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A Reconsideration of Minsky's Financial Instability Hypothesis*

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Abstract

The worst and longest depressions have tended to occur after periods of prolonged, and reasonably stable, prosperity. This results in part from agents rationally updating their expectations during good times and hence becoming more optimistic about future economic prospects. Investors then increase their leverage and shift their portfolios towards projects that would previously have been considered too risky. So, when a downturn does eventually occur, the financial crisis, and the extent of default, become more severe. Whereas a general appreciation of this syndrome dates back to Minsky [1992, Jerome Levy Economics Institute, WP 74] and even beyond, to Irving Fisher [1933, Econometrica 1, 337-357], we model it formally. In addition, endogenous default introduces a pecuniary externality, since investors do not factor in the impact of their decision to take risk and default on the borrowing cost. We explore the relative advantages of alternative regulations in reducing financial fragility, and suggest a novel criterion for improvement of aggregate welfare.

Keywords: Financial Instability, Minsky, Risk-taking, Leverage, Optimism, Procyclicality

JEL Classification: D81, D83, E44, G01, G21

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1 Introduction

The second theorem of Minsky's Financial Instability Hypothesis, (Minsky (1992)), states that over periods of prolonged prosperity and optimism about future economic prospects, financial institutions invest more in riskier assets, which can make the economic system more vulnerable in the case that default materializes. Minsky's hypothesis emphasizes the interaction between optimism and leverage, and the ensuing increase in portfolio riskiness. This triplet lies at the heart of many financial crises that manifest themselves with extensive default.

In this paper, we focus on the role that expectations formation about future states of the economy -in the sense of investment profitability and growth- plays in the borrowing decision of investors, in their portfolio choice, and eventually in the extent of default in the economy.[1] We thus examine the effect of leverage, as a path-dependent process, on financial stability, by linking learning to risk-taking behaviour. In particular, we consider investors that face a multiperiod portfolio problem. In each period, they use their own capital, augmented by accumulated profits, in combination with short-term borrowing to invest in projects, which mature in one period as well. The absence of any maturity mismatch is unimportant for our results, as we shall be abstracting from any fire sales externality, which should be regarded as complementary to our modeling. Credit is raised from a competitive credit market. Given endogenous default, the borrowing rate, which depositors will require, will be endogenously determined by expectations of future repayment on loans, and hence an endogenous default premium emerges in equilibrium. This will depend on depositors' information and understanding of the portfolio choices of investors.

We assume two types of projects and two states of the world that occur in every period, one of which is good, whereas the other is bad. Although both projects are risky, their payoffs differ and one is riskier than the other under any probability distribution, which assigns positive probability to both states. We assume that the outcomes are perfectly correlated in the sense that both do well in the good state and poorly in the bad state, and that both projects are in perfectly elastic supply, thus abstracting from the general equilibrium effects of projects' origination and supply.[2] Investors

[1] We refer to the agents in our model as investors, which can be any financial institution or asset management company in the broad sense, engaging in borrowing in order to invest in projects/assets.

[2] Endogenising project supply can allow the study of asset price bubbles together with variations in the leverage cycle. Adam and Marcet (2010) and Branch and Evans (2011) use a different learning model to explain bubbles and crashes in asset prices. We leave this interesting extension for future work.

choose their portfolio of projects, at each point in time, according to their expectations about the future realization of payoffs and the borrowing rates that they may face. The projects' payoffs do not change over time. However, the perceived belief about the likelihood of good realizations changes over time according to past realizations. Following Cogley and Sargent (2008), we assume that agents have incomplete information about the true probability measure. They learn it over time by observing the past realizations and updating their priors.[3] The fluctuation of portfolio composition and leverage are due to the variability of beliefs in finite time, although they will converge to the true probability measure in the limit. We show that financial institutions will start investing in the riskier project after a number of good past realizations, since their expectations are boosted and, consequently, the risk/return profile from an individual perspective improves.

Our second contribution to an otherwise canonical portfolio problem is the modeling of endogenous default and of a separate market for credit. The interest rate charged on loans depends on the expectations about future repayment. Creditors hold beliefs about the debtors' portfolios and, accordingly, update their expectations about the future realization of payoffs and subsequent defaults, subject to their information set. We thus connect the credit spread to risk taking via the introduction of endogenous default. Low credit spreads allow investors to borrow more. Whereas if they keep increasing their risk taking, the credit spreads will increase. Risk taking is penalized ex-post via penalties for default, and ex-ante via higher borrowing rates. Nevertheless, expectations, in our framework, vary over time and optimism increases after periods of good news. Riskier projects become more attractive for investors since the expected penalty for default decreases. Moreover, expectations about the possibility of default go down and so creditors are willing to offer low borrowing rates even though debtors invest more in the riskier project.

The introduction of endogenous default and endogenous credit spreads yields two additional implications apart from the fact that investment in riskier projects increases with optimism. The first implication is that optimism and risk-taking are accompanied by lower risk premia. This facilitates the build-up of risk, since investors are not penalized with higher borrowing costs when they take on more risk. In particular, our model draws the distinction between observed default rates during good times and implicit default rates when a bad shock realizes. Lower (or zero in our model) default

[3]One might ask why enough time has not elapsed since the innovation of fractional banking to allow the learning process to converge to the true probability measure. The answer is that the learning process is itself flawed. As Reinhart and Rogoff (2008) show, the young think that 'This time it is different', and the old have retired and disappeared.

rates result in optimism and risk-taking. However, conditional on a future bad realization, default and risk premia increase. The empirical implications of our model can be seen in the evolution of market volatility and credit premia before and during the financial crisis of 2007-2008, which we discuss in the conclusions. Moreover, we calibrate our model in section 4 to capture some of the key characteristics of the 2007-2008 subprime crisis. The second implication is that optimism results in excessive and socially inefficient risk taking. Thus, we do not merely describe situations where premia are low and investors take more risk, but also argue that this leads to excessive risk-taking and, thus, there is scope for regulation to improve welfare.

Arguably, there are always going to be cycles of optimism and pessimism, both in finance and elsewhere, and there is not much that we can, or perhaps should, do to prevent them. There are several reasons why banking and finance involve externalities that generate particular amplifications to a potential debt-deflation spiral. The pioneer of such a view was Fisher (1933), who suggested a Debt-Deflation theory of Great Depressions. His analysis was based on two fundamental principles, namely over-indebtedness and deflation. He argued that over-indebtedness can result in deflation in future periods and, subsequently, cause liquidation of collateralised debt. This theory brings financial intermediation to the center of attention. Numerous studies have subsequently built on the debt-deflation theory of financial amplification to analyse the effect of collateral constraints on borrowing, production and, eventually, financial stability. In a seminal paper, Bernanke and Gertler (1989) modeled a collateral-driven credit constraint, which introduced an external finance premium, and analysed interactions between the balance sheet of financial institutions and the real economy. Debt-deflation dynamics may also give rise to another important externality, that of fire sales -to institutions that can, at times, realize less value from such assets- which can act as an amplification mechanism in financial crises. Financially distressed institutions liquidate their assets to meet their debt obligations, and in doing so, they reduce the value of their own and other institutions' portfolios, which exacerbates the fire sale discounts, and further worsens their debt position.

We do not pursue such an analysis here, partly because it has been so thoroughly examined elsewhere.[4] Indeed, there are other approaches to financial crises that we do not cover. In particular,

[4]Mendoza (2006) and Mendoza and Smith (2006) analyse the role of fire sales in Sudden Stops in emerging markets within an RBC model. Geanakoplos (2003) and Fostel and Geanakoplos (2008) show how the arrival of bad news about the future economic prospects results in a reduction in the price of assets used as collateral and leads to a drying up of liquidity and fire-sales externalities. Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) show how the borrowing capacity of agents, i.e. funding liquidity, and the pricing of assets, i.e. market liquidity, interact and how an

we do not model bank run externalities arising from coordination problems (Diamond and Dybvig (1983)), network externalities (Bhattacharya and Gale (1987) , Rochet and Tirole (1996)), or market freezes due to portfolio opaqueness and unduly pessimistic beliefs (Dubey et al. (2005), Stiglitz and Weiss (1981)). Instead, the externality that we do address is that investors do not incorporate the impact of their portfolio and default decisions on borrowing rates since they are price takers in the credit market. Investors take on more downside risk by over-investing in the riskier assets. Therefore, they are likely to default more, and as a result their borrowing rates rise, which causes even more default, and so on. A social planner understands the impact of risk-taking on interest rates and the deadweight loss associated with default, and does not switch to the riskier asset as fast as individual investors do after a series of good realizations due to their higher optimism.

We consider three policy responses to tackle the issue of excessive risk-taking accompanied by high leverage; stricter penalties for default, lower permitted leverage ratios, and finally our novel criterion that puts a lower bound on the ratio of safer minus riskier portfolio holdings over total borrowing. Only the latter is successful in both reducing the amount of default and in increasing aggregate welfare, since it addresses risk taking during optimistic times directly. Compared to the use of a crude leverage ratio, our proposal regarding optimal regulation resembles a combination of leverage ratios on all assets, which is responsive to their relative riskiness in the cross-section. Hence, it is close to the regulatory proposal of Geanakoplos (2010) who suggested higher margin/haircut requirements on bank generated asset holdings during good times. We discuss the difference in more detail in section 5.

Finally, we discuss the empirical implications of our paper. We argue that the underlying reason why commonly used return-based risk measures, such as VIX or the TED spread,[5] failed to capture the build-up of risk before the 2007-2008 crisis is that they were biased by optimistic expectations, as are the credit spreads in our model. Boz and Mendoza (2014) consider a learning model in which agents update their expectations about the leverage constraints they will face, as exogenous

idiosyncratic liquidity shock can lead to fire-sales and the unraveling of the whole market. Other papers, which model fire-sales due to adverse productivity or funding shocks to capture debt-deflationary effects on asset prices, leading to loss spirals and financial instability, include Acharya et al. (2011), Diamond and Rajan (2011), Kiyotaki and Moore (1997), Kyle and Xiong (2001) and Shleifer and Vishny (1992). Adrian and Shin (2009) investigate the importance of this channel empirically for financial institutions.

[5]VIX is the CBOE Volatility Index, created by the Chicago Board Options Exchange as a measure of equity market volatility. The computation of the value of VIX is based on the implied volatility of eight option series on the S&P 100 index. The TED spread is the difference between the interest rates on interbank loans and on short-term U.S. government debt.

multiples of asset values, which will prevail in the future. They examine the interaction between their borrowing constraints and the mispricing of risk. A sequence of periods characterised by lax borrowing constraints induces optimistic expectations about the continuation of such regimes, and leads to the underpricing of risk, high leverage, and over inflated collateral values. A sharp collapse then follows after the realization (exogenously) of a tighter constraint. We regard, in contrast, that our quantity-based measure, capturing the shift in portfolios holdings towards riskier projects in optimistic times, is likely to be more effective in identifying endogenous credit cycles, assuming that projects' relative riskiness can still be correctly evaluated when expectations change, although each of the projects may look safer.

The rest of the paper proceeds as follows. Section 2 presents the model. In section 3, we offer an analytical solution which explores the implications for divergences across private and socially optimal risk-taking. In section 4, we present a calibrated example of our model and extend our arguments. In section 5, we assess quantitatively the relative performance of alternative regulatory regimes. Section 6 discusses the empirical implications of our model and concludes.

2 The Model

Consider a multi-period economy populated by a continuum of identical investors with total mass normalized to 1. Hereafter, we consider the problem of the representative investor. At any date $t = 0,\ldots,T$, the economy can be in one of two states, denoted by u ("up"/good state) and d ("down"/bad state) respectively. For example, the "up" state at time t is denoted by $s_t = s_{t-1}u$. The set of all states is $s_t \in S = \{0,u,d,\ldots,uu,ud,du,dd,\ldots,s_tu,s_td,\ldots\}$. The probability that a good state occurs at any point in time is denoted by θ, which is chosen by nature. For simplicity we assume that $\theta \in \{\theta_1,\theta_2\}$ with $1 > \theta_1 > \theta_2 > 0$. However, agents do not know this probability and try to infer it by observing past realizations of good and bad states. Agents have priors $\Pr(\theta = \theta_1)$ and $Pr(\theta = \theta_2) = 1-\Pr(\theta = \theta_1)$ that the true probability is θ_1 or θ_2 respectively. Their subjective belief in state s_t of a good state occurring at t+1 is denoted by π_{s_t} and that of the bad $1-\pi_{s_t}$. These probabilities depend on the whole history of realizations up to t. In other words, $\pi_{s_t} = Pr_{s_t}(s_{t+1} = s_tu|s_0,\ldots,s_t)$. Given our notation, state s_t completely summarizes the history of realizations up to t. Thus, $\pi_{s_t} = Pr_{s_t}(s_{t+1} = s_tu|s_0,\ldots,s_t) = Pr_{s_t}(s_{t+1} = s_tu|s_t)$. We assume that past realizations of the states of the world are observable by all agents, thus there is no information asymmetry on top of the imperfect information structure.

Consequently, agents' subjective belief is $\pi_{s_t} = Pr_{s_t}(\theta = \theta_1 | s_t) \cdot \theta_1 + Pr_{s_t}(\theta = \theta_2 | s_t) \cdot \theta_2$. Agents are Bayesian updaters and try to learn from past realizations the true probability θ. Their conditional probability given past realizations is:

$$\begin{aligned} Pr_{s_t}(\theta = \theta_1 | s_t) &= \frac{Pr_{s_t}(s_t | \theta = \theta_1) \cdot Pr(\theta = \theta_1)}{Pr(s_t)} \\ &= \frac{Pr_{s_t}(s_t | \theta = \theta_1) \cdot Pr(\theta = \theta_1)}{Pr_{s_t}(s_t | \theta = \theta_1) \cdot Pr(\theta = \theta_1) + Pr_{s_t}(s_t | \theta = \theta_2) \cdot Pr(\theta = \theta_2)} \\ &= \frac{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1)}{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1) + \theta_2^n (1-\theta_2)^{t-n} \cdot Pr(\theta = \theta_2)}, \end{aligned}$$

where n is the number of good realization up to time t. Then,

$$\begin{aligned} \pi_{s_t} = & \frac{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1)}{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1) + \theta_2^n (1-\theta_2)^{t-n} \cdot Pr(\theta = \theta_2)} \theta_1 \\ & + \frac{\theta_2^n (1-\theta_2)^{t-n} \cdot Pr(\theta = \theta_2)}{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1) + \theta_2^n (1-\theta_2)^{t-n} \cdot Pr(\theta = \theta_2)} \theta_2. \end{aligned} \tag{1}$$

As the number of good realizations increases, the subjective probability of the good state realizing in the following period increases as well, i.e. given that $s_t = s_{t-1}u$, then $\pi_{s_t} > \pi_{s_{t-1}}$. Assume that the priors are the same, that is $Pr(\theta = \theta_1) = Pr(\theta = \theta_2)$.

To prove our claim that agents become more optimistic after they observe good outcomes in the past, we just need to show that $Pr_{s_t}(\theta = \theta_1 | s_t) > Pr_{s_{t-1}}(\theta = \theta_1 | s_{t-1})$ and $Pr_{s_t}(\theta = \theta_2 | s_t) < Pr_{s_{t-1}}(\theta = \theta_2 | s_{t-1})$ given that $s_t = s_{t-1}u$.

