibrium of the one-period benchmark.*

[16]The D1 refinement criterion (Cho and Kreps 1987) requires that after any deviation, the support of off-equilibrium beliefs is a subset of those firms that are most likely to make such a deviation. In brief, an equilibrium with $S - s_2 > I$ fails D1 because the off-equilibrium beliefs associated with a deviation $\tilde{s}_2 = S - I$ that are needed to support such an equilibrium heavily weight firms with low a, even though firms with higher values of a gain from this deviation for a strictly largely set of off-equilibrium beliefs. A full proof is available upon request.

4 Analysis of the dynamic model

We now turn to the analysis of the full model, in which firms can engage in capital transactions at multiple dates.

4.1 Existence of a repurchase equilibrium

We first show that there is nothing "special" about the example above. There always exists an equilibrium in which the best firms strictly profit from repurchasing, while worse firms repurchase their stock for more than it is worth—i.e., "buy high"—in order to improve the terms at which they can subsequently issue shares to finance the investment.

Proposition 3 *For either $\alpha = 0$, or $\alpha > 0$ sufficiently small, an equilibrium exists in which a strictly positive mass of firms pool and repurchase at date 1. A strict subset of these firms make strictly positive profits from the repurchase, and do nothing at date 2. The remaining repurchasing firms repurchase their stock for more than it is worth, and then issue enough shares to finance investment at date 2.*

The proof of Proposition **??** is constructive. First, the proof constructs an equilibrium for the case $\alpha = 0$. The equilibrium is either similar to the above example; or else features all firms repurchasing at date 1, with a strict subset then issuing equity to fund investment at date 2. The proof then perturbs the equilibrium constructed for $\alpha = 0$ to construct an equilibrium for the case of α small but strictly positive.

4.2 Necessity of repurchases

As is common in games of asymmetric information, multiple equilibria exist. However, we next show that the properties stated in Proposition **??** are possessed by *any* equilibrium satisfying the refinement "Never Dissuaded Once Convinced" (NDOC) (Osborne and Rubinstein (1990)). Hence the NDOC refinement selects precisely equilibria that feature repurchases.

NDOC is a consistency condition on how beliefs evolve over time. Once investors are 100% sure that the firm's type belongs to some set A, NDOC states that subsequent beliefs put positive probability only on firm types within A. This restriction is highly intuitive and is typically regarded as mild; see, for example, Rubinstein (1985) and Grossman and Perry (1986), or more recently, its use as Assumption 1 in Ely and Valimaki (2003) and as Condition R in Feinberg and Skrzypacz (2005).

More formally, in our context, NDOC states that date 2 investor beliefs after observing off-equilibrium firm actions (s_1, s_2) must satisfy the following: (I) if s_1 is an equilibrium action, then date 2 beliefs assign probability 1 to the firm's type lying in the set of firms who play s_1 in equilibrium, and (II) if s_1 is not an equilibrium action, and date 1 beliefs assign probability 1 to some subset A of firm types, date 2 beliefs likewise assign probability 1 to the same subset A.

Proposition 4 *(I) Any equilibrium satisfying NDOC has the properties stated in Proposition* **??***, and in particular, features strictly profitable repurchases at date* 1. *(II) For* $\alpha > 0$ *sufficiently small, an equilibrium satisfying NDOC exists.*

The economics behind Part (I) of Proposition **??** is as follows. Under Assumption **??**, the best firms do not invest in equilibrium.[17] Suppose that, contrary to the stated result, they do not repurchase either. Consequently, the final payoff of a high-value firm a is simply $S + a$. This implies that repurchases are unattractive for the top firm $\bar{a}$ only if investors charge at least $S + \bar{a}$ to surrender their shares; in turn, this requires investors to believe that repurchase offers come from very good firms. But given these beliefs, a low-value firm could profitably deviate from its equilibrium strategy by repurchasing at date 1, thereby triggering beliefs that it is very good, and then (by NDOC) issue at a high price at date 2.

An important implication of Proposition **??** is that the equilibrium outcome of the one-period benchmark economy is not an equilibrium outcome of the full model under NDOC. At first sight, this might seem surprising: one might imagine that one could take the equilibrium

[17] Formally, this is established in Corollary **??** in the appendix.

of the one-period economy and then assign off-equilibrium beliefs to make other actions, and in particular repurchases, unattractive. However, the dynamic nature of the model makes this impossible. The reason is that, as just illustrated, to deter repurchases, beliefs must assign a large weight to a repurchasing-firm being a high type; but given these beliefs, a deviating firm can issue at attractive terms at date 2. In brief, under NDOC it is impossible to assign beliefs that deter *both* date 1 repurchase and date 2 issue.

4.3 Existence of a repurchase equilibrium satisfying NDOC

Part (II) of Proposition **??** establishes the existence of an equilibrium satisfying NDOC.[18] This is the only result in the paper that requires $\alpha > 0$, i.e., some firms are exogenously unable to conduct capital market transactions at date 1.

To understand the main threat to equilibrium existence, consider again the example of Section **??**, in which $\alpha = 0$. In the equilibrium described, if a firm does nothing at date 1, the NDOC restriction implies that investors believe the firm has a type $a \leq 10.55$, regardless of the firm's date 2 action. This in turn means that any firm that does nothing at date 1 is able to repurchase shares at date 2 for a price of $1 + 10.55 = 11.55$ (or less). In particular, firms with $a > 10.55$ would make strictly positive profits by doing nothing at date 1, and then repurchasing at date 2.

In this example, the act of doing nothing at date 1 carries a lot of signaling power—and arguably, too much signaling power. After all, it is easy to imagine that a firm does nothing at date 1 for some exogenous reason; for example, perhaps its manager fails to get approval for either an issue or repurchase, or is otherwise distracted. It is exactly this issue that is addressed by the possibility that a firm is exogenously inactive at date 1, i.e., $\alpha > 0$. In this case, NDOC does not impose any restriction on investor beliefs about firms that do nothing at date 1.

As an aside, we note that for many parameter values we are able to establish the existence

[18] Madrigal et al (1987) give an example of a game in which no equilibrium satisfies NDOC.

of an equilibrium satisfying NDOC even for the case $\alpha = 0$. The example falls in this class, because doing nothing at date 1 and then repurchasing at date 2 is not a profitable deviation if off-equilibrium beliefs are that a firm that does nothing and then repurchases has $a = 10.55$. (Note that this belief satisfies NDOC.) The reason is that both the deviation profits and equilibrium profits for firms $a > 10.55$ are $\frac{a}{1-\frac{1}{11.55}}$. However, this conclusion depends on the fact that, in equilibrium, firms use all their available cash to repurchase shares. For general parameter values and the case $\alpha = 0$, we have neither been able to establish a general existence result, but nor have we found a counterexample to existence.

5 Repurchases and the incidence of share issues

In the setting we analyze, the same force—namely a firm's superior information about its value—affects both a firm's issuance and repurchase decisions. Importantly, the two decisions are linked, and as such, must be analyzed together. In particular, we show that the possibility of share repurchases reduces the fraction of firms that conduct a large enough issue to finance the investment.

More formally, we compare the equilibria of the benchmark one-period model, in which share repurchases do not arise, with the equilibria of the full model. We show the following:

Proposition 5 *Consider an equilibrium in which (I) a positive mass of firms engage in strictly profitable repurchases, and (II) any inactive firm that invests raises weakly fewer funds than any active firm that invests. There exists an equilibrium of the one-period benchmark economy in which strictly more firms invest.*[19]

Proposition **??** establishes that condition (I) holds in any equilibrium satisfying NDOC. Condition (II) is vacuously satisfied if there are no inactive firms ($\alpha = 0$). It is also trivially satisfied if there are inactive firms ($\alpha > 0$), and any inactive firm that invests raises just

[19]In particular, under Condition **??**, there is a unique equilibrium of the one-period benchmark that satisfies D1, and a greater fraction of firms invest in this equilibrium than in any equilibrium of the full model.

enough funds to invest (i.e., $S - s_2 = I$). One reason to focus on equilibria with this property is that in the one-period benchmark, any equilibrium in which firms raise strictly more funding than required fails D1.[20]

Proposition **??** establishes that the possibility of repurchasing shares reduces share issuance. It is worth highlighting that this effect does not in general stem from firms switching from share issuance to share repurchases. To illustrate this point, consider again the example of Section **??**. In the one-period benchmark, firms with $a \leq 7.42$ issue. In the full model, firms with $a \leq 7.07$ issue, with firms between 7.07 and 7.42 doing nothing.

Instead, the reduction in issuance stems from the fact that, in equilibrium, some high-value firms strictly profit from repurchasing their stock for less than its true value. Because investors break even in expectation, the ultimate source of these profits is low-value firms who initially pool with high-value firms and repurchase, in order to reduce the cost of subsequent issues. Low-value firms lose money on the repurchase leg of this transaction. In the one-period benchmark, repurchases do not arise (Proposition **??**), and low-value firms do not have to endure this loss-making leg. This allows them to issue at better terms, which in turn means that a greater fraction of firms find issuance (and investment) preferable to non-issuance.

In our model, overall social surplus is simply the product of b and the fraction of firms that invest.[21] Hence an immediate corollary of Proposition **??** is:

Corollary 2 *Any equilibrium satisfying the conditions stated in Proposition* **??** *has lower social surplus than some equilibrium of the one-period benchmark model.*

At least since Arrow (1973), it has been understood that the possibility of economic agents signaling their type by undertaking a socially costly action may result in lower welfare

[20]Because our dynamic model is not a standard signaling game (in particular, the informed party takes actions at two separate dates, with a response from the uninformed party in between), strictly speaking D1 and other standard refinements do not apply. Nonetheless, we view the result from the one-period benchmark as providing a good reason to treat with considerable suspicion any equilibrium of the full model in which firms raise financing strictly in excess of I at date 2.

[21]If each investor holds a diversified portfolio of shares, this welfare measure coincides with the Pareto welfare ranking.

relative to a situation in which signaling is prohibited or otherwise impossible.[22] In our setting, however, repurchases carry no deadweight cost, yet welfare is still reduced.

6 Stock price reactions and other model predictions

6.1 Stock price reactions

As discussed in the introduction, an important implication of the benchmark one-period model is that a firm's stock price falls in response to an announcement of an SEO, and this prediction is consistent with a large body of empirical research. Our analysis shows that the same forces that deliver price-drops in response to SEOs also lead to equilibrium share repurchases. Here, we analyze the model's predictions for price reactions.

Recall that, in equilibrium, a subset of repurchasing firms lose money on the repurchase transaction. Such firms are nonetheless happy to repurchase because, by doing so, they improve the terms of the subsequent SEO:

Proposition 6 *Let (s_1, s_2) and (s_1', s_2') be strategies each played by a positive mass of firms, where both allow investment, i.e., $S - s_1 - s_2 \geq I$ and $S - s_1' - s_2' \geq I$. Suppose that (s_1, s_2) is a repurchase strategy, i.e., $s_1 > 0$, while (s_1', s_2') does not entail repurchases, i.e., $s_1', s_2' \leq 0$. Then $P_2(s_1, s_2) \geq P_2(s_1', s_2')$, with strict inequality if only a subset of repurchasing firms invest, i.e., $\Pr(s_2|s_1) < 1$. Moreover, if $s_1' < 0$, then $P_1(s_1') = P_2(s_1', s_2')$.*

An immediate corollary is:

Corollary 3 *The cumulative price change, $P_2 - P_0$, of a firm that repurchases and then issues to fund investment is greater than the cumulative price change over the same period of a firm that issues to fund investment without repurchasing.*

[22] For a recent result along these lines, see Hoppe, Moldovanu and Sela (2009).

These predictions are consistent with the empirical findings of Billett and Xue (2007), who document that the price response to an issue announcement is less negative for firms that previously repurchased in the preceding three years than for firms that did not.

Our model also predicts negative announcement effects for share issues. Note that this prediction depends on the comparison of the issue price with the alternatives of (i) doing nothing and (ii) repurchasing. Comparison (i) is the standard effect studied in existing one-period models. Comparison (ii) is new to our analysis. An announcement of a direct share issue (without prior repurchase) leads to a price drop relative to an announcement of a repurchase because, by Proposition **??**, the issue price following a repurchase is higher than the price associated with direct issue. Our formal result is:

Proposition 7 *Suppose that Condition* **??** *holds, Condition (II) of Proposition* **??** *holds, and all equilibrium strategies are played by a positive mass of firms. (A) Prices fall in response to share issue at date 2: if* $S - s_1 < I$ *then* $E\left[P_2|s_1, s_2 < 0\right] \leq P_1\left(s_1\right)$, *with strict inequality if a positive mass of firms that play* s_1 *do not invest. (B) Prices fall in response to share issue at date 1:* $E\left[P_1|s_1 < 0\right] < P_0$.

Note that the conditions stated in Proposition **??** are required only for Part (B), dealing with the date 1 price response, and are not required for Part (A), dealing with the date 2 price response. The conditions are sufficient to eliminate the possibility of equilibria in which inactive firms issue at very bad terms at date 2, in which case date 1 issue could conceivably generate a price increase because it signals that a firm is active rather than inactive.

Our model also generates cross-sectional predictions between, on the one hand, the *size* of repurchases and issues, and on the other hand, the price response associated with these transactions. These predictions emerge in equilibria of the model in which multiple repurchase and issue levels coexist (in contrast to the example, which features just one repurchase level).[23] As one would expect, larger repurchases are associated with higher repurchase prices. Similarly, larger issues are associated with lower issue prices. Both predictions are

[23] One can show, via numerical simulation, that such equilibria exist.

consistent with empirical evidence: see, for example, Ikenberry, Lakonishok and Vermaelen (1995) or Allen and Michaely (2003) for evidence on repurchases, and Asquith and Mullins (1986) for evidence on issues.

