Decentalization Schemes, Cost-Benefit Analysis, and Net National Product as a Measure of Social Well-Being*

Partha Dasgupta
University of Cambridge; Beijer International Institute for Ecological Economics, Stockholm; and Resources for the Future, Washington, DC

Karl-Göran Mäler
Beijer International Institute for Ecological Economics, Stockholm; Stockholm School of Economics; and Resources for the Future, Washington, DC

Contents:

Abstract
1. Motivation
2. Why NNP?
3. The Model: A Macroeconomic Planning Problem
4. The Pontryagin Maximum Principle and Social Cost-Benefit Analyis
5. Investment Projects and Accounting Profits
6. NNP and the Linearized Hamiltonian
7. Project Evaluation in a Non-Optimizing Economy
8. Comparisons of Social Well-Being Across Time and Space
9. Technical Change and Growth Accounting
10. Further Extensions
11. Conclusions: What should NNP include?
References

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The Suntory Centre
Suntory and Toyota International Centres for Economics and Related Disciplines
London School of Economics and Political Science
Houghton Street
London WC2A 2AE
Tel.: 020-7955 6674

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Abstract

This paper is about net national product (NNP). We are concerned with what NNP means, what it should include, what it offers us and, therefore, why we may be interested in it. We show that NNP, properly defined, can be used as a gauge for project evaluation, but we also show that it should not be used in any of its more customary roles, such as in making intertemporal and cross-country comparisons of social well-being. We develop such indices as would be appropriate for making such comparisons. Writings on the welfare economics of NNP have mostly addressed economies pursuing optimal policies. Our analysis includes not only such economies, but also those where the government is capable of engaging only in policy reforms.

The literature on green NNP has widely interpreted NNP as a 'constant-equivalent consumption stream'. We show that this interpretation offers no purchase. It is the Hamiltonian that equals a constant-equivalent utility stream and we argue that, as the Hamiltonian is typically a non-linear function of consumption and leisure, it is of little practical use.

Keywords: allocative effficiency; cost-benefit analysis; capital; investment; project evaluation.
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1. Motivation

This article is on the concept of net national product (NNP). We are concerned with what NNP means, what it should include, what it offers us and, therefore, why we may be interested in it. The concept is old; even its modern version was developed over sixty years ago (Lindahl, 1934). Nevertheless, it has proved to be sufficiently intriguing to have appeared periodically on the research agenda of theoretical economists. In recent years there has been renewed interest in it because of the need to understand the way environmental pollution and resource depletion ought to find expression in NNP if NNP is to reflect what it is believed to reflect. The term "green NNP" is an expression of this belief. In the space of only a few years the term has gained such currency that it is today a commonplace to say that in estimating NNP deduction ought to be made from gross national product (GNP) of not only the depreciation of physical and human capital, but also the depreciation of natural capital and the social losses that are incurred owing to increases in the stock of environmental pollution. So one might think that there would be a consensus on what NNP measures, what it ought to include, and how that which ought to be included in it should be measured. In fact, there is no consensus.

To be sure, the estimation of NNP is as much an art as a science. Paucity of data and the need for simplicity, reliability, and uniformity together mean that compromises have to be made, that theoretical niceties have sometimes to be jettisoned. Therefore, one should not be surprised that there are disagreements among experts on practical methods for estimating NNP. The puzzle is that there would appear to be disagreement even on theoretical issues.[1] This article addresses those analytical foundations that need to be constructed if useful practical procedures for estimating NNP are to be devised. We demonstrate that NNP comparisons across time and space do not reveal what they are widely thought to reveal. We show, however, that NNP can be useful in a different role, namely, as a gauge in social cost-benefit analysis. We also show that when so used in any well-specified economy, there can be no theoretical disagreement on which items ought to be included in NNP and which items ought to be omitted. In short, we argue that at a conceptual level the question how NNP ought to be measured is not a matter

[1] See, for example, Harrison (1989), Hueting (1989), Hartwick (1990, 1994), Bartelmus, Stahmer and van Tongeren (1991), Dasgupta and Mäler (1991), Usher (1994), Johansson and Löfgren (1996), and Weitzman (1998).

of opinion; it is a matter of fact.

2. Why NNP?

Why should we be interested in such measures as GNP and NNP?

There are at least three reasons. One arises from a need to have an index of aggregate economic activity, of a kind that would help one to summarise a macroeconomy. GNP has been found to be useful in this role. The second reason arises from a desire to estimate the levels of aggregate consumption an economy is capable of sustaining along alternative economic programmes. Early definitions of national income (Lindahl, 1934; Hicks, 1940; Samuelson, 1961; Weitzman, 1976) were designed to address this problem, and the bulk of recent theoretical explorations in green NNP have returned to it.[2]

The third reason arises from the need to have a quantitative measure of social well-being. But economic activity, sustainable consumption, and social well-being are not the same object; so their numerical measures are not necessarily the same. For example, in a market economy the wage bill for labour ought obviously to be included if the required index is to measure aggregate economic activity, as in GNP. However, it is by no means obvious that this particular item ought to be included if the index is to measure social well-being (Nordhaus and Tobin, 1972; see Section 6). The moral is banal: the way an index ought to be defined, let alone estimated, is not independent of the purpose to which it is put. Nevertheless, it has become customary to label the indices that measure social well-being and sustainable consumption, respectively, by the same term: net national product. In this article we study NNP in its role as an index of social well-being.

Now it can be argued that if we seek an index of social well-being, we should measure social well-being directly and not look for a surrogate and give it a different name, NNP or whatever. There is something in this. On the other hand, there could be several reasons for seeking a measure of social well-being, and for many purposes the most convenient index could be something other than the thing itself. For example, we could be interested in some object X, but X may prove especially hard to measure. Suppose now that for some purposes X is known to correlate perfectly with Y and that

[2] See, for example, Solow (1986, 1992), Hartwick (1990, 1994), Asheim (1994, 1997), and Weitzman (1998).

Y is easier to measure than X. Then we would wish to rely on Y for those purposes.

The problem is that there are at least three reasons for seeking an index of social well-being: (a) there may be a need to know if the well-being of a particular group (e.g. citizens of a country) is higher today than it was yesterday; (b) we may wish to compare social well-being in different countries, regions, or wherever, at a particular point in time; and (c) public decision-makers require a criterion on the basis of which social and economic policies can be evaluated. If (c) is the reason, then the index developed could be used by decision-makers to judge the relative merits of alternative policies. Criteria for social cost-benefit analysis of investment projects, such as the present discounted value of the flow of accounting profits, are examples (Section 5).

In this article we focus first on (c). We show that there are a number of analytically-equivalent methods available for evaluating public policies (Sections 4-6). Each such method is appropriate for a corresponding economic decentralization scheme. One of them (Section 6) makes direct use of NNP. We show that it is possible to conduct social cost-benefit analysis by studying the impact of an investment project on NNP, suitably measured, in each period of the project's life. In Sections 4-6 we study an economy where the government optimizes over the choice of economic policies. In Section 7 the analysis is extended to cover the case of an economy where the government does not optimize, but is able to engage in policy reforms. We show how NNP should be defined in such an economy if it is to be used for evaluating reforms. The connection between the analyses of optimizing and reformist governments is then sketched.[3]

In Section 8 we show that, contrary to popular belief and customary practice, NNP comparisons across time, across regions, and across groups tell us next-to-nothing unless the economies being studied are in stationary states. We also argue that, when used as a measure of "sustainable-equivalent consumption", NNP would be uninformative unless we were to adopt an ethically indefensible viewpoint concerning intergenerational distributions of well-being. In short, other gauges need to be invoked if we are to make comparisons of social well-being across time and space. In Section 8 such indices are constructed. It is a commonplace today to use GNP comparisons for all

[3] The analysis presented in Sections 4-7 synthesises and extends considerably the arguments in Dasgupta and Mäler (1991), Mäler (1991), and Dasgupta, Kriström and Mäler (1995, 1998).

these purposes, even though it is frequently remarked that GNP is an inadequate index. Our analysis tells us which indices ought to replace GNP in some of its various customary roles.

In order to keep the exposition simple, we ignore technical change upto and including Section 8. In Section 9, therefore, we extend the analysis to cover the case of economies capable of experiencing such change. Section 10 sketches a number of additional extensions, and Section 11 collates our main conclusions.

3. The Macroeconomic Model: A Planning Problem

Consider a model economy where the production of goods and services requires labour, intermediate goods, manufactured capital, and natural resources. The economy is deterministic. Time is continuous and is denoted by t (≥ 0). Assume that there is an all-purpose, non-deteriorating durable good, whose stock at t is K_t (≥ 0). The good can be either consumed, or spent in increasing the stock of natural resources, or reinvested for its own accumulation. Assume that both population size and the stock of human capital are constant. This means that we may ignore them (but see Sections 10-11). The consumption good can be produced with its own stock (K), labour (L_1) and a non-durable intermediate good (X) as inputs. We write the production function as $F(K, L_1, X)$. Production of the all-purpose durable good at date t is then $F(K_t, L_{1t}, X_t)$. We take it that F is concave and an increasing and continuously differentiable function of each of its variables. Thus, we impose on F all the conditions that would enable traditional price theory to run smoothly.

