SPECTRUM AUCTIONS:

GREED IS GOOD, … IF YOU DO IT WELL!

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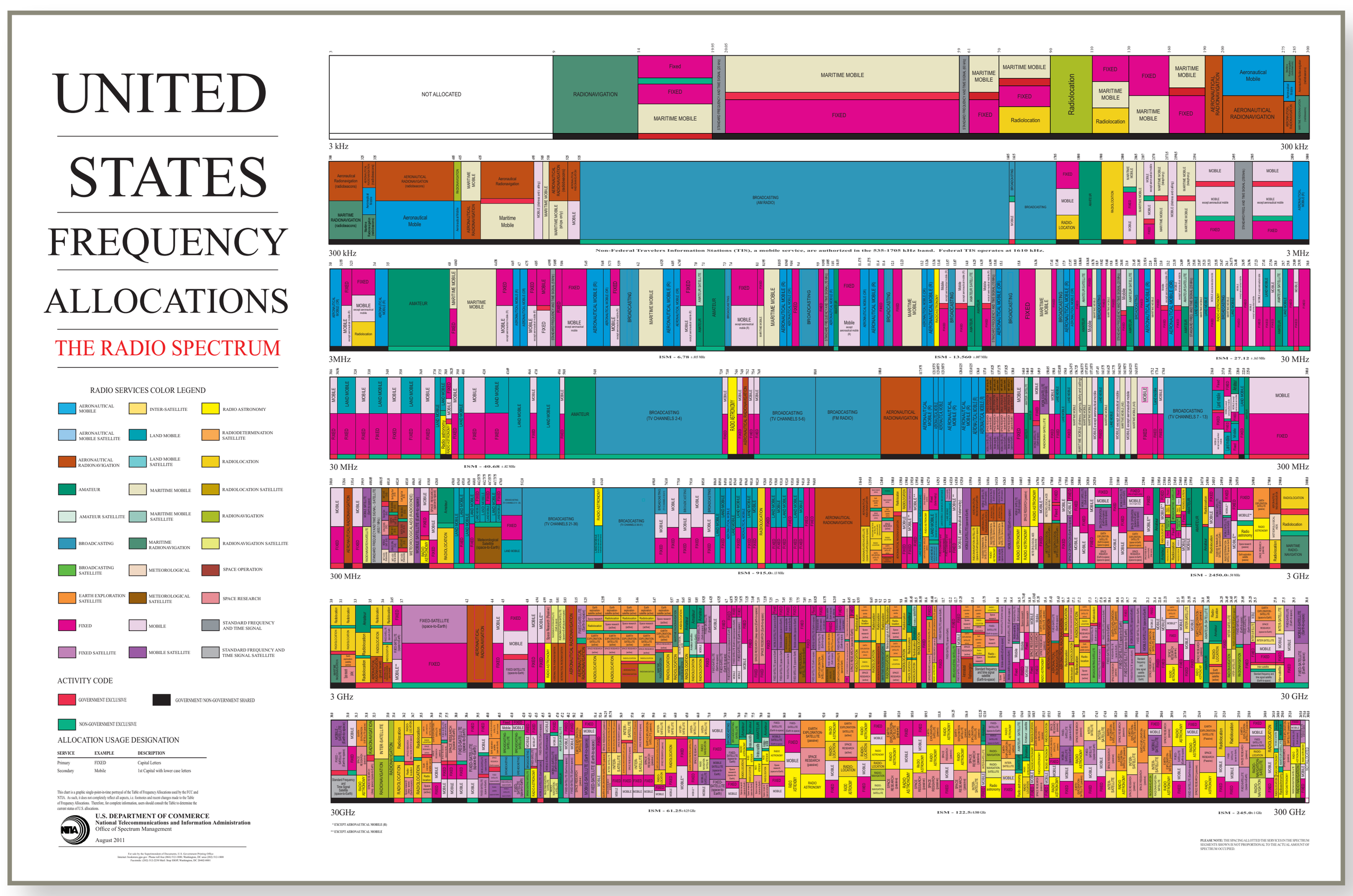
THE PROBLEM

Governments across the globe sell spectrum rights. Imagine it's your job to organise this. How would you do it?

The problem that you face is called a combinatorial auction. You, the auctioneer, have a collection of items (e.g., chunks of spectrum) to sell to a set of buyers. Buyers can be assigned more than one item and different subsets of items have different values for different buyers. As the auctioneer, you do not know those values. Still, you seek to maximise social welfare, that is, the sum of the values that the buyers derive from the items they win.

To this end, you want to run a (sealed-bid) auction: first you ask each buyer to bid on each subset of items they are interested in. Then, based on the bids, you decide on an allocation of the items and subsequent payments from the buyers. This process is called a mechanism.

Which mechanism, i.e., which allocation of items and payments, do you choose?



Frequency allocations in the US as of 2011 (http://www.ntia.doc.gov/)

THE CHALLENGES

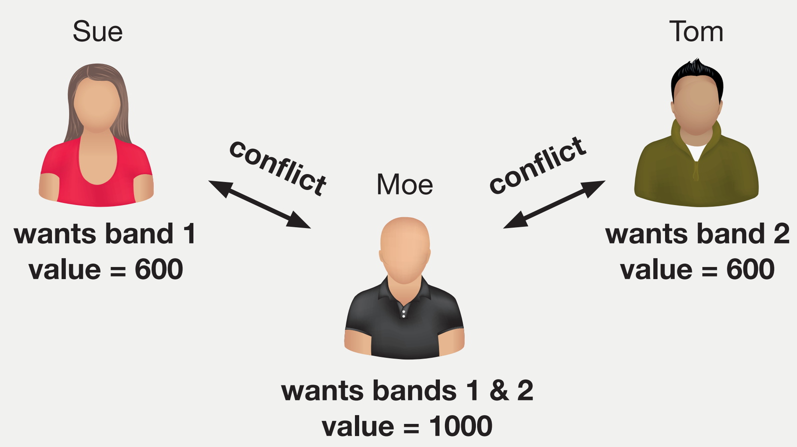
Challenge 1: Auctions are a highly strategic environment
Buyers want to maximise their utility, which is the difference of their value and their payment. To achieve this, they may tactically misreport: give bids that are different from their true values.

