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uncertainty model with credit-, risk-, and  
leverage cycles**

**Working paper**

**Original citation:**

Gerba, Eddie and Zochowski, Dawid (2015) *Macprudential policy in a Knightian uncertainty model with credit-, risk-, and leverage cycles*. ECB working paper, European Central Bank, Frankfurt am Main, Germany.

Originally available from [European Central Bank](#)

This version available at: <http://eprints.lse.ac.uk/62812/>

Available in LSE Research Online: July 2015

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# Macrorpudential Policy in a Knightian Uncertainty 1

## Model with Credit-, Risk-, and Leverage Cycles 2

Eddie Gerba and Dawid Zochowski 3

July 28, 2015 4

### Abstract 5

We study the impact of uncertainty on financial stability and the business cycle. We extend the work of Boz and Mendoza (2014) by endogenizing credit production, modifying learning mechanism into an adaptive set-up, as well as including financial and monetary policies. In our model households are (intrinsically) rational but take economic decisions under incomplete information. The incompleteness is not caused by their cognitive limitations, as in rational inattention theory (Sims, 2003). Households ‘learn by doing’ and once a sufficient number of realizations of the state variable have materialized, and the incomplete information set is completed. This learning set-up is incorporated into a New Keynesian model with credit market frictions, extended to include uncertainty, where a share of households needs external financing to consume. Because of limited enforceability of financial contracts, households are required to provide collateral for their loans, and so the relationship between the bank and household is tightened for many periods ahead. We find in our framework the build up of risk, leverage, increase in consumption and price of collateral takes longer than in other DSGEs with standard financial friction models. We also find that both the frequency and the amplitude of expansions and contractions are asymmetric - recessions are less frequent and deeper than expansions. Moreover, we find that boom-bust cycles occur as rare events. Using the Cogley and Sargent’s (2008) definition of a severe (or systemic) crisis, we find on average two such events per century. Ee also find that, different from standard boom-bust cycles, a systemic crisis can be followed by a sequence of subsequent contractions, as it makes the economy more unstable. The result is asymmetric distributions of key macroeconomic and financial variables, with high skewness and fat tails. Lastly, we also find 30

1 that, by reducing the amount of borrowing and leverage in upturns, the LTV-  
2 ratio regulation is effective in smoothing the cycles and reducing the effects of  
3 a deep contraction on the real-financial variables. We also discuss the role of  
4 macroprudential policy in reducing information incompleteness by generating  
5 information that helps the agent learn faster the new environment, or provide  
6 a smoother transition to the new economic environment.

7 Keywords: uncertainty, financial engineering, deregulation, leverage forecast-  
8 ing, macroprudential policy

9 JEL: G14, G17, G21, G32, E44, E58

# 1 Motivation

The Great Recession has been characterised by two features, which made it distinct from any other recession in the post-war era. First, it was preceded by a significant build-up of leverage, in particular in the household sector. Excessive borrowing, in particular in the mortgage market, was in particular facilitated by securitization, which reached its peak activity in 2007. Second, the depth and length of the recession resulted in a substantial deviation of GDP from its long term trend in the US, UK and in the euro area, which cannot be explained in a standard New Keynesian model set-up even after taking account of financial frictions.

We put forward a model that links these two stylized facts. Financial innovation shocks push the economy into a previously unexplored and unmapped state. In this new state, agents do not know the true riskiness of new financial products and therefore optimize under incomplete information. The incompleteness is not caused by households' cognitive limitations, but because they need to learn the true riskiness of the financial products. This learning process requires sufficient number of realisation of the state variable in order for the information set to be complete. As learning takes time, the economy approaches the new steady state only sluggishly.

The core of the model follows Boz and Mendoza (2014). However, we introduce three important modifications. Following Iacoviello and Neri (2009) we first split households (key agents of the model) into patient, who save and produce land, and impatient, who borrow and consume land. In addition, we introduce a financial intermediary and explicitly model the credit market.

The major friction in the model is uncertainty about new state of the economy after financial innovation shock. In this way, financial innovation interacting with credit/margin constraints can lead to underpricing of the risk associated with a new financial environment. This in turn can lead to the accumulation of leverage and surges in asset prices. Because of limited enforceability of financial contracts, households are required to provide collateral for their loans, and so the relationship between the bank and household is tightened for many periods ahead. Once the agents observe sufficient number of realisations of the new state of the economy and realise that they are overlevered, this can lead to a sudden stop a la Mendoza (2010). More formally, sudden stop is caused by the uncertainty regarding the transition probability of such events. Since systemic crises are rare events, agents inherently misprice the occurrence of such events (see for instance Zeira (1999), Caballero and

1 Krishnamurthy (2008) or Boz and Mendoza (2014)). Uncertainty coupled with  
2 Fisherian deflation mechanism leads to highly volatile and asymmetric distributions  
3 in asset prices, consumption, debt, loan- and deposit rates. Our approach is  
4 loosely linked to the rational inattention theory (Sims, 2010), which recognises that  
5 people have finite information-processing capacity that explains well some of the  
6 frictions.

7 We find that early realizations of the new state result in a much higher (lower)  
8 debt, consumption, price of collateral and risk accumulation (de-leveraging) during  
9 upturns (downturns) compared to standard financial friction models. Moreover,  
10 the loan-to-deposit ratio of banks is rapidly increasing at the onset of the financial  
11 innovation phase, and remains very high until sudden stop has materialized for a few  
12 periods. We also demonstrate that sluggish learning can explain why the economy  
13 can diverge from its long term trend for an extended period of time.

14 Next, we evaluate the efficacy of standard macroprudential tools, such as a cap  
15 on the loan-to-value (LTV) in reducing the leverage of the household sector. We  
16 also discuss the role for a new macroprudential policy in reducing the information  
17 incompleteness related to financial innovation by generating information that helps  
18 the agent learn faster the new environment, or provide a smoother transition to the  
19 new economic environment.

20 The remainder of this paper is organised as follows. In Section 2 we introduce  
21 the model set-up. Section 3 is devoted to the main friction of the model - it discusses  
22 uncertainty and describes the mechanism of learning. Section 4 presents a strategy  
23 for solving the model, while Section 5 presents first order conditions and discusses  
24 their implications. [Finally, we present results in section 6 and conclude in section  
25 7].

## 26 **2 Model**

### 27 **2.1 An overview**

28 The backbone of the model is a standard New Keynesian setup, extended to include  
29 uncertainty, learning and credit market frictions. We extend the work of Boz and  
30 Mendoza (2014) by endogenizing credit production, modify learning mechanism into  
31 an adaptive set-up, as well as include financial and monetary policies. In particular,  
32 we endogenize both the quantity and prices of deposits, loans, bank equity, make

agents learn according to (adaptive) heuristics, and include a central bank who simultaneously sets a (time-varying) policy rate and a macroprudential rule.

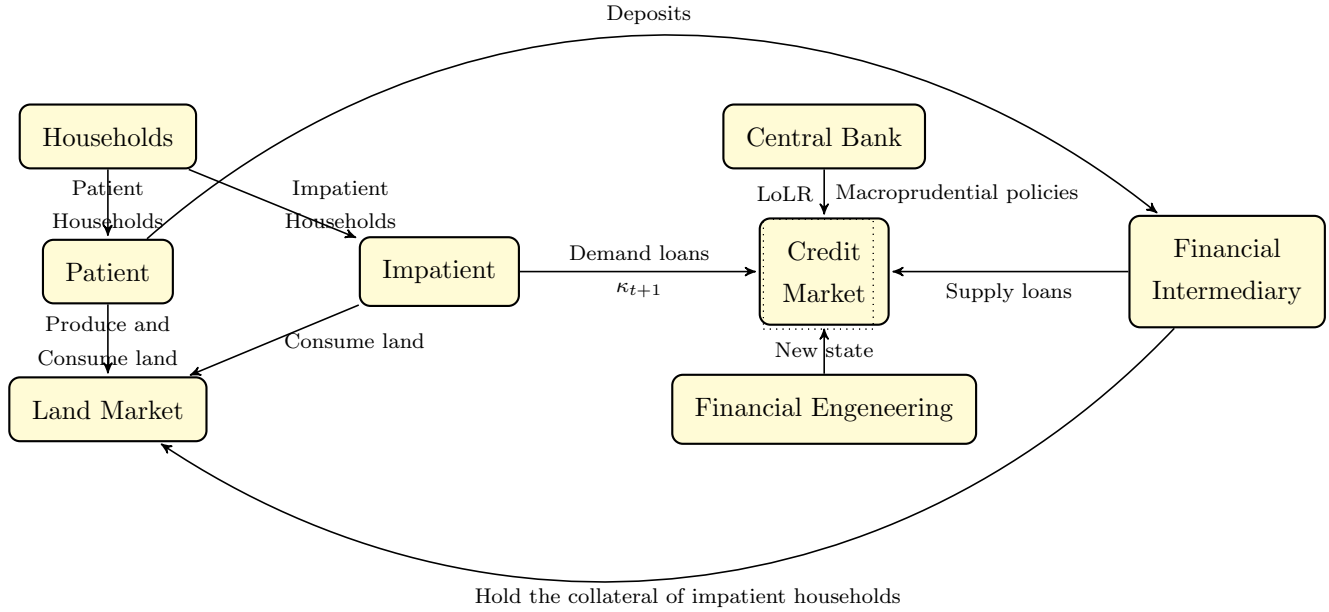
Our model economy is populated by three agents: households, financial intermediaries, and government. Moreover, we divide households into two categories: the patient and the impatient types. What differentiates them is the degree of patience. The discount factor  $\beta$  of patient households is higher than those of the impatient. This forces the latter to complement their internal funds with loans from the credit market. While patient households both produce and consume land, impatient only consume it. Therefore we explicitly model two markets: market for land and market for credit.

Nevertheless, what differentiates this model from most other financial friction frameworks is that this one incorporates uncertainty. Financial sector developments such as financial engineering, de-regulation of markets, and increased competition amongst financial intermediaries has meant that the new market structure is unknown and unexplored to the participants in financial transactions. As a result, agents do not know the true risks, leverage and price of collateral in the ‘new’ economy and therefore optimize under incomplete information. Our take on uncertainty is that agents are (intrinsically) rational insofar that they efficiently optimize over time, but do so under incomplete information regarding two variables in the model: the leverage ratio, and the price of collateral. One is exogenous while the other is endogenous, but dependent on the realization of the first. They engage in adaptive learning and learn about the ‘true’ values of leverage and asset prices only after observing a sufficiently long set of realizations of both variables. Note that this learning is, however, slow since they only learn from their practical experiences.

With respect to Boz and Mendoza (2014), the learning in this framework is more active, since one of the learning variables is endogenous. However, this variable is dependent on the exogenous (the shadow value of collateral constraint) variable which facilitates the tractability of the dynamic solution. Therefore, while agents can partially benefit from experimenting with the dynamic optimization to induce the endogenous land price, the exogenous component of this price will make such experimenting slow and costly. In other words, the values of the ‘learning variables’ cannot be directly deduced by recursively solving the remaining part of the model. Moreover, we will make their learning contingent on two rules, which will make the learning dynamics even more tractable.

Figure 1 provides an overview of the model.

1



2

## 3 2.2 Households

4 The consumption sector is populated by two types of infinitely lived households,  
5 each with a unit mass and they act atomistically in competitive markets.<sup>1</sup> Both  
6 types optimize under uncertainty. The key factor which differentiates them is the  
7 degree of impatience. The discount factor  $\beta$  of impatient households ( $I$ ) is lower  
8 than the one of patient ( $P$ ). This will ‘force’ the impatient households to engage  
9 in external credit market. For the sake of simplicity and tractability, we explicitly  
10 omit the labour supply decision of households which means that they only derive  
11 income from land and saving/borrowing.<sup>2</sup>

### 12 2.2.1 Patient Households

13 The representative patient risk-averse household chooses consumption  $c_t$ , land hold-  
14 ings  $l_{t+1}$ , and deposits  $d_t$ , taken as given the price of land  $q_t$ , the deposit rate  $R_t^d$ , and  
15 the gross real interest rate  $R_t$  so as to maximize a standard CRRA utility function:

<sup>1</sup>One could equivalently assume that in each period households die and are born with a constant probability so that on aggregate there is a unit mass of households.

<sup>2</sup>It would be straightforward to extend the model to include a labour market, as in for instance Gerali et al (2010).

$$E_0^s[\sum_{t=0}^{\infty} \beta^t u(c_t)] \quad (1)$$

where  $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ , and  $\sigma$  is the parameter of relative risk aversion of households. Because of their relative patience, these households are made natural lenders, and face the following budget constraint:

$$d_{t+1} + c_t^P \leq z_t g(l_t) + q_t l_t - q_t l_{t+1} + (1 + R_t^d) d_t + e_t \quad (2)$$

The share of patient households in the population is  $\theta$  (and is time invariant). The production function  $g(l_t) = l^\alpha$  is a standard neoclassical one and is subject to a stochastic productivity shock  $z_t$ , which is known to all agents.<sup>3</sup> Because we are interested in uncertainty regarding financial frictions, we omit from imperfect beliefs regarding the productivity shock. However, an immediate extension could be to also introduce macroeconomic uncertainty.

It is crucial to note that  $E_t^s$  in the utility function above represents expectations subject to agents' (subjective) beliefs using information available up to period  $t$  (inclusive). These beliefs will differ from the ones formulated under rational expectations.

## 2.2.2 Impatient Households

The impatient risk-neutral households (with the share of the total population equal to  $1 - \theta$ ) maximize the same type of CRRA utility function:<sup>4</sup>

$$E_0^s[\sum_{t=0}^{\infty} \beta^t u(c_t)] \quad (3)$$

where  $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ , but face a different budget constraint due to their impatient nature:

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<sup>3</sup>As is standard in this literature, we will assume that the TFP shock has an autoregressive process. However, we could have equivalently assumed the TFP shock to follow a Markov process, without changing much (or even at all) the results.

<sup>4</sup>Note that we depart here from the representative agent assumption and make the impatient households heterogeneous by subjecting them to different initial land holding (or wealth). Aside from this initial wealth heterogeneity, which will generate a wealth distribution in period  $t = 0$ , the constrained optimization problem is equal for all agents within this category. We simply need this initial heterogeneity to motivate the endogenous learning dynamics within this group, and the (possible) reason for switches between one rule and the other. The learning dynamics will be explained in further detail at a later stage.



$$c_t^I \leq q_t l_t - q_t l_{t+1} - \frac{b_{t+1}}{R_t^b} + b_t \quad (4)$$

1 where  $b_t$  are the holdings of one-period discount loans (or bonds). Because of  
 2 imperfections in the credit market (due to limited state-verification a la Townsend),  
 3 impatient households face restrictions in the quantity of external financing obtained  
 4 and must provide a collateral as a security.<sup>5</sup> Therefore, the LTV that the agent must  
 5 satisfy limits the value of credit  $\frac{b_{t+1}}{R_t^b}$  to a time-varying ratio of the market value of  
 6 their land holdings,  $\kappa_t$  according to:

$$E_t^s[\kappa_{t+1}]q_t l_{t+1} \leq -\frac{b_{t+1}}{R_t^b} \quad (5)$$

7 From a microeconomic perspective,  $\kappa$  can be seen as the proportional cost of  
 8 collateral repossession (or liquidation share) in case of default. Debt contract with  
 9 margin clauses are also captured by this relation (Mendoza, 2010). A relaxation  
 10 (tightening) of this constraint can either come from an increase (decrease) in the  
 11 borrowing capacity  $\kappa_t$  or from an increase (decrease) in the value or quantity of the  
 12 collateral  $q_t l_{t+1}$ . From a macroeconomic perspective, this relation can be interpreted  
 13 as the LTV ratio (or leverage) set by the macroprudential authority. This interpre-  
 14 tation will become evident later on when we study the impact of macroprudential  
 15 policies on the model dynamics.

16 The random variable  $\kappa_t$  is continuous with an upper bound at 1 and a non-  
 17 negative lower-bound. It is also time-varying. The framework is flexible enough  
 18 to capture asymmetric regime-switching probabilities between high and low lever-  
 19 age capacities. This is one of the variables that impatient agents have incomplete  
 20 information about, and which they will need to forecast.

21 Notice, again that the expectations operator is dependent on beliefs regarding  
 22 state  $s$ . This uncertainty (or ‘ignorance’) regarding the *true* state applies to the  
 23 entire population equally. Therefore, agents are rational in the sense that they use  
 24 all available information (and models) at time  $t$ , but form subjective beliefs because  
 25 they act under (evenly distributed) incomplete information.<sup>6</sup> However, agents engage

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<sup>5</sup>See, for instance, Bernanke, Gertler and Gilchrist (1999), or Christiano, Motto and Rostagno (2013) for background information and microfoundations of the state-verification problem in external lending. We use the outcomes from that problem to motivate our collateral constraint, but because of the similarity with the aforementioned frameworks, we abstain from providing full microfoundations of that problem.