The production of the intermediate good requires labour (L_2) and the use of natural resources. If R_t is the rate at which the resource is extracted and used in production at t, output of the intermediate is assumed to be given by the function $G(L_{2t}, R_t)$, where G is concave and an increasing and continuously differentiable function of each of its variables. It follows that

$$G(L_{2t}, R_t) \geq X_t \geq 0.^4 \qquad (1)$$

Let C_t (≥ 0) denote aggregate consumption at t, and E_t (≥ 0) the expenditure on

[4] If the intermediate good were durable, we would need to consider the possibility that inventories are created. In the model economy to be studied here, there would be no inventories along an optimum economic programme. Whether or not inventories are accumulated is extraneous to our concern here. So we take it that the intermediate good is non-durable. This explains inequality (1).

increasing the natural-resource base. Net accumulation of physical capital, therefore, satisfies the condition:

$$dK_t/dt = F(K_t, L_{1t}, X_t) - C_t - E_t. \quad (2)$$

It helps to interpret natural resources in broad terms; this enables us to consider a number of issues. We should certainly include in the natural-resource base the multitude of capital assets that provide the many and varied ecosystem services upon which life is based. But we should add to this minerals and fossil fuels. Note too that environmental pollution can be viewed as the reverse side of environmental resources. In some cases the emission of pollutants amounts directly to a degradation of ecosystems (e.g. loss of biomass); in others it amounts to a reduction in environmental quality (e.g. deterioration of air and water quality), which also amounts to degradation of ecosystems. This means that for analytical purposes there is no reason to distinguish resource economics from environmental economics, nor resource management problems from pollution management problems (Dasgupta, 1982). To put it crudely, "resources" are a "good", while "pollution" (the degrader of resources) is a "bad". So we will work with an aggregate stock of natural resources, whose size at t is denoted by S_t (≥ 0). For simplicity of exposition we assume that resource extraction is costless.

Let the natural rate of regeneration of the resource base be $M(S_t)$, where $M(S)$ is a concave and continuously differentiable function.[5] We suppose that the base can also be augmented by expenditure E_t (exploration costs in the case of minerals and fossil fuels, clean-up costs in the case of polluted water, and so forth). Define

$$Z_t = \int_{-\infty}^{t} E_\tau d\tau.\text{[6]} \quad (3)$$

In certain applications of the model, Z_t would be a measure of the stock of knowledge at t. This interpretation enables us to connect our model with one where there is endogenous technical progress, a matter to be discussed in Section 9. Let us now re-express equation (3) in the more useable form,

$$dZ_t/dt = E_t. \quad (4)$$

There are a number of ways in which one can model the process by which the

[5] If the resource in question were minerals or fossil fuels, S_t would denote known reserves at t and we would have $M(S) = 0$ for all S.

[6] Z_0 is part of the data of the economy. Like K_0 and S_0, it is an "initial condition".

resource base is deliberately augmented. Let $N(E_t, Z_t, S_t)$ denote the rate at which this augmentation occurs, where N is taken to be a concave and continuously differentiable function. It is natural to imagine that N is non-decreasing in both E and Z. So we suppose that it is so. We consider two special forms of N in Section 11.

The dynamics of the resource base can then be expressed as:

$$dS_t/dt = M(S_t) - R_t + N(E_t, Z_t, S_t). \quad (5)$$

We formulate the idea of social well-being in a conventional manner and ignore those many matters that arise when households are heterogeneous. We do this so as to keep the notation tidy. The implications of household heterogeneity and other extensions are sketched in Section 10. Let $L_t = L_{1t} + L_{2t}$. Assume that at t=0 notions of intergenerational justice are captured in the "utilitarian" form, ${}_0\int^{\infty} U(C_t, L_t)e^{-\delta t}dt$, where U is strictly concave, increasing in C, decreasing in L (at least at large enough values of L), and continuously differentiable in both C and L. δ (≥ 0), a constant, is the "utility" discount rate. In what follows, we refer to ${}_0\int^{\infty} U(C_t, L_t)e^{-\delta t}dt$ as <u>social well-being</u> at t=0 and, more generally, ${}_t\int^{\infty} U(C_\tau, L_\tau)e^{-\delta(\tau-t)}d\tau$ as social well-being at t ($\geq$ 0). The government's task is to maximize social well-being at t=0 subject to feasibility constraints.

In order to formulate the government's planning problem, let $(C_t, L_{1t}, L_{2t}, X_t, R_t, E_t, K_t, Z_t, S_t)_0^{\infty}$ denote an <u>economic programme</u>, from the present (t = 0) to the indefinite future. Until Section 7 it will be assumed for expositional ease that the economy is capable of attaining a first-best. This means that, given (K_0, Z_0, S_0), an economic programme is <u>feasible</u> if it satisfies conditions (1)-(2) and (4)-(5).[7]

We now collect conditions (1)-(2) and (4)-(5) to express the central planner's optimization problem as:

Choose the control variables $(C_t, L_{1t}, L_{2t}, X_t, R_t, E_t)_0^{\infty}$ so as to maximize

${}_0\int^{\infty} e^{-\delta t}U(C_t, L_{1t}+L_{2t})dt$, subject to the conditions:

$dK_t/dt = F(K_t, L_{1t}, X_t) - C_t - E_t$,

$dZ_t/dt = E_t$,

[7] The first-best is sometimes called the "full optimum". As will become evident, the analysis extends to second- or third-best economies, albeit in suitably amended forms. See Section 7, where a non-optimizing government is postulated. Anything short of the first-best would imply that there are certain constraints additional to equations (1)-(2) and (4)-(5) which are binding.

$dS_t/dt = M(S_t) - R_t + N(E_t, Z_t, S_t)$,

$G(L_{2t}, R_t) \geq X_t$,

where K_0, Z_0, and S_0 are given as initial conditions. (6)

We will refer to the planning problem summarized in (6) as $\mathbf{P_1}$.

Let us assume, without further elaboration, that there is a solution to $\mathbf{P_1}$. Let us also suppose, for notational ease, that U, F, and G are such that, at the optimum, C_t, L_{1t}, L_{2t}, E_t, and R_t (and therefore X_t) are positive and continuous at all t. We now study methods for solving $\mathbf{P_1}$.

4. The Pontryagin Maximum Principle and Social Cost-Benefit Analysis

Social cost-benefit analyses can be regarded as methods for locating optimum economic programmes (i.e. solving problems such as $\mathbf{P_1}$). For $\mathbf{P_1}$ the algorithms involve social cost-benefit analysis of investment projects. Recall that cost-benefit analysis involves calculations on counter-factuals: the typical question asked is what would happen if this, rather than that, economic project were chosen. By investment projects we mean perturbations of macroeconomic programmes. Social cost-benefit analysis can be interpreted as the evaluation of alternative perturbations. Social well-being itself is not useful as a gauge for evaluating projects because it is non-linear in dated consumption and labour (i.e. U is non-linear in C_t and L_t). This is where the fundamental decentralization theorem of welfare economics is useful. The theorem states that, provided certain technical assumptions are met, associated with any conception of social well-being and any set of technological, transaction, information, and ecological constraints, there exists a set of accounting prices that can be used to implement the optimum economic programme in a decentralized manner, where the various agencies are required to maximize their accounting profits.[8] By the accounting profit accruing to an agency we mean accounting revenue minus the accounting cost of the inputs chosen by the agency in question.

It is worth stressing here two advantages afforded by such decentralization schemes. One is that profits are linear functions of economic variables, whereas social

[8] The circumstances are those in which the Kuhn-Tucker theorem holds. For a simple proof, see Dasgupta (1993, ch. 7*). Accounting prices are sometimes called "shadow prices" and also, by the technocratically minded, "Lagrange multipliers".

well-being is not. The other is that in market economies many of the accounting prices can be approximated by market prices, or by suitable conversions of market prices. So if markets function reasonably well, it is possible to know much about the structure of optimum accounting prices even in the absence of a knowledge of the optimum programme itself. Old though it is, this observation has frequently been lost sight of in the current literature; but it remains of fundamental importance.[9]

So, for simplicity of exposition, we begin by considering an extreme (and unrealistic) situation, in which the entire set of optimum accounting prices is known, even though the optimum economic programme itself is not. In this and the following two sections we develop two contrasting algorithms in such a world, both of which rely on optimum accounting prices to locate the optimum programme. We show that each algorithm is appropriate for a corresponding decentralization scheme. We call any such algorithm a method of cost-benefit analysis.

However, for a theory of cost-benefit analysis to be useful, it must be extendable to the case where optimum accounting prices are <u>not</u> known in advance. So in Section 7 we develop a general method of cost-benefit analysis, which starts from an arbitrary economic programme, and uses what we call local prices to identify projects that increase social well-being.

Before doing any of this, we rehearse a well-known scheme in which the public decision-maker is asked to maximize a <u>non</u>-linear function of economic quantities in each period. In Sections 5-6 we will see how the non-linearity can be circumvented. In what follows, starred values of variables denote their values along the optimum economic programme.

Recall the Pontryagin Maximum Principle (Arrow and Kurz, 1970; Seierstad and Sydsaeter, 1987). Let U be the numeraire and let p_t^*, q_t^*, and r_t^* (all ≥ 0) denote the <u>optimum</u> accounting prices, at t, of K_t, Z_t, and S_t, respectively. Next, denote by w_t^* the accounting wage rate and by s_t^* ($t \geq 0$) the Lagrange multiplier associated with the inequality constraint in (6). From the Kuhn-Tucker theory, we know that along the optimum economic programme the multiplier s_t^* and its corresponding constraint satisfy the complementary-slackness condition

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[9] The classic on this observation is Little and Mirrlees (1974).

$$s_t^*(G(L_{2t}^*, R_t^*) - X_t^*) = 0, \qquad \text{for } t \geq 0. \tag{7}$$

Thus, if $s_t^* > 0$, the constraint is binding; and if the constraint is non-binding, then $s_t^* = 0$. Given our assumptions though, we know in advance that $p_t^*, q_t^*, r_t^*, s_t^* > 0$.

