The impossibility of a Paretian republican? Some comments on Pettit and Sen

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The Impossibility of a Paretian Republican? Some Comments on Pettit and Sen[1]

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Abstract. Philip Pettit (2001) has suggested that there are parallels between his republican account of freedom and Amartya Sen's (1970) account of freedom as decisive preference. In this paper, I discuss these parallels from a social-choice-theoretic perspective. I sketch a formalization of republican freedom, and argue that republican freedom is formally very similar to freedom as defined in Sen's "minimal liberalism" condition. In consequence, the republican account of freedom is vulnerable to a version of Sen's liberal paradox, an inconsistency between universal domain, freedom, and the weak Pareto principle. I argue that some standard escape-routes from the liberal paradox – those via domain restriction – are not easily available to the republican. I suggest that republicans need to take seriously the challenge of the impossibility of a Paretian republican.

1. Introduction

Philip Pettit (1997) argues that, within the long tradition of republican thought – from the Roman Republic to the United States of America – a particular notion of freedom can be seen as a unifying theme: the notion of freedom as the opposite of subordination or slavery. Such freedom, on Pettit's account, requires more than the absence of "interference", that is, the absence of certain actual constraints on an individual's actions or choices. It requires the absence of "domination", that is, the absence of the possibility of arbitrary interference. Arbitrary interference is the sort of interference that a master can exercise over a slave capriciously, and which the slave is constantly vulnerable to,

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even if the master happens to be going through a spell of goodwill towards the slave. Domination, Pettit reminds us, is a common grievance:

> "The grievance … is that of having to live at the mercy of another, having to live in a manner that leaves you vulnerable to some ill that the other is in a position arbitrarily to impose … It is the grievance expressed by the wife who finds herself in a position where her husband can beat her at will, and without the possibility of redress; by the employee who dare not raise a complaint against an employer, and who is vulnerable to any of a range of abuses, some petty, some serious, that the employer may choose to perpetrate; by the debtor who has to depend on the grace of the moneylender, or the bank official, for avoiding utter destitution and ruin; and by the welfare dependant who finds that they are vulnerable to the caprice of a counter clerk for whether or not their children will receive meal vouchers." (Pettit 1997, 4-5)

On Pettit's republican account, freedom is the absence of domination. As Pettit argues, this can be defined, in more precise terms, as the absence of interference not just in the actual world, but also in all relevant possible worlds. A slave may not suffer interference in the actual world, because his master presently has some goodwill towards him. But there is a nearby possible world in which his master ceases to have such goodwill and interferes with the slave's actions. So the slave is unfree because his master, although favourably disposed towards him in the actual world, can arbitrarily interfere with the slave's actions in a nearby possible world.

In a recent paper, Pettit (2001) suggests that there are parallels between this republican account of freedom, and Amartya Sen's liberal account of freedom, first presented in Sen's famous paper "The impossibility of a Paretian liberal" (1970), where freedom is defined as decisive preference. The parallel between the two accounts, Pettit suggests, lies in their "emphasis on the connection between freedom and non-dependence" (Pettit 2001, 18). Pettit argues that on Sen's account – as on the republican account – for an individual to be free, the absence of interference – or the decisiveness of the individual's preferences – must be content-independent and context-independent, as discussed in more detail below.

> "Non-interference is not sufficient for freedom under Sen's view, because an agent might enjoy non-interference – might even enjoy content-independently decisive preference – without enjoying preference that is decisive in the full sense: in particular, without enjoying favour-[or context]-independently decisive preference." (Pettit 2001, 18)

Pettit attributes the *content*-independence requirement to Sen himself, but says that the *context*-independence requirement "is not explicitly marked by Sen" (Pettit 2001, 6).

In this paper, I sketch a social-choice-theoretic formalization of Pettit's republican account of freedom and his reading of Sen, and argue that:

(i) Sen's own definition of freedom (as opposed to capability) already satisfies both the content-independence requirement and the context-independence requirement.

(ii) The contrast between classical liberal and republican accounts of freedom can be captured by this formalization.

(iii) That contrast lies not in the fact that one account considers only the actual world whereas the other considers also possible worlds, but rather in how large the class of possible worlds is that each account considers.

(iv) The republican account of freedom (at least if stated demandingly) is affected by a version of Sen's liberal paradox – an inconsistency between universal domain, freedom, and the weak Pareto principle. And:

(v) Some standard escape-routes from the liberal paradox – namely those via domain restriction – may not (easily) be available to the republican.

Hence, depending on the reading and on how demandingly the republican account is stated, we may be faced with the impossibility of a Paretian republican.[2] This raises the question of whether relaxing the weak Pareto principle so as to preserve individual freedom is consistent with the republican position.

2. Freedom as decisiveness

Suppose there are n individuals, 1, 2, ..., n. Each individual has a preference ordering R_i over a set of social alternatives X. For any pair of alternatives x, y in X, xR_iy means that "individual i weakly prefers x to y".[3] The ordering R_i also induces a strict preference ordering P_i, and an indifference relation I_i, defined as follows:

> xP_iy if and only if xR_iy and not yR_ix;
> xI_iy if and only if xR_iy and yR_ix.

A *profile of preference orderings* (in short: *profile*) is a vector $\underline{R} = \langle R_1, R_2, ..., R_n \rangle$. Each profile represents precisely one possible assignment of preference orderings to the n individuals. Let **U** be the set of all logically possible profiles. If – in a simple model – each relevant possible world is characterized by the preferences all individuals hold in that world, then each such possible world can be represented by a suitable profile. Under this interpretation, **U** is the set of all such possible worlds.

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[2] Dowding and van Hees (2003) have suggested that some such impossibility results stem from an *in principle* problem of standard social-choice-theoretic or game-theoretic definitions of freedom (e.g. in terms of decisiveness), namely that such definitions entail *either* the non-compossibility of the freedoms of different individuals, *or* that the freedoms that individuals have are non-existent or vanishingly small. Instead of requiring robust decisiveness for freedom, they suggest that an individual's freedom may sometimes be overruled by other considerations. Freedom should therefore not be interpreted as an unconditional trump, but rather as carrying a certain characteristic threshold probability of being respected. That threshold probability may vary from context to context. A discussion of Dowding and van Hees's alternative proposal is beyond the scope of this paper, as the main focus here is the exchange between Pettit and Sen.

