Redistributive Taxation and Public Expenditures

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Abstract‡

We introduce a model of redistributive income taxation and public expenditure. Besides redistributing personal income by means of taxes and transfers, the government supplies goods and services. The government chooses the tax schedule that is found acceptable by the largest share possible of the population. We show that there is a unique income tax schedule that is universally acceptable. The progressivity of the income tax is shown to depend on the composition of the public expenditure and on the substitutability between the goods and services supplied by the government and the consumption goods privately obtained through the market. We test the empirical implications of the model. Specifically, we use OECD data to observe the relationship between marginal tax rates and the distribution over the taxpayers of the benefits produced by the specific composition of the government expenditure in the provision of goods and services. We confirm that for lower elasticities of substitution between public and private goods, there is a negative relationship between marginal tax rates and pro-taxpayer-bias, and for higher elasticities, there is a positive relationship.

Keywords: Government policy, Income Taxation, Public Expenditure

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1 Introduction

In this paper we study the interdependence between the income redistribution policies and the composition of public expenditure. The literature on income taxation has mostly focused on the disincentive effect of taxation on labour supply. In contrast, we emphasise that, besides redistributing income, the income tax also finances the public provision of a set of goods and services that are valued by individuals. Higher taxes reduce private consumption but increase the supply of the goods furnished by the government. Therefore, the individual valuation of alternative tax and expenditure policies depends on the balance between private consumption and public supply of goods. We also depart from the standard literature in that, instead of assuming that the income taxes are selected by majority voting, we focus on the tax schedules that are found universally acceptable, i.e. nobody wants to change their steepness. We show that there is a unique tax schedule that satisfies this acceptability criterion and examine its properties. The progressivity of the income tax turns out to depend on the composition of the public expenditure and on the substitutability between the goods and services supplied by the government and the consumption goods privately obtained through the market.

We test the empirical implications of the model. Specifically, we use OECD data to observe the relationship between marginal tax rates and the distribution over the taxpayers of the benefits produced by the specific composition of the government expenditure in the provision of goods and services. We use threshold regressions to confirm that for lower elasticities of substitution between public and private goods, there is a negative relationship between marginal tax rates and pro-taxpayer-bias, and for higher elasticities, there is a positive relationship.

We divide the commodities into two sets as to whether they are obtained through the market or through the government. The government, besides redistributing income through taxes and cash transfers, supplies goods and services such as general administration, education, health, law-and-order, infrastructures, culture, or defense. These expenditures are financed with the net revenue of the income tax (net of the social transfers). We wish to investigate the relationship between the redistribution of money income

\footnote{The redistributive task of the government has been the object of extensive studies by Alesina and Glaeser (2004) and Moene and Wallerstein (2001a), (2001b) and (2003). Their concern is the relationship between the pre-tax income inequality, the progressivity of the income tax and the size of the social transfers. They leave aside the public provision of goods and services.}
through taxes and transfers and the composition of public expenditure in goods and services. These two aspects have mostly been studied in separate realms of public economics, and not in conjunction.\(^2\)

The supply of goods and services by the government constitutes a large share of the public budget compared to social cash or near-cash transfers.\(^3\) Moreover, it is well-known that different types of commodities and services have differential redistributive impact along the income distribution.\(^4\) Health tends to uniformly benefit the entire population and education is strongly redistributive.\(^5\) Of course, there are other lines in the public budget, such as foreign service, culture or law-and-order, that give benefits increasing with income (or income taxes).\(^6\) Different compositions of government spending can be interpreted as different ways of distributing its

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\(^2\)The incidence and efficiency of income taxation has usually been analyzed in abstraction from public expenditures. Guesnerie and Robert (1984) and Cremer and Galvani (1997) show that welfare can be improved by some public supply of goods. Meltzer and Richard (1985), Bergstrom and Blomquist (1996), Pirtil and Tuomala (2002), and Blomquist and Christiansen (2007) focus on the publicly supplied good being complementary to labour (e.g. day care). An increase in the supply will increase the productivity and the tax collection. Besley and Coate (1991), Bergstrom and Blomquist (1996), and Blomquist and Christiansen (2005) study the case of publicly provided goods whose quantity/quality cannot be supplemented through the private market, for example public vs private education. Boadway and Marchand (1995) study a system that allows supplementation. Blomquist and Christiansen (1998) compare the relative merits of the two systems. In Epple and Romano (1996) individuals vote over the level of the public provision of a good and the budget balancing proportional income tax. Since the only redistribution possible is through the publicly supplied good this is (nearly) tantamount to choosing a budget balanced linear income tax. The macroeconomics literature has also given some attention to the impact of government spending on individual behaviour. The key issue here has been whether public and private expenditures are complements or a substitutes. Barro (1981), Aschauer (1985), Christiano and Eichenbaum (1992), Baxter and King (1993), Karras (1994), Ahmed and Yoo (1995), Ambler and Cardia (1997), Amano and Wirjanto (1998), and Cardia, Kozhaya, and Ruge-Murcia (2003) all are relevant contributions.

\(^3\)For the UK 2004/2005 the average yearly non-contributory social cash and near-cash benefits were 40 percent of the benefits in kind received from the public provision of education and health (Jones, 2006).


\(^6\)In Adam Smith’s words: “The rich, in particular, are necessarily interested to support that order of things, which can alone secure them in the possession of their own advantages. (. . .) Civil government, so far as it is instituted for the security of property, is, in reality, instituted for the defence of the rich against the poor, or those who have some property
benefits over the income distribution. A more pro-rich expenditure policy may have a positive effect on those who bear the heavier part of the tax burden. Rich people can see that a significant fraction of their taxes comes back to them in the form of the kind of goods and services they value most and this may soften their attitude towards income taxation. In our empirical test, we proxy the pro-taxpayer bias of public expenditure by the inverse of the share of public primary education.

Our work contains a second major novelty. This is how taxes are chosen. There are two major lines of research of how the current literature deals with the choice of taxation. One is that a benevolent government chooses that tax function that maximizes social welfare. This tax has nice efficiency properties, but is not representative of the observed behaviour by tax authorities. The second line – known as positive, in contrast with the previous normative approach – conceives taxation as chosen by majority voting. Individuals have preferences over the available tax functions and, under some assumptions, the tax that obtains a majoritarian support is the one preferred by the median voter. This positive approach, while appealing because of its Politico-Economic flavour, has no efficiency properties and yields predictions that seem refuted by facts. The median voter model appears to fail on three of its empirical implications. The first implication is that higher income inequality will lead to higher progressivity in taxation because of the increased distance between mean and median income. The second implication is that individuals with incomes higher than the mean would support zero income taxation, while people below the mean would support redistributive taxation, the more progressive the poorer they are.

7Modern literature on optimal, normative income taxation originates in Mirrlees (1971). A modern rigorous account of optimal income taxation can be found in Myles (1995), Chapter 5.

8The key references are Romer (1975), Roberts (1977) and Meltzer and Richards (1981)

9Perotti (1996) finds no evidence of a significant relationship between inequality and redistribution in democracies. The implication that the median voter is decisive in the choice of the degree of redistribution has recently been empirically tested by Milanovics (2001) and Wong (2004). They find no evidence of such a decisive role of the median voter.