The Pontryagin Maximum Principle states that the accounting price system supports the optimum economic programme in the sense that, in addition to the feasibility constraints (1)-(2) and (4)-(5), the accounting prices satisfy the equations:

$$dp_t^*/dt = p_t^*(\delta - F_K); \quad dq_t^*/dt = q_t^*(\delta - (r_t^*N_Z/q_t^*)); \text{ and}$$
$$dr_t^*/dt = r_t^*(\delta - (M'(S) + N_S)).[10] \tag{8}$$

Now the current-value Hamiltonian of (6) can be expressed as:

$$H_t^* = U(C_t, L_{1t}+L_{2t}) + p_t^*(F(K_t, L_{1t}, X_t) - C_t - E_t) + q_t^*E_t + r_t^*(M(S_t) - R_t + N(E_t, Z_t, S_t)) + s_t^*(G(L_{2t}, R_t) - X_t). \tag{9}$$

The Maximum Principle instructs the planner to choose the control variables at each date, <u>without restriction</u>, so as to maximize H_t^*. This yields the first-order conditions:

$$U_C = p_t^*; \; U_L = -w_t^* = -p_t^*F_L = -s_t^*G_L;$$
$$p_t^* = q_t^* + r_t^*N_E; \; r_t^* = s_t^*G_R; \text{ and } p_t^*F_X = s_t^*. \tag{10}$$

The feasibility constraints (1)-(2) and (4)-(5), and equations (8) and (10), taken together, enable one to find the optimum economic programme.

When we come to develop the concept of NNP in Section 6, we will discuss the current-value Hamiltonian further. Before doing that, however, it will prove instructive to re-cast $\mathbf{P}_1$ in the form of a decentralization scheme which yields the most commonly-practised method of project evaluation. It is based on the famous "inverse-optimum" theorem of welfare economics (Debreu, 1959).

5. Investment Projects and Accounting Profits

The idea is to decentralize by splitting $\mathbf{P}_1$ into four constituent maximization problems. To see how this can be done, denote by ρ_t^*, σ_t^*, and μ_t^* the accounting rental rates, at t, on manufactured capital (K_t), cumulative expenditure on resource augmentation (Z_t), and natural capital (S_t), respectively. Each of the rental rates is expressed in utility numeraire. It is simple to confirm that

[10] $M'(S)$ is the first derivative of M with respect to S, F_K is the first partial derivative of F with respect to K, and so forth.

$$\rho_t^* = p_t^* F_K,\ \sigma_t^* = r_t^* N_Z, \text{ and } \mu_t^* = r_t^*(M'(S_t) + N_S).$$ [11] (11)

We now observe that $\mathbf{P_1}$ can be replaced by optimization problem $\mathbf{P_2}$, which consists of the following decentralization scheme:

Install a consumption sector, two production sectors (1 and 2 respectively), and a knowledge-cum-natural resource sector, with the task of solving the following four maximization problems, without restriction on their controls:

Consumption sector: This sector owns all capital assets, it supplies whatever quantities are demanded by producing firms on a rental basis, receives payment for resources that are extracted and for the labour it supplies, and purchases the consumption good from the production sector, all at accounting prices. Formally, given K_0, Z_0, and S_0 as the initial stocks of capital, the consumption sector is instructed to choose

$(C_t, L_{1t}, L_{2t})_0^\infty$, without restriction, so as to maximize

$$\int_0^\infty e^{-\delta t}(U(C_t, L_{1t}+L_{2t}) + \rho_t^* K_t^* + \sigma_t^* Z_t^* + \mu_t^* S_t^* + w_t^*(L_{1t} + L_{2t}) + r_t^* R_t^* - p_t^* C_t)dt.$$ [12] (12a)

Production sector 1: This sector is involved in the production of the final good. It rents physical capital and hires labour from the consumption sector, and purchases the intermediate good from production sector 2 (which is in charge of producing the intermediate). All receipts and payments are computed at accounting prices. Production sector 1 is instructed to maximize the present value of the flow of accounting profits. In short, it chooses $(K_t, L_{1t}, X_t)_0^\infty$, without restriction, so as to maximize

$$\int_0^\infty e^{-\delta t}(p_t^* F(K_t, L_{1t}, X_t) - w_t^* L_{1t} - \rho_t^* K_t - s_t^* X_t)dt. \quad (12b)$$

Production sector 2: This is the intermediate goods sector. It hires labour from the consumption sector and purchases natural resources so as to produce the intermediate good, which it sells to producton sector 1. Production sector 2 is instructed to maximize the present-value of the flow of accounting profits. In short, it chooses $(L_{2t}, R_t)_0^\infty$, without restriction, so as to maximize

$$\int_0^\infty e^{-\delta t}(s_t^* G(L_{2t}, R_t) - w_t^* L_{2t} - r_t^* R_t)dt. \quad (12c)$$

Natural-Resource and Knowledge sector: This sector manages both natural

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[11] Note too that $p_0^* = \int_0^\infty e^{-\delta t}\rho_t^* dt$, and so forth for the accounting prices at $t = 0$ for the other two assets.

[12] The capital stocks and the rate of resource extraction have been starred to signify that they are evaluated at their optimum values. These receipts are made on a lump-sum basis.

resources and "knowledge". It rents natural capital and knowledge capital from the consumption sector and incurs expenditure for the augmentation of knowledge. It regards net increases in knowledge and the resource base (dZ_t/dt and dS_t/dt, respectively) as its products. It is instructed to maximize the present-value of the flow of accounting profits. In short, it chooses $(Z_t, S_t, E_t)_0^\infty$, without restriction, so as to maximize

$$_0\int^\infty e^{-\delta t}(q_t^*E_t + r_t^*(M(S_t) - R_t^* + N(E_t, Z_t, S_t)) - p_t^*E_t - \lambda_t^*Z_t - \mu_t^*S_t)dt. \qquad (12d)$$

The interpretation of (12a)-(12d) should be familiar, as should the argument that supports it: the costs of breaking the constraints in $\mathbf{P_1}$ have been incorporated in each of the four "managerial" objective functions in (12a)-(12d), so as to make it unprofitable for the respective managers to break them. This is why each sector is permitted to choose its control variables without restriction. One can now confirm that the first-order conditions arising from the four separate maximization problems (12a)-(12d) are identical to the first-order conditions arising from the combined maximization problem (9). In other words, the first order conditions of problems (12a)-(12d) are the same as those provided in (10)-(11). Note too that if the integrands in expressions (12a)-(12d) were added, at the optimum their sum would equal the value of the Hamiltonian in (9).[13]

We will now concentrate on the production sectors in (12b)-(12c). For ease of exposition, let us combine them to create a vertically integrated sector. The present-value of the flow of accounting profits earned by it can then be obtained by adding (12b) and (12c):

$$_0\int^\infty e^{-\delta t}(p_t^*F(K_t, L_{1t}, X_t) - w_t^*L_{1t} - \rho_t^*K_t - s_t^*X_t + s_t^*G(L_{2t}, R_t) - w_t^*L_{2t} - r_t^*R_t)dt. \qquad (13)$$

The integrand in (13) is the production sector's accounting profit at t. The production manager would be asked to maximize (13) by choosing $(K_t, L_{1t}, L_{2t}, X_t, R_t)_0^\infty$, without restriction. It follows that the profit-maximizing intertemporal production plan is a stationary point of (13). At the optimum, however, the intermediate good would vanish from the production sector's accounts.[14]

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[13] Note also that each of the sectors could be broken down into a continuum of bits, one for each t. In short, we could decentralize even further by having a continuum of decision-makers in each sector. We will explore this interpretation in Section 6.

[14] This is because $s_t^*[G(L_{2t}^*, R_t^*) - X_t^*] = 0$.

Thus far the conditions an optimum macro-economic programme must satisfy. We now proceed to develop techniques for locating the optimum programme. As we observed earlier, social cost-benefit analysis of projects involves the use of such techniques. Note though that the notion of a project is not independent of the decentralization scheme. Under the scheme we have constructed above, a project would be defined as a perturbation $(\Delta K_t, \Delta L_{1t}, \Delta L_{2t}, \Delta X_t, \Delta R_t, (G_L\Delta L_{2t} + G_R\Delta R_t), (F_K\Delta K_t + F_L\Delta L_{1t} + F_X\Delta X_t))_0^\infty$ and would appear in the project's technical feasibility report. In this description, ΔK_t, ΔL_{1t}, ΔL_{2t}, ΔX_t, and ΔR_t would be the project's inputs at t, and $(G_L\Delta L_{2t} + G_R\Delta R_t)$ and $(F_K\Delta K_t + F_L\Delta L_{1t} + F_X\Delta X_t)$ would be the project's intermediate and final outputs, respectively, at t. The project evaluator's task would be to identify desirable projects. One way to do this would be to estimate each project's accounting profit. Let us see how this may be accomplished.

The project's accounting profit at t, measured in terms of utility, is

$$p_t^*(F_K\Delta K_t + F_L\Delta L_{1t} + F_X\Delta X_t) + s_t^*(G_L\Delta L_{2t} + G_R\Delta R_t) - w_t^*(\Delta L_{1t} + \Delta L_{2t}) - \rho_t^*\Delta K_t - s_t^*\Delta X_t - r_t^*\Delta R_t. \quad (14)$$

It follows that the present-value of the flow of accounting profits to this project is

$$\int_0^\infty e^{-\delta t}(p_t^*(F_K\Delta K_t + F_L\Delta L_{1t} + F_X\Delta X_t) + s_t^*(G_L\Delta L_{2t} + G_R\Delta R_t) - w_t^*(\Delta L_{1t} + \Delta L_{2t}) - \rho_t^*\Delta K_t - s_t^*\Delta X_t - r_t^*\Delta R_t)dt. \quad (15)$$

Since the optimum programme (i.e. the solution of $\mathbf{P_1}$) is a stationary point of (13), expression (15) would be zero for the marginal project.

In practice projects are not evaluated in terms of utility numeraire: consumption (Dasgupta, Marglin and Sen, 1972) and investment (Little and Mirrlees, 1974) are frequently in use.[15] Note, however, that the choice of numeraire does not affect project evaluation. To confirm this, observe that if the integrand in expression (15) were divided by, say, the conversion factor p_0^* (which is the price of consumption at $t = 0$ in terms of utility numeraire), we would obtain an expression for the project's present-value of the flow of accounting profits, expressed in consumption numeraire. At the optimum this too would be zero.