A social aggregation function F assigns to each profile $\underline{R}$ a corresponding social preference ordering $R := F(\underline{R})$ on the set of social alternatives X.[4] For any pair of alternatives x, y in X, xRy means that "x is socially weakly preferred to y". The social ordering R also induces a strong ordering P and an indifference relation I. On an outcome-orientated interpretation, we might interpret xPy not merely as "x is socially strictly preferred to y", but rather as "whenever there is choice between x and y, the social outcome will be x rather than y".

What are the criteria for saying that an individual is free? I will discuss three possible criteria, preference satisfaction, content-independent decisiveness, and context- and content-independent decisiveness. I will argue, with Pettit, that each of the first two alone is insufficient for freedom. According to Pettit (2001), republican freedom requires a version of the third criterion: context- and content-independent decisiveness. Below I state the most demanding version of that criterion, but this demanding version is arguably also the most compelling one. Thus, depending on how demandingly freedom is defined on the republican account, republican freedom either requires, or is at least implied by, context- and content-independent decisiveness as defined in this paper. Moreover, I will show that this criterion yields exactly Sen's original definition of freedom in terms of decisive preference, as given in his paper "The impossibility of a Paretian liberal" (1970). Hence, regardless of how demandingly the republican account is defined, individual freedom according to Sen's (1970) definition is sufficient for individual freedom according to the republican account.

Preference satisfaction. Individual i's preferences over x and y are *satisfied* at the profile $\underline{R}$ if (at $\underline{R}$) [if xP_iy then xPy] and [if yP_ix then yPx].

Claim 1. Preference satisfaction alone is insufficient for freedom.

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[3] R_i is assumed to be reflexive, transitive and connected.
[4] We also require R to be reflexive, transitive and connected.

Suppose that there are two individuals, the master (individual 1) and the slave (individual 2). Suppose, further, that the social aggregation function is a lexicographic dictatorship, where individual 1's preferences take lexicographic priority over individual 2's preferences. This is a technical way of saying that the master's preferences always take priority over the slave's in determining the social outcome. The slave's preferences act (at most) as tie-breakers. Formally, the social preference ordering is defined as follows. For any profile $\underline{R} = \langle R_1, R_2 \rangle$ and any pair of social alternatives x, y in X,

$$xPy \quad \text{if and only if} \quad xP_1y \text{ or } [xI_1y \text{ and } xP_2y].$$

Suppose, for example, there are two social alternatives,

x: individual 2 travels;
y: individual 2 does not travel.

First consider the profile $\underline{R} = \langle R_1, R_2 \rangle$, where yP_1x and xP_2y. The slave wants to travel, whereas the master does not want him to. The resulting social preference ordering is yPx. So the slave is prevented from travelling. Formally, individual 2's preferences over x and y are not satisfied at $\underline{R}$. Clearly, and intuitively, the slave is unfree in this situation. So far, this is captured by the preference satisfaction account. But next consider an alternative profile $\underline{R}^* = \langle R^*_1, R^*_2 \rangle$, where individual 1 has the same preferences as before, that is, yP^*_1x, but individual 2 changes his preference to yP^*_2x. In other words, the slave no longer wants to travel. The resulting social preference ordering remains the same as before, that is, yP^*x. This time individual 2's preferences over x and y *are* satisfied. Suppose preference satisfaction is sufficient for freedom. Then individual 2, the slave, is free at $\underline{R}^*$. But this violates our intuitions about freedom. In particular, individual 2's preferences over x and y are satisfied at $\underline{R}^*$ only because individual 2 happens to have the same preferences as individual 1, and individual 1's preferences are still dictatorial. The social preference ordering over x and y *in no way tracks* individual 2's preferences. If individual 2 were to change his preferences back to the original ones in $\underline{R}$ – other things remaining equal – then his preferences over x and y would no longer be satisfied.

Freedom requires more robust tracking of an individual's preferences. But different accounts of freedom disagree on what kind of robustness freedom requires.

As I will now argue, several rival accounts of freedom – including liberal and republican ones – can be formalized in terms of a common scheme. They all belong to the same family of concepts, but they differ in one parameter: the parameter will be called N.

Let N be a function which maps each “actual” profile-individual pair $<\underline{R}, i>$ to a class of “possible” profiles $N(\underline{R}, i)$, interpreted as the relevant neighbourhood of the actual profile $\underline{R}$, relative to individual i.[5]

***N*-decisiveness.** Individual i is *N-decisive* over x and y at the profile $\underline{R}$ if, for all $\underline{R}^*$ in $N(\underline{R}, i)$, [if xP^*_iy then xP^*y] and [if yP^*_ix then yP^*x].

Suppose we have given some definition of N. The concept of N-decisiveness then induces a corresponding definition of freedom.

Freedom as *N*-decisivenss. Individual i is free with regard to the choice between x and y at the profile $\underline{R}$ if i is N-decisive over x and y at $\underline{R}$.

Defining freedom as N-decisiveness means that an individual is free with regard to the choice between x and y at some *actual* profile if the individual is decisive over x and y in a class of *possible* profiles that lie in the relevant neighbourhood of the given actual profile. Now the disagreement between different accounts of freedom can be characterized as a disagreement over how broadly or narrowly that relevant

[5] The function N may not always depend on i – especially, as we will see, in the case of versions of the republican account of freedom, where N is the set of all logically possible profiles.

neighbourhood should be defined – that is, as a disagreement over what the parameter N (the "neighbourhood function") should be.

The preference satisfaction account, as discussed above, defines N most narrowly. For each $\underline{R}$ and each i, $N(\underline{R}, i)$ is simply the set containing only $\underline{R}$ itself. As we have seen, preference satisfaction alone is insufficient for freedom. From this observation we can learn the following point:

Claim 2. A necessary condition for a satisfactory definition of freedom as N-decisiveness is that $N(\underline{R}, i)$ is (at least sometimes) a proper superset of $\{\underline{R}\}$; that is, $N(\underline{R}, i)$ contains not only the *actual* profile $\underline{R}$, but also some *possible* profile(s) other than $\underline{R}$.

For liberal freedom to be distinct from preference satisfaction, the liberal definition of freedom must satisfy this necessary condition. So if liberal freedom is defined as N-decisiveness, then (at least some) $N(\underline{R}, i)$ must contain not only the actual profile $\underline{R}$, but also some other possible profiles. Thus the distinction between the classical liberal account of freedom and the republican account lies not in the fact that the former considers only the actual world, whereas the latter also considers certain possible worlds; both consider the actual world *and* certain possible worlds. Rather, the distinction lies in how large the class of relevant possible worlds is that each account considers.