<sup>6</sup>This is very different from model settings where one agents has more information than the other (asymmetric), or where agents use heterogeneous information and/or models (due to their

in (aggregate) learning and become fully aware of the *true* transition probabilities as they approach time  $t = T$ . We will describe the learning mechanism in further detail once we have outlined the full model.<sup>7</sup>

## 2.3 Financial intermediary

The representative financial intermediary operates in a perfectly competitive market and uses deposits from patient households to give out as loans to impatient households. As in Gerali et al (2010) they are owned by patient households (captured by the patients' discount factor  $\beta_p^t \lambda_t^p$ ), and maximize the discounted sum of cash flows:

$$E_0^s \sum_{t=0}^{\infty} \beta_p^t \lambda_t^p [(1+R^b)B_t - B_{t+1} + D_{t+1} - (1+R_t^d)D_t + (E_{t+1}^b - E_t^b) - \frac{\kappa_{E^b}}{2} [\frac{E_t^b}{B_t} - \nu^b]^2 E_t^b] \quad (6)$$

subject to the balance sheet constraint:  $B_t = D_t + E_t^b$ .  $B_t$  is the total amount of loans issued at time  $t$ ,  $D_t$  the aggregate number of deposits received from patient households,  $E_t^b$  the bank capital, and  $\nu^b$  is the long-term capital-to-asset ratio. The last term in the above maximization problem represents the cost of operating the financial intermediary. To motivate an undesirable social cost (externality) from excessive intermediary leverage from the point of view of the macroprudential policy maker, we impose a quadratic cost function whenever the intermediary's capital-to-asset ratio  $\frac{E_t^b}{B_t}$  moves away from the target value  $\nu^b$ . Because of the high number of competitors in the banking industry, the individual intermediary takes the deposit  $R_t^d$  and the loan rates  $R_t^b$  as given when maximizing its profits.<sup>8</sup>

The aggregate bank capital evolves according to:

$$E_{t+1}^b = (1 - \delta^b)E_t^b + \pi_t^b \quad (7)$$

cognitive restrictions) to infer the true states (irrationality).

<sup>7</sup>Preston (2005) pointed out that infinite horizon microfounded learning models fail to produce optimal dynamic consumption allocations while violating their intertemporal budget constraint, resulting in an inconsistency in the microfoundations. In defense, Hokaohja and Mittra (2011) showed that the intertemporal accounting consistency holds along the (infinite) sequence of temporary equilibria and that this model can be derived as a special case of Preston's framework.

<sup>8</sup>The intermediary also acts under incomplete information. That is why we have conditioned its expectations on the state  $s$  beliefs. However, their beliefs are of second order importance since they do not optimize with respect to  $\kappa_t$  nor do they engage in learning.  $\kappa_t$  is instead assumed to be out of direct control by either household or intermediary, and plays a key role only for the optimization of households. Therefore we will omit intermediary's subjective beliefs and in what follows, approximate its beliefs with the RE expectations operator.

1 where  $\delta^b$  measures the resources used in managing bank capital and  $\pi_t^b$  are overall  
 2 real profits made by the financial intermediary at date- $t$ . These are described by  
 3 the following relation:

$$\pi_t^b = R_t^b B_t - R^d D_t - \frac{\kappa_{E^b}}{2} \left[ \frac{E_t^b}{B_t} - \nu^b \right]^2 E_t^b - Adj_t^b \quad (8)$$

4 with  $Adj_t^b$  denoting the adjustment costs for changing interest rates on deposits.  
 5 This definition of profits is a narrow one as it coincides with the net interest rate  
 6 margin. It does not include any other items from the income statement in order  
 7 to maintain a closed-form solution for intermediary's optimization problem while  
 8 keeping it simple.

## 9 2.4 Credit Market

10 Next we need to derive the lending and deposit rates that financial intermediaries  
 11 charge. Iterating the balance-sheet constraint of financial intermediaries at date  $t$   
 12 and  $t + 1$  and inserting it into the cash-flow expression in equation 6, we get that  
 13 the intermediary's objective is to maximize:

$$R_t^b B_t - R_t^d D_t - \frac{\kappa_{E^b}}{2} \left[ \frac{E_t^b}{B_t} - \nu^b \right]^2 E_t^b \quad (9)$$

14 Taking first-order conditions with respect to  $B_t$  and  $D_t$  and combining them, we  
 15 get that the spread charged on loans is equal to:

$$R_t^b = R_t^d - \kappa_{E^b} \left[ \frac{E_t^b}{B_t} - \nu^b \right] \left[ \frac{E_t^b}{B_t} \right]^2 \quad (10)$$

16 Since patient households are risk-averse, they will ask for a safe rate on their  
 17 deposits, that by no-arbitrage condition, will equal to the real rate  $R_t$  (see for  
 18 instance Bernanke, Gertler and Gilchrist (1999) or Christiano, Motto and Rostagno  
 19 (2013) for a microfoundation behind this result).<sup>9</sup>We can thus re-write the above  
 20 expression as:

$$R_t^b - R_t = \kappa_{E^b} \left[ \frac{E_t^b}{B_t} - \nu^b \right] \left[ \frac{E_t^b}{B_t} \right]^2 \quad (11)$$

21 This expression represents the trade-offs that the financial intermediary faces in

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<sup>9</sup>Following Gerali et al (2010), we could equivalently assume that the financial intermediary has continuous and risk-free access to central bank liquidity at the safe rate  $R_t$ , which by arbitrage would make the deposit rate equal to the safe rate.

setting the lending rate. The left-hand side represents the marginal benefit from increasing lending meanwhile the right-hand side represents the costs of increasing leverage (by deviating from the  $\nu^b$  target). The final lending rate will be set where the two are equal.

## 2.5 Land Market

We can show that the effects of the collateral constraint on asset pricing can be derived by combining the Euler equations of land for the two households.<sup>10</sup> Solving the equations forward in which the future stream of land dividends is discounted at the stochastic discount factor and adjusted for the shadow value of the credit constraint:

$$q_t = E_t^s \sum_{j=0}^{\infty} \left[ \prod_{i=0}^j \frac{\beta u'(c_{t+1+i})}{u'(c_{t+i}) - \mu_{t+i} \kappa_{t+i}} \right] z_{t+1+j} g'(l_{t+1+j}) \quad (12)$$

This condition equalizes the equilibrium price of land with the marginal cost of investment. Looking at the denominator of I.22, we see that the collateral constraint lowers land prices since it increases the rate of return at which future land dividends are discounted. It is forward-looking since not only will a binding constraint at  $t$  reduce the value of land, but also if agents expect that the constraint can bind at any future date  $E_t^s[\mu_{t+i} \kappa_{t+i}]$  for any  $i > 0$ , the value of land will fall.

If we further define the next period marginal utility of consumption as  $\lambda_{t+1} \equiv \beta u'(c_{t+1})$  and return on land as:

$$R_{t+1}^q = \frac{z_{t+1+j} g'(l_{t+1+j}) + q_{t+1}}{q_t} \quad (13)$$

we can define the (subjective) premium on land as (Mendoza, 2010):

$$E_t^s [R_{t+1}^q - R_t] = \frac{(1 - \kappa_t) \mu_t - Cov_t^s(\lambda_{t+1}, R_{t+1}^q)}{E_t^s(\lambda_{t+1})} \quad (14)$$

The land premium rises in every state in which the collateral constraint binds because of these three effects:

- The direct effect,  $(1 - \kappa_t) \mu_t$ , is due to a rise in the shadow value of the collateral constraint (with an upper bound determined by  $\kappa_t$ , the amount of the collateral that can be turned into debt).

<sup>10</sup>We follow the method described in Mendoza (2010).

- 1 • The indirect effect, represented by a lower  $Cov_t^s(\lambda_{t+1}, R_{t+1}^q)$  and a higher  
2  $E_t^s(\lambda_{t+1})$ .
- 3 • Because of the collateral constraint, the household's ability to smoothen her  
4 consumption is limited, leading her to transfer the consumption into the future.

5 To see the effects of this on the price of tangible, we can write the land price as  
6 a function of the return according to:

$$q_t = E_t^s \sum_{j=0}^{\infty} \left[ \prod_{i=0}^j \frac{1}{E_t^s[R_{t+1}^q]} \right] z_{t+1+j} g'(l_{t+1+j}) \quad (15)$$

7 since the expected land return satisfies the condition  $q_t E_t^s[R_{t+1}^q] = E_t^s[z_{t+1+j} g'(l_{t+1+j})]$ .  
8 Then, as Aiyagari and Gertler (1999) show, an increase in expected return will lead  
9 to lower equity prices in the current period, since the discount rate of future divi-  
10 dends will increase due to the binding collateral constraint, in the current or next  
11 period. Hence, if the collateral constraint binds at least occasionally in the stochas-  
12 tic steady state, the entire equilibrium asset pricing function will be distorted by  
13 the constraint (independent of whether the constraint binds in the current period  
14 or not).

15 If these effects are at work under rationally formed expectations (with the knowl-  
16 edge of the true state of  $\kappa$ ), these effects are further accentuated if we in addition  
17 introduce learning into this framework. To understand how, one needs to exam-  
18 ine the interactions between the collateral constraint and learning regarding the  $\kappa_t$   
19 variable. Suppose that the constraint was binding at  $t$ . In booms (or states with  
20 high leverage possibilities), the price of asset is higher, which will relax the LTV  
21 constraint. From equation 15 it implies that the land return is lower. So assuming  
22 that beliefs are optimistic (pessimistic) in a boom (bust), impatient households will  
23 assign a higher probability to lower (higher) future land returns than under RE. This  
24 will push land price further up (down), which via the LTV-constraint, will result in  
25 higher (lower) indebtedness.

26 Taking into account the tight and procyclical link between leverage and asset  
27 prices, and considering that the the value of  $\kappa_t$  (which is an argument of the land  
28 price  $q_t$ ) is unknown and therefore forecasted, it is reasonable to also make the value  
29  $q_t$  uncertain (and state contingent). Hence households will have to forecast the  
30 values of  $\kappa$  as well as  $q$ .

Following Boz and Mendoza (2014), for simplicity we will assume that the aggregate land supply is fixed and equal to 1. Consequently, the market clearing condition in the land market:

$$1 = \theta l_t^P + (1 - \theta) l_t^I \quad (16)$$

implies that the land holdings of the representative household must at each  $t$  satisfy  $l_t = 1$ , as well as the production function will be reduced to  $z_t g(1)$ .<sup>11</sup>

## 2.6 Central Bank

To close the model, we separately model the two policies of the central bank. Assuming that the variables without time subscripts denote their steady state values, we can characterize the monetary policy of the central bank with a standard Taylor-rule (expressed in deviations-from-the-target terms):

$$\frac{R_t}{R} = \frac{R_{t-1}}{R}^{\gamma_R} \left[ \frac{\pi_t}{\pi} \frac{y_t}{y} \right]^{\gamma_y} e^{\epsilon_{R,t}} \quad (17)$$

where  $\epsilon_{R,t}$  is a monetary policy shock.

On the other end, macroprudential policy is modeled as a set of *ex ante* rules that the intermediary sector must obey to. The first rule is a cap on the LTV ratio (independent of the state):

$$\kappa_t = \bar{\kappa} \quad (18)$$

Alternatively, we will test a more elaborate version of the above LTV-rule. Recently, several papers (Lambertini et al (2013), Angelini et al (2014)) have proposed Taylor-type macroprudential rules as a good approximation of the Basel II/III-style of regulatory requirements. We will therefore perform an alternative scenario where the central bank uses:

$$\kappa_t = \rho_\kappa * \kappa_{t-1} + (1 - \rho_\kappa) * \kappa^* + (1 - \rho_\kappa) * (b_t - b_{t-1}) \quad (19)$$

,where  $\kappa^*$  is the steady state value for the LTV-ratio. We calibrate it to 2 in line with the above rule in order to facilitate the comparison between a static (state-independent) and a dynamic (state-dependent) version.

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<sup>11</sup>Hence all the variation in land will come in its value, which is a function of the intertemporal consumption smoothing of households, as well as the shadow value of collateral constraint.

## 1 **3 Uncertainty and Learning**

2 Now that we outlined the key decision makers in the model, we need to devote  
3 some attention to the non-standard aspects of our model. In particular, we wish to  
4 describe the environment and the processes that govern the learning of our agents.

### 5 **3.1 The general outline**

6 Following Boz and Mendoza (2011, 2014), we model a situation in which financial  
7 engineering and market de-regulation lead to an increase in credit, leverage and risks.  
8 Agents know therefore that the environment (and the value of all these variables)  
9 has changed, but they don't know exactly by how much. Thus, the uncertainty  
10 concerns the 'true' values of the LTV-ratio  $\kappa$ , and the land price  $q$ .<sup>12</sup>Therefore, in  
11 contrast to Boz and Mendoza (2011, 2014), we assume that there are more than two  
12 possible future regimes as the values of land and leverage can have many different  
13 realizations. Moreover, in our framework agents are adaptive learners and use simple  
14 heuristics to forecast the two variables. In the (very) long-run, their beliefs converge  
15 to rational expectations. In the short run, however, their beliefs will be different  
16 from the equilibrium with full information. They learn only from past experience  
17 and fully 'understand' the riskiness of the new financial environment only after they  
18 have observed a sufficiently large sample of realizations. As a result, agents are slow  
19 learners and their learning process is strongly history dependent.<sup>13</sup>

20 Cecchetti et al (2000) and Cogley and Sargent (2008b) show that CRRA utility  
21 functions with Markov process for the consumption growth can generate asymmetric  
22 behaviour in consumption. High-growth states in consumption are persistent and  
23 common. However, once a low-growth state has been reached, the contractions are  
24 severe, with a mean decline of 6.785% p.a. Moreover, once the economy is in the  
25 low-growth state, there is a certain positive probability of running into a sequence of  
26 contractions, with a total decline in consumption amounting to 25% (assuming the

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<sup>12</sup>Equivalently, and using the approach by Boz and Mendoza (2014), one could say that the uncertainty is regarding the transition probability to a new state. This state is a subset to a bounded set between 0 and 1.

<sup>13</sup>In contrast, agents are Bayesian learners in Boz and Mendoza (2014) and their learning space is constrained to only two realizations of the 'learning variable': High or Low leverage states. In addition, the uncertainty concerns leverage only (and not land prices, despite the fact that uncertainty will enter the land price function via the shadow value of collateral constraint.) Therefore the speed of learning and convergence is expected to be higher in their model compared to ours once we acknowledge that the probability space of the (learning) variables in their model is much smaller.

contraction lasts for 4 years with a probability of 7.1%). We will use this threshold to identify *ex post* severe contractions (or systemic crises) in our model.

The current learning set-up means that agents learn quickly about the leverage/land price states that occur more frequently. Therefore, taking into account that severe contractions are rare, learning about them will also be slower and asymmetric with respect to expansions. Moreover, because the ergodic probability of a contraction is as small as 0.0434 (Cogley and Sargent, 2008b), the time elapsed before a sufficiently large sample of contractions has been observed is very large. This retards the learning of contractions significantly.

The type of uncertainty we model in this paper generates fat tails. In addition, the tails are asymmetric since contractions are more rare than expansions, and so the lower tail (low (or negative) values of consumption, credit and bank equity, or high values of leverage and interest rates) is significantly fatter than the upper tail (as the uncertainty regarding it is higher). This leaves open the possibility for serious downward spirals in contractions.

### 3.2 Specification of the learning process

Let us next formalize the learning described above. Our approach is similar to De Grauwe and Macchiarelli insofar that we use the same type of heuristics and updating of beliefs.

Under rational expectations, the forecasted variable will equal its realized value in the next period, i.e.  $E_t X_{t+1} = X_{t+1}$ , denoting generically by  $X_t$  any variable in the model. However, as anticipated above, we depart from this assumption in this framework by making the forecast contingent on imperfect information, but allowing the agents to learn. Expectations are replaced by a convex combination of heterogeneous expectation operators  $E_t \kappa_{t+1} = E_t^s \kappa_{t+1}$  and  $E_t q_{t+1} = E_t^s q_{t+1}$ . In particular, agents forecast the LTV-ratio and the land price using two alternative forecasting rules: *fundamentalist* vs. *extrapolative rule*. Under the fundamentalist rule, agents are assumed to use the steady-state value of the LTV-ratio -  $\kappa^*$ , against a naive forecast based on the LTV's latest available observation (extrapolative rule). Equally for the value of land, *fundamentalist* agents are assumed to base their expectations on the steady-state value -  $q^*$  against the *extrapolatists* who naively base their forecast on the latest available observable.<sup>14</sup> Defining  $i = (\kappa, q)$  we can formally

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<sup>14</sup>The latest available observation is the best forecast of the future, i.e. a random walk approach



1 express the fundamentalists as:

$$E_t^{s,f} i_{t+1} = i^* \quad (20)$$

2 and the extrapolative (or adaptive) rule as:

$$E_t^{s,e} i_{t+1} = \theta i_{t-1} \quad (21)$$

3 This particular form of adaptive expectations has previously been modelled by  
4 Pesaran (1987), Brock and Hommes (1997, 1998), and Branch and McGough (2009),  
5 amongst others, in the literature. Setting  $\theta = 1$  captures the "naive" agents (as they  
6 have a strong belief in history dependence), while a  $\theta < 1$  or  $\theta > 1$  represents an  
7 "adaptive" or an "extrapolative" agent (Brock and Hommes, 1998). For reasons  
8 of tractability, we set  $\theta = 1$  in this model, but the model dynamics would not be  
9 significantly altered with any of the other parameter values.

10 Note that for the sake of consistency with standard RE DSGE model, all variables  
11 here are expressed in gaps. Focusing on their cyclical component makes the model  
12 symmetric with respect to the steady state (see Harvey and Jaeger, 1993). Moreover,  
13 this facilitates the interpretation of the model as the fundamentalists can be seen as  
14 'benchmarking' the variable values, meanwhile the problem of extrapolists is pinned  
15 down to guessing the deviation of these values from their benchmark (or steady  
16 state).

