Appendix For Online Publication

Hot and Cold Seasons in the Housing Market, by Ngai and Tenreyro.

The Appendix is organized as follows. Section A provides supporting figures and tables and a supplementary description of housing price seasonality. Section B studies alternative models of the housing market. It starts with the simplest (frictionless) model, carrying out back-of-the-envelope calculations using the implied asset-pricing relations. It then examines the main canonical models in the housing market. Section C provides micro-foundations for the key assumption in the model, that is, the stochastic dominance of distribution functions for match qualities with higher vacancies. Section D discusses the efficiency properties of the model and studies its robustness to different modelling assumptions; in particular, it studies the case with moving costs and their role as alternative triggers of seasonality, and a different searching procedure, that allows the buyer and seller to contemplate their second-best offers. Section E presents all the derivations and proofs. Section F presents the model when the quality of the match is not observed by the seller and investigates different pricing mechanisms, including price posting by the seller. Section G describes additional statistics generated by the model.

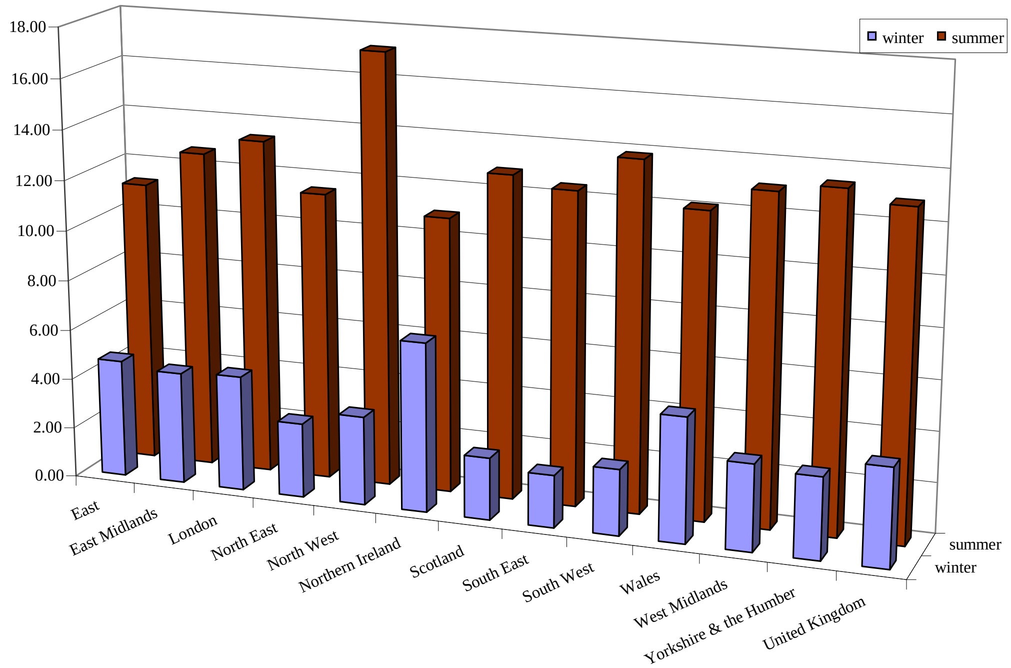
A Supporting Empirical Evidence

This Section of the Appendix first provides supporting figures and tables referred to in the text. It then provides an alternative description of the seasonality.

A.1 Supporting Material

Figure $A1$ shows similar results as Figure 1, for the period 1983-2007 using the constant-quality price index provided by the Department of Communities and Local Government (DGLG).

Figure $A1$: Average Annualized House Price Increases in Summer and Winter, 1983-2007



Note: Annualized price growth rates in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. DCLG, 1983-2007.

Figure $A2$ shows the average annualized real house price increases using the Land Registry data for 1996-2012. (The difference from Figure 1 in the text is that this shows real prices.)

Figure $A2$: Average Annualized Real House Price Increases in Summer and Winter, Land Registry 1996-2012

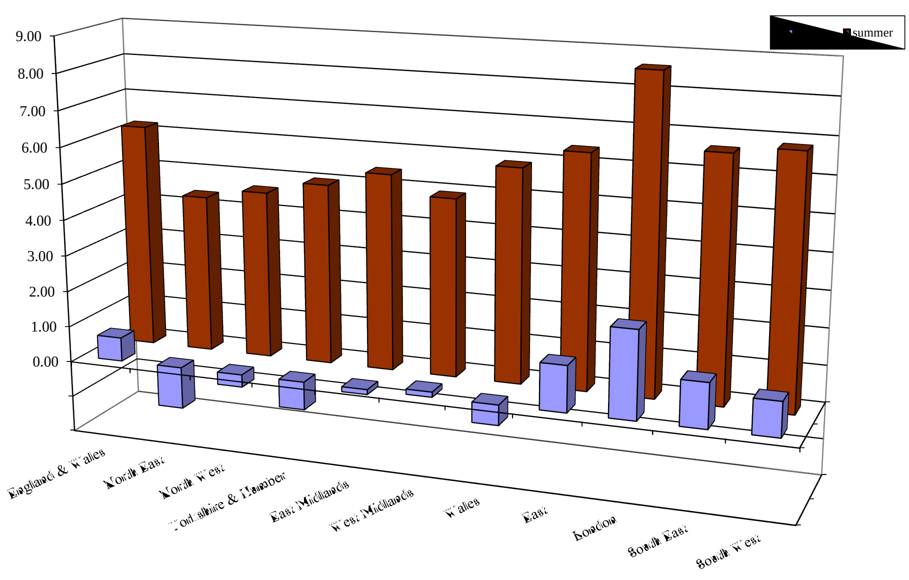


Table $A1$ shows the difference in annualized price growth rates between summers and winters in the United Kingdom differentiating among existing houses and new houses, and buyers who were former owner occupiers, and first time buyers. The data are available from Halifax for 1983 through 2005 (note the disaggregated data are not available for later years).

Table $A1$. Differences in price growth rates between summers and winters.

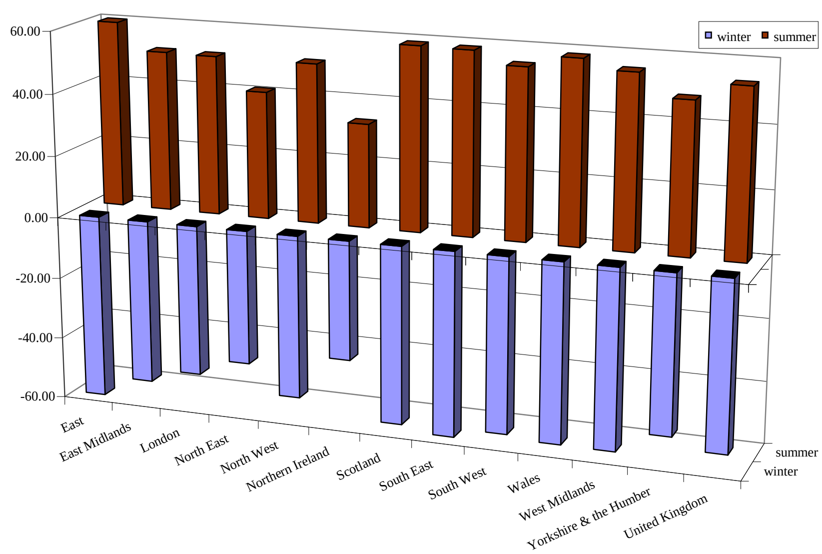
By type of house, buyer, and region. UK Halifax data. 1983-2005.

	All Houses (All buyers)		Existing houses (All buyers)		New houses (All buyers)		Former owner occupiers (All houses)		First-time buyer (All houses)	
	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error	Coef.	Std. Error
E. Anglia	9.7**	(3.1)	9.1**	(3.3)	3.0	(5.7)	11.1**	(3.1)	4.8	(3.8)
E.Midlands	10.6**	(2.9)	11.2**	(3.0)	2.8	(4.8)	11.8**	(3.0)	8.2**	(3.1)
Gr. London	5.4*	(2.6)	5.7*	(2.6)	13.3	(7.4)	4.6	(2.5)	5.4	(2.8)
N. West	7.8**	(2.4)	8.9**	(2.5)	-1.7	(6.1)	9.1**	(2.6)	4.9	(2.6)
North	2.0	(2.7)	2.3	(2.8)	2.7	(4.6)	1.0	(2.8)	3.3	(3.1)
S. East	7.0**	(2.6)	7.4**	(2.6)	2.7	(3.6)	8.0**	(2.6)	3.8	(2.7)
S. West	9.5**	(2.9)	9.7**	(3.0)	6.6	(4.4)	10.3**	(3.0)	5.6	(3.3)
W. Midlands	6.4*	(3.0)	6.3*	(3.0)	10.6	(6.2)	7.1*	(3.1)	5.7	(3.0)
Yorkshire&Humb	6.8*	(2.6)	7.5**	(2.7)	1.6	(5.0)	7.5**	(2.6)	6.3*	(3.0)
N. Ireland	8.2**	(2.9)	10.0**	(3.4)	9.0	(5.3)	6.6	(3.9)	8.6*	(4.3)
Scotland	9.8**	(2.3)	12.1**	(2.6)	12.4*	(5.6)	11.6**	(2.4)	4.8	(2.7)
Wales	9.0**	(3.1)	8.9**	(3.1)	0.7	(6.1)	9.4**	(3.1)	6.8*	(3.4)
U.K.	7.5**	(2.1)	8.1**	(2.1)	5.1*	(2.2)	8.3**	(2.1)	5.2*	(2.0)

Note: The Table shows the coefficients (and standard errors) on the dummy variable (Summer) in the regression g_t=a+b×Summer$_t$+e$_t$, where g_t is the first diference in the log-house price. The equations use quarterly data from 1983 to 2005. Robust standard errors in parentheses. * Significant at the 5%; ** significant at 1%.

Figure $A3$ shows the growth rates in the number of mortgages (a proxy for the number of transactions) in the two seasons from 1983 to 2007 for different U.K. regions. The data are compiled by the Council of Mortgage Lenders (CML). As the figure shows, the number of transactions increases sharply in the summer term and accordingly declines in the winter term.

Figure $A3$: Average Annualized Increases in the Number of Transactions in Summer and Winter, 1983-2007



Note: Annualized growth rates in the number of transactions in summers (second and third quarters) and winters (fourth and first quarters) in the U.K. and its regions. CML, 1983-2007.

Tables $A2a$ and $A2b$ complement Table 1 in the text, showing the differences in annualized nominal and real percentage changes in prices and transactions at more disaggregated levels of aggregation. The data come from the Land Registry and correspond to the period 1996 to 2012.

Table *A2a*. Difference in annualized percentage changes in house prices and sales volumes between semesters in the UK, by County/Unitary Authority.

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
Bath And North East Somerset	7.8***	(2.9)	6.9**	(3.1)	168.4***	(17.3)
Bedford	4.2	(2.8)	3.3	(3.0)	135.6***	(18.1)
Blackburn With Darwen	2.6	(4.3)	1.7	(4.5)	108.1***	(17.8)
Blackpool	0.1	(3.9)	-0.8	(4.0)	109.8***	(17.7)
Blaenau Gwent	-0.3	(5.9)	-1.2	(5.9)	89.3**	(18.5)
Bournemouth	8.6***	(3.0)	7.7**	(3.1)	152.6***	(17.8)
Bracknell Forest	7.5**	(2.9)	6.6**	(3.1)	152.7***	(22.5)
Bridgend	7.4**	(3.4)	6.4*	(3.6)	116.1***	(21.7)
Brighton And Hove	9.6***	(3.1)	8.7***	(3.3)	153.4***	(15.3)
Buckinghamshire	6.3**	(2.4)	5.4**	(2.6)	171.0***	(16.6)
Caerphilly	7.9**	(3.5)	6.9*	(3.7)	109.8***	(18.8)
Cambridgeshire	6.5***	(2.4)	5.6**	(2.6)	153.1***	(15.3)
Cardiff	6.7**	(2.6)	5.8**	(2.8)	151.5***	(16.8)
Carmarthenshire	11.2***	(3.7)	10.3**	(3.9)	139.4***	(20.0)
Central Bedfordshire	6.2**	(2.6)	5.3*	(2.8)	151.6***	(20.1)
Ceredigion	10.4**	(4.4)	9.5**	(4.5)	172.7***	(18.9)
Cheshire East	5.8**	(2.4)	4.8*	(2.6)	164.4***	(16.9)
Cheshire West And Chester	2.9	(2.5)	1.9	(2.8)	151.6***	(17.4)
City Of Bristol	8.1***	(3.0)	7.2**	(3.2)	146.0***	(16.5)
City Of Derby	9.4***	(3.1)	8.5**	(3.3)	130.0***	(13.8)
City Of Kingston Upon Hull	7.2*	(3.6)	6.3*	(3.7)	108.3***	(18.0)
City Of Nottingham	8.4**	(3.2)	7.5**	(3.4)	142.7***	(14.2)
City Of Peterborough	5.7*	(3.0)	4.8	(3.2)	117.2***	(19.9)
City Of Plymouth	8.0**	(3.3)	7.1**	(3.4)	138.0***	(17.9)
Conwy	11.3***	(3.8)	10.4***	(3.9)	117.1***	(18.3)
Cornwall	6.5**	(2.8)	5.6*	(3.0)	135.2***	(16.7)
Cumbria	5.9**	(2.7)	5.0*	(2.9)	145.4***	(16.3)
Darlington	8.1**	(3.5)	7.1*	(3.6)	124.3***	(18.5)
Denbighshire	5.2	(3.4)	4.3	(3.6)	104.2***	(14.4)
Derbyshire	5.8**	(2.7)	4.9*	(2.9)	138.3***	(16.4)
Devon	6.1**	(2.5)	5.2*	(2.8)	157.6***	(17.0)
Dorset	5.7**	(2.7)	4.7	(2.8)	161.3***	(14.8)
Durham	6.0*	(3.5)	5.1	(3.7)	122.9***	(19.0)
East Riding Of Yorkshire	6.1**	(2.9)	5.1*	(3.1)	150.5***	(19.6)
East Sussex	6.3**	(2.7)	5.4*	(2.9)	146.4***	(15.5)
Essex	5.0**	(2.5)	4.1	(2.7)	141.8***	(16.1)
Flintshire	4.3	(3.2)	3.3	(3.4)	128.7***	(20.0)

Table $A2a$ continued.

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
Gloucestershire	7.2***	(2.5)	6.3**	(2.7)	141.6***	(17.5)
Greater London	7.1***	(2.5)	6.2**	(2.6)	125.0***	(15.0)
Greater Manchester	6.5**	(2.7)	5.6**	(2.8)	119.6***	(15.5)
Gwynedd	8.0**	(3.9)	7.0*	(4.1)	138.6***	(16.9)
Halton	8.9**	(4.1)	8.0*	(4.3)	127.8***	(22.1)
Hampshire	5.6**	(2.4)	4.6*	(2.6)	162.4***	(17.2)
Hartlepool	10.7**	(5.2)	9.8*	(5.4)	110.2***	(19.3)
Herefordshire	7.1**	(2.8)	6.2**	(3.0)	164.1***	(15.5)
Hertfordshire	6.8***	(2.4)	5.8**	(2.6)	146.6***	(16.3)
Isle Of Anglesey	3.7	(4.3)	2.8	(4.4)	144.9***	(19.3)
Isle Of Wight	5.2*	(2.8)	4.3	(3.1)	124.8***	(16.1)
Kent	6.0**	(2.5)	5.1*	(2.7)	140.3***	(15.8)
Lancashire	5.8**	(2.8)	4.9	(3.0)	130.5***	(17.1)
Leicester	9.0***	(3.2)	8.1**	(3.4)	120.4***	(17.3)
Leicestershire	5.2**	(2.6)	4.3	(2.8)	150.3***	(19.1)
Lincolnshire	7.4***	(2.7)	6.5**	(2.9)	137.8***	(17.0)
Luton	4.5	(3.4)	3.6	(3.6)	117.4***	(17.8)
Medway	3.9	(2.8)	3.0	(3.1)	119.8***	(17.3)
Merseyside	6.2**	(2.9)	5.2*	(3.1)	125.5***	(15.4)
Merthyr Tydfil	1.6	(8.3)	0.6	(8.4)	107.9***	(24.8)
Middlesbrough	5.2	(4.6)	4.3	(4.7)	127.5***	(19.8)
Milton Keynes	5.3*	(2.8)	4.4	(3.0)	113.5***	(17.3)
Monmouthshire	9.3***	(3.4)	8.3**	(3.7)	183.6***	(21.3)
Neath Port Talbot	5.7	(4.7)	4.7	(4.7)	97.0**	(21.1)
Newport	5.2	(3.9)	4.3	(4.1)	125.8***	(20.5)
Norfolk	6.2**	(2.5)	5.3*	(2.8)	158.7***	(17.1)
North East Lincolnshire	8.4**	(3.6)	7.4**	(3.7)	127.2***	(17.6)
North Lincolnshire	5.3	(3.6)	4.4	(3.7)	145.4***	(16.8)
North Somerset	4.3	(2.6)	3.3	(2.9)	153.7***	(20.3)
North Yorkshire	7.5***	(2.6)	6.5**	(2.8)	156.4***	(18.6)
Northamptonshire	5.2**	(2.6)	4.3	(2.8)	136.3***	(18.9)
Northumberland	8.1**	(3.2)	7.2**	(3.4)	145.1***	(17.8)
Nottinghamshire	6.4**	(2.6)	5.5*	(2.8)	141.0***	(16.7)
Oxfordshire	6.7***	(2.3)	5.7**	(2.6)	176.1***	(16.1)
Pembrokeshire	7.3*	(3.9)	6.4	(4.1)	142.0***	(17.1)
Poole	5.8**	(2.8)	4.9	(3.0)	146.0***	(17.8)
Portsmouth	7.9***	(3.0)	7.0**	(3.0)	153.2***	(18.5)

Table $A2a$ continued.