The standard way in which liberal freedom is defined – "freedom is the absence of interference *in the actual world*" – may thus seem somewhat misleading. But the definition can be made consistent with claim 2 by interpreting "*interference* in the actual world" as "violation of preference *satisfaction* in the actual world or in a nearby possible world". Under this reading, the liberal account defines freedom as decisiveness in the actual world and in nearby possible worlds, but casts its net fairly narrowly across possible worlds.[6] The republican account also defines freedom as decisiveness in the

[6] Using a similar substitution, we can state two equivalent definitions of the republican account: (i) freedom is the absence of domination in the actual world, and (ii) freedom is the absence of interference in the actual world and all relevant possible worlds. The two

actual world and in a class of possible worlds, but casts its net more widely across possible worlds. As we will see, the question of how widely the net is cast across possible worlds – that is, how broadly *N* is defined – has crucial implications for whether or not an account of freedom is vulnerable to (a version of) Sen's liberal paradox.

Pettit notes the shortcomings of the preference satisfaction account, and points out that, to define freedom plausibly as decisive preference, it is necessary that the individual's decisiveness be independent of content.

> "[I]t will not be enough for freedom that I get A if my preference is for A, when it is not the case that I get B if my preference is for B. Freedom requires that my preference is empowered in a content-independent way: it is decisive, regardless of which of the relevant options is preferred." (Pettit 2001, 5)

Pettit attributes to Berlin (1969) what he takes to be the best argument for the content-independence requirement: "If we reject it, we must say that a person can make themselves free just by adapting their preference appropriately." And he adds that Sen explicitly endorses the content-independence requirement.

We can formalize *content-independence* as the requirement that an individual's preferences be satisfied not only at the actual profile, but also at all possible profiles that result from the actual one if that individual changes his preferences, but all other individuals' preferences are held fixed. On this formalization, *content* is interpreted as individual *i*'s preference ordering.

Define two profiles $\underline{R}$ and $\underline{R}^*$ to be *i-variants* if, for all $j \neq i$, $R_j = R^*_j$. ("Nobody, except possibly individual *i*, changes their preferences.")

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definitions can be made equivalent by interpreting "*domination* in the actual world" as "*interference* in the actual world or in a relevant possible world".

Content-independent decisiveness. For each $\underline{R}$ and each i, $N(\underline{R}, i)$ is the set of all i-variants of $\underline{R}$.

Given the remarks above, content-independent decisiveness might be taken to be the narrowest version of a liberal account of freedom, as distinct from a mere preference satisfaction account.

Claim 3. Content-independent decisiveness alone is insufficient for freedom.

Suppose the individuals, the social alternatives and the social aggregation function are exactly as in the master and slave example above. Consider the profile $\underline{R} = <R_1, R_2>$, where xI_1y and xP_2y. By the definition of content-independence, $N(\underline{R}, 2)$ contains all 2-variants of $\underline{R}$. $\underline{R}$ has precisely three 2-variants: (i) $\underline{R}$ itself, (ii) $\underline{R}^* = <R^*_1, R^*_2>$, where xI^*_1y and yP^*_2x, (iii) and $\underline{R}^{**} = <R^{**}_1, R^{**}_2>$, where xI^{**}_1y and xI^{**}_2y. The social preference orderings in cases (i), (ii) and (iii) are, respectively, xPy, yP^*x, and $xI^{**}y$. Thus individual 2 is N-decisive – that is, content-independently decisive – at $\underline{R}$. Suppose content-independent decisiveness is sufficient for freedom. Then individual 2 is free at $\underline{R}$. However, individual 2's decisiveness is not very robust. Suppose that individual 1, the master, suddenly changes his preferences from indifference over x and y to ranking y strictly above x. Then individual 2's preferences will no longer be satisfied. If the profile is $\underline{R}^{***} = <R^{***}_1, R^{***}_2>$, where yP^{***}_1x and xP^{***}_2y, then the social preference ordering is $yP^{***}x$. Individual 2's decisiveness at $\underline{R}$ is not independent of the goodwill of individual 1. In Pettit's terms, individual 2 is only favour-dependently decisive at $\underline{R}$.

Pettit argues that the required additional robustness is captured by a context-independence requirement, which he says "is not explicitly marked by Sen".

> "It is … possible for preference to be just context-dependently decisive and so insufficient for freedom. In particular it is possible for preference … to be just favour-dependently decisive: to be decisive only so far as the person enjoys the

> gratuitous favour of certain others – the sort of favour that can be bestowed or withdrawn at the pleasure of the giver." (Pettit 2001, 6).

I here formalize context-independent decisiveness as follows. An individual is *context-independently decisive* if the individual's preferences are satisfied not only at the actual profile, but also at *all* possible profiles that result from the actual one if that individual's preferences are held fixed, but some other individuals change their preferences. On this formalization, *context* is interpreted as the preference orderings of all individuals *other than* individual *i*. This version of context-independent decisiveness is a demanding one: It requires that the individual be decisive across *all* possible profiles that result from the actual one if the individual's own preferences are fixed, but some other individuals change their preferences. Less demanding versions of context-independent decisiveness might require decisiveness only across *some suitably defined but not all* such profiles.

The requirements of context-independence and content-independence, as stated here, are independent from each other. However, it can be argued that context-independence alone, just as content-independence alone, is insufficient for freedom. On Pettit's account, it is the *combination* of context-independence and content-independence that is necessary and sufficient for freedom. That combination – given the present demanding version of context-independent decisiveness – is the following requirement. An individual is *context- and content-independently decisive* if the individual's preferences are content-independently decisive not only at the actual profile, but also at *all* possible profiles that result from the actual one if that individual's preferences are held fixed, but some other individuals change their preferences. (In a less demanding definition, the italicized "all" might be replaced with "some suitably defined but not all".) Thus an individual is context- and content-independently decisive if the individual's preferences are satisfied not only at the actual profile, but also at *all* possible profiles that result from the actual one if that individual or some other individuals (or both) change their preferences. This means that individual *i* is context- and content-independently decisive over *x* and *y* (at $\underline{R}$)

if and only if the individual is decisive across *all* profiles in **U**.[7] A less demanding version of context- and content-independent decisiveness would require decisiveness only across a sufficiently large subset of **U**, depending on $\underline{R}$ and i. We will see below that, to avoid an impossibility result, the republican faces a choice between *either* opting for such a less demanding decisiveness requirement *or* relaxing the weak Pareto principle.

