

# **The Spatial Decay in Commuting Probabilities: Employment Potential vs. Commuting Gravity**

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**Abstract**

We show that an employment potential capitalisation model, which establishes a spatial relationship between the price of land and the spatial distribution of employment through a transport matrix, produces estimates of the spatial decay in bilateral commuting probabilities that are very close to the decay observed in commuting data.

Keywords: Accessibility, commuting, employment, gravity, potential

JEL Classifications: R12, R28, R38, R48

# 1 Introduction

The standard urban economics framework predicts that *ceteris paribus* the price of land will mirror the cost of commuting. In classic stylized models, the price of land must decline as a compensation for the increase in commuting costs to the central business district (CBD), the destination of all commuting trips, in order to maintain a spatial equilibrium with no relocation incentives (Alonso, 1964; Mills, 1967; Muth, 1969). In reality, employment is dispersed and there is a remarkable degree of cross-commuting within most metropolitan areas. Recent models that rationalise these empirical observations feature agglomeration economies and idiosyncratic worker preferences for location (Ahlfeldt et al., 2015) or probabilistic household and firm location choice (Wrede, 2015). A central theoretical implication of such models is that the price of residential land at a given location depends on the proximity to employment at all locations in a labour market area, not just proximity to the CBD.

Recent empirical literature uses employment potentials to capture the labour market accessibility effects on the price of real estate in cities with a polycentric or dispersed employment distribution (Ahlfeldt, 2011, 2013; McArthur et al., 2012; Osland and Thorsen, 2008). Borrowing from Harris' (1954) market potential concept, an employment potential capitalisation model establishes a spatial relationship between the price of land at a given location and employment at all locations in the city. Briefly summarised, the employment potential at a given location is the sum of employment across all potential commuting destinations, weighted by the bilateral transport costs. Crucially, employment at closer locations receives a higher weight in the employment potential, with the exact rate of spatial decay being subject to estimation. Although the evidence base is fairly limited, it is notable that the existing studies tend to find a similar spatial decay (Ahlfeldt, 2013).

Employment potentials tend to be successful in empirically establishing a relationship between land prices and the spatial distribution of employment (Ahlfeldt, 2011). It is less clear, however, to which extent the estimated spatial decays in employment potential capitalisation models reflect the declining commuting probability between two locations as travel costs increase. Theoretically, an employment potential may also capture the benefits from access to the amenities that often correlate with the distribution of employment in space, such as retail services or gastronomic establishments. Locational fundamentals – such as access to parks or water spaces – that simultaneously determine the distribution of population at residence and employment at workplace could also affect the estimated

spatial decay in an employment potential capitalisation model. This raises the question of how to interpret the capitalisation effect captured by an employment potential.

In this short paper we seek to shed light on this question by comparing estimates of the decay parameter in an employment capitalisation model to the actual decay in bilateral commuting probabilities observed in commuting data. To benchmark the estimates from the employment potential capitalization model we draw on a separate literature that has estimated the decay in the commuting probability between two localities as a function of effective distance (e.g. travel time) using commuting gravity equations (e.g. Ahlfeldt et al., 2015; McArthur et al., 2011).<sup>1</sup> This approach provides credible estimates of the rate at which the bilateral commuting probability declines in travel cost as it is directly estimated from commuting data. Commuting gravity models share many similarities with gravity models of trade (e.g. Camarero et al., 2014) or immigration (e.g. Lewer and Van den Berg, 2008), but micro-geography data on commuting flows is usually more difficult to find. If the employment potential captures the cost of commuting in a polycentric environment, the estimated decay parameters in the employment potential capitalisation model and the commuting gravity model will be of similar magnitude.

Our contribution to the literatures is twofold. First, we estimate an employment potential capitalization model using a large polycentric metropolitan region, which has never been analysed using these methods before. Thus, we add to the growing, but still small evidence base in this literature. Second, we compare the estimated spatial decay from the employment potential capitalisation model to the spatial decay in commuting probabilities estimated from a commuting gravity model using actual commuting data from the same region. Our results substantiate the claim in the literature that the estimated spatial decay in employment potential capitalisation models is reflective of the cost of commuting in polycentric regions. As collateral we find that commuting decays can be inferred from the spatial distribution of land prices and employment where suitable data for the estimation of a commuting gravity model are not available.

