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Condorcet meets Ellsberg

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The Condorcet Jury Theorem states that given subjective expected utility maximization and common values, the equilibrium probability that the correct candidate wins goes to 1 as the size of the electorate goes to infinity. This paper studies strategic voting when voters have pure common values but may be ambiguity averse—exhibit Ellsberg-type behavior—as modeled by maxmin expected utility preferences. It provides sufficient conditions so that the equilibrium probability of the correct candidate winning the election is bounded above by $\frac{1}{2}$ in at least one state. As a consequence, there is no equilibrium in which information aggregates.

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JEL classification. D72, D81.

1. Introduction

The Condorcet Jury Theorem shows that elections can reduce uncertainty by aggregating the electorate's private information. Roughly, it states that if voters maximize subjective expected utility (henceforth, SEU) and have common values, then there exists an equilibrium to the voting game in which all private information is revealed for a large enough electorate.[1] However, the correct policy often depends on events about which the electorate has only vague information—ambiguous events—and the literature, e.g., Ellsberg (1961), argues that SEU does not accurately describe preferences under ambiguity. This paper studies whether elections aggregate information with ambiguity averse voters.

My main result shows that many pure common values voting games with ambiguity averse voters have no equilibrium in which information aggregates, regardless of the size of the electorate. Information aggregation fails when each voter's private information is not precise enough to reduce her prior uncertainty sufficiently. A rational voter

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[1]For instance, see Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1997, 1999), Myerson (1998), or Wit (1998).

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takes into account the probability that her vote changes the outcome of the election, and her equilibrium strategy may differ from the action that her private information alone would suggest is best. If voters are SEU, then each vote noisily reveals private information, and with enough voters, information aggregates. An ambiguity averse voter may instead minimize the probability of casting a pivotal vote because she overweights the probability that her choice alters the outcome in favor of the worse candidate. To do so, she plays a mixed strategy, and in equilibrium, no vote reveals information, precluding information aggregation.

Many important policy decisions are made under ambiguity.[2] A policy to cap carbon emissions deals with poorly understood costs, base case emissions, and tails of the probability distribution of temperature changes. The recession of 2008–2009 resulted at least in part from an unprecedented event (systematic default in AAA rated bonds) in the credit market. The Federal Reserve decided whether to bail out banks and hedge funds based on their beliefs about the poorly understood connection between this default, these companies, and the financial system as a whole. Many foreign policy decisions must be made despite possessing only poor quality information, such as that leading to the 2003 invasion of Iraq.

Agents are typically ambiguity averse—they prefer betting on unambiguous events to ambiguous ones. For example, a bet on an event E, which is known to occur with probability $\frac{1}{2}$, may be preferred both to a bet on the event F and a bet on its complement F^c when no information is provided about F. Ambiguity aversion explains evidence from asset markets that contradicts SEU (see, e.g., Epstein and Schneider 2010). To accommodate ambiguity aversion, I assume voters conform to maxmin expected utility (henceforth, MEU; axiomatized in Gilboa and Schmeidler 1989). Each voter considers a set of probability measures and evaluates an act by taking its minimum expected utility with respect to every measure in that set, i.e., has a utility function of the form $U(f) = \min_{\pi\in\Pi} \mathbb{E}_\pi[u \circ f]$, where Π is a set of probability measures and $u(\cdot)$ is a utility index.

The paper proceeds as follows. Section 2 gives an example illustrating the mechanism by which ambiguity aversion precludes information aggregation. Section 3 introduces the model, and Section 4 presents the paper's main results. Theorem 1 shows that ambiguity aversion can preclude the existence of any equilibrium that aggregates information. Theorem 2 provides sufficient conditions for the existence of an equilibrium that aggregates information. Section 5 explores the robustness of the result. Theorem 3 shows that information may fail to aggregate even if voters strategically abstain. Section 6 concludes by relating the main results to other works that show failure of information aggregation in voting games. Proofs are collected in the Appendix. Additional information and calculations are available in a supplementary file on the journal website, http://econtheory.org/supp/1284/supplement.pdf.

[2]Previous work that addresses political economy questions with ambiguity averse voters or candidates include Berliant and Konishi (2005), Ghirardato and Katz (2006), and Bade (2011), though none considers strategic interaction between voters.

2. Sincere voting and ambiguity

This section offers an example illustrating the mechanism by which ambiguity aversion alters voter behavior. In Section 4, I show that this mechanism precludes information aggregation as an equilibrium outcome in many voting games with ambiguity averse players. I defer formal details to Section 3.

Consider an election with 101 voters who vote for one of two candidates, A and B. The candidate with the most votes wins. There are two states of the world, a and b, and all voters agree that A's policy is better in state a but B's policy is better in state b. Voters are uncertain about which state obtains and conform to MEU with the set of priors, Π, that contains measures assigning a marginal probability of a ranging from $\underline{p}$ to $\bar{p}$, where $\underline{p} = 1 - \bar{p}$. Before voting, each voter observes a signal from the set $\{1, 2\}$. Signal 1 occurs with probability 0.6 in state a, signal 2 occurs with probability 0.6 in state b, and signals are independently distributed conditional on the state of the world. After observing signal t, each voter considers the set of posteriors Π_t consisting of the Bayesian update of each measure in Π. Each voter gets utility equal to 1 if the correct candidate is elected and 0 otherwise. Letting σ denote the strategy profile, all voters have the same ex ante preference, represented by

$$\min_{\pi \in \Pi} \pi(\text{correct candidate wins}|\sigma).$$

Because of the symmetry of beliefs about states and signals, a voter (strictly) prefers to bet on a over b if she observes signal 1 and vice versa if she observes signal 2. If all voters who observe 1 vote for A and all those who observe 2 vote for B, then information aggregates.[3] If voters are SEU ($\underline{p} = \bar{p} = \frac{1}{2}$), then Theorem 1 of McLennan (1998) shows that this sincere voting strategy is an equilibrium. However when $\underline{p} < 0.4$ and $0.6 < \bar{p}$, this strategy is *not* an equilibrium because all players best respond by voting for both A and B with equal probability.

For instance, suppose that $\underline{p} = 0.39$ and $\bar{p} = 0.61$. After updating, a player who observes signal 1 (respectively, 2) considers the marginal probability of state a (b) to be in the interval $[0.49, 0.7]$. Consider the problem of an arbitrary voter when all others vote sincerely. If this voter observes signal 1, then she picks her strategy to maximize

$$\min_{p \in [0.49, 0.7]} [p \Pr(A \text{ wins}|a) + (1 - p) \Pr(B \text{ wins}|b)].$$

She affects the outcome only when she is pivotal, or when exactly 50 of the others vote for A. Since all others vote sincerely,

$$\Pr(A \text{ has 50 votes}|a) = \Pr(B \text{ has 50 votes}|b) = \binom{100}{50} 0.6^{50} 0.4^{50} = \rho$$

and

$$\Pr(51+ \text{ votes for } A|a) = \Pr(51+ \text{ votes for } B|b) = \sum_{j=51}^{100} \binom{100}{j} 0.6^{j} 0.4^{100-j} = \theta.$$

[3] The probability the correct candidate wins is about 0.979 in either state.

If she votes for A with probability α, then

$$\Pr(A \text{ wins}|a) = \theta + \rho\alpha \quad \text{and} \quad \Pr(B \text{ wins}|b) = \theta + \rho(1-\alpha).$$

Therefore, this voter's utility from voting for A with probability α is

$$V(\alpha) = \min_{p \in [0.49, 0.7]} p[\alpha\rho + \theta] + (1-p)[(1-\alpha)\rho + \theta].$$

It is easy to see that $V(\cdot)$ is a continuous, piecewise linear function with slope equal to $(0.4)\rho$ when $\alpha < \frac{1}{2}$ and equal to $-(0.02)\rho$ when $\alpha > \frac{1}{2}$. Since $V(\cdot)$ increases with α when $\alpha < \frac{1}{2}$ and decreases with α when $\alpha > \frac{1}{2}$, voting for A and B with equal probability maximizes her utility.

This randomization insures her against ambiguity, and she strictly prefers it to any pure strategy, an impossibility under SEU. Intuitively, randomizing minimizes the probability that she makes a mistake. Conditional on being pivotal, she thinks that she makes a mistake with probability as high as 0.51 by voting for A or as high as 0.7 by voting for B, but by mixing, she makes a mistake with precisely probability 0.5. Ambiguity aversion implies that she strictly prefers the mixed strategy. Hence, her best response is to randomize between voting for A and B regardless of the signal she observes, and sincere voting fails to be an equilibrium. A symmetric argument to the above shows that the voter also prefers to mix in this way after observing signal 2. Should the whole electorate play this strategy, information could not aggregate because no individual's vote reveals the underlying signal. Indeed, all voters randomizing as above is an equilibrium to the game (Proposition 1).

That sincere voting fails to be an equilibrium is not in itself surprising: Austen-Smith and Banks (1996) show this is typically the case even with SEU voters. However, Theorem 1 shows that there is *no* equilibrium to the above game in which information aggregates. If there were a strategy profile in which information aggregates, then each voter best responds to it as above, by mixing to minimize the probability that she makes a mistake.

3. The model

This section formally introduces the class of games studied and defines the equilibrium concept. I then discuss the modeling assumptions, focusing on the set of priors considered. Finally, I show how the behavior of MEU voters differs from SEU voters.

Candidates A and B have committed to distinct policies in an unmodeled stage before the game begins. There are two payoff relevant states, $S = \{a, b\}$, and there is a finite set $I = \{1, \dots, 2n+1\}$ of voters, where n is a nonnegative integer. All voters agree that the policy of A is better in state a but that the policy of B is better in state b. Voter i receives utility $u_i(c, s)$ if candidate c wins the election and $s \in S$ obtains, where

$$u_i(A, a) = u_i(B, b) = 1$$

and

$$u_i(A, b) = u_i(B, a) = 0$$

for each i. The realized state of the world is initially unknown to all voters, but before taking any action, each observes a signal from a finite set T that has at least two elements. Voters simultaneously either cast a vote for candidate A (take action A) or cast a vote for candidate B (take action B). The candidate with the most votes wins the election and implements the policy to which she committed.

Up to this point, all assumptions follow previous work closely. I depart from the literature by allowing voters to perceive ambiguity regarding the payoff relevant state. Each voter conforms to MEU and considers a common set of priors Π, a closed and convex set of probability distributions over the underlying state space $\Omega = S \times T^I$.[4] Voters assign a marginal probability to a between $\underline{p}$ and $\bar{p}$, where $0 < \underline{p} \leq \bar{p} < 1$. Conditional on state s, the signal that voter i observes is distributed according to the distribution r_s. Formally, $\pi \in \Pi$ if and only if there exists a $p \in [\underline{p}, \bar{p}]$ so that

$$\pi(a, t) = p \prod_{i=1}^{2n+1} r_a(t_i) \quad \text{and} \quad \pi(b, t) = (1-p) \prod_{i=1}^{2n+1} r_b(t_i)$$

for all $(s, t) \in \Omega$. I discuss properties of and justification for this set of priors in Section 3.1. Any collection $\Gamma = (I, [\underline{p}, \bar{p}], T, r_a, r_b)$ as above defines an *ambiguous voting game*. An ambiguous voting game where $\bar{p} = \underline{p}$ corresponds to a standard SEU voting game.

Voters form a set of posteriors by updating each measure in Π according to Bayes rule. For any event E, let $\Pi(\cdot|E) = \{\pi(\cdot|E) : \pi \in \Pi\}$, so if voter i observes signal t_i, then she forms the set of posteriors $\Pi(\cdot|t_i)$. Denoting the vector of signals seen by other voters as t_{-i}, Bayes rule gives that π_{t_i} is a member of $\Pi(\cdot|t_i)$ if and only if there exists

$$p \in \left[\underline{p}_{t_i} = \frac{r_a(t_i)\underline{p}}{r_a(t_i)\underline{p} + r_b(t_i)(1-\underline{p})}, \bar{p}_{t_i} = \frac{r_a(t_i)\bar{p}}{r_a(t_i)\bar{p} + r_b(t_i)(1-\bar{p})}\right]$$

so that

$$\pi_{t_i}(a, t_i, t_{-i}) = p \prod_{j \neq i} r_a(t_j) \quad \text{and} \quad \pi_{t_i}(b, t_i, t_{-i}) = (1-p) \prod_{j \neq i} r_b(t_j)$$

for every $(s, t) \in \Omega$.[5]

After observing signal t_i, voter i chooses the strategy $\sigma_i(t_i)$, a probability distribution over $\{A, B\}$ so that voter i votes for candidate c with probability $\sigma_i(t_i)(c)$. A strategy profile is a vector denoting the strategy of every player conditional on every signal; as is standard, denote by σ_{-i} the vector $(\sigma_j)_{j \in I \setminus \{i\}}$. Her strategy combines with σ_{-i} and the conditional distribution of signals to yield a set of probabilities of electing A in a and of electing B in b. She evaluates $\sigma_i(t)$ by the minimum probability of electing the correct candidate according to these probabilities and her beliefs; specifically, she gets

[4]That is, Ω encapsulates both the payoff relevant state as well as the signal that each voter receives.

