ON THE PERFORMANCE OF SOCIAL BENEFITS SYSTEMS*

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Abstract

The paper analyses the performance of unemployment benefit systems in a search-theoretic framework. The criteria of evaluation comprise the alleviation of poverty and the reduction in income inequality, whilst the diversity of opinions about these is taken into account. Also, the trade-off between the attainment of social objectives and work incentives is examined.

**Keywords:** unemployment insurance, pay-roll taxation, poverty, inequality, welfare.

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1 Introduction

Social insurance is a prominent institution in developed economies, designed to protect economic agents against income risk. The form of social insurance schemes found in practice varies considerably: unemployment benefit in the UK is a flat rate, whereas it is earnings related on the Continent and the US; the replacement rates and the methods of finance differ. This paper analyses the performance of alternative unemployment benefit systems in a search-theoretic framework. The relative performance of flat-rate and earnings-related unemployment benefits will be assessed in the light of policy objectives such as the reduction in inequality and the alleviation of poverty. However, these policy objectives may not command a universal consensus because of either a diversity of opinion or an intrinsic arbitrariness in the parameters characterising the social welfare function \(^1\). So we might ask whether the ranking of the benefit regimes depends on the parameters of the social objective; although people may disagree about parameters, could they agree on a ranking?

There is also a potential trade-off between the equity objective of poverty alleviation and the efficiency consideration of work incentives. A greater benefit might increase the income of an unemployed beneficiary and thereby reduce the difference between his income and the poverty line. For a person who remains

\(^1\) As Atkinson (1993) observes in the context of poverty alleviation: "Such a 'sharp' representation of the social objective may not, however, be universally accepted. There may well be disagreement about the location of the poverty line ... Alternatively, there may be agreement about the location of ..[the poverty line], but concern for the non-poor, or the group close to the poverty line" (p.17).
in poverty, this increase reduces his poverty. But an increase in benefits reduces the incentives to work (particularly for those persons with a current low job productivity). Consequently, unemployment might rise, possibly increasing the numbers of the poor, raising aggregate poverty. This potential trade-off is examined in a general equilibrium setting. Again, does the resolution of this trade-off depend on the parameters of the social objective, or is it unambiguous for all admissible parameters?

The paper is structured as follows: Section 1.2 briefly reviews the related literature. The model is presented in section 2. It juxtaposes a flat-rate and an earnings-related unemployment benefit regime in a general equilibrium framework. Incentive problems of the benefit regimes are also examined. Section 3 assesses the relative performance of the benefit regimes, and analyses the conditions under which an initial ranking of the benefit regimes is reversed. In section 3.1 the evaluation criterion is poverty, and in section 3.2 it is inequality. Within the (limited) context of the current search-theoretic framework, it is shown that no benefit regime dominates its competitor in all circumstances. Section 3.3 examines whether there is an equity-efficiency trade-off. As this section makes clear, preserving incentive compatibility for its own sake is a value judgement, and its normative character ought to be made explicit. Section 4 concludes.

1.1 The context of the problem

The nature of the risk analysed in this paper is that of a temporary job loss, engendering the institutional response of unemployment insurance. Pension schemes attempting to counter the risk of a permanent job loss through age or disability have
been examined elsewhere (e.g. Diamond and Mirrless (1986)). Poynter and Martin (1995), Habib (1995), and Schluter (1995) provide extensive examinations of the complexities governing the British, the French, and the German social insurance system.

These institutional features have been relatively neglected in the recent literature on social insurance schemes. Some papers completely ignore the institutional rules, whilst taking them explicitly into account dramatically reverses the implications of some popular models (Atkinson (1990)). An example is the treatment of eligibility conditions in the Shapiro and Stiglitz (1984) efficiency wage model. Since, in practice, shirking automatically disqualifies the claimant from benefit entitlement for a non-trivial period, their shirking condition is simply not applicable. Atkinson and Micklewright (1991) develop this criticism of neglecting institutional considerations in economic modelling. The incentive problems engendered by the benefit system are further analysed in Besley (1990), who compares the relative performance of a means-tested benefit, which tops up incomes in order to reach a pre-defined poverty line, with a universal benefit, paid to all persons in the economy, even to the richest person. Some authors, like Besley and Coate (1992), model benefit institutions in a historical fashion. They design a revelation mechanism by subjecting the benefit claimant to a sufficiently large work requirement, the result of which bears a strong resemblance to the Poor Laws in Britain and practices in colonial India (Dreze (1990)).

Easley, Kiefer, and Possen (1985) use a two-state, two-period general equilibrium model with two heterogeneous risk averse agents in order to analyse the (relative and joint) performance of unemployment insurance and negative income tax systems. Their numerical simulations suggest that under certain parame-
ter configurations and functional forms, both programmes bring about Pareto improvements. Other researchers have incorporated efficiency wages into general equilibrium models of unemployment: when workers' effort depends on the relative remuneration of capital and labour, Agell and Lundberg (1992) show that any tax policy which increases the wage-rental rate leads to a reduction in unemployment.