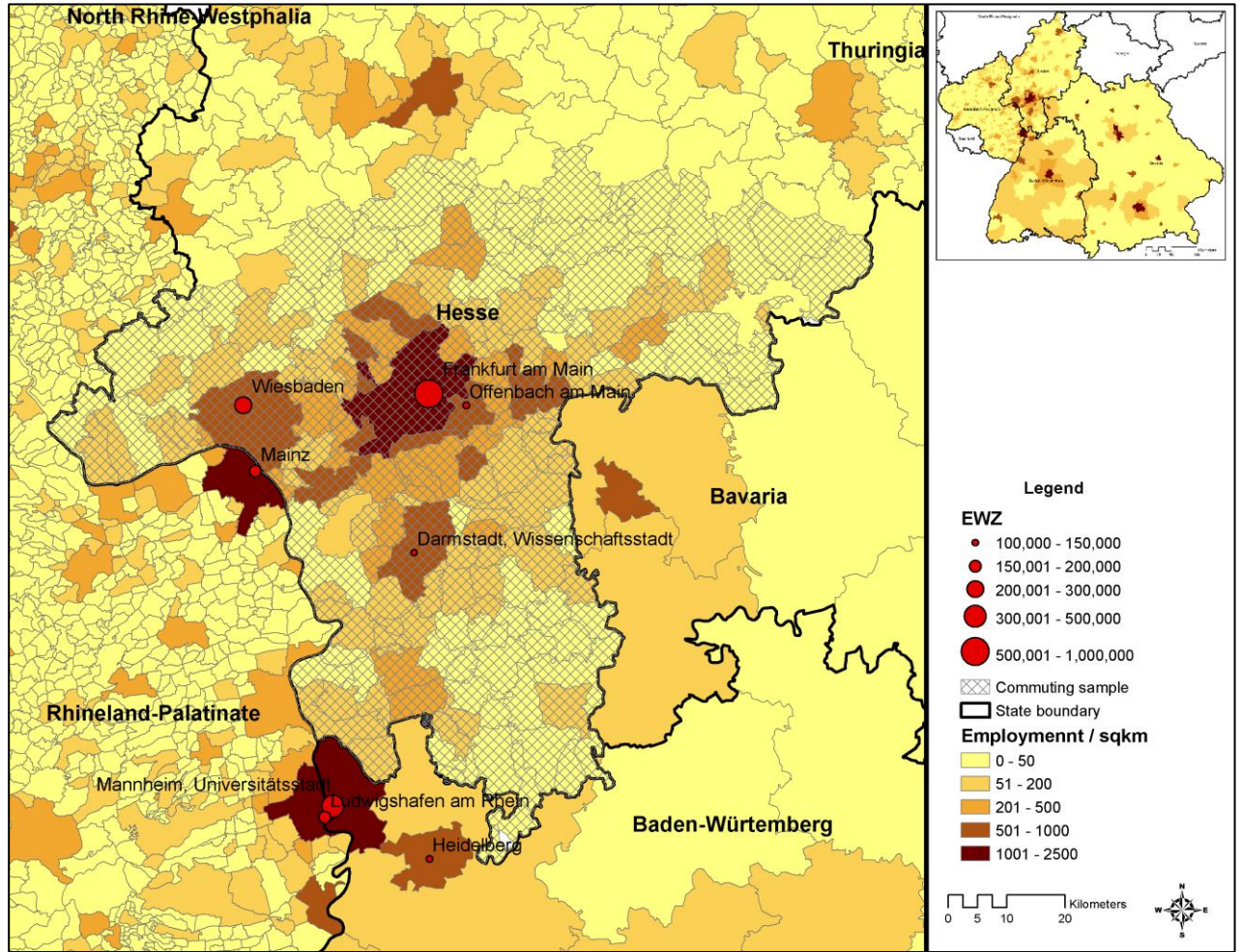
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<sup>1</sup> For a recent review on the reduced form literature on commuting gravity estimates see McDonald and McMillen (2010).

## 2 Data

Our core study area is the planning region South-Hesse (Planungsregion Südhessen), which roughly corresponds to the wider Frankfurt (Main) metropolitan area. comprises of 185 municipalities (Gemeinden) within the hatched area in Figure 1. With a 2009 population of slightly less than 3.8m and a 2009 GDP of slightly more than €150bn this region belongs to the economically more prosperous regions in Germany. From the Federal Employment Agency we obtained bilateral commuting data for all combinations of municipalities within this region with at least 10 commuters on the reporting day (30/6/2011).