6. NNP and the Linearized Hamiltonian

In fact there is another way to solve $\mathbf{P_1}$. It involves a different decentralization

[15] At a first-best, which is what we are studying here, the accounting prices of consumption and investment are the same. At a second-best they need not be the same.

scheme from the one in $\mathbf{P}_2$, and so a different technique for social cost-benefit analysis. It relies on net national product as the basis for project evaluation and uses the Pontryagin Maximum Principle directly. It does not involve the use of accounting profits in the sense profits were defined in the previous section. Instead, it replaces the Hamiltonian in $\mathbf{P}_1$ by its linearized form and then applies the Pontryagin Maximum Principle to the linearized Hamiltonian. It transpires that the linearized Hamiltonian is what we should call NNP.

To see this, recall the well-known fact (Little and Mirrlees, 1974, p. 299) that the solution of $\mathbf{P}_1$ is also the solution of the following optimization problem:

Choose the control variables $(C_t, L_{1t}, L_{2t}, X_t, R_t, E_t)_0^\infty$ so as to maximize

$\int_0^\infty e^{-\delta t}(p_t^*C_t - w_t^*(L_{1t} + L_{2t}))dt$, subject to the conditions:

$dK_t/dt = F(K_t, L_{1t}, X_t) - C_t - E_t$,

$dZ_t/dt = E_t$,

$dS_t/dt = M(S_t) - R_t + N(E_t, Z_t, S_t)$,

$G(L_{2t}, R_t) \geq X_t$,

where K_0, Z_0, and S_0 are given as initial conditions. (16)

We will refer to the optimization problem summarized in (16) as $\mathbf{P}_3$.

Compare $\mathbf{P}_1$ and $\mathbf{P}_3$. $\mathbf{P}_3$ has been obtained from $\mathbf{P}_1$ by the supporting hyperplane theorem. So we know that the solution of $\mathbf{P}_3$ is the same as the solution of $\mathbf{P}_1$. The reason we are interested in $\mathbf{P}_3$ is that it is simpler to solve than $\mathbf{P}_1$; and the reason it is simpler to solve is that, unlike the integrand in $\mathbf{P}_1$, the integrand in $\mathbf{P}_3$ is a <u>linear</u> function of the controls C_t, L_{1t}, and L_{2t}.

Denote the current-value Hamiltonian of $\mathbf{P}_3$ by $\hat{\wp}_t^*$. It follows that

$\hat{\wp}_t^* = p_t^*C_t - w_t^*(L_{1t} + L_{2t}) + p_t^*(F(K_t, L_{1t}, X_t) - C_t - E_t) + q_t^*E_t + r_t^*(M(S_t) - R_t + N(E_t, Z_t, S_t)) + s_t^*(G(L_{2t}, R_t) - X_t)$.[16] (17)

Define $n_t^* \equiv w_t^*/p_t^*$, $u_t^* \equiv q_t^*/p_t^*$, $v_t^* \equiv r_t^*/p_t^*$, $z_t^* \equiv s_t^*/p_t^*$, and $\wp_t^* \equiv \hat{\wp}_t^*/p_t^*$. Write

$I_t^K \equiv dK_t/dt$; $I_t^Z \equiv u_t^*dZ_t/dt$; and $I_t^S \equiv v_t^*dS_t/dt$, (18)

[16] This is the linearized Hamiltonian of equation (9). Note that an alternative way of arriving at expression (17) would be to "linearize" the Hamiltonian of $\mathbf{P}_1$ (H_t in expression (9)) at each t by making use of the optimum marginal social rate of substitution between C_t and L_t (Dasgupta and Mäler, 1991).

vhich represent the value of investment in the three types of capital assets, respectively. Ve can then re-express (17) in terms of consumption numeraire as:

$$\wp_t^* = C_t - n_t^* L_t + I_t^K + I_t^Z + I_t^S + z_t^*(G(L_{2t}, R_t) - X_t). \qquad (19)$$

$\wp_t^*$ is NNP at t, evaluated at prices supporting the optimum economic rogramme. When applied to $\mathbf{P_3}$, the Pontryagin Maximum Principle states that the ontrol variables (C_t, L_{1t}, L_{2t}, E_t, R_t, X_t) should be chosen, without restriction, so as to naximize $\wp_t^*$ at each t (≥ 0). This suggests that there is yet another method available for roject appraisal, where perturbations to an economic programme are evaluated at each ate separately in terms of their contributions to current NNP. We look into this.

Observe that any intertemporal project $(\Delta C_t, \Delta L_{1t}, \Delta L_{2t}, \Delta E_t, \Delta R_t, \Delta X_t)_0^\infty$ can be ecomposed into a continuum of elementary projects, where an elementary project at is an instantaneous perturbation $(\Delta C_t, \Delta L_{1t}, \Delta L_{2t}, \Delta E_t, \Delta R_t, \Delta X_t)$.[17] Since any project can e regarded as a composite of elementary projects, we can decompose the evaluation f a project into separate evaluations of the elementary projects that comprise it. Consider therefore date t. Stocks of various types of capital will have been inherited rom the past. They are to be taken as given. We now wish to choose the controls at t. 'he Pontryagin Maximum Principle implies that, if an elementary project at t increases decreases) $\wp_t^*$ in (19), it should be accepted (rejected). Along an optimum economic rogramme the marginal elementary project would contribute nothing to $\wp_t^*$; which is nother way of saying that, at each t, elementary projects should be so chosen as to naximize $\wp_t^*$.[18]

. Project Choice as Reform in a Non-Optimizing Economy

In the previous three sections we developed, successively, three distinct methods or identifying optimum economic programmes in convex economies. The analysis was onducted in the context of a first-best. But it is obvious that the methods can as well be

[17] For a rigorous account of the use of what we are calling elementary projects in social cost-benefit nalysis of investment projects, see Seierstad and Sydsaeter (1987: 221-2).

[18] The statement in the penultimate sentence of the paragraph is informal. Strictly, we should think f the perturbation in question to be applied over a short interval of time. A limiting argument can hen be applied under the assumptions we have made about the various functions. On this see, for xample, Seierstad and Sydsaeter (1987). Formally, one can show that if a small perturbation at t to an ptimal economic programme increases (decreases) what is called the "value function" (see Section 7 elow), then it increases (decreases) $\wp_t^*$.

used to locate second- or third-best programmes.[19]

This said, a theory of project evaluation capable of speaking only to optimizing governments would be of very limited interest. For it to be of practical use, a theory should be able to cover economies where governments not only do not optimize, but perhaps cannot even ensure that economic programmes resulting from its policies are intertemporally efficient. Consider then such an economy. To have a problem to discuss, imagine that even though the government does not optimize, it can bring about small changes to the economy by altering its existing, sub-optimal policies in minor ways. The change in question may, for example, consist of small adjustments to the prevailing structure of taxes, or it could be minor alterations to the existing set of property rights, or whatever. We call any such change a policy reform. The question is, how should policy reforms be evaluated? We turn to this.[20]

For concreteness, consider an economy facing the technological constraints in equations (1)-(2) and (4)-(5). In addition, it faces institutional constraints (sometime called transaction and information constraints) which we will formalize presently. The initial capital stocks (K_0, Z_0, S_0) are given and known. Assume now that the institutional structure of the economy (by which we mean market structures, the structure of property-rights, tax rates, and so forth) is given and known. If in addition we knew the behavioural characteristics of the various agencies in the economy (i.e. those of households, firms, the government, and so on) it would be possible to make a forecast of the economy, by which we mean a forecast of the economic programme $(C_t, L_{1t}, L_{2t}, X_t, R_t, E_t, K_t, Z_t, S_t)_0^\infty$ that would be expected to unfold. We will call this relationship a resource allocation mechanism. So, a resource allocation mechanism is a mapping from initial capital stocks (K_0, Z_0, S_0) into the set of economic programmes $(C_t, L_{1t}, L_{2t}, X_t, R_t, E_t, K_t, Z_t, S_t)_0^\infty$ satisfying equations (1)-(2) and (4)-(5).

We now formalise this. Write

$$\Omega_\tau \equiv (K_\tau, Z_\tau, S_\tau), \text{ and} \tag{20a}$$

[19] In second- or third-best problems the set of constraints would be greater.

[20] Social cost-benefit rules for a reformist government are developed in Dasgupta, Marglin and Sen (1972), Ahmad and Stern (1990), and Dreze and Stern (1990). If the policy change being envisaged were large (e.g. a large, irreversible change in tax rates) consumer and producer surpluses would need to be estimated: a linear index would not suffice.

$$(\xi_t)_\tau^\infty \equiv (C_t, L_{1t}, L_{2t}, X_t, R_t, E_t, K_t, Z_t, S_t)_\tau^\infty, \text{ for } \tau \geq 0. \qquad (20b)$$

Next let $\{\Omega_\tau\}$ denote the set of possible Ω_τs and $\{\tau, (\xi_t)_\tau^\infty\}$ the set of pairs of dates, τ, and economic programmes from τ to the indefinite future. A resource allocation mechanism, α, can then be expressed as a mapping

$$\alpha: \{\tau, \Omega_\tau\} \rightarrow \{(\xi_t)_\tau^\infty\}. \qquad (21)$$

α would depend on calender time if knowledge, or population, or terms of trade were to change autonomously over time.[21] If they were not to display any exogenous shift, α would be independent of τ. For reasons to be discussed in Section 9, we will pay particular attention to the case where α is autonomous. So let us now assume that α does not depend on calendar time.

It bears re-emphasis that we do <u>not</u> assume α to sustain an optimum economic programme, nor even do we assume that it sustains an efficient programme. The analysis that follows is valid even if α is riddled with economic distortions and inequities.