Proof.

$$\begin{aligned} & Pr_{s_t}(\theta = \theta_1 | s_t) > Pr_{s_{t-1}}(\theta = \theta_1 | s_{t-1}) \\ & \Rightarrow \frac{\theta_1^{n+1}(1-\theta_1)^{t+1-(n+1)}}{\theta_1^{n+1}(1-\theta_1)^{t+1-(n+1)} + \theta_2^{n+1}(1-\theta_2)^{t+1-(n+1)}} > \frac{\theta_1^n(1-\theta_1)^{t-n}}{\theta_1^n(1-\theta_1)^{t-n} + \theta_2^n(1-\theta_2)^{t-n}} \\ & \Rightarrow \left(\frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\right)^n \left(\frac{1-\theta_2}{1-\theta_1}\right)^t > \left(\frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\right)^{n+1} \left(\frac{1-\theta_2}{1-\theta_1}\right)^{t+1} \\ & \Rightarrow 1 > \frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\frac{1-\theta_2}{1-\theta_1} \Rightarrow 1 > \frac{\theta_2}{\theta_1}. \end{aligned}$$

$\square$

To simplify notation, we will use hereafter simply t to denote histories of shocks (s_t), and we will refer to these as simply histories or nodes. Using this notation, $t-1$ will denote the history up to the previous period, and $t+1$ will denote all the possible histories following node t.

At t the representative investor faces two investment opportunities; a safer project, denoted by L (standing for "*low*" risk), and a riskier one, denoted by H (standing for "*high*" risk). Both projects are in perfectly elastic supply, with their prices normalized to 1, and expire in one period. The safer project yields a payoff X_u^L in the good state and X_d^L in the bad. Equivalently, the payoffs for the riskier project are X_u^H and X_d^H. We assume that $X_u^H > X_u^L > 1 > X_d^L > X_d^H > 0$, such that the riskier project is more profitable if the good state realizes. Ex-post payoffs are independent of the history of past realizations.

Each investor has the following payoff/utility function at t:

$$U_t = \Pi_t - \gamma \cdot (\Pi_t)^2, \tag{2}$$

where γ is the risk aversion coefficient and Π_t are the distributed profits in t.

The amount of funds available for investment by investors is equal to their equity capital, plus funds borrowed from credit markets, plus the profits from the previous period's investment that are not distributed as profits and consumed. We consider a general portfolio problem under which investors decide how much of the available funds to invest in the safer project and how much in the riskier one. We denote by w_t^j the portfolio holdings of the investor in project $j \in \{L, H\}$ at t. For example, the riskier project's holdings in the second period after a good state realization at $t = 1$ are denoted by w_u^H. The interest rate for borrowing from the credit market is denoted by r_t at t.

We allow for default in the credit market. The amount repaid is an endogenous decision by the investor, who weighs the benefits from defaulting against a deadweight loss. The latter is assumed to be a linear function of the amount that the investor chooses not to deliver.[6] Denoting by $1 - v_t$ the

[6] In the event of default, investors can extract a private benefit, which is pinned down by the exogenously set non-pecuniary default penalty, given the linearity of the disutility of default. As the marginal penalty for default increases, the private benefit investors can extract decreases and they have a lower incentive to default. Shubik and Wilson (1977), Dubey et al. (2005) and Zame (1993) are canonical models of such choice processes. Many papers have followed the seminal contribution by Dubey et al. (2005) to study the implications of default in different markets settings. For example, Acharya and Bisin (2014) study the effect of default and transparency on counterparty risk externalities in centralized and over-the-counter markets. Athreya et al. (2009) study the extent to which unsecured credit markets have altered the

percentage default on one unit of borrowed funds, the deadweight loss is equal to $\lambda(1-v_t)(1+r_{t-1})$, where λ is the default penalty, v_t is the percentage repayment on one unit of owned debt and r_{t-1} is the interest rate set at the node preceding state t. We assume risk-neutral creditors who break even in expectation and their valuation of the debt is independent of the position they take. Thus, the interest rate will be inversely related to their expectation about future percentage delivery. The amount of funds that the investor chooses to borrow is denoted by w_t and his initial capital at $t=0$ by $\bar{w}_0$. Without loss of generality, investors are not endowed with additional capital for $t=1,\ldots,T$.

Each investor tries to maximize his lifetime expected utility by choosing the amount he invests in the safer and riskier projects at each point in time $\left(w_t^j,\ \mathrm{j} \in \{\mathrm{L},\mathrm{H}\}\right)$, the amount that he borrows from the credit market (w_t), the percentage repayment on past loans (v_t), and the amount of realized profits that he reinvests (I_t), i.e.,

$$\max_{w_t^j, v_{t+1}, \Pi_{t+1}, I_{t+1}} \sum_t \mathbb{E}_t \left[U_{t+1} - \lambda \max\left[(1-v_{t+1})w_t(1+r_t), 0\right]\right],$$

where $\mathbb{E}_t$ is the expectations operator in state t, under the probability measure π_t, when the investment decision is made, and U_{t+1} is given by equation 2. Note that investors do not derive any utility at $t=0$, thus all the funds go to investment, i.e., $w_0^L + w_0^H \leq f(I_0) + w_0$ and $f(I_0) = \bar{w}_0$. $f(\cdot)$ is a concave function which captures a cost of retaining high levels of realized earnings and guarantees boundedness of the solution for probabilities for the good state approaching one. Every investor optimizes the payoff function above subject to the following budget constraints:

$$\Pi_{t+1} + I_{t+1} \leq w_t^L X_{t+1}^L + w_t^H X_{t+1}^H - v_{t+1} w_t (1+r_t)$$

i.e., distributed + retained profits $\leq$ safer and riskier investments' payoff - loan repayment in $t+1$.

$$w_{t+1}^L + w_{t+1}^H \leq f(I_{t+1}) + w_{t+1}$$

i.e., investment in the safer and the riskier projects $\leq$ reinvested profits + leverage in t.

The second type of agents in our economy are the suppliers of credit. There is a continuum of creditors who are risk-neutral and have an infinite supply of funds, Hence, they will supply credit in

transmission of increased income risk to consumption variability. Goodhart et al. (2006) and De Walque et al. (2010) have applied utility penalties for default to study banking stability, while Peiris and Vardoulakis (2013) to study the interaction between default and the precautionary motive for savings.

every period at a rate that breaks even in expectation with their outside risk-free option given their beliefs. We denote by $r^{outside}$ their outside option. Also, creditors do not have additional information and hence they will form the same beliefs as investors at each point in time. In equilibrium, they require a borrowing rate r_t, such that

$$\mathbb{E}_t\left[v_{t+1}\right] \cdot (1+r_t) = 1 + r^{outside} \tag{3}$$

One can observe the reverse relationship between the interest rate and expected percentage delivery. When the latter increases, the interest rate charged falls. This provides some intuition for the seemingly counterintuitive result that when expectations are optimistic, investors increase their leverage, paying lower than otherwise expected interest rates, though at the end their percentage repayment is lower in a bad state and default is higher. The result follows from the fact that the perceived probability that a good state realizes is higher, since expectations are optimistic. Thus, overall expected delivery is higher, though loss given default is higher as well.

Equilibrium is reached when investors optimize given their constraints and the credit and projects' markets clear. Interest rates are determined endogenously by equation 3 and are taken as fixed by agents. Credit market clearing requires that equation 3 holds. The above modeling has assumed a perfectly elastic supply of projects. Equilibrium purchases are determined by investors' demand at a given price of 1 for each project. The analysis of equilibrium and our main result that leverage, investment in the riskier project and realised default all increase when expectations become more optimistic would not have changed had we assumed an upward sloping supply curve. One can find endowments of projects that support the price of 1 in equilibrium. Thus, endogenising asset prices as well would have allowed the joint investigation of changes in leverage and asset prices during a boom or a bust. We consider this an interesting extension for future work.

The variables determined in equilibrium and taken by agents as fixed are, thus, given by $\eta = \{r_t\}$, $\mathrm{t} = 0,\ldots,\mathrm{T}$. The choices by investors are given by $\square = \left\{w_t^j, w_t, v_{t+1}, \Pi_{t+1}, I_{t+1}\right\}$. We say that $(\eta, \square)$ is an equilibrium of the economy if and only if:

i. $(\square) \in Argmax_{\square \in B(\eta)}\ \mathbb{E}_t U_t$.

ii. $\mathbb{E}_t\left[v_{t+1}\right] \cdot (1+r_t) = 1 + r^{otuside}$.

iii. $w_t^j, w_t \geq 0$ for $j \in \{L, H\}$.

iv. Subjective probabilities π_t are determined by equation 1.

v. Creditors expectations are rational, i.e. they anticipate correctly the delivery v_t.

Condition (i) states that investors optimize; (ii) says that the credit market clears; (iii) says that investors cannot go short on the credit and projects' markets; (iv) says that all agents update their expectations using Bayes rule, and (v) says that creditors are correct about their expectations of loan delivery or default.

3 Optimism, Risk-taking and Externalities

In this section we present an analytical solution for this model, the intuition for which is Minsky's financial instability hypothesis. In section 3.1 we solve for the endogenous variables of the model and derive propositions about the effect that an increasing probability of a good outcome has on them. For tractability, we assume that the initial capital of investors is zero and that investors do not reinvest any of the realized profits at any time $t > 0$. Thus, their portfolio is fully debt financed. Under these assumptions, solving for equilibrium reduces to a static problem for which we can derive a closed form solution. Finally, we assume for simplicity that $r^{outside} = 0$.

A closed form solution is important to show explicitly the externality inherent in our model and to enable us to perform welfare analysis. Default twists investors utilities by adding a component which depends on downside risk and represents the disutility of default. Investing in the riskier project increases downside risk. However, profits in the case of default is pinned down by the default penalty and does not depend on the probability of a bad realization. This can be easily seen from the first order condition with respect to the repayment rate, v_t, which is $\lambda = 1 - 2\gamma\Pi_t$, i.e. investors equate the marginal loss from defaulting to the marginal benefit of an additional unit of profits/consumption. As expectations improve the (total) expected disutility from default decreases, while the private benefit in the event of default remains fixed. There is a probability threshold after which investors start investing in the riskier asset and shift consumption to the good state of the world.

Increased risk-taking results in higher default, which puts upwards pressure on the borrowing rates and the amount owed. Investors do not take into consideration the impact that their portfolio decisions have on the optimizing decisions of creditors and the resulting borrowing rates. This is a type of pecuniary externality along the lines of Stiglitz (1982). We show in section 3.2 that a social planner, who incorporates this externality in her decisions, ceases to invest in the riskier asset. In particular, borrowing rates go down as does default, and the welfare of investors increases in the social planner's solution. Creditors' welfare is unaffected, since they are risk-neutral and break even in expectation. Hence, we get a Pareto improvement. Section 3.3 establishes that optimism exacerbates the risk-taking externality and that it requires distinct regulatory treatment.

3.1 Optimism and Risk-taking

Under the assumptions made above, the investors' problem reduces to maximizing

$$\max_{\Pi_u,\Pi_d,w,v} \mathbb{U} = \mathbb{E}\Pi - \gamma \cdot \mathbb{E}\Pi^2 - \lambda(1-\pi)\max\left[(1-v),0\right] wR,$$

subject to the budget constraints

$$\Pi_u \leq a \cdot wX_u^L + (1-a)\cdot wX_u^H - wR = w\left[a(X_u^L - X_u^H) + X_u^H - R\right] \quad (\mu_u),$$

$$\Pi_d \leq a \cdot wX_d^L + (1-a)\cdot wX_d^H - vwR = w\left[a(X_d^L - X_d^H) + X_d^H - vR\right] \quad (\mu_d)$$

and the short-sale constraints $0 \leq a \leq 1$, where a stands for the percentage of borrowed funds invested in the safer asset, L, and w is the amount of borrowing. Thus, agents cannot go short on any of the two projects. The good and the bad state occur with probabilities π and $1-\pi$ respectively. If the bad state occurs, investors choose to repay a fraction v of the amount owed, wR, where $R(=1+r)$ is the gross borrowing rate. μ_u and μ_d are the Lagrange multipliers associated with the budget constraints. Finally, the credit market clears when

$$R = \frac{1}{\pi + (1-\pi)v}. \tag{4}$$

The Lagrangian is

$$\begin{aligned}\mathbb{L} =& \mathbb{E}\Pi - \gamma\mathbb{E}\Pi^2 - \lambda(1-\pi)\max\left[(1-v),0\right]wR - \mu_u\left[\Pi_u - w\left[a(X_u^L - X_u^H) + X_u^H - R\right]\right] \\ & - \mu_d\left[\Pi_d - w\left[a(X_d^L - X_d^H) + X_d^H - vR\right]\right] - \phi\left[a-1\right] + \psi\cdot a.\end{aligned}$$

The Lagrange multipliers μ_u and μ_d are positive due to concave utility, thus the budget constraints hold with equality in equilibrium. Investors are price takers and do not factor in the impact of their decision to default on R. They take R as given. Thus, when they optimize they will take $\frac{\partial R}{\partial v} = 0$. However, their decision to default does have a price effect, since from equation 4, $\frac{\partial R}{\partial v} = -(1-\pi)R^2$. Optimizing with respect to the delivery rate, v, we get

$$(1-\pi)\lambda = \mu_d. \tag{5}$$

Moreover, consumption in the event of default satisfies $\lambda = (1-2\gamma\Pi_d)$. Optimizing with respect to borrowing, w, we get

$$\mu_u\left[a(X_u^L - X_u^H) + X_u^H - R\right] + \mu_d\left[a(X_d^L - X_d^H) + X_d^H - vR\right] - \lambda(1-\pi)(1-v)R = 0. \tag{6}$$

Combining the last equation together with equation 5 yields

$$\frac{\mu_u}{\mu_d} = -\frac{a(X_d^L - X_d^H) + X_d^H - R}{a(X_u^L - X_u^H) + X_u^H - R}. \tag{7}$$

The complementary slackness conditions $\phi\left[a-1\right] = 0$ and $\psi\cdot a = 0$ yield the following three (candidate) equilibrium solutions for percentage investment in the safer asset:

1. The portfolio consists of both assets, i.e., $0 < a < 1$. This implies $\phi = 0$ and $\psi = 0$.

2. The portfolio consists solely of the riskier asset, i.e., $a = 0$. This implies $\phi = 0$ and $\psi > 0$.

3. The portfolio consists solely of the safer asset, i.e., $a = 1$. This implies $\phi > 0$ and $\psi = 0$.

We consider these three cases in turn and evaluate the range of exogenous parameters such that each of them holds. We denote by π^H, derived below, the probability threshold after which investors choose only the riskier asset (case 2). The region for which there is investment for both assets is

$\pi \in [\pi^*, \pi^H\ )$ (case 1). Finally, π^L stands for the probability threshold after which it is profitable to invest in the safer asset. Thus, the portfolio consists solely of the safer asset for the region of $\pi \in [\pi^L, \pi^*\ )$ (case 3). Figure 1 presents the regions corresponding the three cases for a.