Challenge 2: Auctions are computationally challenging
For example, finding the allocation which is most beneficial to social welfare may be computationally infeasible even if the values are known and the number of buyers and items is small.

How Maths Can Help You

1. Mathematics lets you capture the important aspects in a simple model.
2. Mathematics lets you reason about strategic behaviour and computation before the auction is run.
3. Mathematics lets you prove that a certain design is optimal!

The Basic Question



Who should get what, and how much should they pay?

GOALS

Maximising the sum of the buyers' bids is recognised as a computationally intractable problem, even if each buyer is interested in a single bundle of items. Hence the best we can hope for is:

Goal 1: Find the optimal social welfare that is computationally feasible

Goal 2: Individual agents should not be able to benefit from misreporting

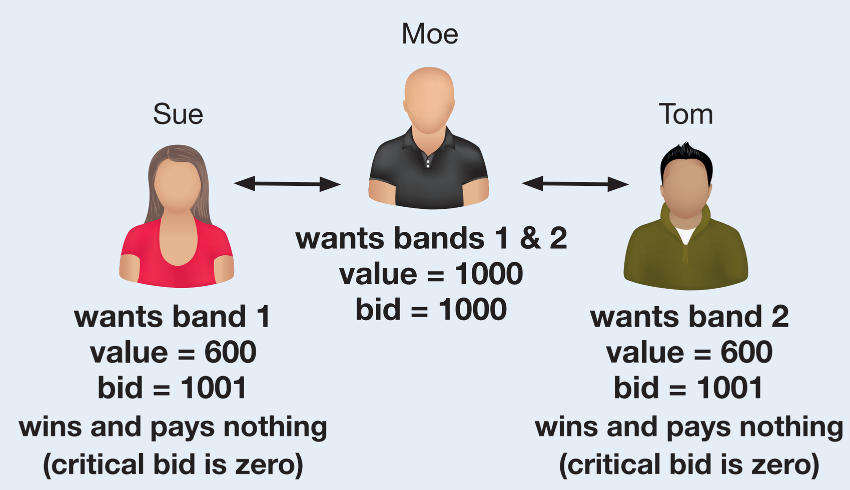
Goal 3: Groups of agents should not be able to benefit from misreporting

Prior Work

The central result in this area is the so-called **(forward) greedy mechanism**. To determine an allocation, it iteratively accepts what looks like the most promising bid. To this end, the mechanism ranks buyers by bids, from high to low. It then goes through the buyers, assigning the next bidder the bundle of items they asked for and removing any conflicting bidders. In the end, the mechanism charges each buyer their critical bid (this is the smallest bid that would ensure that the buyer gets the items they are interested in, holding the bids of the other buyers fixed).

The (forward) greedy mechanism achieves Goals 1 & 2, but it does not achieve Goal 3.

Why it Fails



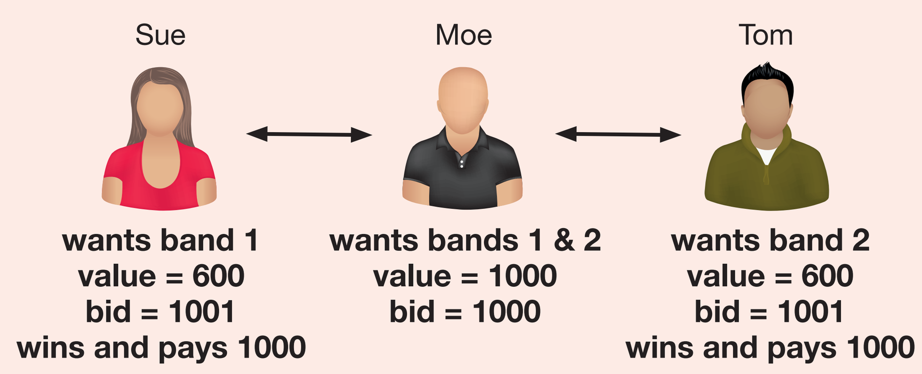
Sue and Tom can benefit from joint misreporting!

OUR RESULT

We show that a **"reverse" greedy mechanism**, which iteratively rejects what looks like the least promising bid, achieves all desired goals. This mechanism obtains the optimal social welfare subject to computability, whilst also ensuring that no individual buyer or group of buyers can benefit from misreporting.

Our mechanism thus achieves Goals 1, 2 and 3. This is extremely important in practice, as joint misreporting is very hard to avoid when it is possible.

Why Our Method Works



Sue and Tom will no longer find misreporting beneficial!

Conclusion

It's OK to be greedy, … But make sure you do it right!

LSE

The Performance of Deferred-Acceptance Auctions.
Paul Dütting, Vasilis Gkatzelis, Tim Roughgarden.
Proceedings of the 15th Conference on Economics and Computation, Palo Alto, California, USA, 2014.