[5]Since $\pi(a, t_i, t_{-i}|t_i) = pr_a(t_i)/(pr_a(t_i) + (1-p)r_b(t_i)) \prod_{j \neq i} r_a(t_j)$ is increasing in p, the maximum occurs at $p = \bar{p}$ and the minimum occurs at $p = \underline{p}$.

utility

$$\begin{aligned} V_{t_i}(\sigma_i(t_i), \sigma_{-i}) &= \min_{\pi\in\Pi} \mathbb{E}_\pi[u_i|t_i, \sigma_i(t_i), \sigma_{-i}] \\ &= \min_{\pi\in\Pi} \pi(\text{correct candidate wins}|\sigma_i(t), \sigma_{-i}, t_i) \end{aligned} \tag{1}$$

conditional on other voters playing the strategy profile σ_{-i} and on observing the signal t_i.

A strategy profile is an equilibrium if each voter chooses the best strategy given her set of posteriors about the play of other voters.

Definition 1. A strategy profile σ^* is an *equilibrium* if for each player i and each signal t,

$$\sigma_i^*(t_i) \in \arg\max_{\hat\sigma} V_{t_i}(\hat\sigma, \sigma_{-i}^*).$$

If σ^* is an equilibrium, then every player chooses a strategy that maximizes her utility, i.e., the minimum probability that the correct candidate wins, given her updated beliefs and that the other players follow the strategy profile σ^*. When Π is singleton, this definition is equivalent to the notion of Bayesian Nash equilibrium. This solution concept specializes Lo's (1996, 1999) "beliefs equilibrium with agreement" so that Nature is a player and in which all players, except Nature, choose an unambiguous strategy. Nature, who gets the same utility from every outcome, chooses an action in Ω, and the set Π describes other player's beliefs about Nature's chosen action.

Since nothing distinguishes one voter from another except information, I focus on symmetric strategy profiles. A strategy profile σ is *symmetric* if $\sigma_i = \sigma_{i'}$ for all players i and i'; when σ is symmetric, I abuse notation slightly by writing $\sigma(t_i)$ instead of "$\sigma_i(t_i)$ for an arbitrary i." My solution concept is a *symmetric equilibrium*, i.e., a strategy profile that is both symmetric and an equilibrium. This is the standard solution concept in voting games (see, e.g., Austen-Smith and Banks 1996, Feddersen and Pesendorfer 1996, 1997, or Myerson 1998). A symmetric equilibrium (and thus an equilibrium) exists for every ambiguous voting game. For example, whenever there are at least three voters, every player voting for candidate A with probability 1, regardless of the signal observed, is a symmetric equilibrium.[6]

I interpret mixed strategies as objective randomization.[7] When each voter selects her strategy, the state of the world is realized but unknown. Ambiguity averse players act as if Nature picked the distribution in Π that minimizes her utility given her strategy. A mixed strategy substitutes objective risk for subjective uncertainty, smoothing utility across states and limiting her exposure to Nature's choice. Consequently, she may

[6]Other equilibria typically exist; see Proposition 1 or Theorem 2.

[7]Alternative interpretations include the Harsanyi (1973) idea that the strategy of player i represents the uncertainty of the other players about the action of player i. The strategy of an SEU player is nondegenerate only if strategies of other players make her indifferent between all actions in its support. An MEU player values hedging, so her optimal strategy may assign positive probability to two actions with different payoffs. This implies that one cannot purify mixed strategies with small utility shocks.

strictly prefer a mixed strategy to any pure strategy.[8] For a more in depth discussion of mixed strategies in games with MEU players, see Lo (1996) or Klibanoff (1996).

3.1 *The set of priors*

I focus on the class of sets of priors above because any member is characterized by the following three properties.

First, voters are symmetric. Formally, both $\Pi(\cdot|a)$ and $\Pi(\cdot|b)$ have the same set of marginals on T_i as on T_j for all $i, j \in I$. This assumption, standard in the literature, makes the games tractable to analyze.

Second, the signals of two voters have known conditional distributions that make each (conditionally) independent of the other. In fact, the model collapses to a SEU voting game conditional on either state of the world. This assumption simplifies exposition and gives information the best chance to aggregate. To see why, note that if all private information were public and $r_a \neq r_b$, then uncertainty vanishes for a large enough electorate. Even under SEU, relaxing conditional independence can lead to failure of information aggregation; see, e.g., Mandler (2012). Additionally, calculating the probability, *conditional on either state*, that she is pivotal or that a given candidate wins the election is tractable for a symmetric strategy profile.

Third, the set of priors has a natural structure in a dynamic setting. Although the model considered is essentially static because agents take no ex ante action, uncertainty has an implicit dynamic component. In voting games, the uncertainty is naturally modeled as a two stage process. In the first stage, $s \in \{a, b\}$ is determined. In the second stage, the signal of each voter is determined through a process that may depend on the realization of s in the first stage.[9] The set of priors is rectangular (Epstein and Schneider 2003) with respect to the natural filtration of an outsider who observes the outcome of each stage in order.[10]

Players perceive information in a different order than that of an outside observer and so do not have rectangular priors. As a consequence, and as in most games with incomplete information and ambiguity, voters violate dynamic consistency.[11] In equilibrium, each voter chooses an interim optimal strategy—the best strategy given her observed signal without consideration of counterfactual signal realizations. I focus on interim optimality rather than ex ante optimality because a voter's first decision takes place after she observes her signal, i.e., no meaningful action takes place ex ante. In this sense, the game is essentially static. While one could extend the model to allow for a voter to commit to a plan of action at an ex ante state, this is undesirable. Signals may not be observable, and even if they are, they may not be contractible. Even if signals are feasible to contract upon, enforcing a (conditional) commitment to vote for a given candidate likely

[8] Raiffa (1961) first observes that objective randomization reduces ambiguity, and the uncertainty aversion axiom of Gilboa and Schmeidler (1989) is often interpreted as a strict preference for randomization.

[9] Although the order of the stages matters for MEU, it does not for SEU.

[10] Specifically, $\mathcal{F}_0 = \{\Omega\}$ and $\mathcal{F}_1 = \{\{a\} \times T^{2n+1}, \{b\} \times T^{2n+1}\}$.

[11] Given a common set of priors, dynamic consistency and consequentialism for all players is a very demanding condition that fails in, among others, Salo and Weber (1995), Lo (1998), Bose et al. (2006), Chen et al. (2007), Bose and Daripa (2009), Bodoh-Creed (2012), and Bose and Renou (2014).

gives rise undesirable social consequences: almost all democratic countries have secret balloting for a reason. In fact, offering a voter a contract that commits her to follow a given plan would almost certainly violate U.S. law.[12]

Special attention needs to be paid to the value of randomization in the absence of dynamic consistency. Given that voters know the outcome of their own randomization before they vote and that uncertainty has not resolved, they may prefer to play the randomization again rather than its realization. However, past realizations provide no new information and thus do not alter her preferences, so should she randomize again, she does so with the same distribution. Provided that she must vote eventually, her final vote should still obey the distribution of the initially chosen optimal randomization. Alternatively, Machina (1989) argues in a different context that unwillingness to revisit the choice is a reasonable property in the absence of expected utility.

3.2 *Pivot probabilities and voting behavior*

When choosing for whom to cast her ballot, voter i cares only about how others vote, not their type. Of particular importance is the event that she is pivotal, i.e., of the $2n$ other voters, exactly n vote for each candidate. If voter i is pivotal, then her vote determines the outcome of the election. If voter i is not pivotal, then her vote affects neither the winner of the election nor the policy implemented.

Given a symmetric strategy profile σ, the probability of a given player voting for candidate c conditional on state s equals

$$\tau(c|\sigma, s) = \sum_{t\in T} r_s(t)\sigma(t)(c),$$

and this probability is independent across voters. This allows decomposition of the utility in terms of the pivot probability conditional on state s,

$$\rho_s(\sigma) = \binom{2n}{n} \tau(c|\sigma, s)^n[1 - \tau(c|\sigma, s)]^n,$$

and the probability that the correct candidate wins the election in state s regardless,

$$\theta_s(\sigma) = \sum_{m=n+1}^{2n} \binom{2n}{m} \tau(c|\sigma, s)^m[1 - \tau(c|\sigma, s)]^{2n-m},$$

where candidate c is the preferred winner in state s. Given this notation, (1) can be rewritten as

$$V_{t_i}(\hat{\sigma}, \sigma_{-i}) = \min_{q\in[\underline{p}_{t_i}, \bar{p}_{t_i}]} \bigl\{q[\hat{\sigma}(A)\rho_a(\sigma) + \theta_a(\sigma)] + (1 - q)[\hat{\sigma}(B)\rho_b(\sigma) + \theta_b(\sigma)]\bigr\}. \quad (2)$$

[12]Section 597 of Title 18 of the U.S. Code states that any person who "makes or offers to make an expenditure to any person, either to vote or withhold his vote, or to vote for or against any candidate" has committed a felony.

As in the SEU case, pivot probabilities affect voter behavior; for instance, if the voter is much more likely to be pivotal in state b, i.e., $\rho_a(\sigma)/\rho_b(\sigma) \approx 0$, then no player votes for candidate A.

Nevertheless, (2) reveals a key difference between the behavior of MEU and SEU voters: the probabilities that the correct candidate wins the election in each state ($\theta_a(\sigma)$ and $\theta_b(\sigma)$) affect the behavior of MEU but not SEU voters. As previous work emphasizes, SEU voters update their beliefs based on the probability that they are pivotal, so $\theta_a(\sigma)$ and $\theta_b(\sigma)$ are conditioned out of the utility function. This is an *as if* result that fails for MEU voters.[13] For an MEU voter, $\theta_a(\sigma)$ and $\theta_b(\sigma)$ play a role in determining which probability measure minimizes the payoff to her strategy. As a consequence, the pivot probabilities in alone are not sufficient to determine her best response.[14]

To illustrate, suppose that there exist two strategy profiles σ_1 and σ_2 so that $\theta_a(\sigma_1) > \theta_b(\sigma_1) + \rho_b(\sigma_1)$, $\theta_a(\sigma_2) + \rho_a(\sigma_2) > \theta_b(\sigma_2)$ and $\rho_a(\sigma_1)/\rho_b(\sigma_1) = \rho_a(\sigma_2)/\rho_b(\sigma_2)$. The best response of an SEU voter is the same when others play either σ_1 or σ_2. However, the behavior of MEU voters may differ. When others play strategy profile σ_1, the minimizing posterior in (2) assigns the smallest allowable probability to a, but when others play strategy profile σ_2, it assigns the smallest allowable probability to b. Because her minimizing posterior differs, her best response differs as well.

This lack of equivalence has some similarity to an experimental finding of Esponda and Vespa (2014). They show that subjects act differently when explicitly informed that they are pivotal than in equivalent situations when uninformed. As their setting includes only risk, the lack of equivalence is attributed to failure to think hypothetically rather than ambiguity aversion. In this setting, conditioning on the pivotal event makes voters more likely to hedge by mixing or abstaining.

4. Information aggregation

This section analyzes the properties of equilibria for ambiguous voting games. The two main results give conditions on the beliefs of voters that either rule out aggregation of information in equilibrium (Theorem 1) or allow construction of a sequence of equilibria as the number of players goes to infinity so that information aggregates (Theorem 2).

4.1 *Information aggregation*

Because there is always some possibility of a mistake in a finite electorate, one cannot require full certainty that voters elect the proper candidate in a given game. I focus on two

[13] Specifically, an SEU voter with prior π who sees signal t maximizes

$$U_t(\hat{\sigma}) = \pi(a|t)[\hat{\sigma}(A)\rho_a(\sigma) + \theta_a(\sigma)] + \pi(b|t)[\hat{\sigma}(B)\rho_b(\sigma) + \theta_b(\sigma)].$$

One can subtract $\pi(a|t)\theta_a(\sigma) + \pi(b|t)\theta_b(\sigma)$ and multiply by $[\pi(a|t)\rho_a(\sigma) + \pi(b|t)\rho_b(\sigma)]^{-1}$ (both positive affine transformations) to yield

$$G_t(\hat{\sigma}) = \hat{\sigma}(A)\pi(a|t, \mathit{pivotal}) + \hat{\sigma}(B)\pi(b|t, \mathit{pivotal}).$$

Consequently, $\hat{\sigma}$ maximizes $U_t(\cdot)$ if and only if $\hat{\sigma}$ maximizes $G_t(\cdot)$. The term $G_t(\cdot)$ is often referred to as the gain from playing a strategy (relative to abstention).

[14] In the Supplement, I show that assuming that voters update each prior conditional on the pivotal event does not alter the main result.

definitions of information aggregation. The first applies to a single game and is a necessary condition for the second, which applies to a sequence of games with population approaching infinity.

The weaker definition requires that A has more expected votes than B in state a and B has more expected votes than A in state b.

