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Article (Accepted version) (Refereed)

Original citation: Freeman, Mark C., Groom, Ben, Panopoulou, Ekaterini and Pantelidis, Theologos (2015) *Declining discount rates and the Fisher Effect: inflated past, discounted future?* Journal of Environmental Economics and Management, 73 . pp. 32-49. ISSN 0095-0696

DOI: 10.1016/j.jeem.2015.06.003

Declining discount rates and the 'Fisher Effect': Inflated past, discounted future?

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June 2, 2015

Abstract

Uncertain and persistent real interest rates underpin one argument for using a declining term structure of social discount rates in the Expected Net Present Value (ENPV) framework. Despite being controversial, this approach has influenced both the Inter-Agency Working Group on Cost-Benefit Analysis and the UK government's guidelines on discounting. We first clarify the theoretical basis of the ENPV approach. Then, rather than following previous work which used a single series of U.S. Treasury bond returns, we treat nominal interest rates and inflation as co-integrated series and estimate the empirical term structure of discount rates via the 'Fisher Effect'. This nests previous empirical models and is more flexible. It also addresses an irregularity in previous work which used data on nominal interest rates until 1950, and real interest rates thereafter. As we show, the real and nominal data have very different time series properties. This paper therefore provides a robustness check on previous discounting advice and updated methodological guidance at a time when governments around the world are reviewing their guidelines on social discounting. The policy implications are discussed in the context of the Social Cost of Carbon, nuclear decommissioning and public health.

JEL: Q48, C13, C53, E43

Keywords: Social Discounting, Declining Discount Rates, Fisher Effect, Real and Nominal Interest Rates, Social Cost of Carbon.

1 Introduction

Despite some puzzles along the way, the burgeoning theoretical literature on discounting distant time horizons points more or less unanimously towards the use of a declining term structure of social discount rates (DDRs) for risk free public projects. [18, 19, 55, 59, 61]. [1] This conclusion is more or less robust to one's stance on the normative-positivist debate provided that the primitives of the discounting problem, growth or the interest rate, exhibit persistence over time [3, 4, 14, 18].[2] Consensus in an area of theory as potentially fraught as social discounting is a rare thing. Perhaps for this reason the literature on DDRs has been highly influential in policy circles, and many governments have either adopted DDRs or are in the process of considering them.[3] Yet, it has become clear that there is no consensus on how to operationalize a schedule

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[1]This is for risk-free discounting of certainty-equivalent future cash flows. See [20] for a discussion about project risk in discounting.

[2]Strictly speaking, the social planner must exhibit 'prudence' (e.g. [18]).

[3]The UK, French and Norwegian governments now recommend DDRs for intergenerational Cost-Benefit Analysis [29, 34, 37]. The U.S. Environmental Protection Agency (USEPA) and U.S. Inter-agency Working Group on the Social Cost of Carbon both recommend lower discount rates for intergenerational projects on the basis of DDRs [56, 31], and DDRs are still being considered as a result of a joint USEPA, Office for the Management of Budgets (OMB), Resources For the Future expert panel meeting [10, 4]. The Dutch government, the UK Treasury, the Cypriot government and the OECD International Transport Forum are reviewing their guidelines on long-term discounting.

of DDRs for use in Cost-Benefit Analysis (CBA). One need only look at the different and occasionally ad hoc motivations for current policies as evidence for this (e.g. [29, 60, 40]). This lack of consensus turns out to be important since evaluation of intergenerational projects is very sensitive to the discount rates deployed. Indeed, the range of policy prescriptions arising from different approaches to estimating the DDR schedule is comparable to that arising from the thorny normative-positive debate that characterized the aftermath of the Stern Review [43, 25, 14].

In this paper we explore the empirical sensitivities of the DDR term structure associated with the Expected Net Present Value (ENPV) approach proposed by Martin Weitzman [59]. Our main objective is to advance new empirical practices which operationalize this approach and thereby inform government guidelines and policy. There are three main contributions in this direction. First, we clarify the theoretical basis of the ENPV approach. Second, we generalize the empirical methods for analyzing historical interest rate data in this context. Specifically we estimate the 'Fisher Effect', which allows the real interest rate to be modeled in terms of its component parts: the nominal interest rate and inflation. Third, our approach addresses an important irregularity in the interest rate series used before. The overall contribution is to develop an empirical method which encompasses and supersedes the approaches used previously.

In the ENPV framework, DDRs are driven by uncertainty over long-term average future Treasury bonds yields. The result stems from the fact that the ENPV approach values the future using the expected discount factor, not the expected discount rate. The associated certainty equivalent discount rate is declining with the time horizon because costs and benefits in future states of the world with persistently high discount rates contribute less to the overall evaluation. The ENPV of the distant future is dominated by the low discount rate states of the world, and hence the appropriate discount rate for the distant future converges to the lowest possible future rate. One way to understand the behavioral underpinnings of this phenomenon is to recognize, through the Ramsey Rule for example, that interest rates are highly positively correlated with the expectation of future economic growth. If expected growth is high, our descendants in the distant future will be substantially wealthier than us, and we will be prepared to save less today on their behalf compared to when low growth is expected. This leads to higher interest rates and lower valuations of projects with intergenerational consequences. Current investments for the distant future are then primarily driven by the possibility that growth, and hence the interest rate, is persistently low in the future. To understand how this works, we need to introduce uncertainty about the future state of the world. A numerical example illustrates the basic idea. Suppose it is equally likely that growth is low or high in the future, and interest rates are either 1% or 7%. In this case the present value of $1 in t years time will either be $P_l = \exp(-0.01t)$ or $P_h = \exp(-0.07t)$. The ENPV approach takes the average of these valuations and calculates the certainty equivalent rate, $r(t)$, which in this case is defined by: $\exp(-tr(t)) = 0.5(\exp(-0.01t) + \exp(-0.07t))$. For time horizons of $t = [1, 30, 200, 400]$, the certainty equivalent rate is [4.0%, 2.8%, 1.7%, 1.2%] respectively. This result can be generalized, with the decline of the term structure increasing with both the persistence over time and the volatility of interest rates.[4]

These are the simple mechanics of the ENPV approach. A deeper story is that the declining term structure reflects a precautionary savings motive that increases the further one looks into the future [23, 18]. With persistence in interest rates coming from, say, different possible states of mean growth, compounding effects make uncertainty appear greater the further one looks into the future. The precautionary savings effect therefore increases with the time horizon and the discount rate term structure is declining. When formalizing this argument, though, the devil is in the detail and careful interpretation is needed [21]. The first contribution of this paper, therefore, is to clarify the theoretical motivation for the ENPV approach and the implications

[4] In this simple framework, and in many applications, the term structure declines from the historic average at $t = 0$, thereby abstracting away from current market interest rates. It is straightforward to include current market conditions by, for example, using the current Treasury bond yield in place of the historic average at the short end of the term structure. This may lead to an increasing, or hump-shaped, term structure, as expected growth effects vie against the precautionary savings motive.

this has for empirical work. We do this by taking ENPV out of its rather ad hoc theoretical origins (e.g. [59]) and placing it within the asset pricing literature (e.g. [9]).

The main contribution of this paper is empirical. The context is that the ENPV approach is operationalized using historical interest rate data, rather than expert opinion [14, 60]. Previous work of this type has illustrated the importance of careful modeling to establish the time series properties of the series, particularly persistence and volatility. Newell and Pizer [40](henceforth N&P) showed that U.S. bond yields have exhibited sufficient persistence in the past two centuries for the empirical term structure of discount rates to exhibit a rapid decline. This decline raises the social cost of carbon (SCC) from \$5.7/tC to between \$6.5/tC and \$10.4/tC in the process (US\$ 2000; see also [39]). Yet, these results were shown to be highly sensitive to the time-series model used to characterize interest rate uncertainty. Subsequent work by Groom, Koundouri, Panopoulou and Pantelidis [25] (henceforth GKPP) showed that more flexible characterizations of interest rate uncertainty lead to a schedule of DDRs that raises the SCC yet further to \$14.4/tC. Similar results have been found in many different countries and also when a global perspective is taken [28, 22]. This work has been important since the ENPV approach and these empirical exercises have strongly influenced the current guidance on long-term discounting in the U.S., U.K. and Norway, among others (e.g. [29, p98.] [31, p 24.] [37, Ch 5, p79.] [56, Ch 6, p23.]).[5]

Our empirical contributions flow from an inspection of the time-series of bond yields used by N&P. Their data have been particularly important as a testing ground for empirical methods for discounting in several other contributions [25, 22, 28]. The main irregularity is that the data used by N&P, i.e. annual market yields for long-term government bonds for the period 1798 to 1999, change from nominal to real interest rates in 1950. Given the history of inflation in the U.S., the data series is likely to have distinct time series properties either side of 1950.[6] Indeed, we show that the persistent uncertainty that underpins DDRs in the ENPV framework is only present in the nominal interest rate series. Real and nominal interest rate volatility also differs. These observations suggest that the shape of the term structures reported in earlier studies may be sensitive to this break in the data, and not robust to different assumptions about the inflation process. For instance, if real interest rates follow a mean-reverting process throughout the sample period, the resulting schedule of social discount rates could be effectively flat, and valuations of the SCC low.

Investigating the sensitivity of current policy recommendations to the treatment of inflation requires a completely different approach which removes the disconnect between nominal and real interest rates in the N&P data. An obvious candidate is to determine the real interest rate series by empirically modeling the theoretical relationship between nominal interest rates and inflation known as the 'Fisher Effect' [12]. This approach addresses the 1950 break in the U.S. data, but also has several advantages over previous work.

First, we show that the statistical model of N&P is a special case of a more general model which includes the Fisher Effect. This has the practical advantage of removing the need for the repeated periods of negative real interest rates to be smoothed out, as they were in previous work. Consequently, the certainty equivalent discount rate need not be restricted to be positive.[7] Our approach is therefore more flexible.