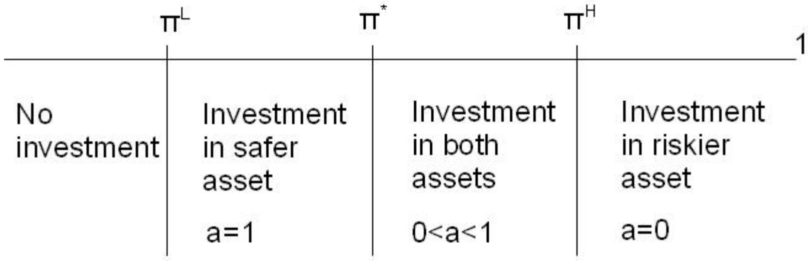

Figure 1: Probability regions and investment in assets

We start with the case that investors choose both assets, i.e., $0 < a < 1$, since we can easily derive a closed form solution to evaluate the corner solutions, which correspond to the other two cases. The first order condition with respect to the allocation, a, of borrowed funds yields

$$\frac{\mu_u}{\mu_d} = -\frac{X_d^L - X_d^H}{X_u^L - X_u^H}. \tag{8}$$

Combining equations 7 and 8 we can calculate the equilibrium borrowing rate, which is given by

$$R = \frac{X_u^H \cdot X_d^L - X_d^H \cdot X_u^L}{X_u^H - X_d^H - (X_u^L - X_d^L)} \tag{9}$$

It is easy to see from the market clearing condition 4 that, as the probability of a good outcome increases, the repayment rate decreases, i.e.,

$$\frac{\partial v}{\partial \pi} = \frac{\partial \frac{1}{1-\pi}\left(\frac{1}{R} - \pi\right)}{\partial \pi} = \frac{1}{(1-\pi)^2}\left(\frac{1}{R} - 1\right) < 0, \tag{10}$$

since $R > 1$; otherwise creditors would not break even and there would be no trade. We, thus, focus on values for asset payoffs that yield a gross rate greater than 1 in equation 9, i.e., $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$.

Next we need to show that the percentage of borrowed funds, $1-a$, invested in the riskier project increases with the probability of a good outcome. Substituting the budget constraints into the first order conditions with respect to consumption,

$$\mu_u = \pi(1 - 2\gamma\Pi_u) \tag{11}$$

and

$$\mu_d = (1-\pi)(1 - 2\gamma\Pi_d), \tag{12}$$

we can solve for a and w, which yields

$$a = \frac{(X_u^H - R)\left(1 - \frac{\mu_d}{1-\pi}\right) - (X_d^H - vR)\left(1 - \frac{\mu_u}{\pi}\right)}{(X_d^L - X_d^H)\left(1 - \frac{\mu_u}{\pi}\right) - (X_u^L - X_u^H)\left(1 - \frac{\mu_d}{1-\pi}\right)} \tag{13}$$

and

$$w = \frac{1}{2\gamma}\frac{1}{a(X_u^L - X_u^H) + X_u^H - R}\left(1 - \frac{\mu_u}{\pi}\right). \tag{14}$$

Equations 4, 5, 8, 9, 13, 14, together with 11 and 12 give the closed form solution for all endogenous variables.

Lemma 1: *As the probability of a good realization increases, investors reallocate their portfolio towards the riskier asset.*

See the Appendix for the proof.

The lemma above holds for any π. To verify that there exists probability regions such that investors choose both assets, the safer or only the riskier one, we need to prove that π^* and π^H are below one (and greater than zero) and that $\pi^* < \pi^H$. The following proposition establishes these two facts.

Proposition 1: *There are thresholds for the probability of the good state π^* and π^H, such that investors choose both assets for $\pi \in [\pi^*, \pi^H\ )$ and only the risker asset for $\pi \in [\pi^H, 1]$.* See the Appendix for the proof.

For proposition 1 we took the limit of equation 28 in the Appendix as $\pi \to 0$ to prove that there is a threshold π^* such that $a(\pi^*) = 1$. However, for $\pi < \pi^*$ investors invest only in the safer asset, thus $a = 1$ and $\phi > 0$. We, thus, need to show that there exist π^L greater than zero and lower than π^* such that the investor chooses just the safer asset for $\pi \in [\pi^L, \pi^*\ )$. We first establish that the equilibrium variables w and R (and hence consumption) are continuous at π^*.

For $\pi < \pi^*$ the solution for the amount of borrowing, the borrowing rate and the percentage default are given by

$$\frac{X_u^L - R}{X_d^L - \frac{1-\pi R}{1-\pi}} = \frac{1 - \frac{1-\pi}{\pi}\frac{R-X_d^L}{X_u^L-R}\lambda}{1-\lambda}, \tag{15}$$

$$\frac{\pi}{1-\pi}\frac{1-2\gamma w(X_u^L - R)}{\lambda} = \frac{R - X_d^L}{X_u^L - R}, \tag{16}$$

$$R = \frac{1}{\pi + (1-\pi)v}. \tag{17}$$

We evaluate these conditions as π approaches π^* from the left (denoted by π^{*-}) and compare them with those as π goes to π^* from the right (denoted by π^{*+}). In the latter region, investors invest in both assets, but as $\pi \to \pi^{*+}$, $a \to 1$. The equivalent of equations 15 and 16 as $\pi \to \pi^{*+}$ are

$$\frac{X_u^L - R}{X_d^L - \frac{1-\pi R}{1-\pi}} = \frac{1 - \frac{1-\pi}{\pi}\frac{X_d^L - X_d^H}{X_u^H - X_u^L}\lambda}{1-\lambda} \tag{18}$$

and

$$\frac{\pi}{1-\pi}\frac{1-2\gamma w(X_u^L - R)}{\lambda} = \frac{X_d^H - X_d^L}{X_u^L - X_u^H}. \tag{19}$$

Evaluating the first order conditions 7 and 8 as $\pi \to \pi^{*+}$ we get that $\dfrac{X_d^L - X_d^H}{X_u^H - X_u^L}$ approaches $\dfrac{R(\pi^{*+}) - X_d^L}{X_u^L - R(\pi^{*+})}$. From equations 15 and 18 we get that $R(\pi^{*-}) = R(\pi^{*+})$. Also, $v(\pi^{*-}) = v(\pi^{*+})$ and $w(\pi^{*-}) = w(\pi^{*+})$ from equations 16 and 19. Hence, there is no discontinuity in equilibrium variables when π crosses the threshold π^* and investors start investing in the riskier project as well.

Investors optimize at π^*. In addition, it is easy to show that utility is increasing at π for $\pi \in (0, \pi^*)$. The proof follows the same steps as in proposition 1. We, thus, need to show that there exists a probability $0 < \pi^L < \pi^*$ such that for $\pi < \pi^L$ the individual rationality of investors violates the participation constraint of creditors. Proposition 2 establishes this result.

Proposition 2: *There exists a probability threshold π^L greater than zero and lower that π^*, such that investors choose only the safer asset for $\pi \in \left[\pi^L, \pi^*\ \right)$.*

See the Appendix for the proof.

Corollary 1: *The rate of default, $1 - v$, is falling as investors become more optimistic for $\pi \in \left(\pi^L, \pi^*\right)$, then gradually increases for $\pi \in \left(\pi^*, \pi^H\right)$, and finally starts falling again for $\pi \in \left(\pi^H, 1\right)$.*

See the Appendix for the proof.

In the analysis above, the perceived probability of a good realization could change continuously. However, agents are Bayesian learners and they update their beliefs in discrete intervals as new information about payoff realizations arrives. The perceived probability of a good realization will exhibit jumps as new information arrives. Moreover, investors can accumulate profits over time, which affects their risk-taking behaviour and the resulting borrowing rates at each point in time. We examine these issues in section 4 where we present a calibrated example of the dynamic model.

The following section discusses the externality induced by optimism and risk-taking in the static framework presented above. The nature of the externality provides intuition about potential regulatory interventions, which are discussed in section 5 for the case of the calibrated example.

3.2 Social Planner's Solution

The ability to default twists investors' preferences and allows them to take on more downside risk. As shown in the previous section, they may do so by starting to invest in the riskier project when $\pi > \pi^*$. Consequently, they will default more. This introduces a pecuniary externality along the lines of Stiglitz (1982) and Korinek (2011). Investors are price takers and do not take into account the effect that their default decision has on the equilibrium borrowing rate. Once they start investing in the riskier project, the borrowing rate can stop being a decreasing function of the probability of a good realization (corollary 1). Investors increase their downside risk by investing more in the riskier project and are charged with a higher rate than otherwise. Investors do not factor in the effect of their default on borrowing rates, and thus, on aggregate default and the deadweight loss/disutility associated with it.[7]

We consider a social planner who takes into account the effect of her decisions on the borrowing rate and aggregate default. The social planner's objective is to maximize the utility of the representative investor by choosing the level of investment, the allocation between the safer and the riskier assets, the rate of default, as well as the borrowing rate, which in the context of the social planner's equilibrium should be thought as the promised return to creditors. The social planner is otherwise constrained in her decisions by the same payoff structure, the same penalties for default and the

[7]The pecuniary externality does not affect creditors' welfare in equilibrium, since they are risk-neutral and break even in expectation. This will not be the case with risk-averse creditors.

borrowing contract she can write with creditors, which cannot be state-contingent. The social planner will try to minimize the deadweight loss from default and she will return the whole investment payoff to creditors in the event of a bad realization. The consumption of investors in the bad state will, thus, be zero. Note that the social planner still defaults on creditors in the bad stare, since the payoffs of both assets L and H are less than one and creditors demand an interest rate higher than one to break even.

The social planner sets the return to creditors equal to $R^{sp} = \dfrac{1-(1-\pi)\left[a^{sp}\left(X_d^L - X_d^H\right) + X_d^H\right]}{\pi}$, such that they break even for any level of investment allocation, a^{sp}. This constraint should always hold with equality and we substitute it directly into the optimization problem. Due to risk-neutral creditors and homogeneous risk-averse investors, aggregation is easy and the social planner maximizes the utility of the representative investor, i.e.,[8]

$$\max_{\Pi_u^{sp}, w^{sp}, a^{sp}} \mathbb{U}^{sp} = \pi\Pi_u^{sp} - \gamma\cdot\pi\Pi_u^{sp2} - \lambda(1-\pi)w^{sp}\left[\frac{1-\left[a^{sp}\left(X_d^L - X_d^H\right) + X_d^H\right]}{\pi}\right],$$

subject to the budget constraints

$$\begin{aligned}
&\Pi_u^{sp} \leq a^{sp}\cdot w^{sp}X_u^L + (1-a^{sp})\cdot w^{sp}X_u^H - w^{sp}R^{sp} \\
\Rightarrow &\Pi_u^{sp} \leq w^{sp}\left[\frac{a^{sp}\left[\pi(X_u^L - X_u^H) + (1-\pi)(X_d^L - X_d^H)\right] + \pi X_u^H + (1-\pi)X_d^H - 1}{\pi}\right] \qquad (\mu_u^{sp}),
\end{aligned}$$

and the short-sale constraints $0 \leq a^{sp} \leq 1$.

The Lagrangian which the social planner maximizes is

$$\begin{aligned}
\mathbb{L}^{sp} = &\pi\Pi_u^{sp} - \gamma\cdot\pi\Pi_u^{sp2} - \lambda(1-\pi)w^{sp}\left[\frac{1-\left[a^{sp}\left(X_d^L - X_d^H\right) + X_d^H\right]}{\pi}\right] \\
&-\mu_u^{sp}\left[\Pi_u^{sp} - w^{sp}\left[\frac{a^{sp}\left[\pi(X_u^L - X_u^H) + (1-\pi)(X_d^L - X_d^H)\right] + \pi X_u^H + (1-\pi)X_d^H - 1}{\pi}\right]\right] \\
&-\phi^{sp}\left[a^{sp} - 1\right] + \psi^{sp}\cdot a^{sp}.
\end{aligned}$$

[8]The superscript *sp* is used to distinguish the equilibrium values from the ones in the competitive equilibrium.

The first order condition with respect to w^{sp} is

$$\mu_u^{sp}\left[a^{sp}\left[\pi(X_u^L - X_u^H) + (1-\pi)(X_d^L - X_d^H)\right] + \pi X_u^H + (1-\pi)X_d^H - 1\right] = \lambda(1-\pi)\left[1 - a^{sp}\left(X_d^L - X_d^H\right) - X_d^H\right]$$

$$\Rightarrow \mu_u^{sp} = \frac{\lambda(1-\pi)\left[1 - a^{sp}\left(X_d^L - X_d^H\right) - X_d^H\right]}{a^{sp}\left[\pi(X_u^L - X_u^H) + (1-\pi)(X_d^L - X_d^H)\right] + \pi X_u^H + (1-\pi)X_d^H - 1} \tag{20}$$

With respect to investment allocation, a^{sp}, the optimizing condition is

$$\mu_u^{sp}\frac{\pi\left(X_u^L - X_u^H\right) + (1-\pi)\left(X_d^L - X_d^H\right)}{\pi} + \lambda(1-\pi)\frac{X_d^L - X_d^H}{\pi} - \frac{\phi^{sp}}{w^{sp}} + \frac{\psi^{sp}}{w^{sp}} = 0. \tag{21}$$

Lemma 2: *The social planner chooses to invest:*

- *in both assets if* $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L = X_u^H - X_d^H - (X_u^L - X_d^L)$,
- *in the safer asset if* $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$,
- *in the riskier asset if* $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L < X_u^H - X_d^H - (X_u^L - X_d^L)$.

See the Appendix for the proof.

We turn to the solution under the assumption that $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$, i.e., $a^{sp} = 1$. The analysis is equivalent for $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L < X_u^H - X_d^H - (X_u^L - X_d^L)$. For $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L = X_u^H - X_d^H - (X_u^L - X_d^L)$, the equilibrium allocation, a^{sp}, is indeterminate. Thus, we will not consider this special case of measure zero.

For $a^{sp} = 1$, the equilibrium level of investment w^{sp} and the promised return to creditors are

$$w^{sp} = \frac{1}{2\gamma}\frac{1}{X_u^L - R^{sp}}\left(1 - \frac{\mu_u^{sp}}{\pi}\right) \tag{22}$$

$$R^{sp} = \frac{1 - (1-\pi)X_d^L}{\pi} \tag{23}$$

where

$$\mu_u^{sp} = \frac{\lambda(1-\pi)\left(1 - X_d^L\right)}{\pi X_u^L + (1-\pi)X_d^L - 1} \tag{24}$$

Proposition 3: *Assume that* $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$. *There exist probability thresholds* $\bar{\pi}^L$ *and* π^* *such that:*

- *Both competitive investors and the social planner invest only in the safer asset for* $\pi \in \left[\bar{\pi}^L, \pi^*\right]$.
- *Competitive investors gradually switch their investment towards the riskier asset for* $\pi > \pi^*$, *while the social planner continues investing in the safer one.*

See the Appendix for the proof.

The assumption about asset payoffs is made to satisfy the creditors' participation constraint in the competitive equilibrium, i.e., $R > 1$ (see equation 9), so that we can compare the solution in the competitive equilibrium with the social planner's solution for the whole range of $\pi' s$. R^{sp}, which is given by equation 23, satisfies the participation constraint for $\pi \leq 1$. The following lemmas establish the change in equilibrium variables in the social planner's solution.

Lemma 3: *Investors consume more in the good state and less in the bad one in the social planner's solution compared to the competitive solution for* $\pi \in \left(\pi^*, \pi^H\right)$.
See the Appendix for the proof.

Lemma 4: *The amount of borrowing increases, whereas the borrowing and default rates decrease in the social planner's solution compared to the competitive equilibrium for* $\pi \in \left(\pi^*, \pi^H\right)$.
See the Appendix for the proof.

We now prove one of our main claims; that the competitive equilibrium is Pareto inefficient, thus there is scope for policy intervention. We consider the cases for which $\pi > \pi^*$, i.e. $a < 1$. We restrict the percentage repayment, v, and set it as an exogenous variable. Our objective is to evaluate the change in investors' utility for small changes in v. We maintain the equilibrium values for consumption in the good and the bad state, and we allow the investment allocation, a, and the level of borrowing, w, to vary in order to neutralize the effect of v on consumption. Thus, the only effect will be on the disutility of default.

Proposition 4: *In a competitive equilibrium, investors shift their portfolio towards the riskier asset once expectations improve sufficiently. This creates an externality, since investors are price takers and do not factor in the impact of their portfolio choices on default and credit spreads in equilibrium. An exogenous restriction on the equilibrium level of default can result in a Pareto improvement.*
See the Appendix for the proof.