10Such "tax resistance" does not emerge from opinion polls. According to the US 1998 Gallup Poll – as cited in Fong (2001) – among American families with incomes of $10,000 or less, 35 percent report that the government should not redistribute wealth by heavy taxes on the rich and 21 percent believe that the poor should help themselves rather than this being the government’s responsibility. For the entire sample these percentages are 53.9 percent and 30.2 percent, respectively. More interestingly, Fong finds that income is a very poor predictor of redistributive attitudes. Wong (2004) also finds no evidence
A third implication is that poor individuals refrain from voting for full expropriation because they are aware that labor supply will elastically respond to high taxation so that increases in progressiveness would reduce the tax collection and harm the transfer-receiving poor.\footnote{There is no evidence of an elasticity of labor supply significantly different from zero. Blundell and MacCurdy (1999) summarize the different existing estimates of the uncompensated wage elasticity (all using different approaches and estimation techniques). The estimates reported there give nearly rigid labor supply functions: (i) US, zero (MacCurdy et al.,1990) and 0.05 (Triest,1990); (ii) UK, 0.09 (Blundell et al.,1988), (iii) France, 0.1 (Bourguignon and Magnac, 1990); (iv) Germany -0.004 (Kaiser et al.,1992); and similar values for Sweden, Netherlands and Finland.}

Our approach to how taxes get selected is different. We simply assume that the government chooses the tax schedule that is considered acceptable to the largest share of the population possible. An individual considers a tax schedule acceptable if she does not wish to modify its steepness by means of an affine transformation that keeps the same net tax revenue. For every individual there is a (large) set of acceptable tax schedules. The government then chooses the tax schedule that is most widely accepted. As we shall see, there is a unique tax schedule that is unanimously acceptable. Furthermore, this tax schedule has the standard efficiency properties: the unanimously acceptable tax maximizes social welfare among the tax functions yielding the same net tax revenue. We thus provide a \textit{positive} basis for a \textit{normative} tax.\footnote{While the voting models restrict to linear tax functions, in our case we place no restriction on neither the functional form of the tax schedules nor the net tax revenue.}

In contrast with the existing literature, our model implies that the main determinants of redistribution are: (i) the composition of the public spending in the different commodities and services supplied by the government, and (ii) the substitutability between the commodities privately obtained through the market and the commodities publicly furnished. Indeed, at the individual level income taxation is also seen as a mechanism by which disposable income is turned into publicly supplied goods.\footnote{Note the similarity with the case of the voluntary provision of public goods. In that case too individuals see the tax paid as the cost necessary to obtain a useful commodity provided by the state. On this see Warr (1983), Bernheim (1986) and Bergstrom et al (1986).} Therefore, the degree of substitutability between the two types of commodities plays a crucial in

\textit{pocketbook voting} in income redistribution. In contrast, Corneo and Gruner (2002) obtain a strong and significant relationship between attitudes towards redistribution and expected benefit from more progressive income taxation. The reason for this divergent result might be the specificities of the set of countries they work with: six former socialist countries, four Anglo-Saxon countries, Norway and West Germany.
determining the attitudes towards taxation. We examine how the changes in substitutability translate into the progressiveness of income taxation.

For the case of constant-elasticity preferences we can obtain explicit, testable results. Specifically we focus on two implications: (i) the (constant) marginal redistributive factor (which we call marginal tax rate for simplicity) is negatively related to the degree of substitutability and (ii) we should observe a negative relationship between the marginal tax rate and the pro-taxpayer bias among countries with low substitutability and a positive relationship among countries with high substitutability. Both implications are empirically validated by our results.

The structure of the paper is as follows. In section 2 we develop the model. Section 3 defines the notion of acceptable taxation, proves the existence and uniqueness of an acceptable income tax and shows that this tax is welfare efficient. Section 4 is devoted to the relation between income tax progressiveness, the pro tax-payer bias of public spending, and the degree of substitutability between the private and public bundles of commodities. Section 5 is devoted to the empirical test of the predictions of the model. Section 6 concludes.

2 The Model

2.1 Individuals

We assume that there is a continuum of individuals. Individual income is denoted by $y$, it is assumed to be exogenous, and distributed over the population accordingly with the cdf $F$ with support $[a, \infty)$.

We shall denote by $\bar{y}$ the average per capita income.

The set of commodities is divided into two bundles, private (denoted $x$) and public (denoted $g$), accordingly with the mechanism by which they are allocated. Individual demand for the private commodities is satisfied through the markets: in view of market prices individuals choose how best to allocate their disposable income. The individual consumption of the publicly supplied commodities is fixed by the government through its expenditure policy.

We assume that all commodity prices are constant. This allows us to just focus on the aggregate expenditure on the two bundles of commodities.

On individual preferences we make the following standard assumptions:

**Assumption 1**: $u_x > 0$, $u_g > 0$, $u_{xx} < 0$, $u_{gg} < 0$ and $u_{xg} > 0$. Further, we assume that for $g > 0$, $\lim_{x \to 0} u_x = \infty$ and $\lim_{x \to \infty} u_x = 0$, and for $x > 0$, $\lim_{g \to 0} u_g = \infty$ and $\lim_{g \to \infty} u_g = 0$. 
\[
\lim_{g \to 0} u_g = \infty \text{ and } \lim_{g \to \infty} u_g = 0.
\]

The elasticity of substitution between the two commodity bundles will play a key role in our analysis. A higher consumption of the commodities supplied by the government can be achieved only by accepting higher taxation. This is nothing but substituting private for publicly provided consumption goods. How much individuals will be willing to give up on private consumption to increase the level of the public bundle depends on their substitutability. Therefore, the individual attitudes towards taxation will be critically influenced by the elasticity of substitution between the two bundles of commodities.

### 2.2 Income Taxation

The government raises taxes/transfers in order to redistribute income across individuals. The net public revenue left after performing the redistribution of incomes is spent in the provision of the public commodity bundle (for instance, education, justice or defence). Individuals expend their disposable income in the purchase of the private commodities. To save on notation we denote the disposable income by \(x\).

We denote by \(t(y)\) the tax (if positive) or the subsidy (if negative) allocated to each individual with income \(y\). Hence,

\[
x(y) = y - t(y).
\]  

Note that disposable income will exceed the pre-tax factor income when \(t(y) < 0\).

We denote by \(t\) the per capita aggregate net surplus/deficit left after income redistribution, i.e.

\[
t = \int t(y) dF(y).
\]  

### 2.3 Public Expenditure

The role of taxing income is not only to achieve a given degree of income redistribution, but also to raise a net revenue to finance the provision of the publicly supplied commodities. This net revenue will be endogenously determined together with the income tax schedule. Therefore, besides analyzing how income redistribution depends on the key factors, we shall also have something to say about their effect on the size of government: the share of the public supply of commodities over aggregate factor income.
Concerning the structure of public spending, we assume that the government can choose the composition of the bundle in order to benefit differently the individuals on the basis of their tax payments. Hence, the benefit from public spending obtained by an individual with income \( y \), \( g(y) \) is \( g(y) = \gamma(t(y), g) \), where \( g \) is the per capita public expenditure. This description of public spending is aimed at capturing the preferences for the different commodities of this bundle at different income levels. Transferring resources from primary education to the support of opera or economic activities is a way of compensating the rich for the burden of the taxes paid, as these commodities are more appreciated the higher is the income (and the tax) of the individual. The geographical location of the public resources may also introduce a bias in favor or different income groups. Think, for instance, on the allocation of the police force by neighborhoods. It may be allocated uniformly or making its numbers be larger the richer the area. All these are instances of how different compositions of public spending produce a bias in the distribution of the benefits favoring the rich taxpayers.

There are different arguments made in support of such policies. All basically boil down to a sort of pro-incentives idea: to give to the tax-payer goods and services that at least partially respond to the size of their contribution. This may be intentional or the outcome of a more intensive—or effective—lobbying by the rich before the government in favor of the publicly provided goods they like best.\(^{14}\) The bottom line is that the tax-payers perceive that the benefits will be proportionate to the tax contributed. Incentive arguments may be used to advocate in favor of such expenditure policies. We would expect tax-payers to look more favorably upon progressiveness in income taxation if they know that they can ripe part of the benefits deriving from their contributions.

The government’s budget is balanced and hence

\[
g \equiv \int g(y) dF(y) = t. \tag{3}\]

In order to make the problem more operational, we shall specialize to the case in which

\[
\gamma(t(y), g) = \gamma t(y) + (1 - \gamma)g = \gamma t(y) + (1 - \gamma)t. \tag{4}\]

When \( \gamma = 1 \) individuals obtain publicly supplied goods by the same amount they have paid for in taxes. At the other extreme, when \( \gamma = 0 \)

\(^{14}\)See Esteban and Ray (2006) for a model of lobbying with imperfect capital markets. The government may be confounded by signals that respond to the size of the benefits as well as to the wealth of the lobbying groups.
public spending will be fully egalitarian. Throughout the analysis we shall assume that there are rigidities in the expenditure policy so that \( \gamma \) cannot be modified by the government in the short run.