Definition 2. A strategy profile σ *has correct expected winners* if

$$\tau(A|\sigma, a) > \tau(B|\sigma, a) \quad \text{and} \quad \tau(B|\sigma, b) > \tau(A|\sigma, b).$$

When voters play a strategy profile that has correct expected winners, the candidate whom voters prefer to win, i.e., A in state a or B in state b, wins with probability greater than $\frac{1}{2}$. Existence of such an equilibrium strategy profile for all games with a large enough population is a necessary condition to apply the law of large numbers and conclude that information aggregates. Nonexistence of such an equilibrium strategy profile implies more than just this inability to apply the law of large numbers. Specifically, either A wins in state a or B wins in state b with probability of at least $\frac{1}{2}$ in any equilibrium, a stronger failure of information to aggregate.

The stronger definition follows the literature in studying sequences of voting games for which the population of voters grows to infinity. Information aggregates if there exists a sequence of equilibria to the games along which the probability of electing the wrong candidate vanishes. Throughout, any sequence of ambiguous voting games is indexed by the number of players, with the understanding that all of the other primitives remain the same.

Definition 3. For a sequence of ambiguous voting games $(\Gamma_n)_{n=1}^{\infty}$, a sequence of strategy profiles $(\sigma_n)_{n=1}^{\infty}$ satisfies *full information equivalence* (FIE) if there exists an $\epsilon > 0$ and an $N \in \mathbb{R}$ so that

$$\tau(A|\sigma_n, a) > \tau(B|\sigma_n, a) + \epsilon \quad \text{and} \quad \tau(B|\sigma_n, b) > \tau(A|\sigma_n, b) + \epsilon$$

for all $n > N$.[15]

By the law of large numbers, the probability of A (resp. B) winning in state a (resp. b) goes to 1 whenever the sequence of strategy profiles satisfies FIE. If σ_n is the member of such a sequence and $n > N$, then σ_n has correct expected winners. Consequently, if each equilibrium strategy profile does not have correct expected winners, then the sequence fails to satisfy FIE.

If all signals were public and $r_a \neq r_b$, then voters would be arbitrarily certain which candidate is correct for a large enough electorate. FIE for a sequence of strategy profiles requires that the outcome of the election reflects this with a probability that goes to 1 as the size of the electorate goes to infinity. That is, the outcome of the election reflects the aggregation of all private information with an arbitrarily high probability.

[15]This definition is adapted from Feddersen and Pesendorfer (1997).

4.2 *Lack of confidence precludes information aggregation*

This subsection describes a set of ambiguous voting games for which no equilibrium aggregates information. Theorem 1 below shows that if voters lack confidence, then no equilibrium aggregates information. Voters lack confidence when the following condition holds.

Definition 4. An ambiguous voting game *has voters who lack confidence* if for each $t_i \in T$, $\underline{p}/(1-\underline{p}) < r_b(t_i)/r_a(t_i) < \bar{p}/(1-\bar{p})$ or, equivalently, $\underline{p}_t < \frac{1}{2} < \bar{p}_t$.

After observing her signal, an SEU voter typically prefers to bet on one state rather than to bet on the other at fair odds, interpreted as a belief that one state is more likely than the other.[16] An MEU voter may strictly prefer to take neither bet, interpreted as a lack of confidence in her likelihood judgment. An ambiguous voting game has voters who lack confidence when every voter strictly prefers to take neither bet, regardless of the signal that she observes.

Voters lack confidence when signals do not provide enough information to reduce the prior ambiguity sufficiently. The likelihood of b given t_i is proportional to the ratio $r_b(t_i)/r_a(t_i)$. A very precise signal structure (there is some t_i so that $r_b(t_i)/r_a(t_i)$ is very high or very low) can offset more ex ante uncertainty (a larger interval $[\underline{p}, \bar{p}]$) than a very imprecise signal structure ($r_b(t_i)/r_a(t_i)$ close to 1 for all $t_i \in T$). If the signal structure is imprecise relative to the degree of ex ante uncertainty, then voters lack confidence. To illustrate concretely, consider the game in Section 2 where $r_a(1) = r_b(2) = 0.6$. Voters lack confidence if and only if $\underline{p} < 0.4$ and $0.6 < \bar{p}$. If instead $r_a(1) = r_b(2) = 0.51$ (a less precise signal structure), then voters lack confidence whenever $\underline{p} < 0.49$ and $0.51 < \bar{p}$. If $\bar{p} = 0.55$ and $\underline{p} = 0.45$, then voters lack confidence when $r_a(1) = r_b(2) = 0.51$ but not when $r_a(1) = r_b(2) = 0.6$.

If voters lack confidence, then each has a preference to hedge her bet. Suppose a voter strictly prefers a bet on a to a bet on b. If voters lack confidence, then she strictly prefers a lottery that yields the bet on a with probability $(1-x)$ and the bet on b with probability x, where x is small and positive, to a bet on a with probability 1. The small probability of a bet on b hedges her bet on a by increasing her expected utility conditional on b occurring, thereby decreasing her exposure to ambiguity. An SEU voter never strictly prefers this type of hedge: if she prefers a bet on a to a bet on b, then the independence axiom implies that a bet on a is at least as good as *any* lottery over the two bets.

This translates into the voting setting as follows. Suppose an arbitrary voter is made a dictator—whichever policy she chooses is implemented. If, irrespective of the signal she receives, she *strictly* prefers to pick a policy implemented by lottery rather than implementing either policy for sure, then, and only then, voters lack confidence.

[16] Specifically, any SEU agent strictly prefers either to bet on a or to bet on b at sufficiently small stakes rather than take no bet whenever her utility index is differentiable and her posterior probability of a is not exactly $\frac{1}{2}$.

Theorem 1. *Any symmetric equilibrium to an ambiguous voting game where voters lack confidence does not have correct expected winners.*

The result shows that if voters lack confidence, then no equilibrium of the game aggregates information. It implies that in *any* symmetric equilibrium, the probability that the correct candidate wins the election is bounded above by $\frac{1}{2}$ in at least one state. The result is valid for any number of voters, so it applies to both small committees and large elections.[17]

Theorem 1 shows that the behavior demonstrated in Section 2, i.e., randomizing in a manner that does not reveal information, precludes the existence of an equilibrium with correct expected winners in *every* game that has voters who lack confidence. Intuitively, when others play a strategy profile for which information would aggregate and voters lack confidence, no voter is confident that voting for either A or B would improve the outcome of the election. Consequently, each prefers to insure herself against altering the outcome for the worse by mixing between voting for A and voting for B. More specifically, pivot probabilities affect an MEU voter's strategy in two ways: her incentive to vote for A rather than for B increases as she becomes more likely to be pivotal in state a relative to state b, and ambiguity aversion gives her an incentive to hedge against casting a pivotal vote for the worse candidate. The former increases the responsiveness of the voter's behavior to her signal, while the latter decreases it. Theorem 1 shows that whenever both expected winners are correct and voters lack confidence, the hedging motive dominates.

Before providing an outline of the proof, I first detail some properties of a voter's best response used therein. Ambiguity aversion qualitatively affects the voter's best response through a strict preference for randomization. If she strictly prefers to randomize between voting for A and B, then the posterior that minimizes her utility of voting for A does not also minimize the utility of voting for B; see, e.g., Theorem 3 of Klibanoff (1996). Thus a necessary condition for a strict preference to randomize is that the *election is close*, in the sense that her strategy determines whether or not her utility conditional on state a exceeds her utility conditional on state b. Given symmetry and independence, either the election is close for all voters or the election is close for no voter.

The best response of a voter who observes signal t_i, henceforth a t_i-voter, depends on the relationship between her set of posteriors and the ratio of the probability that her vote is pivotal in state a to that in state b, $\rho_a(\sigma)/\rho_b(\sigma)$. If the election is close and if signal t_i does not sufficiently favor either a or b (specifically, $(1-\underline{p}_{t_i})/\underline{p}_{t_i} > \rho_a(\sigma)/\rho_b(\sigma) > (1-\bar{p}_{t_i})/\bar{p}_{t_i}$), then a t_i-voter strictly prefers to play the mixed strategy σ_{ins} that equalizes her utility conditional on each state. This strategy does not depend on the voter's signal, and $\sigma_{\text{ins}}(A) \geq \frac{1}{2}$ if and only if $\tau(B|\sigma, b) \geq \tau(A|\sigma, a)$.[18] When signal t_i sufficiently favors a or b, a t_i-voter best responds by voting for A with higher (if it favors a) or lower (if it favors b) frequency than $\sigma_{\text{ins}}(A)$; when the election is close, she best responds by playing A at least as frequently as $\sigma_{\text{ins}}(A)$ if $\rho_a(\sigma)/\rho_b(\sigma) \geq (1-\underline{p}_{t_i})/\underline{p}_{t_i}$ and no more frequently

[17] An earlier version of this paper (Ellis 2012) shows that Theorem 1 is robust to the introduction of population uncertainty via a Poisson distribution.

[18] Specifically, $\sigma_{\text{ins}}(A)/\sigma_{\text{ins}}(B) = (\rho_b(\sigma)+\theta_b(\sigma)-\theta_a(\sigma))/(\rho_a(\sigma)+\theta_a(\sigma)-\theta_b(\sigma))$.

if $(1-\bar{p}_{t_i})/\bar{p}_{t_i} \geq \rho_a(\sigma)/\rho_b(\sigma)$. If the election is not close, then a t_i-voter weakly prefers to vote for A (resp. B) if and only if $\rho_a(\sigma)/\rho_b(\sigma)$ is at least as large as (resp. no larger than) $(1-p)/p$, where p equals $\underline{p}$ if utility is always higher conditional on a than on b and $\bar{p}$ otherwise.

The following text outlines the proof. Suppose the result is false, so there exists a symmetric equilibrium σ with correct expected winners. Since expected winners are correct, there must exist signals t_A and t_B so that a t_A-voter is more likely to vote for A and a t_B-voter is more likely to vote for B, i.e., $\sigma(t_A)(A) > \frac{1}{2}$ and $\sigma(t_B)(B) > \frac{1}{2}$. The first step shows that σ leads to a close election. The second step establishes that either the t_A-voter or the t_B-voter is not best responding to σ.

To see why the first step holds, suppose to the contrary that utility is always higher conditional on a than on b.[19] Utility conditional on a is proportional to the probability that A wins in state a. Since the probability that candidate c wins is proportional to c's vote share, utility is always higher conditional on a than on b only if $\tau(A|\sigma, a) > \tau(B|\sigma, b)$. As with SEU, the pivot probability in state s is inversely proportional to the expected margin of victory for the winning candidate, i.e., $|\tau(A|\sigma, s) - \tau(B|\sigma, s)|$. Since $\tau(A|\sigma, a) > \tau(B|\sigma, b)$ and expected winners are correct, $\tau(A|\sigma, a) - \tau(B|\sigma, a) > \tau(B|\sigma, b) - \tau(A|\sigma, b) > 0$, implying that pivot probability in state b exceeds that in a, i.e., $\rho_b(\sigma) > \rho_a(\sigma)$. Because voters lack confidence, $(1-\underline{p}_{t_A})/\underline{p}_{t_A} > 1 > \rho_a(\sigma)/\rho_b(\sigma)$. However, a t_A-voter best responds with $\sigma(t_A)$ only if she thinks state a is likely relative to the ratio of pivot probabilities, specifically $(1-\underline{p}_{t_A})/\underline{p}_{t_A} \leq \rho_a(\sigma)/\rho_b(\sigma)$. Hence, $\sigma(t_A)$ is not a best response for a t_A-voter, a contradiction.

To see why the second step holds, consider separately $\tau(B|\sigma, b) \geq \tau(A|\sigma, a)$ and $\tau(A|\sigma, a) > \tau(B|\sigma, b)$. On the one hand, if $\tau(B|\sigma, b) \geq \tau(A|\sigma, a)$, then $\sigma_{\text{ins}}(A) \geq \frac{1}{2}$ and the above relationship between pivot probability and vote share implies that $\rho_a(\sigma)/\rho_b(\sigma) \geq 1$. However, $1 > (1-\bar{p}_{t_B})/\bar{p}_{t_B}$ because voters lack confidence, and a t_B-voter best responds with $\sigma(t_B)$ only if $\rho_a(\sigma)/\rho_b(\sigma) \leq (1-\bar{p}_{t_B})/\bar{p}_{t_B}$. Hence, $\sigma(t_B)$ is not a best response for a t_B-voter. On the other hand, if $\tau(A|\sigma, a) > \tau(B|\sigma, b)$, then similar arguments show that $\sigma(t_A)$ is not a best response for a t_A-voter. Since either case results in a contradiction, σ must not have correct expected winners, completing the proof.

Theorem 1 only rules out equilibria, leaving open the question of which equilibria exist. Proposition 1 shows that all players voting as if flipping a fair coin is an equilibrium. Neither asserts uniqueness of this equilibrium.