To make the dependence of the economic forecast on α explicit, let $(C_t(\alpha), L_{1t}(\alpha), L_{2t}(\alpha), X_t(\alpha), R_t(\alpha), E_t(\alpha), K_t(\alpha), Z_t(\alpha), S_t(\alpha))_0^\infty$ denote the forecast at $t = 0$. Consider date t (≥ 0). For the remainder of this section, we assume that $\delta > 0$. Now use (20a,b) and (21) to define

$$V_t(\alpha, \Omega_t) \equiv \int_t^\infty e^{-\delta(\tau - t)} U(C_\tau(\alpha), L_{1\tau}(\alpha) + L_{2\tau}(\alpha)) d\tau. \qquad (22)$$

The right-hand-side (RHS) of equation (22) is social well-being at t. In optimum programming V_t is called the <u>value function</u> at t (Bellman, 1957).[22]

The crucial assumption we now make is that V_t is differentiable in each of the three components of Ω. We apologise for imposing a technical condition on something that is endogenous, but space forbids we explore here the various conditions on an economy's fundamentals (for example, behavioural characteristics of the various agencies and properties of the various production functions and ecological processes)

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[21] There are exceptions to this statement in extreme cases, namely, closed economies where production is subject to constant-returns-to-scale, population changes exponentially, technical change is Harrod-neutral, there are no environmental resources, and social well-being is based on classical utilitarianism (Mirrlees, 1967). In such an economy α would be a mapping from the set of capital assets per efficiency unit of labour into the set of economic programmes, where the programmes are expressed in efficiency units of labour.

[22] In all this, we take it that V_t is well defined. The assumption that $\delta > 0$ is crucial for this.

which would guarantee a differentiable value function.

It is not easy to judge if differentiability of V_t is a strong assumption. What is required is that α itself be a differentiable mapping. To be sure, this would rule out threshold effects in ecological processes and other such discontinuities in the neighbourhood of the economic forecast, but it would not appear to rule out much else. For example, differentiability of V_t almost everywhere would be consistent with smooth non-convexities in ecological processes and production possibilities. Consequently, the analysis that follows is valid for a considerably more general set of environments than we have considered so far in this paper.

We will now re-trace the arguments of the previous section, but with a crucial difference: we will not work with accounting prices which support optimum programmes, we will instead work with prices that reflect social scarcity values along the economic forecast. Thus, define

$$p_t(\alpha) \equiv \partial V_t(\alpha)/\partial K_t; \; q_t(\alpha) \equiv \partial V_t(\alpha)/\partial Z_t; \text{ and } r_t(\alpha) \equiv \partial V_t(\alpha)/\partial S_t. \quad (23)$$

For want of a better terminology, we call them local prices. They measure the social scarcity of the economy's capital assets.

How might they be estimated? If households are not rationed in any market and externalities are negligible, then market prices would be the right estimates. However, when households are rationed, or externalities are rampant, estimating local prices involves more complicated work. For example, in the presence of externalities market prices need to be augmented by the external effects. This involves extending the notion of commodities to "named goods" (Section 10). If households are rationed, one has to estimate "willingness-to-pay". There are known techniques for estimating local prices in each of these circumstances.

It will now be argued that NNP, computed on the basis of local prices, can be used to evaluate policy reform.

Recall that α is being assumed not to depend on calendar time. We may think of a policy reform as a perturbation to α over the short interval $[0, \tau]$. The perturbation is expressed as $\Delta\alpha$. During $[0, \tau]$ the resource allocation mechanism is denoted as $(\alpha + \Delta\alpha)$. From τ onwards the economy is assumed to be governed by α again. Note now that, if the policy reform were undertaken, the economic variables during $[0, \tau]$ would be slightly perturbed ($(C_t + \Delta C_t)$ rather than C_t, and so forth). Note too that at τ stocks of capital assets would be slightly different from what they would have been had the reform not been undertaken.[23] Let the stocks at τ be $(\Omega_\tau + \Delta\Omega_\tau)$ as a consequence of the reform. As before, write $L = L_1 + L_2$.

The change in V_0 arising from the reform can be expressed as

$$\Delta V_0 = V_0(\alpha+\Delta\alpha, \Omega_0) - V_0(\alpha, \Omega_0)$$

$$= {}_0\int^\tau e^{-\delta t}[U(C(\alpha+\Delta\alpha), L(\alpha+\Delta\alpha)) - U(C(\alpha), L(\alpha))]dt + e^{-\delta\tau}[V_\tau(\alpha, \Omega_\tau+\Delta\Omega_\tau) - V_\tau(\alpha, \Omega_\tau)]. \quad (24)$$

On using equation (23) and the accumulation equations in (6), equation (24) can be expressed as:

$$\Delta V_0 = \tau e^{-\delta\tau}(U_C\Delta C + U_L\Delta L) + e^{-\delta\tau}(V_K\Delta K_\tau + V_Z\Delta Z_\tau + V_S\Delta S_\tau) + \varepsilon(\tau), \quad (25)$$

where $\varepsilon(\tau)$ is an error term with the property that $\varepsilon(\tau)/\tau \rightarrow 0$ as $\tau \rightarrow 0$.[24]

Consider now the perturbations to the capital assets at τ as a consequence of the reform. Observe that

$$\Delta K_\tau = {}_0\int^\tau \Delta(dK_t/dt)dt = \tau\Delta(dK_t/dt)_{t=0} + \gamma(\tau),$$

where $\gamma(\tau)$ is an error term with the property that $\gamma(\tau)/\tau \rightarrow 0$ as $\tau \rightarrow 0$. Perturbations to Z_τ and S_τ can be estimated in a similar manner. Therefore, equation (25) can be re-written as

$$\Delta V_0/\tau = e^{-\delta\tau}(U_C\Delta C + U_L\Delta L + p_0\Delta(dK_t/dt)_{t=0} + q_0\Delta(dZ_t/dt)_{t=0} + r_0\Delta(dS_t/dt)_{t=0}) + \theta(\tau), \quad (26)$$

where $\theta(\tau)$ is an error term with the property that $\theta(\tau) \rightarrow 0$ as $\tau \rightarrow 0$. The left-hand-side (LHS) of (26) is the change in social well-being per unit of time during $[0, \tau]$. As we are interested in small perturbations, we let $\tau \rightarrow 0$. The LHS of equation (26) then becomes the change in social well-being occasioned by the reform, and the right-hand-side (RHS) tends in the limit to:

$$U_C\Delta C_0 + U_L\Delta L_0 + p_0\Delta(dK_t/dt)_{t=0} + q_0\Delta(dZ_t/dt)_{t=0} + r_0\Delta(dS_t/dt)_{t=0}. \quad (27)$$

Choose consumption as numeraire and write

$n_0 = -U_L/U_C$; $m_0 = p_0/U_C$; $u_0 = q_0/U_C$; and $v_0 = r_0/U_C$.[25]

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[23] It is here that we are invoking the assumption that α is a differentiable mapping.

[24] U_C and U_L are evaluated at t=0. V_K is the partial derivative of V with respect to K at t=0, and so forth. In what follows we do not write the dependence of the economic forecast on α. This saves on notation.

[25] Since the economic programme sustained by α is not a first-best, m_0 is typically not equal to 1.

On dividing expression (27) by U_C, we obtain

$$\Delta C_0 - n_0\Delta L_0 + m_0\Delta(dK_t/dt)_{t=0} + u_0\Delta(dZ_t/dt)_{t=0} + v_0\Delta(dS_t/dt)_{t=0}. \quad (28)$$

If expression (28) is positive, the reform increases social well-being, so it is desirable; if it is negative, the reform decreases social well-being, so it is undesirable. Define

$$\hat{\wp}_t \equiv U_C C_t - U_L L_t + p_t dK_t/dt + q_t dZ_t/dt + r_t dS_t/dt, \quad (29a)$$

and thereby

$$\wp_t \equiv C_t - n_t L_t + m_t dK_t/dt + u_t dZ_t/dt + v_t dS_t/dt. \quad (29b)$$

If the RHSs of equations (29a,b) have a familiar ring to them, it is because they represent NNP at t (in utility and consumption numeraires, respectively), measured in local prices. Observe now that expression (28) is the change in NNP at t = 0 occasioned by the acceptance of the elementary project at t = 0. We conclude that NNP, measured in local prices, can be used to evaluate policy reforms. It is the main result of this section.

Note that autonomous changes in α would not affect our result. Being exogenous, such changes are unaffected by policy reform, so they are irrelevant for social cost-benefit analysis.

What are the dynamics of local prices? To study this, note that the current-value Hamiltonian associated with α can be expressed as

$$H_t = U(C_t, L_t) + p_t(F(K_t, L_{1t}, X_t) - C_t - E_t) + q_t E_t + r_t(M(S_t) - R_t + N(E_t, Z_t, S_t)). \quad (30)$$

Recall equation (22), which we re-write here:

$$V_t(\alpha, \Omega_t) \equiv \int_t^{\infty} e^{-\delta(\tau-t)} U(C_\tau, L_{1\tau}+L_{2\tau})d\tau. \quad (31)$$

V_t is social well-being at t. Differentiating V_t with respect to t we obtain

$$dV_t/dt = \delta V_t - U(C_t, L_t). \quad (32)$$

But $V_t = V_t(\alpha, \Omega_t)$. Using (23), we conclude also that

$$dV_t/dt = p_t dK_t/dt + q_t dZ_t/dt + r_t dS_t/dt + \partial V_t/\partial t. \quad (33)$$

Now combine equations (30), (32) and (33) to obtain

$$H_t = \delta V_t - \partial V_t/\partial t. \quad (34)$$

As α has been assumed not to depend on calendar time, V_t does not depend on it either. So equation (34) reduces to

$$H_t = \delta V_t. \quad (35)$$

Equation (35) is fundamental in intertemporal welfare economics. It says that the Hamiltonian equals the return on social well-being.

We can use equations (23) and (35) to conclude that

$dp_t/dt = - \partial H_t/\partial K_t + \delta p_t$; $dq_t/dt = - \partial H_t/\partial Z_t + \delta q_t$; and $dr_t/dt = - \partial H_t/\partial S_t + \delta r_t$. (36)

Equations (23) and (36) mirror equations (10) and (8), respectively. We therefore have a complete characterization of the welfare economics of policy reform, one that parallels the welfare economics of optimizing economies.