Proposition 8 *(A, repurchases) Let s' and $s'' > s' > 0$ be equilibrium repurchases, with associated prices P' and P'', such that there exist firms that repurchase s' (respectively, s'') and do not conduct any other capital transaction at any other date. Then $P'' \geq P'$.*

(B, issues) Let $(s_1' \neq 0, s_2')$ and $(s_1'' \neq 0, s_2'')$ be equilibrium strategies such that $S - s_1'' - s_2'' > S - s_1' - s_2' \geq I$. Then $P_2(s_1', s_2') > P_2(s_1'', s_2'')$, i.e., greater cumulative issue is associated with lower date 2 prices.[24]

6.2 Repurchase size versus SEO size

In our model, firms that repurchase and then issue do so in order to finance an investment that they initially lacked the resources to undertake. Consequently, a very basic prediction of our model is that when firms repurchase and then conduct an SEO, the revenue from the SEO exceeds the cash distributed via prior repurchases. Consistent with this prediction, Billett and Xue (2007) report that the median and mean ratio of repurchases to subsequent SEOs is 6% and 38% respectively.

6.3 Repurchases, financial constraints, and investment opportunities

Our model is one in which firms (i) possess profitable investment opportunities, but (ii) lack easy access to outside financing to undertake these investments, and moreover, these facts are publicly observed. Our main results show that, under such circumstances, a subset of firms repurchase. In contrast, if either assumptions (i) or (ii) does not hold, no repurchases would occur within our model. As such, our analysis predicts that repurchase activity

[24]It is also possible to establish that $s_1' > s_1''$, i.e., greater cumulative issue is associated with smaller initial repurchases. A proof is available upon request.

should be greater among firms that are credit constrained, and have profitable investment opportunities. Note that this prediction holds even if firms have other motives to repurchase besides the one identified in our analysis, as is likely to be the case.

Grullon and Michaely (2002) report evidence consistent with this prediction. In a comparison between firms that repurchase (but do not issue dividends) with those that issue dividends (but do not repurchase), repurchasing firms have a higher Tobin's Q, which is a standard proxy for investment opportunities, and are both smaller and younger, which are standard proxies for credit constraints.

A related prediction of our model is that, among repurchasing firms, firms that subsequently conduct an SEO have better investment opportunities and are more credit constrained than those that do not. This prediction is obtained as follows. From the discussion immediately above, repurchase-SEO firms should *all* have good investment opportunities and be credit constrained. In contrast, firms may also repurchase for reasons outside our model, and there is little reason to think these other repurchasing firms would *all* be credit constrained with good investment opportunities. Consequently, firms that only repurchase without a subsequent SEO are a mix of firms with these characteristics (i.e., the high a firms in our model), and firms not necessarily possessing these characteristics. Therefore, conditional on repurchase, firms that subsequently issue equity are more likely to have good investment opportunities and be credit constrained.

Billett and Xue (2007) provide evidence for this second prediction. They compare repurchase-SEO firms with repurchase firms that do not conduct an SEO in the following three years. Repurchase-SEO firms have a higher Tobin's Q, and are more credit constrained (by standard proxies).

7 Extension: Investment timing

In our main model, the investment project can only be undertaken at date 2. Here, we consider an extension in which the investment can be undertaken at either date 1 or date 2 (though not both). We write I_t and b_t for the investment size and net present value if the investment is undertaken at date t.

In our analysis above, we normalize the discount rate to 0; or more precisely, the objects S, s_1, s_2, I, b, a are all expressed as date 3 future values. Consequently, in the benchmark case in which the project available is exactly the same at dates 1 and 2, it follows that $\frac{I_1}{I_2} = \frac{b_1}{b_2} =$ gross interest rate. Accordingly, in the discussion below we assume that $b_1 > b_2$: this nests the benchmark case of identical projects, but also allows for time-variation in investment opportunities.

The flexibility of investment timing introduces an additional dimension in which firms can signal their type. In particular, delaying investment is costly because $b_1 > b_2$. Because of this, there may exist equilibria in which bad firms issue and invest at date 1, while good firms signal their type by waiting until date 2 to issue and invest.[25] However, when b_1 and b_2 are sufficiently close (which corresponds to a low discount rate in the benchmark case of identical investment opportunities) one can show that no equilibrium of this type exists, and the best firms never invest in equilibrium. Intuitively, waiting to invest is not a strong enough signal to support separation. In this case, the economic forces behind our result that any equilibrium satisfying NDOC features repurchases remain unchanged, and Proposition **??** continues to hold.

Consequently, when b_1 and b_2 are relatively close, the extension of our model to endogenous investment timing leaves our main results unchanged. At the same time, endogenous investment timing introduces a new effect into our model: namely that repurchases are associated with an inefficient delay of investment. Specifically, if repurchases are exogenously

[25] See Morellec and Schurhoff (2011) for an analysis dedicated to this issue, in a setting where asymmetric information is over the value of a growth option, and investing early is costly because it destroys option value.

ruled out, the one-period benchmark equilibrium remains an equilibrium of the two-period model, with all investment by active firms conducted at date 1. But when repurchases are feasible, any equilibrium satisfying NDOC features at least some investment by active firms at date 2. Hence, there are three distinct costs associated with investment: (i) inefficient delayed investment (the new effect of this section); (ii) the cross-subsidy from investing firms to repurchase-only firms (the effect stressed in the main model); and (iii) the cross-subsidy from better investing firms to worse investing firms (the standard Myers and Majluf effect).

Finally, when b_2 is substantially lower than b_1, then as alluded to above, under some circumstances there exists an equilibrium in which high-a firms invest at date 2 while low-a firms invest at date 1, and no firm repurchases in equilibrium.[26] It is worth emphasizing, however, that the condition that b_2 is sufficiently low relative to b_1 is necessary but not sufficient for the existence of an equilibrium of this type. Specifically, any such equilibrium must satisfy the condition that low-a firms do not mimic high-a firms. But this condition is relatively demanding, because when low-a firms mimic they gain by selling overpriced shares, and these profits may exceed the value lost by delaying investment. We leave a fuller analysis of equilibrium outcomes when b_2 is low relative to b_1 for future research.

8 Robustness

We have restricted attention to the case in which firms can only signal via equity repurchases. However, we do not believe this restriction is critical, as follows.

Our main equilibrium characterization result is that that any equilibrium satisfying NDOC must feature repurchases (Proposition **??**). A key ingredient in this result is that in any candidate equilibrium without repurchases, the best firms would obtain their reservation payoff of $S + a$. As discussed, this property implies that repurchases can only be deterred

[26] An equilibrium of this type exists for an economy in which a is drawn from a binary distribution, and parameter values are as follows: $S = 2$, $I_1 = I_2 = 4$, $b_1 = 1$, $b_2 = 0.2$, $a \in \{4, 9\}$. (Note that Assumption **??** is satisfied provided $\Pr(a = 4)$ is sufficiently large.) It is straightforward to perturb this example to obtain a similar equilibrium for an economy in which a is continuously distributed and $\alpha > 0$.

in equilibrium if off-equilibrium beliefs associate a repurchase offer with a high firm type. The dynamic setting, combined with NDOC, then implies that a firm that deviates and repurchases could issue at very good terms the following period, thereby undercutting the proposed equilibrium without repurchases.

This argument still works even if additional signaling possibilities are introduced,[27] provided that any candidate equilibrium without repurchases has the best firms receiving their reservation payoffs. Indeed, the extension of Section **??** in which investment timing potentially serves as a signal illustrates exactly this. Note that in this generalization firms may repurchase a *different* security from equity; however, under the conditions described, some firms will repurchase some form of risky security.

At the same time, it is important to acknowledge that in some asymmetric information environments, high-value firms possess enough signaling avenues that they can separate themselves from low-value firms. Again, one potential example is (as discussed in the prior section) when investment opportunities decay sufficiently fast that good firms can signal their type by delaying investment. Other examples are when firms are able to post sufficient collateral; or when project-financing is feasible; or when firms can issue callable convertible bonds, as discussed in Chakraborty and Yilmaz (2011). In all these cases, in a benchmark model in which repurchases are exogenously ruled out there exist equilibria in which high-value firms obtain strictly more than their reservation payoff $S+a$. Moreover, these equilibria continue to exist even when repurchases are feasible.[28,29]

[27]For existing analysis of multi-dimensional signaling models, see, e.g., John and Williams (1987), Ofer and Thakor (1987), Williams (1988), and Vishwanathan (1995).

[28]To deter repurchases at date 1, off-equilibrium beliefs place high probability on a firm having high a, together with a low probability on a firm having low a. Off-equilibrium beliefs at date 2 then place high probability on a firm having low a if a firm repurchases at date 1 and then issues at date 2. These off-equilibrium beliefs are similar to those used in the constructive proof of equilibrium existence (Proposition **??**). These beliefs ensure that repurchase is an unattractive deviation for high-a firms, whose equilibrium payoff in this case strictly exceeds $S+a$.

[29]Recall also that good firms are able to separate in both Brennan and Kraus (1987) and Constantinides and Grundy (1989) by repurchasing *some* security. In these papers, it is important that the repurchased security is different from the security issued. As noted, in a more general version of our model, firms may repurchase a security different from equity, but unlike in Brennan and Kraus, and Constantinides and Grundy, it is not necessary that the repurchased security and issued security differ.

To summarize the above discussion, our results are robust to many perturbations of the model that expand the signaling possibilities of high-a firms. At the same time, there are certainly environments in which high-a firms are able to separate from low-a firms and profitably invest, and in such cases there exist equilibria without repurchases.

9 Conclusion

Share prices generally fall when a firm announces an SEO. A standard explanation of this fact is that an SEO communicates negative information to investors. We show that if repeated capital market transactions are possible, this same asymmetry of information between firms and investors implies that some firms repurchase shares in equilibrium. A subset of these firms directly profit from the repurchase transaction, consistent with managerial accounts. The ultimate source of these profits is that other firms buy "high" in order to improve the terms of a subsequent SEO, consistent with the empirical findings of Billett and Xue (2007). The possibility of repurchases reduces both SEOs and investment. Our analysis also suggests that firms that are credit constrained and have good investment opportunities are more likey to repurchase. Overall, our analysis highlights the importance of analyzing SEOs and repurchases in a unified framework.

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Appendix

Proof of Lemma ??: Suppose to the contrary that $a' \geq a''$. Since firms a' and a'' follow different strategies, $a' > a''$. Let P_1' and P_2' (respectively, P_1'' and P_2'') be the prices associated with s_1' and s_2' (respectively, s_1'' and s_2''). Also, let $\mathbf{1}'$ and $\mathbf{1}''$ be the investment decisions of firms a' and a''.

Because both firms are either active or inactive, they can potentially mimic each other's strategy. From the equilibrium conditions,

$$\frac{S - s_1'' - s_2'' + a'' + b\mathbf{1}''}{1 - \frac{s_1''}{P_1''} - \frac{s_2''}{P_2''}} \geq \frac{S - s_1' - s_2' + a'' + b\mathbf{1}'}{1 - \frac{s_1'}{P_1'} - \frac{s_2'}{P_2'}}. \tag{A-1}$$

By supposition, and given optimal investment decisions, the numerator of the LHS is strictly smaller than the numerator of RHS. Hence the denominator of the LHS must also be strictly smaller, i.e.,

$$1 - \frac{s_1''}{P_1''} - \frac{s_2''}{P_2''} < 1 - \frac{s_1'}{P_1'} - \frac{s_2'}{P_2'}. \tag{A-2}$$

Also from the equilibrium conditions,

$$\frac{S - s_1' - s_2' + a' + b\mathbf{1}'}{1 - \frac{s_1'}{P_1'} - \frac{s_2'}{P_2'}} \geq \frac{S - s_1'' - s_2'' + a' + b\mathbf{1}''}{1 - \frac{s_1''}{P_1''} - \frac{s_2''}{P_2''}}.$$

From (**??**),

$$\frac{a'-a''}{1-\frac{s_1'}{P_1'}-\frac{s_2'}{P_2'}} < \frac{a'-a''}{1-\frac{s_1''}{P_1''}-\frac{s_2''}{P_2''}},$$

which implies

$$\frac{S-s_1'-s_2'+a''+b\mathbf{1}'}{1-\frac{s_1'}{P_1'}-\frac{s_2'}{P_2'}} > \frac{S-s_1''-s_2''+a''+b\mathbf{1}''}{1-\frac{s_1''}{P_1''}-\frac{s_2''}{P_2''}},$$

contradicting the equilibrium condition (**??**) and completing the proof.

Proof of Corollary ??: We prove the result for active firms (the proof for inactive firms is identical). Suppose to the contrary that the claim does not hold, i.e., there exists an equilibrium in which there are firms a' and $a'' > a'$ where a'' invests and a' does not invest. Since investment decisions are optimal, the capital transactions of firms a' and a'', say (s_1', s_2') and (s_1'', s_2''), must satisfy $S - s_1' - s_2' < I \leq S - s_1'' - s_2''$. This contradicts Lemma **??**, completing the proof.