The model developed in this paper is based on a standard search-theoretic framework as described in Pissarides (1990). Its principal attraction stems from the fact that it endogenises the risk of losing one's income. This is achieved by modelling trade in the labour market as uncoordinated, time consuming and costly for both workers and firms. A congestion or thin market externality will be present in most equilibrium conditions, their levels depending on the number of workers and firms engaged in search. With the unemployment rate thus endogenised, its level will reflect the incentive structure of the benefit system. It is then possible to examine to which extent unemployment is caused by the incentive structure under operation, rather than by the stochastic nature of the exogenous shocks. Involuntary unemployment can be distinguished from 'voluntary' unemployment. It has been observed that "(t)he optimum taxation models developed to date are not satisfactory in this regard, since the treatment of the labour market is insufficiently developed" (Atkinson (1989), p.42). The model employed in this paper is an attempt at this in the context of social insurance. In using this framework, the present analysis is akin to Atkinson (1990)'s, in that a search-theoretic framework is also employed. However, he analyses these issues within a model of a segmented labour market: a primary sector job offers high wages and unemployment insurance, whereas the low paid jobs in the secondary sector are
uninsured.

2 The model

This section spells out a standard search-theoretic framework with stochastic job matching derived from Pissarides (1990), chapter 5, which is itself an extension of the basic search model of Diamond (1982). The only novelty is introduced by considering explicitly an unemployment benefit \( b \), and the budget constraint faced by the government which levies a pay-roll tax \( \tau \) on workers and employers. Moreover, the benefit might be further constrained by incentive considerations.

The aim of this section is to establish a simultaneous equation system, which permits the determination of the endogenous unemployment rate \( u \), the set of incentive compatible benefit levels, and the level of the pay-roll tax necessary to finance the latter. Stochastic job matching permits the derivation of a non-trivial wage distribution, which then may give rise to a non-trivial benefit distribution.

Assume that workers are identical ex ante, and if they search, they do so with the same intensity, but the productivity \( \alpha \) of a particular job match varies. Its precise value is only revealed upon contact although the distribution of productivities \( G(\alpha) \) is common knowledge. Workers are heterogeneous ex post. Since all workers are identical ex ante, they have the same reservation productivity \( \alpha^r \). If productivity has the distribution \( G \) with support \([\alpha_l, \alpha_u]\), then workers accept all jobs characterised by
\[ \alpha \geq \alpha_r: \]
\[ \int_{\alpha_r}^{\alpha} dG = 1 - G(\alpha_r) \]  
\[(1)\]

Let \( u \) denote the unemployment rate and \( v \) the vacancy rate (being the number of job vacancies over the total labour force). Trade in the labour market is uncoordinated, time consuming and costly. This notion is captured by a matching function, \( x(u, v) \), giving the fraction of job matches \( x \) as a function of the unemployment and vacancy rates. This function is commonly assumed to be homogeneous of degree one. Define \( \theta := v/u \) as a measure of labour market tightness. Thus \( x/v = x(\theta^{-1}, 1) =: x(\theta^{-1}) \). The stochastic processes governing the economy are Poisson processes. A vacant job becomes occupied at a rate \( q(\theta, \alpha_r) := [1 - G(\alpha_r)] x(\theta^{-1}) \) since only jobs are formed which exceed the reservation productivity. Workers transit from unemployment to employment at rate \( \theta q(\theta) \), but they become unemployed at the exogenously given separation rate \( s \).

In equilibrium, the inflow into unemployment equals its outflow, \( \theta q(\theta, \alpha_r) u = s(1 - u) \), whence the Beveridge Curve (BC) in \((v, u)\)-space is arrived at:

\[ u = \frac{s}{s + \theta q(\theta, \alpha_r)} \]  
\[(2)\]

The benefit system

A person may apply for a benefit \( b \) when unemployed, but the institutional conditions of eligibility may be more extensive. For

\( ^2 \)Firms have a reservation productivity, but, as demonstrated below, the Nash bargaining rule governing wage determination implies that both workers and firms agree on a common reservation productivity.
instance, unemployment benefit may be paid for a limited duration only or it may be contingent on the contributions record of the claimant. Below two benefit schedules will be discussed, viz. a flat-rate (FR) and an earnings-related (ER) schedule. The attainment of any policy objective is, however, constrained by the scarcity of resources, and the social budget needs to be balanced. It is a common institutional practice that contributions are shared equally between employer and employee. Here it is implemented as a pay-roll tax $\tau$ on gross earnings $w$, levied in equal proportions on the two parties.