Context- and content-independent decisiveness. For each $\underline{R}$ and each i, $N(\underline{R}, i) = \mathbf{U}$.

As $N(\underline{R}, i)$ is a constant function here – which always takes the value **U** – we can simplify the notation by writing **U**-decisiveness instead of N-decisiveness.

3. Context- and content-independent decisiveness and the liberal paradox

The requirement of context- and content-independent decisiveness, in its strong version, yields exactly Sen's original definition of freedom together with the condition that the relevant domain is **D**:=**U** (as in Sen 1970).

Sen's original definition of freedom. Individual i is free (at some profile $\underline{R}$) with regard to the choice between x and y if, for *all* profiles $\underline{R}$ in **D**, [if xP_iy then xPy] and [if yP_ix then yPx].

If the domain is **U**, then freedom, on Sen's (1970) definition, is **U**-decisiveness. Freedom according to Sen's original definition thus *implies* freedom according to the republican definition. Moreover, on the demanding version of context- and content-independent decisiveness I have defined, the two definitions – Sen's and the republican one – coincide, and their difference is at most linguistic. Under Sen's original definition,

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[7] Formally, define two profiles $\underline{R}$ and $\underline{R}^*$ to be $(N\backslash i)$-*variants* if $R_i = R^*_i$. We can now say that individual i is *context- and content-independently decisive* over x and y at the profile $\underline{R}$ if, for all $\underline{R}^*$ in $N(\underline{R}, i)$, i is content-independently decisive at $\underline{R}^*$, where, for each $\underline{R}$ and each i, $N(\underline{R}, i) = \{\underline{R}^* \in \mathbf{U} : \underline{R}^*$ is an $(N\backslash i)$-variant of $\underline{R}\}$. Then individual i is context- and content-independently decisive over x and y at $\underline{R}$ precisely if i is N^*-decisive at $\underline{R}$, where, for each $\underline{R}$ and each i, $N^*(\underline{R}, i) = \mathbf{U}$.

freedom of an individual with regard to a pair of alternatives is stated as a 'global' property of a social aggregation function. Under my reading of the republican definition, freedom of an individual with regard to a pair of alternatives is stated as a 'local' property that holds *at a particular profile*. But implicitly freedom under the republican definition is also a 'global' property: For an individual to be free at some profile $\underline{R}$, the individual must be decisive across all profiles in a large neighbourhood of $\underline{R}$, where that neighbourhood is – in the limit – all of **U**. Logically, the two definitions are equivalent: reference to a specific profile $\underline{R}$ makes no difference, and can therefore be added or dropped as we wish.

We can now see that Sen's own definition of freedom, given the domain **U**, already satisfies *both* the content-independence requirement *and* the context-independence requirement (and, of course, if either requirement is defined less demandingly, Sen's definition will still satisfy them). The two independence requirements are satisfied precisely because of the universal quantification – reference to *all* profiles $\underline{R}$ in the relevant domain (here **U**) – in Sen's definition of freedom. Whether or not context-independence is "explicitly marked" by Sen, it is *implied* by his original definition of freedom.

Claim 4. Context- and content-independent decisiveness – in its demanding form – leads to Sen's "liberal paradox" – which we might therefore also call a "republican paradox".

Let me state Sen's original result on the impossibility of a Paretian liberal.

The weak Pareto principle. For all profiles $\underline{R}$ in **D** and all pairs of alternatives x and y, if [for all i, xP_iy] then xPy.

Minimal liberalism. There exist at least two individuals, i and j, and two corresponding pairs of alternatives $\langle x_1, x_2\rangle$ and $\langle y_1, y_2\rangle$ such that individual i is **D**-decisive over x_1, x_2 and individual j is **D**-decisive over y_1, y_2.

Theorem. (Sen 1970) Let **D** := **U**. Then there exists no social aggregation function *F* defined on **D** which satisfies the weak Pareto principle and minimal liberalism.

Under the demanding reading of republican freedom, we can restate Sen's theorem by replacing the condition of minimal liberalism with the logically equivalent condition of minimal republicanism.

Minimal republicanism. There exist at least two individuals, *i* and *j*, and two corresponding pairs of alternatives $<x_1, x_2>$ and $<y_1, y_2>$ such that, for *at least one* profile $\underline{R}$, individual *i* is context- and content-independently decisive over x_1, x_2 at $\underline{R}$ and individual *j* is context- and content-independently decisive over y_1, y_2 at $\underline{R}$.

Sen's theorem then implies that, given *any* social aggregation function *F* satisfying the weak Pareto principle, there exists *not even one* profile at which *two* individuals are each free (in the republican sense, demandingly interpreted) with regard to the choice between *at least one* pair of alternatives.

4. Are escape-routes from the liberal paradox via domain restriction open to the republican?

It is well known that, if we suitably restrict the domain of a social aggregation function – that is, we define *F* not on **U**, but on a suitable proper subset **D** of **U** – then there exist social aggregation functions (defined on **D**) which satisfy both the weak Pareto principle and minimal liberalism (as well as stronger versions of that condition).

I will briefly review three such domain restriction conditions (see Sen 1983 for a more extensive discussion; see also Blau 1975; Craven 1982; Gigliotti 1986).

Tolerant preferences. D is the set of all profiles $\underline{R}$ for which the following holds. For any pair of alternatives over which some individual has a decisiveness right, all other individuals are indifferent over that pair.

Empathetic preferences. **D** is the set of all profiles $\underline{R}$ for which the following holds. For any pair of alternatives over which some individual has a decisiveness right, all other individuals' preferences over that pair mirror those of the individual who has that decisiveness right.

Non-meddlesome or liberal preferences. **D** is the set of all profiles $\underline{R}$ for which the following holds. There exists no individual who has a more intense preference on a pair of alternatives over which some other individual has a decisiveness right than on those pairs of alternatives over which she herself has a decisiveness right.[8]

For each of these domain restriction conditions, there exist social aggregation functions (defined on **D**) which satisfy the weak Pareto principle *and* make each individual **D**-decisive over at least one pair of alternatives. Under **D**-decisiveness, for each actual profile $\underline{R}$ (in **D**) and each individual i, the relevant neighbourhood of possible profiles, $N(\underline{R}, i)$, is **D**. Thus **D-**decisiveness, under the domain conditions just described, defines $N(\underline{R}, i)$ more broadly than the preference satisfaction account of freedom – where $N(\underline{R}, i)$ is only $\{\underline{R}\}$ – but less broadly than the most demanding version of the republican account – where $N(\underline{R}, i)$ is all of **U**.