How is policy reform related to optimum planning? Consider an indefinite sequence of policy reforms at every t, each of which increases NNP at t, where NNP is computed at the prevailing local prices. We take it that the entire sequence is conducted in a counter-factual manner; that is, as a tatônnement. Such an adjustment process is called a gradient process (it is also called the "hill-climbing method"). Provided the economy has a strong convex structure, such a sequence of project selections (i.e. policy reforms) leads the decision-maker to identify the optimum economic programme. Put another way, if the economy is sufficiently convex, the gradient process converges to the optimum.[26]

8. Comparisons of Social Well-Being Across Time and Space

We have shown that NNP, appropriately defined, can be used as a gauge for evaluating small elementary projects. But the practice has been different. It is GNP (per head) that has routinely been in use, and to a different end: it has been used to make welfare comparisons across time, across groups, and across countries.[27] Admittedly, GNP per head has also been criticized routinely for its narrowness of scope (see the annual publications of the United Nations Development Programme, for example, UNDP, 1994), but it has become customary to augment the index by such current measures of well-being as life expectancy at birth, the under-5 survival rate, the infant survival rate, and so forth. Being current measures, they capture nothing of what the future portends.

The bulk of the recent literature on green NNP has been perspicacious on this matter. It has focussed on sustainable economic development, and in so doing has explored the thought that NNP is the constant-consumption equivalent of optimum

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[26] See Arrow and Hurwicz (1958) for a formal demonstration in the context of a finite-dimensional economy.

[27] See, for example, the annual World Development Report of the World Bank.

economic programmes.[28] In this section we scrutinize these ideas, demonstrate that NNP is of no practical use in intertemporal and inter-regional comparisons of social well-being, and then develop indices that can be used for such purposes. We will note that comparisons of social well-being, be they across time or space, involve the use of measures of wealth.

8a. Intertemporal Comparisons of Well-Being and the Concept of Sustainability

World Commission (1987) defined "sustainable development" as an economic programme in which, loosely speaking, the well-being of future generations is not jeopardized. There are a number of possible interpretations of this (see Pezzey, 1992). Consider the following:

(i) An economic programme is sustainable if $dU_t/dt \geq 0$, where $U_0 \geq \lim U_t$ as $t \rightarrow -0$.

(ii) An economic programme is sustainable if $dU_t/dt \geq 0$.

(iii) An economic programme is sustainable if $dV_t/dt \geq 0$, where $V_t(\alpha, \Omega_t) \equiv \int_t^{\infty} e^{-\delta(\tau-t)} U(C_\tau, L_{1\tau}+L_{2\tau}) d\tau$.

It is clear that (i) lacks ethical foundation. For example, it may be desirable to reduce U in the short run in order to accumulate assets in order that the flow of U is still higher in the future. In this sense (ii) offers greater flexibility in ethical reasoning: it permits initial sacrifices in current well-being U (a burden assumed by the generation engaged in the reasoning), but requires that future generations should not have to experience a decline in their own well-being.

In contrast, the focus of (iii) is social well-being, V. The criterion permits initial sacrifices in V, but requires that social well-being should never decline in the future. Note that, while (ii) implies (iii), (iii) does not imply (ii). In short, (iii) is more general. In what follows, we will adopt (iii) as our notion of sustainable development and develop criteria for judging if any given economic programme is sustainable.

Consider the resource allocation mechanism α. The mechanism allows one to make an economic forecast. Assume that $\partial V_t/\partial t = 0$ and $\delta > 0$. Differentiating both sides

[28] On the latter see, for example, Solow (1986, 1992), Hartwick (1990, 1994), Asheim (1994, 1997), Aronsson, Johansson and Löfgren (1997), and Weitzman (1998). The inspiration has been Weitzman (1976), although in that paper Weitzman did not include environmental resources in his model. Constant-equivalent consumption is sometimes called "sustainable-equivalent consumption".

of equation (35) with respect to time, we have

$$dH_t/dt = \delta dV_t/dt. \tag{37}$$

Use (29b) to define

$$I_t^K \equiv p_t dK_t/dt;\ I_t^Z \equiv q_t dZ_t/dt;\ \text{and}\ I_t^S \equiv r_t dS_t/dt, \tag{38}$$

which are net investments in the three types of capital assets, respectively, expressed in utility numeraire. We may then define aggregate net investment as,

$$I_t = I_t^K + I_t^Z + I_t^S. \tag{39}$$

It follows from equations (30), (33) and (39) that

$$U_C dC/dt + U_L dL/dt + dI_t/dt = \delta I_t. \tag{40}$$

Equation (40) enables us to obtain two alternative indicators of sustainable development.

The first is obtained from the RHS of equation (40). It says that α results in sustainable development if, and only if, under α net investment in the economy's capital assets is non-negative at each date.[29] The result has intuitive appeal. It says that social well-being is higher today than it was yesterday if the economy enjoys greater wealth today. Here, an economy's "wealth" is interpreted as the accounting value of all its capital assets. Samuelson (1961) argued in connection with national income accounting that welfare comparisons should deal with "wealth-like" entities. Our result formalizes that insight.

Note, however, that what we have obtained is an equivalence result; the result cannot on its own tell us if sustainable development is feasible. Whether the economy is indeed capable of growing wealthier indefinitely depends, among other things, on the extent to which different assets are substitutable in production.[30]

A second way of stating this equivalence result can be obtained directly from the LHS of equation (40). It says that social well-being increases (decreases) over a short interval of time if, during the interval, the value of the change in the flow of consumption services plus the change in the value of investment is positive (negative).

[29] This result, shown to be a property of optimum economic programmes, originated in Solow (1974) and Hartwick (1977), who determined the investment rule that would sustain the maximum constant utility stream. Pearce and Atkinson (1993) suggested the use of the rule we have obtained in the text for practical purposes, but offered no proof that the suggestion is valid. Serageldin (1995) has reported empirical work done at the World Bank on the use of the rule.

[30] For an account of this, see Dasgupta and Heal (1979, ch. 7).

Notice that this would not amount to NNP comparisons across time unless the economy were stationary (i.e. $dp_t/dt, dq_t/dt, dr_t/dt = 0$). We conclude that intertemporal NNP comparisons tell us nothing about changes in social well-being excepting under empirically uninteresting circumstances.

8b. The Hamiltonian as Constant-Equivalent Utility

In the theoretical literature, however, NNP is today widely used toward a diferent end: as an index of "constant-equivalent consumption". Let us look into this interpretation.

In what follows, we will continue to assume that $\partial V_t/\partial t = 0$. Since $\delta[{}_t\int^{\infty} e^{-\delta(\tau-t)}d\tau] = 1$, equation (35) can be written as

$$H_t = H_t\{\delta[{}_t\int^{\infty} e^{-\delta(\tau-t)}d\tau]\} = \delta[{}_t\int^{\infty} e^{-\delta(\tau-t)}H_t d\tau] = \delta V_t,$$

from which we have

$$H_t[{}_t\int^{\infty} e^{-\delta(\tau-t)}d\tau] = {}_t\int^{\infty} e^{-\delta(\tau-t)}H_t d\tau = V_t \equiv {}_t\int^{\infty} e^{-\delta(\tau-t)}U(C_\tau, L_\tau)d\tau. \quad (41)$$

Equation (41) says that the Hamiltonian at each date is equal to the constant-equivalent stream of U starting from that date. This result was proved for optimum economic programmes by Weitzman (1976), who restricted his analysis to linear utility functions, specifically that $U(C,L) = C$. Since in this case the Hamiltonian <u>is</u> NNP, Weitzman interpreted NNP as the constant-equivalent stream of consumption. This interpretation is today in wide usage.

But a linear utility function is ethically flawed: it is insensitive to distributional issues. Since it may seem that Weitzman's interpretation of NNP does not extend to non-linear utility functions, one might think that his equivalence result tells us nothing about the meaning of NNP. However, in an interesting paper Weitzman (1998) has argued that, by suitably calibrating U, the interpretation of NNP as a constant-equivalent stream of utility can be restored even when U is strictly concave.

To see how this can be done, recall that the ethical ordering of economic programmes represented by V_t is invariant under positive affine transformations of U. Thus, if U is the utility function, one could as well use $(aU + b)$, where a and b are constants and $a > 0$. This means that there are two degrees of freedom when U is calibrated. For simplicity of exposition, consider the special case where utility depends solely on consumption; that is, $U = U(C)$, with $U'(C) > 0$ and $U''(C) < 0$. Let α be the resource allocation mechanism and C_0 the initial rate of aggregate consumption resulting from it. Choose a and b so that $aU'(C_0) = 1$ and $(aU(C_0) + b) = C_0$. The idea, therefore, is to so calibrate U that initial utility equals initial consumption (expressed in utility numeraire) and initial marginal utility equals unity. This makes the Hamiltonian at $t = 0$ equal to NNP at $t = 0$. It follows from equation (41) that at $t = 0$ NNP can indeed be interpreted as the constant-equivalent utility stream associated with α.

So far so good. The irony is that the very feature of U which makes Weitzman's particular calibration possible is also his result's undoing. For note that a high or low value of U in itself carries no significance (a and b are freely chooseable, remember). So to be told that today's NNP, expressed in utility numeraire, is high (or low) because the constant-equivalent utility is high (or low), in itself has no meaning. What would have meaning would be comparisons of U; across time, or space, or groups of people, or whatever. It would certainly be informative if we could be told, say, that because NNP is expected to be greater tomorrow than it is today, tomorrow's constant-equivalent utility can be expected to be greater than what it is today. If we were to be told that, we would be able to infer that social well-being tomorrow should be expected to be higher than what it is today. Unfortunately, we cannot be told that. Since we have already proved that NNP comparisons across time are not equivalent to intertemporal comparisons of social well-being, we should clearly not expect Weitzman's calibration to permit us to make them be equivalent. Nevertheless, let us confirm by a direct argument that the calibration does not work.