Proof of Proposition ??: Suppose otherwise. Let $s_2(a)$ be the strategy of firm a, and $A^{rep} = \{a : s_2(a) > 0\}$ be the set of firms that repurchase in equilibrium. By supposition, $\mu(A^{rep}) > 0$. On the one hand, a firm prefers repurchasing s_2 to doing nothing if and only if $\frac{S-s_2+a}{1-\frac{s_2}{P_2(s_2)}} \geq S + a$, or equivalently, $P_2(s_2) \leq S + a$. Moreover, note that one cannot have $P_2(s_2(a')) = S + a'$ and $P_2(s_2(a'')) = S + a''$ for $a'' > a'$ and $a', a'' \in A^{rep}$, since this would imply $P_2(s_2(a'')) > P_2(s_2(a'))$, and hence that firm a'' strictly prefers repurchase $s_2(a')$ to the supposed equilibrium repurchase $s_2(a'')$. Consequently, $P_2(s_2(a)) < S + a$ for almost all firms in A^{rep}, and so

$$E\left[P_2(s_2(a)) - (S+a)\,|a \in A^{rep}\right] < 0.$$

On the other hand, investors only sell if $P_2(s_2) \geq E\left[\frac{S-s_2+a}{1-\frac{s_2}{P_2(s_2)}}|s_2\right]$, or equivalently, $P_2(s_2) \geq S + E[a|s_2]$. By the law of iterated expectations, this implies

$$E\left[P_2(s_2(a)) - (S+a)\,|a \in A^{rep}\right] \geq 0.$$

The contradiction completes the proof.

Proof of Proposition ??: Fix an equilibrium. From Proposition **??**, there cannot be a positive mass of firms that repurchase and obtain $P_3 > S + a$. By a parallel proof, there cannot be a positive mass of firms who issue, do not invest, and obtain $P_3 > S + a$. By (**??**), any issue s_2 that is enough for investment is associated with the price $P_2(s) = S + E[a|s_2] + b$. Given these observations, standard arguments then imply that there exists some $\varepsilon > 0$ such that almost all firms in $[\underline{a}, \underline{a} + \varepsilon]$ issue and invest: if an equilibrium does not have this property, then these firms certainly have the incentive to deviate and issue and invest, since this is profitable under any investor beliefs. So by Corollary **??**, there exists $a^* > \underline{a}$ such that all firms in $[\underline{a}, a^*)$ issue and invest.

Finally, suppose that contrary to the claimed result that different firms in $[\underline{a}, a^*)$ issue different amounts. Given Lemma **??**, it follows that there exists $\check{a} \in (\underline{a}, a^*)$ such that any firm in $[\underline{a}, \check{a})$ issues strictly more than any firm in $(\check{a}, a^*)$. Hence there must exist firms $a' \in [\underline{a}, \check{a})$ and $a'' \in (\check{a}, a^*)$ such that

$$P_2(s_2(a')) \leq S + a' + b < S + a'' + b \leq P_2(s(a'')).$$

Since $-s_2(a') > -s_2(a'')$, this combines with the equilibrium condition for firm a' to deliver the following contradiction, which completes the proof:

$$\frac{S - s_2(a'') + a' + b}{1 - \frac{s_2(a'')}{P_2(s_2(a''))}} \leq \frac{S - s_2(a') + a' + b}{1 - \frac{s_2(a')}{P_2(s_2(a'))}} \leq \frac{S - s_2(a'') + a' + b}{1 - \frac{s_2(a'')}{P_2(s_2(a'))}} < \frac{S - s_2(a'') + a' + b}{1 - \frac{s_2(a'')}{P_2(s_2(a''))}}.$$

Proof of Lemma ??: We establish that the equation $\frac{S - s^* + a^* + b}{1 - \frac{s^*}{S + E[a|a \in [\underline{a}, a^*] + b]}} = S + a^*$ has a unique solution in a^*. To do so, note that

$$\begin{aligned}
& sign\left(\frac{S-s^*+a^*+b}{1-\frac{s^*}{S+E[a|a\leq a^*]+b}}-(S+a^*)\right) \\
= \quad & sign\left((S+E\left[a|a\leq a^*\right]+b)\frac{S-s^*+a^*+b}{S-s^*+E\left[a|a\leq a^*\right]+b}-(S+a^*)\right) \\
= \quad & sign\left((S+E\left[a|a\leq a^*\right]+b)(S-s^*+a^*+b)-(S+a^*)(S-s^*+E\left[a|a\leq a^*\right]+b)\right) \\
= \quad & sign\left((S+E\left[a|a\leq a^*\right]+b)(-s^*+b)+s^*(S+a^*)\right) \\
= \quad & sign\left(s^*(a^*-E\left[a|a\leq a^*\right]-b)+b(S+E\left[a|a\leq a^*\right]+b)\right) \\
= \quad & sign\left(\frac{E\left[a|a\leq a^*\right]+b-a^*}{S+E\left[a|a\leq a^*\right]+b}-\frac{b}{s^*}\right).
\end{aligned}$$

The sign of the above expression is positive at $a^*=\underline{a}$, and negative (by Assumption **??**) at $a^*=\bar{a}$. The result is then immediate from Condition **??**.

Proof of Proposition ??:

Preliminaries:

Given any date 1 repurchase level $s_1>0$, define an auxiliary function on $[\underline{a},\overline{a}]$:

$$H(x)=\frac{1}{1+\frac{I-S+s_1}{S-s_1+E[a|a\leq x]+b}}(I+x+b)-(S-s_1+x)\,, \tag{A-3}$$

and let $a^*(s_1)=\max\{a: H(\tilde{a})\geq 0, \forall\tilde{a}\leq a\}$. Intuitively, $a^*(s_1)$ is the smallest zero of $H(x)$, beyond which the function first becomes negative. We first show that $a^*(s_1)$ is well-defined, strictly decreasing in s_1, and lies in $(\underline{a},\bar{a})$. The proof is as follows. First, $H(\underline{a})>0$. In addition, $H(\cdot)$ is strictly decreasing in s_1 for any $a^*>\underline{a}$. Consequently, Assumption **??** implies $H(\overline{a})<0$. Existence of $a^*(s_1)$ in $(\underline{a},\bar{a})$ follows by continuity. Monotonicity also follows immediately.

Observe that at $s_1=0$ and $a_1=\underline{a}$,

$$\frac{1}{1-\frac{s_1}{S+\bar{a}}}\frac{1}{1+\frac{I-S+s_1}{S-s_1+E[a|a\leq a_1]+b}}(I+a_1+b)>S+a_1. \tag{A-4}$$

By continuity, choose $\bar{a}_1 > \underline{a}$ and $\bar{s}_1 > 0$ such that inequality (**??**) holds for all $(a_1, s_1) \in [\underline{a}, \bar{a}_1] \times [0, \bar{s}_1]$. Consequently, $a^*(s_1) > \bar{a}_1$ for any $s_1 \leq \bar{s}_1$.

Fix $s_1 \in (0, \min\left\{\bar{s}_1, \frac{S}{2}\right\}]$ such that

$$S + a^*(s_1) \neq S + E[a] + b\Pr(a \leq a^*(s_1)), \tag{A-5}$$

and sufficiently small such that

$$\max\left\{\frac{I - S + s_1}{S - s_1 + E\left[a|a < a^*(\frac{S}{2})\right] + b}, \frac{I - S + s_1}{S - s_1 + E\left[a|a < \bar{a}_1\right] + b}\right\} \leq \frac{I - S}{S + \underline{a} + b}. \tag{A-6}$$

To show that such a choice of s_1 is possible, we show that there is no subinterval of $[0, \bar{s}_1]$ over which (**??**) instead holds with equality. Suppose to the contrary that such a subinterval exists. Choose s_1 from the interior of this subinterval such that a^* is continuous at s_1, and such that $bf(a^*(s_1)) \neq 1$: such a choice is possible since a^* is a strictly decreasing function, and hence has at most countably many points of discontinuity, combined with Assumption **??**. Then the supposition that (**??**) holds with equality over a subinterval containing s_1 implies that $1 = bf(a^*(s_1))$, giving a contradiction.

Given s_1, we explicitly construct an equilibrium. There are two cases, corresponding to whether $S + a^*(s_1)$ is strictly larger or strictly smaller than $S + E[a] + b\Pr(a \leq a^*(s_1))$. In the first case, all firms repurchase s_1 at date 1, and then a strict subset of firms issue $I + s_1 - S$ at date 2. In the second case, some firms repurchase s_1 at date 1, with a strict subset then issuing $I + s_1 - S$ at date 2; while other firms do nothing at date 1, with a strict subset then issuing $I - S$ at date 2. Off-equilibrium beliefs for both cases are specified at the end of the proof.

For use throughout the proof, note that if firm a plays (s_1, s_2) with $s_2 = -(I - S + s_1)$, then by (??) its payoff is

$$\frac{I+a+b}{1-\frac{s_1}{P_1(s_1)}+\frac{I-S+s_1}{P_2(s_1,s_2)}}=\frac{1}{1-\frac{s_1}{P_1(s_1)}}\frac{I+a+b}{1+\frac{I-S+s_1}{S-s_1+E[a|s_1,s_2]+b}}. \tag{A-7}$$

Case 1: $S+a^*(s_1)>S+E[a]+b\Pr(a\leq a^*(s_1))$.

We construct an equilibrium in which: At date 1, all active firms other than some subset A_0 repurchase an amount s_1. At date 2, firms that repurchased at date 1 and have $a\leq a_1$ issue $I-S+s_1$ and invest, while firms that did not repurchase at date 1 and have $a\leq a_3$ issue $I-S$ and invest. The set A_0 and cutoffs a_1 and a_3 satisfy $A_0\subset[\underline{a},a_1]\subset[\underline{a},a_3]$.

The date 1 repurchase price is

$$P_1=S+E[a|a\notin A_0]+b\Pr(a\leq a_1|a\notin A_0)$$

and the equilibrium indifference conditions are

$$\begin{aligned}
\frac{1}{1-\frac{s_1}{P_1}}\frac{I+a_1+b}{1+\frac{I-S+s_1}{S-s_1+E[a|a\in[\underline{a},a_1]\setminus A_0]+b}} &= \frac{S-s_1+a_1}{1-\frac{s_1}{P_1}}\\
\left(1-\frac{s_1}{P_1}\right)\left(1+\frac{I-S+s_1}{S-s_1+E\left[a|a\in[\underline{a},a_1]\setminus A_0\right]+b}\right) &=\\
1+\frac{I-S}{S+\frac{\alpha\mu([\underline{a},a_3])E[a|a\leq a_3]+(1-\alpha)\mu(A_0)E[a|A_0]}{\alpha\mu([\underline{a},a_3])+(1-\alpha)\mu(A_0)}+b}\\
\frac{I+a_3+b}{1+\frac{I-S}{S+\frac{\alpha\mu([\underline{a},a_3])E[a|a\leq a_3]+(1-\alpha)\mu(A_0)E[a|A_0]}{\alpha\mu([\underline{a},a_3])+(1-\alpha)\mu(A_0)}+b}} &= S+a_3.
\end{aligned}$$

Respectively, these three conditions say that: firm a_1 is indifferent between repurchase-issue and repurchase-do-nothing; firms are indifferent between repurchase-issue and direct-issue; firm a_3 is indifferent between direct issue and do-nothing.

Notationally, define $\gamma_0\equiv\frac{\mu(A_0)}{\mu([\underline{a},a_1])}$ and $E_0\equiv E[a|A_0]$. Note that $E[a|a\notin A_0]=\frac{E[a]-\gamma_0\mu([\underline{a},a_1])E_0}{1-\gamma_0\mu([\underline{a},a_1])}$ and $\Pr(a\leq a_1|a\notin A_0)=\frac{(1-\gamma_0)\mu([\underline{a},a_1])}{1-\gamma_0\mu([\underline{a},a_1])}$ and $\mu([\underline{a},a_1]\setminus A_0)=(1-\gamma_0)\mu([\underline{a},a_1])$ and $E[a|a\in[\underline{a},a_1]\setminus A_0]=\frac{E[a|a\leq a_1]-\gamma_0E_0}{1-\gamma_0}$.

Hence

$$P_1(a_1, E_0) = S + \frac{E[a] - \gamma_0 \mu([\underline{a}, a_1]) E_0 + b(1-\gamma_0)\mu([\underline{a}, a_1])}{1 - \gamma_0 \mu([\underline{a}, a_1])} \quad \text{(A-8)}$$

and the equilibrium indifference conditions are

$$\begin{aligned}
\frac{I + a_1 + b}{1 + \frac{I - S + s_1}{S - s_1 + \frac{E[a|a \le a_1] - \gamma_0 E_0}{1-\gamma_0} + b}} &= S - s_1 + a_1 && \text{(A-9)}\\
\left(1 - \frac{s_1}{P_1(a_1, E_0)}\right)\left(1 + \frac{I - S + s_1}{S - s_1 + \frac{E[a|a \le a_1] - \gamma_0 E_0}{1-\gamma_0} + b}\right) &= \\
1 + \frac{I - S}{S + \frac{\alpha\mu([\underline{a}, a_3]) E[a|a \le a_3] + (1-\alpha)\gamma_0 \mu([\underline{a}, a_1]) E_0}{\alpha\mu([\underline{a}, a_3]) + (1-\alpha)\gamma_0 \mu([\underline{a}, a_1])} + b} & && \text{(A-10)}\\
\frac{I + a_3 + b}{1 + \frac{I - S}{S + \frac{\alpha\mu([\underline{a}, a_3]) E[a|a \le a_3] + (1-\alpha)\gamma_0 \mu([\underline{a}, a_1]) E_0}{\alpha\mu([\underline{a}, a_3]) + (1-\alpha)\gamma_0 \mu([\underline{a}, a_1])} + b}} &= S + a_3. && \text{(A-11)}
\end{aligned}$$

For any $\gamma_0 > 0$, let $E_0(a_1; \gamma_0)$ be the value of E_0 that solves (**??**) given a_1. (The LHS of (**??**) is strictly decreasing in E_0 for $\gamma_0 > 0$, so if a solution exists, it is unique.) Note that $E_0(a^*(s_1); \gamma_0) = E[a|a \le a^*(s_1)]$. Recall that $a^*(s_1)$ lies strictly between $\underline{a}$ and $\bar{a}$. Hence, for $\gamma_0 > 0$, the LHS of (**??**) strictly exceeds the RHS at $a_1 = a^*(s_1)$ and $E_0 = \frac{\underline{a} + E[a|a \le a^*(s_1)]}{2}$. Define $\bar{a}^*(\gamma_0) \ge a^*(s_1)$ by

$$\bar{a}^*(\gamma_0) = \max_{a_1} \left\{ \begin{array}{r} \frac{I + \tilde{a}_1 + b}{1 + \frac{I - S + s_1}{S - s_1 + \frac{E[a|a \le \tilde{a}_1] - \gamma_0 \frac{\underline{a} + E[a|a \le \tilde{a}_1]}{2}}{1-\gamma_0} + b}} - (S - s_1 + \tilde{a}_1) \ge 0 \\ \text{for all } \tilde{a}_1 \in [a^*(s_1), a_1] \end{array} \right\}.$$

Note that, by the definition of $a^*(s_1)$, $\bar{a}^*(0) = a^*(s_1)$. In addition, because the expression in the above definition is strictly increasing in γ_0, so $\bar{a}^*(\gamma_0) > a^*(s_1)$ for $\gamma_0 > 0$. Moreover, $\bar{a}^*(\gamma_0) \to a^*(s_1)$ as $\gamma_0 \to 0$.