For the same set of municipalities we collected standard land values (Bodenrichtwerte) of residential land from the respective local Committees of Valuation Experts (Gutachterausschüsse). Land values assessed by such committees, which exist throughout Germany, capture the fair market value of a square metre of land if it was undeveloped. The assessment by these committees is based on recent market transactions and is generally considered reliable (Weiss, 2004). Ahlfeldt et al. (2015), who use similar data, show that the standard land values tend to closely follow market prices. We further use data on the resident population 2009 and employment at workplace from the German Federal Statistical Office. To avoid border effects in the employment potential we collect employment at workplace for a much larger region, namely 2,872 administrative units in Hesse and Rhineland Palatinate (both at municipality level), Baden-Württemberg and Bavaria (both at county level) illustrated in the small map in the upper-right corner of Figure 1. Shares of the 2009 population with completed A-levels, apprenticeship, polytechnic degree, and university degree were also available from the Federal Statistical Office at the county level (Kreise und kreisfreie Städte).

**Fig. 1 Study area**

Notes: Own illustration using electronic maps from the Federal Agency for Cartography and Geodesy and data from the Federal Statistical Office. Hatched area is the sample of municipalities for which commuting data is available. The small map in the upper right corner shows the coverage of employment data in the potential (the states of Baden-Württemberg, Bavaria, Hesse, and Rhineland-Palatinate).

### 3 Empirical strategy

Our employment potential capitalisation model takes the following form:

$$\ln(P_i) = \alpha + \beta \ln\left(\sum_j E_j e^{\tau^P t_{ij}}\right) + X_i b + \varepsilon_i \quad (1)$$

, where  $P_i$  is the residential land price at location  $i$ ,  $E_j$  is employment at workplace  $j$ ,  $X_i$  is a vector of locational control variables, and  $t_{ij}$  is the travel time in minutes between the geographic centroids of



$i$  and  $j$ .<sup>2</sup> The vector of corresponding implicit hedonic prices (Rosen, 1974)  $b$ ,  $\alpha$ , and  $\tau^P$  are the parameters to be estimated and  $\varepsilon_i$  is a random error term. The capitalisation effect of the employment potential is jointly determined by  $\beta$ , the elasticity of land price with respect to employment potential, and  $\tau^P$ , the spatial decay parameter, the latter being the parameter of primary interest in this research. Economically sensible combinations of parameter values satisfy  $\beta > 0$  and  $\tau^P < 0$ . Following the standard in the literature we estimate this non-linear model using a non-linear least squares estimator. The exponential cost function in the employment potential has received empirical support in the literature (Ahlfeldt, 2013) and is directly comparable to the decay function used in our gravity equation (1).

To obtain a reliable approximation of the true spatial decay in the bilateral commuting probabilities we estimate the following commuting gravity equation:

$$\ln(c_{ij}) = \tau^C t_{ij} + o_i + d_j + \epsilon_{ij} \quad (2)$$

, where  $c_{ij}$  is the probability that a commuter commutes from locality  $i$  to locality  $j$ , which we empirically approximate as the share of commuters in  $i$  commuting to  $j$  ( $C_{ij}$ ) at all commuters in  $i$ . ( $C_{ij} / \sum_j C_{ij}$ ).  $o_i$  and  $d_j$  are origin and destination fixed effects which capture all push and pull factors specific to individual locations in a flexible manner,  $t_{ij}$  is defined as above,  $\epsilon_{ij}$  is an error term and  $\tau^C < 0$  is the parameter of interest capturing the spatial decay in commuting probabilities. We adopt the semi-log form for the gravity equation, which Ahlfeldt et al. (2015) demonstrate to represent a good approximation for Germany.<sup>3</sup> We estimate the specification using OLS and the Poisson PML estimator used by Silva and Tenreyro (2006; 2010).

## 4 Empirical results

We report the results of the estimation of equation (1) in Table 1. We begin with a model excluding controls in column (1), then add a number of exogenous geographic controls in column (2) and some additional endogenous controls capturing the density and education of the resident population in

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<sup>2</sup> Travel times are computed using MS MapPoint 2010. We approximate the internal travel time (min) for  $i=j$  using the following formula:  $t_{ij} = 4/3\sqrt{A\pi}$ , where  $A$  is the geographic area of a spatial unit in km.

<sup>3</sup> According to Forthringham and O’Kelly (1989) the semi-log specification has emerged as a consensus in the related literature. See for another recent application McArthur et al. (2011).

column (3). While in theory the last model could be subject to a “bad control problem” (Angrist and Pischke, 2009) it turns out that the estimates are highly consistent across specifications. The estimated decay parameters are also within the range of previous estimates of employment potential capitalisation models (Ahlfeldt, 2011, 2013; Osland and Thorsen, 2008).