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
Powys	6.1*	(3.6)	5.1	(3.8)	172.4***	(16.6)
Reading	6.2**	(2.9)	5.2*	(3.1)	124.4***	(17.5)
Redcar And Cleveland	3.0	(4.2)	2.0	(4.3)	125.4***	(19.2)
Rhondda Cynon Taff	5.4	(3.5)	4.5	(3.7)	114.0***	(18.9)
Rutland	6.3*	(3.5)	5.3	(3.7)	162.2***	(20.1)
Shropshire	7.2***	(2.6)	6.2**	(2.8)	161.5***	(17.5)
Slough	5.6*	(3.1)	4.7	(3.1)	120.1***	(19.0)
Somerset	7.1***	(2.6)	6.2**	(2.8)	154.9***	(19.0)
South Gloucestershire	7.0**	(2.9)	6.0*	(3.1)	143.0***	(19.7)
South Yorkshire	5.6**	(2.8)	4.7	(3.0)	124.6***	(16.5)
Southampton	8.1***	(2.8)	7.2**	(3.0)	141.5***	(15.2)
Southend-On-Sea	6.6**	(3.0)	5.7*	(3.1)	117.2***	(17.8)
Staffordshire	5.4**	(2.3)	4.4*	(2.5)	136.7***	(18.3)
Stockton-On-Tees	9.5**	(3.6)	8.5**	(3.7)	139.5***	(22.4)
Stoke-On-Trent	5.0	(3.6)	4.0	(3.8)	101.0***	(17.0)
Suffolk	8.1***	(2.6)	7.1**	(2.7)	149.2***	(16.7)
Surrey	7.0***	(2.5)	6.1**	(2.6)	165.8***	(15.9)
Swansea	8.1**	(3.4)	7.2**	(3.6)	138.8***	(18.3)
Swindon	6.4**	(2.7)	5.5*	(2.9)	131.2***	(20.5)
The Vale Of Glamorgan	9.1***	(2.9)	8.2***	(3.0)	149.3***	(19.2)
Thurrock	2.7	(2.8)	1.7	(3.1)	128.7***	(19.6)
Torbay	6.4**	(3.0)	5.5*	(3.1)	138.3***	(17.3)
Torfaen	6.0	(4.2)	5.1	(4.3)	142.8***	(23.9)
Tyne And Wear	6.0**	(2.9)	5.1	(3.1)	143.0***	(17.2)
Warrington	4.9	(3.0)	4.0	(3.2)	142.5***	(21.4)
Warwickshire	6.3***	(2.4)	5.4**	(2.5)	150.4***	(16.9)
West Berkshire	6.2**	(2.6)	5.3*	(2.7)	163.7***	(20.0)
West Midlands	5.3**	(2.5)	4.4	(2.7)	123.3***	(14.7)
West Sussex	7.1***	(2.6)	6.2**	(2.8)	152.4***	(16.4)
West Yorkshire	6.6**	(2.7)	5.7*	(2.9)	135.6***	(16.3)
Wiltshire	7.2***	(2.4)	6.3**	(2.6)	172.3***	(19.0)
Windsor And Maidenhead	8.4***	(2.6)	7.5***	(2.6)	169.0***	(18.0)
Wokingham	4.0	(2.7)	3.1	(2.8)	167.1***	(18.1)
Worcestershire	6.0**	(2.4)	5.1*	(2.6)	160.1***	(17.6)
Wrekin	4.2	(3.1)	3.3	(3.3)	126.4***	(19.3)
Wrexham	7.1*	(3.8)	6.2	(3.9)	133.7***	(22.8)
York	7.1***	(2.7)	6.2**	(2.8)	169.1***	(19.4)

Note: Average differences (and standard errors), by county for 1995-2012.

*Significant at 10%; ** 5%; *** 1%. Source: Land Registry.

Table $A2b$. continued. Difference in annualized percentage changes in house prices and sales volumes between semesters, by London Borough

	Nominal house price		Real house price		Volume of Sales	
Region	Difference	Std. Error	Difference	Std. Error	Difference	Std. Error
Barking And Dagenham	5.7	(3.5)	4.8	(3.7)	105.5***	(19.1)
Barnet	5.3**	(2.5)	4.4	(2.7)	131.2***	(15.7)
Bexley	3.6	(2.6)	2.7	(2.8)	123.9***	(16.2)
Brent	2.6	(2.9)	1.6	(3.0)	111.1***	(18.2)
Bromley	6.6**	(2.6)	5.7*	(2.8)	132.8***	(16.1)
Camden	6.9**	(3.3)	6.0*	(3.4)	131.2***	(16.8)
City Of Westminster	12.2***	(2.9)	11.2***	(3.0)	102.5***	(17.3)
Croydon	4.5	(2.8)	3.6	(2.9)	107.5***	(15.8)
Ealing	5.9**	(2.7)	5.0*	(2.9)	117.6***	(15.9)
Enfield	5.6**	(2.7)	4.6	(2.8)	111.5***	(16.6)
Greenwich	4.6	(2.8)	3.7	(2.9)	124.9***	(18.0)
Hackney	8.1**	(3.2)	7.1**	(3.4)	120.8***	(21.1)
Hammersmith And Fulham	10.4***	(3.1)	9.5***	(3.2)	150.3***	(19.5)
Haringey	6.0**	(3.0)	5.1	(3.1)	130.1***	(17.0)
Harrow	9.1***	(3.0)	8.1**	(3.1)	129.6***	(15.9)
Havering	5.8**	(2.6)	4.8*	(2.7)	122.2***	(17.7)
Hillingdon	5.9**	(2.6)	5.0*	(2.8)	126.9***	(16.6)
Hounslow	9.6***	(2.9)	8.6***	(3.0)	123.2***	(16.6)
Islington	9.0***	(3.0)	8.1**	(3.1)	138.8***	(17.1)
Kensington And Chelsea	10.4***	(3.4)	9.5***	(3.5)	93.3***	(17.6)
Kingston Upon Thames	7.8**	(3.2)	6.9**	(3.3)	144.3***	(16.6)
Lambeth	9.5***	(3.0)	8.5***	(3.2)	140.7***	(17.1)
Lewisham	7.5***	(2.8)	6.6**	(3.0)	140.3***	(16.9)
Merton	8.2***	(3.0)	7.2**	(3.1)	138.9***	(15.8)
Newham	3.4	(3.9)	2.4	(4.1)	50.9**	(19.3)
Redbridge	4.2	(2.9)	3.3	(3.1)	116.9***	(14.4)
Richmond Upon Thames	7.4**	(3.0)	6.5**	(3.1)	173.6***	(17.1)
Southwark	8.7***	(3.1)	7.8**	(3.2)	127.6***	(17.9)
Sutton	4.3	(2.9)	3.4	(3.0)	129.0***	(16.3)
Tower Hamlets	6.1**	(3.0)	5.2*	(3.1)	130.3***	(20.2)
Waltham Forest	4.0	(3.1)	3.1	(3.3)	108.0***	(15.1)
Wandsworth	11.5***	(3.1)	10.6***	(3.2)	151.1***	(17.2)

Table $A3$ complements Table 1 in the text, showing the differences in annualized nominal and real percentage changes in prices between summers and regions in the United Kingdom using the DCGL and Halifax datasets, as well as the corresponding figures for transactions, using CML. The data cover the period 1983-2007.

Table $A3$: Difference in Annualized Percentage Changes in U.K. House Prices (Nominal and Real) and Transactions between Summer and Winter, by Region. using DCGL, Halifax (prices) and CML (transactions). 1983-2007.

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	6.5	(3.6)	4.9	(3.5)
East Midlands	8.2*	(3.1)	6.4*	(3.1)
Gr. London	8.8**	(3.3)	7.0*	(3.4)
North East	8.5*	(4.0)	6.8	(3.9)
North West	13.7***	(3.3)	12.6**	(3.2)
Northern Ireland	4.2	(3.4)	2.4	(3.5)
Scotland	10.4***	(2.8)	8.6**	(2.7)
South East	10.4**	(3.5)	8.7*	(3.3)
South West	11.2**	(3.4)	9.4**	(3.5)
Wales	7.2*	(3.5)	5.4	(3.4)
West Midlands	9.6**	(3.1)	7.8*	(3.1)
Yorkshire & the Humber	10.1**	(3.1)	8.3**	(3.1)
United Kingdom	9.0***	(2.3)	7.2**	(2.3)

	Nominal house price		Real house price	
Region	Difference	Std. Error	Difference	Std. Error
East Anglia	9.9**	(3.6)	8.1*	(3.7)
East Midlands	10.2**	(3.4)	8.4*	(3.4)
Gr. London	5.7	(3.0)	3.9	(3.2)
North East	2.2	(2.9)	0.4	(2.9)
North West	8.0**	(2.7)	6.2*	(2.5)
Northern Ireland	6.1	(3.4)	4.3	(3.5)
Scotland	9.3***	(2.3)	7.5**	(2.3)
South East	7.1*	(3.0)	5.3	(3.1)
South West	9.3*	(3.5)	7.5*	(3.5)
Wales	7.8*	(3.3)	6.0	(3.3)
West Midlands	6.0	(3.5)	4.2	(3.5)
Yorkshire & the Humber	7.3*	(2.9)	5.5	(2.8)
United Kingdom	7.6**	(2.4)	5.8*	(2.4)

Region	Difference	Std. Error
East Anglia	119.4***	(11.8)
East Midlands	104.3***	(11.2)
Gr. London	99.8***	(11.6)
North East	84.1***	(9.8)
North West	103.5***	(9.0)
Northern Ireland	71.5***	(12.2)
Scotland	116.2***	(9.8)
South East	118.0***	(9.7)
South West	111.0***	(8.8)
Wales	115.9***	(13.9)
West Midlands	112.9***	(9.5)
Yorkshire & the Humber	98.9***	(8.2)
United Kingdom	107.7***	(8.4)

A.2 Moving patterns

A reading of the empirical literature (Goodman, 1991), suggests then that the school calendar might be a likely trigger; as noted by Goodman, however, parents of school age children are less than a third of total movers, and hence one needs an amplification effect. Our model provides such mechanism.

Figure $A4$ illustrates the fact that most people (not just parents of school-age children) move in the summer months. The data are based on the American Housing Survey (1999 and 2001).

Figure $A4$: Monthly Distribution of Moves, by Life Cycle Stage

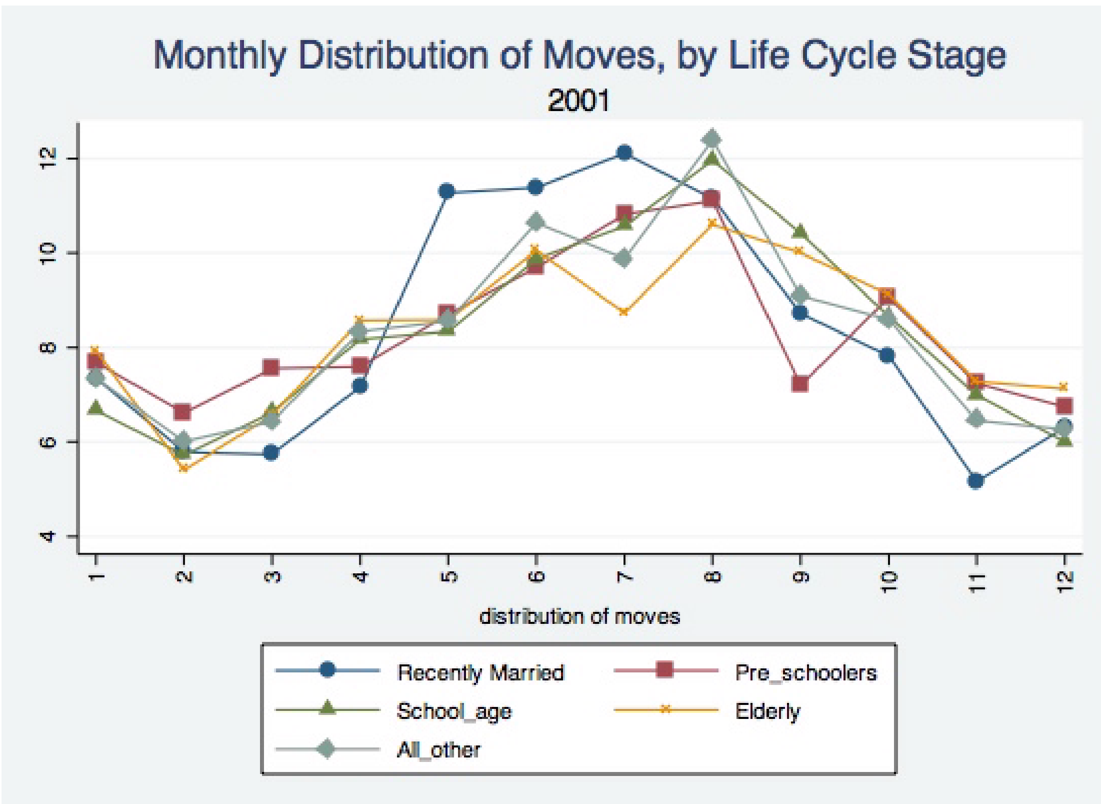
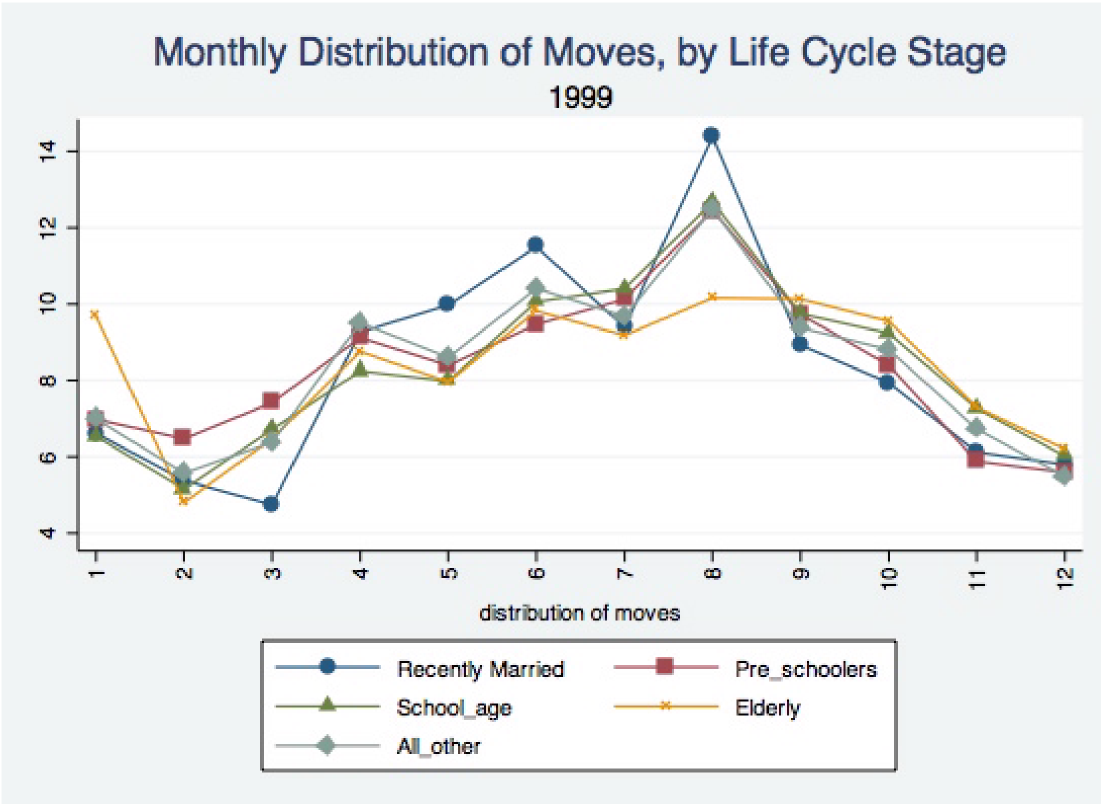