17 Next, agents' preference for one forecast over the other depends on the (historical)  
18 performance of the two rules given by a publically available fitness measure,  
19 the mean square forecasting error (MSFE). After time  $t + 1$  realization is revealed,  
20 the two predictors are evaluated *ex post* using MSFE and new fractions of agent  
21 types are determined. These updated fractions are used to determine next period  
22 (aggregate) forecasts of LTV-and land prices, and so on. Agents' rationality consists  
23 therefore in choosing the best-performing predictor using the updated fitness  
24 measure. There is a strong empirical motivation for inserting this type of switching  
25 mechanism amongst different forecasting rules (see DeGrauwe and Macchiarelli  
26 (2015) for a brief discussion of the empirical literature, Frankel and Froot (1990)  
27 for a discussion of *fundamentalist* behaviour, and Roos and Schmidt (2012), Cogley  
28 (2002), Cogley and Sargent (2007) and Cornea, Hommes and Massaro (2013) for  
29 evidence of *extrapolative* behaviour in the context of microeconomic and financial  
30 decision-making.).

The aggregate market forecasts of the LTV-ratio and land price are obtained as  
a weighted average of each rule ( $i = \kappa, q$ ):

$$E_t^s i_{t+1} = \alpha_t^f E_t^{s,f} i_{t+1} + \alpha_t^e E_t^{s,e} i_{t+1} \quad (22)$$

where  $\alpha_t^f$  is the weighted average of fundamentalists, and  $\alpha_t^e$  that of the ex-  
trapolists. These shares are time-varying and based on the dynamic predictor se-  
lection. The mechanism allows to switch between the two forecasting rules based  
on MSFE / utility of the two rules, and increase (decrease) the weight of one rule  
over the other at each  $t$ . Assuming that the utilities of the two alternative rules  
have a deterministic and a random component (with a log-normal distribution as  
in Manski and McFadden (1981) or Anderson et al (1992)), the two weights can be  
defined based on each period utility  $U_{i,t}^x$ ,  $i = (\kappa, q)$ ,  $x = (f, e)$  according to:

$$\alpha_{i,t}^f = \frac{\exp(\gamma U_{i,t}^f)}{\exp(\gamma U_{i,t}^f) + \exp(\gamma U_{i,t}^e)} \quad (23)$$

$$\alpha_{i,t}^e \equiv 1 - \alpha_{i,t}^f = \frac{\exp(\gamma U_{i,t}^e)}{\exp(\gamma U_{i,t}^f) + \exp(\gamma U_{i,t}^e)} \quad (24)$$

, where the utilities are defined as:

$$U_{i,t}^f = - \sum_{k=0}^{\infty} w_k [i_{t-k-1} - E_{t-k-2}^{s,f} i_{t-k-1}]^2 \quad (25)$$

$$U_{i,t}^e = - \sum_{k=0}^{\infty} w_k [i_{t-k-1} - E_{t-k-2}^{s,e} i_{t-k-1}]^2 \quad (26)$$

and  $w_k = (\rho^k(1 - \rho))$  (with  $0 < \rho < 1$ ) are gemoetrically declining weights  
adapted to include the degree of forgetfulness in the model (DeGrauwe, 2012).  $\gamma$  is  
a parameter measuring the extent to which the deterministic component of utility  
determines actual choice. A value of 0 implies a perfectly stochastic utility. In  
that case, each agent decides to be one type or the other simply by tossing a coin,  
implying a probability of each type equalizing to 0.5. On the other hand,  $\gamma = \infty$   
implies a fully deterministic utility, and the probability of using the fundamentalist  
(extrapolative) rule is either 1 or 0. Another way of interpreting  $\gamma$  is in terms of  
learning from past performance:  $\gamma = 0$  implies zero willingness to learn, while it  
increases with the size of the parameter, i.e.  $0 < \gamma < \infty$ .

As mentioned above, agents will subject the performance of rules to a goodness-

1 of-fit measure and choose the one that generates least errors. In that sense, agents  
2 are 'boundedly' rational and learn from their mistakes. More importantly, this  
3 discrete choice mechanism allows to endogenize the distribution of heterogeneous  
4 agents over time with the proportion of each agent using a certain rule (parameter  
5  $\alpha$ ). The approach is consistent with the empirical studies (Cornea et al, 2012) who  
6 show that the distribution of heterogeneous agents varies in reaction to economic or  
7 financial volatility (Carroll (2003), Mankiw et al (2004)).

### 8 **3.3 Learning - stability remark**

9 One valid critique of learning environments in macroeconomics has been that they  
10 often become unstable. Unless strict boundaries are imposed on the learning process  
11 (i.e. a (high) degree of bounded rationality is imposed which ensures that system  
12 will, locally, converge to rational expectations equilibrium), the model often turns  
13 explosive.<sup>15</sup> In our framework, however, we do impose strict limits on the learning  
14 process and trace the learning dynamics throughout the entire simulations. More-  
15 over, using the results of Bullard and Suda (2011) that macroeconomic systems with  
16 Bayesian learning are non-explosive as expectations are locally stable in the sense  
17 of Evans and Honkapohja (2001) and the insights from Camerer and Ho (1999) or  
18 Payzan-LeNestour and Bossaerts (2014) that adaptive and Bayesian learning are  
19 'close cousins', we can conclude that our framework does not suffer from explosive  
20 paths or instability.<sup>16</sup>

### 21 **3.4 The recursive solution method and the numerical set-up**

22 We formulate the model in recursive form and solve using recursive methods (see  
23 DeGrauwe (2012) for further details). This allows for non-linear effects and is a  
24 more cost-effective alternative to the standard Bellman equation approach since we  
25 avoid using aggregate states and iterations to converge on the representative agent  
26 condition, matching individual and aggregate laws of motion for credit.

27 The model has eleven endogenous variables: land price, leverage, consumption,  
28 loans, interest rate on loans, deposits, interest rate on deposits, bank profits, bank

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<sup>15</sup>A very popular example of a bounded rationality environment in the literature has been the recursive least square learning.

<sup>16</sup>For future work, however, it would be interesting to computationally test the long-run dynamics of our learning framework, and compare with other (similar) versions of adaptive or Bayesian updating set-ups.

equity, land, and the interest rate. The first four are obtained after solving the following reduced equilibrium system that iterates on the policy and pricing functions using households' FOCs and the forecasting rules:

$$\begin{aligned}
& \begin{bmatrix} 1 & \mu^{fx} & \bar{c} & 0 \\ 1 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} q_t \\ \kappa_t \\ c_t \\ b_t \end{bmatrix} = \\
& = \begin{bmatrix} \alpha\nu & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_t^s[q_{t+1}] \\ E_t^s[\kappa_{t+1}] \\ E_t^s[c_{t+1}] \\ E_t^s[b_{t+1}] \end{bmatrix} + \\
& + \begin{bmatrix} 1 - \nu & -\mu^{fx} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_{t-1} \\ \kappa_{t-1} \\ c_{t-1} \\ b_{t-1} \end{bmatrix} + \\
& + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 + r_{t-1}^d \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{t-1}^b \\ l_{t-1} \\ d_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \epsilon_t^z \\ \epsilon_t^{Eb} \\ \psi_t \\ \epsilon_t^r \end{bmatrix}
\end{aligned}$$

Using matrix notation, we can write this as:  $\mathbf{AZ}_t = \mathbf{B}\tilde{\mathbf{E}}_t\mathbf{Z}_{t+1} + \mathbf{CZ}_{t-1} + \mathbf{DX}_{t-1} + \mathbf{Ev}_t$ . We can solve for  $\mathbf{Z}_t$  by inverting:  $\mathbf{Z}_t = \mathbf{A}^{-1}(\mathbf{B}\tilde{\mathbf{E}}_t\mathbf{Z}_{t+1} + \mathbf{CZ}_{t-1} + \mathbf{DX}_{t-1} + \mathbf{Ev}_t)$  and assuring  $\mathbf{A}$  to be non-singular.

Once these optimal values for the policy functions have been found, they are then inserted into the remaining general equilibrium system and the values of the remaining model variables are recursively solved. So, the solution for land, the interest rate on borrowings, deposits, bank profits, bank equity, and the interest rate are recursively obtained using the solutions obtained for land price, leverage, consumption and loans iterated above.

Expectation terms with an  $s E_t^s$  implies that we derive the optimal solution using the subjective beliefs governed by the learning process specified above.

Note that for the forecasts of land price and leverage, the expectation terms in equations 5, 12 and 15 are substituted by the discrete choice mechanism in 22.

1 We have four shocks in this model.  $\epsilon_t^z$  is a standard TFP shock in the land  
2 production function.  $\epsilon_t^{Eb}$  is a shock to bank capital (or equity),  $\psi_t$  denotes a shock  
3 to income (or collateral value), whereas  $\epsilon_t^r$  is a standard monetary policy shock.  
4 Their parametrizations will be discussed in the next subsection.

### 5 **3.5 Calibration and simulations**

6 We will divide the discussion in three parts. First, we will discuss the parameters  
7 related to the general equilibrium set-up. We will continue with the parameters  
8 related to the learning dynamics in the second part, followed by the calibration of  
9 the four shocks in the model. A full list of parameters and their values are reported  
10 in Table 1.

11 For the calibration of parameters related to the general equilibrium, we use the  
12 parameters calibrated or estimated in a number of closely related DSGE models. In  
13 particular, the (constant) risk aversion coefficient  $\sigma$  in households' utility function  
14 is, following Boz and Mendoza (2014), set to 2. We set the share of impatient  
15 households in the total economy to 0.61, as in Brzoza-Brzezina et al (2014), in order  
16 to match the micro data on the share of liquidity constrained consumers reported  
17 in the Survey of Consumer Finances (SCF). This is also in line with the number  
18 reported in Justiniano et al (2015). The discount factor  $\beta$  of patient households is  
19 higher and set to 0.9943 in order to obtain an annualized average real interest rate  
20 of slightly below 3%. This is in line with much of the literature, including Gerali  
21 et al (2010) and Brzoza-Brzezina et al (2014). The discount factor of the impatient  
22 types is lower and set to 0.975, as in Gerali et al (2010).

23 For parameters related to financial intermediaries, we use the estimation results  
24 from Gerali et al (2010) and De Grauwe and Macchiarelli (2015). In particular, we  
25 set the share of bank profits in bank equity equation  $\omega^b$  to 1, the cost for managing  
26 banks' capital position  $\delta^k$  to 0.1049, the adjustment costs of changing the interest  
27 rate on deposits  $Adj^b$  to 0 (since the unlimited access to liquidity from the central  
28 bank makes this process costless) and the target capital-to-loans ratio  $\nu^b$  (or the  
29 inverse of the leverage target ratio) to 0.09. In order to make the deviation from  
30 this target value costly, we calibrate the cost parameter  $\kappa^{Eb}$  to 11.49, which is the  
31 value obtained from estimations in Gerali et al (2010).

32 Turning to the land market, we use the values obtained in Boz and Mendoza  
33 (2014). In particular, we calibrate the factor share of land in the production  $\alpha$  to

0.025, and we set the supply of land  $l$  fixed at 1. The (fixed) Lagrange multiplier  $\mu^{fx}$  in the credit constraint, which is used to derive the shadow value of collateral in the land price function in equation 12, is set to 0.30.

Following Boz and Mendoza (2014), we set the consumption-GDP ratio in the aggregate resource constraint to 0.670, or two-thirds of the total output. Meanwhile, the remaining third is split between land and bank equity, where land-GDP ratio is set to 0.20 and bank equity-GDP to 0.13.

For the Taylor-rule parameters, we use the values estimated in Gerali et al (2010). In particular, the interest rate smoothing (AR) coefficient is set to 0.77, the response to inflation in the Taylor rule to 2.01, meanwhile the response to output is set to 0.35. Equally, for macroprudential policy, we set the target (or cap) on household leverage  $\bar{\kappa}$  to 2, and the response of LTV to credit growth  $\rho^\kappa$  in the Taylor-type macroprudential rule to 0.75, as in Lambertini et al (2013).

We turn to the parameters governing the learning process. The initial fraction of fundamentalists and extrapolists,  $\alpha_0^f$  and  $\alpha_0^e$  are each set to 0.5. The switching parameter,  $\gamma$  in equations 23 and 24 is set to 1, as in Brock and Hommes (1998).  $\rho$ , or the geometrically declining weight adapted to include a degree of forgetfulness in the learning dynamics in 25 and 26, is set to 0.5. For fundamentalists, we set the SS value of LTV,  $\kappa^*$  to 0.93 (as in Brzoza-Brzezina et al, 2014), and for the land price  $q^*$  simply to 1. To conclude this part, we make the land price highly contingent on its forecasted value by households, and therefore set the weight of the forecasted land price in the land price function  $\nu$  equal to 0.7. That is in order to capture the uncertainty regarding its future value in the aggregate land dynamics.

We are considering four shocks in this model. A shock to TFP (or technology), (bank) capital quality, household income, and a monetary policy shock. The standard deviation of all shocks is normalized to 1 to facilitate the interpretation of the impulse responses. In line with the literature, the TFP and monetary policy shocks include an AR component equal to 0.90. (Bank) capital quality and income shocks, on the other hand, are each modelled as a white noise (with no AR component) since they lack a theoretical grounding for incorporating inertias into their process.

We simulate the model for 2000 periods, or 500 years.

Table 1: Parameters in the model and their descriptions

Parameter	Description	Value
<u>Calibrated</u>		<u>Calibration</u>
$\sigma$	Constant risk aversion parameter in CRRA utility function	2
$\omega$	Share of impatient households in the economy	0.61
$\beta^P$	Discount factor of patient households	0.9943
$\beta^I$	Discount factor of impatient households	0.975
$\omega^b$	Share of bank profits in bank equity accumulation	1
$\delta^k$	Cost for managing banks' capital position	0.1049
$Adj^b$	Adjustment cost for changing the deposit rate	0
$\nu^b$	Target capital-to-asset ratio	0.09
$\kappa^{Eb}$	Cost of deviating from target capital-to-asset ratio	11.49
$\alpha$	Factor share of land in production	0.025
$l$	Aggregate land supply	1
$\mu^{fx}$	(Fixed) Lagrangian multiplier of the credit constraint	0.3
$\bar{c}$	Consumption-GDP ratio	0.67
$\bar{l}$	Land-GDP ratio	0.20
$\bar{c}$	Bank equity-GDP ratio	0.130
$\gamma^r$	Interest rate smoothing parameter	0.77
$\gamma^\pi$	Response to inflation in the Taylor rule	2.01
$\gamma^y$	Response to output in the Taylor rule	0.35
$\bar{\kappa}$	Cap on household LTV-ratio	2
$\rho^k$	Response of LTV to credit growth	0.75
$\alpha_0^f$	Initial fraction of fundamentalists	0.5
$\alpha_0^e$	Initial fraction of extrapolators	0.5
$\gamma$	Switching parameter in MSFE	1
$\kappa^*$	SS LTV-ratio	0.93
$q^*$	SS land price	1
$\nu$	Weight of forecasted land price in the land price function	0.7
$\tilde{c}$	SS consumption parameter in CRRA geometric series	0.125
$\epsilon^z$	Standard deviation of the TFP shock	1
$\epsilon^{Eb}$	Standard deviation of the capital quality shock	1
$\psi$	Standard deviation of the income quality shock	1
$\epsilon^r$	Standard deviation of the monetary policy shock	1
$\rho_z$	AR parameter in the TFP shock process	0.9
$\rho_{Eb}$	AR parameter in the capital quality shock process	0
$\rho_\psi$	AR parameter in the income shock process	0
$\rho_r$	AR parameter in the monetary policy shock process	0.9

## 1 4 Quantitative results

2 Our analysis consists of four parts. The first part is a depiction and analysis of the  
3 nature of the model variables over the business cycle, with a particular focus on the  
4 systemic events. The second part is an examination of the (model generated) second-  
5 , and higher-order moments. In the third part, we will analyze (model consistent)  
6 impulse responses to the four shocks described above. To end, in the final part  
7 we will examine the effects of a macroprudential policy in terms of smoothening

the business cycles, reducing the asymmetries and fat tails of model variables, and improving the welfare.

## 4.1 Forcing variables

The four shocks we will examine are:

- (Positive) TFP (or technology) shock,  $\epsilon_{z_t}$

$$y_t = z_t \epsilon_{z_t} g(l_t) \quad (27)$$

,where the TFP shock has an AR component,  $\rho^z$  calibrated to 0.9:

$$\epsilon_{z_t} = \rho^z + \eta \quad (28)$$

- (Negative) capital quality shock,  $\epsilon_{Eb_t}$

$$E_{t+1}^b = (1 - \delta^b) E_t^b + \pi_t^b + \epsilon_{Eb_t} \quad (29)$$

,where  $\epsilon_{Eb_t}$  is a white noise shock to the evolution of bank equity stock.