**Firms**

The firm has a standard neo-classical production function, exhibiting constant returns to scale, but because of the reservation productivity rule it has to be written as $\hat{F} = \tilde{F}(K, N\alpha_f^y)$ with conditional expectations $\alpha_f^y = \mathcal{E}[\alpha|\alpha \geq \alpha_f]$, since firms have to forecast productivities. $\alpha_f$ is the reservation productivity of the firm below which workers are rejected. The production function can be re-written as $f(k)$, where $k := K/N\alpha_f^y$. The value of an occupied job $J$ or a vacancy $V$ are captured by asset value (or "no arbitrage") equations. The value of a job $J$ is

$$r(J + \alpha k) = \alpha [f(k) - \delta k] - w(1 + \tau) + s(V - J)$$

(3)

since it produces $\alpha [f(k) - \delta k]$ but the firm has to pay a wage $w$ and a tax $w\tau$, and loses a worker at the exogenously given separation rate $s$. $r$ is the interest rate and $\delta$ the depreciation
rate. The value of a vacancy $V$ is

$$rV = -\Gamma + q(J^e - V)$$  \hspace{1cm} (4)$$

where $\Gamma$ is the search cost of the vacancy, and the vacancy becomes occupied at rate $q(\theta, \alpha_f)$. In equilibrium the value of the vacancy must be zero, $V = 0$, which implies $J^e = \Gamma/q$, since otherwise the firm would change its behaviour. The condition $J = 0$ yields the reservation productivity, since firms employ workers as long as the job is profitable, whilst reaping the surplus from the intra-marginal worker. In consequence,

$$\alpha_f = \frac{(1 + \tau)w}{f'(k) - (\delta + r)k}$$  \hspace{1cm} (5)$$

Firms choose $k$ optimally, which implies $f'(k) = \delta + r$. After taking conditional expectations of the valuation of an occupied job $J$ and imposing $V = 0$, the equilibrium condition on vacancy supply by firms becomes:

$$\alpha^e [f'(k) - (\delta + r)k] - w^e (1 + \tau) - \frac{(r + s)\Gamma}{q} = 0$$  \hspace{1cm} (6)$$

**Workers**

Being unemployed implies a value $U$ to the unemployed because of the receipt of a benefit $b$ and a chance of $\theta q(\theta)$ to become employed. The value of employment $E$ is derived from a net wage $w(1 - \tau)$, but the worker may lose her job with an exogenous probability $s$. Consequently the asset value equations
\[ rE = w(1 - \tau) - s(E - U) \quad (7) \]
\[ rU = b + \theta q(\theta, \alpha_r)(E - U) \]

\textbf{Wages}

The occupied job creates a surplus which must cover the search costs of both parties. It is commonly assumed in the search literature that the surplus bargained over by the worker and the firm is divided according to the Nash bargaining rule (Binmore, Rubinstein, and Wolinsky (1986)). This Nash rule implies that workers and firms have a common reservation productivity, \( \alpha_r = \alpha_f \). Suppose that this surplus is divided equally \(^3\): the chosen wage then maximises \((E - U)^{0.5}(J - V)^{0.5}\), which implies the wage equation

\[ 2(1 - \tau)w = b + [(1 - \tau) / (1 + \tau)] \theta \Gamma + \alpha [f(k) - (\delta + \tau)k] \quad (8) \]

A higher productivity is remunerated by a higher wage.

\textbf{Flat-rate (FR) benefits and eligibility}

If the benefit is a flat rate, it is most conveniently formalised as a constant fraction of expected wages \( b = \lambda w^e \) where \( \lambda \in (0; 1) \). Following Pissarides (1990, p.99), it is also convenient to formalise the firms' search costs in a similar manner, \( \Gamma = \gamma w^e \).

\(^3\)Note that the resulting returns will, in general, not be efficient, since neither party obtains its respective marginal product.
If the social budget is balanced, expected expenditures have to equal expected incomes

$$\int_{0}^{a} (1-u) 2\tau w d\tilde{G} = 2(1-u)\tau w^e = bu$$  \(9\)

where $\tilde{G}$ is the conditional productivity distribution. This formulation implies that all unemployed receive a benefit $b$, even those whose productivity falls below the generally accepted reservation productivity $\alpha_r$. Solving (9) yields

$$\lambda = \left[ \frac{1-u}{u} \right] 2\tau$$  \(10\)

Some algebraic manipulations yield the equation

$$(1+\tau) - \frac{1+\tau}{1-\tau} \frac{1-u}{u} 2\tau - \gamma u \left[ \frac{1}{u} - \frac{\tau + s}{s} \frac{1}{1-u} \right] = 0$$  \(11\)

This is the so-called Vacancy Supply curve ($VS$), which, like the Beveridge curve, is usually analysed diagrammatically in $(u,v)$-space. Also an expression for the reservation productivity can be derived

$$\frac{\alpha_r}{\alpha^e} = \left( \frac{1+\tau}{1-\tau} \lambda + \theta \gamma \right) \left( 2(1+\tau) - \frac{1+\tau}{1-\tau} \lambda - \theta \gamma \right)^{-1}$$  \(12\)

where $\alpha^e$ denotes the expected productivity.