Figure $A4$ continued



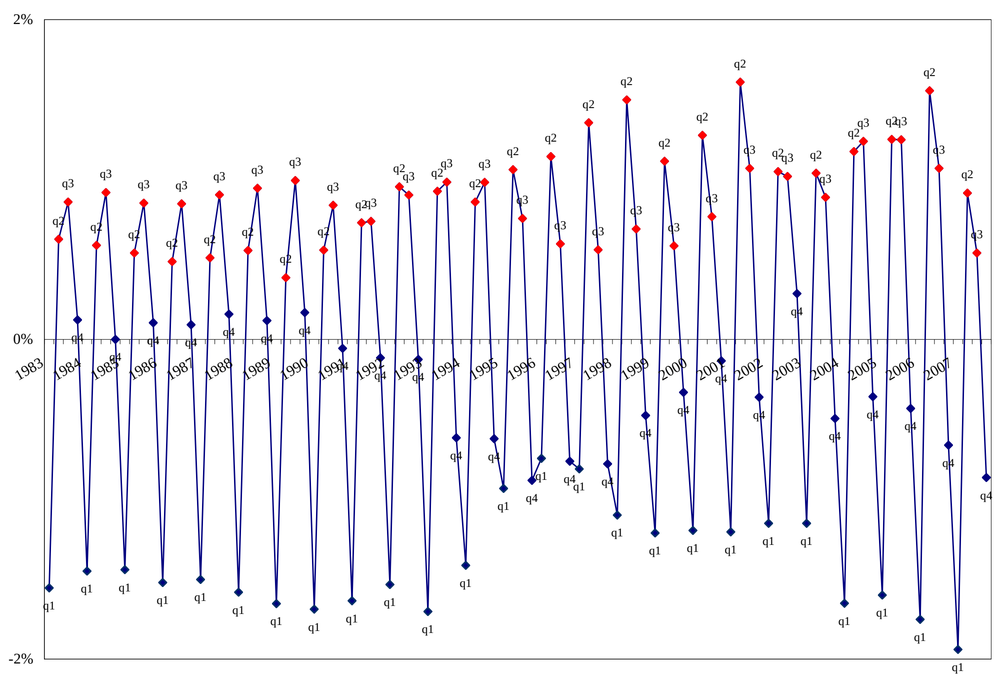
A.3 Aggregate Seasonality (as Reported by Publishers of House Price Indexes)

A first indication that house prices display seasonality comes from the observation that most publishers of house price indexes directly report SA data. Some publishers, however, report both SA and NSA data, and from these sources one can obtain a first measure of seasonality, as gauged by the publishers. For example, in the United Kingdom, Halifax publishes both NSA and SA house price series. Using these two series we computed the (logged) seasonal component of house prices as the ratio of the NSA house price series, P_t, relative to the SA series, P_t^*, from 1983:01 to 2007:04, $\left\{\ln \frac{P_t}{P_t^*}\right\}$. This seasonal component is plotted in Figure $A3$. (Both the NSA and the SA series correspond to the United Kingdom as a whole.)

In the United States, both the Office of Federal Housing Enterprise Oversight (OFHEO)'s house price index and the Case-Shiller index published by Standard & Poor's (S&P) are published in NSA and SA form. Figure $A4$ depicts the seasonal component of the OFHEO series for the US as a whole, measured as before as $\left\{\ln \frac{P_t}{P_t^*}\right\}$, from 1991:01 through to 2007:04. And Figure $A5$ shows the corresponding plot for the Case-Shiller index corresponding to a composite of 10 cities, with the data running from 1987:01 through to 2007:04. (The start of the sample in all cases is dictated by data availability.)

All figures seem to show a consistent pattern: House prices in the second and third quarters tend to rise above trend (captured by the SA series), and prices in the fourth, and particularly in the first quarter, tend to be in general at or below trend. The figures also make it evident that the extent of price seasonality is more pronounced in the United Kingdom than in the United States as a whole, though as shown in the text, certain cities in the United States seem to display seasonal patterns of the same magnitude as those observed in the United Kingdom. (Some readers might be puzzled by the lack of symmetry in Figure $A4$, as most expect the seasons to cancel out; this is exclusively due to the way OFHEO performs the seasonal adjustment;[1] for the sake of clarity and comparability across different datasets, we base our analysis only on the "raw", NSA series and hence the particular choice of seasonal adjustment by the publishers is inconsequential.)

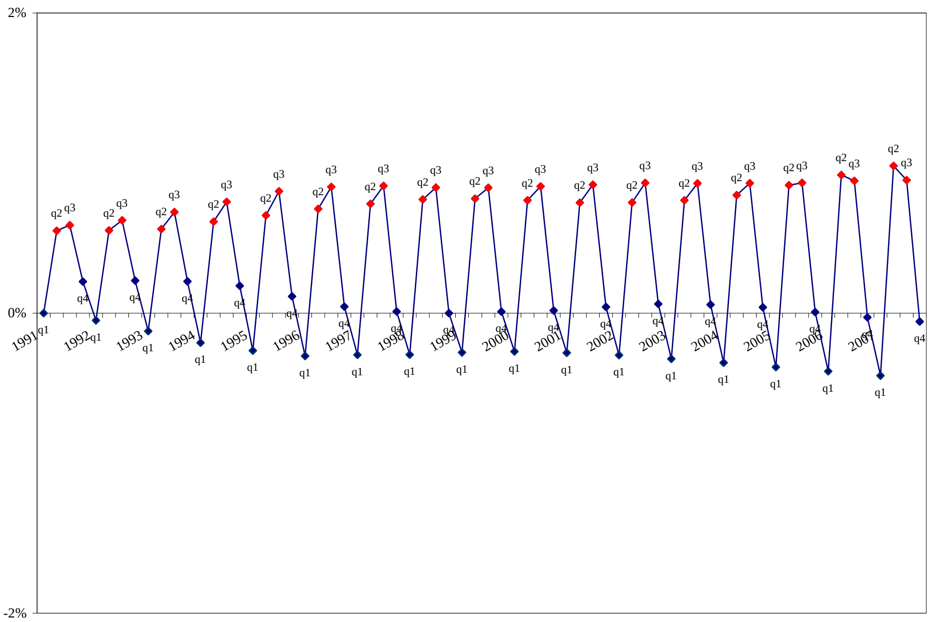
Figure $A3$: Seasonal Component of House Prices in the United Kingdom, 1983-2007



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$. P_t is the NSA and P_t^* the SA index. Source: Halifax.

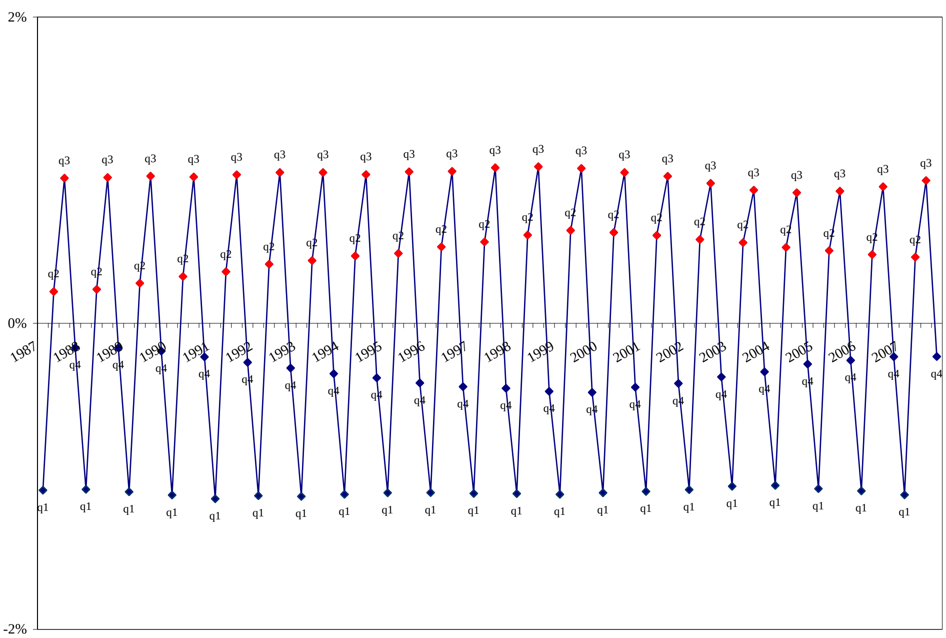
[1] OFHEO uses the Census Bureau's X-12 ARIMA procedure for SA; it is not clear, however, what the exact seasonality structure chosen is.

Figure $A4$: Seasonal Component of House Prices in the United States, 1991-2007



Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$; P_t is the NSA and P_t^* the SA index. Source: OFHEO.

Figure $A5$: Seasonal Component of House Prices in U.S. cities, 1987-2007

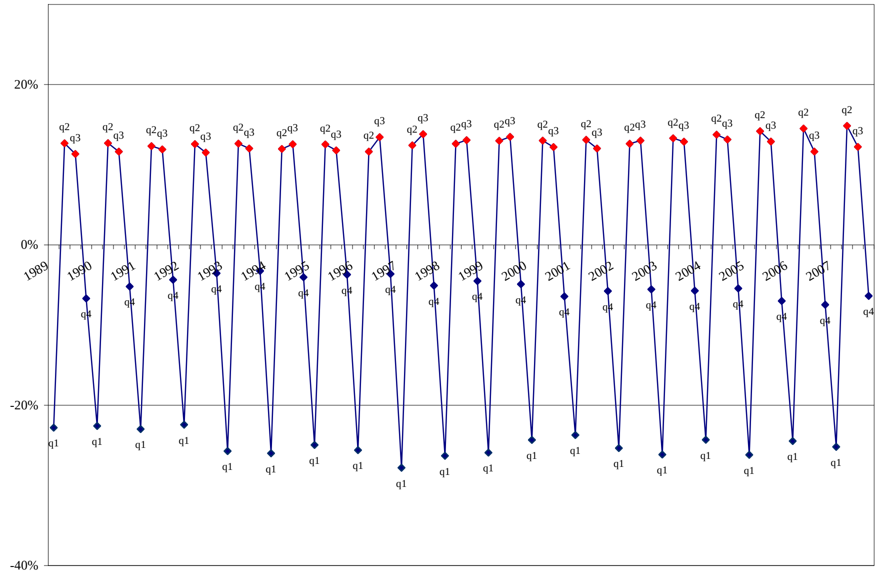


Note: The plot shows $\left\{\ln \frac{P_t}{P_t^*}\right\}$. P_t is the NSA and P_t^* the SA index.

Source: Case-Shiller 10-city composite.

Last, but not least, the U.S. National Association of Realtors (NAR) publishes data on transactions both with and without seasonal adjustment. Figure $A6$ plots the seasonal component of house transactions, measured (as before) as the (logged) ratio of the NSA number of transactions Q_t, divided by the SA number of transactions Q_t^*:$\left\{\ln \frac{Q_t}{Q_t^*}\right\}$.

Figure $A6$: Seasonal Component of Housing Transactions in the United States, 1989-2007



Note: The plot shows $\left\{\ln \frac{Q_t}{Q_t^*}\right\}$; Q_t is the NSA and Q_t^* the SA number of transactions.

Source: NAR.

The seasonal pattern for transactions is similar to that for prices: Transactions surge in the second and third quarters and stagnate or fall in the fourth and first quarters. (In the United Kingdom only NSA data for transactions are available from the publishers.)

B Alternative Models of the Housing Market

We argued previously that the predictability and size of the seasonal variation in housing prices pose a challenge to existing models of the housing market. We discuss the key challenge using a simple, frictionless model and then we turn to the canonical models in the housing literature.

B.1 Frictionless Model

The equilibrium condition embedded in most dynamic general equilibrium models states that the marginal benefit of housing services should equal the marginal cost. Following Poterba (1984) the asset-market equilibrium conditions for any seasons $j = s$ (summer), w (winter) at time t is:[2]

$$d_{t+1,j'} + (p_{t+1,j'} - p_{t,j}) = c_{t,j} \cdot p_{t,j}, \tag{B.1}$$

[2] See also Mankiw and Weil (1989) and Muellbauer and Murphy (1997), among others.

where j' is the corresponding season at time $t+1$, $p_{t,j}$ and $d_{t,j}$ are the real asset price and rental price of housing services, respectively; $c_{t,j} \cdot p_{t,j}$ is the real gross (gross of capital gains) $t-$period cost of housing services of a house with real price $p_{t,j}$; and $c_{t,j}$ is the sum of after-tax depreciation, repair costs, property taxes, mortgage interest payments, and the opportunity cost of housing equity. Note that the formula assumes away risk (and hence no expectation terms are included); this is appropriate in this context because we are focusing on a "predictable" variation of prices.[3] As in Poterba (1984), we make the following simplifying assumptions so that service cost rates are a fixed proportion of the property price, though still potentially different across seasons ($c_{t,j} = c_{t+2,j} = c_j$, $j = s, w$): 1) Depreciation takes place at rate δ_j, $j = s, w$, constant for a given season, and the house requires maintenance and repair expenditures equal to a fraction κ_j, $j = s, w$, which is also constant for a given season. 2) The income tax-adjusted real interest rate and the marginal property tax rates (for given real property prices) are constant over time, though also potentially different across seasons; these rates are denoted, respectively as r_j and τ_j, $j = s, w$ (in the data, as seen, these are actually constant across seasons; we shall come back to this point below).[4] This yields $c_j = \delta_j + \kappa_j + r_j + \tau_j$, for $j = s, w$.

Subtracting (B.1) from the corresponding expression in the following season and using the condition that there is no seasonality in rents ($d_w \approx d_s$), we obtain:

$$\frac{p_{t+1,s} - p_{t,w}}{p_{t,w}} - \frac{p_{t,w} - p_{t-1,s}}{p_{t-1,s}} \frac{p_{t-1,s}}{p_{t,w}} = c_w - c_s \cdot \frac{p_{t-1,s}}{p_{t,w}}. \tag{B.2}$$

Using the results from the Department of Communities and Local Governments (DCLG), real differences in house price growth rates for the entire United Kingdom are $\frac{p_s - p_w}{p_w} \simeq 8.25\%$, $\frac{p_w - p_s}{p_s} \simeq 1.06\%$,[5] the left-hand side of (B.2) equals $7.2\% \approx 8.25\% - 1.06\% \cdot \frac{1}{1.0106}$. Therefore, $\frac{c_w}{c_s} = \frac{0.072}{c_s} + \frac{1}{1.0106}$. The value of c_s can be pinned-down from equation (B.1) with $j = s$, depending on the actual rent-to-price ratios in the economy. In Table $B1$, we summarize the extent of seasonality in service costs $\frac{c_w}{c_s}$ implied by the asset-market equilibrium conditions, for different values of d/p (and hence different values of $c_s = \frac{d_w}{p_s} + \frac{p_w - p_s}{p_s} = \frac{d_w}{p_s} + 0.0106$).

[3] Note that Poterba's formula also implicitly assumes linear preferences and hence perfect intertemporal substitution. This is a good assumption in the context of seasonality, given that substitution across semesters (or relatively short periods of time) should in principle be quite high.

[4] We implicitly assume the property-price brackets for given marginal rates are adjusted by inflation rate, though strictly this is not the case (Poterba, 1984): inflation can effectively reduce the cost of homeownership. This, however, should not alter the conclusions concerning seasonal patterns emphasized here. As in Poterba (1984) we also assume that the opportunity cost of funds equals the cost of borrowing.