Proposition 1. *If an ambiguous voting game Γ has voters who lack confidence, then the symmetric strategy profile $\hat{\sigma}$ defined by $\hat{\sigma}(t_i)(A) = \frac{1}{2}$ for all $t_i \in T$ is an equilibrium for Γ.*

To see why $\hat{\sigma}$ is an equilibrium, fix a voter i and suppose that all others play $\hat{\sigma}$. Voter i is equally likely to be pivotal in either a or b, so being pivotal does not give her any additional information. Moreover, if she did not vote, then her expected utilities conditional on each state equal one another. Because she lacks confidence, her best response given only her signal is to randomize fifty–fifty. Consequently, this strategy profile constitutes an equilibrium.

[19] The argument when utility is always higher conditional on b than on a is very similar.

In this equilibrium, both candidates are elected with equal probability regardless of the state. Knowing the winner of the election does not provide any additional information about which state obtains. Theorem 1 implies that an equilibrium resulting in a higher probability of electing the correct candidate in one state of the world than $\hat{\sigma}$ must result in a lower probability of electing the correct candidate in the other state of the world than $\hat{\sigma}$.

4.3 *Sufficient condition for FIE*

An implication of Theorem 1 is that FIE fails for any sequence of ambiguous voting games in which voters lack confidence. In contrast, as long as the signal structure is informative (the conditional distribution of signals varies with the state), any sequence of SEU voting games satisfies FIE. Since SEU is a special case of MEU, some ambiguous voting games satisfy FIE. However, SEU is not necessary for information to aggregate. Theorem 2 proves the existence of a sequence of equilibria that aggregates information whenever the game has disjoint posteriors.

Definition 5. An ambiguous voting game has *disjoint posteriors* if $r_a(t_i), r_b(t_i) > 0$ for each $t_i \in T$ and for every distinct $t_i, t_i' \in T$,

$$\frac{r_b(t_i)}{r_a(t_i)} \le \frac{r_b(t_i')}{r_a(t_i')} \implies \frac{(1-\bar{p})r_b(t_i)}{\bar{p}r_a(t_i)} \le \frac{(1-\underline{p})r_b(t_i')}{\underline{p}r_a(t_i')}$$

or, equivalently, that $(\underline{p}_{t_i}, \bar{p}_{t_i}) \cap (\underline{p}_{t_i'}, \bar{p}_{t_i'})$ is empty.

Disjoint posteriors requires that signals are sufficiently precise relative to the degree of ex ante uncertainty. In an ambiguous voting game with disjoint posteriors, the modeler can unambiguously rank signals by how likely a voter who sees it views state a. If all voters are SEU, then the game has disjoint posteriors.[20] Moreover, a sufficiently small amount of ambiguity implies disjoint posteriors. Specifically, for any SEU voting game with prior of a equal to π and signal distributions r_a and r_b such that $r_b(t_i)/r_a(t_i) \neq r_b(t_i')/r_a(t_i')$ for every distinct $t_i, t_i' \in T$, there is an $\epsilon > 0$ so that an ambiguous voting game with the same signal distributions and a set priors that satisfies $\underline{p} = \pi - \epsilon$ and $\bar{p} = \pi + \epsilon$ has disjoint posteriors. In this sense, disjoint posteriors is close to SEU.

An ambiguous voting game cannot have both disjoint posteriors and voters who lack confidence.[21] One can distinguish between SEU, disjoint posteriors, and voters who lack confidence using Lemma 1 (in the Appendix). Consider an ambiguous voting game Γ. If Γ has singleton posteriors, then all voters act to maximize SEU and none strictly prefers to randomize for any strategy profile. If Γ has disjoint posteriors, then for

[20] If $\bar{p} = \underline{p}$, then $\underline{p}_t = \bar{p}_t$ for every $t \in T$, implying that $(\underline{p}_t, \bar{p}_t) = \varnothing$ so $(\underline{p}_t, \bar{p}_t) \cap (\underline{p}_{t'}, \bar{p}_{t'}) = \varnothing$.

[21] Note that an ambiguous voting game may have neither disjoint posteriors nor voters who lack confidence.

any strategy profile at most one type of voter strictly prefers to randomize. If Γ has voters who lack confidence, then there exists a strategy profile such that all voters strictly prefer randomizing to playing any pure strategy.

Any sequence of ambiguous voting games with disjoint posteriors and an informative signal structure satisfies FIE.

Theorem 2. *For any sequence of ambiguous voting games with disjoint posteriors, there exists a sequence of symmetric equilibria satisfying FIE.*

The result shows the possibility of information aggregation under disjoint posteriors. A sufficiently small amount of ambiguity does not preclude information aggregation. Because ambiguous voting games with disjoint posteriors are close to SEU voting games, there exists an equilibrium similar to that in those games. The proof adapts the construction in Theorem 2 of Myerson (1998) to find a "step strategy" equilibrium. Specifically, there is a p^* that serves as a cutoff posterior of a: any voter who sees a signal that leads to a set of posteriors that all assign a larger (resp. smaller) probability of a than p^* votes for A (resp. B). Otherwise, the voter plays a strategy—typically mixed—that incentivizes all voters to follow the strategy profile. If σ^* is the limit of the sequence of equilibrium strategies, then $\tau(A|\sigma^*, a) = \tau(B|\sigma^*, b) > \frac{1}{2}$, establishing FIE. Note that the limiting σ^* is also the limit of the sequence of equilibrium strategy profiles for an SEU voting game with the same signal structure and an arbitrary prior.

For a class of symmetric ambiguous voting games that includes the game in Section 2, Proposition 2 gives a complete characterization of those that satisfy FIE.

Proposition 2. *If $(\Gamma_n)_{n=1}^{\infty}$ is a sequence of ambiguous voting games where $\bar{p} \geq \frac{1}{2}$, $\underline{p} = 1 - \bar{p}$, $T = \{1, 2\}$, and $r_a(1) = r_b(2) > \frac{1}{2}$, then there exists a sequence of symmetric, equilibrium strategy profiles that satisfy FIE if and only if $r_a(1) \geq \bar{p}$.*

In words, information aggregates if and only if the information structure is sufficiently precise relative to the degree of ex ante ambiguity. Proposition 2 follows from Theorem 1 and Theorem 2. If $r_a(1) \geq \bar{p}$, then each Γ_n has disjoint posteriors. Applying Theorem 2 implies that $(\Gamma_n)_{n=1}^{\infty}$ satisfies FIE. However, if $\bar{p} > r_a(1)$, then every Γ_n has voters who lack confidence. Theorem 1 then implies that no symmetric equilibrium strategy profile has correct expected winners, precluding FIE.

5. Robustness

This section explores the robustness of Theorem 1. I consider first allowing voters to choose to abstain from voting. I also briefly discuss voters who perceive ambiguity about the distribution of signals and the strategies of others. Neither variation affects the conclusion of Theorem 1.

5.1 *Abstention*

In SEU voting games, abstention typically improves the outcome of the election. This is due to the "swing voter's curse": given two SEU voters who observe different signals, the voter whose signal conveys less information about the state of the world is more likely to abstain in equilibrium (e.g., Feddersen and Pesendorfer 1996, 1999 or Bouton and Castanheira 2009). Because the percentage of votes cast by more informed voters is higher in an election with abstention compared to one with mandatory voting, allowing abstention improves the expected outcome of the election. Ambiguous voting games explicitly rule out the possibility of strategic abstention, leaving open the possibility that the conclusion of Theorem 1 fails when voters can choose to abstain.

The main result of this subsection shows that if signals are distributed symmetrically, then Theorem 1 holds when voters may abstain strategically. So as to allow for abstention, modify the ambiguous voting games from Section 3 by allowing voters to abstain (take action $\varnothing$) in addition to casting a vote for either of the two candidates. As before, the candidate with the most votes wins, but abstention leaves open the possibility of a tie. I assume that when candidates have the same number of votes, a fair coin flip determines the winner. I define the equilibrium to such a game as in Section 3, except that each voter's strategy is a probability distribution over $\{A, B, \varnothing\}$ rather than over $\{A, B\}$. For tractability, I only analyze the game in which $T = \{1, 2\}$ and $r_a(1) = r_b(2)$. Call such a game an *ambiguous voting game with abstention*.

Theorem 3. *Any symmetric equilibrium to an ambiguous voting game with abstention where voters lack confidence does not have correct expected winners.*

The proof follows the same steps as Theorem 1. However, the possibility of abstention complicates the argument, particularly the first step of showing that any equilibrium with correct expected winners is close. The proof establishes this for symmetric distributions of signals. An earlier version of this paper (Ellis 2012) in which the number of voters followed a Poisson distribution extends Theorem 3 to many more distributions.

Theorem 3 provides insight into the mechanism behind Theorem 1, in that equilibrium behavior can be interpreted as an extreme swing voter's curse. Each voter prefers to minimize the chance that she casts a pivotal vote. If she abstained, then she would never be pivotal, which would be better than any available strategy. However, Theorem 1 assumes that she must vote. Among her available choices, her best option is to mimic abstention through a mixed strategy. When abstention is allowed and voters lack confidence, even the voters who see more informative signals may abstain in equilibrium, and allowing abstention need not alter the composition of vote shares.

Proposition 3. *If Γ is an ambiguous voting game with abstention that has voters who lack confidence, then for any $s \in [0, 1]$, the symmetric strategy profile σ^* defined by $\sigma^*(t_i)(\varnothing) = s$ and $\sigma^*(t_i)(A) = \sigma^*(t_i)(B) = (1 - s)/2$ for every $t_i \in T$ is an equilibrium for Γ.*[22]

[22] This result generalizes immediately to the case where T is any finite set and r_a and r_b are unrestricted.

Proposition 3 shows that the equilibrium expected turnout with MEU voters can be anywhere between 0 and 100 percent. Of particular interest are the equilibrium where $\sigma^*(t_i)(\varnothing) = 0$ for all $t_i \in T$ and the equilibrium where $\sigma^*(t_i)(\varnothing) = 1$ for all $t_i \in T$. The former equilibrium is the same as in Proposition 1, i.e., each player votes for every candidate with equal probability. In the latter equilibrium, all voters abstain with probability 1. Despite the different strategies, the expected outcome is the same for *all* of the equilibria shown to exist by Proposition 3: each candidate is elected with equal probability, regardless of the state of the world that obtains. Consequently, the payoffs are the same for each voter, as is the information that observing the outcome would provide to an observer.

The equilibrium in which all voters abstain contrasts with Propositions 2 and 3 of Feddersen and Pesendorfer (1996) and Proposition 5 of Feddersen and Pesendorfer (1999). In these papers, the fraction of voters who do not abstain remains bounded away from 0 along any sequence of equilibria. These results are a consequence of SEU preferences: unlike when voters who lack confidence, if signals are information, one of them will induces an SEU voter to prefer either a bet on a (voting for A) or a bet on b (voting for B) to not making a bet (abstaining), at least for small stakes.

5.2 *Ambiguity about signal distributions or strategies*

As currently formulate, the model does not permit ambiguity about the conditional distribution of signals. Nonetheless, results extend as long as signals are conditionally independent, adapted as follows from Gilboa and Schmeidler (1989). Say that the signal of voter i is $\Pi(\cdot|s)$-*independent* if for any acts f and g that are measurable with respect to $\{\{(s,t) : t_i = \hat{t}\} : \hat{t} \in T\}$ and $\{\{(s,t) : t_j = \hat{t}_j \ \forall j \neq i\} : \hat{t} \in T^{I\setminus i}\}$, respectively, (i) there exists $P_0 \in \Pi(\cdot|s)$ so that $P_0 \in \arg\min_{\pi\in\Pi(\cdot|s)} \int u \circ f \, d\pi$ and $P_0 \in \arg\min_{\pi\in\Pi(\cdot|s)} \int u \circ g \, d\pi$, and (ii) $u \circ f$ and $u \circ g$ are stochastically independent random variables with respect to any extreme point of $\Pi(\cdot|s)$. Proposition 4.2 of that paper shows that independence requires that the set of priors consists of product measures. Formally, *signals are conditionally independent* if the signal of every voter i is $\Pi(\cdot|s)$-independent for each $s \in S$. Many papers studying games with incomplete information under ambiguity implicitly or explicitly assume independence, such as Bose et al. (2006), Chen et al. (2007), and Bose and Daripa (2009).[23]

With this definition in mind, consider the set of priors defined by $\pi \in \Pi$ if and only if there exists a $p \in [\underline{p}, \bar{p}]$ and an r_s in the convex hull of $\{\bigotimes_{i\in I} r_{s,i} : r_{s,i} \in R_s \ \forall i \in I\}$ for each $s \in S$ so that

$$\pi(a,t) = pr_a(t) \quad \text{and} \quad \pi(b,t) = (1-p)r_b(t)$$

for all $(s,t) \in \Omega$, where each R_s is a closed, convex, nonempty set of probability distributions over T. Conditional on state s, the signal that voter i observes is distributed

[23] Other papers, such as Lo (1999) or Bodoh-Creed (2012), instead consider sets of priors with the form $\{\mu \otimes \mu \otimes \cdots \otimes \mu : \mu \in \Delta\}$. This reflects that all types are distributed independently according to the same distribution. The results mentioned below also hold if both sets of posteriors have this form.

according to one of the distributions in the set R_s. Again, the above notions of utility, strategies, and equilibrium generalize naturally. In the Supplement, I prove the analog of Theorem 1: no equilibrium strategy profile has correct expected winners if voters lack confidence, when both properties are suitably generalized.