For the remainder of the proof, and given that we are in Case 1, fix $\gamma_0 > 0$ sufficiently small such that for all $a_1 \in [a^*(s_1), \bar{a}^*(\gamma_0)]$

$$S + a_1 > S + \frac{E[a] - \gamma_0 \mu([\underline{a}, a_1]) \underline{a} + b(1-\gamma_0)\mu([\underline{a}, a_1])}{1 - \gamma_0 \mu([\underline{a}, a_1])}, \quad \text{(A-12)}$$

and moreover, such that $H(\cdot)$ (defined in (**??**)) is strictly negative on $(a^*(s_1), \bar{a}^*(\gamma_0)]$. Having fixed $\gamma_0 > 0$, we omit the γ_0 arguments in $\bar{a}^*$ and $E_0(a_1)$ for the remainder of the proof.

By the definition of $a^*(s_1)$ and $\bar{a}^*$, the LHS of (**??**) is strictly less than the RHS for $a_1 \in (a^*(s_1), \bar{a}^*]$ and $E_0 = E[a|a \le a_1]$. Hence $\frac{\underline{a}+E[a|a\le a_1]}{2} \le E_0(a_1) < E[a|a \le a_1]$ for $a_1 \in (a^*(s_1), \bar{a}^*]$, with $E_0(\bar{a}^*) = \frac{\underline{a}+E[a|a\le \bar{a}^*]}{2}$. Therefore the function $E_0(a_1)$ is well-defined and continuous over $[a^*(s_1), \bar{a}^*]$.

Define $a_3(a_1;\alpha)$ as the value of a_3 that solves (**??**), given a_1 and $E_0 = E_0(a_1)$. Observe that the LHS of (**??**) strictly exceeds the RHS at $\alpha = 0$, $E_0 \ge \underline{a}$ and $a_3 = \underline{a}$. Moreover, by Assumption **??**, the LHS is strictly less than the RHS at $\alpha = 0$, $E_0 \le E[a]$ and $a_3 = \bar{a}$. Hence for $a_1 \in [a^*(s_1), \bar{a}^*]$, $E_0 = E_0(a_1)$, and $\alpha = 0$, there is a unique value of a_3 solving (**??**). When α is small, applying the fact that the density f is bounded and continuous, simple calculation shows that the difference between the LHS and the RHS of (**??**) is strictly decreasing in a_3. So, $a_3(a_1;\alpha)$ is uniquely defined when $a_1 \in [a^*(s_1), \bar{a}^*]$, $E_0 = E_0(a_1)$, and α sufficiently small. Note that $a_3(a_1;\alpha)$ is continuous in both a_1 and α.

For use below, we next establish that $a_3(a^*(s_1);0) > a^*(s_1)$. By definition,

$$\frac{I + a^*(s_1) + b}{1 + \frac{I-S+s_1}{S-s_1+E[a|a\le a^*(s_1)]+b}} = S - s_1 + a^*(s_1).$$

Since $a^*(s_1) > E[a|a \le a^*(s_1)]$, it is straightforward to show that

$$\frac{I + a^*(s_1) + b}{1 + \frac{I-S}{S+E[a|a\le a^*(s_1)]+b}} > S + a^*(s_1).$$

By the definition of $a_3(a_1;\alpha)$ from (**??**), this last inequality is at equality if the $a^*(s_1)$'s in the numerator and in the LHS are replaced by $a_3(a^*(s_1);0)$. Consequently, $a_3(a^*(s_1);0) > a^*(s_1)$.

We now turn to (**??**). Since $E_0(\bar{a}^*) = \frac{\underline{a}+E[a|a\le\bar{a}^*]}{2} < E[a|a \le \bar{a}^*]$, and $E[a|a \le \bar{a}_1] < E[a|a \le a^*(s_1)] < E[a|a \le \bar{a}^*]$,

$$\frac{I-S+s_1}{S-s_1+\frac{E[a|a\leq\bar{a}^*]-\gamma_0 E_0(\bar{a}^*)}{1-\gamma_0}+b}<\frac{I-S+s_1}{S-s_1+E\left[a|a\leq\bar{a}^*\right]+b}<\frac{I-S+s_1}{S-s_1+E\left[a|a\leq\bar{a}_1\right]+b}.$$

Since certainly $1-\frac{s_1}{P_1(a_1,E_0)}<1$, it follows from (**??**) that the LHS of (**??**) is strictly less than the RHS at $(a_1,E_0)=(\bar{a}^*,E_0(\bar{a}^*))$ and $\alpha=0$.

Next, we show that the LHS of (**??**) strictly exceeds the RHS at $(a_1,E_0)=(a^*(s_1),E_0(a^*(s_1)))$ and $\alpha=0$. The proof is by contradiction. Suppose to the contrary that, at $(a_1,E_0)=(a^*(s_1),E_0(a^*(s_1)))$,

$$\left(1-\frac{s_1}{P_1(a_1,E_0)}\right)\left(1+\frac{I-S+s_1}{S-s_1+\frac{E[a|a\leq a_1]-\gamma_0 E_0}{1-\gamma_0}+b}\right)\leq 1+\frac{I-S}{S+E_0+b}. \tag{A-13}$$

First, we show that at $(a_1,E_0)=(a^*(s_1),E_0(a^*(s_1)))$,

$$P_1(a_1,E_0)<S+a_1. \tag{A-14}$$

To establish (**??**), note that because $(a_1,E_0)=(a^*(s_1),E_0(a^*(s_1)))$ satisfies (**??**), the combination of (**??**) and (**??**) implies

$$\frac{S-s_1+a_1}{1-\frac{s_1}{P_1(a_1,E_0)}}\geq\frac{I+a_1+b}{1+\frac{I-S}{S+E_0+b}}.$$

Substituting in for $a_3(a_1)$, and using the earlier observation that $a_3(a^*(s_1);0)>a^*(s_1)=a_1$, we have

$$\frac{S-s_1+a_1}{1-\frac{s_1}{P_1(a_1,E_0)}}\geq\frac{(I+a_1+b)\left(S+a_3(a^*(s_1);0)\right)}{I+a_3(a^*(s_1);0)+b}>S+a_1,$$

which is equivalent to (**??**).

Second, straightforward algebra implies

$$
\begin{aligned}
P_1\left(a_1, E_0\right) &= \frac{(1-\gamma_0)\mu([\underline{a},a_1])}{1-\gamma_0\mu([\underline{a},a_1])} \frac{I+\frac{E[a|a\leq a_1]-\gamma_0 E_0}{1-\gamma_0}+b}{\left(1-\frac{s_1}{P_1}\right)\left(1+\frac{I-S+s_1}{S-s_1+\frac{E[a|a\leq a_1]-\gamma_0 E_0}{1-\gamma_0}+b}\right)} \\
&+ \frac{1-\mu([\underline{a},a_1])}{1-\gamma_0\mu([\underline{a},a_1])} \frac{S-s_1+E[a|a\geq a_1]}{1-\frac{s_1}{P_1}}.
\end{aligned}
$$

Then (**??**) and (**??**) imply that at $(a_1, E_0) = \left(a^*\left(s_1\right), E_0\left(a^*\left(s_1\right)\right)\right)$,

$$
\begin{aligned}
P_1\left(a_1, E_0\right) &> \frac{\left(1-\gamma_0\right)\mu\left(\left[\underline{a}, a_1\right]\right)}{1-\gamma_0\mu\left(\left[\underline{a}, a_1\right]\right)} \frac{I+\frac{E[a|a\leq a_1]-\gamma_0 E_0}{1-\gamma_0}+b}{1+\frac{I-S}{S+E_0+b}} + \frac{1-\mu\left(\left[\underline{a}, a_1\right]\right)}{1-\gamma_0\mu\left(\left[\underline{a}, a_1\right]\right)}\left(S+E\left[a|a\geq a_1\right]\right) \\
&= \frac{\left(1-\gamma_0\right)\mu\left(\left[\underline{a}, a_1\right]\right)}{1-\gamma_0\mu\left(\left[\underline{a}, a_1\right]\right)}\left(S+E\left[a|a\leq a_1\right]+b\right) + \frac{1-\mu\left(\left[\underline{a}, a_1\right]\right)}{1-\gamma_0\mu\left(\left[\underline{a}, a_1\right]\right)}\left(S+E\left[a|a\geq a_1\right]\right) \\
&= S+\frac{E\left[a\right]-\gamma_0\mu\left(\left[\underline{a}, a_1\right]\right)E\left[a|a\leq a_1\right]+\left(1-\gamma_0\right)\mu\left(\left[\underline{a}, a_1\right]\right)b}{1-\gamma_0\mu\left(\left[\underline{a}, a_1\right]\right)} \\
&= P_1\left(a_1, E_0\right),
\end{aligned}
$$

where the equality is simply (**??**). The contradiction completes the proof that the LHS of (**??**) strictly exceeds the RHS at $(a_1, E_0) = \left(a^*\left(s_1\right), E_0\left(a^*\left(s_1\right)\right)\right)$ and $\alpha = 0$.

By continuity, it follows that, for $\alpha = 0$, there exists $a_1^{**} \in \left(a^*\left(s_1\right), \bar{a}^*\right)$ such that $(a_1, E_0, a_3) = \left(a_1^{**}, E_0\left(a_1^{**}\right), a_3\left(a_1^{**};\alpha\right)\right)$ satisfies the required conditions (**??**), (**??**) and (**??**). By continuity, the same statement holds true for all $\alpha > 0$ sufficiently small.

Finally, to complete the proof of Case 1, we must show that all firms prefer the equilibrium action described to doing nothing. It suffices to show this for firm a_1. We must show that firm a_1 indeed profits from repurchasing its own stock, i.e., $S + a_1 > P_1\left(a_1, E_0\right)$. This follows from (**??**), together with the fact that P_1 satisfies (**??**), $E_0 > \underline{a}$, and $a_1 \in \left[a^*\left(s_1\right), \bar{a}^*\right]$.

Case 2: $S + a^*\left(s_1\right) < S + E(a) + b\Pr\left(a \leq a^*\left(s_1\right)\right)$.

In this case, we show there exists a_1, a_2 along with a partition A_0, A_1 of $[\underline{a}, a_1]$, such that the following is an equilibrium: At date 1 firms $A_1\cup[a_2, \bar{a}]$ repurchase s_1, while other firms do nothing; and at date 2 firms A_1 issue $I - S + s_1$ and invest, firms A_0 directly issue $I - S$ (without previously repurchasing), along with inactive firms $[\underline{a}, a_1]$, and the remaining firms do nothing.

In such an equilibrium, the date 1 repurchase price P_1 and date 2 issue price P_2 following repurchase are

$$\begin{aligned} P_1 &= S + \frac{E\left[a|A_1\right]\mu\left(A_1\right) + E\left[a|a \geq a_2\right]\mu\left(\left[a_2, \bar{a}\right]\right) + b\mu\left(A_1\right)}{\mu\left(A_1\right) + \mu\left(\left[a_2, \bar{a}\right]\right)} \\ P_2 &= \frac{S - s_1 + E\left[a|A_1\right] + b}{1 - \frac{s_1}{P_1}}. \end{aligned}$$

We show that there exist $a_1, a_2 \in [\underline{a}, \bar{a}]$, together with a partition A_0, A_1 of $[\underline{a}, a_1]$, that solve the following system of equations (where P_1 is as defined immediately above):

$$S + a_1 = \frac{1}{1 - \frac{s_1}{P_1}} \frac{1}{1 + \frac{I - S + s_1}{S - s_1 + E[a|A_1] + b}} (I + a_1 + b) \tag{A-15}$$

$$S + a_2 = P_1 \tag{A-16}$$

$$\frac{1}{1 - \frac{s_1}{P_1}} \frac{1}{1 + \frac{I - S + s_1}{S - s_1 + E[a|A_1] + b}} = \frac{1}{1 + \frac{I - S}{S + \frac{\alpha\mu([\underline{a}, a_1])E[a|a \leq a_1] + (1-\alpha)\mu(A_0)E[a|A_0]}{\alpha\mu([\underline{a}, a_1]) + (1-\alpha)\mu(A_0)} + b}} \tag{A-17}$$

Condition (??) states that firm a_1 is indifferent between repurchase-issue and do-nothing. Condition (??) states that firm a_2 is indifferent between repurchase-do-nothing and do-nothing. Condition (??) states that firms are indifferent between repurchasing and then issuing, and issuing directly.