In summary, the system consists of four equations, viz. (2),(6),(11), and (12). Given a pay-roll tax rate $\tau$, the unknowns are $u, v, k$ and $\alpha_r$. Differentiating the $VS$ curve (11) shows that, as usual,
$VS$ is upward sloping in $(v, u)$-space. As regards the Beveridge curve (2), the problem is more complicated because of the presence of $\alpha_r$. But, following Pissarides(1990), making the assumption $(\partial \alpha^e/\partial \alpha_r) \alpha_r/\alpha^e < 1$ - at the optimum a rise in the reservation productivity increases the conditional mean proportionately less- the Beveridge curve can be shown to be downward sloping. This follows since the assumption implies that a change in the labour market tightness $\theta$ has a stronger direct effect on the probability of leaving unemployment, which exceeds the indirect effect through the reservation productivity. Finally, the unemployment rate is determined by the intersection of the two curves $VS$ and $BC$ depicted in Figure 1, and $k$ is derived recursively from $f'(k) = \delta + r$.\(^4\)

**The effect of a change in the pay-roll tax**

An increase in the pay-roll tax may be examined diagrammatically, analysing the behaviour of (2) and (11) separately. Holding $u$ constant and differentiating (11) totally gives an equation in $d\tau$ and $dv$

$$A d\tau = \gamma \left[ \frac{1}{u} - \frac{r + s}{s} \frac{1}{1 - u} \right] dv$$

where $A(\tau) := 1 - 2 \frac{1 - u}{u} \left( \frac{1 + \tau}{1 - \tau} + \frac{\tau}{(1 - \tau)^2} \right)$

At $\tau = 0$, $A (0) < 0$ and $A$ falls monotonically with $\lim_{\tau \to 1} A(\tau) = -\infty$. Thus $dv/d\tau < 0$ and $VS$ shifts down. Concerning the Beveridge curve (2), the same reasoning which showed that $BC$ is downward sloping leads to the conclusion that it shifts to the

\(^4\)Observe that Pissarides'(1990) partial equilibrium model is nested within this general equilibrium framework, as can be seen by setting $\tau = 0$.  

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right. As \( \tau \) increases both returns to the employed workers and to firms fall and the reservation productivity thus increases. In consequence, an increase in \( \tau \) unambiguously leads to a higher unemployment rate. The size of the increase depends on the distribution of productivities \( G \). This situation is depicted in Figure 1.

Flat-rate benefits and incentive compatibility

We have considered a (non-negative) value of the pay-roll tax below 100\%, but the domain may be further constrained by considerations of incentive compatibility. Is the benefit sufficiently low that all unemployed beneficiaries have an incentive to search and to accept any given job offer? The incentive constraint is

\[
b = \lambda w^e \leq (1 - \tau) w(\alpha_r)
\]  

(14)

since \( w(\alpha_r) \) is the lowest wage in the economy, associated with the lowest admissible job productivity, viz. the reservation productivity \( \alpha_r \). Using the wage equation (8) and the equation for the reservation productivity yields

\[
0 \leq \left[ \frac{1 - \tau}{1 + \tau} \right] \theta \gamma w^e
\]  

(15)

which holds for all \( \tau \in [0; 1) \). The flat-rate benefit does not create an incentive problem because wages are always sufficiently high.

Earnings-related benefits (ER) and eligibility conditions

When the benefit is earnings-related, \( b = \rho w \) with replacement ratio \( \rho \), the inter-temporal structure of the economy be-
comes important since benefits are determined by past earnings. Assume then that agents are infinitely lived (an implicit assumption so far), but that benefits last for one period only so that persons continuously unemployed for more than one period are ineligible for the benefit. The balanced budget becomes

$$\int_{\alpha_r}^{\alpha_u} (1 - u) 2\tau wdG = \int_{\alpha_r}^{\alpha_u} upwdG$$

(16)

since all those whose productivity falls below the reservation productivity, $G(\alpha_r)$, are not entitled to an unemployment benefit. In order to receive a benefit one must have been separated from the job at most in the last period. This is a realistic assumption because most unemployment benefit programmes (as distinct from unemployment assistance) make eligibility conditional on a work or contributions record.\(^5\) Simplifying (16)