Second, the estimators we use accommodate both endogeneity and co-integration of nominal interest rates and inflation. Doing so allows for the fact that policy decisions affect the future path of the economy and these decisions are reflected in both nominal interest rates and

[5]For instance, the Norwegian Guidelines conclude: "Beyond 40 years, it is reasonable to assume that one will be unable to secure a long-term rate in the market, and the discount rate should accordingly be determined on the basis of a declining certainty-equivalent rate as the interest rate risk is supposed to increase with the time horizon. A rate of 3 percent is recommended for the years from 40 to 75 years into the future. A discount rate of 2 percent is recommended for subsequent years."

[6]Prior to 1950 the United States went through periods of both highly positive and highly negative inflation [15] and [6].

[7]N&P argue that short-term fluctuations are not strictly relevant to the time horizons that are the focus of their paper. Furthermore, negative real rates do not appear in their data, the argument being that these are transitory phenomena [39, p 10].

inflation rates. This relationship is a typical feature of discounting theory and captures the idea that discount rates are dependent on current economic conditions and policy uncertainty. Our approach is the first to reflect these aspects when estimating the term structure of discount rates (e.g. [20, Ch 4]).

What is surprising, but perhaps heartening, about our results is that the term structure of discount rates in the U.S. case is largely robust to our more rigorous treatment of inflation. The term structures we describe generally decline more sharply at long horizons than either N&P or GKPP. However, at the short end, social discount rates are higher than those described by GKPP. As a consequence, the estimated present value of long-term cash flows, such as those associated with the damages from carbon emissions, lies between the estimates of N&P and GKPP in our preferred case, but closer to the latter than the former. We conclude that the methods and results in this paper provide a robust basis upon which to base discounting policy rooted in the ENPV approach.

2 A Theory of Declining Discount Rates

In this section we clarify the theoretical basis for the ENPV approach and the implications for the use of market data to operationalize it. When using market bond yields to inform the discount rate, policy makers are taking a positive, rather than normative, approach to social discounting. A project with a consumption certainty equivalent future cash flow V_t at future time t and zero at all other times is then, from a valuation perspective, economically equivalent to a zero-coupon default risk-free bond with maturity t. The appropriate valuation approaches can therefore be taken directly from the asset pricing literature.[8]

A well-known result from financial economics (see, for example, [2, Equation 16]) is that the present value at time h of the project cash flow at time t, $P(h,t)$, is given by:

$$P(h,t) = E_h\left(V_t \exp\left(-\sum_{\tau=h}^{t-1} r_\tau\right)\right) \tag{1}$$

where r_τ is defined as the logarithmic expected single-period return for holding a claim on V_t over the interval $[\tau, \tau+1]$: $\exp(r_\tau) = E_\tau\left[P(\tau+1,t)/P(\tau,t)\right]$. The derivation of equation (1) emerges simply from repeated iteration of the single-period Net Present Value equation.

Using equation (1), define the variable $r(t)$ by: $P(0,t) = E_0[V_t]\exp(-tr(t))$. Now assume that V_t is non-stochastic, or at least uncorrelated with $\sum_{\tau=0}^{t-1} r_\tau$ so that $P(0,t)$ can be written as $P(0,t) = E_0[V_t]\, E_0\left[\exp\left(-\sum_{\tau=0}^{t-1} r_\tau\right)\right]$. This assumption is essentially what Martin Weitzman refers to as a 'pragmatic-decomposition' [60]. It then follows that:

$$r(t) = -\frac{1}{t}\ln\left(E_0\left(\exp\left(-t\bar{r}_t\right)\right)\right) \tag{2}$$

where $\bar{r}_t = t^{-1}\sum_{\tau=0}^{t-1} r_\tau$ is the average value of r_τ over the horizon of the project. Following Weitzman [59] we call $r(t)$ the *certainty equivalent discount rate*, and the corresponding *certainty-equivalent forward rate,* $\widetilde{r}_t$, for discounting between adjacent periods at time t is:

$$\widetilde{r}_t = \frac{P(0,t)}{P(0,t+1)} - 1 \tag{3}$$

Taken together, equations 1, 2 and 3 effectively define the Expected Net Present Value (ENPV) approach.

Crucially, exponential functions are convex and so, by Jensen's inequality, $r(t) < E_0(\bar{r}_t)$. The magnitude of this inequality is driven by two parameters; the value of t and the uncertainty over $\bar{r}_t$. The inequality becomes more pronounced as t increases, and, ceteris paribus, this causes the term structure of certainty equivalent discount rates to decline with the horizon of the

[8] See also the technical Appendix B of [14] for a related discussion.

project. The inequality also becomes more pronounced as the uncertainty over $\overline{r}_t$ increases. Hence, understanding the volatility of average future costs of capital is the critical empirical task facing those who wish to operationalize the ENPV approach.

One way to understand the behavioral underpinnings of this result is to think about how interest rates relate to expected growth, and how consumption-saving decisions are made in the face of uncertain growth. A helpful theoretical structure for this might be the Ramsey Rule, through which there would be is strong positive correlation between interest rates and growth. If growth is expected to be high, savings will be low and interest rates high and we value future benefits less. The opposite would be the case for low expected growth. Suppose, in the extreme, we would be prepared to invest nothing now for the future if we knew growth was going to be high, so our valuation is $P_h = 0$. Alternatively, if we knew growth was going to be low, under a standard net present value argument we would invest $P_l = \exp(-tr_l)$ for each \$1 we will receive at time t, where r_l reflects the (low) interest rate in the low-growth state. If we are uncertain about which state may prevail, we might assign a probability $\pi > 0$ to the low growth state. The ENPV approach then says that we should invest $P = \pi P_l$, the expected discount factor, to receive \$1 at time t. The certainty equivalent discount rate $r(t)$ is defined by $\exp(-tr(t)) = P$, and is the discount rate that, if applied to the time horizon t, would give the same valuation as the expected discount factor, P. Simple rearrangement gives $r(t) = -ln(\pi)/t + r_l$. This declines smoothly from $-ln(\pi) + r_l$ at $t = 1$, to r_l as the time horizon extends to infinity: $t \rightarrow \infty$.

From an empirical perspective, when parameterizing equation (2), N&P and others estimate the statistical properties of $\overline{r}_t$ from a historic time-series of yields on long-term Treasury bonds. However, it is not immediately obvious that the yield on long bonds, with horizons of a few decades, and the single period return to a many-century $t-$period default risk-free fixed income security should be the same. In general, empirical estimates of the Treasury yield curve are upward sloping, suggesting that $\overline{r}_t$ is likely to be higher than an average long-term bond yield. However, the literature on social discount rates generally ignores these yield curve issues by assuming that the liquidity premium on bonds of all horizons is zero. We retain this assumption here, the motivation for which is two-fold.

Within the social discounting literature, it has been common to justify the ENPV approach through the original thought experiment of Weitzman [59]. He assumes that future interest rates are currently unknown but that, in one instant, all uncertainty will be removed. The true value of r_0 will be revealed and $r_\tau = r_0$ with certainty for all future τ. In this case, the ENPV approach with r_τ proxied by a short-term risk-free rate has been justified through the literature on the so-called 'Weitzman-Gollier puzzle' [9] Alternatively, in fixed income pricing, the use of the ENPV equation in the absence of liquidity premiums is proposed by [9] in continuous time and [16] in discrete time. Here equation (2) is referred to as the Local Expectations Hypothesis. In this case, rather than all uncertainty being removed in one instant, a less restrictive 'local certainty' equivalent is required. By having consumption at time $\tau + 1$ fully known at time τ, all assets have a zero consumption beta and therefore all risk and liquidity premiums are also zero.

The social planner's current uncertainty over the far-horizon average future Treasury long-bond yield will depend on two things; the volatility of r_τ itself and the persistence of shocks to this series. Even if interest rates are highly volatile, provided that these shocks are transitory then the long-term average of r_τ will be relatively stable, leading to a slowly declining schedule of social discount rates. However, if shocks are persistent, then these will remain important into the distant future. In essence, Weitzman's original thought experiment embodied an extreme assumption about persistence by assuming that once interest rates were realized they remained the same forever [23]. The Local Expectations Hypothesis allows a more flexible interpretation which lends itself to an empirical investigation of the time series properties of the interest rate, in particular, the presence or absence of persistence.

In this vein, N&P used their data to estimate an AR(3) model and compare this to a fully persistent Random Walk specification. The uncertainty in the discount rate was then simulated

[9] This literature starts with [17] and thus far culminates with [23] and [55] via [27], [8] and [13].

using multiple predictions of possible future paths. In both cases persistence is found to be sufficient to cause a rapidly declining term structure. N&P could not distinguish between the two models on statistical grounds. For this reason, the USEPA guidelines on discounting take the average of these two models and consequently recommend a lower 2.5% rate for intergenerational projects (e.g. [56]). Subsequent work showed that the empirical schedule of DDRs based on N&P's data is not robust to different empirical models, making model selection crucial for informing policy.[10] Later work looked at the international context [22] [28].

This paper is focused on the time series properties of the components of the real interest rate: inflation and nominal interest rates. As discussed above, the disconnect between real and nominal interest rate data in 1950, that is a feature of all the previous U.S. studies, is potentially problematic when determining the term structure of DDRs. More generally, this disconnect points to the need for methods that can accommodate the fact that real interest rates are dependent on both inflation and nominal interest rates, which have different data generating processes.