The proposition above showed that an exogenous decrease in the level of default can result in a Pareto improvement given that creditors are risk-neutral and break even in expectation. We proved this by keeping the level of consumption constant and varying the level and the allocation of investment. However, in the social planner's solution the level of consumption will be different from the competitive equilibrium levels as shown in lemma 3. Figure 2 reports the welfare gains in the social planner's solution measured as the compensating variations in terms of profits.[9] We have parameterized the model by setting $X_u^L = 1.4$, $X_d^L = 0.8$, $X_u^H = 2.1$, $X_d^H = 0.2$, $\lambda = 0.9$ and $\gamma = 0.035$. The parameters are such that the social planner chooses $a^{sp} = 1$ (lemma 2). The social planner's solution strictly dominates the competitive solution even for $\pi < \pi^*$ where there is only investment in the safer asset for both. The reason is that the social planner takes into consideration her impact on the deadweight loss and minimizes it. We also report the welfare gains when the social planner chooses the riskier asset to demonstrate that it is Pareto dominated.

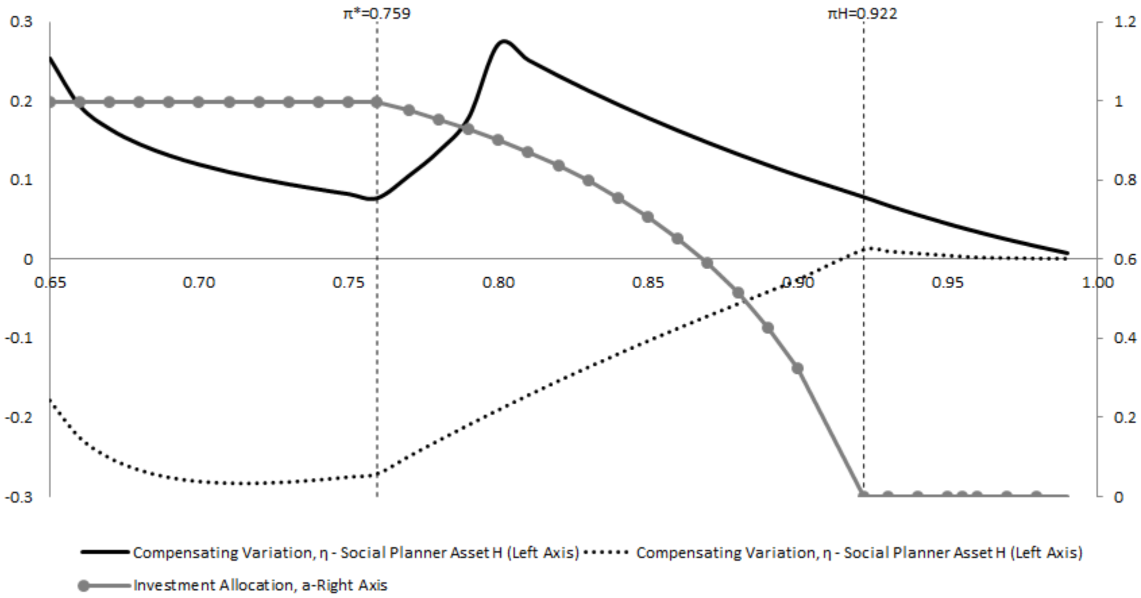

Figure 2: Welfare gains in social planner's solutions versus the probability of a good realization

[9]The figure shows the constant fraction of profits, η, at each date, and state that investors would need to be paid (or forgo) to get the same utility they would obtain in the social planner's solution. η is the solution to equation $\sum_t \mathbb{E}_t \left[\Pi_{t+1}^* (1+\eta) - \gamma \cdot \left(\Pi_{t+1}^* (1+\eta) \right)^2 - \lambda [(1 - v_{t+1}^*) w_t^* (1 + r_t^*), 0] \right] = \overline{\mathbb{U}}$, where the superscript * denotes the equilibrium values in the competitive solution and $\overline{\mathbb{U}}$ is the equilibrium level of utility for the social planner.

3.3 Externality from Optimism

The planner corrects for the externality that arises because investors do not take into account the impact of their default decision or portfolio allocation on the equilibrium borrowing rate and hence they tend to default too much and take excessive risk. The welfare gains presented above come from eliminating this externality. The planner is assumed to be imperfectly informed like investors. Hence, the planner eliminates the excessive risk-taking given the prevailing beliefs about the state of the world.

This risk-taking externality is present without incomplete information and learning as well. The fundamental question is whether optimism as a result of learning exacerbates the risk-taking externality and leads to even higher excessive risk. To answer this question we look at the wedge between the optimality conditions of investors and the planner. We show that the wedge does not depend only on the usual distortion arising from risk taking, but there is also a component that depends on the level of optimism measured by the portfolio loading on the riskier asset (lemma 1 establishes that a is a decreasing function of the good state's subjective probability).This has important policy implications, since the planner would need distinct tools to correct for both externalities associated with excessive risk-taking; the one emanating from endogenous default and endogenous borrowing rates and the other arising from optimism.

Given lemma 2, the optimality condition with respect to borrowing for the planner (equation 20) is written as:

$$\mu_u^{sp}\left[\pi X_u^L + (1-\pi)X_d^L - 1\right] + \lambda(1-\pi)\left(X_d^L - 1\right) = 0 \tag{25}$$

Let us now consider the competitive economy. Solving for v from the budget constraint in the down state and substituting in equation 20 we derive:

$$R = \frac{1-(1-\pi)\left[a\left(X_d^L - X_d^H\right) + X_d^H - \frac{\Pi_d}{w}\right]}{\pi}$$

Moreover, substituting the previous in the optimality condition 7 and using equation 5, we conclude:

$$\begin{aligned}&\mu_u\left[\pi X_u^L + (1-\pi)X_d^L - 1\right] + \lambda(1-\pi)\left(X_d^L - 1\right)\\ &-(1-\pi)\left[\mu_u + (1-\pi)\lambda\right]\frac{\Pi_d}{w} - (1-a)\left[\mu_u \cdot \pi\left(X_u^L - X_u^H\right) + (\mu_u+\lambda)(1-\pi)\left(X_d^L - X_d^H\right)\right] = 0\end{aligned} \tag{26}$$

As compared with the optimality condition 25 of the planner, equation 26 is distorted in two ways. The first distortion is caused by the term $(1-\pi)\left[\mu_u+(1-\pi)\lambda\right]\dfrac{\Pi_d}{w}$ and is associated with the excessive risk-taking because investors do not internalize the borrowing rate and the cost of default in their decisions. The second distortion is generated by investors when they start investing in the riskier asset as they become more optimistic. This in turn introduces a wedge $(1-a)\left[\mu_u\cdot\pi\left(X_u^L-X_u^H\right)+(\mu_u+\lambda)(1-\pi)\left(X_d^L-X_d^H\right)\right]$ in the optimality condition. Similarly, we can consider the two regions for good state's subjective probability defined in proposition 3. For $\pi\in\left[\bar{\pi}^L,\pi^*\right]$, the last term in equation 26 cancels out. The planner can decentralize her solution by imposing default penalties such that $\Pi_d=(1-\lambda)/(2\gamma)$ equals zero. Put differently, the default penalty can be adjusted so that investors fully internalize the risk-taking externality.

We hasten to add that this is not the case when optimism results in excessive risk-taking, i.e., for $\pi>\pi^*$. We see from equation 28 in the Appendix, that a can be less than 1 even for default penalties that eliminate the aforementioned externatility (i.e., $\lambda=1$). In particular, for the parametrization used in figure 2, $\dfrac{\partial a}{\partial\lambda}<0$, which means that whenever the planner increases the default penalty she exacerbates the externality stemming from optimism and its effect on risk-taking. Section 5 presents how regulation can change economic outcomes and what types of regulation result in a Pareto improvement using a calibrated example of our economy described in the following section.

The upshot of the argument is that the two externalities generated by default and optimism are not "collinear", or reinforcing each other in an identical fashion, and, hence, different policy tools are warranted to account for them.

4 Calibrated example

In this section, we calibrate the model to match some key characteristics of the period leading up to the subprime crisis of 2007-2008.[10] During this period financial institutions were increasing their holdings of high-risk mortgage-backed financial products, which promised a higher return if market conditions did not considerably deteriorate. However, elements of a crisis started to become visible during 2007 as delinquencies increased. In the first half of 2007 a number of institutions continued

[10]Our model is not specific to this crisis episode and can be used to study the connection between optimism, risk-taking and financial stability in other cases, such as the U.S. dot-com bubble in the end of the 20th century, or the Spanish/Irish housing crisis recently. We choose the subprime crisis because of its importance in the Great Recession that followed and because of the data availability to calibrate our model.

to increase their holdings of mortgage backed securities (MBS) and the stock market was improving until the end of July 2007. The events that unfolded after August 2007, including the collapse of two hedge funds of Bear Stearns, resulted in widespread distress in financial markets, big losses through the course of 2008 and an economic recession.

In terms of our model, the period of low volatility and sustainable growth until the summer of 2007 corresponds to realizations of good states of the world, which induce agents to improve their expectations given that it is more probable to be in the higher growth regime where the probability of a good realization is θ_1.[11] On the contrary, the events that unfolded thereafter and throughout 2008 correspond to bad realizations, which increase the weight put of the low growth regime θ_2 in forming expectations and most importantly result in losses and potentially default for levered financial institutions. We set the transition probabilities for high- and low-growth states, θ_1 and θ_2, to 0.98 and 0.48 respectively following Checchetti et al. (2000) and Cogley and Sargent (2008). We will calibrate the prior belief, $Pr(\theta = \theta_1)$, to match the initial portfolio holdings of the riskier asset observed in the data as discussed below.

Our objective is not to match the data over a long period, but rather to examine whether optimistic expectations can account for the increase in MBS held by financial institutions before the crisis and compute how the implied default rates evolve. Our model draws the distinction between realized and implied default rates during times of optimism. Default rates are low (or zero given that we consider two states of the world) during boom periods when optimism builds and, then, jump after the realization of bad news. This is consistent with our model, where realized low default rates result in optimism and excessive risk-taking. Should a bad shock realize, default is much more severe as suggested by the implied default rates at each point in time. We, thus, consider a three period version of our model where the first period corresponds to 2006, the second to 2007 and the third to 2008. The year of 2006 was a period of low volatility and low default rates. Financial institutions started increasing their holdings of MBS compared to other assets classes. Good market conditions and optimism continued in 2007 and financial institutions increased their holdings of MBS even further. The first signs of market slowdown appeared in August 2007 and became more

[11] Figure 8 in the Appendix reports the VIX index for the period leading up to the crisis. Market volatility was at historical low levels before the crisis and jumps quickly thereafter. We discuss in the conclusions how our model can generate this “volatility" paradox, which is also analyzed in Fostel and Geanakoplos (2012) and Brunnermeier and Pedersen (2014).

apparent in 2008 leading to the collapse of Lehman Brothers in September 2008. In terms of our model, the year 2007 corresponds to a good state realization and the year 2008 to a bad one. We will further assume that investors consume their private benefit in the case that the bad shock is realized in the second period and they chose to default, i.e. we will not consider the investment decisions that would have followed a bad realization in the second period. The reason is that we want to focus on paths present in the data and restrict ourself in calibrating the model. The model would allow for higher risk-taking if we considered additional paths, because the continuation value at the bad state in the second period would be higher.

We choose the riskier and safer holdings as well as the leverage to match the balance sheet of broker dealers' over this period taken from the U.S. Flow of Funds data. We choose broker dealers to represent investors in our model for two main reasons. First, broker dealers adjust their balance sheet faster and more dynamically when news arrive, as documented by Adrian and Shin (2009). Second, the returns for the asset classes they invest in can be computed by publicly available macro-level data. This exercise would be more complicated if we considered traditional banking institutions, because a large portion of their assets consists of bank specific loans for which data are not publicly available.

Since we consider a representative investor, we use aggregate asset holdings and liabilities for broker dealers. Table 1 shows the aggregate balance sheet of broker dealers for 2006 and 2007. The total financial assets are divided in five main categories: 1) cash, 2) credit market instruments, 3) equities, 4) security credit, and, 5) miscellaneous assets.

The credit market instruments and the equities on the asset side of the balance sheet provide information about the investment strategies of broker dealers, while the data on miscellaneous assets provide a proxy for investments by broker-dealer clients via their cash collateral positions (see Adrian et al. (2013) for a discussion of broker dealers' asset portfolios). Broker dealers holdings of MBS are assumed to correspond to investment in the riskier asset, w_t^H; while all other asset holdings (equities and remaining credit market instruments), which presumably correspond to a diversified portfolio, will capture the investment in the safer asset, w_t^L.[12] In the rest of the section, we describe how we compute the portfolio holdings of riskier and safer assets, as well as leverage, to match the

[12]We also exclude syndicated loans because of lack of data for their returns. This does not bias our analysis given that syndicated loans are a very small part of broker dealers' balance sheet.

Table 1: Assets & Liabilities of Security Brokers and Dealers as of the end of Q4/2006 and Q4/2007. The amounts are in billion of dollars, not seasonally adjusted (Source: U.S. Flow of Funds)

	2006	2007
Total Financial Assets		
Checkable deposits and currency	80.5	105.0
Credit market instruments	583.4	803.1
-Open market paper	64.3	87.1
-Treasury securities	-67.0	-85.0
-Agency- and GSE-backed securities	138.0	315.2
-Municipal securities and loans	50.9	50.1
-Corporate and foreign bonds	355.5	382.8
-Syndicated loans to nonfinancial corporate business	41.7	52.8
Equities	186.4	224.8
Security credit	292.1	325.5
Miscellaneous assets	1599.4	1633.7
Total liabilities and equity	2765.6	3131.3
Total liabilities	2669.1	3019.4
Equity	96.6	111.9
Leverage	28.6	28.0

data, and how we calibrate the remaining parameters in the model, which are the prior belief in the initial period, $Pr(\theta = \theta_1)$, the initial equity capital, $\bar{w}_0$, the function $f(\cdot)$, the default penalty, λ, the risk-aversion coefficient, γ, and the outside option of creditors, $r^{outside}$. We conclude by discussing the model predictions for the remaining endogenous variables.

Holdings of MBS by broker dealers are not broken-out separately in the Flow of Funds data, and are included in the "Agency- and GSE-backed securities" category. Following Greenlaw et al. (2008), we estimate the broker dealers' MBS holdings by extrapolating from the asset-backed securities in their balance sheet and the share of MBS in the total amount of outstanding asset-backed securities, which is available in the Flow of Funds. In particular, MBS were 57% and 60% of the total asset-backed securities in 2006 and 2007, respectively.

Table 2 shows the holdings for each asset class as a percentage of total investment in equities and credit market instruments in 2006 and 2007, together with the calibrated payoffs for a good and a bad state realization. We calibrate the payoffs for fixed-income instruments to their average yield over the preceding year, i.e. for the payoff in the good state (realized in 2007) we average monthly yields in 2006 and for the payoff in the bad state (realized in 2008) we use the 2007 monthly yield. For *corporate and foreign bonds* we take the average yield between AAA and BAA bonds taken from the FRED database. The three-month T-bill rate is used to calibrate the payoffs of *treasury securities*, while the payoffs on the *open market paper* holdings are calibrated using the 60-Day AA Financial Commercial Paper Interest Rate (both taken from FRED). We use the average of 2, 5, and 10 year Fannie Mae constant maturity par-yield to calibrate the payoffs on *asset-backed securities* other than MBS, while the Bond Buyer Go 20-Bond Municipal Bond Index, which uses 20 general obligation bonds with 20 year maturities and arithmetic average Moody's rating of A1, is used for *municipal securities and loans* (taken from Bloomberg).