\(^5\)Restricting benefit eligibility in this way is analytically convenient since "productivity in the last period" is the state variable in terms of which the subsequent welfare analysis can easily be carried out diagrammatically. Moreover, if $\tau \leq \tau^*$, the $ER$ benefit is a mean-preserving spread of the $FR$ benefit. However, as pointed out by an anonymous referee, the $ER$ benefit differs from the $FR$ benefit also in terms of its temporal nature. Suppose the eligibility restriction is removed so that the benefit depends on the wage of the last job. In this case the analytical details become more awkward. For instance, the balanced budget equation becomes contingent on the entire earnings history of the population. In order to satisfy the incentive compatibility constraint, $\tau$ has to be selected such that the highest possible benefit does not exceed the lowest possible wage offer. The former is attained when $\alpha = \alpha_u$ and the latter when $\alpha = \alpha_r$ in
yields an expression for the replacement ratio $\rho$

$$\rho = \frac{1 - u^2 2\tau}{u^2}$$

(17)

(10) and (17) look similar but tax rates (and thus unemployment rates) may differ because of incentive problems. The subscripts have been added to emphasise the potential difference.

Earnings-related benefits and incentive problems

An incentive problem occurs in this regime if the person is entitled to a high benefit which exceeds a current low net wage offer. The benefit may be high because of a high previous productivity level $\alpha$ and a high pay-roll tax $\tau$. Thus, to prevent this from happening, $\tau$ must be sufficiently low. How low? Examining the wage equation (8), the lowest wage is achieved when $\alpha = \alpha_r$ and the person is not entitled to the benefit. The highest wage, and thus the highest benefit entitlement, is attained when productivity is at its highest, $\alpha = \alpha_u$, and the entitlement is in turn the highest. Putting these together yields the incentive constraint

$$\rho \left[ 1 + \left( 1 - \tau \frac{\alpha_u F + \theta \Gamma}{\theta \Gamma} \right) \right] \leq 2 (1 - \tau) \quad \text{where} \quad F := f(k) - (\tau + \delta) k$$

(18)

two consecutive periods. But although the eligibility restriction is removed, the logic of the subsequent welfare analysis remains unchanged (cf footnote 7). Therefore the restriction remains for expositional clarity.

\*These are given by $w_{\min} = \theta \Gamma / (1 + \tau)$ and $w_{\max} = [2 (1 - \tau) - \rho]^{-1} [(1 - \tau) / (1 + \tau)] [\alpha_u F + \theta \Gamma]$. 

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Unfortunately one cannot derive a closed form solution since (18) depends on the unemployment rate \( u \), which can be conveniently examined only diagrammatically as the intersection of the VS and the BC curves. However, examining the boundaries of (18) for \( \tau = 0 \) and \( \tau = 1 \) shows that at low levels of \( \tau \) the constraint is satisfied, but at high levels the constraint is violated. By continuity there exists a critical level \( \tau^* \) (when (18) holds with equality), below which tax rates are incentive compatible but above which they are not.

What happens if the eligibility rules are relaxed for the earnings-related benefit, so that receipt of the benefit is only contingent on being unemployed? For instance, a previously ineligible unemployed person could receive a flat rate below the lowest earnings-related benefit. The following argument demonstrates that the critical level of \( \tau \), below which all tax rates are incentive compatible, exceeds \( \tau^* \) in this new regime. Set \( \tau = \tau^* \). First observe that, if the unemployment rate is held constant, awarding previously ineligible unemployed persons a benefit increases the number of beneficiaries, which reduces the replacement rate. Now let \( u \) vary. The outside option for the previously ineligible person increases, raising labour costs. The reservation productivity and thus unemployment rise whilst the replacement rate falls further. As only more productive jobs are formed, the lowest offered wage has risen. The discrepancy between the highest benefit entitlement and the lowest offered net wage increases, the incentive constraint becomes a bit more relaxed, and the tax rate can be increased whilst remaining incentive compatible.

**Summary**

Figure 2 summarises the preceding discussion of the model in \((u,v)\)-space. As long as \( \tau \leq \tau^* \), the earnings-related benefit is
incentive compatible, and the respective unemployment rates in the two benefit regimes are the same. But if the incentive compatibility constraint for the earnings-related benefit is violated, the unemployment rate increases and exceeds the one for the flat-rate regime. An increase in the pay-roll tax rate \( \tau \) increases the unemployment rate \( u \), but the precise increase depends on the distribution of productivities \( G \).

What do the benefit schedules look like? In Figure 3, they are depicted as a function of \( \alpha \), the last productivity of the unemployed claimant. It is assumed that \( \tau \leq \tau^* \), so the unemployment rates associated with the two regimes are equal, which implies that the two benefit parameters satisfy \( \lambda = \rho \). The diagram also shows the different eligibility conditions for the different regimes (but this argument will be generalised below). The position of the \( ER \) schedule depends on the location of \( \alpha_r \) and two possibilities are arise: either \( \alpha_r \) is sufficiently low, so that the \( ER \) schedule intersects the \( FR \) schedule, or it is so high that the \( ER \) benefit always exceeds the \( FR \) benefit.