3 Historical Data on U.S. Interest Rates

N&P base their analysis on nominal long-bond Treasury yields for the period 1798 to 1999. The series of bond yields was compiled from Homer and Sylla's monumental 'History of Interest Rates' and used their assessments to determine the best instrument among high-quality, long-term government bonds available each year [30]. Based on these nominal rates, starting in 1950, the Livingston Survey of professional economists is used to construct a measure of expected inflation, which is then used to create real interest rates. No adjustment to nominal yields is made before 1950. The interest rates are then converted to their continuously compounded equivalents and estimations are made using a three-year moving average of this series. Finally, N&P used logarithms of the series which preclude negative rates and makes interest rate volatility more sensitive to the level of interest rates. A trend correction is also required [39].

N&P have an extremely thorough description of their methods and the treatment of their data, as well as a convincing justification for the steps taken (see also [39]). Nevertheless, there are certain features of their 202 year series and the transformations undertaken which are worthy of further investigation given the sensitivity of the schedule of DDRs to different empirical treatments. Tables 1-3 show some descriptive statistics and statistical tests on the N&P series, including its comparison with nominal and real interest data sourced from Global Financial Data (GFD) for the duration of overlap: 1820-1999. Each serves to motivate our closer scrutiny of the N&P data and our subsequent alternative methodological approach.

First, Figures 1 to 4 show the result of a rolling estimates of an AR(4) model of interest rates similar to the type used by N&P, together with the associated p-value of the Augmented Dickey Fuller (ADF) test for a unit root.[11,12] Figures 1 and 2 use unsmoothed data, while Figures 3 and 4 use the three year moving average data used by N&P. Figure 1 uses a 50 year window for the rolling estimation and shows that the null hypothesis of a unit root is rejected in the set of 50 year windows starting from 1945 until 1950. The latter set of windows are made up predominately of the real data series. The pattern remains in Figure 2 in which a 100 year window is used for the rolling estimation. In both cases, persistence in the unsmoothed data series is shown to be largely a pre-1950 phenomenon associated with the nominal but not the real interest rate data.

Figures 3 and 4 show the results of a similar exercise for the smoothed data used by N&P, for 50 and 100 year windows respectively. Qualitatively speaking, Figures 3 and 4 show that persistence again declines towards the more recent windows of data containing a greater proportion of the post-1949 real interest rate series. More importantly, when the data is smoothed

[10]See GKPP for the US and [24] for the UK.

[11]The ADF test contains the lagged difference terms appropriate for the AR(4) model.

[12]N&P used an AR(3), however we found that the AR(4) model minimizes the Akaike Information Criterion. The analysis of the data set is similar whichever approach is taken.

there is no 50 or 100 year window in which the null hypothesis of a unit root can be rejected.[13] A comparison of Figures 1 and 2 with Figures 3 and 4 shows that, whatever the theoretical justification, smoothing the data inevitably increases persistence in the series.

[PLACE FIGURES 1-4 HERE]

Additional evidence for the existence of a unit root in the nominal interest rate data, but not the real interest rate data, can be found in Table 1. Here an ADF test is undertaken on the pre-1950 nominal data and the post-1949 real data, unsmoothed and smoothed. The unit root hypothesis is rejected for the unsmoothed real (post-1949) data. This underpins the rejection of the null when the entire unsmoothed series is tested. For the smoothed data, we fail to reject the null hypothesis of a unit root in either period.

As further evidence of the likely importance of the use of nominal rather than real interest rate data, Figure 5 illustrates how the unsmoothed N&P series compares with the GFD data on real and nominal interest rates since 1820. The first thing to notice is that the N&P series has smoothed away three periods during which real interest rates were negative: the early 1900s, the late 1930s to early 1940s and the late 1960s to early 1970s. Second, the GFD real interest rate data is much more volatile than the N&P data, particularly pre-1950 when the N&P data is nominal. Table 2 further shows that the correlation of the N&P data with the GFD data is much stronger pre-1950 for both real and nominal GFD series. The N&P series is more strongly correlated with the nominal GFD data than the real. Lastly, Table 3 shows that the autocorrelation coefficients for each data source: N&P, GFD nominal and GFD real, are quite different.

[PLACE FIGURE 5 HERE]

Much of this analysis is merely descriptive of course. However, from a theoretical and empirical perspective it seems clear that some of the assumptions underpinning the series used by N&P and GKPP are not completely satisfactory. Smoothing, the removal of negative real interest rates and, in particular, the disconnect between nominal and real interest rates before 1950 appear to be driving some of the time-series properties of the data that are important from the perspective of deriving the term structure of the certainty equivalent discount rate. There may also be some conceptual problems with the use of the Livingston Survey of Professionals data on the CPI since the interest rate data is for a long-bond, while the survey is typically concerned with one-year inflation estimates. It is also worth noting that a fairly dim view of the Livingston survey is taken in some quarters.[14]

In summary, the previous empirical studies of DDRs in the US used data which was a mixture of real and nominal interest rates. These studies also used smoothing techniques and removed periods of negative interest rates. Each of these features is likely to have affected the estimated time series properties in ways which will affect the empirical term structure of discount rates. These data issues only serve to highlight the need to treat inflation and nominal interest rates separately in the analysis of real interest rates. In the following section we propose an alternative empirical and theoretical approach for estimating the term structure of discount rates which addresses all of the issues raised here.

[PLACE TABLES 1 - 3 HERE]

[13] The rolling ADF test is undertaken without a trend component, although similar results arise when the trend is included.

[14] It has been described as being "poorly designed throughout most of its history, having been intended more for journalistic than scientific purposes." [54, p.127]

4 A Bivariate Model for Calculating the Declining Discount Rates

The key issues highlighted in the previous section were the disconnect between nominal and real interest rates in the N&P data, the smoothing of the data, and the fact that these two operations might ultimately be driving the estimated decline in the term structure of social discount rates. Our solution to these issues involves separating real interest rates into its component parts. The method we propose allows real interest rates to be estimated using data on nominal interest rates and expected inflation, without the need for data smoothing.

4.1 A model of nominal interest rates and inflation: The 'Fisher effect'

The relationship between nominal interest rates and inflation is often analyzed in the context of the 'Fisher' relationship [12]. Specifically, let $y_t(m)$ denote the $m-$period nominal interest rate at time t, $x_t^e(m)$ denote the expected rate of inflation from time t to $t+m$, and $r_t^e(m)$ denote the ex-ante $m-$period real interest rate. The 'Fisher effect' can be expressed as follows:[15]

$$y_t(m) = x_t^e(m) + r_t^e(m) \tag{4}$$

The additional assumption of rational expectations (see, e.g. [36]) allows us to link realized inflation to expected inflation, $x_t(m) = x_t^e(m) + \upsilon_t$, where υ_t is a white noise process, orthogonal to $x_t^e(m)$. Finally, if we further assume that the real interest rate is a white noise process with a mean value r, we end up with the following equation:

$$y_t(m) = r + \theta x_t(m) + u_{1t} \tag{5}$$

In the literature, there are alternative theories about the magnitude of the θ parameter in the above equation. The traditional Fisher hypothesis suggests that $\theta = 1$. However, there are different approaches that suggest a θ that is either greater than unity (e.g. [11]) or less than unity (e.g. [38]). The empirical findings are also mixed. Mishkin was one of the first researchers to point out the problem of spurious regression when examining the relationship between nominal interest rates and inflation due to the non-stationarity of the series, and to suggest that co-integration techniques are necessary to investigate the Fisher effect [36]. However, even if the appropriate co-integration methods are applied, uncertainty remains about the ability of the co-integration estimators to reveal the true value of θ in small samples. To take account of this it is important not to impose any restrictions on the value of θ and to work in a framework that embodies the uncertainty surrounding the value of θ. This is the approach we take in what follows.[16]

The empirical model is organized around a Data Generating Process (DGP) for the relationship between nominal interest rates and inflation that was first proposed by Phillips [48, 49]. This provides a general framework for the dynamics of the variables under scrutiny and is often used in the literature to examine the finite sample properties of co-integrating estimators (see, [45, 53]).

4.2 The triangular data generating process

We consider the triangular DGP for the I(1) vector $\mathbf{z}_t = [y_t, x_t]^\top$ given in equation (5), and:[17]

$$\Delta x_t = u_{2t} \tag{6}$$

[15] This is an approximate Fisher model. The exact relationship being: $(1 + y_t(m)) = (1 + x_t^e(m))(1 + r_t^e(m))$. The approximation works well when $x_t(m) r_t^e(m)$ is small.

[16] Our focus on the co-integrating relationship between nominal interest rates and inflation means that we are not interested in modeling the real interest rate directly, and hence we do not follow the procedures associated with previous models of the certainty equivalent discount rate found in GKPP.

[17] For expository purposes, we drop m.

The co-integrating error, u_{1t}, and the error that drives the regressor, u_{2t}, are each assumed to be I(0) processes, $\mathbf{u}_t = [u_{1t}, u_{2t}]^\top$, described by the following VAR(1) model:

$$\mathbf{u}_t = A\mathbf{u}_{t-1} + \mathbf{e}_t \tag{7}$$

where A is a 2×2 parameter matrix and $\mathbf{e}_t$ is a white noise process. More specifically, $\mathbf{u}_t$ is given by:

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{pmatrix} u_{1t-1} \\ u_{2t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \tag{8}$$

and

$$\begin{pmatrix} e_{1t} \\ e_{2t} \end{pmatrix} \sim NIID\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \Sigma_e = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix}\right] \tag{9}$$

Note that this DGP suggests that the co-integration parameter θ is time-invariant. In what follows we provide empirical tests which find support for this assumption in our context.