The payoff of MBS in the good state is calibrated to the average 30-Year conventional mortgage rate (taken from FRED). The bad state in this calibrated example corresponds to the increased mortgage defaults and the drop in the market value of MBS, which induced considerable losses on leveraged institutions. We follow Greenlaw et al. (2008) and calibrate the payoff of MBS in the bad state of the world to match the drop in ABX index.[13] We use the ABX prices on June 11th, 2008, which was the lowest point for the S&P 500 before the collapse of Lehman Brothers. We want to exclude this event from our analysis, because it can be considered as an additional bad shock and because other mechanisms not present in our model, such as fire-sales dynamics, were potentially operating. We use the realized return on the S&P 500 index from January to June 11th, 2008 to calibrate the payoff on equity holdings in the bad state of the world, while the payoff in the good state is set to the realized return over 2007.

MBS will capture the investment in the riskier asset in our model, while the portfolio of the remaining assets will be used as a proxy for the safer asset. The payoffs for the safer asset are computed as the value-weighted average of the payoffs of the individual asset classes. Using these

[13] The Markit.com's ABX index represents a basket of credit default swaps linked to subprime mortgages. The indices are constructed by pooling mortgages with similar (initial) credit ratings. ABX indices differ by vintage and credit ratings. We will consider a weighted average of the five separate ABX indices based on the rating of the underlying security, from AAA to BBB-. We use the same weights as in Greenlaw et al. (2008), which rely on analysis from Moody's that maps subprime originations into a distribution of mortgage-backed securities with various credit ratings. In particular, 80.8% of all (subprime) mortgages are AAA, 9.6% are AA, 5.0% are A, 3.5% are BBB and 1.1% are BBB-. Finally, we will use the ABX 06-01 indices, which were launched in January 2006 and covered mortgages originated in 2005. These yield a weighted value of 0.79 or equivalently a 21% loss. If we additionally used the ABX 06-02 and ABX 07-01 indices for subsequent vintages the calibrated payoff would be 0.63, which would result in higher defaults and would require a higher degree of optimism to explain the actual holdings of MBS by broker dealers.

Table 2: % portfolio holdings and payoffs by asset class

	% holdings in 2006	% holdings in 2007	Payoff in good state	Payoff in bad state
MBS	**9.73%**	**17.51%**	**1.0641**	**0.7850**
Rest of portfolio	**90.27%**	**82.49%**	**1.0481**	**0.9927**
-Checkable deposits and currency	*9.95%*	*9.72%*	*1.0000*	*1.0000*
-Open market paper	*7.95%*	*8.07%*	*0.0795*	*0.0807*
-Treasury securities	*-8.29%*	*-7.87%*	*1.0473*	*1.0435*
-Other asset-backed securities	*7.34%*	*11.67%*	*1.0510*	*1.0488*
-Municipal securities and loans	*6.29%*	*4.64%*	*1.0440*	*1.0440*
-Corporate and foreign bonds	*43.97%*	*35.44%*	*1.0603*	*1.0602*
- Equities	*23.05%*	*20.81%*	*1.0443*	*0.8376*

data, the model yields borrowing rates of 4.4% for the first and second period using equation 9, which is lower than the average 3-Month London Interbank Offered Rate (LIBOR) during 2006 and 2007 (5.20% and 5.30%). We later evaluate the model predictions about the spread between the borrowing rate and the outside option of creditors by comparing it to the LIB-OIS spread, which is the spread between the 3-month LIBOR and the overnight index swap (OIS). Gorton and Metrick (2012) also use the LIB-OIS spread as a proxy for fears about financial institutions solvency.

The effective equity capital invested in the asset classes in table 2 can be approximated by dividing their total dollar amount by the leverage of broker dealers in 2006, shown in table 1, and is equal to 28.2 bn dollars. We rescale this amount and set the initial capital of investors $\bar{w}_0 = 0.3$. Given the initial capital, we compute the amount of total assets and investment in each asset class in the first period by using the leverage ratio from table 1 and the percentage holdings from table 2. Similarly, we compute the effective equity capital, total assets and investment by asset class in the second period using the respective ratios for 2007.

We set the following parameters, which are the prior of investors in the initial period, the risk-aversion coefficient, the default penalty, the function transforming retained earnings to available funds for investment, and the outside option of creditors, such that the model matches the investment in the riskier and safer assets in the initial and the second period, as well as the equity capital in the second period, which are observed in the data.[14] Table 3 below shows the calibrated parameter

[14] We assume that the function that transforms retained earnings to reinvested funds is $f(x) = \delta \cdot \sqrt{x}$ and we calibrate the parameter δ.

values as well as the portfolio allocations that we match to the data.

Table 3: Calibrated example

Calibrated parameters		Model predictions	
θ_1	0.980	π_0	0.968
θ_2	0.480	π_u	0.974
$Pr(\theta = \theta_1)$	0.975	r_0	0.044
γ	0.700	r_u	0.044
λ	0.843	Π_{uu}	0.505
δ	0.683	Π_{ud}	0.112
X_g^L	1.048	Π_d	0.112
X_b^L	0.993	w_0	8.292
X_g^H	1.064	w_u	11.030
X_b^H	0.785	v_d	0.952
Endogenous variables		v_{uu}	1.000
matched to data		v_{ud}	0.941
$\bar{w}_0$	0.300	LGD_d	0.414
w_0^L	7.756	LGD_{ud}	0.682
w_0^H	0.836	$r - r^{outside}$	16bps
$f(I_u)$	0.410	$\mathbb{E}_0(v_1)$	0.998
w_u^L	9.468	$\mathbb{E}_u(v_2)$	0.998
w_u^H	1.973		

Given this parameterization, our model makes the following predictions. The percentage default rate, $1 - v_d$ is 0.05 if the bad state occurs in the second period. The default rate is higher than this, at $1 - v_{ud} = 0.06$, when the bad state occurs in the final period following a previous good realization. This is due to the fact that investors increase their holdings of the riskier asset in the second period given that expectations are updated upward ($\pi_u > \pi_0$). This risk-taking is accompanied by higher borrowing, which results in much higher loss given default. However, creditors are willing to increase their lending to investors because their expectations are also updated upwards as indicated by the fact that expected percentage default is the same in the initial period and in the second period where the good state of the world realizes. Moreover, the model predicts a credit spread of 16 basis point, which is close to the steady value of 10 basis points for the LIBOR-OIS spread before the beginning of the crisis. Our analysis suggests that low expected default rates and credit spreads during good times are consistent with higher risk-taking and higher ex-post default when a bad shock hits.

As we argue in section 3.3, optimism induces an externality which results in excessive risk-taking over and beyond to what would be expected with just the limited liability assumption. The next section consider the calibrated economy described above and examines the ability of regulatory interventions to mitigate this externality and increase welfare.

5 Policy Responses

The driving force behind an increase in borrowing and increased risk taking is the optimism that comes after the realization of good news. The expectations formation mechanism is exogenous in our model and is implemented through Bayesian updating. Agents have imperfect information about the real world probability of a good state occurring and they try to infer it by observing past realizations. They are Bayesian learners. There is also no additional asymmetry of information. Every agent knows and observes the same information. Thus, regulation cannot control optimism in the markets. Agents are rational and none have more information than others. Regulation cannot affect optimism, but it can control its consequences by affecting the incentives and ability of investors to take on risk. Simplified as it is, our model can be used to evaluate regulatory policies to control borrowing and mitigate excessive risk-taking and default. The purpose of regulation is not to eliminate risk-taking altogether, but to discourage investors from taking excessive risk due to optimistic expectations. A first type of policy is to enforce more severe default penalties for investors, while a second is to control their leverage ratios. Finally, the regulator could impose direct limits on the amount of safer-to-riskier investment. We use the equilibrium outcomes calculated in section 4 to evaluate the effect of policy interventions on equilibrium variables and welfare.

The social planner's solution (see section 3.2) suggests that there can exist policy interventions which increase the welfare of investors, while not affecting creditors' welfare. The competitive equilibrium is Pareto suboptimal. We have shown in lemmas 3 and 4 that consumption is higher in the good and lower in the bad state in the social planner's solution, while the amount of borrowing is higher than in the competitive equilibrium. These equilibrium variables do not move in the same way as in the social planner's solution when stricter default penalties and leverage ratios are imposed in the competitive equilibrium (sections 5.1 and 5.2 respectively). Although they are successful in reducing borrowing and loss given default, they result in lower welfare for investors, while not changing the welfare of the creditors. We, thus, construct a different regulatory ratio, which better captures the social planner's solution. This policy ratio is equal to safer minus riskier holdings per unit of borrowing and captures the excessive risk-taking for leveraged investors. Imposing a stricter regulatory threshold for this ratio results in lower risk-taking and higher welfare. We discuss in section 5.3 how such a tool differs from risk-weighted capital regulation. Figure 3 presents the welfare gains/losses over the unregulated competitive equilibrium, measured as the compensating variations in terms of profit (η), for higher default penalties, stricter leverage requirements and stricter values for the regulatory ratio we propose (see footnote 9 for a definition of η).

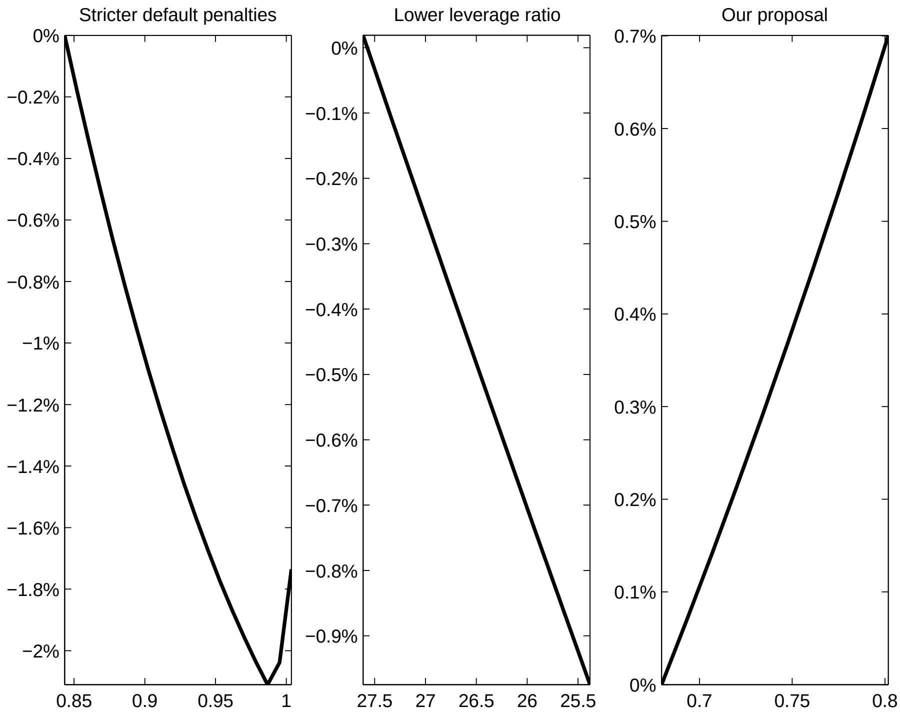

Figure 3: Regulation and Welfare: Compensating variations in terms of profit

5.1 Stricter Default Penalties

A way to correct for the risk-taking externality could be to make default more costly by setting a higher default penalty.[15] We argued in section 3.3 that such a policy would exacerbate the excessive

[15]Given limited liability, direct ways for the legal system or regulation to affect these penalties are lengthy bankruptcy processes or time-consuming investigations to discover fraudulent behaviour, in which case default penalties are higher

risk-taking due to optimistic expectations. In this section we discuss the mechanism through which higher default penalties operate using the calibrated economy as the benchmark equilibrium. Figure 4 in the Appendix reports the changes in selected endogenous variables as we increase the default penalty from its calibrated value to a value of one, which is the maximum value such that profits in the default states are positive (recall that the latter are given by $(1-\lambda)/2\gamma$).

The first order effect of increasing the default penalty would be a reduction in borrowing and lower profits in the default states, d and ud. The fact that investors are discouraged to leverage up as much as before also results in lower profits when the good state realizes in the final period. This creates an adverse effect whereby investors switch their portfolio holdings towards the riskier asset in both the initial and the second period to compensate for the loss in profits due to lower leverage. The reason for this risk-shifting is that the borrowing rate continues to be determined by equation 9 and, thus, investors are not penalized ex-ante with higher borrowing costs which would diminish the profit margin in state uu. Harsher penalties are effective in reducing the loss given default due to optimistic expectations, but they are doing so by discouraging borrowing rather than mitigating excessive risk-taking. They reduce profits in all states and they result in lower welfare (figure 3 above).

5.2 Leverage Requirements

Another policy response would be to restrict leverage, which is defined by $\frac{w_t^L + w_t^H}{f(I_t)}$ at each point in time. In our calibration, leverage is slightly higher in the initial period than in the good state in the second period (table 1). Thus, a time invariant leverage requirement would bind in both periods. The first-order effects of this regulation is to reduce borrowing in both periods, which can be balanced by an increase in own funds invested. However, a leverage requirement does not distinguish between the safer and the riskier projects, and the optimal portfolio decision continues to yield $\mu_{uu} = \frac{X_d^L - X_d^H}{X_u^H - X_u^L}\mu_{ud}$. Given that the default penalty is the same, μ_{ud} does not change with leverage requirements and, thus, investors are optimizing to maintain the same level of consumption in state uu. Since the ability to borrow is lower, this requires that the reinvested funds in the second

due to legal sanctions. Regulation can affect this component of default penalties and make default more costly during good times. An alternative way to impose penalties in a more quantifiable way is through renumeration reforms involving deferred managerial compensation, which would allow clawback of accrued past bonuses in the case of bad outcomes and default.

period increase or that investors deleverage internally by substituting the safer asset with the riskier one. Figure 5 shows how investment in the safer asset goes down together with the amount of borrowing, while at the same time riskier asset holdings increase in both periods. Since the leverage requirements binds in the initial period as well, investors will risk-shift in order to increase the reinvested profits in the second period and make up for the reduction in gearing. This results in higher default when a bad shock realized and the loss given default in state *ud* goes up, which result in lower welfare (figure 3 above).

The capital requirements in the Basel Accords try to avoid such a situation by imposing requirements on equity capital in relation to risk-weighted assets. The idea is that the capital requirement becomes tighter if banks invest in riskier assets. Both Basel I and II regulation have the feature that equity capital is higher after good realizations due to the higher returns from previous investments. However, when expectations become more optimistic, risk-weights based on internal models decrease and, hence, capital requirements are less likely to bind. This is true for both safer and riskier assets. Section 5.3 below discusses this drawback of risk-weighted capital requirement and examines a type of regulation that could tackle the externalities from optimism and excessive risk-taking more directly.

5.3 An alternative requirement

As expectations become more optimistic, investors switch towards the riskier investment. As shown in section 3.3, this creates an externality leading to excessive default relative to the social optimum, since investors do not factor in the impact that a riskier portfolio has on default. It is also true that higher investment in the safer asset and borrowing are desirable when the prospects of the economy improve (lemma 4). Regulation can provide incentives to investors to behave in a socially optimal way by restricting the level of borrowing that is shifted from safer to riskier investment. Such a requirement can be specified in variety of ways. Herein, we consider a requirement that is equal to the difference between safer and riskier holdings per unit of borrowed funds, i.e.,

$$w_u^L - w_u^H \geq \zeta \cdot w, \tag{27}$$

where ζ is the regulatory requirement.[16]

Stricter requirements of this type result in lower borrowing costs in the second period and a lower rate of default in state *ud*, since investors reduce their investment in the riskier asset (figure 6 in the Appendix). Moreover, this kind of regulation provides incentives to increase investment in the safer asset, in contrast to leverage regulation whereby investors would reduce borrowing and de-lever internally. Affecting behaviour in a way that resembles the social planner's solution, the regulatory constraint in equation 27 results in higher borrowing, but lower deadweight loss from default (LGD_{ud} decreases). In contrast to the prior types of regulation, this one addresses the externality from excessive risk-taking due to optimism more successfully and results in an increase in welfare (figure 3).