3 Evaluating the performance of the benefit regimes

The criteria according to which the benefit regimes will be assessed are poverty, inequality, and a more general social welfare function. Does the ranking of the benefit regimes depend on which of these criteria is chosen? Moreover, these criteria incorporate value judgments and a certain degree of arbitrariness. For instance, not everyone may agree on the location of the poverty line. Or there may be disagreement about the sensitivity parameters of these criteria. Can this diversity of opinion
be accommodated, so that the ranking of the benefit regimes does not change as these parameters change? Finally, to which extent does the ranking depend on the eligibility conditions?

3.1 Poverty

The properties of the conventional poverty indices are well known, but the choice of a particular poverty index may be quite arbitrary. The particular choice may be defended in the light of the special question posed, and for the present analysis the decomposable poverty index proposed in Foster, Greer, and Thorbecke (1984) is convenient. Let \( z \) denote an exogenous poverty line, \( F \) is the distribution of incomes \( y \), and \( \beta \) a sensitivity parameter. The poverty index \( P_\beta \) only takes into account the income of the poor (all \( y \leq z \)), and weighs their (percentage) income shortfall from \( z \), i.e. the gravity of poverty, by the sensitivity parameter \( \beta \):

\[
P_\beta = \int_0^z \left( \frac{z - y}{y} \right)^\beta dF(y) = \sum_{k=1}^{K} v_k P_{\beta_k}, \quad \text{where } \beta \geq 0 \quad (19)
\]

The poverty index can be decomposed as follows. Partition the population into \( K \) groups with respective population share \( v_k \), and let \( P_{\beta_k} \) denote the computed poverty index \( P_\beta \) for group \( k \). Then the index is expressible as the weighted sum of poverty over the \( K \) subgroups of the population.

The natural partition in the model is to group the employed and the unemployed. If \( \tau = \tau^* \), the population and the wages of the employed are the same for the two benefit regimes. In
consequence, in an assessment of the relative performance of the two benefit regimes, one can concentrate on the poverty of the unemployed.

Does a change in the poverty line $z$ change the ranking of the benefit regimes whilst keeping the sensitivity parameter fixed? If $\beta = 0$, then (19) becomes the head count index. If $z = z_1$ in Figure 3 then $FR$ dominates $ER$ but if $z = z_2$ then $ER$ dominates $FR$. In fact $z = z_1$ is the trivial case since $FR$ dominates $ER$ for all sensitivity parameters $\beta$.

Does the ranking change when the sensitivity parameter $\beta$ changes whilst the poverty line remains unchanged at $z = z_2$? Is there a trade-off between the incidence and the gravity of poverty? For instance, two situations may emerge. In situation (a) a certain number of people live below the poverty line, but the income shortfall is not large. In situation (b) fewer are poor, but they suffer from a more severe income shortfall. Which situation is deemed worse is captured by the sensitivity parameter $\beta$. This trade-off can be examined when the poverty line exceeds the flat-rate benefit. A reversal of the initial ranking can be made by means of a continuity argument. If $\beta = 0$ then $ER$ dominates $FR$, but if $\beta = \infty$ then $FR$ dominates $ER$. Given the monotonicity of the poverty index, a critical level of exists $\beta, \beta^*$, such that for $\beta \geq \beta^*$ all $FR$ dominates $ER$.

What happens to the ranking when eligibility conditions change?\footnote{If the temporal eligibility condition is removed as suggested in the previous footnote, the logic of the above arguments remains unchanged. Mapping the incomes of the unemployed in (benefit,income)-space, the $FR$ is a horizontal line which is cut by the $ER$ benefit schedule. This crossing is sufficient to guarantee the existence of a critical $\beta$ at which a reversal of the poverty ranking of the two regimes occurs (when the poverty}
Assume the earnings-related benefit is extended to cover all unemployed by awarding the previously ineligible a flat rate. In this case the income shortfall of the poorest is less severe which translates into a higher critical level above which the ranking is reversed. On the other hand, FR may be restricted to the same beneficiary population as ER. Yet, the same continuity argument applies, leading to a reversal of the initial ranking.

### 3.2 Inequality

Another assessment criterion is income inequality. Similar decomposition considerations lead to the choice of the Generalised Entropy measure, defined by

$$GE_\beta = \frac{1}{\beta^2 - \beta} \left[ \frac{1}{n} \sum_i \left( \frac{y_i}{\mu} - 1 \right) \right]^\beta = \sum_k v_k^{1-\beta} s_k^\beta GE_{\beta k} + GE_{\beta B} \quad (20)$$

where $\beta$ is a sensitivity parameter, $y_i$ income of person $i$, and $\mu$ average income. The index decomposes, so that $GE_{\beta k}$ measures inequality within group $k$, where $v_k$ is its population share, and $s_k$ its income share. $GE_{\beta B}$ measures the inequality between groups when each person within group $k$ is assigned the average income of the group.