A budget balanced fiscal policy is fully characterized by \( \gamma \) and the tax function \( t(.) \).

One of the main points of this paper is that individual attitudes with respect to taxation and redistribution critically depend on the expenditure policy followed by the government. In the next section we shall study how the tax policies chosen by society depend on the given redistributive bias of the public expenditure as captured by \( \gamma \).

We have already mentioned that the substitutability between the private and the public bundles of commodities will play a key role in determining the individual attitudes with respect to taxation. This substitutability obviously depends on the nature of the commodities included in each bundle, but it can also be influenced by government policy decisions. Allowing for private security – instead of keeping it as a monopoly of the government – increases the substitutability between the two bundles. The same can be said of health, education, or even defense or prisons, for instance.

For reasons of tractability we shall not address the issue of the choice of substitutability and will consider the elasticity of substitution as an exogenous parameter.

3 Acceptable Income Taxation

3.1 Definition

Let us start by putting the concept of acceptable taxation in perspective. In the politico-economic literature on income taxation individuals observe the existing distribution of income and consider affine transformations of this distribution yielding the same (nil) net tax revenue. Individuals establish a preference orderings over the set of feasible affine transformations. The socially chosen linear income tax is the one corresponding to the affine transformation earning a majoritarian support.

In this paper, for each tax function individuals consider the different feasible affine transformations yielding the same net tax revenue (not necessarily nil). In other words, they consider all possible variations in steepness. For each \( t(.) \) every individual associates the affine transformation she prefers the most. A tax function \( t(.) \) will be acceptable to an individual with income \( y \) if the most preferred affine transformation consists of leaving the steepness unchanged. The social support for a given tax function is given
by the share of the population that finds it acceptable. A tax function will be collectively acceptable if it earns unanimous support.

Let us now be more precise. Consider a particular $t(.)$ with net tax revenue $t$, as defined in (2). An individual with income $y$ considers alternative affine transformations $\tilde{t}$ such that

$$\tilde{t}(y) = \alpha + \beta t(y).$$

(5)

with

$$\int \tilde{t}(y)dF(y) = \int [\alpha + \beta t(y)]dF(y) = \int t(y)dFy).$$

(6)

Because of (6) we obtain that

$$\tilde{t}(y) = t + \beta [t(y) - t].$$

(7)

The parameter $\beta$ defines the degree of progressiveness of $\tilde{t}(y)$ relative to $t(y)$. Choosing $\beta > 1$ implies that all the individuals contributing below average will see their contribution diminished while the ones with incomes above will contribute more. The opposite will happen when $\beta < 1$. Therefore, $\beta > 1$ increases [and $\beta < 1$ decreases] the progressiveness of $\tilde{t}(y)$ relative to $t(y)$.

We shall place very weak restrictions on the tax functions. We shall work with the set $\Theta$ of all functions from $\mathbb{R}$ to $\mathbb{R}$ that are strictly increasing.

Consider any arbitrary $t(.) \in \Theta$ and any given $\gamma$. The valuation of a change in progressiveness by $\beta$ will be

$$u(y - [t + \beta(t(y) - t)], \gamma [t + \beta(t(y) - t)] + (1 - \gamma) t].$$

(8)

Given a tax function $t(.)$ we denote by $\beta(t, y)$ the progressiveness change preferred by an individual with income $y$.

**Definition 1** A tax function $t(.)$ is individually acceptable to a person with income $y$ if $\beta(t, y) = 1$.

A tax function $t(.)$ will be individually acceptable to $y$ if that individual would not benefit from changing its progressiveness [by means of a constant net tax collection affine transformation]. We denote by $\mathcal{Y}(y)$ the set of all tax functions $t(.) \in \Theta$ that are individually acceptable to earners of income $y$.

We assume that the government seeks to maximize support. To this effect, the government chooses the tax function that is acceptable to the largest share possible of the population. We explore here the most demanding acceptability requirement: unanimity.
Definition 2 A tax function $t(.)$ is collectively acceptable, $t(.) \in \Xi$, if it is unanimously accepted; that is, if $t(.) \in \bigcap_y \Xi(y)$.

A tax function will be acceptable if every individual agrees not to vary its progressiveness. As we shall now see such a stringent requirement does not yield an empty set.

3.2 Existence of Acceptable Income Taxation

We start by noting that the notion of "acceptable taxation" is not meant to describe any explicit institutional process. Rather, we wish to capture the idea that the income tax schedule has been chosen so as to avoid any significant rejection on either side of the income ladder. A substantial change in the progressiveness of the income tax does not appear to rank high atop the agenda of political parties in OECD countries. Accordingly, we view the existing tax as the result of an informal evolution aiming at gaining the most widespread support over the population.

In short, we simply pose the following question: is there a tax schedule such that a vast majority of the population would not feel compelled to change its steepness? If the answer were in the affirmative one would expect to observe tax schedules close to the one attracting such widespread acceptance.

Surprisingly, the requirement of unanimous acceptance is neither too stringent nor too loose. For any given distribution of income there is always one and only one tax function in the set $\Theta$ that satisfies this property.

Theorem 1 The set $\Xi$ is non-empty and contains one single element only.

Proof Consider any arbitrary $t(y)$, $t$ and $\gamma$. The valuation of a $\beta$ affine transformation, as in (8), will be

$$u(y - [t + \beta (t(y) - t)], \gamma [t + \beta (t(y) - t)] + (1 - \gamma)t).$$

(9)

It can be readily verified that the utility valuation is concave in $\beta$. Hence, the first order condition fully characterizes the preferred $\beta$.

Differentiating with respect to $\beta$ we obtain

$$\frac{\partial u}{\partial \beta} = (t(y) - t) \left[ -u_x \left( y - \tilde{t}(y), \gamma(y, \tilde{g}) \right) + \gamma u_y \left( y - \tilde{t}(y), \gamma(y, \tilde{g}) \right) \right] .$$

11
Note that for all $t(.) \in \Theta$, $(t(y) - t) \neq 0$ except for at most one value of $y$. Hence, $\beta(y, t(y))$ is implicitly characterized by the condition

$$\frac{u_x(y - [t + \beta t(y) - t]), \gamma t + \beta t(y) + (1 - \gamma)t}{u_g(y - [t + \beta t(y) - t]), \gamma t + \beta t(y) + (1 - \gamma)t} = \gamma. \quad (10)$$

If $t^*(y)$ is universally acceptable it has to be that $\beta(y, t^*(y)) = 1$ for all $y$.

We start with an arbitrary parameter $t$ and with the implicit definition of $t(y)$ by

$$\frac{u_x(y - t(y), \gamma t(y) + (1 - \gamma)t)}{u_g(y - t(y), \gamma t(y) + (1 - \gamma)t)} = \gamma. \quad (11)$$

Because of Assumption 1, the left-hand-side of (11) is strictly increasing in $t(y)$, it goes to infinity as $t(y) \to y$ and to zero as $t(y) \to -\frac{1 - \gamma}{\gamma}t$. Hence, for each $t$ and $y$ there exists a unique $t(y)$ satisfying (11). We can thus write

$$t(y) = \psi(y, t, \gamma). \quad (12)$$

It can be readily verified that $\psi$ is continuous and strictly increasing in $y$ and continuous and strictly decreasing in $t$.

For an arbitrary $t$, the average tax collection $\bar{t}$ is

$$\bar{t} = \int \psi(y, t, \gamma)dF(y) = \phi(t, \gamma).$$

The socially acceptable tax-transfer policy $t^*(.)$ is given by (12) evaluated at $t^*$, where $t^*$ satisfies $t^* = \phi(t^*, \gamma)$.