**Tab. 1 Employment potential capitalisation model estimates**

	(1) Ln land price	(2) Ln land price	(3) Ln land price
Ln employment potential	0.566*** (0.049)	0.529*** (0.053)	0.558*** (0.079)
Spatial decay	-0.119*** (0.013)	-0.137*** (0.014)	-0.097*** (0.018)
Estimation	NLS	NLS	NLS
Exogenous controls	-	Yes	Yes
Endogenous controls	-	-	Yes
N	185	185	185
r <sup>2</sup>	0.684	0.729	0.766

Notes: Exogenous controls are geographic surface area of the municipality, the latitude and the longitude of the geographic centroid, the elevation, and the distance from the nearest river. Endogenous controls are population density as well as shares of the population with completed A-levels, apprenticeship, polytechnic degree, and university degree (county level). Standard errors for ln employment potential are heteroscedasticity robust in (1–2) and clustered on counties in (3), in each case estimated in auxiliary OLS regression using an employment potential constructed based on the NLS decay parameter estimate. Decay parameter standard errors are from the original NLS estimation. \*  $p < 0.1$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

The elasticity of land price with respect to the employment potential is large, which is in line with the prominent role commuting plays in standard urban economics models. To facilitate a comparison with the existing literature we translate our estimated land price capitalisation elasticity into a house price capitalisation elasticity by multiplying the former by 0.25 following Ahlfeldt et al. (2015) and Combes et al. (2013). This transformation follows from assuming a competitive construction sector with a Cobb-Douglas housing production function, with 0.25 being the land share. The implied elasticity of house prices with respect to employment potential of about 0.14 is somewhat below previous estimates for Berlin, Germany, Rogaland, Norway, and London, UK, which are within the range of 0.25–0.3 (Ahlfeldt, 2011, 2013; Osland and Thorsen, 2008).<sup>4</sup>

We report the results of the estimation of equation (2) in Table 2. Our preferred model is the Poisson PML model using all available data reported in column (3). The estimates imply that the commuting

<sup>4</sup> The German statistical office recently reported that the expenditure share on housing has risen to 35% (Statistisches Bundesamt, 2015). With this share, the implied elasticity of housing rent price with respect to employment potential is about 0.2 and, thus, closer to the existing estimates.

probability declines by 11.2% per minute of travel time between two locations.<sup>5</sup> This estimate is within the range of recent estimates in the literature (Ahlfeldt et al., 2015; McArthur et al., 2011). The estimated parameter is not particularly sensitive to changes in sample size as shown by the estimates in column (4) which use less than one fourth of all available observations. The estimates are also reasonably close to OLS estimates (reported in columns 1 and 2). We note that a visual inspection of the partial correlation between bilateral commuting probabilities and travel times suggests that the semi-log gravity equation provides a decent fit to the data (see Figure A1 in the appendix).

**Tab. 2 Commuting gravity model estimates**

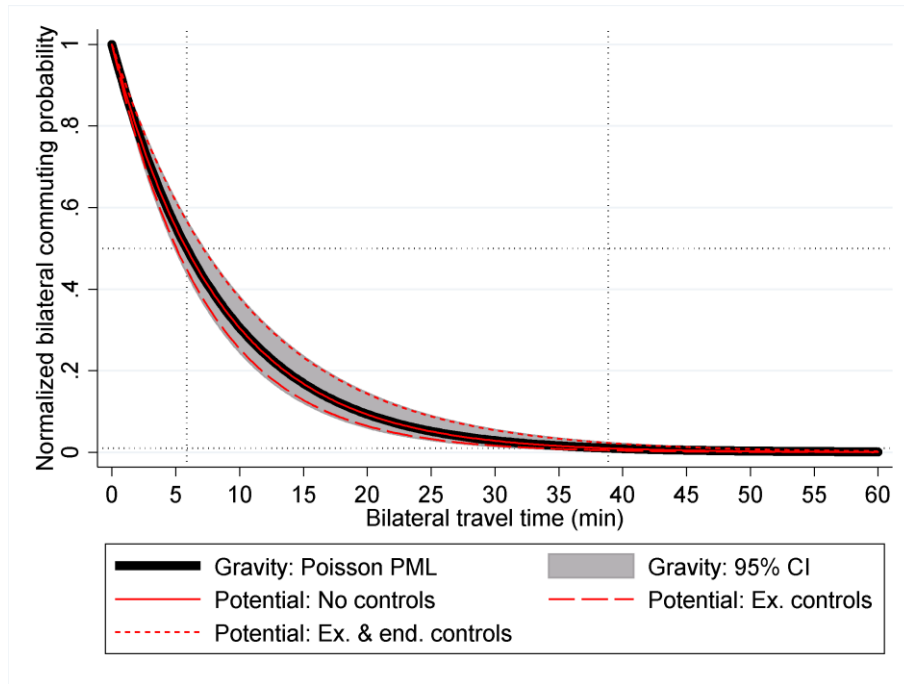
	(1) Ln bilateral commuting probability	(2) Ln bilateral commuting probability	(3) Ln Bilateral commuting probability	(4) Ln Bilateral commuting probability
Travel time from <i>i</i> to <i>j</i> in minutes	-0.074*** (0.002)	-0.088*** (0.003)	-0.119*** (0.011)	-0.105*** (0.011)
Fixed effects	Yes	Yes	Yes	Yes
Estimation	OLS	OLS	Poisson PML	Poisson PML
Commuters	≥10	≥100	≥10	≥100
r <sup>2</sup>	0.790	0.872	0.791	0.803
N	6,918	1,507	6,918	1,507