Now consider ambiguity about the strategies of other players. Studying this requires the introduction of a new equilibrium concept, and one must make a crucial modeling decision: whether ambiguity about another player's strategy arises completely endogenously (as in Lo 1996) or at least partially exogenously (as in Klibanoff 1996). Such an extension is beyond the scope of this paper, but incorporating ambiguity about the strategies of others seems unlikely to change the main conclusions. Intuitively, the Condorcet Jury Theorem shows that the strategy played by an SEU voter *given her correct perception of other voters' strategies* leads to information aggregation, and if other voters' strategies are ambiguous, then a player no longer correctly perceives them. This decreases the informational content of the pivotal event even further, so one should not expect information to aggregate. Moreover, it increases the uncertainty and thus the incentive for a voter to hedge against ambiguity.

6. Conclusion

Theorem 1 shows that rational but ambiguity averse voters may find it optimal to insure themselves by minimizing the chance they cast a pivotal vote. This leads to a failure of information aggregation not documented by previous work. The literature shows that the dimensionality of the uncertainty and the degree of commonality between voters are important in evaluating the efficiency of the election. In contrast, this paper suggests that how familiar the electorate is with the issues at stake also matters. By way of conclusion, this section reviews some of these results and contrasts them with Theorem 1.

Feddersen and Pesendorfer (1997) prove that if the distribution of preferences is unknown, then FIE fails generically. The problem is one of dimensionality; namely, each voter must infer both the distribution of signals and the distribution of preferences from these votes. Even if a voter knew which votes others cast and the electorate were large, she could not infer the state of the world. In contrast, this paper assumes common knowledge of the distribution of preferences. However, the distribution of votes may not vary with the state (Proposition 1) because voters insure themselves against ambiguity by abstaining or randomizing.

Mandler (2012) shows that if the conditional distribution of signals is unknown, then FIE may fail. If all the signals were observed by each voter, then uncertainty would remain as to which state is correct even as the size of the electorate goes to infinity. In this paper, if all signals were observed, then the true state would be known with probability approaching 1 despite the prior ambiguity.

Bhattacharya (2013) drops the assumption of common values and characterizes the distributions of preferences for which FIE fails. For instance, FIE fails when any voter who receives information in favor of the Condorcet winner with perfect information is very likely to strongly prefer the other candidate.[24] In contrast, this paper maintains pure common values.

Finally, the result in this paper relates to work that studies the effect of ambiguous information in other contexts. For instance, Condie and Ganguli (2011) demonstrate a failure of information transmission with ambiguity averse agents in general equilibrium. They show that a rational expectations equilibrium for an exchange economy may be partially revealing when agents are ambiguity averse; in contrast, fully revealing equilibria are generic with SEU agents. Two differences are worth pointing out. First, in their model agents do not act strategically—they are price takers. Second, they assume that only a subset of agents are ambiguity averse, while an ambiguous voting game has voters who lack confidence only if all voters are ambiguity averse.

Appendix: Proofs

A.1 *Proof of Theorem 1*

A.1.1 *Preliminary results for the proof of Theorem 1* This section contains three preliminary results that will be used to prove Theorem 1 and Theorem 2. Lemma 1 establishes the form of a voter's best response correspondence. Lemmas 2, 3, and 4 establish how a change in vote share alters $\theta_s(\sigma)$ and $\rho_s(\sigma)$. When it will not cause confusion, the strategy profile is suppressed as an argument in the function θ_s and ρ_s (defined in Section 3). Throughout, a strategy is indexed solely by the probability of playing A. This is without loss of generality since $\sigma_i(t)(B) = 1 - \sigma_i(t)(A)$.

Lemma 1. *If σ^* is a symmetric equilibrium, $\sigma^*(t_i)(A) \in BR_{t_i}(\sigma^*)(A)$ for all $t_i \in T$, where*

$$BR_{t_i}(\sigma)(A) = \begin{cases} \{0\} & \text{if } \theta_a \geq \theta_b + \rho_b \text{ and } \frac{\rho_a(\sigma)}{\rho_b(\sigma)} < \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} \\ & \text{or } \theta_b \geq \theta_a + \rho_a \text{ and } \frac{\rho_a(\sigma)}{\rho_b(\sigma)} < \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \\ [0,1] & \text{if } \theta_a \geq \theta_b + \rho_b \text{ and } \frac{\rho_a(\sigma)}{\rho_b(\sigma)} = \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} \\ & \text{or } \theta_b \geq \theta_a + \rho_a \text{ and } \frac{\rho_a(\sigma)}{\rho_b(\sigma)} = \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \\ \{1\} & \text{if } \theta_a \geq \theta_b + \rho_b \text{ and } \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} < \frac{\rho_a(\sigma)}{\rho_b(\sigma)} \\ & \text{or } \theta_b \geq \theta_a + \rho_a \text{ and } \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} < \frac{\rho_a(\sigma)}{\rho_b(\sigma)} \\ \hat{BR}_{t_i}(\sigma)(A) & \text{otherwise} \end{cases}$$

[24]Additionally, the nonaggregation result in this paper is stronger because of his more demanding definition of FIE, which requires the definition of FIE from this paper to hold for *every* sequence of symmetric, Bayesian Nash equilibria in undominated strategies. Unlike Theorem 1, his conditions do not rule out the existence of a different equilibrium in which information would aggregate. For example, the game depicted by his Figure 1 fails his definition of FIE but satisfies the definition in this paper.

and

$$\hat{BR}_{t_i}(\sigma)(A) = \begin{cases} \{0\} & \text{if } \frac{\rho_a(\sigma)}{\rho_b(\sigma)} < \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \\ [0, \bar{s}(\sigma)] & \text{if } \frac{\rho_a(\sigma)}{\rho_b(\sigma)} = \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \\ \{\bar{s}(\sigma)\} & \text{if } \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} < \frac{\rho_a(\sigma)}{\rho_b(\sigma)} < \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} \\ [\bar{s}(\sigma), 1] & \text{if } \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} = \frac{\rho_a(\sigma)}{\rho_b(\sigma)} \\ \{1\} & \text{if } \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} < \frac{\rho_a(\sigma)}{\rho_b(\sigma)}, \end{cases}$$

where $\bar{s}(\sigma) = (\theta_b - \theta_a + \rho_b)/(\rho_a + \rho_b)$.

Proof. A player of type t_i has a best response to σ of playing A with probability s if s maximizes $V_{t_i}(s, \sigma)$.

Consider the case where $\theta_a \geq \theta_b + \rho_b$. Note that

$$\begin{aligned} V_{t_i}(s, \sigma) &= \min_{p \in [\underline{p}_{t_i}, \bar{p}_{t_i}]} \{p[s\rho_a + \theta_a] + (1-p)[(1-s)\rho_b + \theta_b]\} \\ &= \underline{p}_{t_i}[s\rho_a + \theta_a] + (1 - \underline{p}_{t_i})[(1-s)\rho_b + \theta_b] \end{aligned}$$

because for every s, $s\rho_a + \theta_a \geq (1-s)\rho_b + \theta_b$. Clearly $V_{t_i}(\cdot, \sigma)$ is increasing if and only if $\underline{p}_{t_i}\rho_a > (1 - \underline{p}_{t_i})\rho_b$, decreasing if and only if $\underline{p}_{t_i}\rho_a < (1 - \underline{p}_{t_i})\rho_b$, and constant if and only if $\underline{p}_{t_i}\rho_a = (1 - \underline{p}_{t_i})\rho_b$. Hence, $BR_{t_i}(\sigma)$ has the desired form, and dual arguments establish the claim when $\theta_a + \rho_a \leq \theta_b$.

Now, suppose that $\theta_a < \theta_b + \rho_b$ and $\theta_a + \rho_a > \theta_b$. The term $V_{t_i}(\cdot, \sigma)$ is not differentiable everywhere, but since it is concave (as a minimum of a set of linear functions), the superdifferential (see, e.g., Aliprantis and Border 1999) exists everywhere. Adapted to this setting, the superdifferential is given by

$$\partial V_{t_i}(s, \sigma) = \left\{x \in \mathbb{R} : V_{t_i}(y, \sigma) \leq V_{t_i}(s, \sigma) + \sum_{\omega}(y - s)x \ \forall y \in [0, 1]\right\}.$$

By Lemma 7.10 of Aliprantis and Border (1999), the best response correspondence is the set of all s such that $0 \in \partial V_{t_i}(s, \sigma)$.

There exists an $\bar{s} \in (0, 1)$ so that the conditional utilities in A and B are equal. Further, if $s > \bar{s}$, then conditional utility in state a is larger than that in state b, and if $s < \bar{s}$, then the utility in state B is larger than that in state A. Algebra shows that

$$\bar{s} = \frac{\theta_b - \theta_a + \rho_b}{\rho_a + \rho_b}.$$

Since for all $s \in (0, \bar{s})$ and every $s \in (\bar{s}, 1)$ the minimizer is unique, the derivative of $V_{t_i}(\cdot, \sigma)$ exists and coincides with the superdifferential whenever $s \notin \{0, \bar{s}, 1\}$. If $s \in (\bar{s}, 1)$, then

$$\partial V_{t_i}(s, \sigma) = \left\{\frac{\partial}{\partial s} V_{t_i}(s, \sigma)\right\} = \{\underline{p}_{t_i}\rho_a - (1 - \underline{p}_{t_i})\rho_b\},$$

so any $s \in (\bar{s}, 1)$ is an optimum only if $\underline{p}_{t_i}\rho_a = (1 - \underline{p}_{t_i})\rho_b$. Similarly, any $s \in (0, \bar{s})$ is an optimum when $\bar{p}_{t_i}\rho_a = (1 - \bar{p}_{t_i})\rho_b$. Otherwise the optimum must be in $\{0, \bar{s}, 1\}$.

When $s = 1$, the superdifferential exists and equals

$$\partial V_{t_i}(1, \sigma) = \{x \in \mathbb{R} : V_{t_i}(y, \sigma) - V_{t_i}(1, \sigma) \leq (y - 1)x \ \forall y \leq 1\}.$$

Since $V_{t_i}(y, \sigma) - V_{t_i}(1, \sigma)$ is equal to

$$(y - 1)(\underline{p}_{t_i}\rho_a - (1 - \underline{p}_{t_i})\rho_b),$$

$0 \in \partial V_{t_i}(1, \sigma)$ if and only if

$$V_{t_i}(y, \sigma) - V_{t_i}(1, \sigma) \leq 0 \quad \iff \quad \underline{p}_{t_i}\rho_a > (1 - \underline{p}_{t_i})\rho_b.$$

Hence $s = 1$ is an optimum only if $\underline{p}_{t_i}\rho_a > (1 - \underline{p}_{t_i})\rho_b$. Similar arguments show that $s = 0$ is an optimum if and only if $0 \in \partial V_{t_i}(0, \sigma) \iff \bar{p}_{t_i}\rho_a < (1 - \bar{p}_{t_i})\rho_b$.

Now consider the strategy $\bar{s}$. In this case,

$$\partial V_{t_i}(\bar{s}, \sigma) = \{p\rho_a - (1 - p)\rho_b : p \in [\underline{p}_{t_i}, \bar{p}_{t_i}]\}$$

so $0 \in \partial V_t(\bar{s}, \sigma)$ if and only if $\bar{p}_{t_i}\rho_a - (1 - \bar{p}_{t_i})\rho_b \geq 0$ and $\underline{p}_{t_i}\rho_a - (1 - \underline{p}_{t_i})\rho_b \leq 0$. Combining the above statements yields the desired form of $BR_{t_i}(\sigma)$. □

I establish following three facts that will be used in the proof of Theorem 1.

Lemma 2. *For any symmetric strategy profile* σ, $\theta_a(\sigma) \geq \theta_b(\sigma) \iff \tau(A|\sigma, a) \geq \tau(B|\sigma, b)$.

Proof. Write $\theta_a(\sigma) = \sum_{m=n+1}^{2n} \binom{2n}{m} \tau(A|\sigma, a)^m (1 - \tau(A|\sigma, a))^{2n-m}$ and note that

$$\frac{\partial \theta_a}{\partial \tau(A|\sigma, a)} = (2n) \binom{2n - 1}{n} \tau(A|\sigma, a)^n (1 - \tau(A|\sigma, a))^{n-1} > 0.$$

See the Supplement for the algebra. Similarly

$$\frac{\partial \theta_b}{\partial \tau(B|\sigma, b)} = (2n) \binom{2n - 1}{n} \tau(B|\sigma, b)^n (1 - \tau(B|\sigma, b))^{n-1} > 0.$$

Since $\theta_a(\sigma) = 0$ if $\tau(A|\sigma, a) = 0$ and $\theta_b(\sigma) = 0$ if $\tau(B|\sigma, b) = 0$, conclude that $\theta_a(\sigma) \geq \theta_b(\sigma) \iff \tau(A|\sigma, a) \geq \tau(B|\sigma, b)$. □

Lemma 3. *For any symmetric strategy profile* σ, $\rho_a(\sigma) \geq \rho_b(\sigma) \iff |\tau(A|\sigma, a) - \frac{1}{2}| \leq |\tau(B|\sigma, b) - \frac{1}{2}|$.