This type of regulation can be used as a countercyclical tool, because it will first bind when expectations become more optimistic and the relative investment in riskier projects goes up. Depending on the level on the regulatory minimum ζ it can discourage excessive risk-taking behavior in the early or later stages of the boom period during which expectations improve.

6 Summary and Concluding Remarks

The perceived risk profile of investment opportunities changes over time. Agents are Bayesian learners and update their beliefs about future realisations by observing the sequence of past ones. After a prolonged period of good news, expectations are boosted and investors find it profitable to shift their portfolios towards projects that are on average riskier, but promise higher expected returns. Creditors are willing to provide them with funds, since their expectations have improved as well. Consequently, the level of borrowing increases, risk premia do not increase and portfolios

[16] Basel II capital requirements are inadequate for this task, since (internal) model based risk-weights go down during good times (Catarineu-Rabell et al. (2005) and Pederzoli et al. (2010)). Figure 9 in the Appendix presents average risk weights, calculated as the ratio of the aggregate risk weighted assets over aggregate assets, for a panel of 33 international big banks. The panel includes the National Bank of Australia, ANZ, Macquarie, Dexia, China Merchants Bank, BNP Paribas, Credit Agricole, Societe Generale, Natixis, Deutsche Bank, Commerzbank, Unicredit, Monte dei Paschi, ING, Santander, BBVA, Nordea, SEB, Svenska Handelsbanken, UBS, Credit Suisse, Royal Bank of Scotland, Barclays, HSBC, Lloyds, Standard Charted, JP Morgan Chase, Citigroup, Bank of America, Wells Fargo, Bank of NY Mellon, State Street and PNC. Source: Bloomberg. Our proposal results in lower risk-taking accompanied by higher borrowing and higher investment in safer projects. Although relative risk is sometimes not accurately measured, for example when the top tranches of CDOs and MBSs were given too high a rating before the 2007 financial crisis, so that banks which were subject to an RWA, but not a leverage ratio, tended to expand their leverage enormously on the basis of such supposedly risk-free assets, which were not so. To account for such circumstances, our proposal could be augmented by a leverage restriction, in order not only to mitigate risk-taking, but also to reduce investment in projects that are mistakenly perceived to be safer.

comprise of relatively riskier projects. When bad news does occur, default is higher than it would otherwise be and the consequences for financial stability are more severe. This creates an externality, since investors do not take into account the level of default when making their portfolio decisions. We examine three regulatory tools to correct for the inefficiency caused by optimism and leverage, which are stricter default penalties for default, tighter leverage requirements and a novel criterion capturing the relative risk-taking per unit of borrowed funds. Our results suggest that only the latter results in a Pareto improvement.

Our analysis has empirical implications for the identification of points in the leverage cycle where there is a higher risk of future financial instability. In particular, we have constructed a theoretical model to highlight the variables that can potentially be used to develop an index, which could act as a leading indicator for financial distress. In our framework, as expectations become more optimistic due to good realizations, investors start investing in riskier projects and increase their borrowing. Although the loss given default increases under a riskier portfolio composition, expected default and credit spreads do not adjust commensurately. Thus, not only credit growth, but also portfolio switches to riskier projects should be used to identify the point in the leverage cycle in conjunction with low (ex-ante) risk premia.

An important element of the identification strategy would be expectations formation. The effectiveness of capturing time-varying transition probabilities between good and bad regimes should be the main objective in model selection for empirical work. One of the conjectures to be tested is that the riskiness of the financial system increases as people become more optimistic.

Measuring the riskiness of financial portfolios over the leverage cycle is not an easy task. As has been highlighted in this paper, although investors engage in riskier behavior after a period of good realizations, this results from expectations becoming more optimistic. The commonly used measures to capture risk build-up, such as the volatility of returns on assets or credit spreads, fail to do so due to the fact that they are biased by optimistic expectations. It is evident that market volatility as measured by the VIX index was below its long-term trend before the financial crisis. The same holds for the TED spread (figure 8 in the Appendix).[17]

The index, which we propose, is the difference between safer and riskier portfolio holdings per

[17] Various other indicators, such as credit growth to GDP or housing prices growth, have been proposed in the literature. For a broad overview that differentiates between system-wide and bank-specific variables, see Drehmann et al. (2010).

unit of leverage. Once expectations become optimistic riskier projects are perceived to be less risky; the same holds for safer ones, which are assessed as being even more safe. Although absolute riskiness goes down for both types their ranking is (as assumed) preserved. For example risk weighted assets (RWA) as defined by the Basel Accord II, under which risk weights follow an Internal Ratings Basel (IRB) approach and change over the cycle. We hasten to add that the literature on procyclicality has shown that all risk weights go down in good times, as empirical data also suggest (figure 9 in the Appendix). Thus, RWA do not increase as much as they should when banks shift their portfolio towards projects previously regarded as too risky. This procyclicality in measured risk is mitigated once we focus on the difference between projects with a lower and higher risk-weights, assuming that their relative rankings are preserved. Finally, we normalize by leverage, because it is default on debt that causes a financial crisis, a tightening of credit and forced liquidations. In figure 7 in the Appendix, we simulate our model for different levels of optimism and show how the proposed index can predict risk-taking and financial instability, whereas the more commonly used volatility measures fails. As a proxy for VIX we calculate the volatility of banking portfolios, which instead moves in the opposite direction. Our analysis suggests that quantity based measures, which capture the risk-taking behaviour of leveraged investors, could be valuable as leading indicators for subsequent financial instability.

References

Acharya, V.V. and A. Bisin (2014), 'Counterparty risk externality: Centralized versus over-the-counter markets', *Journal of Economic Theory* **149**, 153–182.

Acharya, V.V., D. Gale and T. Yorulmazer (2011), 'Rolloever risk and market freezes', *Journal of Finance* **66**(4), 1177–1209.

Adam, K. and A. Marcet (2010), 'Booms and busts in asset prices', *IMES Discussion Paper Series 10-E-02, Institute for Monetary and Economic Studies, Bank of Japan* .

Adrian, T., E. Etula and T. Muir (2013), 'Financial intermediaries and the cross-section of asset returns', *Federal Reserve Bank of New York Staff Reports, Number 464* .

Adrian, T. and H.S. Shin (2009), ‘Money, liquidity, and monetary policy’, *American Economic Review: Papers & Proceedings* **99**(2), 600–605.

Athreya, K., X.S. Tam and E.R. Young (2009), ‘Unsecured credit markets are not insurance markets’, *Journal of Monetary Economics* **56**(1), 83–103.

Bernanke, B. and M. Gertler (1989), ‘Agency costs, net worth and business fluctuations’, *American Economic Review* **79**(1), 14–31.

Bhattacharya, S. and D. Gale (1987), ‘Preference shocks, liquidity, and central bank policy’, *New Approaches to Monetary Economics: Proceedings of the Second International Symposium in Economic Theory and Econometrics, Cambridge University Press, Cambridge, New York, and Melbourne* pp. 69–88.

Boz, E. and E.G. Mendoza (2014), ‘Financial innovation, the discovery of risk, and the U.S. credit crisis’, *Journal of Monetary Economics* **62**, 1–22.

Branch, W.A. and G.W. Evans (2011), ‘Learning about risk and return: A simple model of bubbles and crashes’, *American Economic Journal: Macroeconomics* **3**(3), 159–191.

Brunnermeier, M.K. and L. Pedersen (2009), ‘Market liquidity and funding liquidity’, *Review of Financial Studies* **22**(6), 2201–2238.

Brunnermeier, M.K. and L. Pedersen (2014), ‘A macroeconomic model with a financial sector’, *American Economic Review* **104**(2), 379–421.

Catarineu-Rabell, E., P. Jackson and D.P. Tsomocos (2005), ‘Procyclicality and the new basel accord-banks’ choice of loan rating system’, *Economic Theory* **26**(3), 537–557.

Checchetti, S.G., P.-S. Lam and N.C. Mark (2000), ‘Asset pricing with distorted beliefs: are equity returns too good to be true?’, *American Economic Review* **90**(4), 787–805.

Cogley, T. and T.J. Sargent (2008), ‘The market price of risk and the equity premium: A legacy of the great depression’, *Journal of Monetary Economics* **55**, 454–476.

De Walque, G., O. Pierrard and A. Rouabah (2010), ‘Financial (in)stability, supervision and liquidity injections: A dynamic general equilibrium approach’, *The Economic Journal* **120**, 1234–1261.

Diamond, D.W. and P.H. Dybvig (1983), ‘Bank runs, deposit insurance, and liquidity’, *Journal of Political Economy* **91**(3), 401–419.

Diamond, D.W. and R.G. Rajan (2011), ‘Fear of fire sales, illiquidity seeking, and credit freezes’, *Quarterly Journal of Economics* **127**(2), 557–591.

Drehmann, M., C. Borio, L. Gambacorta, G. Jimenez and C. Trucharte (2010), ‘Countercyclical capital buffers: Exploring options’, *BIS Working Paper No.317* .

Dubey, P., J. Geanakoplos and M. Shubik (2005), ‘Default and punishment in general equilibrium’, *Econometrica* **73**(1), 1–37.

Fisher, I. (1933), ‘The debt-deflation theory of great depressions’, *Econometrica* **1**, 337–357.

Fostel, A. and J. Geanakoplos (2008), ‘Leverage cycles and the anxious economy’, *American Economic Review* **98**(4), 1211–1244.

Fostel, A. and J. Geanakoplos (2012), ‘Why does bad news increase volatility and decrease leverage?’, *Journal of Economic Theory* **147**(2), 501–525.

Geanakoplos, J. (2003), ‘Liquidity, default, and crashes: Endogenous contracts in general equilibrium’, *Advances in Economics and Econometrics: Theory and Applications, Eighth World Conference, Volume II, Econometric Society Monographs* pp. 170–205.

Geanakoplos, J. (2010), ‘The leverage cycle’, *NBER Macro Annual* pp. 1–65.

Goodhart, C.A.E, P. Sunirand and D.P. Tsomocos (2006), ‘A model to analyse financial fragility’, *Economic Theory* **27**, 107–142.

Gorton, G. and A. Metrick (2012), ‘Securitized banking and the run on repo’, *Journal of Financial Economics* **104**(3), 425–451.

Greenlaw, D., J. Hatzius, A. Kashyap and H.S. Shin (2008), ‘Leveraged losses: Lessons from the mortgage market meltdown’, *US Monetary Policy Forum Report No. 2.* .

Gromb, D. and D. Vayanos (2002), ‘Equilibrium and welfare in markets with financial constrained arbitrageurs’, *Journal of Finacial Economics* **66**(2-3), 361–407.

Kiyotaki, N. and J. Moore (1997), ‘Credit cycles’, *Journal of Political Economy* **105**(2), 211–248.

Korinek, A. (2011), ‘Systemic risk-taking: Amplifications effects, externalities and regulatory responses’, *ECB WP 1345* .

Kyle, A.S. and W. Xiong (2001), ‘Contagion as a wealth effect’, *Journal of Finance* **56**, 1401–1440.

Mendoza, E. G. (2006), ‘Lessons from the debt-deflation theory of sudden stops’, *American Economic Review* **96**(2), 411–416.

Mendoza, E. G. and K.A. Smith (2006), ‘Quantitative implications of a debt deflation theory of sudden stops and asset prices’, *Journal of International Economics* **70**, 82–114.

Minsky, H.P. (1992), ‘The financial instability hypothesis’, *Working Paper 74, Jerome Levy Economics Institute, Annandale on Hudson, NY* .

Pederzoli, C., C. Torricelli and D.P. Tsomocos (2010), ‘Rating systems, procyclicality and basel ii: an evaluation in a general equilibrium framework’, *Annals of Finance* **6**(1), 33–49.

Peiris, M.U. and A.P. Vardoulakis (2013), ‘Savings and default’, *Economic Theory* **54**(1), 153–180.

Reinhart, C.M. and K. Rogoff (2008), ‘Is the 2007 us sub-prime financial crisis so different? a international historical comparison’, *American Economic Review* **98**(2), 339–344.

Rochet, J.-C. and J. Tirole (1996), ‘Interbank lending and systemic risk’, *Journal of Money, Credit and Banking* **28**(4), 733–762.

Shleifer, A. and R.W. Vishny (1992), ‘Liquidation values and debt capacity: A market equilibrium approach’, *Journal of Finance* **47**, 1343–1366.

Shubik, M. and C. Wilson (1977), ‘The optimal bankruptcy rule in a trading economy using flat money’, *Journal of Economics* **37**(3-4), 337–354.

Stiglitz, J.E. (1982), ‘The inefficiency of the stock market equilibrium’, *Review of Economic Studies* **49**(2), 393–410.

Stiglitz, J.E. and A. Weiss (1981), ‘Credit rationing in markets with imperfect information’, *American Economic Review* **71**, 241–261.

Zame, W. (1993), ‘Efficiency and the role of default when security markets are incomplete’, *American Economic Review* **83**(5), 1142–1164.

Appendix

Proof of Lemma 1

Proof. It suffices to show that $\partial a/\partial\pi < 0$. After substituting, equation 13 becomes

$$a = \frac{(X_u^H - R)(1-\lambda) - (X_d^H - \frac{1-\pi R}{1-\pi})\left(1 - \frac{1-\pi}{\pi}\bar{A}\lambda\right)}{(X_d^L - X_d^H)\left(1 - \frac{1-\pi}{\pi}\bar{A}\lambda\right) - (X_u^L - X_u^H)(1-\lambda)}, \tag{28}$$

where $\bar{A} = \dfrac{X_d^L - X_d^H}{X_u^H - X_u^L} > 0$. The gross rate R is fixed by equation 9 and is greater than 1. Also, we choose $\lambda \leq 1$, otherwise agents would never choose to default as they are on-the-verge of defaulting when $\lambda = (1 - 2\gamma\Pi_d)$. The second term in the numerator is increasing with π, thus the numerator is decreasing. Moreover, the first term in the denominator is increasing in π, thus the denominator is increasing. Combining the two we get that $\partial a/\partial\pi < 0$. □

Proof of Proposition 1

Proof. From lemma 1 we know that $\partial a(\pi)/\partial\pi < 0$. Also, $a(\pi)$ is a continuous function of π for $0 < \pi < 1$. We need to show that there exist $\pi^{'}$ and $\pi^{''}$, where $\pi^{'} < \pi^{''}$, such that $a(\pi^{'}) > 1$ and $a(\pi^{''}) < 1$. Then, by the intermediate value theorem there exists π^*, $\pi^{'} < \pi^* < \pi^{''}$, such that $a(\pi^*) = 1$. The limit of $a(\pi)$ as $\pi \to 0$ is $\lim\limits_{\pi\to 0} a(\pi) = \lim\limits_{\pi\to 0} \dfrac{1 - X_d^H}{X_d^L - X_d^H} + \dfrac{\pi^2}{(1-\pi)^2}\dfrac{1-R}{(X_d^L - X_d^H)\bar{A}\lambda} = \dfrac{1 - X_d^H}{X_d^L - X_d^H} > 1$ given that $0 < X_d^H < X_d^L < 1$. Also, $\lim\limits_{\pi\to 1} a(\pi) = -\infty$ given that $R > 1$. The probability $\pi^*(X_d^L, X_u^L, X_d^H, X_u^H, \lambda)$ is given by setting equation 28 equal to 1 and solving for π^*.