Notes: Observations are bilateral pairs of 185 workplace and residence localities. Only pairs with more than 10 commuters are included. Travel time is measured in minutes. Fixed effects are workplace locality fixed effects and residence locality fixed effects. The specifications labelled ≥10 (≥100) commuters restrict attention to bilateral pairs with 10 (100) or more commuters. Poisson PML is Poisson Pseudo Maximum Likelihood estimator. Standard errors in parentheses are heteroscedasticity robust.

\* p<0.1, \*\* p<0.05, \*\*\* p<0.01.

Figure 2 illustrates the estimated decay parameters reported in Table 1 and the benchmark estimate from Table 2, column (3). Consistently, the estimated parameters point to a half-live distance of slightly more than five minutes. After slightly less than 40 minutes the commuting probability is reduced to 1% of its value at a hypothetical travel time of zero. Notably, all estimates of the employment potential capitalisation models fall within the 95% confidence interval of our benchmark gravity estimate.

<sup>5</sup> We use the standard transformation that applies to semi-log models  $(\exp(-0.119) - 1) \times 100 = -11.2\%$ .

**Fig. 2 Decay parameters: Employment potential vs. commuting gravity**

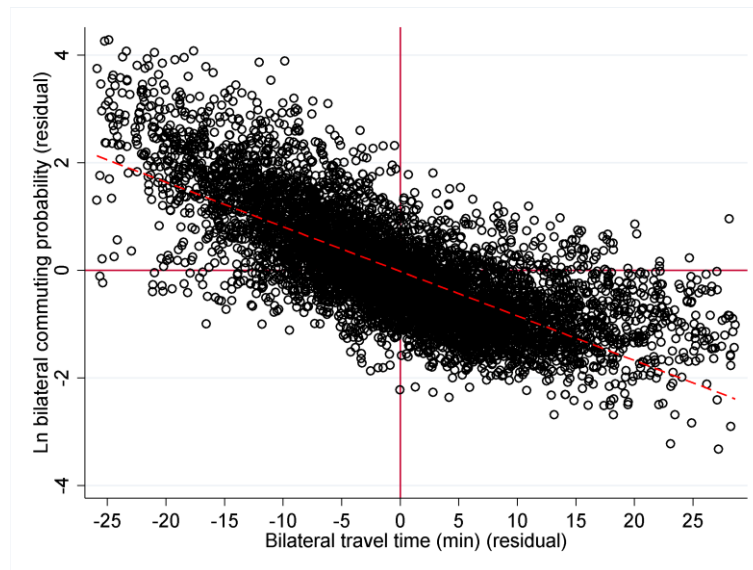
Notes: Figure plots the function  $f(d) = e^{\tau^A d_{ij}}$ ,  $A = (C, P)$  using the decay parameter estimates from Table 1, column (3) and Table 2, columns (1–3). The intersections of the dotted lines define where the commuting probability declines to 50% and 1% of its value at a travel time of zero.

## 5 Conclusion

Using a combination of spatial data sets for the polycentric Rhein-Main-Neckar region in Germany we show that the estimated spatial decay in an employment potential capitalisation model is very close to the spatial decay in observable commuting data. These results suggest that the estimated spatial decay in employment potential capitalization models is reflective of the cost of commuting in polycentric regions as typically assumed in the literature. A collateral finding is that with the help of such a model the decay in commuting probabilities can indirectly be inferred from the spatial distribution of land prices and employment if bilateral commuting data are not available. More generally, our findings support the use of employment potentials as a means to capture labour market accessibility effects in polycentric regions.

## Appendix

**Fig. A1: Partial correlation between bilateral commuting probabilities and travel time**



Notes: Figure illustrates the residuals from separate regressions of ln bilateral commuting probabilities and travel time against origin fixed effects and destination fixed effects.

Our results, thus, suggest that employment potentials are good means to capture labour market accessibility effects in polycentric regions.

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