[5] In the empirical Section we computed growth rates using difference in logs; the numbers are very close using $\frac{p_{t+1,j'} - p_{t,j}}{p_{t,j}}$ instead. We use annualized rates as in the text; using semester rates of course leads to the same results.

Table $B1$: Ratio of Winter-To-Summer Cost Rates

(annualized) d/p Ratio	Relative winter cost rates $\frac{c_w}{c_s}$
1.0%	448%
2.0%	334%
3.0%	276%
4.0%	241%
5.0%	218%
6.0%	201%

As the table illustrates, a remarkable amount of seasonality in service costs is needed to explain the differences in housing price inflation across seasons. Specifically, assuming annualized rent-to-price ratios in the range of 2 through 5 percent, total costs in the winter should be between 334 and 218 percent of those in the summer. Depreciation and repair costs ($\delta_j + \kappa_j$) might be seasonal, being potentially lower during the summer.[6] But income-tax-adjusted interest rates and property taxes ($r_j + \tau_j$), two major components of service costs are not seasonal. Since depreciation and repair costs are only part of the total costs, given the seasonality in other components, the implied seasonality in depreciation and repair costs across seasons in the UK is even larger. Assuming, quite conservatively, that the a-seasonal component ($r_j + \tau_j = r + \tau$) accounts for only 50 percent of the service costs in the summer ($r + \tau = 0.5c_s$), then, the formula for relative costs $\frac{c_w}{c_s} = \frac{\delta_w + \kappa_w + 0.5c_s}{\delta_s + \kappa_s + 0.5c_s}$ implies that the ratio of depreciation and repair costs between summers and winters is $\frac{\delta_w + \kappa_w}{\delta_s + \kappa_s} = 2\frac{c_w}{c_s} - 1$.[7] For rent-to-price ratios in the range of 2 through 5 percent, depreciation and maintenance costs in the winter should be between 568 and 336 percent of those in the summer. (If the a-seasonal component ($r + \tau$) accounts for 80 percent of the service costs ($r + \tau = 0.8c_s$), the corresponding values are 1571 and 989 percent). By any metric, these figures seem extremely large and suggest that a deviation from the simple asset-pricing equation is called for. Similar calculations can be performed for different regions in the US; as expressed before, though the extent of price seasonality for the US as a whole is lower than in the UK, seasonality in several US cities is comparable to that in the UK and would therefore also imply large seasonality in service costs, according to condition (B.1).

[6] Good weather can help with external repairs and owners' vacation might reduce the opportunity cost of time—though for this to be true it would be key that leisure were not too valuable for the owners.

[7] Call λ the aseasonal component as a fraction of the summer service cost rate: $r + \tau = \lambda c_s$, $\lambda \in (0,1)$ (and hence $\delta_s + \kappa_s = (1-\lambda)c_s$). Then: $\frac{c_w}{c_s} = \frac{\delta_w + \kappa_w + \lambda c_s}{\delta_s + \kappa_s + \lambda c_s} = \frac{\delta_w + \kappa_w + \lambda c_s}{c_s}$. Or $c_w = \delta_w + \kappa_w + \lambda c_s$. Hence: $\frac{c_w - \lambda v_s}{(1-\lambda)c_s} = \frac{\delta_w + \kappa_w}{(1-\lambda)c_s}$; that is $\frac{\delta_w + \kappa_w}{\delta_s + \kappa_s} = \frac{c_w}{(1-\lambda)c_s} - \frac{\lambda}{1-\lambda}$, which is increasing in λ for $\frac{c_w}{c_s} > 1$.

B.2 Other search models of housing

We focus on the canonical models of Krainer (2001), Novy-Marx (2009) and Piazzesi and Schneider (2009). In these models, variations in reservation prices depend, correspondingly, on three factors: (1) variations in the value of houses common to all buyers, (2) variations in the ratio of the number of buyers to the number of sellers in the housing market, and (3) variations in the buyer's belief about the house price-to-dividend ratio. The three papers are also different in how they model search frictions. In general there are two types of search frictions: (1) finding a house or buyer, modelled as an aggregate matching function; and (2) how much a buyer likes the house, modelled as a stochastic match-specific housing utility.

Krainer (2001) focuses on the second search friction and assumes housing utility is $d^i = \varepsilon^i + x$, where the stochastic value ε^i is match-specific but x is common to all buyers. He analyzes how house prices vary when x follows a Markov chain between high x_H and low x_L, with a persistent parameter λ. His model implies a negative correlation between price and time-to-sell across 'hot' ($x_t = x_H$) and 'cold' ($x_t = x_L$) markets. The case in which λ equal to zero delivers a deterministic periodic steady state with x switching between x_L and x_H. In other words, the model has a prediction for 'hot' and 'cold' seasons when $\lambda = 0$. However, Figure 3*b* and 5*b* of his paper show that there is virtually no change in price when $\lambda = 0$. In fact Krainer has noted the small change in price in his discussion of Figure 5*b*. Quoting from his paper, "in this model, prices are sticky in that they do not drop too far in down markets. Rather liquidity dries up. The reverse is true in up markets. Prices do not rise as high, but liquidity improves." The intuition in Krainer (2001) is similar to that in the frictionless model presented above. In the absence of a thick-market effect, if prices are too high in a given season, buyers prefer to wait, as the option value of waiting is high. Moreover, as agents know that in the next period the housing utility of owning a house will be low again, they are not willing to pay a high price. (One would need huge seasonality in the house dividend x_H/x_L to generate any seasonality in prices.) Like in Krainer (2001), we also focus on the second friction but unlike it, fluctuations in price in our model are driven by the thick-market-effect where the draws ε^i are stochastically higher in the season with more buyers and sellers. In order to generate seasonality, it is critical to have both i) persistence in the match quality (otherwise the increase in prices due to a temporary high house dividend will be small) and ii) a mechanism whereby the quality of transacted houses are seasonal.

Piazzesi and Schneider (2009) focuses on the first search friction and use an aggregate matching function as in Pissarides (2000, chapter 1). They analyze house prices when there is an exogenous change in buyers' beliefs about houses' price-to-dividend ratios. In other words, their mechanism can deliver any change in price levels as it is specified by the exogenous change in beliefs. For this to explain seasonality in prices, buyers' beliefs have to shift up and down regularly across seasons. We think this mechanism is thus unlikely to generate the seasonality in the data.

Novy-Marx (2009) has both types of search frictions (as in Pissarides (2000, chapter 6)), except that he uses different entry conditions for buyers and sellers. He analyzes house prices when the ratio of buyers to sellers (using his notation, θ) varies due to changes in buyers' relative propensity to enter. This mechanism could potentially generate 'hot' and 'cold' seasons if θ is higher in the hot season. We extend his model to allow for a seasonal cycle and study its quantitative implications. More specifically, we derive a periodic steady state where θ alternates between high θ^s and low θ^w deterministically. We then examine the implied seasonality in prices.

B.2.1 Seasonal Cycle in Novy-Marx (2009)

The original Novy-Marx model can be summarized as follow. There are two sources of search frictions. (1) An aggregate matching function that implies encounter rates for buyers and sellers given by:

$$\lambda_b(\theta) = \lambda\theta^{\eta}; \qquad \lambda_s(\theta) = \theta\lambda_b(\theta) = \lambda\theta^{\eta+1}, \eta > -1 \tag{B.3}$$

where $\theta = \frac{m_b}{m_s}$ is the buyer-to-seller ratio, and (2) there is a match-specific transaction value ε with cdf $\Phi(.)$. A transaction is an absorbing state. The total surplus created by a transaction is equal to $\varepsilon - V_b^* - V_s^*$ where V_b^* and V_s^* are the value for buyer and seller while searching, so the threshold for a transaction satisfies $\varepsilon^* \equiv V_b^* + V_s^*$ and the Bellman equation are

$$rV_i^* = -c_i + \lambda_i [1 - \Phi(\varepsilon^*)] E[(V_i(\varepsilon) - V_i^*) \mid \varepsilon > \varepsilon^*]$$

for $i = b, s$, where c_i is search cost and $V_i(\varepsilon)$ is the value for agent i after the transaction ε goes through, so $V_b(\varepsilon) + V_s(\varepsilon) = \varepsilon$. The total surplus is divided between the buyer and seller via Nash bargaining according to their bargaining power β_i, for $i = b, s$

$$V_i(\varepsilon) - V_i^* = \beta_i [V_b(\varepsilon) + V_s(\varepsilon) - (V_b^* + V_s^*)] = \beta_i(\varepsilon - \varepsilon^*) \tag{B.4}$$

which reduces the Bellman equation, $i = b, s$

$$rV_i^* = -c_i + \lambda_i\beta_i\nu_{\varepsilon}(\varepsilon^*) \tag{B.5}$$

Summing it up across buyers and sellers, and using the definition of ε^* implies an implicit function for the threshold ε^*

$$\Lambda(\theta)\nu(x) = rx + c_b + c_s \tag{B.6}$$

where

$$\begin{aligned}\Lambda(\theta) &= \beta_b\lambda_b(\theta) + \beta_s\lambda_s(\theta) \\ \nu(x) &\equiv [1-\Phi(x)]E(\varepsilon - x \mid \varepsilon > x) = \int_x (z-x)\,d\Phi(z)\end{aligned} \tag{B.7}$$

The model is analyzed in two steps. First, given θ, he solves for equilibrium ε^* using equation (B.6). Then, he uses the Bellman equation (B.5) to derive the equilibrium values (V_b^*, V_s^*). Time-to-sell is the expected duration for seller to exit the market $E(T_s) = \frac{1}{[1-\Phi_\varepsilon(\varepsilon^*)]\lambda_s}$. The reservation/minimum price $p(\varepsilon^s) = V_s^*$ is given by (B.5).The transaction price $p(\varepsilon) = V_s(\varepsilon)$ is given by equation (B.4). The main result of the paper is Figure 4 which reports a negative correlation between $E(T_s)$ and V_s^* across markets with different θ. Finally he specifies entry conditions for buyers and sellers to solve for equilibrium θ^*.

We now introduce a seasonal cycle into the model where θ alternates between θ^s and θ^w deterministically. As in the original Novy-Marx, θ^s and θ^w are determined independently through the entry conditions. So we can proceed to study the seasonality in prices for any given (θ^s, θ^w).

Let U_i^j be the value of agent i searching in season $j = s, w$. The choice to denote this value using a different notation is very important. Here we are studying a seasonal cycle where tightness switches between θ^s and θ^w deterministically whereas in Novy-Marx the value $V_i^*(\theta^s)$ refers to the case that tightness remains at θ^s for all periods. This distinction is very important as it will become clear very soon that U_i^s/U_i^w is much smaller than $V_i^*(\theta^s)/V_i^*(\theta^w)$ for any given levels of (θ^s, θ^w).

Let ε^j be the corresponding threshold for season $j = s, w$:

$$\varepsilon^j = U_b^j + U_s^j \tag{B.8}$$

The Bellman equation for the value of search for agent $i = b, s$ in season s is

$$U_i^s = \delta U_i^w + \lambda_i^s \beta_i \delta \nu(\varepsilon^s) - c_i, \tag{B.9}$$

where δ is the discount factor between seasons (6 months). A similar Bellman equation holds for season w. So equation (B.9) is a set of four equations. Summing up the value for $i = b$ and s, and using definition of threshold (B.8), we have two equations to solve for equilibrium $(\varepsilon^s, \varepsilon^w)$ for given (θ^s, θ^w):

$$\begin{aligned} \varepsilon^s &= \delta\varepsilon^w + \Lambda(\theta^s)\,\delta\nu(\varepsilon^s) - (c_b + c_s) \\ \varepsilon^w &= \delta\varepsilon^s + \Lambda(\theta^w)\,\delta\nu(\varepsilon^w) - (c_b + c_s)\,. \end{aligned} \tag{B.10}$$

Given (θ^s, θ^w), equilibrium $(\varepsilon^s, \varepsilon^w, U_s^s, U_s^w, U_b^s, U_b^w)$ jointly satisfy the set of 6 equations given by (B.9) and (B.10).

We next derive a few analytical results that are useful in addressing the question of whether seasonal variations in θ can explain the observed seasonality in prices. We focus on the case of $\theta^s > \theta^w$, i.e. the summer season has higher buyer-to-seller ratios.

Lemma 1 *If* $\Lambda'(\theta) \geqslant 0$, *then* $\varepsilon^s > \varepsilon^w$.

Proof. Suppose not, i.e. $\varepsilon^s \leqslant \varepsilon^w$. Given $\Lambda'(\theta) \geqslant 0$, so $\Lambda(\theta^s) \geqslant \Lambda(\theta^w)$, and $\nu'(.) < 0$ implies $\nu(\varepsilon^s) > \nu(\varepsilon^w)$, but equation (B.10) implies

$$(1+\delta)(\varepsilon^s - \varepsilon^w) = \Lambda(\theta^s)\,\delta\nu(\varepsilon^s) - \Lambda(\theta^w)\,\delta\nu(\varepsilon^w) \tag{B.11}$$

hence we have $\varepsilon^s > \varepsilon^w$. Contradiction. ■

The average price of a transaction in season j is

$$P^j = E\left[p^j(\varepsilon) \mid \varepsilon > \varepsilon^j\right]$$

given the price equation (B.4) holds for $j = s, w$ we obtain:

$$P^j = U_s^j + \beta_s E\left[\left(\varepsilon - \varepsilon^j\right) \mid \varepsilon > \varepsilon^j\right]. \tag{B.12}$$

The first term is the reservation price $p^j(\varepsilon^j)$ in season j and the second term is any surplus the seller expects to receive if ε is above the threshold ε^j. This price function is similar to that of the original Novy-Marx equilibrium $\theta = \theta^j$. However, the concepts are very different. In the seasonal model, (P^s, P^w) are jointly determined in the periodic steady state whereas in Novy-Marx $P(\theta^s)$ and $P(\theta^w)$ are values for two different steady states.

It is clear from the price function that introducing thick-market effects will substantially increase P^s/P^w (by shifting up $E[(\varepsilon - \varepsilon^j) \mid \varepsilon > \varepsilon^j]$). The question is whether the model can deliver seasonality in price without the thick-market effect. First note that the second term depends on the distribution which is log-concave in Novy-Marx (both Normal and Uniform distribution are log-concave).

Lemma 2 *If $\Lambda'(\theta) \geqslant 0$ and the p.d.f. for ε is log-concave,*

$$\frac{P^s}{P^w} < \frac{U_s^s}{U_s^w},$$

i.e. average price is less seasonal than the reservation price.

Proof. It follows from Lemma 1 that $\varepsilon^s > \varepsilon^w$ and $E\left[\left(\varepsilon - \varepsilon^j\right) \mid \varepsilon > \varepsilon^j\right]$ is decreasing in ε^j if p.d.f for ε is log-concave(see Burdett (1996)). ■

Lemma 2 shows that seasonality in reservation price U_s^j is the upper bound for the seasonality in price P^j. Next we turn to seasonality in the reservation price U_s^j. Iterating forward, the Bellman equation (B.9) for $i = s$ implies that

$$(1+\delta)\left(U_s^s - U_s^w\right) = \delta\beta_s\left[\lambda_s^s v\left(\varepsilon^s\right) - \lambda_s^w v\left(\varepsilon^w\right)\right]. \tag{B.13}$$

Recall from (B.7), $\Lambda(\theta)$ is the weighted average of arrival rates for buyers and sellers. The arrival rate of buyers to sellers, $\lambda_s(\theta)$, is increasing in θ. So $\Lambda'(\theta) \geqslant 0$ as long as $\lambda_b(\theta)$ does not fall too much in θ. Novy-Marx assumes $\lambda_b(\theta) = \lambda$, so this condition always holds. We now proceed the analysis under the case $\Lambda'(\theta) \geqslant 0$, thus $\varepsilon^s > \varepsilon^w$. It follows from (B.13) that the reservation price is higher in the summer if the direct effect of higher arrival rate $\lambda_s(\theta^s) \geqslant \lambda_s(\theta^w)$ dominates the equilibrium effect of higher thresholds $\varepsilon^s \geqslant \varepsilon^w$. We next study its magnitude.