In order to prove that $\pi^H < 1$, we just need to show that $a(\pi)$ crosses zero for π^H as π goes from π^* to one, since $a(\pi^*) = 1$ and $a(\pi)$ is continuous and decreasing. The RHS of 28 as $\pi \to 1$ goes to $-\infty$. Thus, there exists a threshold π^H greater than π^* and lower than one, such that the short sales constraint $a \geq 0$ is hit. For $\pi > \pi^H$ the complementary slackness condition implies that $a = 0$ and $\psi > 0$. The probability $\pi^H(X_d^L, X_u^L, X_d^H, X_u^H, \lambda)$ is given by setting equation 28 equal to 0 and solving for π^H.

To complete the proof, we need to show that investors do actually prefer the riskier asset for $\pi \in \left[\pi^H, 1\right]$ over their outside option, which is zero investment and borrowing yielding a utility value of zero. We first show that the equilibrium variables are continuous at π^H as investors choose $a = 0$. For $\pi > \pi^H$ the solution for the amount of borrowing, the borrowing rate and the percentage default are given by

$$\frac{X_u^H - R}{X_d^H - \frac{1-\pi R}{1-\pi}} = \frac{1 - \frac{1-\pi}{\pi}\frac{R-X_d^H}{X_u^H-R}\lambda}{1-\lambda}, \tag{29}$$

$$\frac{\pi}{1-\pi}\frac{1-2\gamma w(X_u^H - R)}{\lambda} = \frac{R - X_d^H}{X_u^H - R}, \tag{30}$$

$$R = \frac{1}{\pi + (1-\pi)v}. \tag{31}$$

We evaluate these conditions as π approaches π^H from the left (denoted by π^{H-}) and compare them with those as π goes to π^H from the right (denoted by π^{H+}). In the former region, investors invest in both assets, but as $\pi \to \pi^{H-}$, $a \to 0$. The equivalent of equations 29 and 30 as $\pi \to \pi^{H-}$ are

$$\frac{X_u^H - R}{X_d^H - \frac{1-\pi R}{1-\pi}} = \frac{1 - \frac{1-\pi}{\pi}\frac{X_d^L-X_d^H}{X_u^H-X_u^L}\lambda}{1-\lambda} \tag{32}$$

and

$$\frac{\pi}{1-\pi}\frac{1-2\gamma w(X_u^H - R)}{\lambda} = \frac{X_d^H - X_d^L}{X_u^L - X_u^H}. \tag{33}$$

Evaluating the first order conditions 7 and 8 as $\pi \to \pi^{H-}$ we get that $\dfrac{X_d^L - X_d^H}{X_u^H - X_u^L}$ approaches $\dfrac{R(\pi^{H-}) - X_d^H}{X_u^H - R(\pi^{H-})}$. From equations 29 and 32 we get that $R(\pi^{H+}) = R(\pi^{H-})$. Also, $v(\pi^{H+}) = v(\pi^{H-})$ and $w(\pi^{H+}) = w(\pi^{H-})$ from equations 30 and 33. Hence, there is no discontinuity in equilibrium variables when π crosses the threshold π^H and investors choose only the riskier asset.

Investors optimize at π^H. Given the continuity in equilibrium variables it suffices to show that investors' utility is increasing as the probability π goes from π^H to one. The deadweight loss from defaulting in the bad state, $(1-\pi)(1-v)\lambda wR$, can be written as $\lambda w(R-1)$ given that $v = \dfrac{1-\pi R}{R(1-\pi)}$. Investors utility can be written as

$$U = \pi\left(\Pi_u - \gamma\Pi_u^2\right) + (1-\pi)\left(\Pi_d - \gamma\Pi_d^2\right) - \lambda w(R-1). \tag{34}$$

Its derivative with respect to π is

$$\frac{\partial U}{\partial \pi} = \Pi_u - \gamma\Pi_u^2 + \pi(1-2\gamma\Pi_u)\frac{\partial \Pi_u}{\partial \pi} - \left(\Pi_d - \gamma\Pi_d^2\right) - \lambda\frac{\partial\left(w(R-1)\right)}{\partial \pi}, \tag{35}$$

since Π_d is fixed at $\dfrac{1-\lambda}{2\gamma}$. Moreover, $\Pi_u > \Pi_d$, since investors do not default in the good state, and $\pi(1-2\gamma\Pi_u) = \mu_u > 0$. To evaluate $\dfrac{\partial \Pi_u}{\partial \pi}$, we will use equation 7 for $a = 0$ and the derivative of R with respect to π.

For $\pi > \pi^H$, total differentiate equation 29 and recall that $vR = \dfrac{1-\pi R}{1-\pi}$. Then, we get

$$
\begin{aligned}
-dR(1-\lambda) = & -dvR + \lambda X_d^H \frac{1}{\pi^2} dR \left[\frac{(R - X_d^H) + (X_u^H - R)}{(X_u^H - R)^2} \right] \\
& + dvR \frac{1-\pi}{\pi} \frac{R - X_d^H}{X_u^H - R} \lambda - vR\lambda \frac{1}{\pi^2} dR \left[\frac{(R - X_d^H) + (X_u^H - R)}{(X_u^H - R)^2} \right]
\end{aligned}
$$

$$
\Rightarrow dR \left[1 - \lambda + \frac{\pi}{(1-\pi)^2} + \lambda \frac{1}{\pi^2} (X_d^H - vR) \left[\frac{(R - X_d^H) + (X_u^H - R)}{(X_u^H - R)^2} \right] \right] = - \left[1 - \frac{1-\pi}{\pi} \frac{R - X_u^H}{X_u^H - R} \lambda \right].
$$

Thus, $dR/d\pi < 0$ for $\pi > \pi^H$. Thus, equation $\mu_u \left(X_u^H - R \right) + \mu_d \left(X_u^L - R \right) = 0$ implies that $\dfrac{\partial \mu_u}{\partial \pi} = \dfrac{\partial \left(\pi (1 - 2\gamma \Pi_u) \right)}{\partial \pi} < 0 \Rightarrow \dfrac{\partial \Pi_u}{\partial \pi} > \dfrac{1 - 2\gamma \Pi_u}{2\gamma \pi} > 0$. Finally, $w = \dfrac{1-\lambda}{2\gamma} \dfrac{1}{X_d^H - \frac{1-\pi R}{1-\pi}}$, thus $\dfrac{\partial \left(w(R-1) \right)}{\partial \pi} = \dfrac{1-\lambda}{2\gamma} \dfrac{\left(X_d^H - \frac{1-\pi R}{1-\pi} \right) - (R-1)\frac{\pi}{1-\pi}}{\left(X_d^H - \frac{1-\pi R}{1-\pi} \right)^2} = \dfrac{X_d^H - 1}{\left(X_d^H - \frac{1-\pi R}{1-\pi} \right)^2} < 0$. Combining the above we get that investors utility is increasing at π, for $\pi > \pi^H$. □

Proof of Proposition 2

Proof. The optimizing behaviour of investors yields equation 7, which, for $a = 1$, becomes

$$
\mu_u \left(X_u^L - R \right) + \mu_d \left(X_d^L - R \right) = 0 \Rightarrow R = \frac{\mu_u X_u^L + \mu_d X_d^L}{\mu_u + \mu_d}
$$

The participation constraint of the creditors requires $R > 1$, thus $\dfrac{\mu_u}{\mu_u + \mu_d} X_u^L + \dfrac{\mu_d}{\mu_u + \mu_d} X_d^L > 1$ or

$$
\frac{\pi u'(\Pi_u)}{\pi u'(\Pi_u) + (1-\pi) u'(\Pi_d)} X_u^L + \frac{(1-\pi) u'(\Pi_d)}{\pi u'(\Pi_u) + (1-\pi) u'(\Pi_d)} X_d^L > 1. \tag{36}
$$

Investors optimize when consumption in the good state is higher than in the bad one otherwise they would default in the good state as well. This implies that $\dfrac{\pi u'(\Pi_u)}{\pi u'(\Pi_u) + (1-\pi) u'(\Pi_d)} < \pi$ and $\dfrac{(1-\pi) u'(\Pi_d)}{\pi u'(\Pi_u) + (1-\pi) u'(\Pi_d)} > (1-\pi)$. Thus, a necessary, but not sufficient, condition for inequality 36 to hold is

$$
\pi X_u^L + (1-\pi) X_d^L > 1 \Rightarrow \pi > \frac{1 - X_d^L}{X_u^L - X_d^L}. \tag{37}
$$

For $\pi < \dfrac{1 - X_d^L}{X_u^L - X_d^L}$ either the participation constraint of creditors is violated or investors' individual rationality is not satisfied. Hence, there is no investment and investors are left with their outside option, which yields a utility value of zero.

Moreover, $\dfrac{\partial R}{\partial \pi} < 0$ for $\pi \in \left(\dfrac{1 - X_d^L}{X_u^L - X_d^L}, \pi^* \right)$. This is obtained by total differentiating equation 15.

Also, investors' utility is increasing at π. The proof is equivalent to the one outlined in proposition 1. Finally, investors optimize at π^* with positive investment in the safer asset. Hence, there exists a $\pi^L \in \left(\frac{1-X_d^L}{X_u^L - X_d^L}, \pi^*\right)$ such that investors start investing in the safer asset for $\pi > \pi^L$. The probability threshold π^L is computed by setting investors' indirect utility to the outside option, i.e., zero, and solving for π. The equilibrium solution is obtained by solving equations 15, 16 and 17 for the endogenous variables, and requiring that $R > 1$. □

Proof of Corollary 1

Proof. We showed in propositions 1 and 2 that $dR/d\pi < 0$ for $\pi \in (\pi^L, \pi^*)$ and $\pi \in (\pi^H, 1)$. Also, $v = \frac{1-\pi R}{R - \pi R}$. The numerator increases faster than the denominator as π increases and $dv/d\pi > 0$ for $\pi \in (\pi^L, \pi^*)$ and $\pi \in (\pi^H, 1)$. Combining this with equation 10, which holds for $\pi > \pi^*$, and the fact that $v(\pi)$ is continuous at π^* we get the desired result.

□

Proof of Lemma 2

Proof. Assume first that $0 < a^{sp} < 1$, hence $\phi^{sp} = \psi^{sp} = 0$. Combining equations 20 and 21 we get that:

$$\begin{aligned}
&\frac{\left(\pi\left(X_u^L - X_u^H\right) + (1-\pi)\left(X_d^L - X_d^H\right)\right)\cdot\left(1 - a^{sp}\left(X_d^L - X_d^H\right) - X_d^H\right)}{a^{sp}\left[\pi(X_u^L - X_u^H) + (1-\pi)(X_d^L - X_d^H)\right] + \pi X_u^H + (1-\pi)X_d^H - 1} + X_d^L - X_d^H = 0 \\
\Rightarrow &\pi\left(X_u^H \cdot X_d^L - X_u^L \cdot X_d^H - X_u^H + X_d^H + X_u^L - X_d^L\right) = 0
\end{aligned}$$

Thus, there is an interior solution for a only if $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L = X_u^H - X_d^H - (X_u^L - X_d^L)$.

We now turn to the two corner solutions $a^{sp} = 1$ and $a^{sp} = 0$. Consider that the social planner chooses to invest in asset L. Then, $a^{sp} = 1$, $\phi^{sp} > 0$ and $\psi^{sp} = 0$. Equations 20 and 21 yields that:

$$\frac{\left(\pi\left(X_u^L - X_u^H\right) + (1-\pi)\left(X_d^L - X_d^H\right)\right)\cdot\left(1 - X_d^L\right)}{\pi X_u^L + (1-\pi)X_d^L - 1} + X_d^L - X_d^H = \pi\frac{\phi^{sp}}{w^{sp}} > 0$$

$$\pi\left(X_u^H \cdot X_d^L - X_u^L \cdot X_d^H - X_u^H + X_d^H + X_u^L - X_d^L\right) > 0 \tag{38}$$

For $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L > X_u^H - X_d^H - (X_u^L - X_d^L)$, equation 38 is satisfied for all π. Similarly, we can show that $\psi^{sp} > 0$ only if $X_u^H \cdot X_d^L - X_d^H \cdot X_u^L < X_u^H - X_d^H - (X_u^L - X_d^L)$. □

Proof of Proposition 3

Proof. The second part of the proposition derives for proposition 1 and lemma 2. To show the first part, we need to compute the probability threshold such that the social planner chooses to invest in the safer asset. Denote this by π^{Lsp}. Then, $\bar{\pi}^L = \max\left\{\pi^L, \pi^{Lsp}\right\}$, where π^L is given by proposition 2. The Lagrange multipliers $\mu_u^{sp} = \pi(1 - 2\gamma\Pi_u^{sp})$ needs to be positive. This is satisfied for $\pi > \pi^{Lsp} = \frac{1-X_d^L}{X_u^L - X_d^L}$.

□

Proof of Lemma 3

Proof. The result for the bad state is obvious, as investors enjoy a fixed consumption in the competitive equilibrium, which is pinned down by the default penalty and their risk-aversion, while in the social planner's solution consumption is zero. Regarding the good state, it suffices to show that $\mu_u > \mu_u^{sp}$ for every $\pi > \pi^*$, where π^* is given by equation 13 for $a = 1$.