- (Positive) income shock,  $\psi_t$ :

$$\kappa_t q_t l_{t+1} \psi_t \leq -\frac{b_{t+1}}{R_t^b} \quad (30)$$

,where  $\psi_t$  is a white noise shock to the collateral constraint of impatient households. And a

- Standard (negative) monetary policy shock ( $\epsilon^r$ ):

$$r_t = \gamma^r r_{t-1} + \gamma^\pi \pi_t + \gamma^y y_t + \epsilon^r \quad (31)$$

and  $\epsilon^r$  is a white noise shock to monetary policy. In our simulations, we calibrate the interest rate smoothing parameter  $\gamma^r$  to 0.9. The standard deviation of all shocks is normalized to 1.



## 1 4.2 The nature of cycles in the model

2 Figures II.1 to II.4 depict the business cycle evolution of key model variables, such  
3 as consumption, price of land, loans to households, interest rate on borrowings,  
4 bank equity, nominal interest rate, learning variables, and market sentiment. We  
5 are interested in examining two things in this section. On one end, we wish to  
6 understand the type of boom-bust cycles that are captured in this model, focusing  
7 special attention to whether the model is capable of generating systemic crises as  
8 rare events. These are depicted in Figures II.1 to II.3. On the other end, we wish to  
9 track the learning process and analyze its impact on the model dynamics. Those are  
10 depicted in Figure II.4. Remember that the model is simulated over 2000 quarters  
11 (or 500 years).

### 12 4.2.1 Macro-financial cycles

13 In all graphs, the zero-line represents the trend and the area above (below) it repre-  
14 sents the positive (negative) cyclical deviation from the trend. The series should be  
15 interpreted as the filtered cyclical component of a time-series with an independent  
16 time-varying (or time-invariant) trend.

17 Looking at consumption, the first thing to note is the asymmetric cycles that the  
18 model generates. While there are several episodes of heavy boom in consumption  
19 ( $t=100,550,950,1100$  or  $1600$ ), these are followed by even sharper falls in the level  
20 of consumption. So while the heaviest boom in consumption is around 20 % above  
21 the trend, the sharpest contractions lie around 30 % below the trend. Moreover,  
22 the persistence in booms is higher than the persistence in busts. Hence both the  
23 frequency and the amplitude of expansions and contractions are asymmetric.

24 Next, the model is capable of generating small as well as large boom-bust cycles.  
25 While the majority of the cycles are small, several of them can be considered ‘large’.  
26 Using Cogley and Sargant’s (2008) definition that a severe (or systemic) crisis is one  
27 where consumption contracts by at least 25 %, we find 10 such contractions during the  
28 500 year simulation exercise. They are marked by a grey arrow in Figure II.1. If we  
29 take an average over the entire sample, than two systemic crises occur every century.  
30 In most of these, the contraction in consumption is higher than the 25%, which makes  
31 them clear candidates for a truly systemic contraction. Note, moreoverover that the  
32 contractions are preceeded by a heavy surge in consumption. This is in particular  
33 true for the second, seventh and tenth contraction, where consumption increases by

30-40% before it drastically reverses. Also, the build-up phase is much longer than the subsequent bust. Hence, this allows enough time for risks and credit to build up before they materialize.

In addition, there are two key stylized facts that the model captures well. First, Cogley and Sargent (2008b) note that once an economy is in a low-growth state, there is a certain positive probability of running into a sequence of contractions. That is what we see after contraction 2. While the economy tries to recover from the first downfall, in ten years (on average) it runs into the next systemic crisis. As a matter of fact, five systemic contractions occur in less than 300 quarters (75 years). The second stylized fact is that a long build-up of stocks, risks and liquidity in the (financial) system makes the entire economy much more unstable and prone to heavy reversals than an economy where the long build-up phase is controlled and shortened. In our model, that is exactly what occurs. Prior to  $t=700$ , the economy only experiences one systemic crisis. However, after the exceptionally long build-up phase in  $t=[180, 700]$ , the economy suffers 9 crises in around 1000 quarters (or 6 crises in 700 quarters). That is a 6-fold increment during the same time interval. Hence this confirms the fact that a long period with high and sustained build-up of risks, credit, leverage (and speculation via asset prices) with only minor contractions changes the entire structure of the economy over the longer-run. This is because the heavy and sustained accumulation of stocks and optimism make the economy more susceptible to shocks and significantly increases the probability of future sharp downturns.

This observation is further confirmed by the business cycle evolution of loans in Figure II.1, where 6 out of the 10 systemic crises result in the historically heaviest contraction in lending. Total lending to households decreases by between 15-20% during those systemic crises, representing a highly significant downturn. Moreover, the preceding build-up of credit before systemic crisis 2, and the following cycle of contractions are clearly visible also on the same graph.

The same pattern is observed for land prices. During the same episode that consumption sharply contracts, the land price falls by between 30 and 45%. Via wealth and credit channels, it significantly contracts consumption, which is why we observe the systemic crises above. Moreover, the speculation hypothesis mentioned above is confirmed. Because agents have to forecast the land price, and they can only do it based on the historical evidence on realizations of the same (or on the steady state value normalized in this case to 1), a market sentiment cycle will inherently take

1 over in the valuation process. The more that the agents observe positive realizations  
 2 of the land price, the higher the probability of them forecasting a higher land price for  
 3 the next period. Such sentiments re-inforce themselves even more when a long and  
 4 sustained series of increasing values have realized and the sample of systemic crisis  
 5 is small. That is exactly what happens between  $t=[180, 700]$ . During this period,  
 6 the land price has an increasing trend and the economy had only experienced one  
 7 systemic contraction so far. Therefore, while the land price takes a while to start  
 8 this positive feed-back loop of increases, once in  $t=270$ , enough positive realizations  
 9 of land price have occurred and the systemic contraction is sufficiently far behind in  
 10 order for the agents to turn truly optimistic and put a heavy weight on an increasing  
 11 future land price at the same time as it puts a negligible probability on a future fall  
 12 in the price. But, after the second crisis, the weight they put on future contractions  
 13 is significantly higher, and since we include memory in their learning, the agents  
 14 also become more ‘insecure’ in their forecast and thus put a higher probability of  
 15 such events re-occurring in the near future.<sup>17</sup>Hence why we see a downward spiral in  
 16 prices after the second systemic crises has realized.

17 Turning to bank equity and interest rates on borrowing, we also see that the  
 18 systemic crises affect the profitability of the bank. Taking into account that bank  
 19 equity and the interest rate on borrowing have the opposite signs in equation I.13,  
 20 once the price of household collateral (land) starts to contract, the financial inter-  
 21 mediary is obliged to increase its borrowing rate, as the probability of default of  
 22 impatient households has increased and so it is more risky to lend to them. How-  
 23 ever, that will reduce the amount of total borrowing, and thus the profits of the  
 24 bank (since the fall in lending is higher than the rise in the interest rate margin).  
 25 This will subsequently lead to a fall in bank equity, as governed by equation 7. On  
 26 contrary, the higher the bank equity that a financial intermediary holds (a feature  
 27 in upturns), the more leeway the bank has to extend its lending, and so it reduces  
 28 its lending rate. This is why we see the opposite business cycle evolution for bank  
 29 equity and the lending rate. Note also how sensitive the interest rate setting is to  
 30 movements in bank equity. Roughly a 1% drop (rise) in bank equity from its long-  
 31 term trend generates a 10% rise (drop) in the lending rate from its trend. This is  
 32 due to the quadratic composite social cost imposed on excessive leverage in equa-  
 33 tion I.13, which pushes the interest rate up by more than proportionally. Due to

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<sup>17</sup>Remember that the parameters  $\rho$  and  $\gamma$  in the learning set-up in equations 25 and 23 guide the memory of the agents in our learning framework.

this heavy (de)-leveraging (or rebalancing) over the cycle that the financial sector becomes a powerful propagator of shocks, originated within the financial sector as well as outside. Thus a sufficiently high de-leveraging can, via the lending channel, cause a severe downturn in the real economy. We will analyze this mechanism in more depth in the impulse response section.

#### 4.2.2 Learning cycles and model dynamics

The model allows us to track the learning dynamics explicitly and examine how imperfect information generates market sentiment in the market for land.

The first graph in Figure II.4 depicts the proportion of the households that use the extrapolist rule to forecast the leverage ratio. A value of 1 means that the entire population uses that rule, whereas 0 means that the entire population uses the fundamentalist rule. The same applies for the forecast of land price in the second graph. Finally, the graph on market sentiment depicts the (subjective) valuation of the land price that all agents expect will be in the next period. A value of 1 means that all households believe that the price will rise, i.e. *optimism* while a value of 0 means that all households believe the price will fall, i.e. *pessimism*.

The first thing to note is that there are cycles, or regime-switches, in the usage of forecast rules. There are, however, more regime-switches in the forecast of land price compared to leverage. Taking into account that the switching parameter  $\gamma$  is calibrated to 1 in both cases, the difference between the two is endogenously generated by the model. This means that the goodness of fit of the fundamentalist rule in the case of leverage is, on average, higher (in relative terms) compared to the same rule in the forecast of land price. We believe this is due to the (more urgent) necessity of households to use a fixed point reference when conditionally optimizing on the leverage ratio. Since leverage is an exogenous process at time  $t=0$  and it is dependent on the price of risks (which in this set-up agents do not fully know in real time), a ‘benchmarking’ approach in forecasting becomes much more attractive (and efficient) as agents can use a fixed reference to rationally optimize conditional on this simple heuristics.

Nevertheless, note that in both forecast games, there is significantly more inertia in the extrapolative rule. This means that once households use the extrapolative rule, they are very likely to remain under that regime. As a result, most of the time the agents prefer to use history to forecast the future. We should therefore expect to observe strong market sentiment building up over time.

1 That is confirmed by the third graph in Figure II.4. Most of the time, agents are  
 2 optimists. However, at some point the sentiment reverts and agents turn pessimists.  
 3 Just as in Boz and Mendoza (2014), this occurs once they have observed one nega-  
 4 tive realization of the variable they forecast. Since in period  $t=0$  households start  
 5 in a *tabula rasa* environment, the first observation of the realized land price will  
 6 determine their forecast of the future price. The more of price rises they observe,  
 7 the lower the probability they will put on a future price decrease, and therefore they  
 8 will turn more and more *optimistic* over time. However, as soon as a reverse in  
 9 price is observed, households will put a very low (if not zero) probability on future  
 10 price increase, and will therefore forecast a price decrease. Since booms are longer  
 11 and more sustained than busts, agents will remain in the *optimistic* interval for  
 12 longer periods of time compared to the *pessimistic* interval. But as agents observe  
 13 more of the price decreases, they learn about them and put a higher probability of  
 14 them occurring in the future. Thus that is why we see an increase in the number  
 15 of sentiment-switches from *optimism* to *pessimism* as time goes by. The speed of  
 16 this is governed by how much weight they put on the historical realizations ( $\theta$  pa-  
 17 rameter) and how far back they remember, ( $\rho$  and  $\gamma$  parameters, i.e. how adaptive  
 18 their learning is). Because we have calibrated the parameters in such a way that  
 19 households are slow learners, the time elapsed before more frequent switches occur  
 20 is considerable. Note also that exactly for this reason, persistent *optimism* build-up  
 21 can also occur after 1000 quarters of simulation since positive observations from  
 22 recent history outweigh the learning from more distant past if a sufficient sequence  
 23 of price increases has realized.

24 Our sequence of beliefs is in many ways similar to the ones obtained in Boz and  
 25 Mendoza (2011, 2014), but with some important deviations. Comparing the third  
 26 graph in their Figure 6 to our third graph in Figure II.4, in both frameworks the *op-*  
 27 *timistic* interval is more persistent. Moreover, as the number of low leverage regimes  
 28 is observed, the number of switches to *pessimism* increases. However, because the  
 29 state-space is dichotomous in their world, the reversals are also more abrupt. This  
 30 should result in sharper turning-points over the business cycle, not because of the  
 31 model dynamics but because of the constrained model learning construction. In our  
 32 case, on the other hand, households ‘guess’ a full continuous state-space of values,  
 33 and so the reversals are more gradual. Hence, if sharp declines are observed in the  
 34 business cycles, they are entirely generated by the endogenous model dynamics (via  
 35 the interaction between learning and financial frictions), and not by a demarca-

tion of the state-space. Further to that, learning in their framework is significantly faster than in ours, which means that convergence to a RE model is achieved after a relatively short period of time. Looking at the first two graphs of Figure 6, the subjective transition probability is very close to the actual probability already before 300 quarters (or 75 years). That is possible because the state-space is reduced and because agents engage in restricted Bayesian learning (which has been shown to converge faster). In our model, on the other hand, the environment is more uncertain and learning is slower. Because systemic crises are rare, learning about them is also slow, and that is why uncertainty regarding leverage and land prices remains in the model dynamics for a much longer period of time. Lastly, while in Boz and Mendoza (2014) agents know the land price and forecast only the transition probability of leverage, we extend it to include land prices, since it directly depends on the leverage (via the shadow value of collateral). We think that our approach is more realistic under the asset pricing of Mendoza (2010) since a complete knowledge of the land price would allow households to learn the ‘true’ value of leverage by solving the rest of the model and recursively extract the value of leverage.

### 4.3 Distributions and statistical moments over the business cycle

The second part of the model evaluation consists of analysing and validating the model-generated distribution and statistical moments over the business cycle. These are generated using the entire sample of 2000 quarters. For our purposes, we will use the data on (auto)-correlations in Table II.1, the data on second and higher moments in Table II.2, as well as histograms of a selection of these variables in Figures II.5 to II.7.

#### 4.3.1 Correlations

The model is capable of generating high persistence in the evolution of key variables. Except for the nominal interest rate, the rest has an autocorrelation coefficient above 0.95. That is much higher than in most DSGE models. Following the discussion in De Grauwe and Macchiarrelli (2015), we believe that the learning framework is the underlying cause of this high persistence in the model since beliefs are self-enforcing and therefore persistent over time. This pattern has also been documented in other medium-size learning DSGE models, such as Slobodyan and Wouters (2012), where

1 they show that learning improves the fit of the model to the data compared to the  
2 benchmark Smets and Wouters (2007) set-up. Moreover, we note that the persis-  
3 tence of the financial friction variables is slightly higher than that of the macroe-  
4 conomic ones (consumption, land price, policy rate). Hence, there is an additional  
5 layer of persistence coming from the credit friction, albeit the difference is small.

6 Cross-correlations between the model variables is also high. Most variables are  
7 positively correlated, except the ones including leverage and the interest rate on  
8 borrowing. A higher leverage reduces (future) consumption ( $\rho = -0.975$ ) since it  
9 reduces the value of land ( $\rho = 0.985$ ) and it restricts the amount the impatient  
10 households can borrow. Similarly, a higher interest rate means that households can  
11 borrow less, and therefore consume less ( $\rho = -0.99$ ). Equally, the interest rate  
12 on borrowings and the land price are negatively correlated since a lower value of  
13 collateral increases the cost of borrowing (via the collateral channel), or a higher  
14 borrowing rate increases the discounting of today's value of land, by generating lower  
15 consumption ( $\rho = -0.99$ ).

### 16 **4.3.2 Distributions and statistical moments**

17 The key thing to note in this section is that all model variables captured in Figures  
18 II.5 to II.7 and Table II.2 are non-Gaussian, skewed and with fatter tails compared  
19 to variables in standard DSGE models. Normality tests have lead us to reject the  
20 assumption of normal distributions.

21 The variance of almost all variables (except for bank equity and market senti-  
22 ment) is very high, which means that there are high fluctuations in these variables  
23 over the business cycles (i.e. the amplitude of the cycles is high). Moreover, we see  
24 two patterns in the skewness and tails. While consumption, credit, land price, and  
25 bank equity have a negative skew (to the left) and with a fatter tails on the left  
26 side, leverage, the interest rates and market sentiment have a positive skewness (to  
27 the right) and a fatter tail on the right. This means that while there are signifi-  
28 cant booms (i.e. a high probability mass on the right of the median), with a rise  
29 in credit, leverage, consumption and drop in borrowing rate, the reversals are so  
30 sharp that the drop in these variables (or rise in borrowing rate) outperforms the  
31 booms, and that is why we see those shapes in the distributions. Hence systemic  
32 crises drive the probability mass much further from the mean, and the 'true' prob-  
33 ability of such events becomes significantly above zero at any point in time. In RE  
34 frameworks, households would know all this information which would lead them to

price in the probability of systemic shutdowns in their optimizations, which most probably would mean that credit, leverage and consumption would not build-up as much. But, because of uncertainty in our framework, agents have a harder task to define (and understand) these distributions (in particular for leverage and land price and so missprice the ‘true’ tail-risks including an *ex ante* underpricing of systemic crises events.<sup>18</sup>

## 4.4 Impulse response analysis

Figures II.8 to II.9 depict the impulse responses to a positive TFP shock and Figures II.10 to II.11 to an expansionary monetary policy shock.

The numbers on the x-axis indicate number of quarters. All the shocks are introduced in  $t=100$  and we observe the responses over a period of 50 quarters (or 12.5 years). Note that in these figures we depict the *median* impulse response in black amongst a distribution of impulse responses generated with different initializations of the learning parameters. The red lines in the Figures represent the 95% confidence intervals, or a full distribution of impulse responses. For the sake of clarity and focus in the discussions, we will only concentrate on the median impulse response, which is a good representation of the overall distribution. Moreover, we will only concentrate on the standard TFP and monetary policy shocks in the text since these are standard in the literature and are modelled with persistence. However, should you want to see the discussion of the other two shocks (a negative bank capital quality and a positive (financial) wealth shock) do not hesitate to contact the authors.