Which benefit system is associated with higher income inequality? The population partitions into the set of the employed, unemployed beneficiaries and unemployed non-beneficiaries. Let the eligibility rules for FR be (9) and for ER (16). If the pay-roll tax satisfies $\tau \leq \tau^*$, then the group size and the inequality of the employed are the same for the two benefit regimes. For line exceeds the flat-rate benefit).
FR, there is perfect equality amongst the unemployed beneficiaries. But with the eligibility rule (16) there is always a group of unemployed non-beneficiaries, whose income share is zero. This implies that FR always dominates ER, irrespective of β.

What happens if the eligibility rule (9) is changed so that the FR benefit covers exactly the same population as ER? Choosing again in an incentive compatible manner, the various income groups have the same size. The between-group component, GE_{βB}, will also be the same, since ER then is a mean-preserving spread of FR. However, there is perfect equality amongst the group of beneficiaries when the benefit is FR. Thus, even under these new eligibility rules, FR always dominates ER.

3.3 Social welfare and work incentives: a trade-off?

There might be an equity efficiency trade-off between social welfare and work incentives. Let the welfare criterion be the poverty index (19). A socially desirable pay-roll tax, then, is the solution to the programme \( \min \tau, P_\beta \). The problem is not a trivial one, for although an increase in the benefit reduces the gravity of poverty, it might increase its incidence as the unemployment rate rises. Moreover, the initial benefit increase might be eroded away by a rise in the number of beneficiaries. Finally, is the socially desirable \( \tau \) incentive compatible?

If the benefit is a flat rate, differentiating \( \lambda \) with respect to \( \tau \) yields \( d\lambda/d\tau = [2/u] [(1-u) - (du/d\tau) \tau/u] \). The sign of this expression is ambiguous and depends on the elasticity of unemployment, and thus on the distribution of productivities \( G \) and the reservation productivity \( \alpha_r \). Two polar cases are imaginable: In case (a) the labour force is highly skilled (skill has to be loosely interpreted here since it is ex ante unobservable), where
most frequency mass is concentrated on high productivity levels. A sufficiently small increase in \( \alpha \), leads to a small increase in \( u \) and \( \lambda \) increases. In case (b) the labour force is badly skilled, and most frequency mass concentrates on low productivity levels. The same increase in \( \alpha \) leads to a large increase in \( u \) and \( \lambda \) falls. If the poverty line \( z \) is sufficiently low, so that no employed workers are deemed to be in poverty, and after defining \( z \) relative to \( z = \pi w^e \), the social welfare criterion (19) reduces to \( P_\beta = u [(\pi - \lambda) / \pi]^{\beta} \). As \( \tau \) increases, so does \( u \). Fewer persons are taxed at a higher rate, and the revenue is distributed amongst more persons. But in case (a) the increase in \( \lambda \) can outweigh the increase in \( u \), an effect which becomes stronger the higher is \( \beta \).

A further question is raised by the issue of incentive compatibility: is the socially desirable pay-roll tax incentive compatible? For the flat-rate benefit regime this is trivially true, since all pay-roll taxes are incentive compatible. But for the earnings-related benefit regime, to which a similar analysis applies, there is a non-trivial incentive constraint. Whether the socially desirable \( \tau \) satisfies this constraint depends again on the distribution of productivities \( G \). The principal insight is, however, that the attainment of incentive compatibility is a value judgment which needs to be justified. An incentive compatible pay-roll tax might not be the socially desirable one. In particular it may be socially desirable that some low skilled persons face the wrong set of incentives, since the aggregate welfare effect exceeds the welfare loss caused by the latter.
4 Conclusion

Flat-rate (FR) benefits always produce lower inequality than earnings-related (ER) benefits, but poverty outcomes depend on the parameters of the poverty index. By changing these parameters, most initial rankings of the two benefit regimes can be reversed. Moreover, a trade-off between equity and efficiency might occur, which makes clear that the attainment of incentive compatibility for its own sake is a value judgment which needs to be justified.

However, one important assumption of the model is that agents are risk neutral, so that insurance has no role to play since agents only care about mean returns. This risk neutrality is a major cause for the negative results characterising the earnings-related benefit. Yet insurance considerations are important in the design of actual tax-benefit systems: how would the welfare assessments change? Furthermore, some normative question arise: what would be the optimal insurance contract? If workers are assumed to be risk averse, this change destroys the linearity of the no arbitrage conditions for the worker and renders the model analytically intractable. All the same, some qualitative observations may be made, which point to ingredients a useful model of (social) insurance should incorporate. An earnings-related benefit seems to perform well with a proportional pay-roll tax since it achieves a desirable stabilisation of incomes in every state of the world. In the absence of incentive considerations, Yaari (1976) shows that the optimal consumption policy converges to mean consumption for an agent exposed to an iid income risk, when the rate of interest is zero and no borrowing constraints are imposed. However, this stochastic process needs justification by means of a labour market model. The
issues raised by these considerations warrant further research.
Figure 1: Increasing the pay-roll tax \( \tau \) increases the unemployment rate.
Figure 2: Unemployment rates and taxes
Figure 3: Possible benefit schedules
5 Appendix: Risk aversion, tax-financed benefits, and Pareto improvements