Is such an escape-route from the liberal paradox open to a republican? To avoid a republican paradox, the republican must find a domain restriction condition that meets the following two requirements: (i) N-decisiveness captures a sufficiently strong notion of context- and content-independent decisiveness, and (ii) minimal republicanism (defined in terms of N-decisiveness) is consistent with the weak Pareto principle (supposing, for the moment, we accept that principle).

The challenge for the republican is a difficult one, as I will explain now. Arguably, the three domain restriction conditions I have reviewed satisfy requirement (ii) at the expense

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[8] Preference intensity is here defined purely ordinally. Individual i is said to have a more intense preference for x over w (xP_iw) than for y over z (yP_iz) if [xP_iy, yP_iz and zR_iw] or [xR_iy, yP_iz and zP_iw].

of violating requirement (i). Under each domain restriction condition, an individual's decisiveness is – in part – dependent on the context, that is, on the preferences of other individuals. Under tolerant preferences, each individual's decisiveness over a pair of alternatives depends on the other individuals' attitude of "tolerant" indifference over that pair. Under empathetic preferences, the individual's decisiveness depends on the other individuals' "empathetic" mirroring of that individual's preferences over the relevant pair. Under non-meddlesome or liberal preferences, the individual's decisiveness depends on the other individuals' "liberal" attitude of not holding more intense preferences over that individual's sphere of rights than over their own such spheres.

In each case, if the context – that is, the preferences of *other* individuals – changes and the relevant favourable conditions cease to hold, then an individual may cease to be decisive. Hence **D**-decisiveness (for each of the reviewed domain restriction conditions) does not imply context-independent decisiveness in the strong form defined above.[9]

Now suppose that an individual's decisiveness is *contingent* on the fact that actually occurring profiles fall into the domain **D**, but the occurrence of profiles outside **D** is still possible. From the republican perspective, this kind of *contingent decisiveness* would be insufficient for freedom. If, on the other hand, it could somehow be shown that actually occurring profiles will *robustly* fall into the domain **D**, then the republican might consider **D**-decisiveness sufficient for freedom. This would seem to be the most promising route for the republican. But establishing the required *robustness* is a difficult challenge. First, an argument that the occurrence of profiles outside **D** is *empirically unlikely* might be

[9] An alternative domain restriction condition, proposed by Krüger and Gaertner (1981, 1983), called *self-supporting preferences*, has the following peculiar property. If an individual with self-supporting preferences is **D**-decisive (where **D** is the set of all profiles satisfying that domain restriction condition), the individual may be *context-independently* decisive, but not *content-independently* decisive. As Krüger and Gaertner put it, "an individual who preserves an ordering of his (her) own feature-alternatives [that is, whose preferences are self-supporting] can secure social protection for his (her) choice between them". Thus the individual can win "[context-independent] social decisiveness over private-sphere alternatives at the price of revealing self-supporting preferences [that is, at the price of revealing preferences with a specific content]" (Krüger and Gaertner 1983, 214).

insufficient for establishing that actually occurring profiles will *robustly* fall into **D**. Even if the occurrence of profiles outside **D** were empirically rare, their occurrence would still remain a possibility. Second, an arrangement that would *coercively* prohibit the occurrence of such profiles – by explicitly *forcing* people not to hold certain combinations of preferences – is unlikely to satisfy the republican because of its coercive nature (unless such coercion could somehow be justified as being both non-arbitrary and consistent with individual freedom).

The more broadly $N(\underline{R}, i)$ is defined, the harder it is to make N-decisiveness consistent with the weak Pareto principle. In other words, the more robust we want an individual's freedom to be, the more prone we are to encountering a liberal (or republican) paradox. Since the republican account of freedom demands greater robustness than the liberal account, it is also more vulnerable to the paradox. Unless the republican is prepared to relax the weak Pareto principle, avoiding the paradox may require sacrificing some robustness.

5. Are escape-routes from the liberal paradox via relaxing the weak Pareto principle open to the republican?

If we do not restrict the domain of the social aggregation function or give up minimal liberalism, the only remaining escape-route from the liberal paradox is one where the weak Pareto principle is relaxed. Several authors have explored this route (e.g. Sen 1976; Hammond 1982); I will here sketch one version of it.

Note that Sen's theorem on the impossibility of a Paretian liberal arises from the fact that, for some profiles, (i) the (transitive closure of the) ordering on some alternatives induced by individual decisiveness rights conflicts with (ii) the ordering on these alternatives induced by the weak Pareto principle. A way of solving this conflict is to give one of these two conflicting (partial) orderings priority over the other. To give priority to (i) over (ii), we can use the following definition.

For any profile $\underline{R}$ in **D**, we define the corresponding social preference ordering $R := F(\underline{R})$ in two steps. First, for any pair of alternatives x and y, we define xPy if

either (A) xPy is determined by the exercise of an individual decisiveness right over x and y;

or neither xPy nor yPx is determined by the exercise of an individual decisiveness right over x and y *but* (B) xPy is *implied* by the exercise of individual decisiveness rights over *other* pairs of alternatives together with transitivity;

or neither xPy nor yPx is determined, or implied, by the exercise of individual decisiveness rights *but* (C) *all* individuals have the preference xP_iy.

Second, let R be a (suitably defined) reflexive, transitive and connected extension of the strong (partial) ordering P defined in the first step.

In this definition, the three conditions, (A), (B) and (C), are stated in an order of lexicographic priority. Condition (A) corresponds to respecting individual decisiveness rights. Condition (B) corresponds to respecting transitivity after the exercise of such rights. Condition (C) corresponds to respecting a restricted weak Pareto principle for those remaining pairs of alternatives (if any) that have been left unranked by conditions (A) and (B).

Clearly, this social aggregation function F is defined on the domain **U** and satisfies minimal liberalism. It does not satisfy the weak Pareto principle, but only the following restriction of that principle:

Restricted weak Pareto principle. For all profiles $\underline{R}$ in **D** and all pairs of alternatives x and y, if [neither xPy nor yPx is determined, or implied, by the exercise of individual decisiveness rights] and [for all i, xP_iy], then xPy.

Is such a relaxation of the weak Pareto principle consistent with the republican position? The answer to this question depends on how we interpret the original weak Pareto principle. If we interpret that principle as a welfarist requirement, then relaxing it may be unproblematic, as it is unsurprising that any position that attaches a great value to freedom – whether liberal or republican – will sometimes have to accept securing freedom at the expense of a welfare loss. If, on the other hand, we take (a version of) the original weak Pareto principle to be implied by core republican principles, then relaxing it may involve a substantial sacrifice from the republican perspective.