Lemma 3 *If $c_b = c_s = 0$,*

$$\frac{U_s^s}{U_s^w} - 1 = \left(\frac{\lambda_s^s v\left(\varepsilon^s\right)}{\lambda_s^w v\left(\varepsilon^w\right)} - 1\right)\left(\frac{1-\delta}{\frac{\lambda_s^s \delta \nu\left(\varepsilon^s\right)}{\lambda_s^w v\left(\varepsilon^w\right)} + 1}\right). \tag{B.14}$$

Proof. Iterating the Bellman equation forward to obtain

$$\left(1-\delta^2\right) U_s^s = \lambda_s^w \beta_s \delta^2 \nu\left(\varepsilon^w\right) + \lambda_s^s \beta_s \delta v\left(\varepsilon^s\right) - (1+\delta)\, c_i$$

together with (B.13), the result follows. ■

Note that given that δ is the discount factor between the two seasons (6 months), it is very close to 1, so the ratio $\frac{U_s^s}{U_s^w}$ is substantially smaller than the $\frac{V_s^*(\theta^s)}{V_s^*(\theta^w)}$ in Figure 2&4 of Novy-Marx which from equation (B.5) under $c_b = c_s = 0$ is

$$\frac{V_s^*(\theta^s)}{V_s^*(\theta^w)} = \frac{\lambda_s^s v(\varepsilon^*(\theta^s))}{\lambda_s^w v(\varepsilon^*(\theta^w))}.$$

The result is intuitive as $\frac{V_s^*(\theta^s)}{V_s^*(\theta^w)}$ is across steady states whereas $\frac{U_s^s}{U_s^w}$ is across seasons along the periodic steady state.

Time-to-sell in season j is

$$E\left(T_s^j\right) = \frac{1}{\lambda_s^j\left[1-\Phi\left(\varepsilon^j\right)\right]} \tag{B.15}$$

Similar to that of U_s^j, there are two effects: (1) higher θ^s increases the arrival rate $\lambda_s(\theta^s)$, which decreases time-to-sell; and (2) a higher threshold lowers the probability of a transaction conditional on meeting, thus increases time-to-sell.

To summarize, under Novy-Marx assumptions of $\eta = 0$, $c_b = c_s = 0$ and given a logconcave distribution for ε, conditions for Lemma 1-3 are satisfied. We have higher price, shorter time-to-sell and higher transactions in the hot season relative to the cold. Novy-Marx consider the case $\varepsilon \sim N(\mu, \sigma^2)$, so

$$\nu(x) = (\mu - x) N\left(\frac{\mu - x}{\sigma}\right) + \sigma n\left(\frac{\mu - x}{\sigma}\right)$$

where $N(.)$ is the c.d.f. and $n(.)$ is the p.d.f for the Normal distribution. Define $y^j = \frac{\varepsilon^j - \mu}{\sigma}$, using the implicit equations (B.10), equilibrium (y^s, y^w) jointly satisfy

$$\begin{aligned}
\delta y^w &= y^s - \frac{(1-\delta)\mu}{\sigma} + \Lambda(\theta^s)\delta\left[y^s N(y^s) + n(y^s)\right] \\
\delta y^s &= y^w - \frac{(1-\delta)\mu}{\sigma} + \Lambda(\theta^w)\delta\left[y^w N(y^w) + n(y^w)\right].
\end{aligned}$$

Given values (y^s, y^w), $\varepsilon^j = \mu + \sigma y^j$, $i = s, w$, thus $[U_s^s, U_s^w, P^s, P^w, E(T_s^s), E(T_s^w)]$ are obtain from (B.4), (B.9) and (B.15).

We derive the quantitative results using Novy-Marx parameters except we use the 6% annual interest, consistent with Blake (2011) and our own calibration.

Novy-Marx reports quantitative results for various levels of θ as he is interested in comparing across steady states. For our interest of studying seasonality, we need to specify the average level $\bar{\theta}$ and then look at the seasonal cycle around it. We set $\bar{\theta} = 10$ so that the average time-to-sell is around 6.5 months. Note that this number is larger than that is required in Figure 2 of Novy-Marx due to the lower interest rate used here. We then compute the periodic steady state with $\theta^s = (1+a)\bar{\theta}$ and

$\theta^w = (1-a)\,\bar{\theta}$.

We compute the extent in seasonality of a variable X as in our paper: $4 * \ln\left(\frac{X^s}{X^w}\right)$. Given time-to-sell is counter-seasonal, its seasonality is a negative number. It is not surprising that both the extent of seasonality in price and time-to-sell is increasing in the driver of the seasonality $\frac{\theta^s}{\theta^w}$. To have a sense of how large $\frac{\theta^s}{\theta^w}$ should be, we turn to the implied seasonality in transaction. Transaction in season j is related to time-to-sell in season j by

$$Q^j = m_s^j * \{\underbrace{\lambda_s^j \left[1 - \Phi\left(\varepsilon^j\right)\right]}_{\text{prob. of sale}}\} = \frac{m_s^j}{E\left(T_s^j\right)}$$

where m_s^j is measure of sellers/houses in season j. Thus seasonality in transaction is approximately equal to seasonality in m_s^j minus the seasonality in $E\left(T_s^j\right)$, where the later is a negative number given time-to-sell is counter-seasonal. Novy-Marx's model can predict any level of seasonality in m_s^j depending on the entry conditions (similarly, it can predict any level of θ^s/θ^w). Since our objective is to understand whether the model can predict the level of price seasonality in the data, we focus on the case where the model matches the seasonality in m_s^j in the data which is about 28%. The predicted seasonality in transaction is simply equal to (28% minus predicted seasonality in time-to-sell)

The results are reported in Table $B2$ where the bargaining power for seller is $\beta_s = 0.8$ as in Novy-Marx. The results demonstrate that increasing $\frac{\theta^s}{\theta^w}$ has a much larger effect on the time-to-sell ratio than price ratio across seasons. In other words, Figure 4 of Novy-Marx is essentially a flat line when it comes to comparing across 'hot season' and 'cold season'. More specifically, when $\theta^s$is 20 percent above and θ^w is 20 percent below the annual average, predicted seasonality in transactions is 150 percent (close to the U.S. level) but seasonality in price is only 1.6 percent (a third of the U.S. level at 4.8). Making θ^s 30 percent above average (and θ^w 30 percent below) increases the seasonality in price to 2.4% but it sharply increases the seasonality in transaction to 215 percent. To summarize, we find that the implied seasonality in price is too small for reasonable levels of seasonality in transactions when the buyer-to-seller's ratio is the driving force: buyer-to-sell ratios affect time-to-sell directly through the arrival rate of buyers while its effect on transaction prices is through seller's reservation prices. Thus, we conclude that seasonal variation in reservation price that is based on variations in buyer-to-seller's ratio alone cannot generate enough seasonality in price.

Table $B2$. Seasonality in Novy-Marx model with $\frac{\theta^s}{\theta^w} = \frac{1+a}{1-a}$						
	Seasonal Ratio			Seasonality $= 4 * \ln\left(\frac{X^s}{X^w}\right)$		
a	$\frac{\theta^s}{\theta^w}$	$\frac{E(T_s^w)}{E(T_s^s)}$	$\frac{P^s}{P^w}$	Time-to-sell	Price	Transaction
0.1	1.2	$\frac{7.0 \text{ months}}{6.0 \text{ months}} = 1.16$	1.002	-60%	0.8%	88%
0.2	1.5	$\frac{7.7 \text{ months}}{5.6 \text{ months}} = 1.36$	1.004	-122%	1.6%	150%
0.3	1.9	$\frac{8.5 \text{ months}}{5.3 \text{ months}} = 1.60$	1.006	-187%	2.4%	215%

C Microfoundations for First-order Stochastic Dominance

In this Section we provide microfoundations for the key assumption in our model, namely:

$$F(., v') \leq F(., v) \Leftrightarrow v' > v, \tag{C.1}$$

where v denotes the stock of houses for sale (or vacancies). The derivation makes explicit the relation between v and the quality of the (best) match.

Suppose the quality of the match between any given person and any given house follows a distribution $G(x)$. Suppose further the actual number of houses viewed by a buyer, denoted by n, is a stochastic Poisson process with arrival rate λ. The arrival rate (per buyer) is the outcome of a homogeneous matching function $m(b, v)$, which depends on the number of buyers and sellers in the market:

$$\lambda = \frac{m(b, v)}{b} = b^{\alpha - 1} m(1, v/b)$$

In equilibrium in our model, $b = v$, so

$$\lambda = v^{\alpha - 1} m$$

where $m = m(1, 1)$ is constant. We assume that $\alpha > 1$, i.e., the arrival rate (which governs the number of viewings) is increasing in the number of vacancies (or houses that can be viewed).

The distribution of quality when n houses are viewed, using the order statistics is given by:

$$F_n(x) = G(x)^n$$

so the distribution of quality for a buyer in a market with v vacancies is:

$$\begin{aligned} F(x) &= \sum_n G(x)^n \left(e^{-\lambda} \frac{\lambda^n}{n!} \right) \\ &= e^{-\lambda} \sum_n \left(\frac{[\lambda G(x)]^n}{n!} \right) \\ &= e^{-\lambda} e^{\lambda G(x)} = e^{-\lambda(1-G(x))} \\ &= e^{-v^{\alpha-1} m (1-G(x))} \end{aligned}$$

Thus $F(x)$ satisfies assumption (C.1) for any given $G(x)$ and it can be interpreted as the distribution of the maximum match quality from a finite sample, whose size follows a Poisson distribution with arrival rate increasing in v.

This derivation is helpful to understand the foundations for the assumption. For calibration purposes, however, this is of little help, as the underlying distribution G is not known and hence we do not know the shape of F. Therefore, to avoid a deeper level of assumptions, in the paper we just use a "generic" F stochastically increasing in v and take stance only at the calibration stage.

D Efficiency Properties of the Model and Robustness

D.1 Efficiency Properties of the model

This section discusses the efficiency of equilibrium in the decentralized economy. For a complete derivation, see Section E.3 of this Appendix. The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The key difference between the planner's solution and the decentralized solution is that the former internalizes the thick-market effect. It is evident that the equilibrium level of transactions in the decentralized economy is not socially efficient because the optimal decision rules of buyers and sellers takes the stock of vacancies in each period as given, thereby ignoring the effects of their decisions on the stock of vacancies in the following periods. The thick-market effect generates a negative externality that makes the number of transactions in the decentralized economy inefficiently high for any given stock of vacancies (transacting agents do not take into account that, by waiting, they can thicken the market in the following period and hence increase the overall quality of matches).[8]

[8]This result is similar to that in the stochastic job matching model of Pissarides (2000, chapter 8), where the reservation productivity is too low compared to the efficient outcome in the presence of search externalities.

The efficient level of seasonality in housing markets, however, will depend on the exact distribution of match quality $F(\varepsilon, v)$. Under likely scenarios, the solution of the planner will involve a positive level of seasonality; that is, seasonality can be an efficient outcome. Indeed, in some circumstances, a planner may be willing to completely shut down the market in the cold season, to fully seize the benefits of a thick market.[9] This outcome is not as unlikely as one may a priori think. For example, the academic market for junior economists is extremely seasonal.[10] Extreme seasonality of course relies on the specification of utility—here we simply assume linear preferences; if agents have sufficiently concave utility functions (and intertemporal substitution across seasons is extremely low), then the planner may want to smooth seasonal fluctuations. For housing services, however, the concern of smoothing consumption across two seasons in principle should not be too strong relative to the benefit of having a better match that is on average long lasting (9 to 13 years in the two countries we analyze).

D.2 Model Assumptions

It is of interest to discuss four assumptions implicit in the model. First, we assume that each buyer only visits one house and each seller meets only one buyer in a given season. We do this for simplicity so that we can focus on the comparison across seasons. One concern is whether allowing the buyer to visit other houses may alter the results.[11] This is, however, not the case here. Note first that the seller's outside option is also to sell to another buyer. More formally, the surplus to the buyer if the transaction for her first house goes through is:

$$\tilde{S}_b^s(\varepsilon) \equiv H^s(\varepsilon) - \tilde{p}^s(\varepsilon) - \{E^s[S_b^s(\eta)] + \beta B^w\}, \tag{D.1}$$

where $E^s[S_b^s(\eta)]$ is the equilibrium expected surplus (as defined in (13)) for the buyer if she goes for another house with random quality η. By definition $S_b^s(\eta) \geqslant 0$ (it equals zero when the draw for the second house η is too low). Compared to (13), the outside option for the buyer is higher because of the possibility of buying another house. Similarly, the surplus to the seller if the transaction goes through is:

$$\tilde{S}_v^s(\varepsilon) \equiv \tilde{p}^s(\varepsilon) - \{\beta V^w + u + E^s[S_v^s(\eta)]\}. \tag{D.2}$$

[9] The same will happen in the decentralized economy when the ratio $(1-\phi^s)/(1-\phi^w)$ is extremely high, e.g. the required ratio is larger than 10 for the calibrated parametrs we use.

[10] And it is perhaps highly efficient, given that it has been designed by our well-trained senior economists.

[11] Concretely, one might argue that the seller of the first house can now only capture part of the surplus of the buyer in excess of the buyer's second house. In this case, for the surplus (and hence prices) to be higher in the summer one would need higher dispersion of match quality in the summer. This intuition is, however, incomplete. Indeed, one can show that higher prices are obtained independently of the level of dispersion.

The key is that both buyer and seller take their outside options as given when bargaining. The price $\tilde{p}^s(\varepsilon)$ maximizes the Nash product with the surplus terms $\tilde{S}_b^s(\varepsilon)$ and $\tilde{S}_v^s(\varepsilon)$. The solution implies $(1-\theta)\tilde{S}_v^s(\varepsilon) = \theta\tilde{S}_b^s(\varepsilon)$, but the Nash bargaining for the second house implies that $(1-\theta)E^s[S_v^s(\eta)] = \theta E^s[S_b^s(\eta)]$, so:

$$(1-\theta)[\tilde{p}^s(\varepsilon) - (\beta V^w + u)] = \theta[H^s(\varepsilon) - \tilde{p}^s(\varepsilon) - \beta B^w],$$

which has the same form as (16); thus it follows that the equilibrium price equation for $\tilde{p}^s(\varepsilon)$ is identical to (17)—though the actual level of prices is different, as the cutoff match-quality is different. Our qualitative results on seasonality in prices continue to hold as before, and quantitatively they can be even stronger. Recall that in the baseline model we find that seasonality in the sum of buyer's and seller's values tends to reduce the quality of marginal transactions in the summer relative to winter because the outside option in the hot season is linked to the sum of values in the winter season: $B^w + V^w$. Intuitively, allowing the possibility of meeting another party in the same season as an outside option could mitigate this effect and hence strengthen seasonality in prices. To see this, the cutoff quality $\tilde{\varepsilon}^s$ is now defined by: $H^s(\tilde{\varepsilon}^s) = \beta(B^w + V^w) + u + E^s[S^s(\eta)]$. Compared to (4), the option of meeting another party as outside option shows up as an additional term, $E^s[S^s(\eta)]$, which is higher in the hot season.

A second simplification in the model is that buying and selling houses involve no transaction costs. This assumption is easy to dispense with. Let $\bar{\tau}_b^j$ and $\bar{\tau}_v^j$ be the transaction costs associated with the purchase $(\bar{\tau}_b^j)$ and sale $(\bar{\tau}_v^j)$ of a house in season j. Costs can be seasonal because moving costs and repairing costs may vary across seasons.[12] The previous definitions of surpluses are modified by replacing price p^j with $p^j - \bar{\tau}_v^j$ in (12) and with $p^j + \bar{\tau}_b^j$ in (13). The value functions (14) and (15), and the Nash solution (16) continue to hold as before. So, the price equation becomes:

$$p^s(\varepsilon) - \bar{\tau}_v^s = \theta[H^s(\varepsilon) - \bar{\tau}_v^s - \bar{\tau}_b^s] + (1-\theta)\frac{u}{1-\beta}, \tag{D.3}$$

which states that the net price received by a seller is a weighted average of housing value net of total transaction costs and the present discounted value of the flow value u. And the reservation equation becomes:

$$\varepsilon^s =: H^s(\varepsilon^s) - \left(\bar{\tau}_b^j + \bar{\tau}_v^j\right) = \beta(B^w + V^w) + u. \tag{D.4}$$

[12] Repair costs (both for the seller who's trying to make the house more attractive and for the buyer who wants to adapt it before moving in) may be smaller in the summer because good weather and the opportunity cost of time (assuming vacation is taken in the summer) are important inputs in construction). Moving costs, similarly, might be lower during vacation (because of both job and school holidays).