Notationally, define $\gamma_0 \equiv \frac{\mu(A_0)}{\mu([\underline{a}, a_1])}$ and $E_0 \equiv E\left[a|A_0\right]$, and note that $E\left[a|A_1\right] = \frac{E[a|a \leq a_1] - \gamma_0 E_0}{1 - \gamma_0}$. The system of equations (??)-(??) has a solution if and only if the following system has a solution in γ_0, E_0, a_1 and a_2:

$$\frac{1}{1-\frac{s_1}{S+a_2}} \frac{I+a_1+b}{1+\frac{I-S+s_1}{S-s_1+\frac{E[a|a\le a_1]-\gamma_0 E_0}{1-\gamma_0}+b}} - (S+a_1) = 0 \quad \text{(A-18)}$$

$$\frac{I+a_1+b}{1+\frac{I-S}{S+\frac{\alpha E[a|a\le a_1]+(1-\alpha)\gamma_0 E_0}{\alpha+(1-\alpha)\gamma_0}+b}} - (S+a_1) = 0 \quad \text{(A-19)}$$

$$\frac{(E[a|a\le a_1]-\gamma_0 E_0)\,\mu([\underline{a},a_1]) + E[a|a\ge a_2]\,\mu([a_2,\bar{a}])}{(1-\gamma_0)\,\mu([\underline{a},a_1])+\mu([a_2,\bar{a}])} + \frac{b(1-\gamma_0)\,\mu([\underline{a},a_1])}{(1-\gamma_0)\,\mu([\underline{a},a_1])+\mu([a_2,\bar{a}])} - a_2 = 0 \quad \text{(A-20)}$$

along with the additional restriction that E_0 is consistent with γ_0 and a_1. (At $\gamma_0 = 0$ this consistency condition is simply that E_0 lies in the interval $[\underline{a}, a_1]$. As γ_0 increases, the lower bound of this interval increases and the upper bound decreases, with both continuous in γ_0.) Note that equations (**??**) and (**??**) are simple rewritings of (**??**) and (**??**), while (**??**) is obtained from combining (**??**) and (**??**).

*Claim (i): There exists $\hat{a} \in [\bar{a}_1, \bar{a}]$ such that for $\gamma_0 = 0$ and $a_1 \in [\hat{a}, a^*(s_1)]$, equation (**??**) has a unique solution in a_2, which we denote $a_2(a_1)$. Moreover, $a_2(a_1)$ is continuous in a_1, with $a_2(\hat{a}) = \bar{a}$ and $a_2(a^*(s_1)) = a^*(s_1)$, and $a_2(a_1) \in [a_1, \bar{a})$ for $a_1 \in (\hat{a}, a^*(s_1))$.*

Proof of Claim (i): The LHS of (**??**) is strictly decreasing in a_2, so if a solution exists it is continuous. By the definition of $a^*(s_1)$, function $H(a_1)$ in (**??**) is weakly positive for all $a_1 \in [\underline{a}, a^*(s_1)]$. Consequently, the LHS of (**??**) evaluated at $a_2 = a_1$ is weakly greater than $\frac{S-s_1+a_1}{1-\frac{s_1}{S+a_1}} - (S+a_1) = 0$. So at $a_1 = a^*(s_1)$ we have $a_2(a_1) = a_1$, while for $a_1 < a^*(s_1)$ any solution to (**??**) must weakly exceed a_1.

Evaluated at $a_1 = \bar{a}_1$ and $a_2 = \bar{a}$, the LHS of (**??**) is strictly positive, by (**??**). Evaluated at $a_1 = a^*(s_1)$ and $a_2 = \bar{a}$, the LHS of (**??**) is

$$\frac{S-s_1+a^*(s_1)}{1-\frac{s_1}{S+\bar{a}}} - (S+a^*(s_1)) = (S+\bar{a})\frac{S-s_1+a^*(s_1)}{S-s_1+\bar{a}} - (S+a^*(s_1)) < 0.$$

So by continuity, there exists $\hat{a} \in (\bar{a}_1, a^*(s_1))$ such that, for all $a_1 \in (\hat{a}, a^*(s_1))$, the LHS of

(**??**) evaluated at $a_2 = \bar{a}$ is strictly negative, while at $a_1 = \hat{a}$ it is exactly zero.

Consequently, for $a_1 \in [\hat{a}, a^*(s_1)]$ and $\gamma_0 = 0$, equation (**??**) has a unique solution in a_2. The solution lies in the interval $[a_1, \bar{a}]$; equals a_1 when $a_1 = a^*(s_1)$; equals $\bar{a}$ when $a_1 = \hat{a}$; and lies in $[a_1, \bar{a})$ otherwise. This completes the proof of the Claim (i).

Claim (ii): There exist constants $\bar{\gamma}_0, \kappa > 0$ such that: If $\gamma_0 \leq \bar{\gamma}_0$, $\alpha \leq \frac{\gamma_0}{\kappa+\gamma_0}$, $a_1 \in [\hat{a}, a^*(s_1)]$, then there exists a unique $E_0(a_1; \gamma_0, \alpha)$ that solves (**??**), and moreover, $E_0(a_1; \gamma_0, \alpha)$ is consistent with a_1 and γ_0.

Proof of Claim (ii): Fix $a_1 \in [\hat{a}, a^*(s_1)]$. As a preliminary, note that, from Claim (i), (**??**) has a unique solution in a_2 when $\gamma_0 = 0$ and $a_1 \in [\hat{a}, a^*(s_1)]$. A necessary condition for (**??**) to have a solution is that the LHS of (**??**) is weakly negative at $a_2 = \bar{a}$. From (**??**), and the fact that $a_1 \geq \hat{a} \geq \bar{a}_1$, we know $\frac{1}{1+\frac{I-S}{S+\underline{a}+b}} < \frac{1}{1-\frac{s_1}{S+\bar{a}}} \frac{1}{1+\frac{I-S+s_1}{S-s_1+E[a|a\leq a_1]+b}}$. Consequently,

$$\frac{I+a_1+b}{1+\frac{I-S}{S+\underline{a}+b}} - (S+a_1) < 0.$$

From this inequality, the LHS of (**??**) is strictly negative when $\alpha = 0$, $\gamma_0 > 0$ and $E_0 = \underline{a}$. Conversely, the LHS of (**??**) is strictly positive when $\alpha = 0$, $\gamma_0 > 0$ and $E_0 = a_1$. The LHS of (**??**) is strictly increasing in E_0. Consequently, for $\alpha = 0$ and any γ_0, there is a unique solution E_0 to (**??**).

Moreover, there exists $\bar{\gamma}_0$ (independent of a_1) such that, for $\gamma_0 \leq \bar{\gamma}_0$, the solution E_0 is consistent with a_1 and γ_0.

By continuity, there exists some lower bound κ such that the same statement is true provided $\frac{1-\alpha}{\alpha}\gamma_0 \geq \kappa$, i.e., $\alpha \leq \frac{\gamma_0}{\kappa+\gamma_0}$, completing the proof of Claim (ii).

Since (**??**) is strictly decreasing in a_2, it follows from Claims (i) and (ii) that there exist functions $a_2(a_1; \gamma_0, \alpha), \hat{a}(\gamma_0), a^*(s_1; \gamma_0)$, continuous in γ_0 and α, such that for all $a_1 \in [\hat{a}(\gamma_0), a^*(s_1; \gamma_0)]$, the unique solution of (**??**) and (**??**) is $(a_2(a_1; \gamma_0, \alpha), E_0(a_1; \gamma_0, \alpha))$; and moreover, $\lim_{\gamma_0 \to 0}(a_2(a_1; \gamma_0, 0), \hat{a}(\gamma_0), a^*(s_1; \gamma_0)) = (a_2(a_1), \hat{a}, a^*(s_1))$. It is straightforward to see that for any $\gamma_0 \in [0, \bar{\gamma}_0]$, $a_2(a_1; \gamma_0, 0)$ is continuous in a_1.

At $\gamma_0 = 0$, the LHS of (**??**) evaluated at $(a_1, a_2, E_0) = (\hat{a}(\gamma_0), a_2(\hat{a}(\gamma_0); \gamma_0, 0), E_0(\hat{a}(\gamma_0)))$ equals $E[a|a \leq a_1] + b - \bar{a}$, which is strictly negative by (**??**); while evaluated at $(a_1, a_2, E_0) = (a^*(s_1; \gamma_0), a_2(a^*(s_1; \gamma_0); \gamma_0, 0), E_0(a^*(s_1; \gamma_0)))$ it equals $E[a] + b \Pr(a \leq a^*(s_1)) - a^*(s_1)$, which is strictly positive since we are in Case 2. By continuity, the same two statements also hold for γ_0 small but strictly positive. Fix any such γ_0. By continuity, there then exists $(a_1, a_2(a_1; \gamma_0, 0), E_0(a_1))$ that satisfies equations (**??**)-(**??**).

By a further application of continuity, for all α sufficiently small, there exists $(a_1, a_2(a_1; \gamma_0, \alpha), E_0(a_1; \alpha))$ that satisfies equations (**??**)-(**??**), completing the proof of Case 2.

Off-equilibrium beliefs

Off-equilibrium beliefs are as follows.[30] Date 2 repurchases $\tilde{s}_2 > 0$ are associated with the best firm $\bar{a}$ and issues $\tilde{s}_2 < 0$ are associated with the worst firm $\underline{a}$. At date 1, repurchases $\tilde{s}_1 > 0$ are associated with the best firm $\bar{a}$ with probability $1 - \varepsilon$ and the worst firm with probability ε; while issues $\tilde{s}_1 < 0$ are associated with the best firm $\bar{a}$ with probability ε and the worst firm $\underline{a}$ with probability $1 - \varepsilon$.

Write $\tilde{P}_1$ and $\tilde{P}_2$ for the associated off-equilibrium prices. Given the stated off-equilibrium beliefs, there exists some $\kappa > 0$ such that

$$\tilde{P}_1 \begin{array}{ccc} \geq & S + \bar{a} - \varepsilon\kappa & \text{if} \quad \tilde{s}_1 > 0 \\ \leq & S + \underline{a} + b + \varepsilon\kappa & \text{if} \quad \tilde{s}_1 < 0 \end{array}. \tag{A-21}$$

Moreover,

$$\tilde{P}_2 = \begin{cases} \frac{S - \tilde{s}_1 + \bar{a} + b\mathbf{1}_{S - \tilde{s}_1 - \tilde{s}_2 \geq I}}{1 - \frac{\tilde{s}_1}{\tilde{P}_1}} & \text{if} \quad \tilde{s}_2 > 0 \\ \frac{S - \tilde{s}_1 + \underline{a} + b\mathbf{1}_{S - \tilde{s}_1 - \tilde{s}_2 \geq I}}{1 - \frac{\tilde{s}_1}{\tilde{P}_1}} & \text{if} \quad \tilde{s}_2 < 0 \end{cases}. \tag{A-22}$$

It is immediate that inactive firms cannot gain by deviating to an off-equilibrium action: a date 2 repurchase delivers a payoff for firm a of at most $S + a$, which is weakly less than its

[30] As we show in the proof of Proposition **??**, the beliefs specified here satisfy the NDOC refinement. If the NDOC refinement is not imposed, the following simpler set of off-equilibrium beliefs deters all deviations: at either date, off-equilibrium repurchase offers trigger investor beliefs that the firm is type $\bar{a}$, while off-equilibrium issue offers trigger beliefs that the firm is type $\underline{a}$.

equilibrium payoff, while an off-equilibrium date 2 issue delivers a payoff strictly less than that associated with the equilibrium date 2 issue size. The remainder of the proof shows that active firms likewise have no incentive to deviate.

By the equilibrium construction, the payoff of any firm $a \in [\underline{a}, \bar{a}]$ strictly exceeds the payoff from direct issue under investor beliefs $\underline{a}$, namely $\frac{I+a+b}{1+\frac{I-S}{S+\underline{a}+b}}$. Moreover, for firms a sufficiently close to $\bar{a}$, the equilibrium payoff also strictly exceeds the payoff from doing nothing, namely $S+a$. (Of course, this relation holds weakly for *all* firms.) Hence it is possible to choose $\varepsilon > 0$ such that, for all firms $a \in [\underline{a}, \bar{a}]$,

$$\max\left\{\frac{I+a+b}{1+\frac{I-S-\varepsilon\kappa}{S+\underline{a}+b+\varepsilon\kappa}}, a\frac{S+\bar{a}-\varepsilon\kappa}{\bar{a}-\varepsilon\kappa}\right\} < \text{equilibrium payoff of firm } a. \tag{A-23}$$

Moreover, and using $b > 0$ and inequality (**??**), choose $\varepsilon > 0$ sufficiently small such that, in addition to inequality (**??**), the following pair of inequalities holds:

$$\frac{a}{\underline{a}+b} \leq \frac{I+a+b}{I+\underline{a}+b} \quad \text{if} \quad a \in [\underline{a}+b, \underline{a}+b+\varepsilon\kappa], \tag{A-24}$$

$$\underline{a}+b+\varepsilon\kappa \leq \bar{a}-\varepsilon\kappa. \tag{A-25}$$

Firm a's payoff from an arbitrary off-equilibrium strategy $(\tilde{s}_1, \tilde{s}_2)$ is

$$\frac{S-\tilde{s}_1-\tilde{s}_2+a+b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2\geq I}}{1-\frac{\tilde{s}_1}{\tilde{P}_1}-\frac{\tilde{s}_2}{\tilde{P}_2}}.$$

First, observe that

$$-\frac{\tilde{s}_2}{\tilde{P}_2} \geq -\frac{\tilde{s}_2}{S-\tilde{s}_1+\underline{a}+b}\left(1-\frac{\tilde{s}_1}{\tilde{P}_1}\right).$$

This follows directly from (**??**) if $\tilde{s}_2 < 0$, and from (**??**) together with (**??**) if $\tilde{s}_2 > 0$. Second, observe that

$$-\frac{\tilde{s}_1}{\tilde{P}_1} \geq -\frac{\tilde{s}_1}{S+\underline{a}+b+\varepsilon\kappa}.$$

This follows directly from (**??**) if $\tilde{s}_1 < 0$, and from (**??**) together with (**??**) if $\tilde{s}_1 > 0$.