This appendix attempts to indicate how the tax-benefit system can bring about Pareto improvements when agents are risk averse. The argument is only a partial equilibrium one, but the qualitative features are taken from the preceding (general equilibrium) model. Assume that workers are homogeneous. The representative agent maximises the expected discounted stream of utility, where utility is solely defined over consumption $E \left[ \sum_0^\infty \beta^t U(c_t) \right]$, where $U(\cdot)$ is increasing and concave, $U(0) = 0$ and $U'(0) < \infty$. With absent capital markets, the agent consumes all income in each period. As before, let $s$ denote the exogenous job separation rate, $\tau$ the tax, $b$ the benefit, and $w$ the wage. $w$ is drawn from the distribution $F$, with density $f$, over support $[0; \bar{w}]$. For notational convenience, ignore $\tau$ and consider only $b$ which may be zero. Below, we compare the welfare situation with no benefits ($b = 0$) to a situation with a small benefit ($b > 0$).

This problem will be analysed recursively. The agent's expected utility is, when accepting a wage offer $w$,

$$U(w) + \beta [(1 - s) v(w) + sv(b)],$$

and if he chooses to search

$$U(b) + \beta \int_b^{\bar{w}} v(x)f(x)dx,$$

where $v$ denotes the value function. The latter becomes

$$v(w) = \max \left[ U(w) + \beta [(1 - s) v(w) + sv(b)]; U(b) + \beta \int_b^{\bar{w}} v(x)f(x)dx \right]$$

(21)

---

This appendix is based on McCall (1970) and Lucas and Stokey (1989), section 10.7
which is well defined for the above problem. It is convenient to define 
\[ A := U(b) + \beta \int_{b}^{\bar{w}} v(x) f(x) dx, \]
and it follows immediately that there is a unique \( w, w^* \), such that \( (1 - \beta)A = U(w^*) \). This already is the optimal stopping rule: \( w^* \) is the reservation wage below which all job offers will be rejected. The value function then becomes

\[
v(w) = \begin{cases} 
A & \text{for } w < w^* \\
\frac{U(w) + \beta s A}{1 - \beta (1 - s)} & \text{for } w \geq w^*
\end{cases} \tag{22}
\]

which is a continuous function at \( w^* \) and depicted in Figure 4 for the case \( b = 0 \).

In order to derive the defining equation of the reservation wage, eliminate \( A \), and break up the integral from \( b \) to \( \bar{w} \) into one from \( b \) to \( w^* \) and one from \( w^* \) to \( \bar{w} \). It then follows, using \( U(w^*) = (1 - \beta)A \), that

\[
U(w^*)[1 + \beta s - \beta F(w^*) + \left( \frac{\beta^2 s}{1 - \beta} + \beta \right) F(b)] = (1 - \beta + \beta s) U(b) + \beta \int_{w^*}^{\bar{w}} U(x) f(x) dx \tag{23}
\]

(Uniqueness of the reservation wage \( w^* \) due to a single crossing of the schedules can be verified by differentiating both sides with respect to \( w^* \)).

Analysing the effect of a 'small' benefit system reduces to examining \( dv(w)/dw \big|_{b=0} \). What happens to the reservation wage defined by equation (23)? The value of the outside option rises whilst the value of the job falls if taxes are levied on the employed. The overall effect is an increase in the reservation wage.
This is the incentive effect analysed in this paper. Since \( U(w^*) = (1 - \beta) A \), \( A \) rises as well. Whether the upper branch of equation (22) increases as well depends on the sign of the expression

\[
-U''(w) \left. \frac{dr}{db} \right|_{b=0} + \frac{\beta s}{1 - \beta} U'(w^*) \left. \frac{dw^*}{db} \right|_{b=0}.
\]

If, for instance, no taxes are levied, then \( dv(w)/dw|_{b=0} > 0 \) unambiguously, otherwise the concavity of the utility function will play an important role.

For such a case, Figure 4 depicts the effects of introducing a ‘small’ unemployment benefit. Since \( v(w) \) is the expected utility given the current state, the figure shows the areas for which the area is better off. It is not surprising that a small unemployment insurance benefit should make a risk averse agent better off; however, the figure shows that the incentive effect – an increase in the reservation wage – may, in fact, reduce the welfare of some agents.
Figure 14: The value functions
References


McCall, J. (1970). The economics of information and job