The extent of seasonality in transactions depends only on total costs ($\bar{\tau}_b^j + \bar{\tau}_v^j$) while the extent of seasonality in prices depends on the distribution of costs between buyers and sellers. One interesting result is that higher transactional costs in the winter do not always result in lower winter house prices. Indeed, if most of the transaction costs fall on the seller ($\bar{\tau}_v^j$ is high relative to $\bar{\tau}_b^j$), prices could actually be higher in the winter for θ sufficiently high. On the other hand, if most of the transaction costs are borne by the buyer, then seasonal transaction costs could potentially be the driver of seasonality in vacancies (and hence transactions and prices). As said, our theoretical results on seasonality in prices and transactions follow from $v^s > v^w$, independently of the particular trigger (that is, independently of whether it is seasonal transaction costs for the buyer or seasonal moving shocks; empirically, they are observationally equivalent, as they both lead to seasonality in vacancies, which we match in the quantitative exercise[13]).

Third, the model presented so far assumed observable match-quality. In Section F of this Appendix we derive the case in which the seller cannot observe the match quality ε. We model the seller's power θ in this case as the probability that the seller makes a take-it-or-leave-it offer; $1 - \theta$ is then the probability that the buyer makes a take-it-or-leave-it offer upon meeting.[14] In that setting, $\theta = 1$ corresponds to the case in which sellers always post prices. When ε is observable, a transaction goes through whenever the total surplus is positive. However, when the seller does not observe ε, a transaction goes through only when the surplus to the buyer is positive. Since the seller does not observe ε, the seller offers a price that is independent of the level of ε, which will be too high for some buyers whose $\varepsilon' s$ are not sufficiently high (but whose ε would have resulted in a transaction if ε were observable to the seller). Therefore, because of the asymmetric information, the match is privately efficient only when the buyer is making a price offer. We show that our results continue to hold; the only qualitative difference is that the extent of seasonality in transactions is now decreasing in θ. This is because when ε is unobservable there is a second channel affecting a seller's surplus and hence the seasonality of reservation quality, which is opposite to the effects from the seasonality of outside option described above: When the seller is making a price offer, the surplus of the seller is higher in the hot season and hence sellers are more demanding and less willing to transact, which reduces the seasonality of transactions; the higher the seller's power, θ, the more demanding they are and the lower is the seasonality in transaction.

[13] Furthermore, empirically, we are unaware of data on direct measures of moving costs or propensities to move, much less so at higher frequency.

[14] Samuelson (1984) shows that in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all.

E Derivation for the model with observable value

E.1 Solving for prices

To derive $p^s(\varepsilon)$ in (17), use the Nash solution (16),

$$[p^s(\varepsilon) - \beta V^w - u](1-\theta) = [H^s(\varepsilon) - p^s(\varepsilon) - \beta B^w]\theta,$$

so

$$p^s(\varepsilon) = \theta H^s(\varepsilon) + \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u. \tag{E.1}$$

Using the value functions (14) and (15),

$$(1-\theta)V^s - \theta B^s = \beta[(1-\theta)V^w - \theta B^w] + (1-\theta)u$$

solving out explicitly,

$$(1-\theta)V^s - \theta B^s = \frac{(1-\theta)u}{1-\beta}$$

substitute into (E.1) to obtain (17).

E.2 The model without seasons

The value functions for the model without seasons are identical to those in the model with seasonality without the superscripts s and w. It can be shown that the equilibrium equations are also identical by simply setting $\phi^s = \phi^w$. Using (7), (18) and (24) to express the average price as:

$$P^s = \frac{u}{1-\beta} + \theta\left[\frac{\beta(1+\beta\phi^s)h^w(\varepsilon^w) + \left(1-\beta^2F^s(\varepsilon^s)\right)(1+\beta\phi^w)E[\varepsilon-\varepsilon^s \mid \varepsilon \geqslant \varepsilon^s]}{\left(1-\beta^2\right)\left(1-\beta^2\phi^w\phi^s\right)}\right], \tag{E.2}$$

Using (5),

$$\frac{\varepsilon}{1-\beta\phi} = u + \frac{\beta\phi}{1-\beta\phi}(1-\beta)(V+B)$$

and $B+V$ from (7),

$$B+V = \frac{u}{1-\beta} + \frac{1}{1-\beta^2}\left\{\frac{1-F}{1-\beta\phi}E[\tilde{\varepsilon}-\varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon] + \beta\frac{1-F}{1-\beta\phi}E[\tilde{\varepsilon}-\varepsilon \mid \tilde{\varepsilon} \geqslant \varepsilon]\right\}$$

which reduces to:

$$B+V=\frac{u}{1-\beta}+\frac{1-F(\varepsilon)}{(1-\beta)(1-\beta\phi)}E\left(\tilde{\varepsilon}-\varepsilon \mid \tilde{\varepsilon}\geqslant\varepsilon\right).$$

It follows that

$$\varepsilon=u+\frac{\beta\phi}{1-\beta\phi}\left[1-F(\varepsilon)\right]E\left(\tilde{\varepsilon}-\varepsilon \mid \tilde{\varepsilon}\geqslant\varepsilon\right),$$

and the law of motion for vacancy implies:

$$v=\frac{1-\phi}{1-\phi F(\varepsilon)}.$$

E.3 Analytical derivations of the planner's solution

The planner observes the match quality ε and is subject to the same exogenous moving shocks that hit the decentralized economy. The interesting comparison is the level of reservation quality achieved by the planner with the corresponding level in the decentralized economy. To spell out the planner's problem, we follow Pissarides (2000) and assume that in any period t the planner takes as given the expected value of the housing utility service per person in period t (before he optimizes), which we denote by q_{t-1}, as well as the beginning of period's stock of vacancies, v_t. Thus, taking as given the initial levels q_{-1} and v_0, and the sequence $\{\phi_t\}_{t=0...}$, which alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j'=s, w$, the planner's problem is to choose a sequence of $\{\varepsilon_t\}_{t=0,..}$ to maximize

$$U\left(\{\varepsilon_t, q_t, v_t\}_{t=0...}\right)\equiv\sum_{t=0}^{\infty}\beta^t\left[q_t+uv_tF\left(\varepsilon_t; v_t\right)\right] \tag{E.3}$$

subject to the law of motion for q_t :

$$q_t=\phi_t q_{t-1}+v_t\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}x\,dF\left(x; v_t\right), \tag{E.4}$$

the law of motion for v_t (which is similar to the one in the decentralized economy):

$$v_{t+1}=v_t\phi_{t+1}F\left(\varepsilon_t; v_t\right)+1-\phi_{t+1}, \tag{E.5}$$

and the inequality constraint:

$$0\leqslant\varepsilon_t\leqslant\bar{\varepsilon}\left(v_t\right), \tag{E.6}$$

where the upper bound $\bar{\varepsilon}$ can potentially be infinite.

The planner faces two types of trade-offs when deciding the optimal reservation quality ε_t: A static one and a dynamic one. The static trade-off stems from the comparison of utility values generated by occupied houses and vacancies in period t in the objective function (E.3). The utility per person generated from vacancies is the rental income per person, captured by $uv_tF(\varepsilon_t)$. The utility generated by occupied houses in period t is captured by q_t, the expected housing utility service per person conditional on the reservation value ε_t set by the planner in period t. The utility q_t, which follows the law of motion (E.4), is the sum of the pre-existing expected housing utility q_{t-1} that survives the moving shock and the expected housing utility from the new matches. By increasing ε_t, the expected housing value q_t decreases, while the utility generated by vacancies increases (since $F(\varepsilon_t)$ increases). The dynamic trade-off operates through the law of motion for the stock of vacancies in (E.5). By increasing ε_t (which in turn decreases q_t), the number of transactions in the current period decreases; this leads to more vacancies in the following period, v_{t+1}, and consequently to a thicker market in the next period. We first derive the case where the inequality constraints are not binding, i.e. markets are open in both the cold and hot seasons.

The Planner's solution when the housing market is open in all seasons

Because the sequence $\{\phi_t\}_{t=0,...}$ alternates between ϕ^j and $\phi^{j'}$ for seasons $j, j' = s, w$, the planner's problem can be written recursively. Taking (q_{t-1}, v_t), and $\{\phi_t\}_{t=0,..}$ as given, and provided that the solution is interior, that is, $\varepsilon_t < v_t$, the Bellman equation for the planner is given by:

$$\begin{aligned} W(q_{t-1}, v_t, \phi_t) &= \max_{\varepsilon_t} \left[q_t + uv_tF(\varepsilon_t; v_t) + \beta W(q_t, v_{t+1}, \phi_{t+1}) \right] \\ s.t. \quad & q_t = \phi_t q_{t-1} + v_t \int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)} x dF(x; v_t), \\ v_{t+1} &= v_t \phi_{t+1} F(\varepsilon_t; v_t) + 1 - \phi_{t+1}. \end{aligned} \tag{E.7}$$

The first-order condition implies

$$\left(1 + \beta \frac{\partial W(q_t, v_{t+1}, \phi_{t+1})}{\partial q_t}\right) v_t(-\varepsilon_t f(\varepsilon_t; v_t)) + \left(\beta \phi_{t+1} \frac{\partial W(q_t, v_{t+1}, \phi_{t+1})}{\partial v_{t+1}} + u\right) v_t f(\varepsilon_t; v_t) = 0,$$

which simplifies to

$$\varepsilon_t \left(1 + \beta \frac{\partial W(q_t, v_{t+1}, \phi_{t+1})}{\partial q_t}\right) = u + \beta \phi_{t+1} \frac{\partial W(q_t, v_{t+1}, \phi_{t+1})}{\partial v_{t+1}}. \tag{E.8}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W\left(q_{t-1},v_t,\phi_t\right)}{\partial q_{t-1}}=\phi_t\left(1+\beta\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial q_t}\right) \tag{E.9}$$

and

$$\begin{aligned}\frac{\partial W\left(q_{t-1},v_t,\phi_t\right)}{\partial v_t} &= \left(u+\beta\phi_{t+1}\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial v_{t+1}}\right)\left(F\left(\varepsilon_t;v_t\right)-v_tT_{1t}\right) \\ &\quad+\left(1+\beta\frac{\partial W\left(q_t,v_{t+1},\phi_{t+1}\right)}{\partial q_t}\right)\left(\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}xdF\left(x;v_t\right)+v_tT_{2t}\right)\end{aligned} \tag{E.10}$$

where $T_{1t}\equiv\frac{\partial}{\partial v_t}\left[1-F\left(\varepsilon_t;v_t\right)\right]>0$ and $T_{2t}\equiv\frac{\partial}{\partial v_t}\int_{\varepsilon_t}^{\bar{\varepsilon}(v_t)}xdF\left(x;v_t\right)>0$. In the periodic steady state, the first order condition (E.8) becomes

$$\varepsilon^j\left(1+\beta\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial q^j}\right)=u+\beta\phi^{j'}\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial v^{j'}} \tag{E.11}$$

The envelope condition (E.9) implies

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}=\phi^j\left[1+\beta\left(\phi^{j'}+\beta\phi^{j'}\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}\right)\right]$$

which yields:

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial q^{j'}}=\frac{\phi^j\left(1+\beta\phi^{j'}\right)}{1-\beta^2\phi^j\phi^{j'}} \tag{E.12}$$

Substituting this last expression into (E.10), we obtain:

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}=\left(u+\beta\phi^{j'}\frac{\partial W^{j'}\left(q^j,v^{j'}\right)}{\partial v^{j'}}\right)A^j+D^j,$$

where

$$A^j\equiv F^j\left(\varepsilon^j\right)-v^jT_1^j;\qquad D^j\equiv\frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\left(\int_{\varepsilon^j}^{\bar{\varepsilon}^j}xdF^j\left(x\right)+v^jT_2^j\right), \tag{E.13}$$

Hence, we have

$$\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}=\left\{u+\beta\phi^{j'}\left[\left(u+\beta\phi^j\frac{\partial W^j\left(q^{j'},v^j\right)}{\partial v^j}\right)A^{j'}+D^{j'}\right]\right\}A^j+D^j,$$

which implies

$$\frac{\partial W^j\left(q^{j'}, v^j\right)}{\partial v^j} = \frac{uA^j\left(1+\beta\phi^{j'}A^{j'}\right)+\beta\phi^{j'}D^{j'}A^j+D^j}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}. \tag{E.14}$$

Substituting (E.12) and (E.14) into the first-order condition (E.11),

$$\varepsilon^j\left(1+\beta\frac{\phi^{j'}\left(1+\beta\phi^j\right)}{1-\beta^2\phi^j\phi^{j'}}\right) = u+\beta\phi^{j'}\frac{uA^{j'}\left(1+\beta\phi^jA^j\right)+\beta\phi^jD^jA^{j'}+D^{j'}}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}$$

simplify to:

$$\varepsilon^j\left(\frac{1+\beta\phi^{j'}}{1-\beta^2\phi^j\phi^{j'}}\right) = \frac{\left(1+\beta\phi^{j'}A^{j'}\right)u+\beta^2\phi^j\phi^{j'}A^{j'}D^j+\beta\phi^{j'}D^{j'}}{1-\beta^2\phi^j\phi^{j'}A^jA^{j'}}, \tag{E.15}$$

and the stock of vacancies, v^j, $j=s,w$, satisfies (8) as in the decentralized economy.

The thick-market effect enters through two terms: $T_1^j \equiv \frac{\partial}{\partial v^j}\left[1-F^j\left(\varepsilon^j\right)\right]>0$ and $T_2^j \equiv \frac{\partial}{\partial v^j}\int_{\varepsilon^j}^{\bar{\varepsilon}^j} xdF^j(x) > 0$. The first term, T_1^j, indicates that the thick-market effect shifts up the acceptance schedule $\left[1-F^j(\varepsilon)\right]$. The second term, T_2^j, indicates that the thick-market effect increases the conditional quality of transactions. The interior solution (E.15) is an implicit function of ε^j that depends on $\varepsilon^{j'}$, v^j, and $v^{j'}$. It is not straightforward to derive an explicit condition for $\varepsilon^j < v^j$, $j=s,w$. Abstracting from seasonality for the moment, i.e. when $\phi^s=\phi^w$, it follows immediately from (8) that the solution is interior, $\varepsilon < v$. Moreover (E.15) implies the planner's optimal reservation quality ε^p satisfies:

$$\frac{\varepsilon^p}{1-\beta\phi} = \frac{u+\frac{\beta\phi}{1-\beta\phi}\left(\int_{\varepsilon^p}^{\bar{\varepsilon}} xdF(x)+vT_2\right)}{1-\beta\phi F\left(\varepsilon^p\right)+\beta\phi vT_1}. \tag{E.16}$$

Comparing (E.16) with (23), the thick-market effect, captured by T_1 and T_2, generates two opposite forces. The term T_1 decreases ε^p, while the term T_2 increases ε^p in the planner's solution. Thus, the positive thick-market effect on the acceptance rate T_1 implies that the number of transactions is too low in the decentralized economy, while the positive effect on quality T_2 implies that the number of transactions is too high. Since $1-\beta\phi$ is close to zero, however, the term T_2 dominates. Therefore, the overall effect of the thick-market externality is to increase the number of transactions in the decentralized economy relative to the efficient outcome. As discussed in the text, comparing the extent in seasonality in the decentralized equilibrium to the planner's solution depends on the exact distribution $F(\varepsilon, v)$. We next derive the case in which the Planner finds it optimal to close down the market in the cold season.