Consequently, firm a's payoff is bounded above by

$$\begin{aligned} &\frac{S-\tilde{s}_1-\tilde{s}_2+a+b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2\geq I}}{\left(1-\frac{\tilde{s}_1}{S+\underline{a}+b+\varepsilon\kappa}\right)\left(1-\frac{\tilde{s}_2}{S-\tilde{s}_1+\underline{a}+b}\right)} \\ &= \frac{S-\tilde{s}_1-\tilde{s}_2+a+b\mathbf{1}_{S-\tilde{s}_1-\tilde{s}_2\geq I}}{S-\tilde{s}_1-\tilde{s}_2+\underline{a}+b} \frac{S-\tilde{s}_1+\underline{a}+b}{S-\tilde{s}_1+\underline{a}+b+\varepsilon\kappa}\left(S+\underline{a}+b+\varepsilon\kappa\right). \end{aligned} \tag{A-26}$$

To complete the proof, by (**??**) it is sufficient to show that expression (**??**) is bounded above by either the LHS of (**??**), or by $S+a$. There are four cases:

If $S-\tilde{s}_1-\tilde{s}_2 \geq I$ it is immediate that (**??**) is bounded above by $\frac{I+a+b}{I+\underline{a}+b}\left(S+\underline{a}+b+\varepsilon\kappa\right)$, which is the first term in the LHS of (**??**).

If $S-\tilde{s}_1-\tilde{s}_2 < I$ and $a \leq \underline{a}+b$ then (**??**) is bounded above by $\left(S+\underline{a}+b+\varepsilon\kappa\right)$.

If $S-\tilde{s}_1-\tilde{s}_2 < I$ and $a \in [\underline{a}+b, \underline{a}+b+\varepsilon\kappa]$ then (**??**) is bounded above by $\frac{a}{\underline{a}+b}\left(S+\underline{a}+b+\varepsilon\kappa\right)$, and the result then follows from (**??**).

Finally, consider the case $S-\tilde{s}_1-\tilde{s}_2 < I$ and $a > \underline{a}+b+\varepsilon\kappa$. Note first that since $S-\tilde{s}_1-\tilde{s}_2 < I$, the off-equilibrium beliefs imply that the firm weakly loses money on its date 2 transactions, so that its payoff is bounded above by

$$\frac{S-\tilde{s}_1+a}{1-\frac{\tilde{s}_1}{\tilde{P}_1}} = \tilde{P}_1\frac{S-\tilde{s}_1+a}{\tilde{P}_1-\tilde{s}_1}.$$

If $\tilde{s}_1 > 0$, this expression is bounded above by $\max\left\{S+a, \frac{a\tilde{P}_1}{\tilde{P}_1-S}\right\}$, which by (**??**) is bounded above by $\max\left\{S+a, a\frac{S+\bar{a}-\varepsilon\kappa}{\bar{a}-\varepsilon\kappa}\right\}$. If instead $\tilde{s}_1 < 0$ this expression is bounded above by $\max\left\{S+a, \tilde{P}_1\right\}$, which by (**??**) is bounded above by $\max\left\{S+a, S+\underline{a}+b+\varepsilon\kappa\right\} = S+a$. This completes the proof .

Lemma A-1 *There is no equilibrium in which almost all active firms invest.*

Proof of Lemma ??: Suppose to the contrary that there is an equilibrium in which almost all active firms invest. By Assumption **??**, there is an active firm a' that invests, has $a' > E[a]$

and

$$S + a' > \frac{1}{1-\alpha} \frac{I + a' + b}{1 + \frac{I-S}{S+E[a]+b}}.$$

Let (s_1, s_2) be the strategy of firm a', and let (P_1, P_2) be the associated prices. So the equilibrium condition for firm a' implies

$$\frac{S - s_1 - s_2 + a' + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}} \geq S + a' > \frac{1}{1-\alpha} \frac{I + a' + b}{1 + \frac{I-S}{S+E[a]+b}} \geq \frac{1}{1-\alpha} \frac{S - s_1 - s_2 + a' + b}{1 - \frac{s_1+s_2}{S+E[a]+b}},$$

where the final inequality uses $S - s_1 - s_2 \geq I$ (since firm a' invests) and $a' > E[a]$. Since any active firm has the option of following strategy (s_1, s_2), it follows that the equilibrium payoff of an arbitrary active firm a is at least

$$\frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1}{P_1} - \frac{s_2}{P_2}} > \frac{1}{1-\alpha} \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1+s_2}{S+E[a]+b}}.$$

But this contradicts the investor rationality condition (**??**) since it implies

$$E[P_3] > (1-\alpha) E\left[\frac{1}{1-\alpha} \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1+s_2}{S+E[a]+b}}\right] = S + E[a] + b.$$

Corollary A-4 *In any equilibrium, there is a non-empty interval* $[\bar{a} - \delta, \bar{a}]$ *of active firms that do not invest.*

Proof of Corollary ??: Immediate from Corollary **??** and Lemma **??**.

Lemma A-2 *For any date 1 issue or small enough repurchase, the associated price is* $P_1(s_1) < S + \bar{a}$.

Proof of Lemma ??: Suppose otherwise, i.e., for any $\delta > 0$ one can find $s_1 \leq \delta$ such that $P_1(s_1) \geq S + \bar{a}$. From (**??**), the beliefs associated with s_1 must be such that

$$E\left[a + b\mathbf{1}_{S - s_1 - s_2 \geq I} | s_1\right] \geq \bar{a}. \tag{A-27}$$

There are two separate cases, which we deal with in turn. In the first case, investor beliefs after s_1 place probability 1 on the firm being $\bar{a}$. Since any active firm can play $(s_1, s_2 = S - s_1 - I)$, and the NDOC refinement implies $E[a|s_1, s_2 = S - s_1 - I] = \bar{a}$, then (using (**??**)), the date 0 share price P_0 is at least

$$(1-\alpha)\frac{I + E[a] + b}{\left(1 - \frac{s_1}{S+\bar{a}+b}\right)\left(1 - \frac{S-s_1-I}{S-s_1+\bar{a}+b}\right)} = (1-\alpha)\frac{I + E[a] + b}{I + \bar{a} + b}(S + \bar{a} + b) > S + E[a] + b$$

where the inequality follows from Assumption **??**. This contradicts (**??**), and completes the proof for this case.

The remainder of the proof deals with the second case, in which $E[a|s_1] < \bar{a}$. In this case, inequality (**??**) implies that $\Pr(s_2 \text{ s.t. } S - s_1 - s_2 \geq I|s_1) > 0$, and hence that there exists s_2 with $S - s_1 - s_2 \geq I$ such that $E[a + b|s_1, s_2] \geq \bar{a}$. So by (**??**), firm a's payoff from playing (s_1, s_2) is weakly greater than

$$\frac{S - s_1 - s_2 + a + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(1 - \frac{s_2}{S-s_1+\bar{a}}\right)}.$$

Since any active firm can play (s_1, s_2), the date 0 share price P_0 is at least

$$(1-\alpha)\frac{S - s_1 - s_2 + E[a] + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(1 - \frac{s_2}{S-s_1+\bar{a}}\right)} \geq (1-\alpha)\frac{I + E[a] + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(\frac{I+\bar{a}}{S-s_1+\bar{a}}\right)}, \tag{A-28}$$

where the inequality follows from (**??**) and $S - s_1 - s_2 \geq I$. If $s_1 < 0$, since $P_1(s_1) \geq S + \bar{a}$ by supposition, the RHS of (**??**) is weakly greater than

$$(1-\alpha)(I + E[a] + b)\frac{S + \bar{a}}{I + \bar{a}}. \tag{A-29}$$

Moreover, by Assumption **??**, expression (**??**) is itself strictly greater than $S + E[a] + b$, contradicting (**??**). If instead $s_1 > 0$, then note that the RHS of (**??**) converges to (**??**) as s_1 approaches 0, and (**??**) is strictly greater than $S + E[a] + b$. So provided that δ is chosen sufficiently small, the RHS of (??) strictly exceeds $S + E[a] + b$, again contradicting (??) and completing the proof of the lemma.

Proof of Proposition ??: Part (II) largely follows from Proposition ??. The only additional step is to check that the date 2 off-equilibrium beliefs specified in the proof satisfy NDOC. This is indeed the case, as follows. In the equilibrium construction, at date 1 firms either repurchase a (common) amount s_1, or do nothing. In both Cases 1 and 2 of the equilibrium construction, the set of repurchasing firms can be chosen to include $\underline{a}$ and $\bar{a}$. Hence the date 2 beliefs satisfy NDOC after the date 1 equilibrium repurchase. Moreover, because $\alpha > 0$, the date 2 beliefs also satisfy NDOC after the date 1 action of doing nothing.

The remainder of the proof deals with Part (I). First, Lemma ?? states that the repurchase price $P_1(s_1) < S + \bar{a}$ for some $s_1 > 0$. So there is a non-empty interval $[\bar{a} - \delta, \bar{a}]$ of active firms that make strictly positive profits, i.e., obtain a payoff strictly in excess of $S + a$. Together with Corollary ??, there exists $\delta' > 0$ such that all active firms in $[\bar{a} - \delta', \bar{a}]$ make strictly positive profits and do not invest. Let $\varepsilon > 0$ be the minimum profits made by a firm in this interval. (Note that the minimum is well-defined because a firm's equilibrium payoff is continuous in a: if this is not the case, there is a profitable deviation for some a.) Then choose $\delta \in (0, \delta')$ sufficiently small such that, for all $a \in [\bar{a} - \delta, \bar{a}]$, $a + \varepsilon > \bar{a}$,$a + b > \bar{a}$, and $(S + a)\frac{\bar{a}}{a} < S + a + \varepsilon$. To complete the proof, we show that almost all active firms in $[\bar{a} - \delta, \bar{a}]$ repurchase, and make strictly positive profits from the repurchase transaction.

Suppose to the contrary that there exists $\tilde{A} \subset [\bar{a} - \delta, \bar{a}]$ such that $\mu\left(\tilde{A}\right) > 0$ and for every $\tilde{a} \in \tilde{A}$, either $s_1(\tilde{a}) \leq 0$, or $s_1(\tilde{a}) > 0$ with $P_1(s_1(\tilde{a})) \geq S + \tilde{a}$. Let $\bar{A} = \left\{\tilde{a} : (s_1(\tilde{a}), s_2(\tilde{a})) = (s_1(a'), s_2(a')) \text{ for some } a' \in \tilde{A}\right\}$. Since $\tilde{A} \subset [\bar{a} - \delta, \bar{a}]$, no firm in $\bar{A}$ invests. By an analogous argument to the proof of Proposition ??, almost all firms $\tilde{a} \in \bar{A}$ obtain a payoff of $\frac{S - s_1(\tilde{a}) + \tilde{a}}{1 - \frac{s_1(\tilde{a})}{P_1(s_1(\tilde{a}))}}$. Choose any firm $a \in \tilde{A}$ with payoff $\frac{S - s_1(a) + a}{1 - \frac{s_1(a)}{P_1(s_1(a))}}$. So in particular, firm a's payoff is bounded above by

$$\frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{P_1(s_1(a))}}. \tag{A-30}$$

To complete the proof, we consider three possibilities in turn. First, if $s_1(a) > 0$ with $P_1(s_1(a)) \geq S + a$, then expression (**??**) is bounded above by

$$\frac{S - s_1(a) + \bar{a}}{1 - \frac{s_1(a)}{S+a}} = (S + a)\frac{S - s_1(a) + \bar{a}}{S - s_1(a) + a} \leq (S + a)\frac{\bar{a}}{a} < S + a + \varepsilon,$$

a contradiction. Second, if $s_1(a) \leq 0$ and $P_1(s_1(a)) \leq S + a + \varepsilon$, then (**??**) and $a + \varepsilon > \bar{a}$ imply that firm a's payoff is bounded above by

$$(S + a + \varepsilon)\frac{S - s_1(a) + \bar{a}}{S - s_1(a) + a + \varepsilon} < S + a + \varepsilon,$$

a contradiction. Third and finally, if $s_1(a) \leq 0$ and $P_1(s_1(a)) > S + a + \varepsilon$, then $a + \varepsilon > \bar{a}$ implies that $P_1(s_1(a)) > S + \bar{a}$, contradicting Lemma **??** and completing the proof.

Proof of Proposition ??: The heart of the proof is the following claim, which uses Condition (I) to establish a lower bound on the cost of raising enough funding to invest.