### 4.4.1 TFP shock

A 1% TFP shock improves the production of land, and therefore increases the land price by 1.5%. Because quantity of land is fixed, all of the efficiency improvement will go to land price, by improving the intertemporal consumption smoothing of households. Since value of household equity goes up, leverage of impatient households decreases by 1.6%, and their external financing possibilities improves. Via the collateral constraint, impatient households are able to borrow more for the same collateral, which initially pushes up the loans they obtain by 0.4%. For financial intermediaries, this leads to a higher bank equity value (0.04%), which gives them

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<sup>18</sup>Only after a sufficient number of crises events have occurred, and assuming that the distributions of these variables do not change over time, they will start to approach their pricing of these systemic events closer to the ‘true’ probability.



1 space to extend their credit line even further since their capital-to-asset ratio has  
2 increased. Via the interest rate margin equation, they reduce the interest rate on  
3 borrowing to households by 0.65%. This lower cost on loan repayment in turn allows  
4 impatient households to extend their borrowing even further in  $t=102$ , resulting in a  
5 peak increase in external financing at 0.65% above the pre-shock level. For financial  
6 intermediaries, this is an additional increase of bank equity by 0.02%, implying a  
7 total of 0.06% expansion in bank equity as a result of the TFP shock. Hence bank  
8 can extend its activity and size as a result of an improvement in the real (production)  
9 sector.

10 However, this extension in credit makes the households gradually more leveraged,  
11 and the opposite mechanism is then set in motion. The higher leverage raises the  
12 value of the left-hand side of equation 5, which reduces the amount of next-period  
13 borrowing (because of their negative relation), which in turn reduces their (future)  
14 consumption possibility, and therefore the price of land. This opposite mechanism  
15 continues until the economy returns to its pre-shock level.

#### 16 4.4.2 Monetary policy shock

17 A reduction of 1% in the (risk-free) interest rate reduces the deposit rate by the  
18 same amount (since  $R_t^d = r_t$ ). Since this reduces the financing cost for banks, they  
19 can therefore reduce their cost of lending in order to extend their asset side and  
20 increase their profitability. The resulting rise in bank lending increases the amount  
21 of credit that households get, and therefore their (expected) consumption possibility.  
22 Via the pricing function of land in equation 12, the land price also increases. This  
23 reduces the leverage of (impatient) households, and via the collateral constraint  
24 allows them to borrow more. The cost of borrowing therefore reduces even further,  
25 and the bank extends its credit even further. As a result, bank equity rises. The  
26 total effect of the expansionary monetary policy shock is that the interest rate on  
27 borrowings falls by 0.45%, the expansion in credit is 0.45%, the rise in land price  
28 is 0.9%, and the fall in leverage 1.55%. The resulting boom in consumption is  
29 first 0.8% followed by 1.1%, and the rise in profitability of intermediaries raises its  
30 total bank equity by 0.04%. Note that while the economy (including the financial  
31 sector) expands following both shocks, the expansion is quantitatively larger for  
32 the supply side (or TFP) shock.<sup>19</sup>That is not surprising since our framework lacks

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<sup>19</sup>Remember that both shocks are calibrated in the same way. The standard deviation of the white noise component is standardized to 1% while AR component is calibrated to 0.9. That is

sticky prices or wages which would make the monetary policy transmission more persistent, as in standard NK-models. Therefore in our framework, policy makers should concentrate on supply-side policies to generate sustained booms rather than using (discretionary) monetary policy. Therefore, we also expect (relatively) a high efficiency of macroprudential policy in smoothening the cycles, since the policy can be viewed as a type of supply-side constraint on the “production” in the financial sector.

## 5 Macroprudential policy

To quantify the (stabilizing) effects that a macroprudential policy can have, in particular on reducing the number (and impact) of systemic crises, we will evaluate one particular type of macroprudential policy. We will concentrate on a cap on (household) LTV, where the central bank will allow households to leverage up to a certain level (but not beyond), and therefore restrict intermediaries to extend their credit supply only up to a certain quantity.

Tables II.3 to II.4 report the statistical moments of the model variables with and without macroprudential policy. Figures II.12 to II.14 depict the same moments in terms of distributions for a selection of key model variables. Lastly, Figure II.15 compares the number of systemic crises compared to the benchmark model, while Figure II.16 shows the number of switches between the extrapolative and fundamentalist rule for land price with and without a macroprudential rule.

For a list of figures for the remaining model variables, please do not hesitate to contact the authors.

In our simulations, we set the LTV cap at 2. The effects are significant.

Starting with learning dynamics, because of the benchmark provided by the cap, the learning with respect to the leverage is rapid. It only takes a few periods for households to understand what their maximum limit is, and therefore their subjective expectations converge to rational expectations. Under this setting, the extrapolative rule becomes the best predictor of the future as no variations in that variable will occur.

For land price in Figure II.16, the learning dynamics also changes. The fundamentalist rule becomes more frequently visited compared to the benchmark model setting. We believe the reason is because agents now only need to learn about one

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why we can directly compare the two effects.

1 variable meanwhile the other has a fixed point. Hence, benchmarking the land price  
2  $q$  becomes more useful in forecasting since fluctuations in the model are reduced and  
3 therefore using a fixed reference point for forecasting becomes more effective.

4 In terms of the macroeconomic and financial model variables, we also see a  
5 significant change. Looking at the statistical moments and distributions, we see that  
6 most variables become more Gaussian. The distributions become more symmetric  
7 and the fat tails are reduced. In practical terms, it means that sharp rises or drops  
8 in these variables are reduced, as well as probabilities of systemic crises. Many  
9 (auto)-correlations are reduced, which implies less of the (market) sentiment driven  
10 cycles that we observed before. In addition, the volatilities and skewness of the  
11 variables are reduced by a factor of 2-4. Meanwhile, the kurtosis increases slightly,  
12 which means that the distribution becomes more centred around its mean/median.  
13 That is clearly visible in the figures for consumption, land price and credit.

14 This is also visible in Figure II.15. Comparing the number of systemic crises in  
15 the benchmark model with the augmented version, we observe a reduction of 50% in  
16 the latter. Instead of the original 10 crises in 500 years, we now get 5 crises over the  
17 same time period. In particular, the sequence of systemic crises that occurs after  
18 the second are (almost) reduced. Moreover, the losses in each of the crises is also  
19 reduced. Noting that the scaling in the second graph is 3 to 4 times smaller, the  
20 losses in each of the crises is reduced by, on average, a factor of 3. Thus, an LTV-cap  
21 does not only reduce the probability of a systemic crises by 50%, but it also reduces  
22 the losses incurred by each. As a result, the business cycles become shorter and the  
23 amplitude of each smaller.

24 While it is clear that the policy smoothens the cycles and reduces the systemic  
25 events, we would like to quantify these effects in terms of households' welfare. In  
26 standard RE DSGE models, one would value the welfare effects by calculating the  
27 (welfare) gains using a second order approximation of households' welfare. However,  
28 in our model, RE is substituted with subjective beliefs, which means that the policy  
29 maker does not know how to weight these beliefs into a general welfare function.  
30 Hence, imperfect information also concerns households' welfare.<sup>20</sup>

31 To overcome this problem, we instead value the welfare using utility (or consump-  
32 tion equivalence) measure of an economy with and without the policy. Knowing the  
33 parameters in the utility function, and the median consumption of households in

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<sup>20</sup>Recently, Brunnermeier et al (2014) are trying to define 'belief-neutral' welfare functions in models with distorted (or imperfect) beliefs. However, more work is necessary before a robust method can be obtained for loss function derivations.

an economy with and without the LTV-cap, we can calculate the utility gains that households will get from imposing the rule.<sup>21</sup>Since household utility only depends on consumption, the gains will be expressed in consumption equivalence terms.

We find that the utility gains from using a cap on household LTV is 6.5%. It means that, on average, a household will consume 6.5% more when a central bank imposes a cap compared to an economy without it. Decomposing this gain, we find that 6% out of the 6.5% derives from an increase in the level of consumption, while 0.5% comes from a reduction in variability (or volatility) of consumption over the cycle. The reason for this heavy gain in level, we believe, comes from the reduction in the systemic crises. Systemic crises are events when most of the consumption level is reduced. In relative terms, this reduction in level is even higher than the reduction in volatility of consumption, since the *ex ante* probability mass of such event is not big. However, once that state becomes absorbing, the reduction in level is very high.

To conclude, we compare our results with a more elaborate version of the LTV-rule. Following the recent literature on macroprudential policy (see, for instance Lambertini et al (2013) or Angelini et al (2014)), we also try a Taylor-type rule specified in equation 19. We find that a more complex LTV-rule generates very similar economic outcomes to the simpler rule we have used before. The only difference is that the Taylor-type rule smoothens the fluctuations in the interest rates by more. We believe that the explanation for this similarity lies in the learning. While a Taylor-type rule increases the information content in the reaction function and allows the central bank to react to more financial variables, it also delays learning since some uncertainty regarding the leverage level remains. In the simpler rule, on the other hand, there are gains from the heavy ‘benchmarking’ that a fixed rule gives to households in their learning process. Having a fixed reference point regarding leverage reduces uncertainty regarding its (future) value compared to a more complex rule. We see that the fluctuations in the usage of extrapolative versus fundamentalist rule for leverage is higher under the Taylor-type rule, which generates additional dynamics. As a result, the simple LTV-rule is, in relative terms, more effective in generating stability at a lower monitoring (or information) cost.

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<sup>21</sup>Note that since model variables have asymmetric distributions in the benchmark model, the median is more representative of the centre of the distribution, rather than the mean. That is why we use the median consumption in our calculations. We could, however, trivially re-run the same experiments using the mean consumption values.

## 6 Discussion and concluding remarks

Deregulation in the financial services industry since 1980's, the increased competition amongst financial intermediaries and the unprecedented expansion in financial engineering since mid-1990's has, in an exceptional manner, increased the size of the financial sector. Their credit lines to the real economy, and the consumption possibility of households has been historically the highest in the period prior to the Great Recession. The US (and to certain extent the EU) economy experienced one of its sharpest booms in early 2000's. On the other end, however, the pricing of risks and leverage became an increasingly difficult task as uncertainty regarding the true accumulation of risks on balance sheets and the true exposure of households increased. The misspricing of risks gave leeway to market speculation and market sentiment-driven cycles. We include these observations in a recent model by Boz and Mendoza (2014), and analyze the effects of dynamic optimization under uncertainty on the macro-financial cycles, and the probability of systemic crises. In particular, we are interested in understanding the role that macroprudential policy plays in reducing the probability of systemic events and the losses generated by these.

Including these facts into a general equilibrium model with adaptive learning result in an increase in the amplitude and frequency of the cycles. The build-up phase of risks, credit, leverage and consumption is much longer and higher than in standard DSGE models. In the same way, once a reversal in lending occurs, the decline in all variables is also much sharper and lasts shorter. The probability of systemic crises is significant, and we find that, on average, 2 such crises occur every century. Moreover, we find that, different from standard boom-bust cycles, a systemic crisis can be followed by a sequence of subsequent contractions, as it makes the economy more unstable. The result is asymmetric distributions of key macroeconomic and financial variables, with high skewness and fat tails.

A simple cap on the LTV-ratio is effective in smoothening the cycles and reducing the effects of a deep contraction on the real-financial variables. The model distributions become much more symmetric and Gaussian. It also reduces the amount of borrowing and leverage in upturns. The number of systemic crises is halved, and the losses at each is reduced by, on average, a factor of 3. The consumption (utility) gains from such a policy are, on average 6.5% compared to an economy without a macroprudential rule. Also the stabilizing role of monetary policy is increased once a macroprudential rule is used. To conclude, a simple LTV-rule is preferred to a

more elaborate Taylor-type version because it provides a strong ‘benchmarking’ to agents in their learning process, while generating same welfare (improving) effects at a lower information cost.

These are promising results in our understanding of the probability of systemic events, and their destabilizing macroeconomic impacts. While the road in reaching a full understanding of such events is long, these should hopefully be seen as a contribution in the right direction. Future research should therefore try to stretch the framework of this paper in multiple directions.

First, a robust comparison is necessary between the learning framework in this model and the Bayesian set-up in Boz and Mendoza (2014). Both are actively used in the literature, and a serious comparison in terms of long-term learning, memory and model dynamics should be welcomed.

Equally, the regime-switching in rules in this framework should be compared to homogenous learning set-ups. A lot of the dynamics in this model comes from the regime switching in switching. It would therefore be interesting to see the type of macroeconomic dynamics we would get if agents use only one rule, possibly a more elaborate adaptive rule such as least-square learning.

It would also be interesting to conduct a robustness exercise to test the model performance for a larger parameter space of the learning variables. On the same lines, it would be highly relevant for policy purposes to find the optimal LTV-cap whereby gains from such a rule are maximized.

Lastly, systemic crises are rare and non-linear events. Therefore, it would be of high interest to zoom-in such periods and only study the dynamics once such event becomes absorbing. In particular, it would be interesting to examine the statistical moments, the distributions and the transmission channels under only such states. That would bring the model closer to the recent but blooming empirical literature on tail-events and hyper correlations.

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# Appendices

## I The FOCs and the full model system

### I.1 Households

Setting up the Lagrangians for the patient households, we get:

$$L_P = E_0 \beta_P^t \frac{(c_t^P)^{1-\sigma}}{1-\sigma} + \lambda_t^P [z_t g(l_t) + q_t l_t - q_t l_{t+1} + (1 + R^d) d_t - d_{t+1} - c_t^P - e_t] \quad (\text{I.1})$$

Taking the first order conditions with respect to  $c_t^P, l_t$  and  $d_t$ , we get:

$$\frac{\partial L_P}{\partial c_t^P} = (1 - \sigma) \frac{(c_t^P)^{-\sigma}}{1 - \sigma} - \lambda_t^P = 0 \Leftrightarrow \lambda_t^P = (c_t^P)^{-\sigma} \quad (\text{I.2})$$

where we set  $\sigma < 5$  in order to find a reasonable solution.

$$\begin{aligned} \frac{\partial L_P}{\partial l_t} &= \lambda_t^P z_t g'(l_t) + q_t - \beta_P E_t^s [\lambda_{t+1}^P q_{t+1}] = 0 \Leftrightarrow \\ q_t &= \beta_P E_t^s [\lambda_{t+1}^P q_{t+1}] - \lambda_t^P z_t g'(l_t) \Leftrightarrow \\ q_t &= \beta_P E_t^s [(c_{t+1}^P)^{-\sigma} q_{t+1}] - (c_t^P)^{-\sigma} z_t g'(l_t) \end{aligned} \quad (\text{I.3})$$

$$\begin{aligned} \frac{\partial L_P}{\partial d_t} &= \lambda_t^P (1 + R_t^d) - \beta^P E_t^s [\lambda_{t+1}^P] = 0 \Leftrightarrow \\ (1 + R_t^d) &= \frac{\beta^P E_t^s [\lambda_{t+1}^P]}{\lambda_t^P} \Leftrightarrow \\ (1 + R_t^d) &= \frac{\beta^P E_t^s [(c_{t+1}^P)^{-\sigma}]}{(c_t^P)^{-\sigma}} \end{aligned} \quad (\text{I.4})$$

Similarly, for impatient households we get the following Lagrangian:

$$L_I = E_0 \beta_I^t \frac{(c_t^I)^{1-\sigma}}{1-\sigma} + \lambda_t^I [q_t l_t - q_t l_{t+1} - \frac{b_{t+1}}{R_t^B} + b_t - c_t^I] + \gamma_t^I [-\kappa_t q_t l_{t+1} - \frac{b_{t+1}}{R_t^B}] \quad (I.5)$$

1 Taking the first order conditions with respect to  $c_t^I, l_t$  and  $b_t$ , we get:

$$\frac{\partial L_I}{\partial c_t^I} = (1-\sigma) \frac{(c_t^I)^{-\sigma}}{1-\sigma} - \lambda_t^I = 0 \Leftrightarrow \lambda_t^I = (c_t^I)^{-\sigma} \quad (I.6)$$

2 where we again set  $\sigma < 5$  in order to find a reasonable solution.

$$\begin{aligned} \frac{\partial L_I}{\partial l_t} &= \lambda_t^I q_t - \beta_I E_t^s [\lambda_{t+1}^I q_{t+1} + \gamma^I \kappa_t q_{t+1}] = 0 \Leftrightarrow \\ &\lambda_t^I q_t = \beta_I E_t^s [\lambda_{t+1}^I q_{t+1} + \gamma^I \kappa_t q_{t+1}] \Leftrightarrow \\ &q_t = \frac{\beta_I E_t^s [\lambda_{t+1}^I q_{t+1} + \gamma^I \kappa_t q_{t+1}]}{\lambda_t^I} \Leftrightarrow \\ &q_t = \frac{\beta_I E_t^s [(c_{t+1}^I)^{-\sigma} q_{t+1} + \gamma^I \kappa_t q_{t+1}]}{(c_t^I)^{-\sigma}} \Leftrightarrow \\ &q_t = \frac{\beta_I E_t^s [((c_{t+1}^I)^{-\sigma} + \gamma^I \kappa_t) q_{t+1}]}{(c_t^I)^{-\sigma}} \Leftrightarrow \\ &q_t = \frac{\beta_I ((c_{t+1}^I)^{-\sigma} + \gamma^I \kappa_t - \gamma^I \kappa_t)}{(c_t^I)^{-\sigma} - \gamma^I \kappa_t} \Leftrightarrow \\ &q_t = \frac{\beta_I (c_{t+1}^I)^{-\sigma}}{(c_t^I)^{-\sigma} - \gamma^I \kappa_t} \end{aligned} \quad (I.7)$$

3 By combining equations I.3 and I.7, we get the expression for land price as  
4 in Boz and Mendoza (2014). However, because we differentiate our households,  
5 the price of land is determined by the Euler equations of both the patient and  
6 impatient households. As a result, the (final) price of land will be a balance of  
7 the two, taking into account the differentiated discount factors ( $\beta^P < \beta^I$ ), i.e.  
8  $\frac{\beta_I c_{t+1}}{c_t - \gamma^I \kappa_t} = \beta_P E_t^s [c_{t+1} q_{t+1}] - c_t z_t g'(l_t) \equiv q_t$ . Because the share of the two household  
9 types in the economy is normalized to unti, we can write:  $c_t^P = c_t^I = c_t$ . That is the  
10 same expression as in I.22.

$$\begin{aligned}
\frac{\partial L_I}{\partial b_t} &= \lambda_t^I - \beta_I E_t^s \left[ \frac{\lambda_{t+1}^I}{R_{t+1}^B} + \frac{\gamma_{t+1}^I}{R_{t+1}^B} \right] = 0 \Leftrightarrow \\
\lambda_t^I &= \beta_I E_t^s \left[ \frac{\lambda_{t+1}^I}{R_{t+1}^B} + \frac{\gamma_{t+1}^I}{R_{t+1}^B} \right] \Leftrightarrow \\
(c_t^I)^{-\sigma} &= \beta_I \frac{1}{R_{t+1}^B} E_t^s [(c_{t+1}^I)^{-\sigma} + \gamma_{t+1}^I]
\end{aligned} \tag{I.8}$$

In addition, we include the three constraints (one for impatient and two for patient households) as first order conditions with respect to  $\lambda_t^P$ ,  $\lambda_t^I$  and  $\gamma_t^I$ .