4.3 Implications of the triangular model

In this section we now explore the theoretical implications of this model for the term structure of discount rates. N&P present a simple AR(1) representation of their model to show the role that persistence, volatility and maturity play in determining the DDR term structure. We undertake a similar task here under the assumption that $\theta = 1$ and are able show that the simple AR(1) model of N&P is a special case of this model. Under this assumption the real interest rate becomes white noise around an uncertain mean in our model: $r_t = y_t - x_t = r + u_{1t}$. If r is currently unknown, but is assumed to be normally distributed: $r \sim N\left(\overline{r}, \sigma_r^2\right)$, then from equation (1),

$$\begin{aligned} P(0,t) &= E\left(\exp(-rt)\right) E\left(\exp\left(-\sum_{\tau=1}^{t} u_{1\tau}\right)\right) \\ &= \exp\left(-\bar{r}t + 0.5t^2\sigma_r^2\right) E\left(\exp\left(-\sum_{\tau=1}^{t} u_{1\tau}\right)\right) \end{aligned} \tag{10}$$

The structure of the summation in (10) depends on the parameters of A and Σ_e. If $\alpha_{12} = 0$, our DGP for the real interest rate becomes a simple mean-reverting process. The model then coincides with that of N&P in the sense that it is solely persistence, measured by α_{11}, and uncertainty, measured by σ_r^2 and σ_{11}, that determine the shape of the term structure of social discount rates. The approach taken by N&P is in this sense nested within the approach taken here. Alternatively, when this assumption is relaxed and $\alpha_{12} \neq 0$, the dynamics become more complicated and additional structure is required to make further progress on the theoretical model.

We proceed by calculating the expected value of $\exp(-\sum_{\tau=1}^{t} u_{1\tau})$ based on the following infinite Moving Average (MA) representation of $\mathbf{u}_t$:

$$\mathbf{u}_t = \sum_{i=0}^{\infty} \Phi_i \mathbf{e}_{t-i} \tag{11}$$

where $\Phi_0 = I_2$ is a 2×2 identity matrix, and $\Phi_i = A^i$, $i = 1, 2, ...$ Given the Cholesky decomposition of $\Sigma_e = BB^\top$, we obtain the following representation:

$$\mathbf{u}_t = \sum_{i=0}^{\infty} \Theta_i \mathbf{w}_{t-i} \tag{12}$$

where $\Theta_i = \Phi_i B$ and $\mathbf{w}_t = B^{-1}\mathbf{e}_t \sim IIDN(\mathbf{0}, I_2)$ [35].

In order to derive a tractable expression for the expected value of the discount factor, one that can be easily compared and contrasted to previous work, we make the simplifying assumption that $\alpha_{21} = 0$. [18] In this case, the eigenvalues of A, denoted as λ_1 and λ_2, are equal to α_{11} and α_{22} respectively. Then, given that the DGP starts at time $t = 1$, we end up with the following expression for the second term in equation (10):

$$E\left[\exp\left(-\sum_{\tau=1}^{t} u_{1\tau}\right)\right] = \exp\{0.5(\sqrt{\sigma_{11}} + R_{1t} + R_{2t})\}, \tag{13}$$

where:

$$R_{1t} = \sum_{\tau=1}^{t-1}\left[\left(\sqrt{\sigma_{11}} + \frac{a_{12}\sigma_{12}}{\sqrt{\sigma_{11}}(\lambda_1-\lambda_2)}\right)\left(\frac{1-\lambda_1^{\tau+1}}{1-\lambda_1}\right) - \frac{a_{12}\sigma_{12}}{\sqrt{\sigma_{11}}(\lambda_1-\lambda_2)}\left(\frac{1-\lambda_2^{\tau+1}}{1-\lambda_2}\right)\right]^2 \tag{14}$$

$$R_{2t} = \sum_{\tau=1}^{t-1}\left[\frac{\sqrt{\sigma_{22}-\frac{\sigma_{12}^2}{\sigma_{11}}}a_{12}\lambda_1}{(\lambda_1-\lambda_2)}\left(\frac{1-\lambda_1^{\tau}}{1-\lambda_1}\right) - \frac{\sqrt{\sigma_{22}-\frac{\sigma_{12}^2}{\sigma_{11}}}a_{12}\lambda_2}{(\lambda_1-\lambda_2)}\left(\frac{1-\lambda_2^{\tau}}{1-\lambda_2}\right)\right] \tag{15}$$

Substituting equation (13) into equation (10), we obtain an expression for the expected value of the discount factor.

$$P(0,t) = \exp\left(-\bar{r}t + 0.5t^2\sigma_r^2\right)\exp\{0.5(\sqrt{\sigma_{11}} + R_{1t} + R_{2t})\}\} \tag{16}$$

from which the instantaneous discount rate at time t is calculated based on the continuous-time equivalent of the certainty equivalent forward rate in equation (3).

4.4 The determinants of the term structure

From the previous expression it can be shown that, just as in the case of the AR(1) model of N&P, it is persistence measured by λ_1 and λ_2, and uncertainty measured by σ_r^2, that determine the speed of decline of the discount rate with the time horizon, t. Also of general importance are the elements of Σ_e. For comparability with N&P, we now develop an expression which illustrates the theoretical relationship between the term structure and these elements of the data generating process.

In our model the general expression for the certainty-equivalent *forward* rate at time t is given by the rate of change of $P(0,t)$ as shown in equation (3), and is equal to:

$$\widetilde{r}_t = \bar{r} - t\sigma_r^2 - \Omega(\lambda_1, \lambda_2, \alpha_{12}, \sigma_{11}, \sigma_{22}, \sigma_{12}, t) \tag{17}$$

Notwithstanding the second term $\left(-t\sigma_r^2\right)$, it is the comparative statics of the expression $\Omega(\lambda_1, \lambda_2, \alpha_{12}, \sigma_{11}, \sigma_{22}, \sigma_{12}, t)$ that determines the term structure. For values of $|\lambda_1|, |\lambda_2| < 1$ the limiting value of this expression is given by:

$$\lim_{t\to\infty}\Omega(\lambda_1, \lambda_2, \alpha_{12}, \sigma_{11}, \sigma_{22}, \sigma_{12}, t) = \frac{0.5(1-\lambda_2)^2\sigma_{11} + \alpha_{12}\left[(1-\lambda_2)\sigma_{12} + 0.5\alpha_{12}\sigma_{22}\right]}{(1-\lambda_1)^2(1-\lambda_2)^2} \tag{18}$$

This expression clarifies several issues.

First, it is easy to show that when $\alpha_{12} = 0$, this expression coincides with the limiting expression for the AR(1) model of N&P, i.e. $\lim_{t\to\infty}\Omega(\lambda_1, t) = \frac{0.5\sigma_{11}}{(1-\lambda_1)^2}$. This is another illustration that N&P is a special case of our empirical model. Second, equation (18) enables us to disentangle the effect of persistence and uncertainty on the term structure of discount rates.[19] This is illustrated in Figures 6 and 7. Figure 6 contains the theoretical term structure of forward rates for three different levels of persistence (measured by λ_1), while keeping the level of uncertainty constant (i.e. all other parameters are the same for all three term structures in the graph). Figure 7 shows three term structures which differ only in the level of uncertainty: σ_r. In Figure

[18] No such assumption is made in the subsequent empirical analysis, where the elements of the A matrix are unrestricted.

[19] We would like to thank an anonymous referee for pointing this out.

6, the decline in the term structure is sensitive to different values of λ_1 for short maturities, but insensitive for long maturities in the sense that the term structures have the same slope for long maturities. In Figure 7, where persistence is held constant, higher uncertainty leads to a more rapid decline in the term structure for longer maturities, with little effect for short maturities. These comparative statics imply that the term structure of the social discount rate, and hence policy recommendations, are sensitive to these features of the time series data. It is therefore extremely important to use appropriate data and robust empirical methods to estimate the DGP and the term structure.

[INSERT FIGURE 6 AND 7 HERE]

5 Empirical Results and Simulation

5.1 Data

In Section 3 we motivated this study by exposing some potential shortcomings of the interest rate data used in previous work. One solution to these shortcomings was to decompose the real interest rate into its component parts. We now discuss these data.

We use the nominal interest rate data on long-term (10 year) Treasury bonds used by N&P and proxy expected inflation by the 10-year average realized inflation rate as calculated from the CPI deflator (CPI data to 2009). This broadly matches the inflation horizon with the bond horizon. However, the choice of 10-year bond data rather than 1-year T-bills is not strictly motivated by the theoretical model in Section 2. In fact, apart from in some limited theoretical circumstances, 1-year T-bill data would be preferred for this exercise. We use the 10 year data for our analysis largely for practical reasons. First, the availability and quality of the 1-year data is problematic, whereas those from longer-maturity bonds are, by contrast, of superior quality [51, 52].[20] Second, estimation attempts using the 1-year GFD nominal bond data gave rise to problems of convergence. Nevertheless, estimating real interest rates using its component parts represents a methodological improvement on the pioneering work undertaken thus far. The use of the 10-year data allows for a crisp comparison with previous work.

5.2 Estimation of the co-integrating parameters

We now turn to empirical estimates of the co-integrating relationship between inflation and nominal interest rates. A number of models are deployed to allow for flexible estimation of the Fisher parameter, θ, and to check for the robustness of the certainty equivalent discount rate to different specifications of the co-integrating relationship. Our results confirm the widely held view that interest rates and inflation rates are I(1) processes and co-integrated.[21] As a result, the first condition for the Fisher hypothesis, i.e. the condition that $y_t(m)$ and $x_t(m)$ are co-integrated processes is satisfied.

It is well-known that in the case of co-integration, standard asymptotic theory does not apply due to the presence of nuisance parameters in the distribution of the OLS estimator. Even when A is diagonal, the contemporaneous correlation between u_{1t} and u_{2t} suggests that the regressor, $x_t(m)$, is correlated with the error u_{1t}. This leads to an endogeneity issue. We

[20] Consider the following quote for instance:

> Treasury bills, or short-term governments, did not exist before 1920. Data on commercial paper rates dating back to the 1830s are available from Macaulay, but during the 19th century commercial paper was subject to a high and variable risk premium, as Figure C shows. These premiums often developed during or just prior to liquidity and financial crises (marked by NBER-designated recessions). There were also defaults on this paper, but there is insufficient information to correct the yield series for these defaults. Despite the obvious shortcomings of the data, there are few other short-term rates available for the early 19th century, and those that are available cover very short periods. [51, p 31]

[21] Detailed tables of unit root tests and co-integration tests are available from the authors upon request.

overcome this by considering various parametric and semi-parametric co-integration estimators, which are asymptotically efficient provided that the conditions of the Functional Central Limit Theorem (FCLT) are satisfied.