Claim: If $\alpha > 0$, then there exist $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ such that, for any (s_1', s_2') with $S - s_1' - s_2' \geq I$ and $s_1' \neq 0$,

$$1 - \frac{s_1'}{P_1(s_1')} - \frac{s_2'}{P_2(s_1', s_2')} \geq 1 - \frac{s_1' + s_2'}{S + E[a|s_1 \neq 0, S - s_1 - s_2 \geq I] + b} + \varepsilon_1, \tag{A-31}$$

and for any (s_1', s_2') with $S - s_1' - s_2' \geq I$ and $s_1' = 0$,

$$1 - \frac{s_2'}{P_2(s_1', s_2')} \geq 1 - \frac{s_2'}{S + E[a|S - s_1 - s_2 \geq I] + b} + \varepsilon_2. \tag{A-32}$$

Likewise, if $\alpha = 0$, there exists $\varepsilon_3 > 0$ such that, for any (s'_1, s'_2) with $S - s'_1 - s'_2 \geq I$,

$$1 - \frac{s'_1}{P_1(s'_1)} - \frac{s'_2}{P_2(s'_1, s'_2)} \geq 1 - \frac{s'_1 + s'_2}{S + E[a|S - s_1 - s_2 \geq I] + b} + \varepsilon_3. \tag{A-33}$$

Proof of claim: We establish the existence of ε_1; the existence of ε_2 and ε_3 follows by similar arguments. The proof is by contradiction. Write π_R for the expected profit of non-investing firms who play $s_1 \neq 0$, i.e.,

$$\begin{aligned} \pi_R &= (E[P_3|s_1 \neq 0, S - s_1 - s_2 < I] - E[S + a|s_1 \neq 0, S - s_1 - s_2 < I]) \\ &\times \Pr(S - s_1 - s_2 < I|s_1 \neq 0). \end{aligned}$$

By Condition (I), we know $\pi_R > 0$. Suppose that, contrary to the claim, for all $\varepsilon_1 > 0$, there exists some (s'_1, s'_2) with $S - s'_1 - s'_2 \geq I$ and $s'_1 \neq 0$ such that (**??**) does not hold. Since any active firm has the option of following strategy (s'_1, s'_2), and also of doing nothing, by supposition

$$\begin{aligned} E[P_3|s_1 \neq 0] &= E[P_3|s_1 \neq 0, S - s_1 - s_2 \geq I]\Pr(S - s_1 - s_2 \geq I|s_1 \neq 0) \\ &+ E[P_3|s_1 \neq 0, S - s_1 - s_2 < I]\Pr(S - s_1 - s_2 < I|s_1 \neq 0). \\ &> \frac{S - s'_1 - s'_2 + E[a|s_1 \neq 0, S - s_1 - s_2 \geq I] + b}{1 - \frac{s'_1 + s'_2}{S + E[a|s_1 \neq 0, S - s_1 - s_2 \geq I] + b} + \varepsilon_1}\Pr(S - s_1 - s_2 \geq I|s_1 \neq 0) \\ &+ E[S + a|s_1 \neq 0, S - s_1 - s_2 < I]\Pr(S - s_1 - s_2 < I|s_1 \neq 0) + \pi_R \\ &= S + E[a|s_1 \neq 0] + b\Pr(S - s_1 - s_2 \geq I|s_1 \neq 0) + \pi_R \\ &- (S + E[a|s_1 \neq 0, S - s_1 - s_2 \geq I] + b)\frac{\varepsilon_1 \Pr(S - s_1 - s_2 \geq I|s_1 \neq 0)}{1 - \frac{s'_1 + s'_2}{S + E[a|s_1 \neq 0, S - s_1 - s_2 \geq I] + b} + \varepsilon_1}. \end{aligned}$$

Since

$$1 - \frac{s'_1 + s'_2}{S + E[a|s_1 \neq 0, S - s_1 - s_2 \geq I] + b} \geq \frac{I + E[a|s_1 \neq 0, S - s_1 - s_2 \geq I] + b}{S + E[a|s_1 \neq 0, S - s_1 - s_2 \geq I] + b} \geq \frac{I + \bar{a} + b}{S + \bar{a} + b},$$

it follows that for ε_1 chosen small enough,

$$E\left[P_3|s_1 \neq 0\right] > S + E\left[a|s_1 \neq 0\right] + b \Pr\left(S - s_1 - s_2 \geq I|s_1 \neq 0\right),$$

contradicting the investor rationality condition (**??**), and establishing inequality (**??**). This completes the proof of the claim.

To complete the proof, we consider three cases separately. Let a^* and a^{i*} be as defined in Corollary **??**.

Case 1: $\alpha > 0$, and almost all active firms that invest do so with $s_1 \neq 0$.

Fix any firm $a < a^*$. By Corollary **??**, firm a invests when active. The payoff to firm a^* from adopting the strategy of firm a is

$$\begin{aligned}
& \frac{S - s_1(a) - s_2(a) + a^* + b}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|s_1\neq 0, S-s_1-s_2\geq I]}} - (S + a^*) \\
= \; & \frac{S - s_1(a) - s_2(a) + a^* + b}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|s_1\neq 0, S-s_1-s_2\geq I]}} - \frac{S - s_1(a) - s_2(a) + a^* + b}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|s_1\neq 0, S-s_1-s_2\geq I]} + \varepsilon_1} \\
+ \; & \frac{a^* - a}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|s_1\neq 0, S-s_1-s_2\geq I]} + \varepsilon_1} + a - a^* \\
+ \; & \frac{S - s_1(a) - s_2(a) + a + b}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|s_1\neq 0, S-s_1-s_2\geq I]} + \varepsilon_1} - (S + a).
\end{aligned}$$

The first line simplifies to

$$\frac{S - s_1(a) - s_2(a) + a^* + b}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|s_1\neq 0, S-s_1-s_2\geq I]}} \frac{\varepsilon_1}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|s_1\neq 0, S-s_1-s_2\geq I]} + \varepsilon_1},$$

which is bounded away from 0 even as a approaches a^*. The second line converges to 0 as $a \to a^*$. The third line is non-negative: this follows from the claim, together with the fact that firms $a < a^*$ invest when active, and have an equilibrium payoff of at least $S + a$. It follows that for $a < a^*$ close enough to a^*,

$$\frac{S - s_1(a) - s_2(a) + a^* + b}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|a\leq a^*]}} = \frac{S - s_1(a) - s_2(a) + a^* + b}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|s_1\neq 0, S-s_1-s_2\geq I]}} > S + a^*, \tag{A-34}$$

where the equality uses the fact that, in this case, $E[a|a \leq a^*] = E[a|s_1 \neq 0, S - s_1 - s_2 \geq I]$.

To complete the proof, first note that the result follows easily if $a^* < a^{i*}$, since in this case, there is an equilibrium of the one-period benchmark economy in which all firms invest if and only if $a \in [\underline{a}, a^{i*}]$. If instead $a^* \geq a^{i*}$, then by (**??**), Assumption **??**, and continuity, find $a^{**} > a^*$ such that

$$\frac{S - s_1(a) - s_2(a) + a^{**} + b}{1 - \frac{s_1(a)+s_2(a)}{S+E[a|a\leq a^{**}]}} = S + a^{**}.$$

Then there is an equilibrium of the one-period benchmark economy in which firms invest if and only if $a \in [\underline{a}, a^{**}]$.

Case 2: $\alpha > 0$, and a positive measure of active firms invest after $s_1 = 0$.

By the arguments of the proof of Proposition **??**, there is a unique s_2^* such that $(0, s_2^*)$ is played in equilibrium and $S - s_2^* \geq I$.

We first show that $a^{i*} \geq a^*$. Suppose to the contrary that $a^{i*} < a^*$. At most a single active firm in (a^{i*}, a^*) plays $(0, s_2^*)$, since otherwise there is an active firm in (a^{i*}, a^*) that strictly prefers $(0, s_2^*)$ to doing nothing, in turn implying that there is an inactive firms above a^{i*} that would strictly gain by deviating and playing $(0, s_2^*)$. So a positive measure of active firms below a^{i*} must play $(0, s_2^*)$. By Lemma **??** and Condition (II), it follows that the strategy $(s_1(a), s_2(a))$ of any active firm $a \in (a^{i*}, a^*)$ satisfies $S - s_1(a) - s_2(a) = S - s_2^*$ and $1 - \frac{s_1(a)}{P_1(s_1(a))} - \frac{s_2(a)}{P_2(s_1(a), s_2(a))} = 1 - \frac{s_2^*}{P_2(0, s_2)}$. But then any inactive firm in (a^{i*}, a^*) would strictly gain by deviating and playing $(0, s_2^*)$, giving a contradiction and establishing $a^{i*} \geq a^*$.

Since $a^{i*} \geq a^*$, it follows that $E[a|a \leq a^{i*}] \geq E[a|S - s_1 - s_2 \geq I]$. The same argument as in Case 1 then establishes

$$\frac{S-s_2^*+a^{i*}+b}{1-\frac{s_2^*}{S+E[a|a\leq a^{i*}]}} \geq \frac{S-s_2^*+a^{i*}+b}{1-\frac{s_2^*}{S+E[a|S-s_1-s_2\geq I]}} > S+a^{i*}.$$

The final step in the proof is exactly the same as the $a^* \geq a^{i*}$ subcase of Case 1.

Case 3: $\alpha = 0$.

The proof is the same as Case 1, but simpler.

Lemma A-3 *Suppose $\hat{s}_1 < 0$ satisfies $S - \hat{s}_1 < I$ and is played by a positive mass of firms. Then there exists $\hat{s}_2$ such that $S - \hat{s}_1 - \hat{s}_2 \geq I$, $\Pr(\hat{s}_2|\hat{s}_1) = 1$, and $P_2(\hat{s}_1, \hat{s}_2) = P_1(\hat{s}_1)$.*

Proof of Lemma ??: By the same argument as the proof of Proposition **??**, almost any firm a that issues $\hat{s}_1$ but does not invest obtains a payoff $\frac{S-\hat{s}_1+a}{1-\frac{\hat{s}_1}{P_1(\hat{s}_1)}}$.

We first show that a strictly positive measure of firms invest after playing $\hat{s}_1 < 0$. Suppose to the contrary that this is not the case. Then $P_1(\hat{s}_1) = S + E[a|\hat{s}_1]$. Hence there exist firms with $a > E[a|\hat{s}_1]$ who play $\hat{s}_1$ but would do strictly better by doing nothing, a contradiction.

By the proof of Proposition **??**, almost all firms that invest after playing $\hat{s}_1$ do so using the same strategy, which we denote $(\hat{s}_1, \hat{s}_2)$.

Next, suppose that, contrary to the claimed result, $\Pr(\hat{s}_2|\hat{s}_1) < 1$. Almost any firm a that plays $\hat{s}_1$ followed by $s_2 \neq \hat{s}_2$ does not invest and obtains a payoff of $\frac{S-\hat{s}_1+a}{1-\frac{\hat{s}_1}{P_1(\hat{s}_1)}}$. Since such a firm could instead play $(\hat{s}_1, \hat{s}_2)$, the equilibrium conditions include

$$\frac{S-\hat{s}_1+a}{1-\frac{\hat{s}_1}{P_1(\hat{s}_1)}} \geq \frac{S-\hat{s}_1-\hat{s}_2+a+b}{\left(1-\frac{\hat{s}_1}{P_1(\hat{s}_1)}\right)\left(1-\frac{\hat{s}_2}{S-\hat{s}_1+E[a|\hat{s}_1,\hat{s}_2]+b}\right)},$$

which simplifies (using $\hat{s}_2 < 0$) to

$$\frac{S-\hat{s}_1+a}{S-\hat{s}_1+E[a|\hat{s}_1,\hat{s}_2]+b} \geq 1-\frac{b}{\hat{s}_2}.$$

Hence almost all firms that play $\hat{s}_1$ but not $\hat{s}_2$ have $a > E[a|\hat{s}_1, \hat{s}_2] + b$. By Lemma **??**, almost all such firms have a higher a than all firms playing $(\hat{s}_1, \hat{s}_2)$. Hence $S + \sup\{a : a \text{ plays } \hat{s}_1\} > P_1(\hat{s}_1)$. Moreover, there exists a positive mass of firms for which $S + a > P_1(\hat{s}_1)$, and who obtain an equilibrium payoff of $\frac{S - \hat{s}_1 + a}{1 - \frac{\hat{s}_1}{P_1(\hat{s}_1)}}$. Such firms would obtain a strictly higher payoff by deviating and doing nothing. The contradiction completes the proof.

Lemma A-4 *Suppose $\hat{s}_1$ satisfies $S - \hat{s}_1 < I$ and is played by a positive mass of firms. Then $E[P_2(\hat{s}_1, s_2) | S - \hat{s}_1 - s_2 \geq I] \leq P_1(\hat{s}_1)$, with strict inequality if a positive mass of firms play $\hat{s}_1$ and do not invest.*

Proof of Lemma ??: Note that

$$\begin{aligned} P_1(\hat{s}_1) &= \Pr(S - \hat{s}_1 - s_2 \geq I | \hat{s}_1) E[P_2(\hat{s}_1, s_2) | S - \hat{s}_1 - s_2 \geq I] \\ &+ \Pr(S - \hat{s}_1 - s_2 < I | \hat{s}_1) E[P_2(\hat{s}_1, s_2) | S - \hat{s}_1 - s_2 < I]. \end{aligned}$$

If $\Pr(S - \hat{s}_1 - s_2 \geq I | \hat{s}_1) = 1$, the result is immediate. Otherwise, the proof of Proposition **??** implies

$$E[P_2(\hat{s}_1, s_2) | S - \hat{s}_1 - s_2 < I] = \frac{S - \hat{s}_1 + E[a | \hat{s}_1, S - \hat{s}_1 - s_2 < I]}{1 - \frac{\hat{s}_1}{P_1(\hat{s}_1)}}. \tag{A-35}$$

To complete the proof, we show that if $(\hat{s}_1, \hat{s}_2)$ is an equilibrium strategy with $S - \hat{s}_1 - \hat{s}_2 \geq I$ then $P_2(\hat{s}_1, \hat{s}_2) < \frac{S - \hat{s}_1 + E[a | \hat{s}_1, S - \hat{s}_1 - s_2 < I]}{1 - \frac{\hat{s}_1}{P_1(\hat{s}_1)}}$. Suppose to the contrary that this is not the case. Since any firm that plays $\hat{s}_1$ has the option of playing $(\hat{s}_1, \hat{s}_2)$, it follows that

$$\begin{aligned} E[P_2(\hat{s}_1, s_2) | S - \hat{s}_1 - s_2 < I] &\geq \frac{S - \hat{s}_1 - \hat{s}_2 + E[a | \hat{s}_1, S - \hat{s}_1 - s_2 < I] + b}{S - \frac{\hat{s}_1}{P_1(\hat{s}_1)} - \frac{\hat{s}_2}{P_2(\hat{s}_1, \hat{s}_2)}} \\ &\geq \frac{S - \hat{s}_1 - \hat{s}_2 + E[a | \hat{s}_1, S - \hat{s}_1 - s_2 < I] + b}{\left(S - \frac{\hat{s}_1}{P_1(\hat{s}_1)}\right)\left(1 - \frac{\hat{s}_2}{S - \hat{s}_1 + E[a | \hat{s}_1, S - \hat{s}_1 - s_2 < I]}\right)} \\ &> \frac{S - \hat{s}_1 + E[a | \hat{s}_1, S - \hat{s}_1 - s_2 < I]}{S - \frac{\hat{s}_1}{P_1(\hat{s}_1)}}, \end{aligned}$$

contradicting (**??**), and completing the proof.