## I.2 Financial Intermediary

For financial intermediaries, once we recognize that patient households own the intermediaries, we get the following Lagrangian:

$$\begin{aligned}
L_F &= E_0 \sum_{t=0}^{\infty} \beta_P^t \lambda_t^P [(1 + R^b)B_t - B_{t+1} + D_{t+1} - (1 + R_t^d)D_t + (E_{t+1}^b - E_t^b) \\
&\quad - \frac{\kappa_{EB}}{2} \left[ \frac{E_t^b}{B_t} - \nu^b \right]^2 E_t^b + \lambda_t^F (D_t + E_t^B - B_t)] \tag{I.9}
\end{aligned}$$

Taking the partial derivatives with respect to  $B_t$  and  $D_t$  gives us:

$$\frac{\partial L_F}{\partial B_t} = \beta_P^t \lambda_t^P [R_t^B - \kappa_{EB} \left[ \frac{E_t^B}{B_t} - \nu^B \right] \left[ -\frac{E_t^B}{B_t^2} \right] E_t^B - \lambda_t^F] = 0 \Leftrightarrow \lambda_t^F = [R_t^B - \kappa_{EB} \left[ \frac{E_t^B}{B_t} - \nu^B \right] \left[ \frac{E_t^B}{B_t} \right]^2] / \beta^P \lambda_t^P \tag{I.10}$$

$$\frac{\partial L_F}{\partial D_t} = \beta_P^t \lambda_t^P [-R_t^D + \lambda_t^F] = 0 \Leftrightarrow \lambda_t^F = R_t^D / \beta^P \lambda_t^P \tag{I.11}$$

Substituting  $\frac{\partial L_F}{\partial B_t}$  into  $\frac{\partial L_F}{\partial D_t}$ , via the Lagrangian multiplier  $\lambda_t^F$ , and multiplying both sides with  $\beta^P \lambda_t^P$  to get:

$$R_t^D = R_t^B - \kappa_{EB} \left[ \frac{E_t^B}{B_t} - \nu^B \right] \left[ \frac{E_t^B}{B_t} \right]^2 \tag{I.12}$$

Using the no arbitrage condition in bank financing, it implies that  $R_t^D \equiv r_t$ , and we can rewrite the last expression as:

$$R_t^B = r_t + \kappa_{EB} \left[ \frac{E_t^B}{B_t} - \nu^B \right] \left[ \frac{E_t^B}{B_t} \right]^2 \quad (\text{I.13})$$

1 This is the same as in expression 11. Moreover, we include the balance sheet  
 2 constraint  $B_t = D_t + E_t^B$  into the list of FOC. We also include the evolution of bank  
 3 capital in the conditions for financial intermediary, which is:

$$E_t^B = (1 - \delta^k) E_{t-1}^B + \omega^B [R_t^B B_t - R_t^D D_t - \frac{\kappa_{EB}}{2} \left[ \frac{E_t^B}{B_t} - \nu^B \right]^2 E_t^B - Adj_t^B] \quad (\text{I.14})$$

### 4 I.3 Government

5 We complement the private sector optimizations with the government policies. In  
 6 particular, the government sets  $r_t$  according to:

$$\frac{r_t}{r} = \frac{r_{t-1}}{r}^{\gamma_R} \left[ \frac{\pi_t}{\pi} \frac{y_t}{y} \right]^{1-\gamma_R} e^{\epsilon_{R,t}} \quad (\text{I.15})$$

7 As a compliment, the government conducts a set of macroprudential policies  
 8 (caps on LTV-ratios and capital requirements) according to:

$$\kappa_t = \bar{\kappa} \quad (\text{I.16})$$

9 with the *true* drift equal to:

$$\kappa_t = \gamma_\kappa \kappa_{t-1} + (1 - \gamma_\kappa) \bar{\kappa} + \epsilon_{\kappa,t} \quad (\text{I.17})$$

10 and a (bank) capital ratio to:

$$\frac{e_t}{b_t} \leq \bar{e} \text{ or } \frac{d_t}{b_t} \geq \underline{d} \quad (\text{I.18})$$

### 11 I.4 Aggregation

12 The clearing condition for the market of goods is:

$$c_t + \frac{b_{t+1}}{R_t^b} + Adj_t^B = b_t + z_t g(l_t) + \delta^k E_t^B \quad (\text{I.19})$$

13 The aggregate resource constraint in this economy is therefore:

$$rc_t = c_t + q_t [l_t - l_{t-1}] + \delta^k E_t^B + Adj_t^B \quad (\text{I.20})$$

where  $c_t = c_t^P + c_t^I$  and  $Adj_t^B$  is a calibrated cost parameter. In the asset (land) market, the clearing condition is given by:

$$1 = l_t \quad (I.21)$$

## I.5 Equilibrium system

Next, we need to define the equilibrium conditions and the state variables which will be the key for solving the learning model. We define the pricing function for land:

$$q_t = E_t^s \left[ \sum_{j=0}^{\infty} \left[ \prod_{i=0}^{\infty} \frac{1}{E_t^s [R_{t+1+i}^q]} \right] z_{t+1+j} g'(l_{t+1+j}) \right] \quad (I.22)$$

where the spread between the return on land and the policy rate is:

$$E_t^s [R_{t+1}^q - R_t] = \frac{(1 - \kappa_t) \gamma_t^I - Cov_t^s(\lambda_{t+1}^I, R_{t+1}^q)}{E_t^s [\lambda_{t+1}^I]}$$

and the return on land conditional on date-t belief is defined as:

$$E_t^s [R_{t+1}^q] = \frac{z_{t+1} g'(l_{t+1}) + q_{t+1}}{q_t}$$

and the (land) production function is  $g(l_t) = l_t^\alpha$ .

The pricing function for the interest rate on loans (bond) is:

$$R_t^B = r_t + \kappa^{EB} \left[ \frac{E_t^B}{B_t} - \nu^B \right] \left[ \frac{E_t^B}{B_t} \right]^2 \quad (I.23)$$

and for the interest rate on deposits it is:

$$(1 + R_t^D) = \frac{\beta^P E_t^s [c_{t+1}^P]}{c_t^P} \quad (I.24)$$

We also have the following policy functions:

$$(c_t^I)^{-\sigma} = \beta^I \frac{1}{R_{t+1}^B} E_t^s [(c_{t+1}^I)^{-\sigma} + \gamma_{t+1}^I] \quad (I.25)$$

$$d_{t+1} + c_t^P \leq z_t g(l_t) + q_t l_t - q_t l_{t+1} + (1 + R_t^d) d_t + e_t \quad (I.26)$$

$$c_t^I \leq q_t l_t - q_t l_{t+1} - \frac{b_{t+1}}{R_t^b} + b_t \quad (I.27)$$

$$\kappa_t q_t l_{t+1} \leq -\frac{b_{t+1}}{R_t^b} \quad (\text{I.28})$$

1 and the state equation:

$$E_t^B = (1 - \delta^k) E_{t-1}^B + \omega^B [R_t^B B_t - R_t^D D_t - \frac{\kappa_{EB}}{2} [\frac{E_t^B}{B_t} - \nu^B]^2 E_t^B - Adj_t^B] \quad (\text{I.29})$$

2 To complete the system we define the shocks that we use. In particular, we  
3 define the (known) TFP-shock process  $z_t$  as:

$$\ln(z_{t+1}) = \rho \ln(z_t) + \epsilon_z \quad (\text{I.30})$$

4 the capital quality shock  $\epsilon_{\kappa^B}$  as:

$$\epsilon_{\kappa^B} \sim N(0, \sigma_{\epsilon_{\kappa^B}}^2) \quad (\text{I.31})$$

5 where the same shock could be interpreted as an equity injection by the macro-  
6 prudential authority. Lastly, we define an income shock  $e^\psi$  according to:

$$\psi \sim N(0, \sigma_\psi) \quad (\text{I.32})$$

7 The income shock can be interpreted as exogenous movements in land prices, or  
8 in return of land.

9 We solve this system according to the method and algorithm described in section  
10 3.

## II Figures and Tables

1

Table II.1: Model (auto)-correlations

Variables	(Auto)-correlations
$\rho(b_t, b_{t-1})$	0.988
$\rho(c_t, c_{t-1})$	0.972
$\rho(E_t^b, E_{t-1}^b)$	0.988
$\rho(q_t, q_{t-1})$	0.986
$\rho(r_t, r_{t-1})$	0.883
$\rho(\kappa_t, \kappa_{t-1})$	0.99
$\rho(r_t^b, r_{t-1}^b)$	0.988
$\rho(b_t, \kappa_t)$	0.99
$\rho(c_t, ms_t)$	0.82
$\rho(b_t, c_t)$	0.989
$\rho(c_t, \kappa_t)$	-0.975
$\rho(b_t, q_t)$	0.99
$\rho(q_t, \kappa_t)$	-0.985
$\rho(r_t, y_t)$	0.17
$\rho(c_t, r_t^b)$	-0.99
$\rho(q_t, r_t^b)$	-0.99

Table II.2: Second and higher moments

Variables	Standard deviation	Skewness	Kurtosis
$b_t$	4.76	-0.09	2.65
$c_t$	9.55	-0.09	2.68
$E_t^b$	0.43	-0.09	2.65
$\kappa_t$	12.03	0.1	2.68
$q_t$	11.24	-0.06	2.64
$ms_t$	0.48	0.26	1.11
$r_t$	2.17	0.24	3.11
$r_t^b$	4.76	0.09	2.65



Table II.3: Model (auto)-correlations comparison

Variables	(Auto)-correlations without macro-pru	(Auto)-correlations with macro-pru
$\rho(b_t, b_{t-1})$	0.988	0.954
$\rho(c_t, c_{t-1})$	0.972	0.836
$\rho(E_t^b, E_{t-1}^b)$	0.988	0.954
$\rho(q_t, q_{t-1})$	0.986	0.952
$\rho(r_t, r_{t-1})$	0.883	0.88
$\rho(\kappa_t, \kappa_{t-1})$	0.99	0.99
$\rho(r_t^b, r_{t-1}^b)$	0.988	0.954
$\rho(b_t, \kappa_t)$	0.99	0.01
$\rho(c_t, as_t)$	0.82	0.76
$\rho(b_t, c_t)$	0.989	0.888
$\rho(c_t, \kappa_t)$	-0.975	-0.01
$\rho(b_t, q_t)$	0.99	0.97
$\rho(q_t, \kappa_t)$	-0.985	-0.01
$\rho(r_t, y_t)$	0.17	0.30
$\rho(c_t, r_t^b)$	-0.99	-0.88
$\rho(q_t, r_t^b)$	-0.99	-0.97

Table II.4: Second and higher moments comparison

Variables	Standard deviations (pre/post)	Skewness (pre/post)	Kurtosis (pre/post)
$b_t$	4.76/1.46	-0.09/-0.04	2.65/2.72
$c_t$	9.55/3	-0.09/-0.03	2.68/2.87
$E_t^b$	0.43/0.13	-0.09/-0.04	2.65/2.72
$\kappa_t$	12.03/0.05	0.1/-44.69	2.68/2000
$q_t$	11.24/5.01	-0.06/0.03	2.64/2.7
$as_t$	0.48/0.43	0.26/0.60	1.11/1.55
$r_t$	2.17/2.17	0.24/0.24	3.11/3.11
$r_t^b$	4.76/1.46	0.09/0.04	2.65/2.72

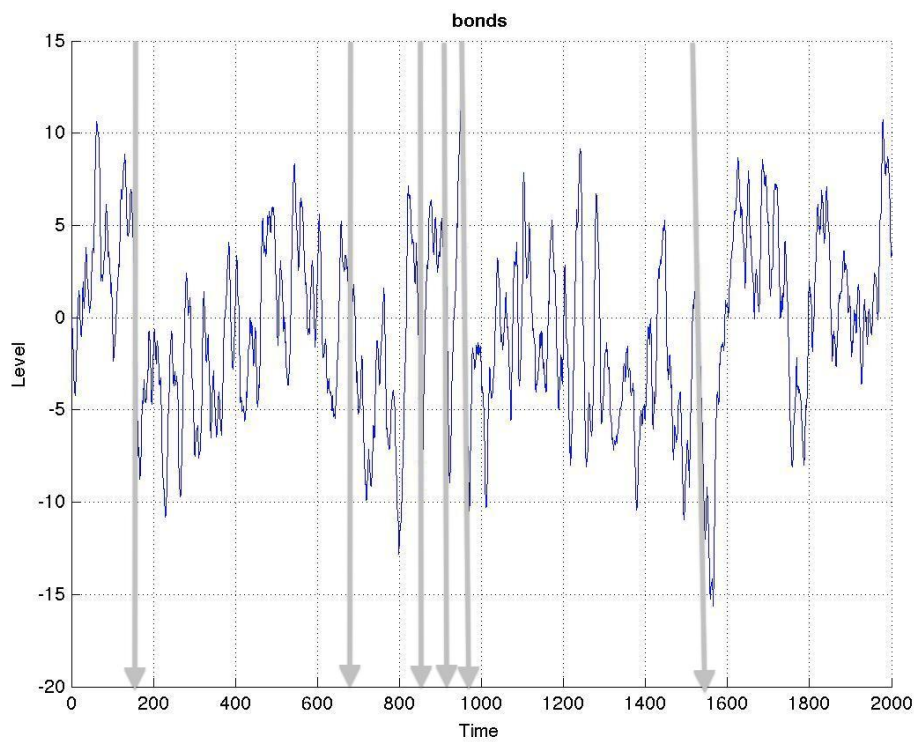
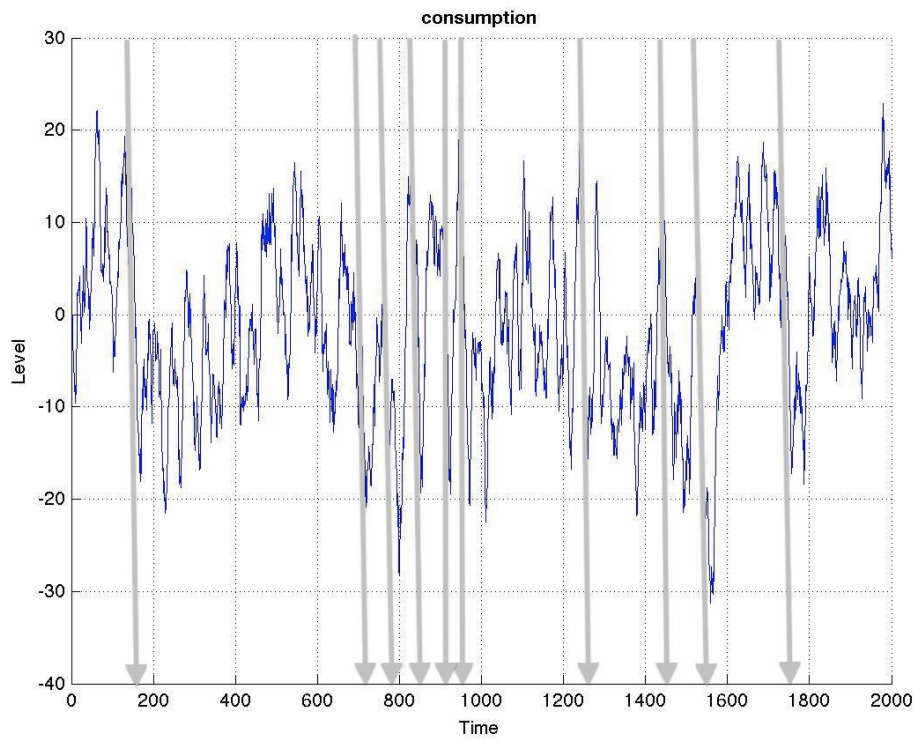