We use five different estimators, each of which addresses the endogeneity issue. This is another advantage of our empirical approach over previous work. We now briefly describe each estimator.

Dynamic OLS (DOLS(p,t)): This estimator has been suggested by several papers [49, 50, 53]. It provides a direct way to estimate the co-integrating relationship and asymptotically leads to valid test statistics. It utilizes the static equation (5), augmented by lags and leads of the first difference of the regressor, i.e.:

$$y_t = \theta x_t + \sum_{i=1}^{p-1} \gamma_i \Delta x_{t-i} + \sum_{j=1}^{t-1} d_j \Delta x_{t+j} + v_t \tag{19}$$

Existence of serial correlation of v_t does not raise any serious problems in the estimation of θ and can be dealt with by consistently estimating the long-run variance of v_t as proposed by Newey and West [41].

Fully Modified Least Squares (FMLS): The FMLS estimation method, proposed by Phillips and Hansen [48], employs semi-parametric corrections for the long-run correlation and endogeneity effects, which fully modify the OLS estimator and its attendant standard error. This estimator is based on consistent estimation of the long-run covariance matrices, which requires the selection of a kernel and the determination of the bandwidth. We employ the Quadratic Spectral kernel and select the bandwidth parameter by applying the Newey-West procedure [42]. Moreover, we consider the "pre-whitened" version of FMLS which filters the error vector $\widehat{\mathbf{u}}_t$ prior to estimating the long-run covariance matrices.[22]

Canonical Co-integrating Regression (CCR): Park's Canonical Co-integrating Regression (CCR) is closely related to FMLS, but instead employs stationary transformations of the data to obtain least squares estimates and remove the long-run dependence between the co-integrating error and the error that drives the regressors [46]. As in FMLS, consistent estimates of the long-run covariance matrices are required. To this end, we consider the "pre-whitened" version of CCR and employ the Quadratic Spectral kernel with the bandwidth selected by the Newey-West procedure.

Augmented Autoregressive Distributed Lag (AADL(q,r,s)): This estimator is based on the following AADL(q,r,s) model [47]:

$$y_t = \theta x_t + \sum_{i=1}^{q-1} a_i \Delta x_{t-i} + \sum_{j=1}^{r-1} b_j \Delta y_{t-j} + \sum_{h=1}^{s-1} a_h \Delta x_{t+h} + \epsilon_t$$

The parameter of interest is equal to the long-run multiplier of y_t with respect to x_t. A direct estimate of the parameter of interest θ along with its standard error may be obtained by transforming the AADL model into the Bewley form (see [7, 58, 5]). Estimates of the coefficients and their standard errors can be obtained by using the Instrumental Variables (IV) estimator, with the original matrix of regressors being the instrumental variables [58].

[22] [45] perform Monte Carlo simulations for a variety of DGPs and show that significant gains can emerge when the "pre-whitened version" of the FMLS estimator is employed.

Johansen's Maximum Likelihood (JOH): This is the well-known system-based maximum likelihood estimator of θ, suggested by Johansen [32, 33]. The order of the JOH estimator corresponds to the lag-order of the Vector Autoregressive model on which this estimator is based. An important difference between this estimator and the other co-integration estimators considered in this study is that it has been developed and proved to be asymptotically optimal in the context of a Gaussian Vector Autoregression which accommodates a rather narrow class of DGPs.

5.3 Stability and estimates of the co-integration vector

Before proceeding to the estimation of the co-integrating regression (5), we first test its stability over the two centuries of data that we employ. Specifically, we employ three tests of stability: the L_c, *MeanF* and *SupF* tests. The null hypothesis in each case is that the co-integrating vector is constant, while the alternative is that parameters either follow a Martingale process (L_c, *MeanF*) or exhibit a single structural break at unknown time t (*SupF*) [26].[23] Each test tends to have power in similar directions and can detect whether the proposed model captures a stable relationship. However, the asymptotic distribution of the test statistics is non-standard and depends on the nature of trends in the co-integrating regression. [26] provides both tabulated critical values and function p-values that map the observed test statistic into the appropriate value in the range of $p \in [0, 1]$ and more specifically into the range of interest: $p \in [0, 0.20]$. Table 4 presents the stability tests for the parameters in the co-integrating regression.[24] A p-value of 0.20 suggests significance at > 0.20 level. Overall, our findings suggest that the co-integrating relationship between the US inflation and the nominal interest rate is stable.

[INSERT TABLE 4 HERE]

We then proceed with the estimation of the parameters in the Fisher equation. Specifically, we employ the five estimators described in Section 5.2 and employ the Akaike Information Criterion (AIC) to choose the lag and lead specification for DOLS and AADL as well as the lag specification for JOH. AIC is also used to determine the optimal lag specification for the estimation of the long-run covariance matrix in the context of FMLS and CCR. Table 5 presents the estimated values of r and θ, together with the standard errors of the estimates for all the estimators under consideration.

[INSERT TABLE 5 HERE]

Our findings suggest that estimates are quite heterogeneous across estimators. Specifically, estimates of θ range from as low as 0.287 (CCR) to 2.259 (JOH). In short there is a high level of uncertainty as can be seen by the standard errors. The same can be said about the estimate of r which range from 0.087 (JOH) to 4.301 (CCR). One obvious question is whether the large difference between the JOH estimator and the others stems from the normality assumption that underpins its asymptotic optimality, compared to the other estimators that require no such assumption. To this end, we employed the Jarque-Bera joint Normality test for the error terms of our DGP and report our findings in Table 1 of the on-line Appendix. In all cases, the test rejects the null hypothesis of normality.[25]

We also applied a battery of diagnostic tests aiming at revealing the single "best" estimator. Specifically, we investigated by means of Monte Carlo simulations, the finite sample performance of the co-integration estimators considered in this study employing Data Generation Processes

[23] The tests are built in the context of fully modified estimation of the co-integrated regression. To save space, we do not give details on the formulation of the tests. The interested reader is referred to [26].

[24] Test statistics are calculated using the Quadratic Spectral kernel and pre-whitened residuals with a VAR(1) model. The bandwidth is selected by means of the Andrews (1991) procedure [1]. P-values are calculated by the function p-value methodology (see [26]). Alternative specifications with respect to the choice of kernel, bandwidth and pre-whitening yielded qualitatively similar results. We thank Prof. Hansen for making the codes available at http://www.ssc.wisc.edu/~bhansen/progs/progs.htm.

[25] We would like to thank an anonymous referee for pointing this out to us.

similar to the ones we have in our empirical application. The estimators were evaluated on the basis of both estimation accuracy and accuracy in terms of statistical inference. Overall, our findings (reported for brevity in Section 2 of the on-line Appendix, Tables 2 and 3) suggest that the AADL estimator outperforms the other estimators considered in our analysis, especially when it comes to their accuracy in terms of statistical inference.

Finally, we conducted a forecasting experiment to compare the predictive power of the five co-integrating estimators considered in our analysis.[26] We applied both rolling and recursive forecasting schemes and also considered four different values for the number of out-of-sample forecasts (30, 50, 70 and 90 observations). We used the Mean Square Forecast Error (MSFE) criterion as a measure of predictive accuracy.[27] Similarly to our Monte Carlo simulations, our out-of-sample forecasting exercise indicates the superiority of the AADL estimator over the remaining ones.

Overall, the comparison of estimators supports the widely held view that the AADL estimator is regarded to have better empirical qualities (see [47] [44] [45]).

5.4 Calculation of certainty-equivalent forward rates

To characterize the uncertainty of future real interest rates, we first simulate multiple future paths of real interest rates and then calculate the certainty equivalent forward rate following the simulation approach proposed by N&P adjusted for our DGP. The estimates (and the corresponding standard errors) of r and θ given in Table 5 are employed to estimate the residual series u_{1t} and u_{2t}. Once the residual series are obtained, we fit a VAR(1) model and get estimates for the elements of the A and Σ_e matrices. The variance-covariance matrix Σ_A of the estimated $vecA$ is also obtained. Table 6 presents the parameter estimates of the A matrix (see equations (7)-(8)). The estimates are similar for all estimators and correspond to a process with high persistence. It is interesting to note that α_{12} is negative in all cases, while α_{21} is positive and close to zero. 300,000 future paths (of 400 years length) are simulated for the nominal interest rate and the inflation rate taking into account: i) the stochastic dynamics of the DGP; ii) the uncertainty surrounding the estimated parameters; and, iii) the in-sample properties of the US real interest rate. The appendix provides a detailed account of the steps taken in the simulation.

Figure 8 shows the term structure of the certainty equivalent forward rates for each of the five co-integration estimators.

[INSERT TABLE 6 & FIGURE 8 HERE]

Strikingly, the empirical schedules of certainty equivalent forward rates arising from our proposed methods appear quite similar irrespective of the choice of the estimator. For comparability with previous work in this area [40, 25], we fix the starting point for each empirical term structure at 4%. From this point the schedules all decline below 3% after 25 years, below 2% after 170 years and below 1% after 400 years. The most rapidly declining in the medium term (50 years) is the JOH estimator. The lowest long-term rate (400 years) is associated with the AADL model which declines to 0.5%. Ultimately, each schedule is qualitatively similar indicating the there is sufficient persistence and uncertainty in the co-integrated series to cause a significant decline in the term structure over a policy relevant time horizon.