We need now to show that $\phi$ has a fix point. That is, we need to show that $\phi$ intersects the 45° line. Since $\psi$ is continuous and strictly decreasing in $t$, so is $\phi$. From (10) we can easily obtain that for $t = 0$, $\psi(y, 0, \gamma) > 0$ for all $y$. Therefore, we have that for $t = 0$, $\phi(0, \gamma) > 0$. Since $\phi$ is continuous and strictly decreasing in $t$, there exists a unique $t^*$ such that $t^* = \phi(t^*, \gamma)$. This completes the proof. $lacksquare$

Theorem 1 has the remarkable implication that there is a unique tax schedule that satisfies the requirement that all the individuals agree on not changing its progressiveness.

A clarifying remark is now in order. When we defined individual acceptability we only considered changes in the steepness of the tax function that did not modify the aggregate tax revenue. This restriction may make the reader wonder whether we have introduced a hidden restriction on the tax functions truly under consideration. The answer is in the affirmative for
what concerns the particular tax function $t(.)$ being evaluated. However, the set $\mathcal{S}(y)$ is obtained after having tested the acceptability of all possible strictly increasing functions with any arbitrary aggregate tax revenue. Therefore, the set $\mathcal{S}(y)$ contains tax functions yielding very different aggregate tax revenues. When we check whether there is a function belonging to all the individually acceptable sets we find one particular function producing its own aggregate tax revenue. The aggregate tax revenue is determined together with the shape of the tax function.

3.3 Properties of Acceptable Income Taxes

We have uniquely characterized an income tax function based on the notion of individual acceptability, combined with a government seeking maximum (unanimous) support. This places our approach within the positive, politico-economic approach to taxation. Yet, in contrast with this literature, the acceptable income tax we have characterized has interesting efficiency properties: the acceptable income tax maximizes Social Welfare among all the tax functions in $\Theta$ that yield the same net tax revenue. Hence, our notion of acceptability and unanimous support establishes a bridge between two quite independent branches of the literature on income taxation: positive and normative.

Before examining the efficiency properties of the acceptable income tax let us first show that the marginal tax rate is positive and does not exceed unity.

**Proposition 1** The marginal tax rate satisfies $0 \leq t'(.) \leq 1$.

**Proof** Totally differentiating (10) with respect to $t(.)$ and $y$ and rearranging we obtain

$$\frac{dt(.)}{dy} = \frac{u_{yg}u_x - u_{xx}u_y}{(u_{yg}u_x - u_{xx}u_y) + \gamma(u_{yg}u_y - u_{gg}u_x)}.$$  

Observe now that the numerator and the first term in the denominator are the same and that they are positive because of Assumption 1. Because of the same Assumption 1 the second term in the denominator is positive as well. Therefore, the marginal tax rate is positive and below unity. ■

We define the (Utilitarian) Social Welfare as the sum of the individual utilities, that is,

$$W(t(.)) = \int u(y - t(y), \gamma t(y) + (1 - \gamma)t)dF(y).$$  

(13)
We shall now show that \( W(t^*(\cdot)) \geq W(t(\cdot)) \) for all \( t(\cdot) \in \Theta \) with net tax revenue \( t^* \).

**Proposition 2** Let \( t^*(\cdot) \) be an acceptable income tax function with net tax revenue \( t^* \). Then, \( t^*(\cdot) \) maximizes the Utilitarian Social Welfare over all tax functions \( t(\cdot) \in \Theta \) with net tax revenue \( t^* \).

**Proof** We can write

\[
W(t^*(\cdot)) - W(t(\cdot)) =
\]

\[
= \int [u(y - t^*(y), \gamma t^*(y) + (1 - \gamma)t^*) - u(y - t(y), \gamma t(y) + (1 - \gamma)t^*)]dF(y).
\]

Since \( u(\cdot, \cdot) \) is concave in the tax function, and using (10), we can write

\[
W(t^*(\cdot)) - W(t(\cdot)) \geq
\]

\[
\geq \int [t^*(y) - t(y)][-u_x(y - t^*(y), \gamma t^*(y) + (1 - \gamma)t^*) + \gamma u_y(y - t^*(y), \gamma t^*(y) + (1 - \gamma)t^*)]dF(y) = 0.
\]

\[\blacksquare\]

We have already mentioned that an acceptable income tax maximizes welfare subject to a net revenue constraint. Note that the revenue is fixed together with the tax function. The implication is that had we fixed an arbitrary exogenous net tax revenue ( diferente from \( t^* \)) we would have found no collectively acceptable tax function yielding this arbitrary revenue.

**3.4 Discussion**

In this paper we have assumed that individual incomes are exogenous. The literature on income taxation, both normative and positive, assumes instead that individual incomes depend on the labor time/effort supplied. This assumption is essential because it is the awareness of the negative effects of taxation on labor supply what prevents the support of the full redistribution of incomes even by the poor. Yet, as already mentioned, there is no solid evidence of an elasticity of labor supply significantly different from zero.

If one assumes that income is exogenous, what prevents poor individuals from accepting full income equalization only? In our case, besides modifying the disposable income, income taxation also has an effect on the amount of the commodities supplied by the government. Therefore, different tax functions imply a different mix of the private and publicly supplied consumption
goods. Changing the progressiveness of an income tax may reduce individual well-being because of an inefficient mix of private/public consumption goods.

The government policy allocates the publicly supplied commodity as an increasing function of one’s contribution in taxes. If a low-income earner demands more progressiveness — and hence a larger transfer— she will be trading-off more private consumption for less public consumption. Likewise, high-income earners might be willing to give up on private consumption in order to obtain higher levels of public consumption. At the equilibrium tax schedule, the marginal rate of substitution between the private and publicly supplied commodities will be equal across the population.

Note that it is precisely this property that makes acceptable tax functions welfare efficient.

Finally, let us informally discuss [a formal analysis in the next section] the degree of progressiveness in an acceptable income tax. Let us start with a linear income tax as the benchmark. Let us consider the effect of an increase by $\Delta$ of an income $y$. Because of the biassed expenditure policy, with a linear income tax private and public consumption will increase at the same rate. Suppose now that preferences have a falling elasticity of substitution of private for public consumption. Individuals with an income increased by $\Delta$ would have preferred a more than proportional increase in the supply of the public good and hence would rather have favored an increasing marginal tax rate. If the elasticity of substitution moves in the opposite way we would have had a preference for declining marginal tax rates. Clearly, whether individuals will unanimously support a tax function with increasing or decreasing marginal tax rates critically depends on the change in the elasticity of substitution as the consumption levels move up. In the next section we shall examine in detail the case of preferences with a constant marginal rate of substitution.

Note that the previous argument is also critical for the effects of increased inequality on the progressiveness of taxation. Consider a discrete distribution and a transfer of income to a richer individual. The donor will decrease the consumption of both types of commodities and the beneficiary will increase both. However the proportional decrease of one and increase of the other will increase depend on the magnitude of the change in the elasticity of substitution. If the proportion is the same for the two, the effects will cancel out and the same (proportional) tax will still be acceptable. However, if this is not the case, the tax will need to be readjusted. The direction of the change obviously depends on whether the beneficiary of the transfer "demands" a more or less than proportional increase in the supply
of the public consumption good [and similarly for the donor].

In the next section we shall examine this issue for the class of CES preferences.

4 Income Tax Progressiveness and Public Expenditure

We shall restrict individual preferences to be of the CES type. This will permit us to examine the effects of income inequality, expenditure bias and elasticity of substitution on the tax schedule and on the size of government.

4.1 CES Preferences

The family of CES utility functions is given by:

$$u(x, g) = \left[ ax^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha)g^{\frac{\sigma - 1}{\sigma}} \right]^\frac{\sigma}{\sigma - 1},$$

(14)

with the elasticity of substitution $\sigma > 0$.