Proof of Proposition ??: We first establish $P_2(s_1,s_2) \geq P_2(s_1',s_2')$. Suppose to the contrary that $P_2(s_1,s_2) < P_2(s_1',s_2')$. The first step is to show

$$S - s_1 - s_2 \leq S - s_1' - s_2'. \tag{A-36}$$

The proof of (??) is by contradiction: suppose instead that $S - s_1' - s_2' < S - s_1 - s_2$. By the proof of Proposition **??**, s_2 is the unique action that is played by a positive mass of firms that repurchase s_1 and later invest. Hence

$$S + E[a|s_1] + b\Pr(s_2|s_1) = P_1(s_1) \geq P_2(s_1,s_2)\Pr(s_2|s_1) + (S + E[a|s_1, \text{not } s_2])(1 - \Pr(s_2|s_1)),$$

where the inequality follows from the equilibrium condition that firm a's final payoff must be at least $S + a$. Consequently, $P_2(s_1,s_2) \leq S + E[a|s_1,s_2] + b$. Consider any firm a that plays (s_1,s_2) and satisfies $P_2(s_1,s_2) \leq S + a + b$. By Lemma **??**, $P_1(s_1) \geq P_2(s_1,s_2)$,[31] and by supposition $-s_1' - s_2' < -s_1 - s_2$,

$$\frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1}{P_1(s_1)} - \frac{s_2}{P_2(s_1,s_2)}} \leq \frac{S - s_1 - s_2 + a + b}{1 - \frac{s_1+s_2}{P_2(s_1,s_2)}} \leq \frac{S - s_1' - s_2' + a + b}{1 - \frac{s_1'+s_2'}{P_2(s_1,s_2)}} < \frac{S - s_1' - s_2' + a + b}{1 - \frac{s_1'+s_2'}{P_2(s_1',s_2')}}.$$

Since the final term above is firm a's payoff from deviating and playing (s_1',s_2'), this sequence of inequalities contradicts firm a's equilibrium condition, and hence establishes inequality (??).

Next, consider any firm $\hat{a} \leq E[a|s_1,s_2]$ that plays (s_1,s_2). The inequalities (**??**), $P_1(s_1) \geq$

[31] For the application of Lemma **??**, note that since (s_1,s_2) is played by a positive mass of firms, the proof of Proposition **??** implies that almost all firms that invest after playing s_1 play (s_1,s_2).

$P_2(s_1, s_2)$, and $P_2(s_1, s_2) < P_2(s'_1, s'_2)$ imply that firm a's equilibrium payoff is

$$\begin{aligned}
\frac{S - s_1 - s_2 + \hat{a} + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(1 - \frac{s_2}{S - s_1 + E[a|s_1, s_2] + b}\right)} &= \frac{S - s_1 - s_2 + \hat{a} + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(\frac{S - s_1 - s_2 + E[a|s_1, s_2] + b}{S - s_1 + E[a|s_1, s_2] + b}\right)} \\
&\leq \frac{S - s'_1 - s'_2 + \hat{a} + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(\frac{S - s'_1 - s'_2 + E[a|s_1, s_2] + b}{S - s_1 + E[a|s_1, s_2] + b}\right)} \\
&= \frac{S - s'_1 - s'_2 + \hat{a} + b}{\left(1 - \frac{s_1}{P_1(s_1)}\right)\left(1 - \frac{s'_1 + s'_2 - s_1}{S - s_1 + E[a|s_1, s_2] + b}\right)} \\
&= \frac{S - s'_1 - s'_2 + \hat{a} + b}{1 - \frac{s_1}{P_1(s_1)} - \frac{s'_1 + s'_2 - s_1}{P_2(s_1, s_2)}} \\
&\leq \frac{S - s'_1 - s'_2 + \hat{a} + b}{1 - \frac{s'_1 + s'_2}{P_2(s_1, s_2)}} < \frac{S - s'_1 - s'_2 + \hat{a} + b}{1 - \frac{s'_1 + s'_2}{P_2\left(s'_1, s'_2\right)}}.
\end{aligned}$$

But then firm $\hat{a}$ would be strictly better off deviating to issue strategy (s'_1, s'_2), giving a contradiction and establishing $P_2(s_1, s_2) \geq P_2(s'_1, s'_2)$.

If $S - s'_1 < I$ and $s'_1 < 0$, then by Lemma **??**, $P_1(s'_1) = P_2(s'_1, s'_2)$. If instead $S - s'_1 \geq I$, then since (s'_1, s'_2) is played by a positive mass of firms, it is straightforward to show that $s'_2 = 0$, and moreover, by the proof of Proposition **??**, that $P_1(s'_1) = P_2(s'_1, s'_2)$.

Finally, if $\Pr(s_2|s_1) < 1$, then Lemma **??** implies $P_1(s_1) > P_2(s_1, s_2)$ in place of $P_1(s_1) \geq P_2(s_1, s_2)$, and a parallel proof to the above then delivers $P_2(s_1, s_2) > P_2(s'_1, s'_2)$.

Proof of Proposition ??: Part (A) is established in Lemma **??**. Here, we establish Part (B). We first establish $E[P_1|s_1 < 0] \leq P_0$, and then show at the end of the proof that the inequality must be strict.

Consider a date 1 issue strategy played by a positive mass of firms, $\hat{s}_1 < 0$. By Lemma **??**, $\hat{s}_1$ is followed almost surely by $\hat{s}_2 \leq 0$ such that $S - \hat{s}_1 - \hat{s}_2 \geq I$, and $P_1(\hat{s}_1) = P_2(\hat{s}_1, \hat{s}_2)$. From Proposition **??** and Lemma **??**, we know that for any repurchase $s_1 > 0$, $P_1(\hat{s}_1) = P_2(\hat{s}_1, \hat{s}_2) \leq P_2(s_1, s_2) \leq P_1(s_1)$.[32] Hence it suffices to show that $P_1(\hat{s}_1) \leq P_1(0)$. There are two cases, which we deal with separately.

[32]For the application of Lemma **??**, note that since (s_1, s_2) is played by a positive mass of firms, the proof of Proposition **??** implies that almost all firms that invest after playing s_1 play (s_1, s_2).

Let a^* and a^{i*} be as defined in Corollary **??**. Let $(0, s_2')$ be a strategy with $s_2' \neq 0$ that is played by a positive mass of firms, if such a strategy exists. By the proof of Proposition **??**, there is at most one such strategy.

Case 1: A positive mass of active firms play $(0, s_2')$.

Because a positive mass of active firms play both $(0, s_2')$ and $(\hat{s}_1, \hat{s}_2)$, the proof of Proposition **??** implies that $\hat{s}_1 + \hat{s}_2 = s_2'$ and $P_1(\hat{s}_1) = P_2(0, s_2')$. Note that $P_1(0) = P_2(0, s_2') \Pr(s_2'|0) + E[S + a|\text{firm does nothing}] \Pr(0|0)$. Certainly $S + a > P_2(0, s_2')$ for any firm a that does nothing, since otherwise the firm would strictly prefer to deviate and play $(0, s_2')$. Hence $P_1(0) \geq P_2(0, s_2') = P_1(\hat{s}_1)$.

Case 2: Any active firm that invests plays $s_1 \neq 0$.

By the proof of Proposition **??**, almost any active firm a that does nothing at date 1 and does not invest obtains a payoff of $S + a$. Hence

$$P_1(0) = (1 - \alpha)(S + E[a|\text{active, do nothing}]) + \alpha \left(S + E[a] + b \Pr\left(a \leq a^{i*}\right)\right).$$

Certainly $P_1(\hat{s}_1) < S + a$ for any active firm a that does nothing, since otherwise the active firm would strictly prefer to deviate and play $(\hat{s}_1, \hat{s}_2)$. If $\alpha = 0$ this immediately establishes $P_1(\hat{s}_1) < P_1(0)$. For the case $\alpha > 0$, it suffices to show $P_1(\hat{s}_1) \leq S + E[a] + b \Pr(a \leq a^{i*})$. Suppose to the contrary that this is not the case. Then

$$E[P_3|\text{active}] > E\left[\max\left\{S + a, \frac{S - \hat{s}_1 - \hat{s}_2 + a + b}{1 - \frac{\hat{s}_1 + \hat{s}_2}{S + E[a] + b \Pr(a \leq a^{i*})}}\right\}\right] \geq S + E[a] + b \Pr\left(a \leq a^{i*}\right). \tag{A-37}$$

Because we are in Case 2, there is no pooling between active and inactive investing firms, so $E[P_3|\text{active}] = S + E[a] + b \Pr(a \leq a^*)$, and hence $a^* > a^{i*}$. Consider any $a \in (a^{i*}, a^*)$ such that $S + a + b > S + E[a|a \leq a^*] + b$. Since $a < a^*$, when firm a is active it plays a strategy (s_1, s_2) satisfying $S - s_1 - s_2 \geq I$, and because we are in Case 2, $s_1 \neq 0$. By inequality (**??**)

of the proof of Proposition **??**, it follows that

$$\frac{S-s_1-s_2+a+b}{1-\frac{s_1+s_2}{S+E[a|a\leq a^*]+b}} \geq \frac{S-s_1-s_2+a+b}{1-\frac{s_1}{P_1(s_1)}-\frac{s_2}{P_2(s_1,s_2)}} \geq S+a.$$

By assumption, $S-s_2'\leq S-s_1-s_2$. Hence

$$\frac{S-s_2'+a^*+b}{1-\frac{s_2'}{S+E[a|a\leq a^*]+b}} \geq S+a^*,$$

which together with Assumption **??** implies that there exists $\tilde{a}\geq a^*>a^{i*}$ such that

$$\frac{S-s_2'+\tilde{a}+b}{1-\frac{s_2'}{S+E[a|a\leq \tilde{a}]+b}} = S+\tilde{a},$$

contradicting Lemma **??** and completing the proof of this case.

Establishing that $E[P_1|s_1<0]\leq P_0$ holds strictly:

From the proof above, the only possibility for $E[P_1|s_1<0]=P_0$ is if every date 1 action $s_1\geq 0$ that is played with positive probability is followed by certain investment. But in this case, almost all active firms invest, contradicting Lemma **??** and completing the proof.

Proof of Proposition ??: *Part (A):* Let a' and a'' be firms that play s' and s'' respectively (with no other capital transactions at other dates). By the equilibrium condition for firm a'', $\frac{S-s''+a''}{1-\frac{s''}{P''}} \geq \frac{S-s'+a''}{1-\frac{s'}{P'}}$. Since $s''>s'$, it follows that $s''/P''>s'/P'$. By the equilibrium condition for firm a',

$$\frac{S-s'+a'}{1-\frac{s'}{P'}} \geq \frac{S-s''+a'}{1-\frac{s''}{P''}}. \tag{A-38}$$

Firm a' also has the choice of doing nothing, and so a separate equilibrium condition implies $S+a'\geq P'$, i.e., firm a' pays weakly less than its stock is worth. Consequently,

$$\frac{S-s''+a'}{1-\frac{s''}{P'}} \geq \frac{S-s'+a'}{1-\frac{s'}{P'}},$$

i.e., if firm a' were able to repurchase more stock at the constant price P', it would weakly prefer to do so. Combined with (**??**), it then follows that $P'' \geq P'$.

Part (B): Taking the expectation over the equilibrium condition for all firms a playing (s_1', s_2'), together with the implication of Lemma **??** that $E\left[a|s_1', s_2'\right] > E\left[a|s_1'', s_2''\right]$, yields

$$\frac{S - s_1' - s_2' + E\left[a|s_1', s_2'\right] + b}{1 - \frac{s_1'}{P_1\left(s_1'\right)} - \frac{s_2'}{P_2\left(s_1', s_2'\right)}} \geq \frac{S - s_1'' - s_2'' + E\left[a|s_1', s_2'\right] + b}{1 - \frac{s_1''}{P_1\left(s_1''\right)} - \frac{s_2''}{P_2\left(s_1'', s_2''\right)}} > \frac{S - s_1'' - s_2'' + E\left[a|s_1'', s_2''\right] + b}{1 - \frac{s_1''}{P_1\left(s_1''\right)} - \frac{s_2''}{P_2\left(s_1'', s_2''\right)}}.$$

Since the first and last terms in this inequality are simply $P_2\left(s_1', s_2'\right)$ and $P_2\left(s_1'', s_2''\right)$ respectively, this establishes $P_2\left(s_1', s_2'\right) > P_2\left(s_1'', s_2''\right)$.