For comparative purposes in Figure 9 we plot the term structure from the AADL model with the certainty equivalent rates of the previous empirical work in this area, alongside the UK Treasury Green Book forward rates. The AADL model is chosen since each of the empirical models is theoretically equivalent, but as already mentioned AADL has better empirical qualities.

[INSERT FIGURE 9 HERE]

[26] We would like to thank an Associate Editor of this Journal for suggesting this approach.

[27] Details of this experiment and the related findings are given in Section 3 and Table 4 of the on-line Appendix.

These results are an important robustness check on previous work and indicate that if a government is to take this positive approach to social discounting long-term time horizons, care is needed not only in model selection, as discussed in GKPP [24, 25], but first and foremost in the treatment of the interest rate data. The term structure that emerges when modeling the Fisher Effect is distinct from those that make more arbitrary assumptions concerning the data. The policy implications of this finding are likely to be important for intergenerational projects. We now address this claim explicitly.

6 Applications to Intergenerational Public Policy

We now present several applications showing the impact that the Fisher Effect results have on long-term public policy. Three such policies are evaluated: i) the Social Cost of Carbon (SCC); ii) the cost of nuclear power decommissioning; and, iii) the benefits of reducing teenage obesity. For the SCC, the marginal damages of an additional ton of carbon are estimated using the DICE model (See [40] for details).[28] The SCC is the present value of the profile of carbon damages, which remains positive for a 400 year horizon at least. For nuclear decommissioning we rely on data from the Nuclear Decommissioning Authority (NDA). The 'NDA' costs are taken from the NDA's annual report and accounts 2012/13. The analysis assumes a delay of 50 years before implementation. 'Teenage Obesity' costs monetize the estimated impacts that are realized between the ages 41 and 65 from being obese at the age of 14. These values are based on the calibration in [57]. The percentage of the cash flows realized at each time horizon is shown in Figure 10.

Table 7 shows the implications of the alternative discounting approaches for each of the policies and each of the discounting approaches. The discounting approaches are placed in ascending order of the present values for each project, in dollars. Columns 3, 5 and 7 indicate the percentage change in the present value compared to the N&P random walk model. With all but the UK Green Book and the USEPA term structures starting at 4% for a maturity of 0, it is obvious that the flat 4% rate consistently yields the lowest valuation. The Fisher Effect approach provides an estimate of the SCC that is 36% (118%) higher than the N&P random walk (mean reverting) model. The Fisher Effect term structure increases the present value of nuclear decommissioning by 49% compared to the N&P random walk, and the present value of ameliorating teenage obesity by 34%. In each case the increase in valuation is close to that arising from the GKPP approach.

In short, disentangling the real interest rate via the estimation of the Fisher Effect not only provides a more robust empirical term structure, it also has policy implications which tend to confirm the larger valuation placed on the future found in GKPP [25].

[INSERT TABLE 7 AND FIGURE 10 HERE]

7 Conclusion

The empirical estimation of Weitzman's [59] declining term structure of the Social Discount Rate using historical interest rate data undertaken by Newell and Pizer [40] (N&P) and Groom, Koundouri, Panopoulou and Pantelidis [25] (GKPP) has directly influenced governmental guidance in the U.S. and indirectly influenced policy in a number of other countries including the U.K. and Norway [29, 37, 31, 56]. Yet the U.S. interest rate data series used by N&P and GKPP reflects nominal interest rates pre-1950 and real interest rates thereafter. A cursory analysis of this series and comparisons to historic real interest rates shows that the time-series properties of the nominal and real data series differ markedly. Properties such as persistence and volatility

[28] There are some potential problems with this approach since the damages stem from the DICE general equilibrium model and so are determined by the discounting assumptions contained therein. The SCC example should be considered as illustrative for these reasons.

are important determinants of the term structure of certainty equivalent discount rates indicating that the term structure will be sensitive to this treatment of the data. This analysis raises more general questions about the appropriate methodology to apply to real interest rate data given that it is a function of the inflation and nominal interest rate series which themselves have distinct time series properties.

By modeling the relationship between expected inflation and nominal interest rates in the U.S. via the Fisher Effect we provide a general methodology for estimating the term structure of social discount rates using readily available data. The approach also has several other advantages. First, it addresses the disconnect between nominal and real interest rate data that occurs in 1950 in the previous applications. Second, the approach allows estimation of the term structure without arbitrarily removing negative real interest rates or smoothing the real interest rate series.

The conclusions are qualitatively similar to N&P in that a declining term structure emerges. Yet the decline with the time horizon is closer to that of GKPP [25] which is more rapid. This results in a social cost of carbon (SCC) which, at $14.2/tC, is 36% (118%) higher than in the case when the term structure is based on the more restrictive random walk (mean reverting) model of N&P, and more than double the SCC when using a flat term structure.

The methods used here are more flexible than N&P in the sense that their simple AR(1) model is a special case of our empirical model. The results provide an important robustness check on previous work and indicate that if a government is to take this positive approach to the social discounting of long-term time horizons, care is needed not only in model selection, but first and foremost in the treatment of the interest rate data. The U.S. guidelines, the ongoing discussions on discounting in the U.K. Treasury, and the review processes taking place in the Netherlands and Cyprus, should all take heed.

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8 Figures

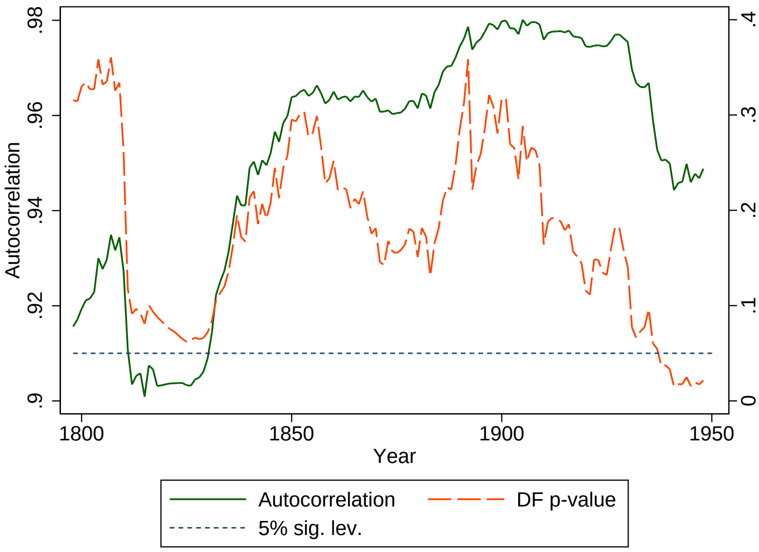

Figure 1: **Tests of Persistence on Unsmoothed N&P Data using a 50 Year Window**: 1) A rolling estimation of the sum of correlation coefficients from the AR(4) N&P model (left hand axis);. 2) The p-value from an Augmented Dickey-Fuller (ADF) on the AR(4) model (right hand axis). The 5% significance level is shown as the dotted horizontal line.

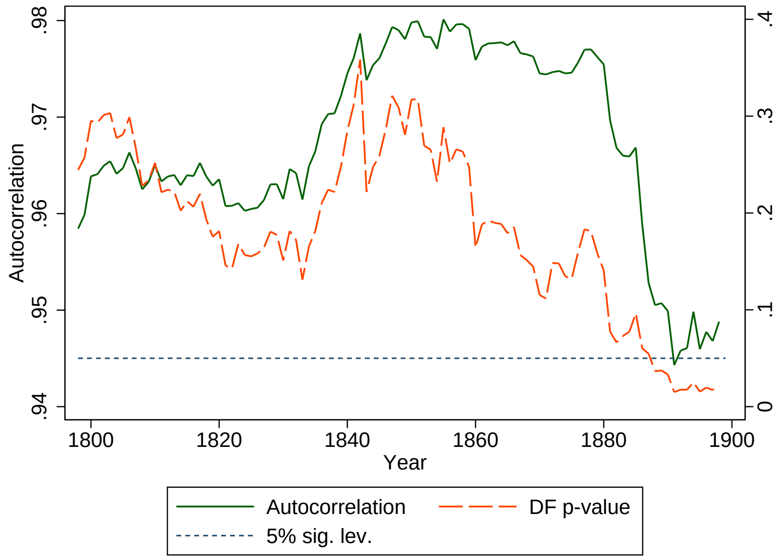

Figure 2: **Tests of Persistence on Unsmoothed N&P Data using a 100 Year Window**: 1) A rolling estimation of the sum of correlation coefficients from the AR(4) N&P model (left hand axis);. 2) The p-value from an Augmented Dickey-Fuller (ADF) on the AR(4) model (right hand axis). The 5% significance level is shown as the dotted horizontal line.

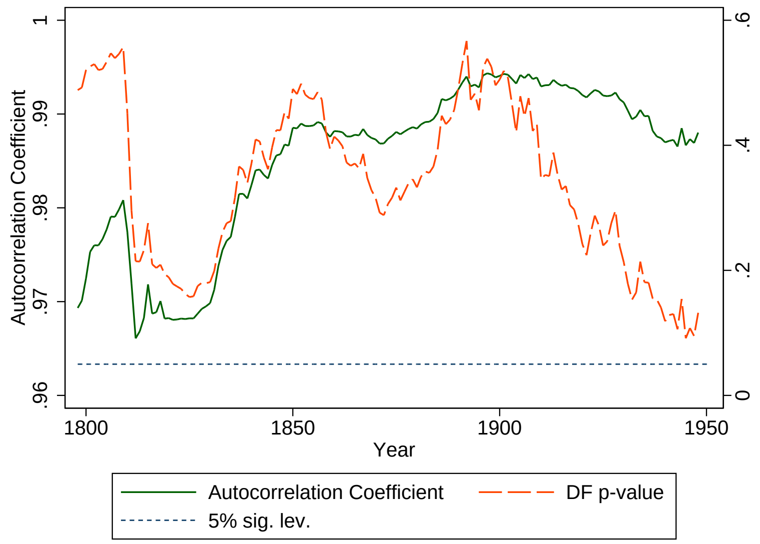

Figure 3: **Tests of Persistence on Smoothed N&P Data using a 50 Year Window**: 1) A rolling estimation of the sum of correlation coefficients from the AR(4) N&P model (left hand axis);. 2) The p-value from an Augmented Dickey-Fuller (ADF) on the AR(4) model (right hand axis). The 5% significance level is shown as the dotted horizontal line.