The point is that once U has been calibrated at $t = 0$, it must not be recalibrated ever again. For to do so would be to alter the underlying ethical ordering of economic programmes, which would render intertemporal comparisons of social well-being meaningless. But unless U were to be constant over time, it would have to be recalibrated continuously if Weitzman's interpretation of NNP were to be preserved at each date. This requirement is fatal for his interpretation.

It was noted earlier that the Hamiltonian is not a useful index in national accounts. Nevertheless, suppose we have obtained an estimate of it. What would it reveal?

Consider the case where $U_t < H_t$. We would then infer that $dV_t/dt > 0$. But from equation (33) we know that $dV_t/dt > 0$ if and only if net investment at t is positive. So there is no need to make a comparison of U_t and H_t to check if $dV_t/dt > 0$. Consequently there is no need to estimate the value of the Hamiltonian. It is worth re-emphasizing that NNP and its component, net investment, are linear functions of economic variables, while the Hamiltonian is not. This is what makes NNP a far more appealing index in social cost-benefit analysis. It is also why, when α does not depend on calendar time, net investment is the more useful measure of sustainable development and also the superior index for making comparisons of social well-being across time.

Thus far theory. In practice NNP could still be a reasonable guide for making intertemporal comparisons of social well-being if the error committed in taking a linear approximation to the Hamiltonian were small. The error-term would depend on the elasticities of demand for various consumption goods and services, and would be small if the various price elasticities of demand were large (Mäler, 1998). So far, though, there has been no work done to determine how the error-term behaves as the parameters of plausible economic models are changed.

But given that exact criteria for making intertemporal comparisons of social well-being exist (equation (40), it is legitimate to ask why one should wish to resort to approximations. Our analysis has pointed to two alternative indicators, both of which are exact. Improving ways of estimating either, or both, should be high on the agenda of applied research.

8c. NNP and Inter-Country Comparisons of Well-Being

Cross-country comparisons of GNP per head are today a commonplace. They are also widely acknowledged to be of little relevance if interpreted as comparisons of social well-being. The question is: what index should be used instead? An analysis identical to the one concerning intertemporal comparisons of social well-being (Section 8a) helps provide an answer.

It is simplest to consider a continuum of closed economies, parametrized by x (a scalar). We may interpret differences among economies in terms of differences in initial endowments, or behavioural characteristics, or the resource allocation mechanisms guiding them. But in order to make meaningful comparisons of social well-being, the same value-function must be ascribed to all countries.

Consider a date when the cross-country comparisons are to be made. To keep the notation simple, we drop the time subscript. Let H_x be the Hamiltonian in country x and V_x the value function there. Recall equation (35). In the present case it reads as $H_x = \delta V_x$.

An argument identical to the one establishing equation (40) then yields

$$\delta[p_x dK_x/dx + q_x dZ_x/dx + r_x dS_x/dx + \partial V_x/\partial x] = U_C dC_x/dx + U_L dL_x/dx + d(I_x)/dx + \partial H_x/\partial x, \quad (42)$$

where I_x is net aggregate investment in x.

For ready tractability, the interesting special case to consider is $\partial V_x/\partial x = \partial H_x/\partial x = 0$.[31] The LHS of equation (42) then says that social well-being in a country is higher (lower) than in any of its immediate neighbours if in the aggregate it has greater (less) wealth. This formalizes Samuelson's suggestion that in making welfare comparisons across countries, one should estimate their wealths.

An equivalent indicator can be obtained from the RHS of equation (42). It says that social well-being in a country is higher (lower) than in any of its immediate neighbours if the value of the difference in the flow of consumption services between them plus the difference in the value of aggregate net investment between them is positive (negative). But this would not amount to NNP comparisons across countries unless local prices were the same (i.e. dp_x/dx, dq_x/dx, $dr_x/dx = 0$). We conclude that cross-country comparisons of NNP tell us nothing about differences in social well-being.

Equation (42) is exact, but the pair of (linear) indicators we have obtained serve their purpose accurately only when $\partial V_x/\partial x = 0$. This, we believe, is a very strong condition. If, as we suspect is the case, $\partial V_x/\partial x$ is not even approximately zero, there are no linear indices to be had. This is one of the morals of our analysis.

9. Technological Change and Growth Accounting

How should NNP be computed in the presence of technical change? Note first that resource augmentation, N, in equation (5) could itself be regarded as a form of technical progress. This said, it must also be granted that the growth and decay of knowledge involve wider considerations. For example, it has been customary in the economics literature to regard technical progress as shifts in production functions. In what follows we will explore this route by introducing technical progress in the production of the final good in the model of Section 3.

We need to extend our notation. Denote by E_{1t} and E_{2t} expenditures on resource

[31] The condition requires that the same resource allocation mechanism prevails in all countries. The condition is strong.

augmentation and on generalized research and development (R & D), respectively. Now define Z_{1t} and Z_{2t} by the equations

$$dZ_{1t}/dt = E_{1t}, \tag{43}$$

and $$dZ_{2t}/dt = E_{2t}. \tag{44}$$

Z_1 and Z_2 can be thought of as two types of knowledge. Denote the resource augmentation function as $N(E_1, Z_1, S)$ and imagine that output of the produced consumption good at t can be expressed as

$$Y_t = e^{\lambda t}Q(Z_{2t})F(K_t, L_{1t}, X_t), \tag{45}$$

where $\lambda \geq 0$ and $Q'(Z_2) \geq 0$. Technical progress in the production of the final good appears here as the term $e^{\lambda t}Q(Z_{2t})$. It combines exogenous factors (λ) with endogenous ones (Z_2).

Consider an optimizing economy. If we were to replace $F(K_t, L_{1t}, X_t)$ in expression (18) by the production function in (45) and the resource augmentation function $N(E, Z, S)$ in (18) by $N(E_1, Z_1, S)$, we would have an expression for NNP in an optimizing economy capable of enjoying technical progress. To see this, let consumption be the numeraire, u_1^* and u_2^* the accounting prices of Z_1 and Z_2, respectively, and the remaining accounting prices denoted as in Section 6. Retracing the arguments in Section 6, it is a simple matter to conclude that NNP in an optimizing economy is:

$$\rho_t^* = C_t - n_t^*L_t + dK_t/dt + u_{1t}^*dZ_{1t}/dt + u_{2t}^*dZ_{2t}/dt + v_t^*dS_t/dt + z_t^*(G(L_{2t}, R_t) - X_t). \tag{46}$$

In a similar manner, one can confirm that the discussion in Section 7 on the evaluation of policy reform remains unchanged in the presence of technical change.

The question remains: what factors contribute to changes in GNP over time? To see what the answer is in the model economy of Section 3, recall from inequality (1) that along an efficient economic programme $X_t = G(L_{2t}, R_t)$. Using this in the expression for GNP in (45), we have:

$$Y_t = e^{\lambda t}Q(Z_{2t})F(K_t, L_{1t}, G(L_{2t}, R_t)). \tag{47}$$

Differentiating both sides of equation (47) with respect to t, re-arranging terms, and dropping the time subscript from variables for the sake of notational simplicity, we obtain the growth accounting identity in our model economy as:

$$(dY/dt)/Y \equiv \lambda + (Q'(Z_2)dZ_2/dt)/H(Z_2) + (F_K dK/dt)/F + (F_L dL_1/dt)/F + F_X[(G_L dL_2/dt) + (G_R dR/dt)]/F. \tag{48}$$

The sum of the first two terms on the RHS of equation (48) measures the percentage rate of change in "total factor productivity", while the remaining terms together represent the contributions of changes in the "factors of production" to the percentage rate of change in GNP. Since λ is an exogenous factor, it is unexplained within the model. For this reason it is called the residual.

In a famous article, Solow (1957) used a reduced-form of the production function in (47) to estimate the contribution of changes in the factors of production to growth of non-farm GNP per "man-hour" in the US economy over the period 1909-1949, and discovered that it was a mere 12 percent of the average annual rate of growth.[32] In other words, 88 percent of the growth was attributable to the residual. (Solow's estimate of λ was 1.5 percent per year.) A significant empirical literature since then has shown that when K is better measured (e.g. by accounting for changes in the utilization of capacity and changes in what is embodied in capital) and when account is taken of human-capital formation, the residual is small for the non-farm sector in the US economy.[33]

This is congenial to intuition. It is hard to believe that serendipity, unbacked by R&D effort and investment in physical capital (learning by doing), can be a continual source of productivity growth. A positive value of λ would imply that the economy is guaranteed a "free lunch" forever. To be sure, such an assumption would guarantee that growth in aggregate consumption was sustainable. In fact, that would be its attraction: it would enable us to assume away problems of environmental and resource scarcities. But there are no theoretical or empirical grounds for presuming that it is a reasonable assumption. At this point in our understanding of the process by which discoveries are made, it makes greater sense to set $\lambda = 0$ in (47). But this would imply that $\partial V_t/\partial t = 0$.

Productivity growth in equation (48) is productivity growth in GNP. It has often been suggested that we should instead be interested in productivity growth in NNP, as defined in equation (46). For example, in their important early work on Indonesia, Repetto et al. (1989) showed that if one were to include deforestation, soil erosion, and the depreciation of oil reserves in the country's national accounts, Indonesia's rate of

[32] Solow assumed in particular that $Q'(Z_2) = 0$.

[33] Jorgenson (1995) contains a masterly account of this complex literature.

growth in NNP during the 1980s would be half the estimated growth rate of her GNP. And there are other environmental and natural resources that Repetto et al. did not consider.

In Section 8a it was shown that NNP comparisons across time tell us nothing about changes in social well-being. It was also shown that we should ask instead if, in the aggregate, net investment is positive. It is possible for an economy's GNP (per head) to increase over a period of time even while, in the aggregate, net investment (per head) is negative. We know of no evidence that in recent years this has not been experienced in a number of countries.[34]

10. Extensions

In developing the concept of NNP we have made use of a series of models of increasing generality. However, of necessity even the most general of the models (Sections 6-9) had important features missing. We comment on a few of them. Readers can easily fill in the details.