Proof. Since $\rho_s(\sigma) = \binom{2n}{n} \tau(A|\sigma, s)^n (1 - \tau(A|\sigma, s))^n$,

$$\frac{\partial \rho_s(\sigma)}{\partial \tau(A|\sigma, s)} = 2n \binom{2n - 1}{n - 1} \tau(A|\sigma, s)^{n-1} (1 - \tau(A|\sigma, s))^{n-1} \big[(1 - 2\tau(A|\sigma, s))\big].$$

See the Supplement for the algebra. Integrating the above derivative from $\frac{1}{2}$ to $\tau(A|\sigma, s)$ yields the result. □

Lemma 4. *If σ is a symmetric strategy profile with correct expected winners, then* $\tau(A|\sigma, a) \geq \tau(B|\sigma, b) \iff \bar{s}(\sigma) \leq \frac{1}{2}$.

Proof. Note that $\bar{s}(\sigma) \leq \frac{1}{2} \iff 2\theta_b(\sigma) + \rho_b(\sigma) \leq 2\theta_a(\sigma) + \rho_a(\sigma)$. Lemma 3 shows that $\partial p_a(\sigma)/\partial\tau(A|\sigma, a), \partial p_b(\sigma)/\partial\tau(B|\sigma, b) > 0$ whenever expected winners are correct. Lemma 2 shows that $\partial\theta_a/\partial\tau(A|\sigma, a), \partial\theta_b/\partial\tau(B|\sigma, b) > 0$. Moreover, when $\tau(A|\sigma, a) = \tau(B|\sigma, b)$, $\partial p_a(\sigma)/\partial\tau(A|\sigma, a)$, $\partial p_b(\sigma)/\partial\tau(B|\sigma, b)$ and $\partial\theta_a/\partial\tau(A|\sigma, a) = \partial\theta_b/\partial\tau(B|\sigma, b)$. Integrating from $\tau(A|\sigma, a) = \tau(B|\sigma, b) = \frac{1}{2}$ establishes that when expected winners are correct, $\tau(A|\sigma, a) \geq \tau(B|\sigma, b)$ if and only if $\bar{s}(\sigma) \leq \frac{1}{2}$. □

A.1.2 *Proof of Theorem 1* Suppose, for the sake of contradiction, that voters lack confidence and that σ is a symmetric equilibrium where $\frac{1}{2} < \tau(A|\sigma, a)$ and $\frac{1}{2} < \tau(B|\sigma, b)$. There must exist $t_A, t_B \in T$ such that $\sigma(t_A)(A) > \frac{1}{2}$ and $\sigma(t_B)(A) < \frac{1}{2}$. The first step is to show that $BR_{t_i}(\sigma) = \hat{BR}_{t_i}(\sigma)$ for all $t_i \in T$. The second step shows that either t_A or t_B will not follow σ.

For the first step, if $BR_{t_i}(\sigma) \neq \hat{BR}_{t_i}(\sigma)$, then either

$$\theta_a \geq \theta_b + \rho_b \tag{A.1}$$

or

$$\theta_b \geq \theta_a + \rho_a \tag{A.2}$$

must hold by Lemma 1.

If (A.1) holds, then $\theta_a > \theta_b$. Lemma 2 implies that $\tau(A|a, \sigma) > \tau(B|b, \sigma)$. Lemma 3 implies $\rho_b(\sigma) > \rho_a(\sigma)$. But $(1 - \underline{p}_{t_A})/\underline{p}_{t_A} > 1$ because voters lack confidence, so $(1 - \underline{p}_{t_A})/\underline{p}_{t_A} > \rho_a(\sigma)/\rho_b(\sigma)$ and Lemma 1 implies $\sigma(t_A)(A) = 0$, a contradiction.

If instead (A.2) holds, then $\theta_b > \theta_a$. Lemma 2 implies $\tau(B|\sigma, b) > \tau(A|\sigma, a)$. Lemma 3 implies $\rho_a(\sigma) > \rho_b(\sigma)$. But $(1 - \bar{p}_{t_B})/\bar{p}_{t_B} < 1$ because voters lack confidence, so $\rho_a(\sigma)/\rho_b(\sigma) \leq (1 - \bar{p}_{t_B})/\bar{p}_{t_B}$ and Lemma 1 implies $\sigma(t_B)(B) = 0$, a contradiction.

For the second step, either $\tau(A|a, \sigma) \leq \tau(B|b, \sigma)$ or $\tau(A|a, \sigma) > \tau(B|b, \sigma)$. In the first case $\tau(A|a, \sigma) \leq \tau(B|b, \sigma)$, so Lemma 4 implies $\bar{s}(\sigma) \geq \frac{1}{2}$ and Lemma 3 implies $\rho_a(\sigma)/\rho_b(\sigma) > 1$. But $(1 - \bar{p}_{t_B})/\bar{p}_{t_B} < 1$ since voters lack confidence, so Lemma 1 implies that $\sigma(t_B)(A) \geq \bar{s}(\sigma) \geq \frac{1}{2}$, a contradiction.

In the second case $\tau(A|a, \sigma) > \tau(B|b, \sigma)$, so Lemma 4 implies $\bar{s}(\sigma) < \frac{1}{2}$ and Lemma 3 implies $\rho_a(\sigma)/\rho_b(\sigma) < 1$. But $1 < (1 - \underline{p}_{t_A})/\underline{p}_{t_A}$ since voters lack confidence, so Lemma 1 implies that $\sigma(t_A)(A) < \bar{s}(\sigma) \leq \frac{1}{2}$, a contradiction. Because both cases result in a contradiction, there cannot exist a strategy profile that has correct expected winners, completing the proof.

A.2 *Proofs of remaining results not in the text*

A.2.1 *Proof of Proposition 1* Suppose σ is played. Clearly, $\theta_a(\sigma) = \theta_b(\sigma)$ and $\rho_a(\sigma) = \rho_b(\sigma)$. Lemma 1 implies that $BR_{t_i}(\sigma) = \hat{BR}_{t_i}(\sigma)$ for all t_i. Further, note that

$\rho_a(\sigma)/\rho_b(\sigma) = 1$ since the vote shares are equal in both states. Since $\rho_a(\sigma)/\rho_b(\sigma) \in ((1-\bar{p}_{t_i})/\bar{p}_{t_i}, (1-\underline{p}_{t_i})/\underline{p}_{t_i})$, voter i of type t_i has a unique best response to play $\sigma_i(t_i)(A) = \hat{s}(A,\sigma) = \frac{1}{2}$. Therefore, σ is an equilibrium, completing the proof.

A.2.2 *Proof of Theorem 2* Relabel $T = \{1, 2, \dots, T\}$ so that

$$\frac{r_a(1)}{r_b(1)} < \frac{r_a(2)}{r_b(2)} < \cdots < \frac{r_a(T)}{r_b(T)}.$$

The labeling and disjoint posteriors imply that $\bar{p}_1 \leq \underline{p}_2 \leq \bar{p}_2 \leq \underline{p}_3 \leq \cdots \leq \bar{p}_T$. Denote by $[h]$ the largest integer less than h and by $\sigma(h)$ for each $h \in [0, T]$ the strategy profile such that $\sigma_i(t)(A) = 0$ if $t \leq [h]$, $\sigma_i(t)(A) = 1$ if $t > [h]$, and $\sigma_i([h])(A) = h - [h]$ for all i. The proof will show that for all n high enough, there is an $h(n)$ so that $\sigma(h(n))$ is an equilibrium and that the expected winner in a is A and the expected winner in b is B.

There exists numbers $I(a) < I(b)$ so that

$$\tau(A|\sigma(I(s)), s) = \tau(B|\sigma(I(s)), s)$$

for each $s \in \{a, b\}$. Since there is some t so that $r_a(t) \neq r_b(t)$,

$$\tau(A|\sigma(I(s)), s) \neq \tau(B|\sigma(I(s)), s)$$

for $s = a, b$. For every $h \in (I(a), I(b))$, $\tau(A|\sigma(h), a) > \tau(B|\sigma(h), a)$ and $\tau(B|\sigma(h), b) > \tau(A|\sigma(h), b)$.

Label ρ_s and θ_s in the game Γ_n (which has $2n + 1$ players) by ρ_s^n and θ_s^n, and define $\beta(h, n) = \rho_a^n(\sigma(h))/\rho_b^n(\sigma(h))$. Note that

$$\beta(I(a), n) = \frac{\rho_a^n(\sigma(I(a)))}{\rho_b^n(\sigma(I(a)))} = \frac{\frac{1}{4}^n}{[\tau(A|\sigma(I(s)), b)\tau(B|\sigma(I(s)), b)]^n} \to \infty$$

as $n \to \infty$. Similarly,

$$\beta(I(b), n) = \frac{\rho_a^n(\sigma(I(b)))}{\rho_b^n(\sigma(I(b)))} = \frac{[\tau(A|\sigma(I(s)), a)\tau(B|\sigma(I(s)), a)]^n}{\frac{1}{4}^n} \to 0$$

as $n \to \infty$. Therefore, there exists n^* such that $\beta(I(a), n^*) > (1 - \underline{p}_1)/\underline{p}_1$ and $\beta(I(b), n^*) < (1 - \bar{p}_T)/\bar{p}_T$.

Consider fixed Γ_n. Define the function $z : [0, T] \to [0, 1]$ by the formula

$$z(h) := \begin{cases} \bar{s}(\sigma(h)) & \bar{s}(\sigma(h)) \in [0, 1] \\ 1 & \bar{s}(\sigma(h)) > 1 \\ 0 & \bar{s}(\sigma(h)) < 0, \end{cases}$$

where $\bar{s}(\sigma(h))$ is defined in Lemma 1 using $\theta_s^n(\sigma(h))$ and $\rho_s^n(\sigma(h))$. If $z(h) < 0$, then

$$\theta_b^n(\sigma(h)) + \rho_b^n(\sigma(h)) < \theta_a^n(\sigma(h))$$

so

$$BR_t(\sigma(h)) = \begin{cases} 1 & \beta(h,n) > \frac{1-\underline{p}_t}{\underline{p}_t} \\ [0,1] & \beta(h,n) = \frac{1-\underline{p}_t}{\underline{p}_t} \\ 0 & \beta(h,n) < \frac{1-\underline{p}_t}{\underline{p}_t} \end{cases}$$

by Lemma 1. Similarly, if $z(h) > 1$, then

$$\theta_a(\sigma(h)) + \rho_a(\sigma(h)) < \theta_b(\sigma(h))$$

so

$$BR_t(\sigma) = \begin{cases} 1 & \beta(h,n) > \frac{1-\bar{p}_t}{\bar{p}_t} \\ [0,1] & \beta(h,n) = \frac{1-\bar{p}_t}{\bar{p}_t} \\ 0 & \beta(h,n) < \frac{1-\bar{p}_t}{\bar{p}_t} \end{cases}$$

by Lemma 1. Otherwise, $BR_t(\sigma)(A) = \hat{BR}_t(\sigma)(A)$.

Given the above notes, Lemma 1 shows that $\sigma(h)$ is an equilibrium if $\beta(h) \in \eta(h)$, where

$$\eta(h) = \begin{cases} [\frac{1-\bar{p}_h}{\bar{p}_h}, \frac{1-\underline{p}_{h+1}}{\underline{p}_{h+1}}] & h \in \mathbb{Z} \\ \frac{1-\bar{p}_{[h]}}{\bar{p}_{[h]}} & h \in ([h]+z(h), [h]+1) \\ [\frac{1-\underline{p}_{[h]}}{\underline{p}_{[h]}}, \frac{1-\bar{p}_{[h]}}{\bar{p}_{[h]}}] & h = [h]+z(h) \\ \frac{1-\underline{p}_{[h]}}{\underline{p}_{[h]}} & h \in ([h], [h]+z(h)). \end{cases}$$

The term $\bar{s}(\sigma(h))$ is a continuous function of h by construction. It follows that $z(\cdot)$ is continuous since it can be written as the minimum of two continuous functions. Therefore $\eta(\cdot)$ is upper hemicontinuous, compact, and convex.

For every $n > n^*$, there exists $h(n) \in [I(a), I(b)]$ so that $\beta(h(n), n) \in \eta(h(n))$ because $\beta(\cdot)$ is continuous and $\eta(\cdot)$ is convex and upper hemicontinuous. Moreover, by the above discussion, this fixed point must be in the interior. Since $h(n) \in (I(a), I(b))$, $\tau(A|\sigma(h(n)), a) > \tau(B|\sigma(h(n)), a)$ and $\tau(B|\sigma(h(n)), b) > \tau(A|\sigma(h(n)), b)$. Conclude that $\sigma(h(n))$ is an equilibrium to Γ_n.