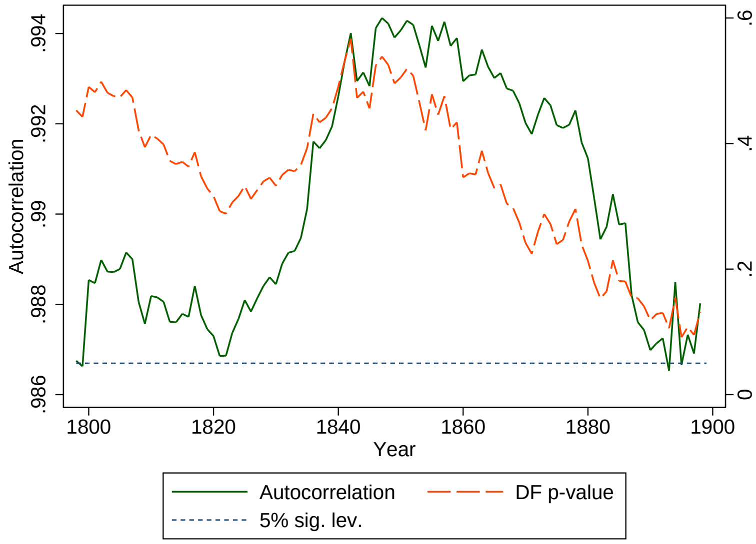

Figure 4: **Tests of Persistence on Smoothed N&P Data using a 100 Year Window**: 1) A rolling estimation of the sum of correlation coefficients from the AR(4) model of N&P (left hand axis);. 2) The p-value from an Augmented Dickey-Fuller (ADF) on the AR(4) model (right hand axis). The 5% significance level is shown as the dotted horizontal line.

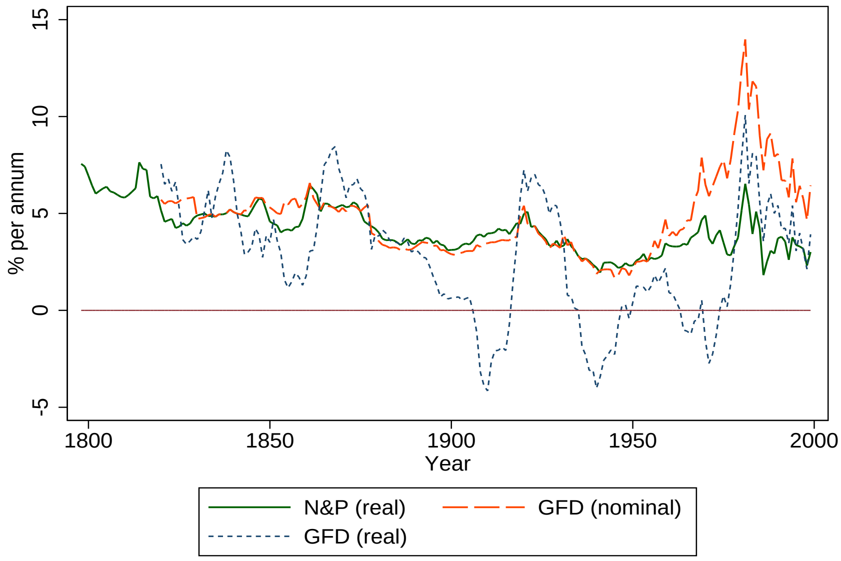

Figure 5: **Real, Nominal and N&P Interest Rate Data:** A graphical comparison of real and nominal interest data from Global Financial Data (GFD) for 10 year bonds (1820 - 1999), with the N&P data series (1798- 1999)

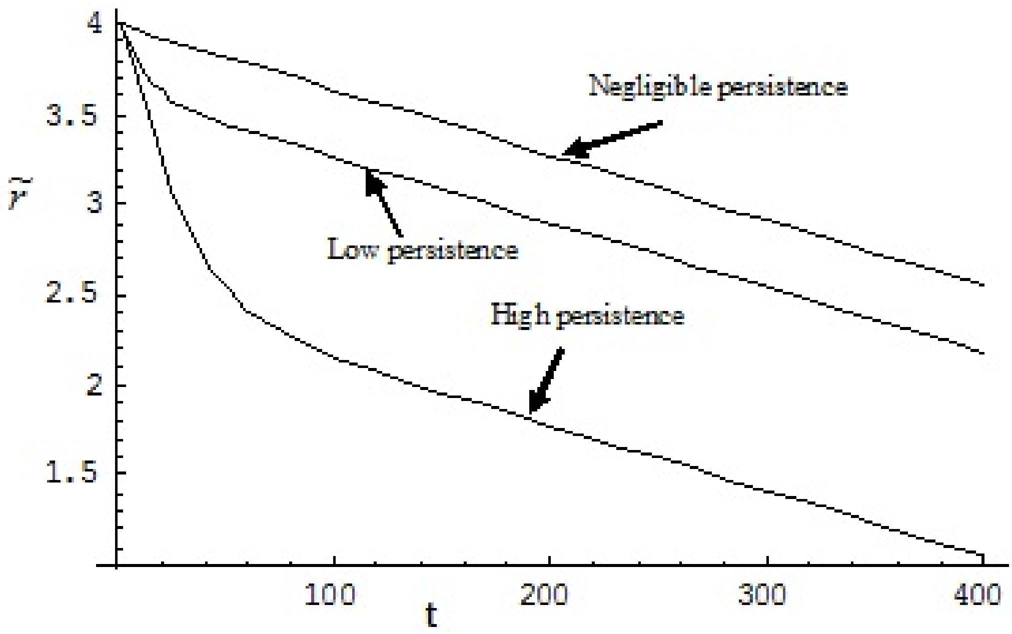

Figure 6: **The effect of persistence, as measured by λ_1 on the term structure**

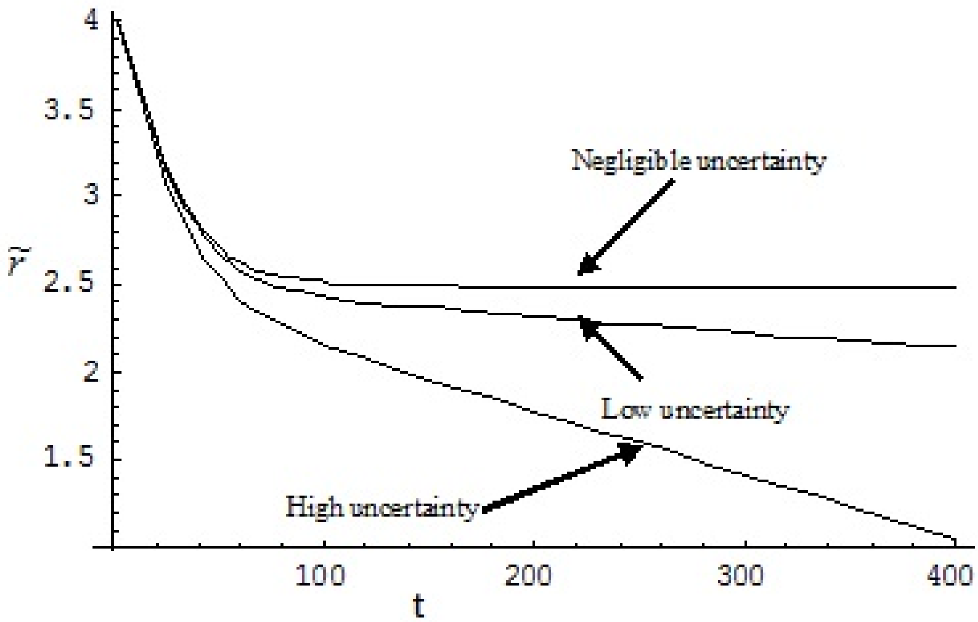

Figure 7: **The role of variance (σ_r^2) on the term structure**

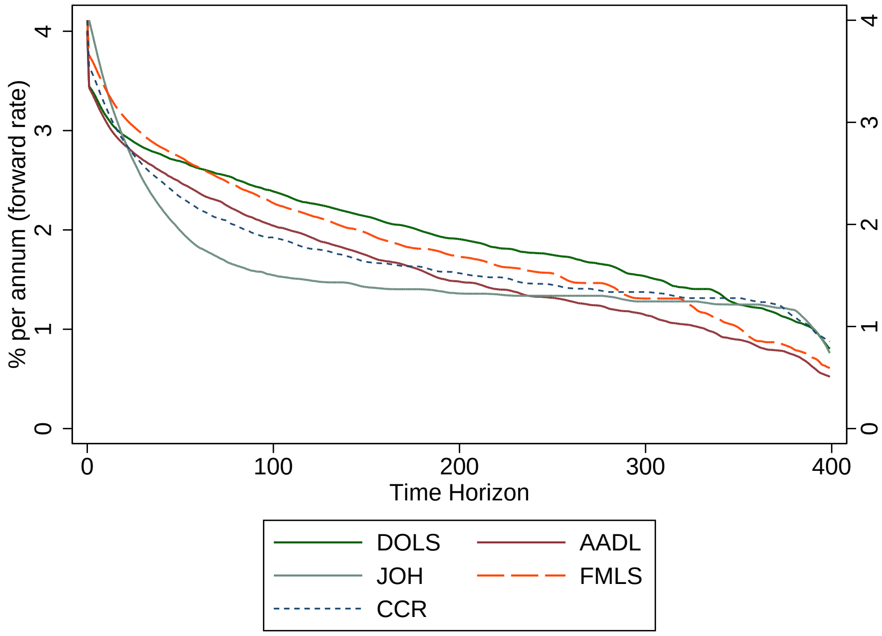

Figure 8: **Comparison of the Empirical Certainty Equivalent Forward Rates**: The five co-integration estimators are shown: the AADL, DOLS, FMLS, CCR and JOH, as described in Section 5 using equation (3).