I am unable to settle the question of whether the outlined relaxation of the weak Pareto principle is consistent with core republican principles. But to sharpen the challenge for the republican, I will briefly review an argument, due to Pattanaik (1988), that suggests that such a relaxation may be inconsistent with core *liberal* principles. I will then explain what challenge I take this argument to pose for the *republican* position.

Pattanaik's argument proceeds as follows. Pattanaik suggests that the same liberal principles that entail the existence of *individual* decisiveness rights also entail the existence of certain *group* decisiveness rights. The original weak Pareto principle might itself be taken to state such a group decisiveness right, namely the right of the group of *all* individuals to determine a social preference for x over y in case *everyone* prefers x to y. But Pattanaik explains that this is *not* the kind of right he means when he refers to group decisiveness rights whose origin lies in the *same* liberal principles that also lead to individual decisiveness rights. Regarding the group right stated by the weak Pareto principle itself, he says that "it is not quite clear what constitutes the intuitive basis of such a group right (for society) over every pair of alternatives, irrespective of the nature of the alternatives" (Pattanaik 1988, 521-522).

Pattanaik describes a different, more specific group decisiveness right. Suppose that each social alternative can be expressed as a vector of components, where some components concern all individuals, whereas others are private, in that they concern only one individual. Suppose components i and j are, respectively, private issues for individuals i and j. Minimal liberalism, in these terms, requires that individual i should be decisive over choices relating to component i (holding other components fixed), while individual j should be decisive over choices relating to component j (again holding other components fixed). Supposing that no other individuals are affected by choices relating to components *i* and *j*, it seems reasonable to require that individuals *i* and *j* – in case they unanimously agree – should also be *jointly* decisive over any choices relating to a *combination* of these two components (holding other components fixed). Pattanaik calls this requirement (or a more precise version of it) the *respect for privacy* condition. It should be evident that this condition can be motivated by the same liberal principles that motivated Sen’s condition of minimal liberalism. Pattanaik then introduces an even less demanding condition, called *weak respect for privacy*. That condition states that, for components *i* and *j* as specified above, the joint preference of individuals *i* and *j* over choices relating to a combination of these two components should prevail *if* individuals *i* and *j* reach unanimous agreement on the choice in question *and* their preference is supported by *all* other individuals.

The key property of the condition of *weak* respect for privacy is that it is implied not only by the slightly stronger condition of respect for privacy, but also by the weak Pareto principle. Pattanaik’s main result shows (roughly) that the liberal paradox occurs even when the weak Pareto principle is restricted to weak respect for privacy: There exists no social aggregation function *F* (defined on the universal domain **U**) which satisfies weak respect for privacy and minimal liberalism. An immediate corollary is that there exists no social aggregation function *F* (defined on **U**) which satisfies respect for privacy and minimal liberalism. Likewise, Sen’s original theorem on the impossibility of a Paretian liberal can be reinterpreted as a corollary. Note that weak respect for privacy is *not* implied by the *restricted* weak Pareto principle introduced above. Nor is it implied by any relaxation of the weak Pareto principle that opens up an escape-route from the liberal paradox.

To summarize, weak respect for privacy is all that is needed for the liberal paradox to occur. But weak respect for privacy is implied by respect for privacy, a condition that seems to be supported by the same liberal principles that support the original condition of minimal liberalism. This suggests that relaxing the weak Pareto principle *beyond* weak respect for privacy is not consistent with these liberal principles. In particular, any relaxation of the weak Pareto principle that opens up an escape-route from the liberal paradox is a relaxation *beyond* weak respect for privacy and thus falls into this category.

I will not take a position on whether Pattanaik's argument is correct. But suppose it is – and Pattanaik's technical result certainly is. If republicanism is simply a particularly demanding form of liberalism, then the fact that the required Pareto relaxation is inconsistent with core *liberal* principles seems to imply, *a fortiori*, that it is also inconsistent with core *republican* principles. Again I will not take a position on whether this *a fortiori* inference is correct. But *if* it is, it leaves the republican in a difficult position. While a liberal who accepts that liberal principles disallow the required Pareto relaxation can still pursue another escape-route from the liberal paradox – namely the one via domain restriction – the republican, depending on how much robustness he demands, may not have such a route available, as we have seen in the previous section. So, to defend his position, the republican must find a way of confronting the trade-off between robust freedom and the weak Pareto principle, and he must explain what his favoured solution to that trade-off is.

6. Concluding remarks

The formalization of freedom as N-decisiveness suggests that "freedom" can be viewed as a *parametrical family* of concepts, all similar in form, but each depending on a specific choice of the *parameter* N. The parameter N – formally a "neighbourhood function" $N(\underline{R}, i)$ – describes the set of possible worlds (or profiles) over which the individual in question is to be decisive. That set of possible worlds typically includes, but may not be confined to, the actual world. The preference satisfaction account, which is arguably insufficient as an account of freedom, defines the set of relevant possible worlds most narrowly, namely to include only the actual world. Decisiveness with respect to this one-member set simply means preference satisfaction in the actual world. The liberal account of freedom defines the set of relevant possible worlds more broadly, but still confines it to a very close neighbourhood of the actual world. Under the narrowest liberal account, that neighbourhood consists just of those possible worlds that are "generated" by the different possible preferences of a given individual, holding the preferences of all other individuals fixed. Decisiveness with respect to this narrow set of possible worlds means content-independent decisiveness, but not context-independent decisiveness. The republican account, finally, defines the set of relevant possible worlds much more broadly. Decisiveness with respect to that larger set of possible worlds means (some version of) context- and content-independent decisiveness. However, what all these accounts of freedom have in common (the preference satisfaction account being the exceptional limiting case) is that they define freedom as decisiveness not just in the actual world, but also in certain possible worlds. Thus they all require a certain degree of robustness; they just differ on how much robustness they require. They all belong to the same parametrical family of concepts; they just differ on what parameter they choose. Consistent with this, Pettit says that "[context]-independence may come in degrees" (Pettit 2001, 7).[10] We can imagine several nested sets of possible worlds such that within the smaller ones an individual is decisive, whereas within the larger ones an individual ceases to be decisive.