Since Γ_n was taken arbitrarily, there is an $h(n)$ that defines an equilibrium for every Γ_n so that $n > n^*$. Define $\sigma_n^* = \sigma(h(n))$ for $n > n^*$; otherwise, let σ_n^* be an arbitrary equilibrium. The sequence $(\sigma_n^*)_{n=1}^\infty$ of strategy profiles is all equilibrium, and both sequences $\tau(A|\sigma_n^*, a)$ and $\tau(B|\sigma_n^*, b)$ must converge and must converge so that $\lim \tau(A|\sigma_n^*, a) = \lim \tau(B|\sigma_n^*, b) > \frac{1}{2}$ (see, e.g., Myerson 1998). Picking $\epsilon \in (0, \lim \tau(A|\sigma_n^*, a) - \frac{1}{2})$ establishes that $(\Gamma_n)_{n=1}^\infty$ satisfies FIE, completing the proof.

A.2.3 *Proof of Theorem 3* Begin by introducing some notation. Write the voter's utility (slightly abusing notation) as

$$V_{t_i}(\sigma_i(t_i);\sigma_{-i}) = \min_{\pi\in\Pi_{t_i}} \mathbb{E}[u_i|t_i,\sigma_{-i},\sigma_i] \equiv V_{t_i}\left(\frac{\sigma_i(t_i)(A)}{\sigma_i(t_i)(A)+\sigma_i(t_i)(B)}, \sigma_i(t_i)(\varnothing);\sigma\right),$$

where (as long as $\tau(\varnothing|\sigma,a) < 1$)

$$\begin{aligned} V_{t_i}(x,\alpha;\sigma) = \min_{\pi\in[\underline{p}_{t_i},\bar{p}_{t_i}]}\big[&\pi\big\{\theta_a(\sigma)+(1-\alpha)[x\rho_{A,a}(\sigma)-(1-x)\rho_{B,a}(\sigma)]\big\} \\ &+(1-\pi)\big\{\theta_b(\sigma)+(1-\alpha)[(1-x)\rho_{B,b}(\sigma)-x\rho_{A,b}(\sigma)]\big\}\big] \end{aligned} \tag{A.3}$$

for $c_a = A$, $c_b = B$, $\tau^*(c|\sigma,s) = \tau(c|\sigma,s)/(1-\tau(\varnothing|\sigma,s))$,

$$\theta_s(\sigma) = \sum_{m=1}^{2n} f(m;1-\tau(\varnothing|\sigma,s),2n)U(\tau^*(c_s|\sigma,s),m)$$

$$\rho_{c,s}(\sigma) = \sum_{m=0}^{2n} f(m;1-\tau(\varnothing|\sigma,s),2n)\gamma(\tau^*(c_s|\sigma,s),m)$$

$$U(p,m) = \begin{cases} 1-F(\frac{1}{2}(m+1);p,m) & \text{if } m \text{ odd} \\ 1-F(\frac{1}{2}m;p,m)+\frac{1}{2}f(\frac{1}{2}m;p,m) & \text{if } m \text{ even} \end{cases}$$

$$\text{and}\quad \gamma(p,m) = \begin{cases} \frac{1}{2}f(\frac{1}{2}(m-1);p,m) & \text{if } m \text{ odd} \\ \frac{1}{2}f(\frac{1}{2}m;p,m) & \text{if } m>0 \text{ and even} \\ 1 & \text{if } m=0, \end{cases}$$

where $f(\cdot)$ and $F(\cdot)$ are the probability mass function and cumulative distribution of the binomial distribution, respectively. If $\tau(\varnothing|\sigma,a) = 1$ (which implies $\tau(\varnothing|\sigma,b) = 1$), $V_{t_i}(x,\alpha;\sigma) = \min_{\pi\in[\underline{p}_{t_i},\bar{p}_{t_i}]}(1-\alpha)[\pi x+(1-\pi)(1-x)]+\frac{1}{2}$. When it will not cause confusion, I write $\rho_{c,s}$ for $\rho_{c,s}(\sigma)$ and θ_s for $\theta_s(\sigma)$.

The function U gives the expected probability of candidate c winning, conditional on m votes cast with a p probability of each being cast for c and on voter i abstaining. The function γ gives the probability that a vote for candidate A (B) would change the election from a tie to winning or from B (A) winning to a tie, conditional on m votes cast with a p probability of each being cast for A (B) and on voter i abstaining.

If $r_a(1) = \frac{1}{2}$, then $r_b(1) = r_a(1)$, so information aggregation is impossible. Therefore, assume without loss of generality that $r_a(1) > \frac{1}{2}$. For the sake of contradiction, assume there exists σ with correct expected winners. In particular, $\sigma(1)(A)/\sigma(1)(B) > 1$ and $\sigma(2)(A)/\sigma(2)(B) < 1$.

Lemma 5. *If σ is an equilibrium with correct expected winners, then*

$$\mathbb{E}[u_i|a,\sigma_{-i},\sigma_i(t)(A)=1] > \mathbb{E}[u_i|b,\sigma_{-i},\sigma_i(t)(A)=1]$$

and

$$\mathbb{E}[u_i|b, \sigma_{-i}, \sigma_i(t)(B) = 1] > \mathbb{E}[u_i|a, \sigma_{-i}, \sigma_i(t)(B) = 1].$$

Proof. Note that $\theta_b(\sigma) + \rho_{B,b}(\sigma) = \mathbb{E}[u_i|b, \sigma_{-i}, \sigma_i(t)(B) = 1]$ and $\mathbb{E}[u_i|a, \sigma_{-i}, \sigma_i(t)(B) = 1] = \theta_a(\sigma) - \rho_{B,a}(\sigma)$. Suppose not, so

$$\theta_b(\sigma) + \rho_{B,b}(\sigma) \leq \theta_a(\sigma) - \rho_{B,a}(\sigma)$$

(the other case follows from reversing the roles of a and b and the roles of A and B). After updating, each voter regards state b as the worst state and a voter of type t_i uses the posterior assigning probability $\underline{p}_{t_i}$ to a. Best responses are as in an SEU game with a prior assigning probability $\underline{p}$ to a.

By standard arguments (e.g., Feddersen and Pesendorfer 1996), no voter mixes between A and B. Voters who observe signal 1 have at least as much incentive to vote for A as those who observe signal 2. Therefore, if expected winners are correct, $\sigma(1)(A), \sigma(2)(B) > 0$. This requires that $1 < (1 - \underline{p}_1)/\underline{p}_1 \leq (\rho_{A,a} + \rho_{B,a})/(\rho_{B,b} + \rho_{A,b}) \leq (1 - \underline{p}_2)/\underline{p}_2$, and the following two facts establish a contradiction.

Given expected winners are correct, $\sigma(1)(B) = 0$, $\sigma(2)(A) = 0$, and $r_a(1) = r_b(1) > \frac{1}{2}$, $(\rho_{A,a} + \rho_{B,a})/(\rho_{B,b} + \rho_{A,b}) \geq 1$ if and only if $\sigma(1)(\varnothing) < \sigma(2)(\varnothing)$. Note that

$$\rho_{A,s} + \rho_{B,s} = \sum_{i=0}^{n} f(2i; \tau_\varnothing, 2n) f(i; \tau_A^*, 2i) + \sum_{i=0}^{n-1} f(2i+1; \tau_\varnothing, 2n)\tfrac{1}{2}[f(i; \tau_A^*, 2i+1) + f(i+1; \tau_A^*, 2i+1)],$$

where $\tau_\varnothing = \tau(\varnothing|\sigma, s)$ and $\tau_A^* = \tau^*(A|\sigma, s)$. As $|\tau^*(A|\sigma, s) - \frac{1}{2}|$ increases, the election gets less close in state s, and $\rho_{A,s} + \rho_{B,s}$ decreases because both $f(i; p, 2i)$ and $f(i; p, 2i+1) + f(i+1; p, 2i+1)$ decrease as $|p - \frac{1}{2}|$ increases. Similarly, as $\tau(\varnothing|\sigma, s)$ decreases, voters become more likely to vote, and $\rho_{A,s} + \rho_{B,s}$ decreases because more weight gets put on higher populations. See the Supplement for the algebra. Since $\tau(A|\sigma, a) = r_a(1) > (1 - \sigma(2)(\varnothing))r_a(1) = \tau(B|\sigma, b) > \frac{1}{2}$ and $\tau(\varnothing|\sigma, b) = \sigma(2)(\varnothing)r_a(1) > \sigma(2)(\varnothing)(1 - r_a(1)) = \tau(\varnothing|\sigma, a)$, $(\rho_{Bb} + \rho_{A,b})/(\rho_{A,a} + \rho_{B,a}) > 1$.

Given expected winners are correct, $\sigma(1)(B) = 0$, $\sigma(2)(A) = 0$, and $r_a(1) = r_b(1) > \frac{1}{2}$, $\theta_a(\sigma) \geq \theta_b(\sigma)$ if and only if $\sigma(1)(\varnothing) < \sigma(2)(\varnothing)$. This follows from observing that $U(\tau(c_s|\sigma, s)/(\tau(B|\sigma, s) + \tau(A|\sigma, s)), m)$ increases in $\tau(c_s|\sigma, s)/(\tau(B|\sigma, s) + \tau(A|\sigma, s))$ and decreases in m. See the Supplement for the algebra. As $\sigma(1)(\varnothing)$ increases, more weight is put on lower values of m and $\tau(A|\sigma, a)/(\tau(B|\sigma, a) + \tau(A|\sigma, a))$ decreases. Simultaneously, $\tau(B|\sigma, b)/(\tau(B|\sigma, b) + \tau(A|\sigma, b))$ increases. This decreases $\theta_a(\sigma)$ relative to $\theta_b(\sigma)$.

The first fact implies that $\sigma(1)(\varnothing) < \sigma(2)(\varnothing)$. The second fact implies that $\theta_b(\sigma) > \theta_a(\sigma)$. But then $\theta_b(\sigma) + \rho_{B,b}(\sigma) \leq \theta_a(\sigma) - \rho_{B,a}(\sigma)$ is impossible, establishing the contradiction. □

Lemma 6. *If σ is a symmetric equilibrium with correct expected winners, then no type mixes between A and B.*

PROOF. If $\underline{p}_{t_i}[\rho_{Aa}+\rho_{Ba}] > (1-\underline{p}_{t_i})[\rho_{Ab}+\rho_{Bb}]$, then differentiating (A.3) shows that $\sigma(t_i)$ such that $\sigma(t_i)(A)=s$ and $\sigma(t)(\varnothing)=1-s$ is strictly better than any distinct $\sigma'(t_i)$ with $\sigma'(t_i)(\varnothing)=1-s$. Similarly if $(1-\bar{p}_{t_i})[\rho_{Ab}+\rho_{Bb}] > \bar{p}_t[\rho_{Aa}+\rho_{Ba}]$, then a $\sigma(t_i)$ with $\sigma(t_i)(B)=s$ and $\sigma(t_i)(\varnothing)=1-s$ is strictly better than any distinct $\sigma'(t_i)$ with $\sigma'(t_i)(\varnothing)=1-s$.

Consider the case where none of these holds, i.e., $(1-\bar{p}_{t_i})/\bar{p}_{t_i} \geq (\rho_{Bb}+\rho_{A,b})/(\rho_{A,a}+\rho_{B,a}) \geq (1-\underline{p}_{t_i})/\underline{p}_{t_i}$. Let $\bar{s}(\alpha,\sigma)$ be the probability of voting for A that equalizes utility in a and b given the voter abstains with probability α, i.e., $\bar{s}(\alpha,\sigma)$ solves

$$[\bar{s}(\rho_{A,a}+\rho_{B,a})-\rho_{B,a}](1-\alpha)+\theta_a=[\rho_{B,b}-\bar{s}(\rho_{B,b}+\rho_{A,b})](1-\alpha)+\theta_b$$

for $\bar{s}$ and $\bar{s}(\alpha,\sigma)=(\theta_b-\theta_a)/((1-\alpha)\sum\rho_{c,s})+(\rho_{Ba}+\rho_{Bb})/\sum\rho_{c,s}$. Note that $s<\bar{s}(\alpha,\sigma)$ is evaluated with $\bar{p}_{t_i}$ and $s>\bar{s}(\alpha,\sigma)$ is evaluated with $\underline{p}_{t_i}$, so in either case, $V_{t_i}(s,\alpha;\sigma) < V_{t_i}(\bar{s}(\alpha,\sigma),\alpha;\sigma)$.