The marginal utilities to the two types of consumption are

$$u_x = ax^{-\frac{1}{\sigma}} \left[ ax^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha)g^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{1}{\sigma - 1}},$$

and

$$u_g = (1 - \alpha)g^{-\frac{1}{\sigma}} \left[ ax^{\frac{\sigma - 1}{\sigma}} + (1 - \alpha)g^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{1}{\sigma - 1}}.$$

Therefore,

$$\frac{u_x (x, g)}{u_g (x, g)} = \frac{\alpha}{1 - \alpha} \left[ \frac{y - t(y)}{\gamma t(y) + (1 - \gamma)t} \right]^{\frac{1}{\sigma}} = \gamma.$$

(15)

We can thus easily obtain that

$$t(y) = \frac{y - (1 - \gamma)\lambda t}{1 + \gamma \lambda},$$

(16)

where

$$\lambda = \left( \frac{\alpha}{1 - \alpha \gamma} \right)^{\sigma}.$$

(17)

Integrating over the incomes $y$ we can obtain

$$t = \frac{\mu - (1 - \gamma)\lambda t}{1 + \gamma \lambda}.$$
Hence,

\[ t = \frac{\mu}{1 + \lambda} = g. \]  

(18)

Therefore, we obtain that the acceptable income tax schedule is linear

\[ t(y) = \tau y - T, \]  

(19)

where

\[ \tau \equiv \frac{1}{1 + \gamma \lambda} \quad \text{and} \quad T \equiv \frac{1 - \gamma}{1 + \gamma \lambda} \frac{\lambda \mu}{1 + \lambda}. \]  

(20)

From (18) we immediately obtain the size of the public sector to be

\[ g = \frac{1}{1 + \lambda}. \]  

(21)

For all OECD countries \( \frac{g}{\mu} < \frac{1}{2} \). Therefore, in view of (21) and of (17) we can deduce that the empirically relevant parameter values satisfy:

\[ \lambda > 1, \quad \text{and} \quad \frac{\alpha}{(1 - \alpha) \gamma} > 1. \]  

(22)

### 4.2 Income Taxation and Public Spending

We can now state our results on income taxation and the size of government.

We start with the effect of \( \gamma \) and \( \sigma \) on the marginal tax rate \( t'(\cdot) \).

**Proposition 3** Let preferences be CES. Then: (i) the unique acceptable income tax is linear; (ii) it is independent of the distribution of income; (iii) the (constant) marginal tax rate, \( t'(\cdot) \equiv \tau \), increases (decreases) with the bias parameter \( \gamma \) if the elasticity of substitution is high (low), \( \sigma > 1 \) (\( \sigma < 1 \)); and (iv) when \( \frac{g}{\mu} < \frac{1}{2} \) (and hence (22) is satisfied) an increase in the elasticity of substitution reduces the marginal tax rate.

**Proof** Statements (i) and (ii) follow immediately from (19).

Differentiating the marginal tax rate in (19) with respect to \( \gamma \) and using (17) we obtain

\[ \frac{d\tau}{d\gamma} = -\frac{1}{(1 + \gamma \lambda)^2} \frac{d\gamma \lambda}{d\gamma} = (\sigma - 1) \frac{\lambda}{(1 + \gamma \lambda)^2}. \]

This proves statement (iii). As for statement (iv) we similarly differentiate with respect to \( \sigma \) and obtain

\[ \frac{d\tau}{d\sigma} = -\frac{\gamma}{(1 + \gamma \lambda)^2} \frac{d\lambda}{d\sigma} = -\frac{\gamma}{(1 + \gamma \lambda)^2} \lambda \ln \frac{\alpha}{(1 - \alpha) \gamma}. \]
Noting that because of (22) \( \ln \frac{\alpha}{(1-\alpha)\gamma} > 0 \), statement (iv) obtains. ■

Without having made any assumption on the tax-transfer function, we obtain that for CES preferences, the unique \( t(.\) that would be universally accepted would be a linear tax function. Therefore, any departure from linearity in taxes requires significant variations in the substitutability of the two bundles of commodities as income varies.

By the same argument, the inequality in the distribution of pre-tax income will not play a major role in determining the degree of income redistribution, \( \tau \), unless individual preferences display a significant variation in the degree of substitutability as real income changes.

The marginal tax rate depends upon the bias of government spending. The effect of a more egalitarian expenditure policy, \( \gamma \to 0 \) on the marginal tax rate \( \tau \) critically depends upon the degree of substitutability between the two bundles of goods. For low substitutability, \( \sigma < 1 \), the marginal tax rate tends to unity and for high substitutability, \( \sigma > 1 \), it tends to zero.

Finally, in economies with a moderate share of government an increase in the substitutability between the two bundles of goods will decrease the marginal tax rate, \( \tau \).

Let us now turn to the effects of \( \gamma \) and \( \sigma \) on the size of government \( \frac{g}{\mu} \). From (21) we can easily obtain the following result.

**Proposition 4** Let preferences be CES. Then, the size of government \( \frac{g}{\mu} \): (i) increases with the bias parameter \( \gamma \); and (ii) decreases with the elasticity of substitution \( \sigma \).

The results in Proposition 4 are not surprising. Yet, they are worth recording. The first result says that the higher the pro-taxpayer bias in the public spending the larger will be the size of the government that the population will consider acceptable. The second result tells us that increasing the substitutability between the market and the publicly supplied goods will induce a demand for a smaller size of the government.\(^{15}\)

\(^{15}\)This result seem in contradiction with Karras's (1994) argument that the larger is the public sector the more the goods and services supplied will be substitutes for the goods provided through the market. Two points are in order. First, Karras does not take into account that the substitutability between public and private goods critically depends on the political decision of allowing or not the private supply of substitutes (e.g. security, mail service, prisons,...). Second, Karras's argument does consider whether such an increase in the size of government would be considered acceptable. This precisely is our point: if the government allows for higher substitutability the policy that will be found acceptable will consist of a smaller size of the government sector.
4.3 Discussion

At the end of the previous section, we have already discussed the role of the elasticity of substitution in the determination of the chosen income tax schedule. Let us now focus on the effect of the bias in public spending, $\gamma$, and of the elasticity of substitution, $\sigma$, on the marginal tax rate.

In order to develop an intuition, imagine that the publicly supplied good is security. The bias in public spending will be the allocation of the number of policemen depending on the tax effort by each neighborhood. An egalitarian policy, $\gamma = 0$, consists of assigning the same number of policemen per person irrespective of the taxes paid. A pro-incentive, $\gamma = 1$, makes the number of policemen proportional to the taxes paid. Accordingly with our assumptions, security is not an inferior good so that high income individuals would like to have more policemen than poor individuals. Since the government has the monopoly of the supply of police force, the rich will be willing to accept to pay more taxes than the poor. If the two goods are complements, the rich will have to finance the increase of police force in all neighborhoods.

Suppose now that the government increases $\gamma$, thus increasing the bias of its expenditure policy (and hence the incentives for the taxpayer). The number of policemen is higher in the rich areas than in poor neighborhoods. The effect of this change will be that the rich paying less than under the egalitarian policy can obtain the desired supply of security. Hence they will accept less taxation than before. As for the poor, they are now receiving less security and because of the bias in spending a demand of more progressiveness will reduce the police force even further (while increasing private consumption). Hence the poor will be willing to accept a less progressive taxation. On the other hand, consider the case in which both commodities are high substitutes. This may correspond to a case in which the government has partially given up the monopoly of the supply of police and allows for a private market of security. The taxation that the rich will be willing to accept will be less because they can satisfy their demand for security through the market. In the limit, when both are perfect substitutes, there is no reason why the rich should accept any effective taxation unless all their money comes back as the supply of security that maximizes their utility.

In our model, the supply of a subset of commodities is the monopoly of the government. This monopoly provides the government with the coercive power to make individuals to accept taxation on incomes. How effective this power is critically depends on the substitutability between this bundle of commodities and the commodities individuals can purchase in the mar-
Hence, our approach suggests that the rich will lobby more strongly for increasing the substitutability between public and private goods by privatizing as many as possible rather than about the shape of the income tax schedule.

5 Empirical results

5.1 Testable Implications

For the case of constant-elasticity preferences, our model has well-defined, testable implications. Specifically, we shall focus on two implications:

1. the (constant) marginal redistributive factor (which we call marginal tax rate for simplicity) \( \tau \) is negatively related to the degree of substitutability and

2. we should observe a negative relationship between the marginal tax rate and the pro tax-payer bias among countries with low substitutability and a positive relationship among countries with high substitutability.

Both implications are empirically validated by our results.