In his reply to Pettit, Sen writes "I … accept Pettit's diagnosis that [an argument for context-independence] can also be seen as an implication of some of the things I have myself said about freedom" (Sen 2001, 53). If my analysis is correct, then it is a little surprising that Sen does not insist more strongly that context-independence is already *satisfied* by his own 1970 definition of freedom. The reason why Sen does not insist on this point might be that, in replying to Pettit, he puts more emphasis on his recent *capability* approach than on his original 1970 definition of freedom. In particular, Sen notes that multiple differentially robust definitions of freedom provide a discriminating power that we would lack if we were to use only a single such definition. To illustrate, let me recapitulate the master and slave example above, in a form similar to Sen's own example (Sen 2001, 54).

[10] Pettit notes that content-independence may also come in degrees (Pettit 2001, 7). Indeed, an individual may be content-independently decisive in a more demanding sense

Case 1. Individual 2 is a slave, and is prevented by individual 1, his master, from travelling.

Case 2. Individual 2 is a slave, but individual 1, his master, *currently* has the goodwill to allow individual 2 to travel whenever he wishes.

Case 3. Individual 2 is a "free" citizen, and can travel whenever he wishes.

As before, consider two alternatives: individual 2 travels (x); individual 2 does not travel (y). Most accounts of freedom will agree that in case 1 individual 2 is not free with regard to the choice between x and y. If freedom is defined as content-independent decisiveness, then individual 2 is free *both* in case 2 *and* in case 3. Sen would say that, in cases 2 and 3, individual 2 has the *capability* to choose between x and y, whereas, in case 1, individual 2 lacks that capability. If freedom is defined as context- and content-independent decisiveness, by contrast, then individual 2 is free only in case 3. From the republican perspective, what matters for freedom is not the *actual capability* to choose between x and y, but rather the individual's *robust decisiveness* over these alternatives. Thus the capability approach and the republican approach draw the boundary between 'unfreedom' and 'freedom' differently. For the capability approach, that boundary lies between case 1 and case 2, whereas for the republican approach it lies between case 2 and case 3. As Sen puts it, each of the two approaches has a particular discriminating power that the other approach lacks.

> "I would argue that we need both the capability approach and the republican approach to point to different aspects of freedom. The former approach concentrates on whether someone is actually free and able to achieve those functionings that she has reason to want, and the latter on whether the capability enjoyed is conditional on the favours and goodwill of others. If through the emendation proposed by Pettit the capability approach becomes just like the republican approach, then we would be one distinction short." (Sen 2001, 55)

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by being decisive for *all* i-variants of a given profile, or in a less demanding sense by being decisive only for a certain subset of the set of all i-variants of a given profile.

Having a parametrical family of differentially robust definitions of freedom rather than a single such definition is, then, a virtue rather than a vice. An interesting implication of this, as I hope to have shown, is that our choice of the parameter *N* determines whether or not we are faced with the impossibility of a Paretian "*N*-liberal". If we choose *N* most broadly, then we will run into that impossibility. The republican therefore faces a trade-off between the robustness of freedom on the one hand and the weak Pareto principle on the other. The proponents of the republican account have two choices. *Either* they must find a definition of *N* that is *both* sufficiently broad to capture the requirement of robust (namely context- and content-independent) decisiveness *and* sufficiently narrow to avoid the impossibility of a Paretian republican – which might ultimately require lowering the standards of robustness. *Or* they must defend the claim that a suitable relaxation of the weak Pareto principle is consistent with core republican principles.

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Appendix: A modal logical formulation of liberal and republican freedom

The social-choice-theoretic formalization of liberal and republican freedom developed in this paper can be restated in terms of modal logic. The resulting notational variant of the formalization supports the claim that the liberal and republican accounts lie at different points within the same continuum, and that the contrast between the two accounts lies not in the fact that one considers only the actual world whereas the other considers also possible worlds, but rather in how large the class of possible worlds is that each considers.

We first define a simple propositional language based on ranking propositions, augmented with modal operators (for an exposition of modal logic, see Priest 2001).

- For any two alternatives $x, y \in X$ and any individual $i \in \{1, 2, \ldots, n\}$, xR_iy, xI_iy, xP_iy are propositions ("representing individual preferences").
- For any two alternatives $x, y \in X$, xRy, xIy, xPy are propositions ("representing social preference").
- If ϕ and ψ are propositions, then so are $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$.
- If ϕ is a proposition, then so are $\Box\phi$ and $\Diamond\phi$.
- There are no other propositions.

An interpretation for this language is given by a triple $<W, N, v>$. Here W is a non-empty set of possible worlds. The function N is an accessibility function that assigns to each world $w \in W$ a subset $N(w) \subseteq W$ – possibly empty, possibly universal – interpreted as the set of all those worlds that are accessible from world w. Finally, v is a truth-function that assigns a truth value (*true* or *false*) to each proposition at each world. For any proposition ϕ and any world $w \in W$, we write $v_w(\phi)$ to denote the truth-value of proposition ϕ at world w. For each $w \in W$ and any ϕ, ψ, the function v_w has the following properties:

- $v_w(\neg\phi) = true$ if and only if $v_w(\phi) = false$;
- $v_w((\phi \wedge \psi)) = true$ if and only if *both* $v_w(\phi) = true$ *and* $v_w(\psi) = true$;
- $v_w((\phi \vee \psi)) = true$ if and only if *at least one of* $v_w(\phi) = true$ *or* $v_w(\psi) = true$;
- $v_w((\phi \rightarrow \psi)) = true$ if and only if *at least one of* $v_w(\phi) = false$ *or* $v_w(\psi) = true$;
- $v_w(\Box\phi) = true$ if and only if, for <u>all</u> $w^* \in N(w)$, $v_{w^*}(\phi) = true$.
- $v_w(\Diamond\phi) = true$ if and only if, for <u>some</u> $w^* \in N(w)$, $v_{w^*}(\phi) = true$.

These are standard definitions. The two operators $\Box$ and $\Diamond$ are typically interpreted as necessity and possibility operators, respectively. So, informally, $\Box\phi$ is true at world w if ϕ is true at *all* worlds w^* that are accessible from world w; and $\Diamond\phi$ is true at world w if ϕ is true at *some* world w^* that is accessible from world w.