Figure II.1: Evolution of the key aggregate variables

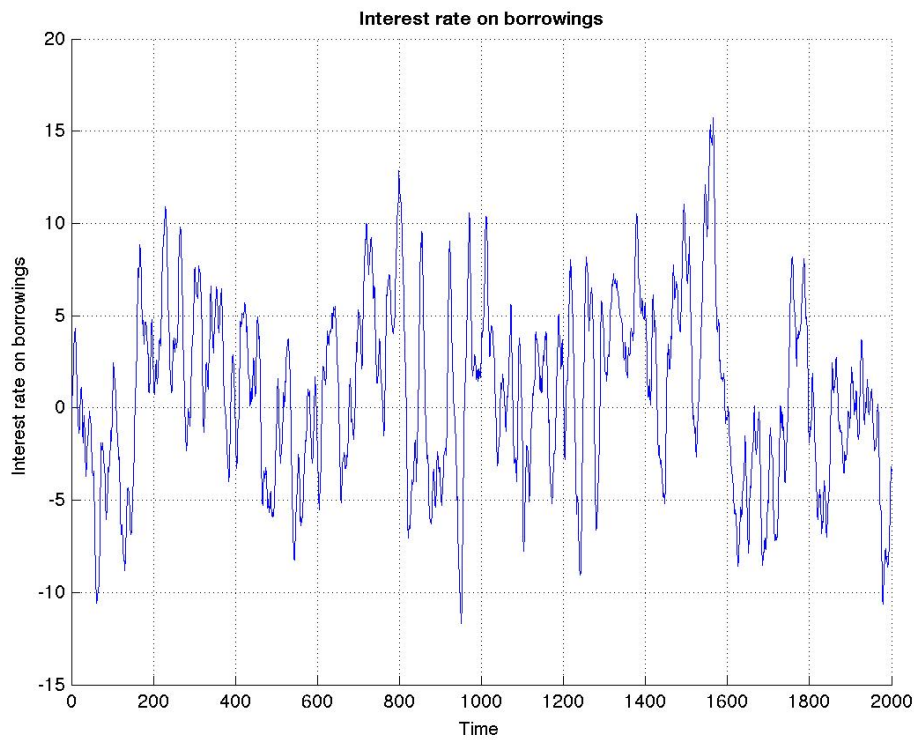
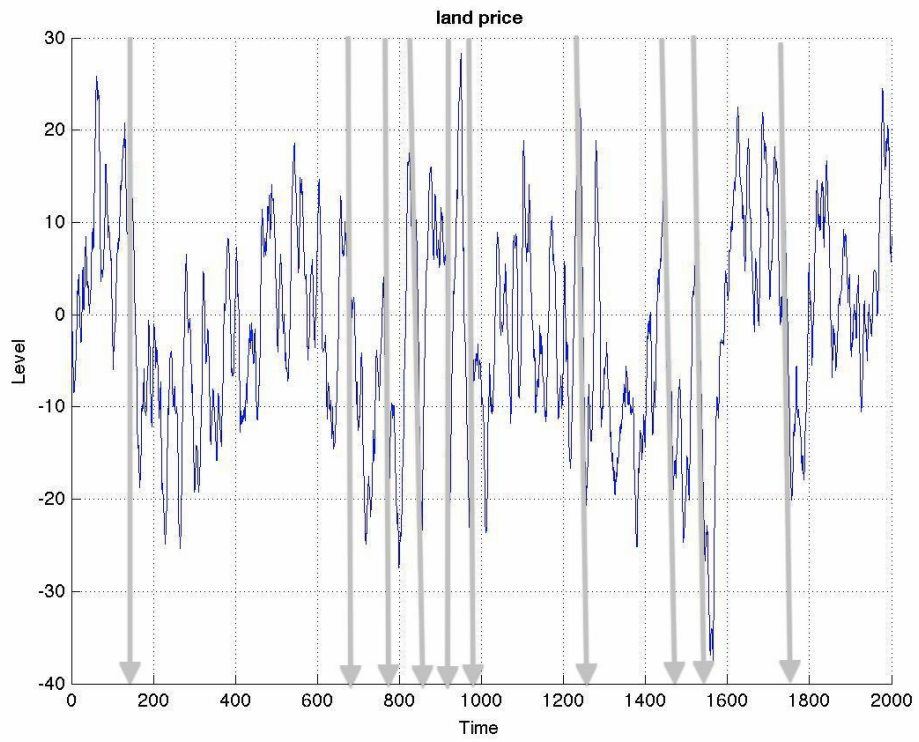


Figure II.2: Evolution of the key aggregate variables 2

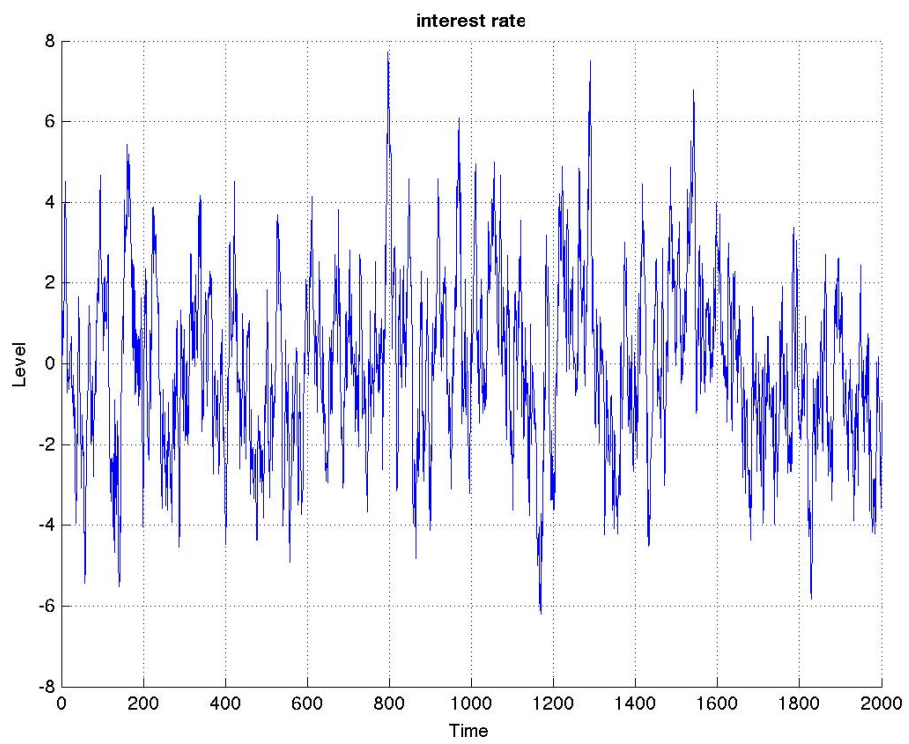
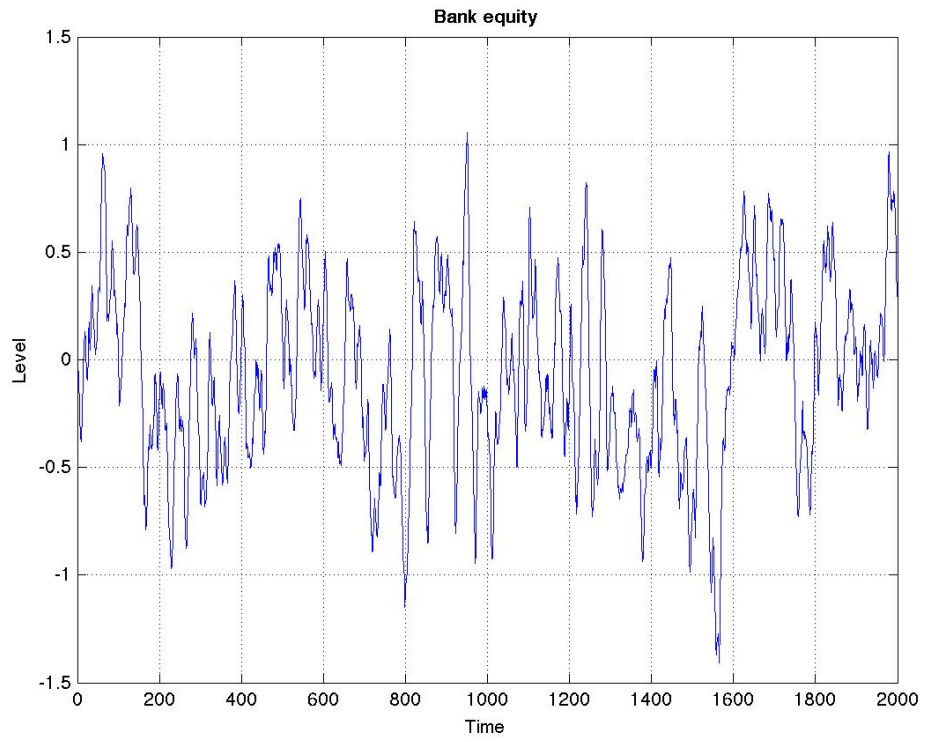
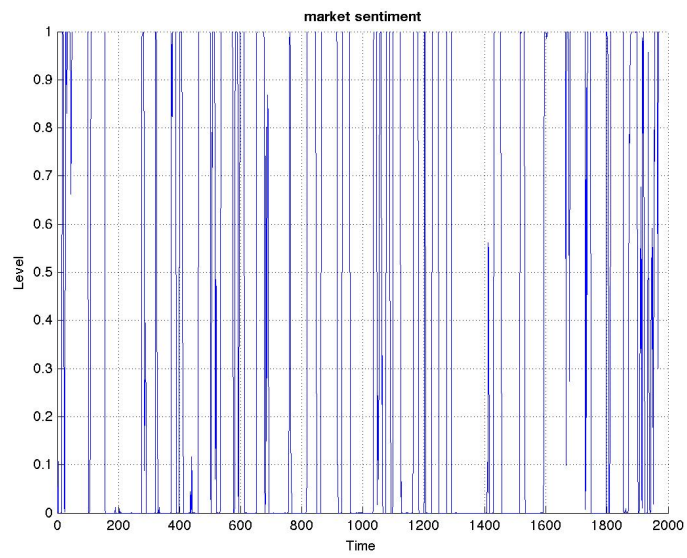
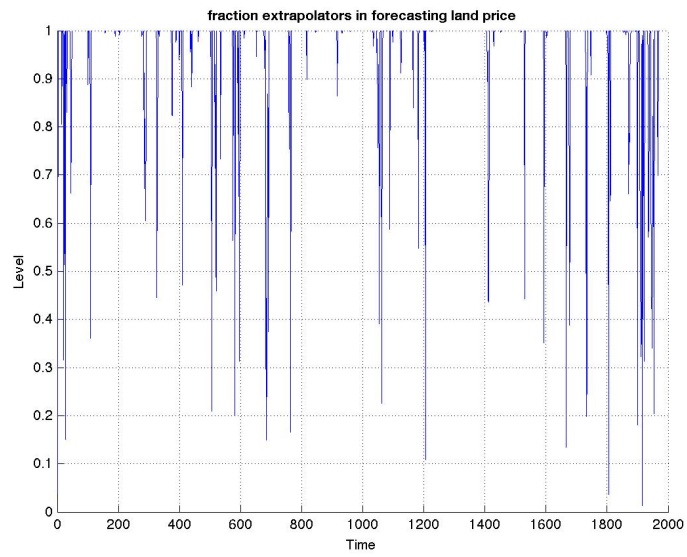
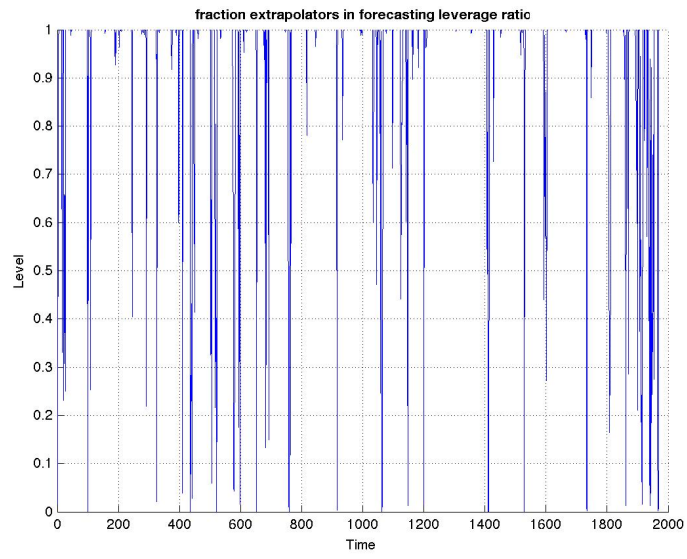


Figure II.3: Evolution of the key aggregate variables



53  
Figure II.4: Learning dynamics

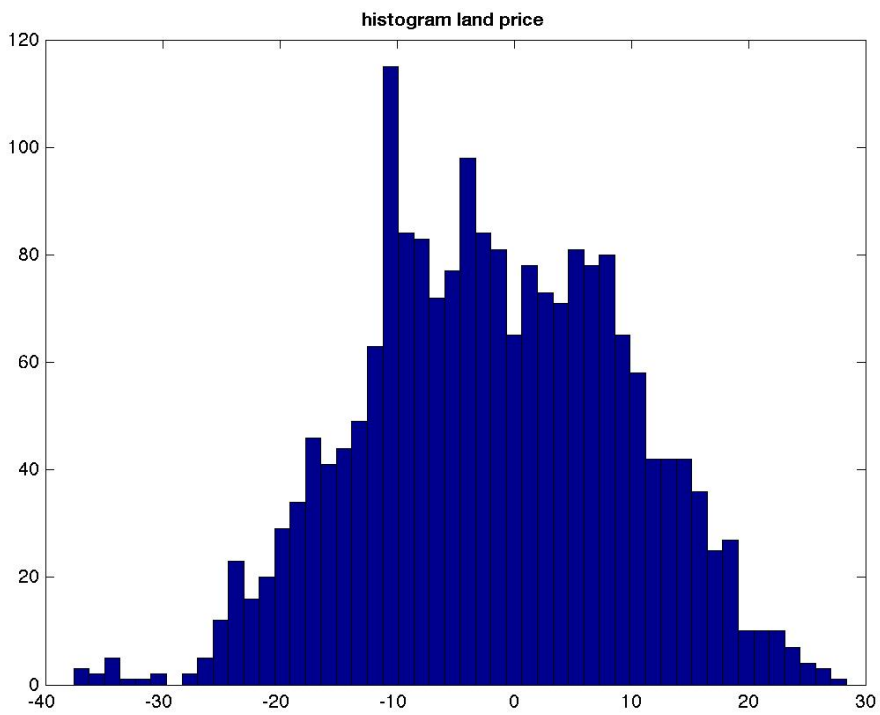
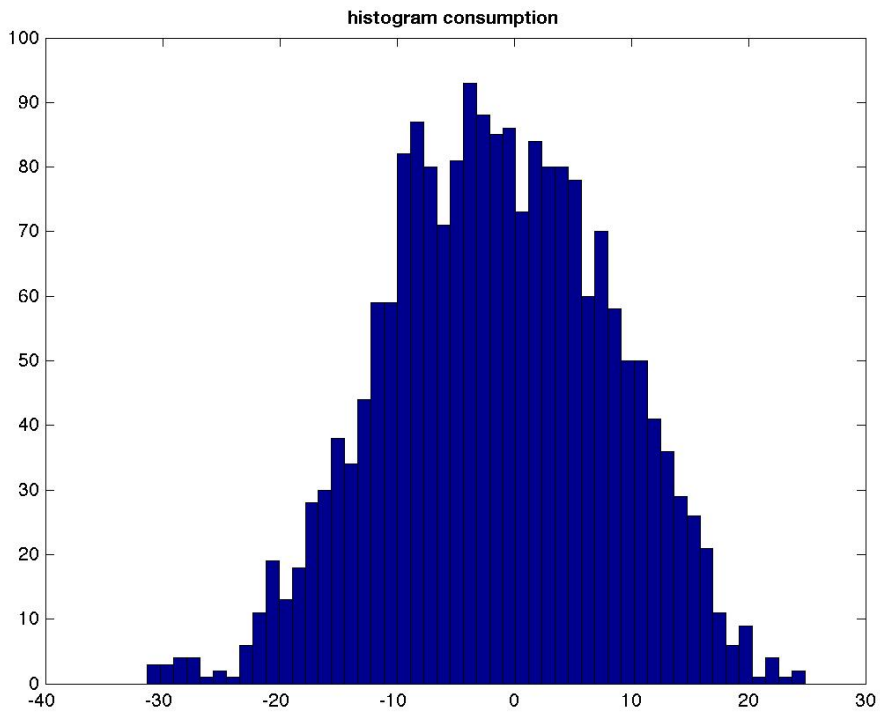