At $\sigma(t_i)(A)=\bar{s}(\alpha,\sigma)$ (when $0\leq\bar{s}(\alpha,\sigma)\leq 1$), utility in each state is equal. Specifically, the utility of playing $\sigma^{\alpha}_{\text{ins}}$ where $\sigma^{\alpha}_{\text{ins}}(A)=(1-\alpha)\bar{s}(\alpha,\sigma)$, $\sigma^{\alpha}_{\text{ins}}(B)=(1-\alpha)[1-\bar{s}(\alpha,\sigma)]$, and $\sigma^{\alpha}_{\text{ins}}(\varnothing)=\alpha$ is

$$V_{t_i}(\sigma^{\alpha}_{\text{ins}},\alpha;\sigma)=\theta_a+\frac{(\theta_b-\theta_a)(\rho_{A,a}+\rho_{B,a})+\rho_{Aa}\rho_{Bb}-\rho_{Ba}\rho_{Ab}}{\sum\rho_{c,s}}+\alpha\frac{\rho_{Ba}\rho_{Ab}-\rho_{Aa}\rho_{Bb}}{\sum\rho_{c,s}}.$$

If expected winners are correct, then $\rho_{Bb}<\rho_{Ab}$ and $\rho_{Aa}<\rho_{Ba}$. Hence $\sigma^{\alpha}_{\text{ins}}$ gives less utility than abstaining with probability slightly higher than α. Consequently, a voter either mixes between A and $\varnothing$ or between B and $\varnothing$. □

In light of Lemma 6, index strategies by $\{A,B\}\times[0,1]\cup\{\varnothing\}$, with (c,α) corresponding to $\sigma(t)(c)=(1-\alpha)$ and $\sigma(t)(\varnothing)=\alpha$, and $\varnothing$ corresponding to $\sigma(t)(\varnothing)=1$.

LEMMA 7. *If $(1-\underline{p}_{t_i})/\underline{p}_{t_i} < (\rho_{Aa}+\rho_{Ba})/(\rho_{Bb}+\rho_{Ab})$ and σ is a symmetric equilibrium with correct expected winners, then $\sigma(t_i)\in BR_{t_i}(\sigma)$, where*

$$BR_{t_i}(\sigma)=\begin{cases}\{\varnothing\} & \text{if } \frac{\rho_{A,a}}{\rho_{A,b}}<\frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}}\\ \{A\}\times[\alpha^*_A,1] & \text{if } \frac{\rho_{A,a}}{\rho_{A,b}}=\frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}}\\ \{(A,\alpha^*_A)\} & \text{if } \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}}<\frac{\rho_{A,a}}{\rho_{A,b}}<\frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}}\\ \{A\}\times[0,\alpha^*_A] & \text{if } \frac{\rho_{A,a}}{\rho_{A,b}}=\frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}}\\ \{(A,0)\} & \text{if } \frac{\rho_{A,a}}{\rho_{A,b}}>\frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}}\end{cases}$$

for $\alpha^*_A=\max\{\min\{1+(\theta_a-\theta_b)/(\rho_{Aa}+\rho_{Ab}),1\},0\}$.

PROOF. Since $(\rho_{Aa}+\rho_{Ba})/(\rho_{Bb}+\rho_{Ab})$ is large enough, (A,α) is better than (B,α). Note that

$$V_{t_i}(1,\alpha;\sigma)=\min_{p\in[\underline{p}_{t_i},\bar{p}_{t_i}]}p[(1-\alpha)\rho_{A,a}+\theta_a]+(1-p)[\theta_b-(1-\alpha)\rho_{A,b}].$$

Utility in a is larger than b if and only if $\alpha \leq 1 + (\theta_a - \theta_b)/(\rho_{A,a} + \rho_{A,b}) = \alpha_A^*$ (algebra given in the Supplement). Consequently, $\alpha < \alpha_A^*$ implies $\partial V_{t_i}(A, \alpha) = \{(1 - \underline{p}_{t_i})\rho_{A,b} - \underline{p}_{t_i}\rho_{A,a}\}$, $\alpha > \alpha_A^*$ implies $\partial V_{t_i}(1, \alpha; \sigma) = \{(1 - \bar{p}_{t_i})\rho_{A,b} - \bar{p}_{t_i}\rho_{A,a}\}$, and $\alpha = \alpha_A^*$ implies $\partial V_{t_i}(1, \alpha; \sigma) = [(1 - \underline{p}_{t_i})\rho_{A,b} - \underline{p}_{t_i}\rho_{A,a}, (1 - \bar{p}_{t_i})\rho_{A,b} - \bar{p}_{t_i}\rho_{A,a}]$. The result follows from α optimal if and only if $0 \in \partial V_{t_i}(1, \alpha; \sigma)$. □

Lemma 8. *If $(1 - \bar{p}_{t_i})/\bar{p}_{t_i} > (\rho_{Aa} + \rho_{Ba})/(\rho_{Bb} + \rho_{Ab})$ and σ is a symmetric equilibrium with correct expected winners, then $\sigma(t_i) \in BR_{t_i}(\sigma)$, where*

$$BR_t(\sigma) = \begin{cases} \{(B, 0)\} & \text{if } \frac{\rho_{B,a}}{\rho_{B,b}} < \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \\ \{B\} \times [0, \alpha_B^*] & \text{if } \frac{\rho_{B,a}}{\rho_{B,b}} = \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \\ \{(B, \alpha_B^*)\} & \text{if } \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} > \frac{\rho_{B,a}}{\rho_{B,b}} > \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \\ \{B\} \times [\alpha_B^*, 1] & \text{if } \frac{\rho_{B,a}}{\rho_{B,b}} = \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} \\ \{\varnothing\} & \text{if } \frac{\rho_{B,a}}{\rho_{B,b}} > \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} \end{cases}$$

for $\alpha_B^* = \max\{\min\{1 + (\theta_b - \theta_a)/(\rho_{Ba} + \rho_{Bb}), 1\}, 0\}$.

Proof. Since $(\rho_{Aa} + \rho_{Ba})/(\rho_{Bb} + \rho_{Ab})$ is small enough, (B, α) is better than (A, α). So the problem is only choosing α. Then

$$V_{t_i}(0, \alpha; \sigma) = \min_{p \in [\underline{p}_{t_i}, \bar{p}_{t_i}]} p[\theta_a - (1 - \alpha)\rho_{B,a}] + (1 - p)[\theta_b + (1 - \alpha)\rho_{B,b}].$$

Utility in a is larger than b if and only if $\alpha_B^* = 1 + (\theta_b - \theta_a)/(\rho_{B,b} + \rho_{B,a}) \leq \alpha$ (algebra given in the Supplement). So $\alpha > \alpha_B^*$ implies $\partial V_{t_i}(0, \alpha; \sigma) = \{(1 - \underline{p}_{t_i})\rho_{B,b} - \underline{p}_{t_i}\rho_{B,a}\}$, $\alpha < \alpha_B^*$ implies $\partial V_{t_i}(0, \alpha; \sigma) = \{(1 - \bar{p}_{t_i})\rho_{B,b} - \bar{p}_{t_i}\rho_{B,a}\}$, and $\alpha = \alpha_B^*$ implies $\partial V_{t_i}(0, \alpha; \sigma) = [(1 - \underline{p}_{t_i})\rho_{B,b} - \underline{p}_{t_i}\rho_{B,a}, (1 - \bar{p}_{t_i})\rho_{B,b} - \bar{p}_{t_i}\rho_{B,a}]$. The result follows from α optimal if and only if $0 \in \partial V_{t_i}(0, \alpha; \sigma)$. □

Lemma 9. *If $(1 - \bar{p}_{t_i})/\bar{p}_{t_i} \leq (\rho_{Aa} + \rho_{Ba})/(\rho_{Bb} + \rho_{Ab}) \leq (1 - \underline{p}_{t_i})/\underline{p}_{t_i}$ and σ is a symmetric equilibrium with correct expected winners, then $\sigma(t) \in BR_t(\sigma)$, where*

$$BR_t(\sigma) = \begin{cases} \{(A, \alpha_A^*)\} & \text{if } \frac{\rho_{A,a}}{\rho_{A,b}} > \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \text{ and } \theta_b > \theta_a \\ \{A\} \times [\alpha_A^*, 1] & \text{if } \frac{\rho_{A,a}}{\rho_{A,b}} = \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \text{ and } \theta_b > \theta_a \\ \{\varnothing\} & \text{if } \theta_a = \theta_b \text{ or } \frac{\rho_{A,a}}{\rho_{A,b}} < \frac{1-\bar{p}_{t_i}}{\bar{p}_{t_i}} \text{ and } \theta_b > \theta_a \\ & \text{or } \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} < \frac{\rho_{B,a}}{\rho_{B,b}} \text{ and } \theta_a > \theta_b \\ \{B\} \times [\alpha_B^*, 1] & \text{if } \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} = \frac{\rho_{B,a}}{\rho_{B,b}} \text{ and } \theta_a > \theta_b \\ \{(B, \alpha_B^*)\} & \text{if } \frac{1-\underline{p}_{t_i}}{\underline{p}_{t_i}} > \frac{\rho_{B,a}}{\rho_{B,b}} \text{ and } \theta_a > \theta_b \end{cases}$$

for α_A^*, α_B^* as defined above.

Proof. Suppose first $\theta_a = \theta_b$. By Lemma 6, abstaining with probability 1 is the only best response.

Now suppose $\theta_b > \theta_a$, so $\alpha^*_A < 1$ and $\alpha^*_B = 1$. By the arguments in Lemma 6, (B, α) cannot be a maximizer unless $\alpha = 1$. If $\alpha > \alpha^*_A$, then the minimizing posterior of $V_{t_i}(1, \alpha; \sigma)$ equals $\bar{p}_{t_i}$. As long as $\alpha \geq \alpha^*_A$, $V_{t_i}(1, \alpha; \sigma)$ is decreasing in α if $\rho_{A,a}/\rho_{A,b} > (1 - \bar{p}_{t_i})/\bar{p}_{t_i}$, increasing in α if $\rho_{A,a}/\rho_{A,b} < (1 - \bar{p}_{t_i})/\bar{p}_{t_i}$, and constant in α if $\rho_{A,a}/\rho_{A,b} = (1 - \bar{p}_{t_i})/\bar{p}_{t_i}$. Hence $\rho_{A,a}/\rho_{A,b} < (1 - \bar{p}_{t_i})/\bar{p}_{t_i}$ implies $V_{t_i}(A, \alpha; \sigma)$ is maximized at $\alpha = 1$, $\rho_{A,a}/\rho_{A,b} = (1 - \bar{p}_{t_i})/\bar{p}_{t_i}$ implies $V_{t_i}(A, \alpha; \sigma)$ is maximized at any $\alpha \in [\alpha^*_A, 1]$, and $\rho_{A,a}/\rho_{A,b} > (1 - \bar{p}_{t_i})/\bar{p}_{t_i}$ implies $V_{t_i}(A, \alpha; \sigma)$ is maximized at $\alpha = \alpha^*$. To see why the latter is a global maximum, Lemma 6 shows that for $\alpha < \alpha^*_A$, $V_{t_i}(x, \alpha; \sigma) \leq V_{t_i}(\bar{s}(\alpha, \sigma), \alpha; \sigma)$ for any $x \in [0, 1]$, $V_{t_i}(\bar{s}(\alpha, \sigma), \alpha; \sigma)$ is increasing and continuous in α, and $\bar{s}(\alpha, \sigma)$ approaches 1 as α approaches α^*_A. Hence, $V_{t_i}(1, \alpha^*_A; \sigma)$ is a global optimum when $\rho_{A,a}/\rho_{A,b} > (1 - \bar{p}_{t_i})/\bar{p}_{t_i}$.

Dual arguments hold when considering $\theta_a > \theta_b$. □

Before completing the proof, note the following statements. First, $r_a(1) > \frac{1}{2} > r_b(1)$ implies $(1 - \bar{p}_1)/\bar{p}_1 < (1 - \bar{p}_2)/\bar{p}_2 < (1 - \underline{p}_1)/\underline{p}_1 < (1 - \underline{p}_2)/\underline{p}_2$. Second, given Lemma 6, if expected winners are correct, then $\sigma(1)(A) > 0$ and $\sigma(2)(B) > 0$. Third, given that $\sigma(2)(A) = \sigma(1)(B) = 0$, $(\rho_{Aa} + \rho_{Ba})/(\rho_{Bb} + \rho_{Ab}) \geq 1 \iff \sigma(1)(\varnothing) \leq \sigma(2)(\varnothing)$; see Lemma 5. Finally, again given $\sigma(2)(A) = \sigma(1)(B) = 0$, $\theta_a(\sigma) \geq \theta_b(\sigma) \iff \sigma(1)(\varnothing) \geq \sigma(2)(\varnothing)$; see Lemma 5.

Consider separately $\sigma(1)(\varnothing) \geq \sigma(2)(\varnothing)$ and $\sigma(1)(\varnothing) < \sigma(2)(\varnothing)$. On the one hand, if $\sigma(1)(\varnothing) \geq \sigma(2)(\varnothing)$, then $(\rho_{Aa} + \rho_{Ba})/(\rho_{Bb} + \rho_{Ab}) \geq 1$ and $\theta_a(\sigma) \leq \theta_b(\sigma)$. But then Lemmas 8 and 9 imply $\sigma(2)(B) = 0$, a contradiction. On the other hand, if $\sigma(1)(\varnothing) < \sigma(2)(\varnothing)$, then $(\rho_{Aa} + \rho_{Ba})/(\rho_{Bb} + \rho_{Ab}) < 1$ and $\theta_a(\sigma) > \theta_b(\sigma)$. But then Lemmas 7 and 9 imply $\sigma(1)(A) = 0$, a contradiction. The conclusion is that expected winners cannot be correct, completing the proof.

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