(a) The analysis in Sections 4-6 can be extended to second- or third-best economies, and so, to economies that harbour unemployment and other forms of resource allocation failure. Section 7 on the evaluation of policy reforms showed that the analysis is widely applicable. The hard work lies in estimating the appropriate accounting prices, not in developing the theory.

(b) Problems associated with intragenerational distribution have been ignored. However, it is theoretically a simple matter to include them. The way to do it would be to enlarge the set of commodities so as to distinguish a good consumed or supplied by one person from that same good consumed or supplied by another person. This means, for example, that a piece of clothing worn by a poor person should be regarded as a different commodity from that same type of clothing worn by a rich person.[35] Such commodities are called named goods (Hahn, 1971). Accounting prices of named goods would typically depend on the "names" attached to them. With this re-interpretation of goods and services, the results we have obtained continue to hold.

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[34] Serageldin (1995) contains a report on the beginnings of this research programme.

[35] We are assuming in this example that income or wealth mal-distribution is the cause of concern. Dasgupta, Marglin and Sen (1972) suggested the use of income distributional weights as a rough-and-ready way to capture such concern.

(c) Environmental externalities can be incorporated by a device identical to (b). To describe who is affected, in which manner, and by whose actions involves the use of named goods and services. It follows that accounting prices would be "named", so as to distinguish private costs from social costs and private benefits from social benefits. Indeed, Pigouvian taxes and subsidies on externalities can be computed on the basis of named accounting prices (Dasgupta and Heal, 1979, ch. 3; Mäler, 1991).

(d) Uncertainty has also been avoided here. Assume then that social well-being at date t=0 is the expected value of the present discounted flow of "utility". The natural move would be to make use of the idea of contingent goods, and therefore of contingent accounting prices. Our analysis would then go through.

(e) The discussion has been restricted to closed economies. However, the analysis can be extended to an economy that trades with the rest of the world. Dasgupta, Kriström and Mäler (1995) and Sefton and Weale (1996) contain an account of this.

(f) Human capital has been absent from our discussion. Analytically it is a simple matter to include it. Since human capital can be thought of as another form of capital, net investment in it would be included in NNP (see Dasgupta, Kriström and Mäler, 1995, for a formulation).

(g) The models studied here have not included demographic change. It is customary in growth accounting to regard changes in population over time as exogenously given. Such an assumption has only convenience to commend it. In many societies parents regard children as both an end in themselves and a means to other things (e.g. income security). Population should be regarded as a stock whose movements over time are, at least in part, endogenously determined.

This said, current understanding of the determinants of, for example, fertility behaviour is weak. Moreover, serious problems arise when one comes to construct intergenerational welfare economics in such a world. There is no received theory. Population ethics is an underdeveloped field of inquiry. For the moment it would seem reasonable to conduct such analyses as we have conducted conditional on specified demographic movements. This has been our approach here.[36]

11. Conclusions: What Should NNP Include?

[36] See Dasgupta (1998) for a discussion of some of the more transparent problems that arise when one thinks about the concept of optimum population.

This paper has been about NNP. We have been concerned with what NNP means, what it should include, what it offers us and, therefore, why we may be interested in it. We began by demonstrating that there are a number of analytically-equivalent methods available for evaluating public policies (Sections 4-6). Each such method is appropriate for a corresponding economic decentralization scheme. One of them (Section 6) makes direct use of NNP. We showed that it is possible to conduct social cost-benefit analysis by studying the impact of an investment project on NNP, suitably measured, in each period of the project's life. In Sections 4-6 we studied an economy where the government optimizes over the choice of economic policies. In Section 7 the analysis was extended to cover the case of an economy where the government does not optimize, but is able to engage in policy reforms. The connection between the two was also noted.

We have shown that while NNP, properly defined, can be used as a gauge for evaluating economic projects, it should not be used in any of its more customary roles. For example, it was shown in Sections 8a,c that comparisons of NNP across time, across countries, and (by similar reasoning) across groups, are not equivalent to what they are widely thought to be equivalent to, namely, comparisons of social well-being across time, across countries, and across groups. So in Section 8a,c we also developed such indices as would be appropriate for making comparisons of social well-being. In particular, we showed that, in cases where the resource allocation mechanism is independent of calendar time, social well-being increases (decreases) over a brief interval of time if during the interval the value of the change in the flow of consumption services plus the change in the value of investment is positive (negative). We also showed that, equivalently, social well-being is an increasing function of time if, and only if, net investment in the economy's capital assets is positive. The latter of the two rules involves comparisons of wealth. In any event, neither amounts to NNP comparisons. A corresponding pair of results was obtained for cross-country comparisons of social well-being.

The recent theoretical literature on the intertemporal welfare economics of NNP (as summarized, say, in Heal, 1998) has focussed on economies pursuing optimal policies. Our analysis has included not only such economies, but also those where the government is capable of engaging only in policy reforms.

Green NNP has widely been interpreted as a constant-equivalent consumption stream. In Section 8b it was shown that this interpretation offers no purchase. It is the Hamiltonian that equals a constant-equivalent utility stream and we argued that, since the Hamiltonian is typically a non-linear function of consumption and leisure, it is of little practical use.

NNP, as defined in in this paper, is not NNP as it is usually defined. Conventional NNP is the sum of aggregate consumption and net investment in physical and human capital. Expressions (19) and (29b) tell us that NNP should incorporate further items. Let us list them:

(i) The accounting value of net investment in the stocks of all durable capital goods (manufactured, natural, human, and knowledge capital) should be included in NNP. The NNP that we have studied here is "green NNP".

(ii) If wages equal the marginal "disutility of work", wages would not be part of NNP: the shadow wage bill ought to be deducted from aggregate consumption. Of course, if labour were supplied inelastically, it would be a matter of indifference whether or not the wage bill is deducted from NNP. Note though that by labour we mean raw labour. If part of the wage bill is a return on past accumulation of human capital, that part would be included in NNP.

(iii) Improvement or deterioration of environmental amenities are formally akin to discoveries or depletion of natural resources. From this one concludes that defensive expenditure against damages to amenities should be included in the estimation of final demand and, therefore, of NNP; assuming, that is, that the instructions under (i) have been fully carried out.

(iv) Investment in capital that acts as a defence against environmental degradation (e.g. terracing agricultural land) ought also to be included in final demand and, therefore, in NNP.[37]

(v) If discoveries of new resource deposits depend on current expenditure, but not on cumulative expenditure (i.e. $N_E > 0$ and $N_Z = 0$, where N is the discovery function), NNP should not include both the value of new discoveries and current exploration costs: only one of the two items should be included. To include both would

[37] (iii) and (iv) are proved formally in Dasgupta and Mäler (1991).

involve double counting (Dasgupta, Kriström and Mäler, 1998).[38]

(vi) If discoveries of new resource deposits depend on cumulative expenditure (i.e. $N_Z > 0$ and $N_E \geq 0$), the value of new discoveries and exploration costs ought both to be included in NNP. However, if current discoveries depend also on current exploration costs ($N_E > 0$), it would be incorrect to add together the monetary values of aggregate consumption and exploration costs for estimating current expenditure: their accounting prices are not the same. It is only in the knife-edge (and unrealistic) case, $N_E = 0$, that to do so would be appropriate (Dasgupta, Kriström and Mäler, 1998).[39]

(vii) A conclusion corresponding to (v) holds in the case where improvements in environmental quality depend only on current expenditure, but not on accumulated expenditure.

(viii) Conclusions corresponding to (vi) hold in the case where improvements in environmental quality depend on accumulated expenditure.

Finally, it is as well to re-stress that this paper been about conceptual matters only. Our findings imply that the estimation of accounting prices should now be a priority. This said, it must be acknowledged that estimating the accounting prices of certain categories of resources will prove to be impossible. In any case, if ecological discontinuities are telling, an exclusive reliance on accounting prices (and therefore NNP) for project evaluation would not suffice. In such situations we would be forced to seek an appropriate combination of NNP, restricted to a subset of goods and services, and quantity indicators for the ones that remain. In short, a single index would not suffice. But this means that tradeoffs would have to be made explicitly (e.g. how much

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[38] To see this, note that in the case being considered, $u_t^* = 0$, and so equation (19) reduces to:

$$\rho_t^* = C_t - n_t^*L_t + dK_t/dt + v_t^*dS_t/dt.$$

But there is an alternative way to express ρ_t^*. Define the function, B_t, by the property, $dB_t/dt \equiv M(S_t) - R_t$. B_t is the size of the stock of natural resources at t excluding discoveries at t. Now linearize the Hamiltonian in (9) round E* as well and express NNP at t as:

$$\rho_t^* = C_t - n_t^*L_t + dK_t/dt + v_t^*dB_t/dt + v_t^*N_E E_t.$$

On using (10), this reduces to:

$$\rho_t^* = C_t - n_t^*L_t + E_t + dK_t/dt + v_t^*dB_t/dt.$$

This proves the claim in the text.

[39] To prove this note that, in the case being considered, expression (9) reduces to:

$$\rho_t^* = C_t - n_t^*L_t + u_t^*E_t + dK_t/dt + v_t^*dS_t/dt.$$

But, from condition (10), we observe that $u_t^* < 1$ if $N_E > 0$ and $u_t^* = 1$ if $N_E = 0$. This proves the claim in the text. See Repetto et al. (1989), who added the monetary values of aggregate consumption and exploration costs in their estimate of current expenditure. Our findings imply that they must have assumed implicitly that $N_Z > 0$ and $N_E = 0$.

biodiversity should be permitted to be destroyed for the sake of so many dollars of aggregate income?). These are hard choices, even tragic choices. But we believe they are unavoidable.

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