The Planner's solution when the housing market is closed in the cold season

Setting $\varepsilon_t^w = \bar{\varepsilon}_t^w$, the Bellman equation (E.7) can be rewritten as:

$$\begin{aligned} W^s\left(q_{t-1}^w, v_t^s\right) &= \max_{\varepsilon_t^s} \left[\begin{array}{c} \phi^s q_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + u v_t^s F_t^s(\varepsilon_t^s) \\ +\beta\left(q_{t+1}^w + u\left[v_t^s \phi^w F_t^s(\varepsilon_t^s) + 1 - \phi^w\right]\right) \\ +\beta^2 W^s\left(q_{t+1}^w, v_{t+2}^s\right) \end{array} \right] \\ & \qquad s.t. \\ q_{t+1}^w &= \phi^w \left[\phi^s q_{t-1}^w + v_t^s \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) \right], \\ v_{t+2}^s &= \phi^s \left[v_t^s \phi^w F_t^s(\varepsilon_t^s) + 1 - \phi^w\right] + 1 - \phi^s. \end{aligned} \tag{E.17}$$

Intuitively, "a period" for the decision of ε_t^s is equal to $2t$. The state variables for the current period are given by the vector $\left(q_{t-1}^w, v_t^s\right)$, the state variables for next period are $\left(q_{t+1}^w, v_{t+2}^s\right)$, and the control variable is ε_t^s. The first order condition is:

$$\begin{aligned} 0 &= v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right) + u v_t^s f_t^s(\varepsilon_t^s) \\ &\quad +\beta\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right) + u v_t^s \phi^w f_t^s(\varepsilon_t^s)\right) \\ &\quad +\beta^2\left[\frac{\partial W^s}{\partial q_{t+1}^w}\left(\phi^w v_t^s\left(-\varepsilon_t^s f_t^s(\varepsilon_t^s)\right)\right) + \frac{\partial W^s}{\partial v_{t+2}^s}\left(\phi^s v_t^s \phi^w f_t^s(\varepsilon_t^s)\right)\right], \end{aligned}$$

which simplifies to:

$$\begin{aligned} 0 &= -\varepsilon_t^s + u + \beta\left(-\phi^w \varepsilon_t^s + u\phi^w\right) \\ &\quad +\beta^2\left[\frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\left(-\phi^w \varepsilon_t^s\right) + \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\phi^s\phi^w\right] \end{aligned}$$

and can be written as:

$$\varepsilon_t^s\left[1 + \beta\phi^w + \beta^2\phi^w \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\right] = \left(1 + \beta\phi^w\right)u + \beta^2\phi^w\phi^s \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \tag{E.18}$$

Using the envelope-theorem conditions, we obtain:

$$\frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial q_{t-1}^w} = \phi^s + \beta\phi^w\phi^s + \beta^2\phi^w\phi^s \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}, \tag{E.19}$$

and

$$\begin{aligned}
&\frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\
= \;& (1+\beta\phi^w)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + v_t^s T_{2t}^s\right) + (1+\beta\phi^w)\, u\left[F_t^s(\varepsilon_t^s) - v_t^s T_{1t}^s\right] \\
& + \beta^2 \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w} \phi^w \left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + v_t^s T_{2t}^s\right) \\
& + \beta^2 \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s} \phi^s \phi^w \left[F_t^s(\varepsilon_t^s) - v_t^s T_{1t}^s\right],
\end{aligned}$$

where $T_{1t}^s \equiv \frac{\partial}{\partial v_t^s}\left[1 - F_t^s(\varepsilon^s)\right] > 0$ and $T_{2t}^s \equiv \frac{\partial}{\partial v_t^s} \int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) > 0$. Rewrite the last expression as:

$$\begin{aligned}
&\frac{\partial W^s\left(q_{t-1}^w, v_t^s\right)}{\partial v_t^s} \\
= \;& \left(1 + \beta\phi^w + \beta^2 \phi^w \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial q_{t+1}^w}\right)\left(\int_{\varepsilon_t^s}^{\bar{\varepsilon}_t^s} x dF_t^s(x) + v_t^s T_{2t}^s\right) \\
& + \left((1+\beta\phi^w)\, u + \beta^2 \phi^s \phi^w \frac{\partial W^s\left(q_{t+1}^w, v_{t+2}^s\right)}{\partial v_{t+2}^s}\right)\left[F_t^s(\varepsilon_t^s) - v_t^s T_{1t}^s\right]
\end{aligned} \tag{E.20}$$

In steady state, (E.19) and (E.20) become

$$\frac{\partial W^s(q^w, v^s)}{\partial q^w} = \frac{\phi^s(1+\beta\phi^w)}{1-\beta^2\phi^w\phi^s}, \tag{E.21}$$

and

$$\begin{aligned}
&\frac{\partial W^s(q^w, v^s)}{\partial v^s}\left(1 - \beta^2\phi^s\phi^w\left[F^s(\varepsilon^s) - v^s T_1^s\right]\right) \\
= \;& \left(1 + \beta\phi^w + \beta^2\phi^w \frac{\phi^s(1+\beta\phi^w)}{1-\beta^2\phi^w\phi^s}\right)\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x) + v^s T_2^s\right) \\
& + (1+\beta\phi^w)\, u\left[F^s(\varepsilon^s) - v^s T_1^s\right].
\end{aligned} \tag{E.22}$$

Substituting into the FOC (E.18),

$$\begin{aligned}
&\varepsilon^s \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s} \\
= \;& (1+\beta\phi^w)\, u + \beta^2\phi^w\phi^s \frac{(1+\beta\phi^w)\, u\left[F^s(\varepsilon^s) - v^s T_1^s\right] + \frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x) + v^s T_2^s\right)}{1 - \beta^2\phi^s\phi^w\left[F^s(\varepsilon^s) - v^s T_1^s\right]}
\end{aligned}$$

which simplifies to

$$\frac{\varepsilon^s}{1-\beta^2\phi^w\phi^s} = \frac{u + \frac{\beta^2\phi^w\phi^s}{1-\beta^2\phi^w\phi^s}\left(\int_{\varepsilon^s}^{\bar{\varepsilon}^s} x dF^s(x) + v^s T_2^s\right)}{1-\beta^2\phi^s\phi^w\left[F^s(\varepsilon^s) - v^s T_1^s\right]}, \tag{E.23}$$

which is similar to the Planner's solution with no seasons in (E.16), with $\beta^2\phi^w\phi^s$ replacing $\beta\phi$.

F Model with unobservable match quality

Assume that the seller does not observe ε. As shown by Samuelson (1984), in bargaining between informed and uninformed agents, the optimal mechanism is for the uninformed agent to make a "take-it-or-leave" offer. The same holds for the informed agent if it is optimal for him to make an offer at all. Hence, we adopt a simple price-setting mechanism: The seller makes a take-it-or-leave-it offer p^{jv} with probability $\theta \in [0,1]$ and the buyer makes a take-it-or-leave-it offer p^{jb} with probability $1-\theta$. ($\theta = 1$ corresponds to the case in which sellers post prices.) Broadly speaking, we can interpret θ as the "bargaining power" of the seller. The setup of the model implies that the buyer accepts any offer p^{sv} if $H^s(\varepsilon) - p^{sv} \geqslant \beta B^w$; and the seller accepts any price $p^{sb} \geqslant \beta V^w + u$. Let S_v^{si} and $S_b^{si}(\varepsilon)$ be the surplus of a transaction to the seller and the buyer when the match quality is ε and the price is p^{si}, for $i = b, v$:

$$S_v^{si} \equiv p^{si} - (u + \beta V^w), \tag{F.1}$$

$$S_b^{si}(\varepsilon) \equiv H^s(\varepsilon) - p^{si} - \beta B^w. \tag{F.2}$$

Note that the definition of S_v^{si} implies that

$$p^{sv} = S_v^{sv} + p^{sb} \tag{F.3}$$

i.e. the price is higher when the seller is making an offer. Since only the buyer observes ε, a transaction goes through only if $S_b^{si}(\varepsilon) \geqslant 0$, $i = b, v$, i.e. a transaction goes through only if the surplus to the buyer is non-negative regardless of who is making an offer. Given $H^s(\varepsilon)$ is increasing in ε, for any price p^{si}, $i = b, v$, a transaction goes through if $\varepsilon \geqslant \varepsilon^{si}$, where

$$H^s(\varepsilon^{si}) - p^{si} = \beta B^w. \tag{F.4}$$

$1 - F^s(\varepsilon^{si})$ is thus the probability that a transaction is carried out. From (2), the response of the reservation quality ε^{si} to a change in price is given by:

$$\frac{\partial \varepsilon^{si}}{\partial p^{si}} = \frac{1 - \beta^2 \phi^w \phi^s}{1 + \beta \phi^w}. \tag{F.5}$$

Moreover, by the definition of $S_b^{si}(\varepsilon)$ and ε^{si}, in equilibrium, the surplus to the buyer is:

$$S_b^{si}(\varepsilon) = H^s(\varepsilon) - H^s(\varepsilon^s) = \frac{1 + \beta \phi^w}{1 - \beta^2 \phi^w \phi^s}\left(\varepsilon - \varepsilon^{si}\right). \tag{F.6}$$

F.1 The Seller's offer

Taking the reservation policy ε^{sv} of the buyer as given, the seller chooses a price to maximize the expected surplus value of a sale:

$$\max_p \left\{ \left[1 - F^s(\varepsilon^{sv})\right] \left[p - \beta V^w - u\right] \right\}$$

The optimal price p^{sv} solves

$$\left[1 - F^s(\varepsilon^{sv})\right] - \left[p - \beta V^w - u\right] f^s(\varepsilon^{sv}) \frac{\partial \varepsilon^{sv}}{\partial p^s} = 0. \tag{F.7}$$

Rearranging terms we obtain:

$$\underbrace{\frac{p^{sv} - \beta V^w - u}{p^{sv}}}_{\text{mark-up}} = \underbrace{\left[\frac{p^{sv} f^s(\varepsilon^{sv}) \frac{\partial \varepsilon^s}{\partial p^s}}{1 - F^s(\varepsilon^{sv})}\right]^{-1}}_{\text{inverse-elasticity}},$$

which makes clear that the price-setting problem of the seller is similar to that of a monopolist who sets a markup equal to the inverse of the elasticity of demand (where demand in this case is given by the probability of a sale, $1 - F^s(\varepsilon^s)$). The optimal decisions of the buyer (F.5) and the seller (F.7) together imply:

$$S_v^{sv} = \frac{1 - F^s(\varepsilon^{sv})}{f^s(\varepsilon^{sv})} \frac{1 + \beta \phi^w}{1 - \beta^2 \phi^w \phi^s}. \tag{F.8}$$

Equation (F.8) says that the surplus to a seller generated by the transaction is higher when $\frac{1 - F^s(\varepsilon^{sv})}{f^s(\varepsilon^{sv})}$ is higher, i.e. when the conditional probability that a successful transaction is of match quality ε^{sv} is lower. Intuitively, the surplus of a transaction to a seller is higher when the house is transacted with a stochastically higher match quality, or loosely speaking, when the distribution of match quality has a "thicker" tail.

Given the price-setting mechanism, in equilibrium, the value of a vacancies to its seller is:

$$V^s = u + \beta V^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv}. \tag{F.9}$$

Solving out V^s explicitly,

$$V^s = \frac{u}{1-\beta} + \theta \frac{\left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}, \tag{F.10}$$

which is the sum of the present discounted value of the flow value u and the surplus terms when its seller is making the take-it-or-leave-it offer, which happens with probability θ. Using the definition of the surplus terms, the equilibrium p^{sv} is:

$$p^{sv} = \frac{u}{1-\beta} + \theta \frac{\left[1 - \beta^2 F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}. \tag{F.11}$$

F.2 The Buyer's Offer

The buyer offers a price that extracts all the surplus from the seller, i.e.

$$S_v^{sb} = 0 \Leftrightarrow p^{sb} = u + \beta V^w$$

Using the value function V^w from (F.10), the price offered by the buyer is:

$$p^{sb} = \frac{u}{1-\beta} + \theta \frac{\beta^2 \left[1 - F^s\left(\varepsilon^{sv}\right)\right] S_v^{sv} + \beta \left[1 - F^w\left(\varepsilon^{wv}\right)\right] S_v^{wv}}{1-\beta^2}. \tag{F.12}$$

The buyer's value function is:

$$\begin{aligned} B^s &= \beta B^w + \theta \left[1 - F^s\left(\varepsilon^{sv}\right)\right] E^s \left[S_b^{sv}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{sv}\right] \\ &\quad + (1-\theta) \left[1 - F^s\left(\varepsilon^{sb}\right)\right] E^s \left[S_b^{sb}\left(\varepsilon\right) \mid \varepsilon \geqslant \varepsilon^{sb}\right], \end{aligned} \tag{F.13}$$

where $E^s[.]$ indicates the expectation taken with respect to the distribution $F^s(.)$. Since the seller does not observe ε, the expected surplus to the buyer is positive even when the seller is making the offer (which happens with probability θ). As said, buyers receive zero housing service flow until they find a successful match. Solving out B^s explicitly,

$$
\begin{aligned}
B^s &= \theta\left[1-F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S_b^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right]+(1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S_b^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right] \\
&\quad +\beta\left\{\theta\left(1-F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S_b^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right]+(1-\theta)\left[1-F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S_b^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\}.
\end{aligned} \tag{F.14}
$$

F.3 Reservation quality

In any season s, the reservation quality ε^{si}, for $i=v,b$, satisfies

$$
H^s\left(\varepsilon^{si}\right)=S_v^{si}+u+V^w+\beta B^w, \tag{F.15}
$$

which equates the housing value of a marginal owner in season s, $H^s\left(\varepsilon^s\right)$, to the sum of the surplus generated to the seller (S_v^{si}), plus the sum of outside options for the buyer (βB^w) and the seller (βV^w+u). Using (2), ε^{si} solves:

$$
\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\varepsilon^{si}=S_v^{si}+u+\frac{\beta\phi^w\left(1-\beta^2\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(B^w+V^w\right)-\frac{\beta^2\phi^w\left(1-\phi^s\right)}{1-\beta^2\phi^w\phi^s}\left(V^s+B^s\right). \tag{F.16}
$$

The reservation quality ε^s depends on the sum of the outside options for buyers and sellers in both seasons, which can be derived from (F.10) and (F.14):

$$
\begin{aligned}
& B^s+V^s \\
= & \frac{u}{1-\beta}+ \\
& \theta\left[1-F^s\left(\varepsilon^{sv}\right)\right] E^s\left[S^{sv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sv}\right]+(1-\theta)\left[1-F^s\left(\varepsilon^{sb}\right)\right] E^s\left[S^{sb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{sb}\right]+ \\
& \beta\left\{\theta\left(1-F^w\left(\varepsilon^{sv}\right)\right) E^w\left[S^{wv}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wv}\right]+(1-\theta)\left[1-F^w\left(\varepsilon^{sb}\right)\right] E^w\left[S^{wb}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{wb}\right]\right\},
\end{aligned} \tag{F.17}
$$

where $S^{si}(\varepsilon) \equiv S_b^{si}(\varepsilon)+S_v^{si}$ is the total surplus from a transaction with match quality ε. Note from (F.16) that the reservation quality is lower when the buyer is making a price offer: $\frac{1+\beta\phi^w}{1-\beta^2\phi^w\phi^s}\left(\varepsilon^{sv}-\varepsilon^{sb}\right)=S_v^{sv}$. Also, because of the asymmetric information, the match is privately efficient when the buyer is making a price offer.

The thick-and-thin market equilibrium through the distribution F^j affects the equilibrium prices and reservation qualities $\left(p^{jv},p^{jb},\varepsilon^{jv},\varepsilon^{jb}\right)$ in season $j=s,w$ through two channels, as shown in (F.11), (F.12), and(F.16)): the conditional density of the distribution at reservation ε^{jv}, i.e. $\frac{f^j\left(\varepsilon^{jv}\right)}{1-F^j\left(\varepsilon^{jv}\right)}$, and the expected surplus quality above reservation ε^{jv}, i.e. $\left(1-F^j\left(\varepsilon^{ji}\right)\right) E^j\left[\varepsilon-\varepsilon^{ji} \mid \varepsilon \geqslant \varepsilon^{ji}\right]$, $i=b,v$. As shown in (F.8), a lower conditional probability that a transaction is of marginal quality ε^{jv} implies higher expected surplus to the seller S_v^{jv}, which increases the equilibrium prices p^{jv} and p^{jb} in (F.11) and (F.12). Similarly as shown in (F.6) and the assumption of first order stochastic dominance, using integration by parts, expected surplus to the buyer $(1 - F^j(\varepsilon^{ji})) E^s [S_b^{si}(\varepsilon) \mid \varepsilon \geqslant \varepsilon^{si}]$, $i = b, v$ is higher in the hot season with higher vacancies. These two channels affect V^j and B^j in (F.10) and (F.14), and as a result affect the reservation qualities ε^{jv} and ε^{jb} in (5).