5.2 Empirical Strategy and Data

We shall now test the empirical validity of the relationship between \( \tau \), \( \gamma \) and \( \sigma \) implied by our results. We do not have direct data on any of the three variables and hence we have to work with reasonable proxies. Furthermore, our choice of proxies has been severely conditioned by the need of a consistent set of basic information available for a sufficiently large number of countries. We have therefore tested our empirical implications using the OECD database\(^{16}\) that includes fifteen countries, listed in the Appendix.

We start with our proxy for \( \tau \). This is the slope of the affine tax function that turns out to be acceptable with CES preferences. In our paper the tax function merges together the income tax schedule and the different money transfers. In other words, \( t(.) \) is the difference between factor income (plus retirement payments and minus retirement contributions) and disposable income.

In purity, we would have had to test whether the difference between the two individual incomes can be represented by an affine function. The only

\(^{16}\) We use the OECD Statistical database obtainable at www.oecd.org/statistics.
data base available with individual information of this kind is the Luxembourg Income Study database. We performed this exercise but discarded this option for two reasons. First, the estimated parameters were unreasonably unstable from year to year, hinting towards some possible deficiencies in the raw data. Secondly, $\gamma$ and $\sigma$ would have to be estimated from a completely different source.

We have opted instead to use the maximum marginal tax rate in each country reported in the OECD database as a proxy for $\tau$. Hence, we have implicitly assumed that there is a stable relationship between the maximum marginal tax rate and the slope of the affine function that would approximate the difference between factor and disposable individual incomes.

Let us now turn to our estimates for $\gamma$. As defined in Section 2, $\gamma$ captures the pro-taxpayer bias in the public provision of goods and services, which we denote by $G$. This bias depends on the share in the government budget of the expenditures that mostly benefit the low incomes versus those that mostly benefit the rich taxpayers. For some countries, discussed earlier, there are estimates of the distribution of the benefits of specific lines in the government expenditure (essentially, education and health).\footnote{There is a large literature which discusses and identifies the redistributiveness of public expenditures. (See Le Grand, 1982 for the redistributive effects of health and education in the UK - he concludes that of the two education is more redistributive). A large debate in the 1980s (Hansen and Weisman 1969, Pechman 1970) contest the redistributive effects of higher education in particular, concluding that the redistributive effects of higher education were debatable, and that existing measurement methodologies were not successful in effectively measuring their effects. We do not perform any statistical analysis to test for the relative redistributiveness of the different types of public expenditures in the OECD countries studied; this would entail a separate econometric exercise beyond the purview of this paper.} We are however interested in the distribution of the benefits of the entire government supply of goods and services (including general administration and law-and-order, among others). Therefore, we have had to estimate our own proxies for $\gamma$.

We first estimate $G$, the total government expenditure in the provision of goods and services, from OECD data sources. $G$ is estimated by subtracting the amounts that are spent on money transfers from the total amount of government expenditure, detailed in the Appendix. All estimates are done at constant 1995 US dollars. Of the total amount of government expenditure, $G$, we focus on two redistributive, pro-poor public expenditures - health and education. For education, we focus on all three types of expenditures - primary, secondary and tertiary.

We estimate $\gamma$ with the following expression:
Finally, we need an estimate for the elasticity of substitution between the two bundles of commodities, private and publicly provided, \( \sigma \). The substitutability between public and private expenditure has been a recurrent topic in Macroeconomics. Since the work of Barro (1981) there have been numerous attempts at estimating the elasticity of substitution. Aschauer (1985) finds a significant degree of substitutability between the two variables for the United States. Karras (1994) finds that they are complementary or unrelated, using data for 30 countries. Evans and Karras (1996) provide additional evidence supporting the complementarity using data for 54 countries. More recently, Amano and Wirjanto (1998) for the US show that the two variables are unrelated or have very weak complementarity. For Japan, Hamori and Asako (1999) find a significant degree of substitutability, while for Okubo (2003) the two bundles are complementary or unrelated. Finally, Bouakez and Rebei (2006) with the same specification of preferences as ours - but with habit formation - estimate \( \sigma = 0.332 \).

Unfortunately, we will not be able to make use of these [quite contradictory] estimates for the following reasons. First, most of the Macro literature has defined substitutability by the sign of the cross derivative and not by the value of the elasticity, \( \sigma \). Second, the models are all inter-temporal and this aspect proves to be critical for the estimates. Ni (1995) empirically finds that when the two expenditures add linearly in the preferences, the estimates indicate substitutability, while if the two expenditures enter the utility function non-separably one obtains complementarity. Third, the estimates are perplexingly contradictory. Finally, most of the literature tries to estimate a world elasticity using panel data, but we are interested in country estimates which can be used to make a ranking across countries. Kwan (2006), for example, using co-integration methods, has found that for nine East Asia countries while the two bundles are substitutes on the average, in some countries they are complements. In sum, we cannot base our empirical work on these estimates.

Our approach to the estimation of \( \sigma \) is therefore as follows. The substitutability between the two bundles of commodities depends on the nature of individual preferences and on the degree of monopoly that the government keeps for itself for some subset of commodities, as discussed earlier. For many OECD countries the postal system or security has been a public monopoly until fairly recently. Today, however, rich people can supplement the public supply of police force, for instance, by purchasing additional pri-
vate security. Similarly, in many countries education and health have high
degrees of publicness while in others, a good share of the demand is satisfied
through the private market. The larger the share of the expenditure channelled
through the market, the higher is the substitutability between the public and the
private provision of these goods. For our purposes, therefore, we estimate a proxy
of the elasticity of substitution using a metric $\lambda$, which equals the ratio of
private over the total of public and private expenditures. Using this ratio, we
estimate a proxy of the elasticity of substitution, $\sigma = \lambda/(1 - \lambda)$. If all is
private, and $\lambda = 1$, then elasticity is infinity. Comparative data of public and
private expenses have been used for both health and education for the OECD
countries in this study\textsuperscript{18}. We estimate $\lambda$ for all three types of
education - primary, secondary and tertiary - and for health, obtained from
OECD sources. Estimates of all four types of $\sigma$s is available
at the location http://darp.lse.ac.uk/expenditures.

Indeed, these estimates are very rough proxies for the "true" elasticities
of substitution. However, our empirical exercise essentially rests on the
"ranking" of the countries by their degree of substitutability more than on its
absolute value.

5.3 Relationship between $\tau$ and $\sigma$

In section 4.2, theory predicts that the relationship between $\tau$ and $\sigma$
should be negative. Here, we investigate this relationship using the data and
estimates that have been discussed in the previous section, for all four types
of elasticities - primary, secondary and tertiary education, and health. We
estimate OLS regressions and also estimate kernel regressions to ascertain
the nature of the relationship between the two variables. The latter method
is used to allow the data to determine the true nature of the relationship,
rather than imposing a known structure.

The model we estimate is given as

$$\tau_{it} = f(\sigma_{it}) + \varepsilon_{it}$$

where $\tau_{it}$ is the marginal tax rate for country $i$ in time $t$, $\sigma_{it}$ is the
elasticity of substitution between the publicly and privately provided good
for country $i$ in time $t$, $f(.)$ is a generic function and $\varepsilon_{it}$ is an error term. We
do not impose any structure on $f(.)$, except for the case when we estimate
the above relationship using OLS.

\textsuperscript{18}The data source for these estimates is the OECD Social Expenditure database. Data
has been available only for specific years - 1995, 2000 and 2004.
The kernel regressions support a negative relationship. This is also supported by OLS regressions as well, except for that of $\sigma_{\text{tertiary}}$, given in 1. All other three $\sigma$s are significant at 1% level of significance.

In the figures below, we present the kernel regressions for each of the four relationships. The Epanechnikov estimator is used for the kernel regression estimates. We present results with a smaller bandwidth (rather than a large one) to reveal the true relationship as closely as possible. For each of the four $\sigma$s it is clear that the dominant relationship between $\tau$ and $\sigma$ is a negative one.