We prove that $\mu_u^{sp} < \mu_u$ by construction. The last inequality can be reduced to $\left(\pi \frac{X_u^L - X_d^L}{1 - X_d^L} - 1\right)^{-1} < \frac{R - X_d^L}{X_u^L - R}$ where R is given by equation 9. The left hand side (LHS) of the inequality is defined for $\pi > \underline{\pi} = \frac{1 - X_d^L}{X_u^L - X_d^L}$, since μ_u^{sp} should be positive. The limit of the LHS as $\pi \to \underline{\pi}^+$ is $+\infty$, while the limit as $\pi \to 1$ is $\frac{1 - X_d^L}{X_u^L - 1} < \frac{R - X_d^L}{X_u^L - R}$, since $R > 1$. Given that $\frac{\partial}{\partial \pi} LHS < 0$, there exists a π^{**} such that $\mu_u^{sp} < \mu_u$ for $\pi > \pi^{**}$. It remains to show that $\pi^{**} < \pi^*$. Substitute $\pi^{**} = \frac{1 - X_d^L}{R - X_d^L}$ in equation 13 and assume that $a(\pi^{**}) < 1$, which reduces to $(-1 + \lambda)(X_u^L - R) > 0$, a contradiction. Thus, $a(\pi^{**}) > 1$. In combination with lemma 1 we get that $\pi^{**} < \pi^*$. For $\pi < \pi^*$ and $\pi > \pi^H$, we do not have an analytical solution for the equilibrium variables and need to resort to numerical approximations to compare equilibrium consumption. □

Proof of Lemma 4

Proof. In the social planner's solution, creditors receive X_d^L in the bad state of the world. In the competitive equilibrium, the repayment is strictly less irrespective of the value for a given that investors enjoy a private benefit, i.e. creditor receive $aX_d^L + (1 - a)X_d^H - \frac{1 - \lambda}{2\gamma}$ in the bad state. Thus, the percentage repayment for a unit of borrowed funds is lower and percentage default and the borrowing rate are higher in the competitive equilibrium. Combining this result with equations 14, 22 and lemma 3 we get that $w^{sp} > w$. □

Proof of Proposition 4

Proof. Consumption in the good and the bad state are given by $\Pi_u = w\left[a(X_u^L - X_u^H) + X_u^H - R\right]$ and $\Pi_d = \left[a(X_d^L - X_d^H) + X_d^H - \frac{1 - \pi R}{1 - \pi}\right]$, respectively. Taking the total derivative with respect to v-recall that $\frac{\partial R}{\partial v} = -(1 - \pi)R^2$-and setting $\frac{\partial \Pi_u}{\partial v} = \frac{\partial \Pi_d}{\partial v} = 0$ we get that:

$$\frac{\partial a}{\partial v} = \frac{\pi \Pi_u + (1 - \pi)\Pi_d}{(X_u^H - X_u^L)\Pi_d + \left(X_d^L - X_d^H\right)} \quad \text{and} \quad \frac{\partial w}{\partial v} = -w \frac{\frac{\partial a}{\partial v}\left(X_d^L - X_d^H\right) - \pi R^2}{a\left(X_d^L - X_d^H\right) + X_d^H - \frac{1 - \pi R}{1 - \pi}}.$$

$\frac{\partial a}{\partial v} > 0$, thus investors have to reduce investment in the riskier asset for a higher delivery. The effect on borrowing depends on the payoffs of assets L and H. If $R^2 > \frac{X_d^L - X_d^H}{X_u^H - X_u^L}$, then $\frac{\partial w}{\partial v} > 0$, otherwise $\frac{\partial w}{\partial v} < 0$. The result is intuitive. If the spread of the two payoffs in the good state is higher than in the bad, i.e. $\frac{X_d^L - X_d^H}{X_u^H - X_u^L} < 1$, then investors have to compensate for the reduction in risky investment with higher borrowing in the safer asset. For $\frac{\partial w}{\partial v} < 0$, it is obvious that an exogenous increase of v results in a lower deadweight loss of default and thus higher welfare given that consumption in both states in preserved. For $\frac{\partial w}{\partial v} > 0$, the deadweight loss of default can be written as $\lambda w(R-1)$ and its derivative with respect to v is $= \lambda \frac{\partial w}{\partial v}(R-1) - w(1-\pi)R^2 =$ $\lambda w \left[\frac{(R-1)\left(\pi R^2 - \frac{\pi \Pi_u + (1-\pi)\Pi_d}{(X_u^H - X_u^L)\Pi_d + \left(X_d^L - X_d^H\right)} \left(X_d^L - X_d^H\right)\right)}{a\left(X_d^L - X_d^H\right) + X_d^H - \frac{1-\pi R}{1-\pi}} (1-\pi) \right]$. The last expression is increasing in π. Denote by $\bar{\bar{\pi}}$, the probability that the derivative becomes zero. It is easy to show that $\bar{\bar{\pi}} < 1$. Then for $\pi \in (\pi^*, \bar{\bar{\pi}})$, an increase in v results in a lower deadweight loss and higher welfare, while for $\pi > \bar{\bar{\pi}}$ this is not the case. □

FIGURES

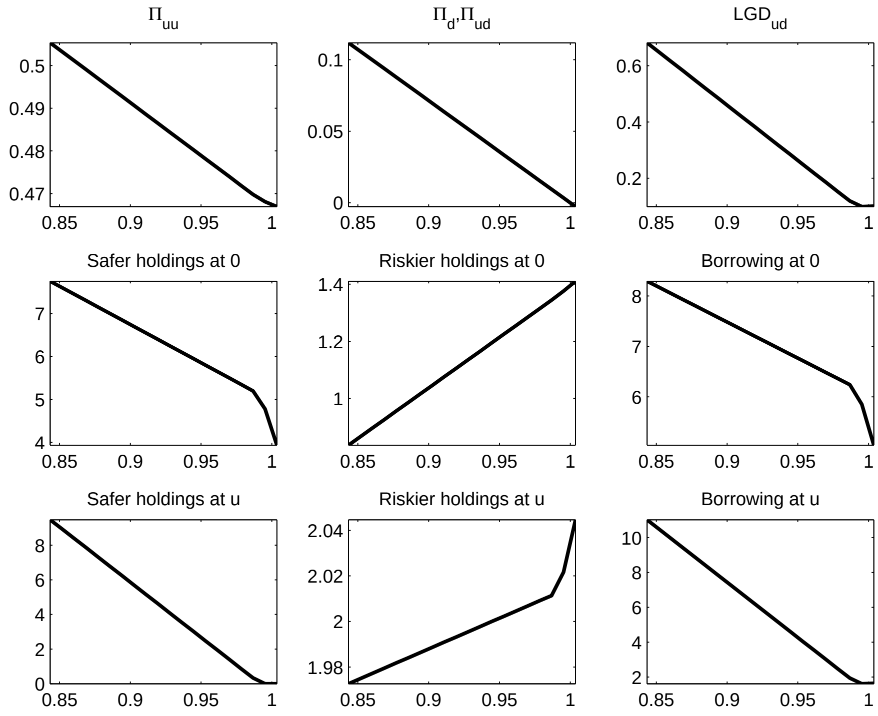

Figure 4: Change in selected variables as the default penalty (x-axis) increases from its value in the benchmark equilibrium to one.

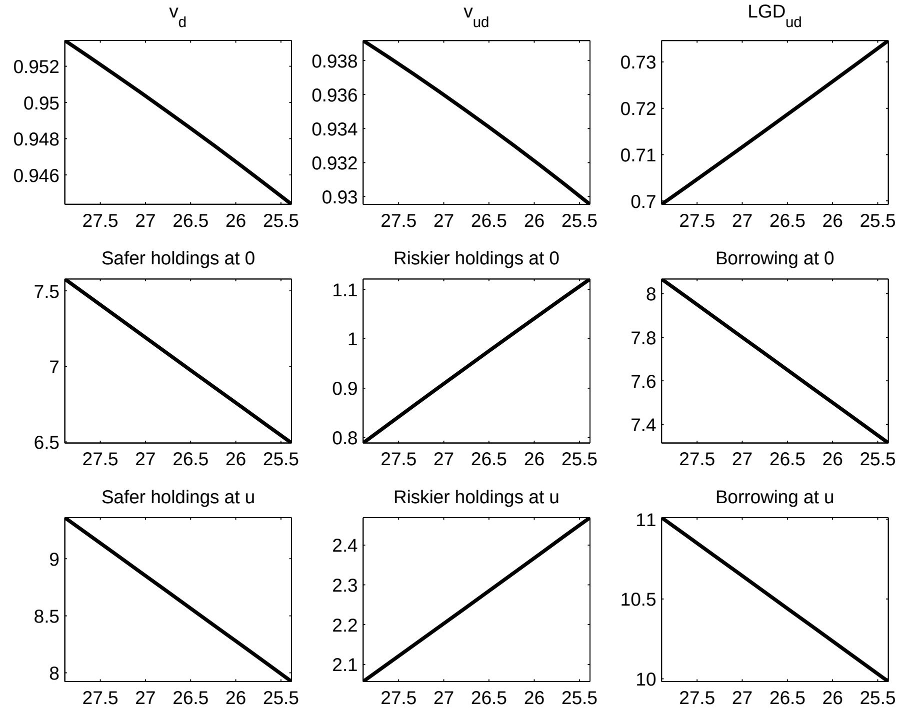

Figure 5: Change in selected variables as the leverage requirement (x-axis) tightens.

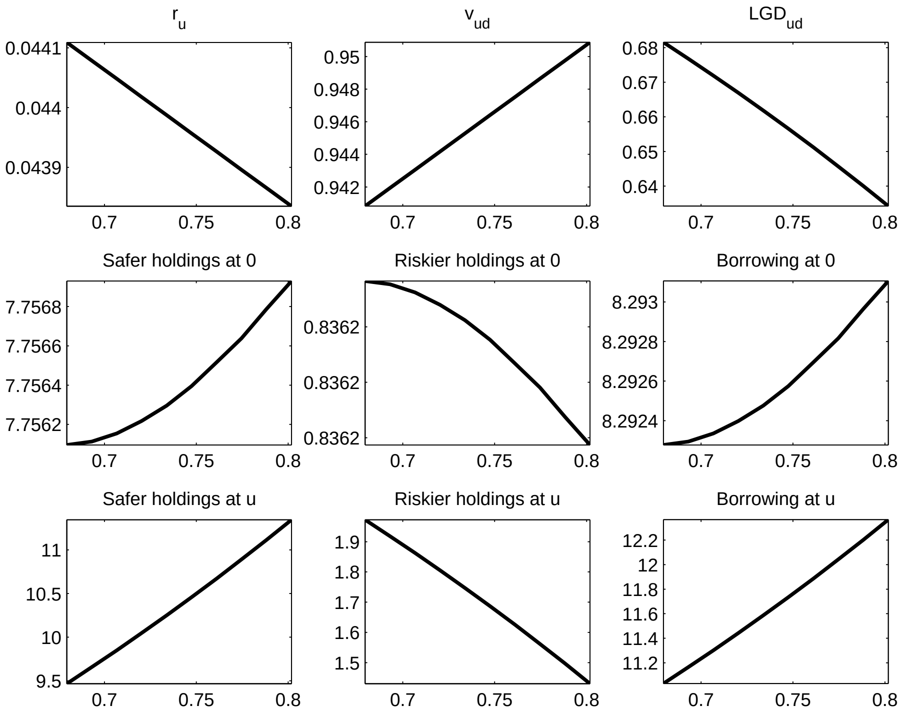

Figure 6: Change in selected variables as ζ (x-axis) increases.

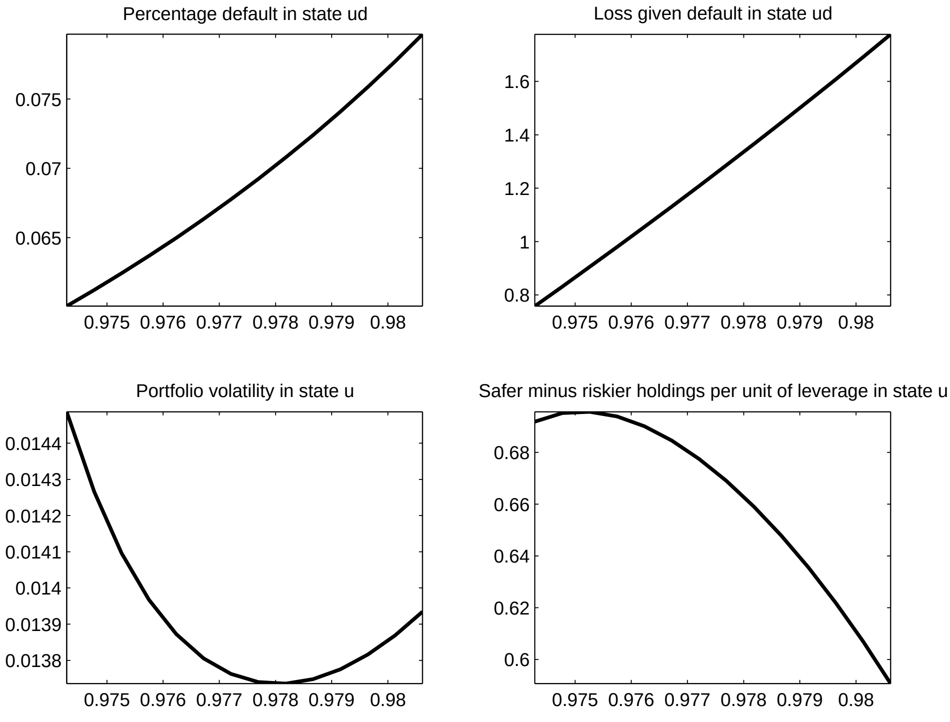

Figure 7: Change in default, volatility and riskiness as π_u (x-axis) increases.

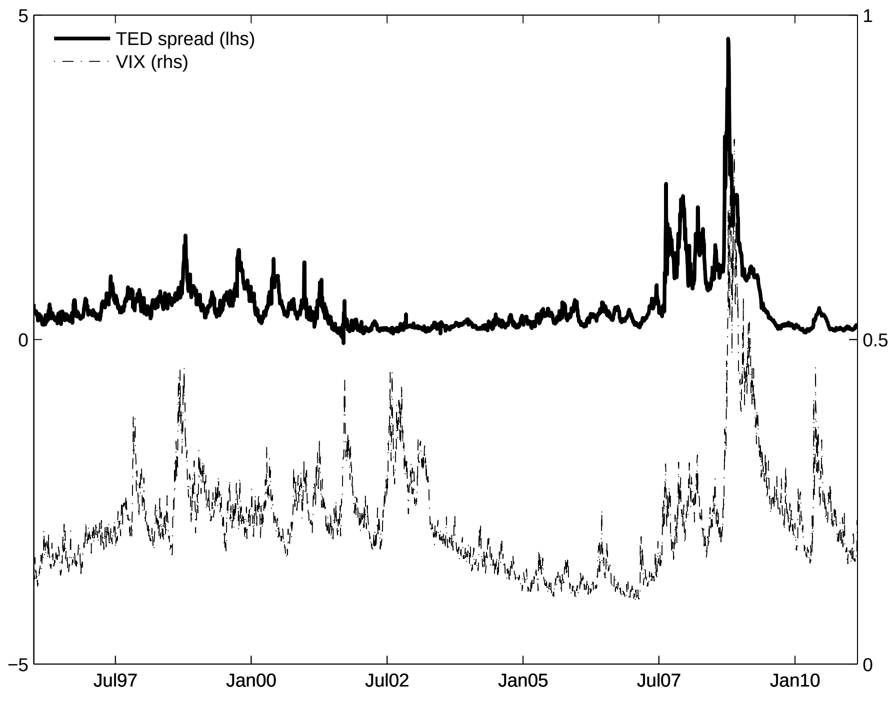

Figure 8: VIX and TED spread evolution over time

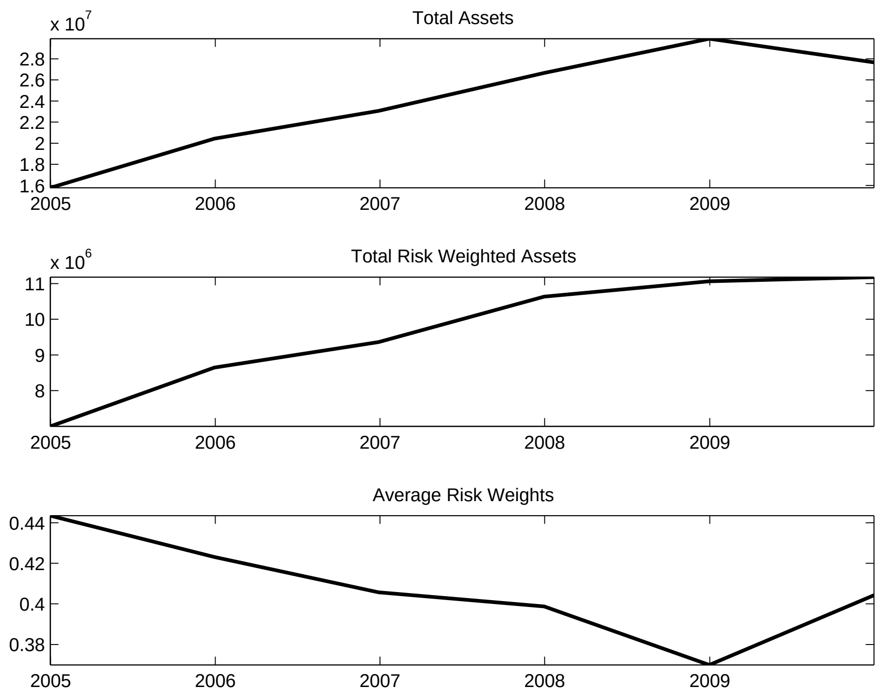

Figure 9: Average risk weights evolution over time