F.4 Stock of vacancies

In any season s, the average probability that a transaction goes through is $\left\{\theta [1 - F^s(\varepsilon^{sv})] + (1-\theta)\left[1 - F^s\left(\varepsilon^{sb}\right)\right]\right\}$, and the average probability that a transaction does not through is $\left\{\theta F^w(\varepsilon^{wv}) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}$. Hence, the law of motion for the stock of vacancies (and for the stock of buyers) is

$$\begin{aligned} v^s &= (1-\phi^s)\left\{v^w\left[\theta(1 - F^w(\varepsilon^{wv})) + (1-\theta)\left(1 - F^w\left(\varepsilon^{wb}\right)\right)\right] + 1 - v^w\right\} \\ &\quad + v^w\left\{\theta F^w(\varepsilon^{wv}) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\}, \end{aligned}$$

where the first term includes houses that received a moving shock this season and the second term comprises vacancies from last period that did not find a buyer. The expression simplifies to

$$v^s = v^w \phi^s \left\{\theta F^w(\varepsilon^{wv}) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\} + 1 - \phi^s, \tag{F.18}$$

that is, in equilibrium v^s depends on the equilibrium reservation quality $\left(\varepsilon^{wv}, \varepsilon^{wb}\right)$ and on the distribution $F^w(.)$.

An equilibrium is a vector $\left(p^{sv}, p^{sb}, p^{wv}, p^{wb}, B^s + V^s, B^w + V^w, \varepsilon^{sv}, \varepsilon^{sb}, \varepsilon^{wv}, \varepsilon^{wb}, v^s, v^w\right)$ that jointly satisfies equations (F.11),(F.14),(F.16), (F.17) and (F.18), with the surpluses S_v^j and $S_b^j(\varepsilon)$ for $j = s, w$, derived as in (F.8), and (F.6). Using (F.18), the stock of vacancies in season s is given by:

$$v^s = \frac{(1-\phi^w)\phi^s\left\{\theta F^w(\varepsilon^{wv}) + (1-\theta) F^w\left(\varepsilon^{wb}\right)\right\} + 1 - \phi^s}{1 - \phi^w \phi^s \{\theta F^s(\varepsilon^{sv}) + (1-\theta) F^s(\varepsilon^{sb})\}\{\theta F^w(\varepsilon^{wv}) + (1-\theta) F^w(\varepsilon^{wb})\}}. \tag{F.19}$$

Given $1 - \phi^s > 1 - \phi^w$, as in the observable case, it follows that, in equilibrium $v^s > v^w$.

F.5 Seasonality in Prices

Let

$$p^s \equiv \frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]p^{sv}+(1-\theta)\,p^{sb}}{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

be the average price observed in season s. Given $p^{sv}=S_v^{sv}+p^{sb}$, we can rewrite it as

$$p^s = p^{sb}+\frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]+1-\theta}$$

using (F.12)

$$\begin{aligned}
p^s &= \frac{u}{1-\beta}+\theta\frac{\beta^2\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}+\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2}+\frac{\theta\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{1-\theta F^s\left(\varepsilon^{sv}\right)} \\
&= \frac{u}{1-\beta}+\theta\left(\frac{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\beta^2+1-\beta^2}{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)}\right)\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}+\frac{\theta\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2}
\end{aligned}$$

we obtain,

$$p^s=\frac{u}{1-\beta}+\theta\left\{\frac{\left[1-\theta\beta^2F^s\left(\varepsilon^{sv}\right)\right]\left[1-F^s\left(\varepsilon^{sv}\right)\right]S_v^{sv}}{\left[1-\theta F^s\left(\varepsilon^{sv}\right)\right]\left(1-\beta^2\right)}+\frac{\beta\left[1-F^w\left(\varepsilon^{wv}\right)\right]S_v^{wv}}{1-\beta^2}\right\}. \tag{F.20}$$

Since the flow u is a-seasonal, house prices are seasonal if $\theta>0$ and the surplus to the seller is seasonal. As in the case with observable match quality, when sellers have some "market power" ($\theta>0$), prices are seasonal. The extent of seasonality is increasing in the seller's market power θ. To see this, note that the equilibrium price is the discounted sum of the flow value (u) plus a positive surplus from the sale. The surplus S_v^{sv}, as shown in (F.8), is seasonal. Given $v^s>v^w$, Assumption 2 implies hazard rate ordering, i.e. $\frac{f^w(x)}{1-F^w(x)}>\frac{f^s(x)}{1-F^s(x)}$ for any cutoff x, i.e. the thick-market effect lowers the conditional probability that a successful transaction is of the marginal quality ε^{sv} in the hot season, that is, it implies a "thicker" tail in quality in the hot season. In words, the quality of matches goes up in the summer and hence buyers' willingness to pay increases; sellers can then extract a higher surplus in the summer; thus, $S_v^{sv}>S_v^{wv}$. As in the case with observable ε, there is an equilibrium effect through the seasonality of cutoffs. As shown in (F.16), the equilibrium cutoff ε^{sv} depends on the surplus to the seller (S_v^{sv}) and on the sum of the seller's and the buyer's outside options, while the equilibrium cutoff ε^{sb} depends only on the sum of the outside options. The seasonality in outside options tends to reduce $\varepsilon^{si}/\varepsilon^{wi}$ for $i=b,v$. This is because the outside option in the hot season s is determined by the sum of values in the winter season: B^w+V^w, which is lower than in the summer. However, the seasonality in the surplus term, $S_v^{sv}>S_v^{wv}$ (shown before), tends to increase $\varepsilon^{sv}/\varepsilon^{wv}$ (the marginal house has to be of higher quality in order to generate a bigger surplus to the seller). Because of these two opposing forces, the equilibrium effect is likely to be small (even smaller than in the observable case.)

Given that θ affects S_v^{sv} only through the equilibrium vacancies and reservation qualities, it follows that the extent of seasonality in price is increasing in θ.

F.6 Seasonality in Transactions

The number of transactions in equilibrium in season s is given by:

$$Q^s = v^s \left[\theta \left(1 - F^w \left(\varepsilon^{wv}\right)\right) + (1-\theta)\left(1 - F^w \left(\varepsilon^{wb}\right)\right)\right]. \tag{F.21}$$

(An isomorphic expression holds for Q^w). As in the observables case, seasonality in transactions stems from three sources. First, the direct effect from a larger stock of vacancies in the summer, $v^s > v^w$. Second the amplification through the thick-market effects that shifts up the probability of a transaction. Third, there is an equilibrium effect through cutoffs. As pointed out before, this last effect is small. As in the case with observable ε, most of the amplification stems from the thick-market effect. What is new when ε is unobservable is that the extent of seasonality in transactions is decreasing in the seller's market market power θ. This is because higher θ leads to higher surplus in the summer relative to winter, S_v^{sv}/S_v^{wv}, which in turn increases $\varepsilon^{sv}/\varepsilon^{wv}$ and hence decreases Q^s/Q^w); the higher is θ, the stronger is this effect (it disappear when $\theta = 0$).

G Model's Additional Statistics

G.1 Time-on-market and Transaction Probabilities

For the baseline seasonal model with $\frac{1-\phi^s}{1-\phi^w} = 1.25$, for the U.S., the steady state transaction probabilities are $1 - F^s(\varepsilon^s) = 0.31$ in the summer and $1 - F^w(\varepsilon^w) = 0.25$ in the winter. The transaction probabilities are seasonal, and indeed the source of the amplification mechanism that makes the volume of transaction more seasonal than the number of houses for sale. Under these probabilities, we can compute the steady state median time-on-market for each season. Let x^s be the number of semester that a house stays on the market if it is put on sale in season s. The distribution for x^s can be computed using Table $G1$.

Table $G1$. Distribution of time-to-sell	
Stays exactly x^s semester	pdf of x^s
0	$(1-F^s)$
1	$F^s(1-F^w)$
2	$F^sF^w(1-F^s)$
3	$(F^s)^2 F^w(1-F^w)$
∞	

Thus given the steady state probabilities, we can derive the distribution of time-to-sell x_s for houses that are put on the market in season $s = s, w$. The numbers are reported in Table $G2$

Table $G2$. Distribution of time-to-sell					
x^s	pdf of x^s	cdf of x^s	x^w	pdf of x^w	cdf of x^w
0	0.31	0.31	0	0.25	0.25
1	0.17	0.48	1	0.23	0.48
2	0.16	0.64	2	0.13	0.61

Thus the median TOM is around 6 months for both seasons, being slightly higher in the winter than summer, this is also the time-to-sell used in Piazzesi and Schneider (2009). Our predicted median time-to-sell is consistent with the median number of months reported in Ungerer (2012) and Diaz and Jerez (2012), where they use the median number of months for newly built and report numbers of 5.2 months for 1974-2011, and 5.7 months for 1960-2012.

(Note that the average TOM in the market in our model is given, correspondingly by $F^s \frac{1+F^w}{1-F^sF^w}$ and $F^w \frac{1+F^s}{1-F^sF^w}$. Given the well known problem with the average TOM reported in the data, we prefer to focus on the median, which is less sensitive to some of these concerns.[15])

G.2 Likelihood of an agent having η houses for sale

A any point in time, we can derive the distribution of houses for sale for a given agent. The support for the distribution is from 0 to infinite. However, given that both v and $(1-\phi)$ are small, the distribution

[15] An example illustrating the problems with the average is described in http://www.manausa.com/how-long-does-it-take-to-sell-a-home/#ixzz2MmFiytY9

Suppose the total time on the market for a house is the sum of 1) 30 days "For Sale By Owner"; 2) 180 days with Broker A; 3) 10 days with Broker B. This is a total of 220 days, yet the MLS would report it as "10 days." The average only informs on the average of the final listing periods for those homes. Most problematic, the average does not include the days on the market of houses that failed to sell.

concentrates around 0 or 1 house for sale. To see this in brief, consider the model without season with steady state $v = 0.17$ and transaction probabilities $[1 - F(\varepsilon)] = 0.28$. There are two reasons why the probability that an agent has more than one house for sale is close zero. First, the steady state v is small so very few houses are for sale. Second and more importantly, the probability of a moving shock is very small because it is set to match the average duration of staying in a house which is 9 years for the U.S., $(1 - \phi) = 0.056$. More specifically, conditional on having $\eta \geqslant 1$ houses for sale it is highly unlikely that an agent can transit to having $\eta + 1$ house for sale. This requires three events: the agent fails to sell, buys a new house but receives a moving shock immediately after; which happens with probability $(1 - F(\varepsilon)) F(\varepsilon)(1 - \phi) = 0.01$. Thus, it is unlikely that agents will have more than one house for sale. The answer for the baseline seasonal model (with $\frac{1-\phi^s}{1-\phi^w} = 1.25$) is very similar because the moving probability in both summer and winter are also very small, $(1 - \phi^s) = 0.062$ and $(1 - \phi^w) = 0.049$; and steady state $v^s = 0.180$ and $v^w = 0.167$.

We next provide more details on how one could derive the full distribution for the number of houses for sale. As in the paper, v_t is the measure of houses for sale, $(1 - F_t)$ is the transaction probability and $(1 - \phi)$ is the probability of a moving shock.

To compute the likelihood of an agent having η houses for sale at any period t, it is useful to divide the population into two broad types: the matched agents (m) and the non-matched agents (n). Within each broad group, agents are also different with regards to the number of houses they have for sale. Thus it is useful to denote the type of an agent at time t as $s_t = (k, i)$ for $k = m, n$ denoting matched or unmatched and $i = 0, 1, ...$ denoting the number of houses owned by the agent.

We next tables describe the probability of the number of houses for sale at the beginning of period $t + 1$ for all types of agents $s_t = (k, i)$ in period t. The probability is different across $k = m, n$ and between $i = 0$ and any $i > 0$. Therefore, there are four tables to report.

Table $G3$ is the distribution of those who are matched to a house and have no house to sell at time t. Tables $G4$, $G5$, and $G6$ show the distributions for the remaining cases. The first column in each table shows the potential number of houses in the next period. The second column shows the corresponding probabilities. The third column explains how the probability is derived, and the last column shows the state in which it transitions to.

Table $G3$: $s_t = (m, 0)$

η_{t+1}	$\Pr(\eta_{t+1} \mid s_t)$	events	s_{t+1}
i	ϕ	stay at $t+1$	$(m, 0)$
$i+1$	$1 - \phi$	move at $t+1$	$(n, 1)$

Table $G4$: $s_t = (m, i)\,, i > 0$			
η_{t+1}	$\Pr(\eta_{t+1} \mid s_t)$	events	s_{t+1}
$i-1$	$(1-F_t)\,\phi$	sold at t, stay at $t+1$	$(m, i-1)$
i	$(1-F_t)(1-\phi)$	sold at t, move at $t+1$	(n, i)
i	$F_t\phi$	didn't sell at t, stay at $t+1$	(m, i)
$i+1$	$F_t(1-\phi)$	didn't sell at t, move at $t+1$	$(n, i+1)$

Table $G5$: $s_t = (n, 0)$			
η_{t+1}	$\Pr(\eta_{t+1} \mid s_t)$	events	s_{t+1}
i	F_t	didn't buy at t	$(n, 0)$
i	$(1-F_t)\,\phi$	bought at t, stay at $t+1$	$(m, 0)$
$i+1$	$(1-F_t)(1-\phi)$	bought at t, move at $t+1$	$(n, 1)$

Table $G6$: $s_t = (n, i)\,, i > 0$			
η_{t+1}	$\Pr(\eta_{t+1} \mid s_t)$	events	s_{t+1}
$i-1$	$(1-F_t)\,F_t$	sold and didn't buy at t	$(m, i-1)$
$i-1$	$(1-F_t)^2\,\phi$	sold and bought at t, stay at $t+1$	$(m, i-1)$
i	$(1-F_t)^2(1-\phi)$	sold and bought at t, move at $t+1$	(n, i)
i	F_t^2	didn't sell and didn't buy at t	(n, i)
i	$F_t(1-F_t)\,\phi$	didn't sell and bought at t, stay at $t+1$	(m, i)
$i+1$	$F_t(1-F_t)(1-\phi)$	didn't sell and bought at t, move at $t+1$	$(n, i+1)$

We aggregate across all types of agents to derive $\Pr(\eta_{t+1} = i)$.

$$\Pr(\eta_{t+1} = i) = \sum_{s_t} \Pr(\eta_{t+1} = i \mid s_t) \Pr(s_t)$$

where given the initial distribution of types, $\Pr(s_0)$, we can compute $\Pr(s_t) = \Pr(s_t \mid s_{t-1}) \Pr(s_{t-1})$ with $\Pr(s_t \mid s_{t-1})$ given in the above four tables. Given that agents can only buy and sell one house in a period, the relevant s_t in the summation includes only those with $(i-1, i, i+1)$ houses for sale. The total number of houses for sale in period $t+1$ is

$$v_{t+1} = \sum_i i \Pr(\eta_t = i).$$

References

[1] Blake, T. (2011). "What's Exile to the Exurbs Worth? The Effect of Commuting Costs on Geographic Sorting in the Housing Market," UC Davis manuscript.

[2] Burdett, K. (1996), "Truncated means and variances," Economics Letters 52, 263-267.

[3] Diaz, A. and B. Jerez (2012) "House Prices, Sales and Time on the Market: A Search-Theoretic Framework, forthcoming in International Economic Review.

[4] Goodman, J. 1993. "A Housing Market Matching Model of the Seasonality in Geographic Mobility." Journal of Real Estate Research 8 (1): 117–137.

[5] Krainer, J. 2001. "A Theory Of Liquidity in Residential Real Estate Markets." Journal of Urban Economics 49 (1): 32–53.

[6] Mankiw, G. and D. Weil. 1989. "The Baby Boom, the Baby Bust, and the Housing Market." Regional Science and Urban Economics 19 (2): 235–258.

[7] Muellbauer, J. and A. Murphy. 1997. "Booms and Busts in the UK Housing Market." The Economic Journal 107 (445): 1701–1727.

[8] Novy-Marx, R. 2009. "Hot and Cold Markets." Real Estate Economics 37, 1-22.

[9] Piazzesi, M. and M. Schneider 2009. "Momentum traders in the housing market: survey evidence and a search model". American Economic Review P&P, V99 (2): 406-411.

[10] Pissarides, C. 2000. Equilibrium Unemployment Theory, second edition. Cambridge, MA: The MIT Press.

[11] Poterba, J. 1984. "Tax Subsidies to Owner-Occupied Housing: An Asset-Market Approach." Quarterly Journal of Economics 99 (4): 729–752.

[12] Samuelson, W. 1984. "Bargaining Under Asymmetric Information." Econometrica 52 (4): 995–1005.

[13] Ungerer, C. (2012) "Monetary Policy, Hot Housing Markets And Household Debt. Manuscript, LSE.