![Kernel Regression of $\tau$ on $\sigma_{\text{primary}}$](a0f471a1e2f2d51ead2461a242a6cf3b.png)

Figure 1: Kernel Regression of $\tau$ on $\sigma_{\text{primary}}$

<table>
<thead>
<tr>
<th>co-efficient</th>
<th>$\sigma_{\text{primary}}$</th>
<th>$\sigma_{\text{secondary}}$</th>
<th>$\sigma_{\text{tertiary}}$</th>
<th>$\sigma_{\text{health}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>374</td>
<td>374</td>
<td>374</td>
<td>374</td>
</tr>
</tbody>
</table>

Table 1: OLS Regressions for Relationship between estimates of Marginal Tax Rate, $\tau$ and Elasticity of Substitutions, $\sigma$. 

Figure 2: Kernel Regression of $\tau$ on $\sigma_{\text{secondary}}$

Figure 3: Kernel Regression of $\tau$ on $\sigma_{\text{tertiary}}$
5.4 Relationship between $\tau$ and $\gamma$

In this section, we investigate the relationship between $\tau$ and $\gamma$. Theory in Section 4.2 predicts that for low values of $\sigma$, there exists a negative relationship between the marginal tax rate and the pro-tax bias; for higher values of $\sigma$, there exists a positive relationship. We use threshold regressions to determine the value(s) of $\sigma$ which splits the sample into two (or more) parts, $\sigma^*$, such that for values of $\sigma$ less than the estimated threshold value, $\sigma^*$, there is a negative relationship between $\tau$ and $\gamma$, and for values of $\sigma$ above the estimated threshold value $\sigma^*$, there is a positive relationship between $\tau$ and $\gamma$. Threshold regressions thus determine the different "regimes" of the relationship between $\tau$ and $\gamma$. Alongside the threshold regressions we estimate kernel regressions, to ascertain the exact nature of the relationship between $\tau$ and $\gamma$. Figures 5 to 8 present the kernel regressions between the marginal tax rate for each observation, $\tau$, and the pro-tax bias, $\gamma$, where the data is sorted for increasing values of $\sigma$. We will first determine the threshold values of $\sigma$.

To estimate the threshold values and the regimes, we estimate the following model:

$$
\tau_{it} = \alpha_1 \gamma_{it} 1(\sigma_{it} \leq \sigma^*) + \alpha_2 \gamma_{it} 1(\sigma_{it} > \sigma^*) + u_{it}
$$

(25)
Table 2: Regimes obtained by Threshold Regression Estimations for Elasticity of Substitution for Primary Education

<table>
<thead>
<tr>
<th>Regimes</th>
<th>$\sigma_{\text{primary}}$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>$\leq 0.21304$</td>
<td>-1.12829***</td>
</tr>
<tr>
<td>Regime 2</td>
<td>$&gt; 0.21304$</td>
<td>1.448073***</td>
</tr>
</tbody>
</table>

where for each $i$, $\tau_{it}$ is the dependent variable (the marginal tax rate corresponding to a country-year), $\gamma_{it}$ is the explanatory variable (the pro-tax bias corresponding to the country-year), $\sigma_{it}$ is the threshold variable (corresponding to the country-year), assumed to be strictly exogenous, $\sigma^*$ is the threshold parameter, $\alpha_1$ and $\alpha_2$ are the slope parameters that will differ according to the value of $\sigma_{it}$, and $u_{it}$ is a random disturbance term. $1(\sigma_{it} \leq \sigma^*)$ is an indicator variable that takes the value 1 if $\sigma_{it} \leq \sigma^*$ and 0 otherwise. The threshold value of $\sigma$ is the estimate at which likelihood function achieves a local minimum.\(^{19}\)

Table 2 presents the regimes that we obtain, using $\sigma_{\text{primary}}$, with the estimates of the slope parameters. For $\sigma_{\text{primary}}$, we observe that for values of $\sigma_{\text{primary}} \leq 0.21304$, there is a negative relationship and for values greater, there is a positive relationship between $\tau$ and $\gamma$.\(^{20}\)

We have also estimated the threshold values for $\sigma_{\text{health}}$, and unlike the case for $\sigma_{\text{primary}}$ we obtain more than two clear regimes. One interpretation of there being several threshold values is that the $\tau$ and $\gamma$ relationship corresponding to $\sigma_{\text{health}}$ is a weak one. We however, present the kernel regressions corresponding to $\sigma_{\text{health}}$ for the highest and lowest threshold values obtained, presented below.

Kernel regressions are estimated to observe the nature of the relationship between $\tau$, the actual marginal tax rate, and $\gamma$, the pro-tax bias. The model estimated is given by

\[^{19}\]There is no asymptotic theory to obtain p-values corresponding to the the threshold value obtained, thus our reported threshold values of $\sigma$ depends on the value obtained by minimising the likelihood function. Estimations were performed using Bruce Hansen’s Gauss programmes obtained from http://www.ssc.wisc.edu/~bhansen/progs/progs_threshold.html

\[^{20}\]We are less interested in the redistributive effects of secondary and tertiary education, as they are least likely of all to be redistributive. For the interested reader, the kernel regressions of the relationship between $\tau$ and $\gamma$ for these both are presented in the Appendix. The relationship is decidedly weaker for tertiary education.
\[ \tau_{it} = k(\gamma_{it}) + v_{it} \]  

where \( \tau_{it} \) is the marginal tax rate for country \( i \) in time \( t \), \( \gamma_{it} \) is the elasticity of substitution between publicly and privately provided good for country \( i \) in time \( t \), \( f(.) \) is a generic function and \( v_{it} \) is an error term. We do not impose any structure on \( k(.) \); we allow the data itself to determine the functional relationship. We again use the Epanechnikov kernel estimator (Silverman 1986). We also provide further kernel estimates using the Nadaraya-Watson estimator, which are available for the interested reader at the extended Appendix at http://darp.lse.ac.uk/expenditures/. The results suggest similar, if not identical, results to those obtained with the Epanechnikov estimator.

The figures suggest that for the values of \( \sigma \leq \sigma^* \), the relationship between \( \tau \) and \( \gamma \) is negative, while for values of \( \sigma_{it} > \sigma^* \), the relationship is positive. The sample size for the full data set is 375 - the data set consists of observations from 15 OECD countries (listed in the Appendix), over the years 1975 to 1999.

The sample is split into two sets on the basis of the threshold value of \( \sigma \) (corresponding to each redistributive expenditure). The relationship between \( \tau \) and \( \gamma \) is then observed for the two samples, one corresponding to values of \( \sigma \leq \sigma^* \) and another corresponding to values of \( \sigma_{it} > \sigma^* \). The threshold values of \( \sigma \) that has been used to split the samples are given below, and the nature of the relationship between \( \tau \) and \( \gamma \) as observed in the kernel regressions are presented in the figures. We have two sets of results - one set estimating the relationship between \( \tau \) and \( \gamma_{\text{primary}} \) (\( \gamma \) corresponding to expenditures on primary education) and another estimating the relationship between \( \tau \) and \( \gamma_{\text{total}} \) (\( \gamma \) corresponding to expenditures on education and health) The results for \( \sigma_{\text{secondary}} \) and \( \sigma_{\text{tertiary}} \) education are presented in the Appendix.

- For \( \sigma \) corresponding to that of primary and pre-primary education expenditures, the threshold values are \( \sigma_{\text{primary}} < 0.21 \) and \( \sigma_{\text{primary}} > 0.21 \)

- For \( \sigma \) corresponding to that of health expenditures, the threshold values are \( \sigma_{\text{health}} < 0.33 \) and \( \sigma_{\text{health}} > 0.59 \).

Figures 5 to 8 present the kernel regressions for the relationship between \( \tau \) and \( \gamma_{\text{primary}} \). It is clear that for values of \( \sigma \) below the threshold value, we
have a negative relationship between $\tau$ and $\gamma_{primary}$, and for values higher, there is a positive relationship.