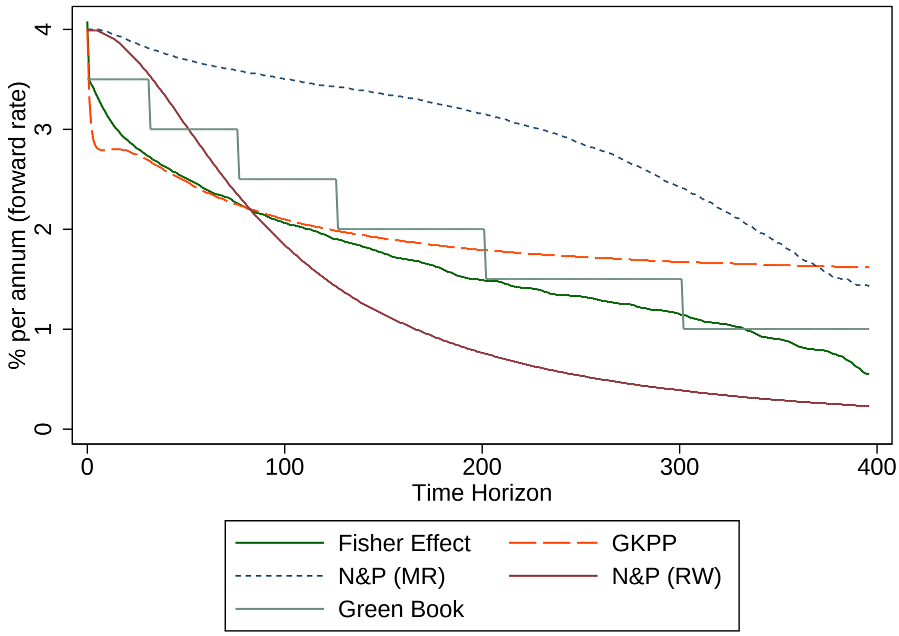

Figure 9: **Comparison of Empirical and Policy Certainty Equivalent Forward Rates**: The 'Fisher Effect' comes from our preferred AADL model. This is compared to the schedules estimated in the empirical literature by GKPP [25] and N&P [40], and used in practice by the U.K. Treasury Green Book guidelines [29].

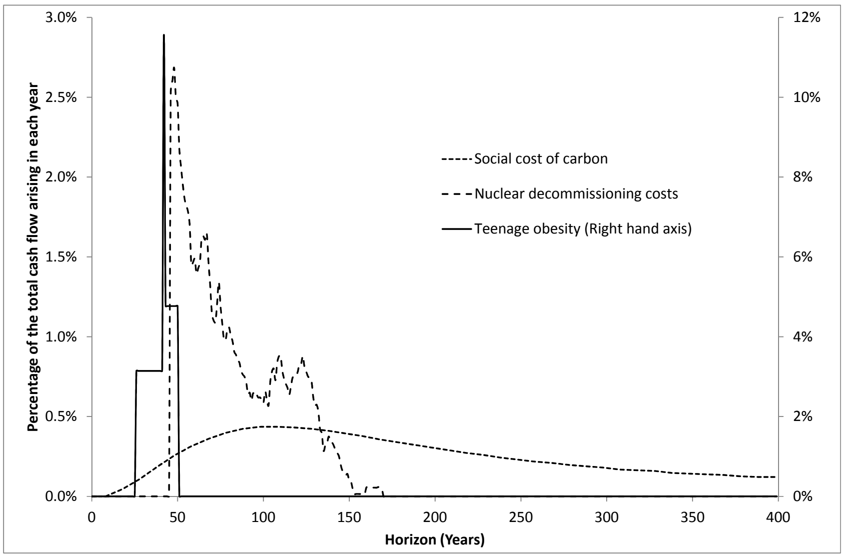

Figure 10: **Percentage of cash flows in each year of project**

9 Tables

Table 1. Augmented Dickey Fuller Tests (AR(4))						
	Smoothed (3yr M.A.)			Unsmoothed		
Test	All	Pre 1950	Post 1949	All	Pre 1950	Post 1949
ADF	-2.46	-1.51	-2.43	-3.29**	-1.68	-3.15**
ADF (trend)	-3.15*	-2.80	-2.24	-3.97**	-3.24	-3.10*

Significance levels: *** = 1%, ** = 5% and * = 10%

Table 2. Correlations (10 year Inflation)		
Years	N&P, GFD real	N&P, GFD nominal
1820 -1949	0.665	0.905
1950 -1999	0.355	0.677

Table 3. Auto-correlation (10 year inflation)

Order	N&P	GFD Nominal	GFD real
1	0.905	0.946	0.947
5	0.601	0.797	0.609
10	0.574	0.616	0.138

Table 4. Parameter stability tests

Test	L_c	*(p-val)*	*MeanF*	*(p-val)*	*SupF*	*(p-val)*
United States	0.151	(0.20)	1.099	(0.20)	2.130	(0.20)

Table 5. Co-integrating Regression Parameter Estimates

	DOLS		FMLS		CCR		AADL		JOH	
	Estimate	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*	*Estimate*	*s.e.*
r	3.652	0.421	3.995	0.889	4.301	0.805	3.173	1.336	0.087	2.062
θ	0.541	0.221	0.434	0.260	0.287	0.187	0.815	0.485	2.259	0.608

Table 6. Parameter estimates of the A matrix

	DOLS	FMLS	CCR	AADL	JOH
α_{11}	0.921	0.924	0.929	0.920	0.932
	(0.025)	(0.025)	(0.025)	(0.023)	(0.019)
α_{12}	-0.327	-0.273	-0.198	-0.464	-1.168
	(0.078)	(0.075)	(0.072)	(0.088)	(0.163)
α_{21}	0.028	0.021	0.010	0.034	0.022
	(0.021)	(0.023)	(0.024)	(0.018)	(0.008)
α_{22}	0.479	0.474	0.468	0.486	0.492
	(0.067)	(0.067)	(0.067)	(0.067)	(0.066)

Table 7. Comparison of Discounting Advice

Discounting Approach	SCC ($/tC)	% Δ N&P (RW)	NDA ($bn)	% Δ N&P (RW)	Teen. Ob. ($m)	% Δ N&P (RW)
Flat 4%	5.7	-42.5.0%	5.80	-35.8%	15.9	-8.7%
N&P [40] (Mean Reverting)	6.5	-38.0%	6.58	-27.1%	16.46	-5.3%
N&P [40] (Random Walk)	10.4	0%	9.0	0%	17.37	0
Green Book [29]	10.53	1.3%	10.21	13.1%	20.24	16.5%
Fisher Effect (AADL)	**14.2**	**36.2%**	**13.5**	**49.3%**	**23.2**	**33.5%**
GKPP [25]	14.4	38.5%	14.6	61.3%	24.9	43.2%
USEPA (Flat 2.5%) [56]	15.1	45.4%	15.5	71.7%	27.5	58.0%

10 Appendix: Simulation Procedure

The following steps are taken to simulate possible future paths of real interest rates and calculate the certainty equivalent discount rate:

1. We generate random values for $\mathbf{e}_t = [e_{1t}, e_{2t}]^\top$ from the bivariate Normal distribution $N(\mathbf{0}, \widehat{\Sigma_e})$ based on the estimated variance-covariance matrix $\widehat{\Sigma_e}$.

2. We obtain random values for the elements of A from the multivariate Normal distribution $N(vec\widehat{A}, \widehat{\Sigma_A})$ and generate random values for $\mathbf{u}_t = [u_{1t}, u_{2t}]^\top$ from equation (7)

3. We generate random values for r and θ from $N(\widehat{r}, se(\widehat{r}))$ and $N(\widehat{\theta}, se(\widehat{\theta}))$ respectively.

4. We use equations (5)-(6) to generate a random path for both the nominal interest rate, y_t, and the inflation rate, x_t. In this way, we calculate a future path for the real interest rate, $y_t - x_t$.

5. We check whether the estimated real interest rate fluctuates between the minimum and maximum values of the observed real interest rate of our sample for the US. If this condition is not satisfied, the simulated sample is discarded. Specifically, the min/max filter discards the entire simulated series if it exceeds 10% or is less than -4.15%, yet without direct restrictions on the underlying series of co-integrated nominal interest rates and inflation. This approach is undertaken in order to purge the simulation of explosive processes and is typical in many simulation exercises.[29] When the error process is highly persistent, the finite-sample distribution of the estimators can be heavily skewed and therefore generating shocks to the estimates from the symmetric normal distribution often results in explosive simulated processes.[30] Judging by the time of execution of the simulation, the number of discarded paths varies with the estimator at hand and is directly related to the persistence of the process determined by the estimates of the elements of the A matrix.[31] We should also note that in order to get a valid path, all real interest rate realizations in the 400-years period should be within the historical bounds.

6. Steps 1-5 are repeated as many times as needed to generate 300,000 simulated samples.

7. Finally, we calculate the *certainty-equivalent* discount factor and the *certainty equivalent* forward rate based on equations (1) and (3) respectively.

[29] N&P do something similar by discarding all simulated paths when the randomly drawn parameters lead to explosive processes.

[30] We would like to thank an anonymous referee for pointing this out to us.

[31] The execution time is approximately 1.9, 2.7, 30.2 and 21.6 times greater than that of AADL for DOLS, FMLS, CCR and JOH, respectively.