As suggested in the paper, each profile $\underline{R}$ can be interpreted as a possible world. Then $W := \mathbf{U}$ is the set of all such possible worlds. Further, each profile $\underline{R}$ can be interpreted as an assignment of truth-values to all individual ranking propositions of the forms xR_iy, xI_iy, xP_iy. Moreover, once we fix an aggregation function F, each profile $\underline{R}$ (together with F) induces an assignment of truth-values to all social ranking propositions of the forms xRy, xIy, xPy. The truth-values of propositions involving $\neg$, $\wedge$, $\vee$, $\rightarrow$ are then determined by the truth-values of all such individual and social ranking propositions (together with the reflexivity, transitivity and connectedness of each R_i and R). Finally, once we fix an individual i, the neighbourhood function $N(\underline{R}, i)$ defined in the paper can be interpreted as an accessibility function assigning to each world a corresponding set of accessible worlds (relative to *i*), that is, $N(\underline{R}) := N(\underline{R}, i)$. Note that the accessibility function *N* has only *one* argument whereas the original neighbourhood function has *two* (since we have fixed individual *i*). The truth-values of propositions involving the modal operators $\Box$ and $\Diamond$ are now determined by the truth-values of all non-modal propositions (together with *N*).

Thus the set **U** together with a fixed *F*, a fixed *i*, and a fixed *N* induces an interpretation of our propositional language.

We can now restate the key definitions of this paper in our propositional language. As required, let *F* and *i* be fixed, and let *N* be some neighbourhood/accessibility function. Then

- *i*'s preferences over *x* and *y* are satisfied at world $\underline{R}$ if and only if

  (1) $((xP_iy \rightarrow xPy) \wedge (yP_ix \rightarrow yPx))$ is *true* at $\underline{R}$;
- *i* is *N*-decisive over *x* and *y* at world $\underline{R}$ if and only if

  (2) $\Box((xP_iy \rightarrow xPy) \wedge (yP_ix \rightarrow yPx))$ is *true* at $\underline{R}$.

Thus, on both the liberal and the republican account, saying that individual *i* is free over *x* and *y* at $\underline{R}$ means that $\Box((xP_iy \rightarrow xPy) \wedge (yP_ix \rightarrow yPx))$ is *true* at $\underline{R}$, for a suitable *N*. So, on both accounts, saying that an individual is free is a *modal* proposition. The difference between the two accounts lies solely in the adopted *interpretation* of that modal proposition. The accounts differ in how broadly or how narrowly they define the accessibility function *N*. For each $\underline{R}$, the liberal account (in a narrow version) defines $N(\underline{R})$ to be the set of all *i*-variants of $\underline{R}$, thus identifying freedom with content-independent decisiveness. The republican account (in a demanding version) defines $N(\underline{R})$ to be **U** or some large subset of **U**, thus identifying freedom with context- and content-independent decisiveness.[11]

[11] The republican might raise the following objection. Defining freedom in terms of proposition (2), which involves only a *single* modal operator, seems to conceal the fact that there are *two* 'dimensions' of modality that the republican is concerned with. One such dimension is given by *content*-independence and the other by *context*-independence.

Similarly, we can formalize the liberal concept of *interference* and the republican concept of *domination* in our propositional language. Consider the negations of the two propositions in (1) and (2) above.

- i's preferences over x and y are *not* satisfied at world $\underline{R}$ if and only if

  (3) $((xP_iy \wedge \neg xPy) \vee (yP_ix \wedge \neg yPx))$ is *true* at $\underline{R}$;

- i is *not* N-decisive over x and y at world $\underline{R}$ if and only if

  (4) $\Diamond((xP_iy \wedge \neg xPy) \vee (yP_ix \wedge \neg yPx))$ is *true* at $\underline{R}$.

Now one might be tempted to think that (3) formalizes *interference* with respect to x and y (in the liberal sense) and (4) formalizes *domination* with respect to x and y (in the republican sense). One might be tempted to think so if one believes – in my view incorrectly – that only domination, but not interference, is a modal notion.

The claim that (4) formalizes domination seems uncontroversial. If the accessibility function N is defined sufficiently broadly – that is, if $N(\underline{R})$ is **U** or some large subset of **U** – then (4) indeed seems to capture the republican concept of domination. Saying that individual i suffers domination with respect to x and y at $\underline{R}$ means that there is a possible world in the relevant neighbourhood of $\underline{R}$ in which individual i's choice over x and y is not respected.

Remember that in the paper I defined *context- and content-independent decisiveness* as the requirement that the individual's preferences be content-independently decisive not only at the actual profile, but also at *all* possible profiles that result from the actual one if that individual's preferences are held fixed, but some other individuals change their preferences. This definition might suggest a logical formalization in terms of a *nested* use of *two* modal operators, where each of these two operators has a different interpretation (that is, in terms of a different corresponding accessibility function). Specifically, one operator might be interpreted as quantifying over different possible *contents* and the other as quantifying over different possible *contexts*. However, in the social-choice-theoretic framework of this paper, such a nested use of these two (more narrowly interpreted) modal operators is equivalent to the use of a single modal operator, where that single operator is interpreted more broadly (compare footnote 6). Therefore the formalization of republican freedom in terms of proposition (2) seems more parsimonious than one in terms of a nested use of two modal operators. But there are clearly avenues for further research here.

But the claim that (3) formalizes interference is not correct, in my view. Note that (3) says only that individual *i*'s *actual* preferences at $\underline{R}$ are not satisfied. Suppose we take Berlin's (1969) demand seriously that an acceptable account of (negative) freedom should not entail "that a person can make themselves free just by adapting their preference appropriately". Then it seems that there are possible instances of *interference* in which (1) holds and therefore (3) does *not* hold. This suggests that interference – even in the liberal sense – should also be formalized in terms of proposition (4). However, the accessibility function N would have to be defined more narrowly than on the republican account, that is, we might define $N(\underline{R})$ to be the set of all *i*-variants of $\underline{R}$ rather than all of **U**, as explained above.

In conclusion, on both the liberal and the republican account, saying that individual *i* is *free* over x and y is to say that $\Box((xP_iy \rightarrow xPy) \wedge (yP_ix \rightarrow yPx))$. Also, on both accounts, saying that individual *i* suffers *interference* or *domination* with regard to x and y is to say that $\Diamond((xP_iy \wedge \neg xPy) \vee (yP_ix \wedge \neg yPx))$. The only difference between the two accounts lies in how narrowly or broadly the accessibility function N is defined, that is, how broadly we define the neighbourhood of possible worlds around the actual world that we consider relevant for assessing an individual's freedom. Finally, if, for each $\underline{R}$, $N(\underline{R})$ is defined to contain only $\underline{R}$ itself, then the corresponding modal account of freedom collapses into the preference satisfaction account.