Figure 5: Kernel Regression of $\tau$ on $\gamma$, for $\sigma_{primary} < 0.21$

We now repeat kernel regression estimates of the relationship between $\tau$, and $\gamma_{total} - \gamma$ estimated for total expenditures on education and health. A cursory glance at the graphs reveals an identical relationship as before - for lower values of $\sigma$, we have a negative relationship between $\tau$ and $\gamma_{total}$ and for higher values of $\sigma$, a positive one. The threshold values for $\sigma_{primary}$ and $\sigma_{health}$ obtained are the same as before, as follows:

- For $\sigma$ corresponding to that of primary and pre-primary education expenditures, the threshold values are $\sigma_{primary} < 0.21$ and $\sigma_{primary} > 0.21$
- For $\sigma$ corresponding to that of health expenditures, the threshold values are $\sigma_{health} < 0.33$ and $\sigma_{health} > 0.59$
Figure 6: Kernel Regression of $\tau$ on $\gamma$, for $\sigma_{\text{primary}} > 0.21$

Figure 7: Kernel Regression of $\tau$ on $\gamma$, for $\sigma_{\text{health}} < 0.33$
Figure 8: Kernel Regression of $\tau$ on $\gamma$, for $\sigma_{\text{health}} > 0.59$

Figure 9: Kernel Regression of $\tau$ on $\gamma_{\text{total}}$, for $\sigma_{\text{primary}} < 0.21$
Figure 10: Kernel Regression of $\tau$ on $\gamma_{total}$, for $\sigma_{primary} > 0.21$

Figure 11: Kernel Regression of $\tau$ on $\gamma_{total}$, for $\sigma_{health} < 0.33$
The relationships observed support a negative relationship between the marginal tax rate $\tau$ and the pro-taxpayer bias (for both $\gamma$s defined for primary education and that for total expenditure on education and health), for lower values of the expenditures’ elasticities between private and public provisions, and a positive relationship for higher values of the elasticities’. The relationship is less clear for the case of expenditure of tertiary education, particularly for lower values of $\sigma_{tertiary}$. As is revealed in Figures 15 and 19. But, this is not surprising since the benefits of tertiary education may befall on mid- and high incomes and hence might not be an appropriate measure of the bias in public spending as it has been defined here. These results are further confirmed by kernel regressions that are estimated using the Nadaraya-Watson estimator, a local polynomial averaging estimator, which are available in the extended Appendix at http://darp.lse.ac.uk/expenditures/ for the interested reader.

6 Conclusion

Let us sum up. In this paper we have jointly treated public taxation and spending. We have proposed a new notion of taxation selection, which is more in line with available evidence of attitudes towards redistribution. This model allows us to address novel issues such as the interdependence between income taxation, the composition of public spending and the substitutability
between public and private goods. The main results are that higher substitutability produces a lower progressivity of the income tax and a smaller size of the public spending over the GDP and that stronger pro-taxpayer bias in public spending goes with a lower/higher progressivity in the tax function as the substitutability is low or high. Both implications turn out to be validated by empirical evidence.

The paper has substantial room for improvement on both counts: theoretical and empirical. While the redistributive activity by the government through taxes and transfers has attracted the interest of researchers, the public spending has been comparatively neglected.

We know too little about the redistributational impact of the different lines of the government budget $G$. Even for the countries where this information is regularly computed such as the UK, the estimates focus on five budget lines only: education, health, housing subsidies, travel subsidies, and school meals. For most countries these estimates simply do not exist. This lack of information is paralleled by a similar lack of modelling on how the change in the structure of government spending affects the consumer behavior and well-being.

The analysis of the substitutability between the private and publicly provided goods and services is in still a much weaker position. We are aware of no empirical work estimating this degree of substitutability nor of any formal modeling of the effect of the regulation of the private substitutive supply of goods and services that are being furnished by the state.

Generally speaking, there is much to be gained by the joint analysis of public taxation and expenditure. Our work is but a first step in this direction.

References


A Countries used in the study - OECD database

The countries which are used for our analysis are as follows. Data has been obtained from the OECD database, at www.oecd.org/statistics

Australia
Belgium
Denmark
Finland
France
Germany
Ireland
Italy
Netherlands
Norway
Poland
Sweden
Switzerland
United Kingdom
United States

The data is available from 1975 to 1999, with no missing years.

A.1 Definition of social transfers, OECD

The social transfers data, and expenditures on education and health, which have been used to estimate $G$, have been obtained from the OECD Social Expenditures database. The variables which are used to estimate the social

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transfers, obtained from the OECD data base, are the following. These social transfers are subtracted from total government expenditures to obtain the total amount of government expenditures on public services. This will constitute our estimate of $G$.

- GD1P: Compensation of employees; payable
- GD3P: Subsidies; payable
- GD62_631XXP: Social benefits and Social transfers in kind (via market producers); payable
- GD7P: Other current transfers; payable
- GD9P: Capital transfers; payable.

All estimates are converted to constant 1995 US dollars.

B  Further kernel regression estimates of $\tau$ and $\gamma$ relationship

In this section we present the set of estimates of the relationship between $\tau$ and $\gamma$, for increasing values of $\sigma$, the elasticity of substitution between private and public provision, for secondary and tertiary education. As in the earlier analysis, we split the sample of the basis of threshold values of $\sigma$ that have been obtained - here as well we obtain a number of threshold values at which the relationship switches (positive and negative) - we use the lowest and highest values of the threshold values obtained. It is difficult to make an economic interpretation of the different threshold values obtained (over which the relationship switches in sign), and deduce that it is indicative of a weak/unstable relationship between $\tau$ and $\gamma$ for these $\sigma_{secondary}$ and $\sigma_{tertiary}$. We present kernel regressions, using the Epanechnikov estimator. The main observation is that for lower values of $\sigma$, the relationship between $\tau$ and $\gamma$ is a positive one; and that for higher values of $\sigma$, the relationship of $\tau$ and $\gamma$ is a negative one. The first set of results correspond to $\gamma$ corresponding to that for primary education.

- For $\sigma$ corresponding to that of primary and secondary education expenditures, the two sets of values are for $\sigma_{prim\&sec} < 0.11$ and $\sigma_{prim\&sec} > 0.16$

- For $\sigma$ corresponding to that of tertiary education expenditures, the two sets of values are for $\sigma_{tertiary} < 0.2$ and $\sigma_{tertiary} > 0.29$

Here we present the results that are obtained using the $\gamma_{primary}$
Figure 13: Kernel Regression of $\tau$ on $\gamma$ for $\sigma_{secondary} < 0.11$

Figure 14: Kernel Regression of $\tau$ on $\gamma$ for $\sigma_{secondary} > 0.16$
Figure 15: Kernel Regression of $\tau$ on $\gamma$ for $\sigma_{\text{tertiary}} < 0.2$

Figure 16: Kernel Regression of $\tau$ on $\gamma$ for $\sigma_{\text{tertiary}} > 0.2$
We now estimate kernel regressions for the relationship between $\tau$ and $\gamma_{health}$, corresponding to $\sigma$ s for secondary and tertiary education.

- For $\sigma$ corresponding to that of primary and secondary education expenditures, the two sets of values are for $\sigma_{prim&sec} < 0.11$ and $\sigma_{prim&sec} > 0.16$.

- For $\sigma$ corresponding to that of tertiary education expenditures, the two sets of values are for $\sigma_{tertiary} < 0.2$ and $\sigma_{tertiary} > 0.29$

![Kernel regression graph](image)

Figure 17: Kernel Regression of $\tau$ on $\gamma_{total}$ for $\sigma_{secondary} > 0.11$
Kernel regression, bw = 4, k = 3
Grid points
55.7684 73.3213
.517491
.598858
tau
gamma

Figure 18: Kernel Regression of $\tau$ on $\gamma_{\text{total}}$ for $\sigma_{\text{secondary}} > 0.16$

Kernel regression, bw = 2.2, k = 3
Grid points
37.4359 93.1
.144121
.748285
tau
gamma

Figure 19: Kernel Regression of $\tau$ on $\gamma_{\text{total}}$ for $\sigma_{\text{tertiary}} < 0.2$
Figure 20: Kernel Regression of $\tau$ on $\gamma_{total}$ for $\sigma_{tertiary} > 0.29$