Figure II.5: Histograms

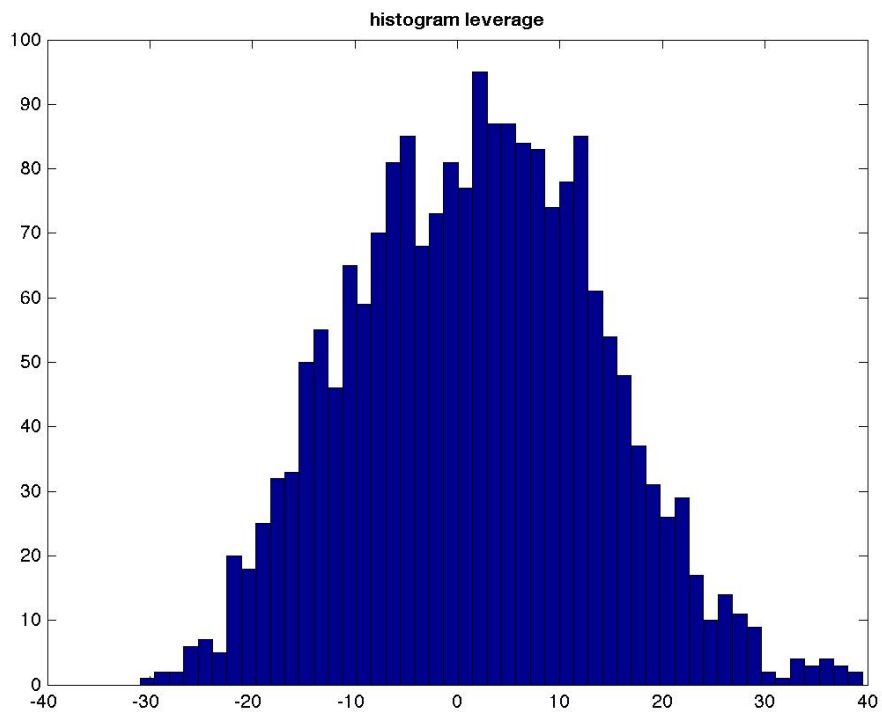
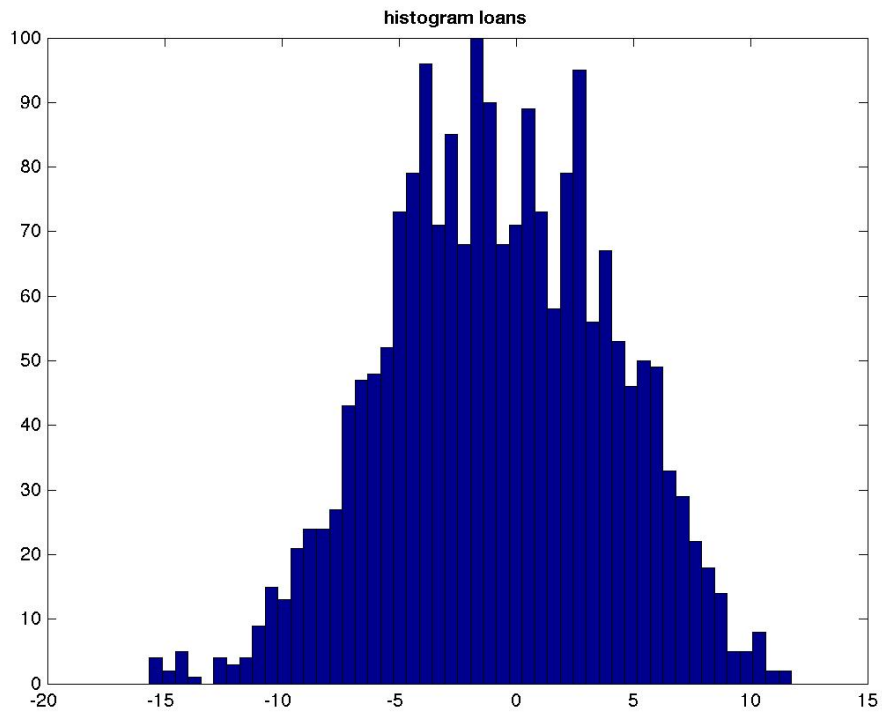
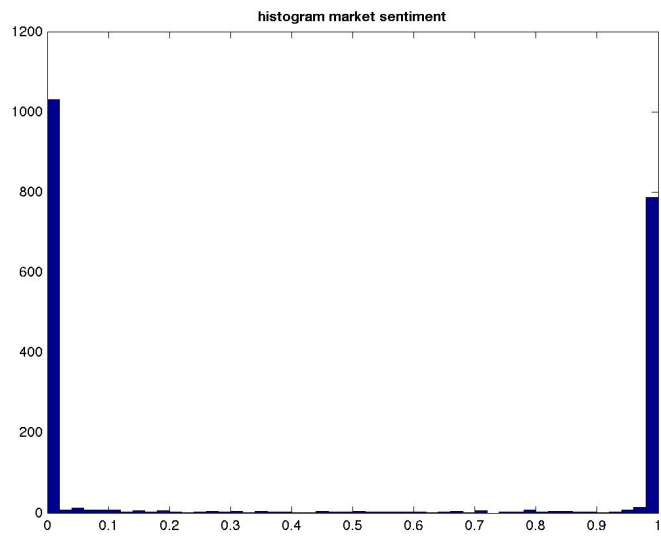
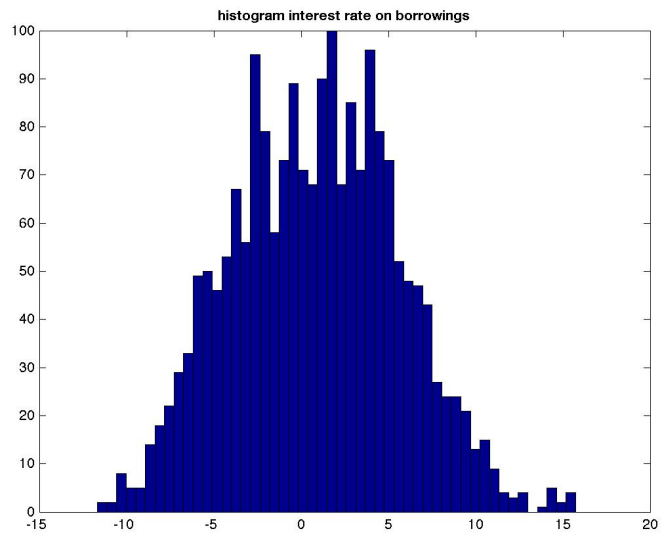
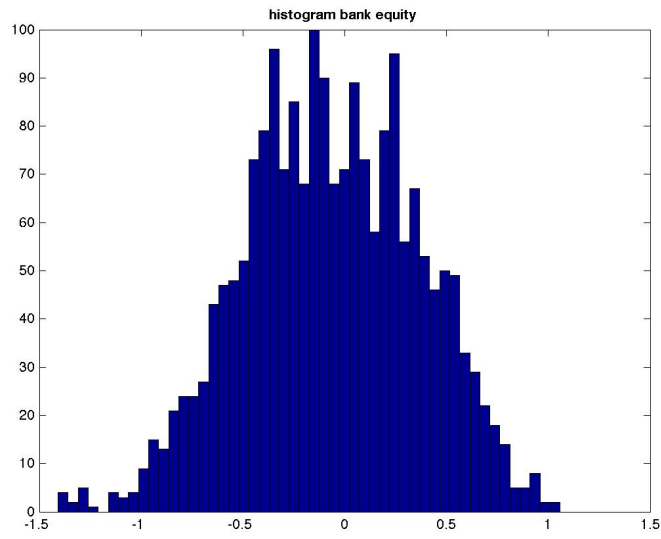


Figure II.6: Histograms 2





56  
Figure II.7: Histograms 3

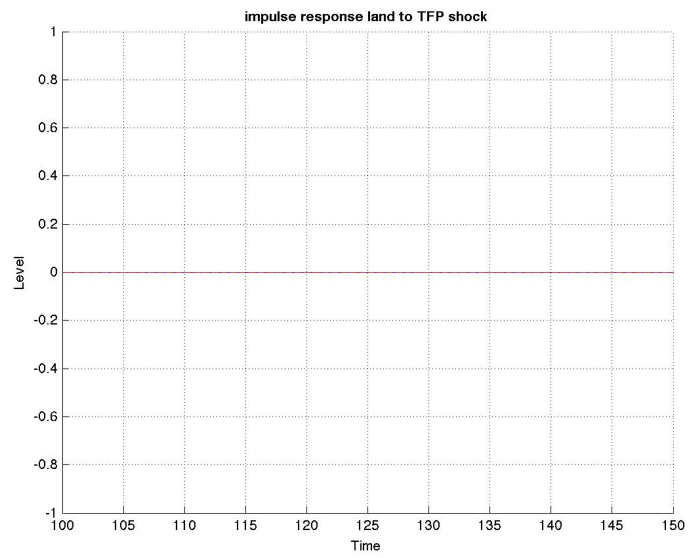
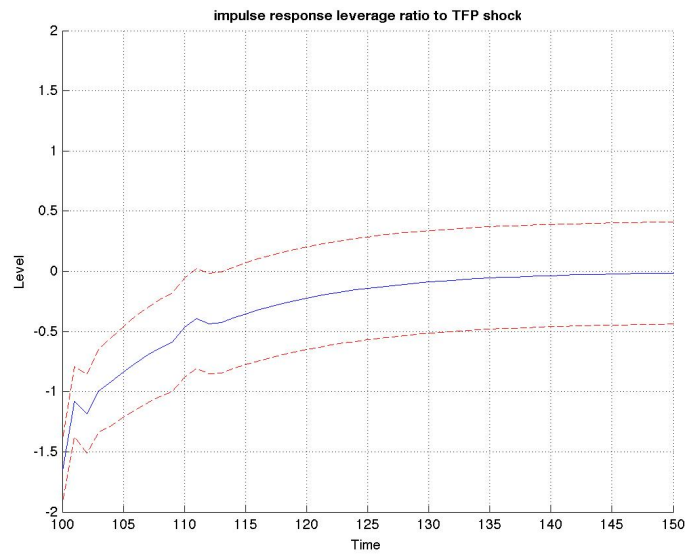
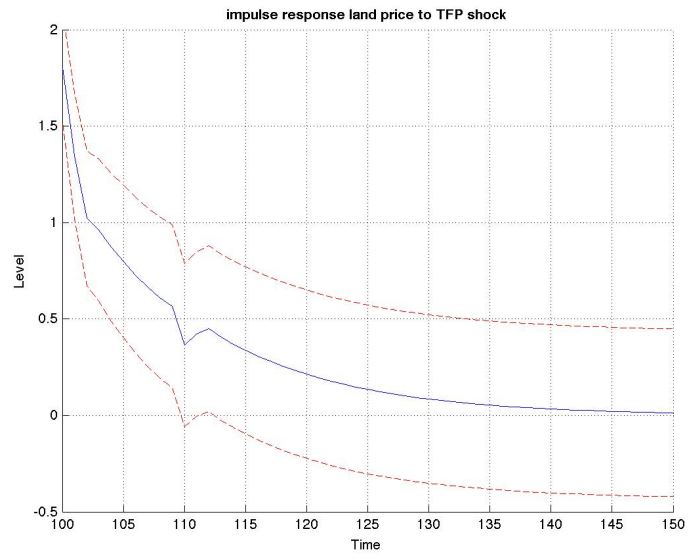


Figure II.8: Impulse responses to a positive TFP shock in  $t=100$

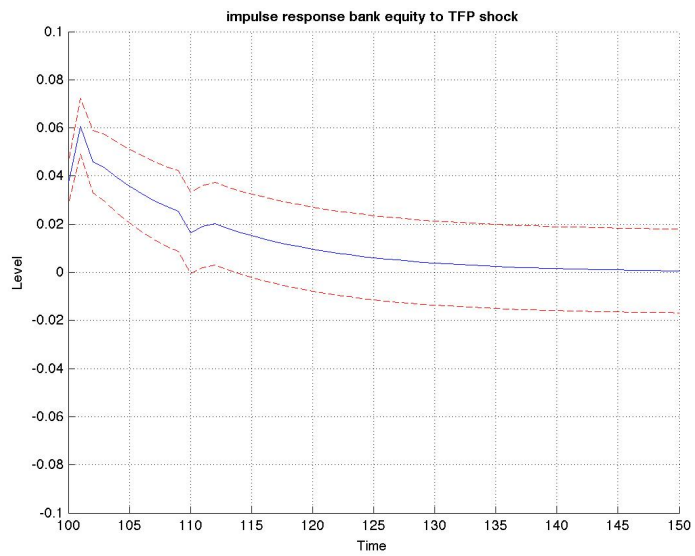
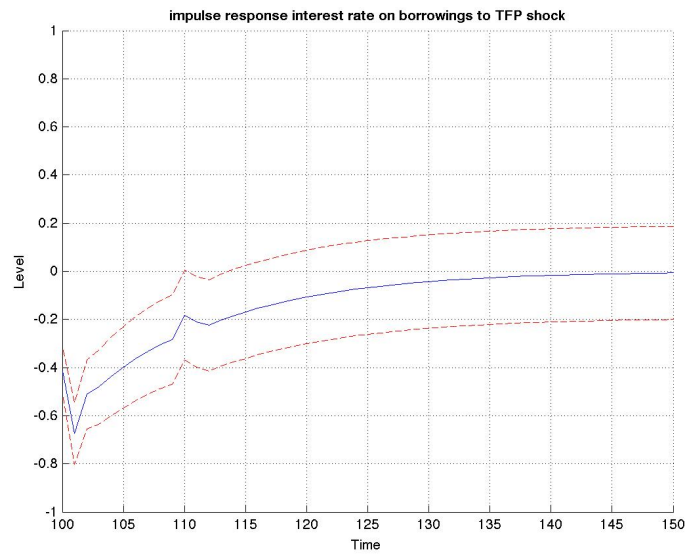
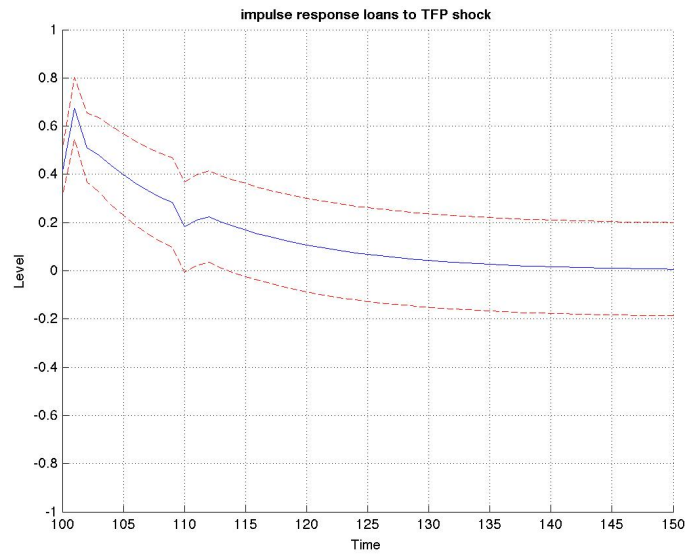


Figure II.9: Impulse responses 2<sup>58</sup> to a positive TFP shock in t=100

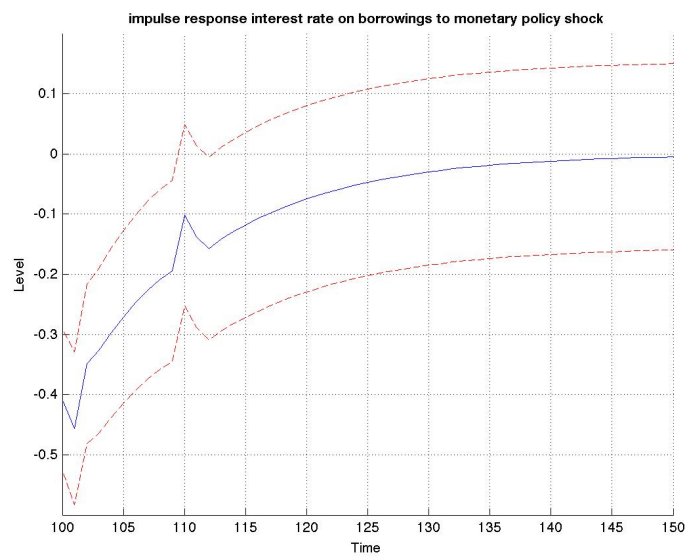
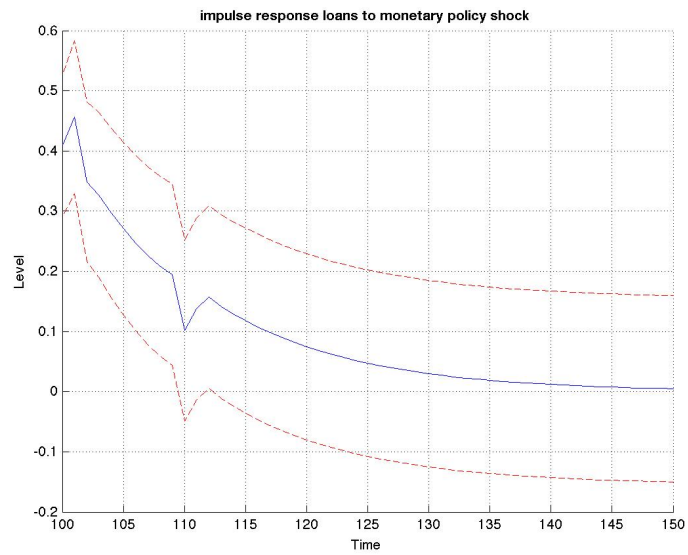
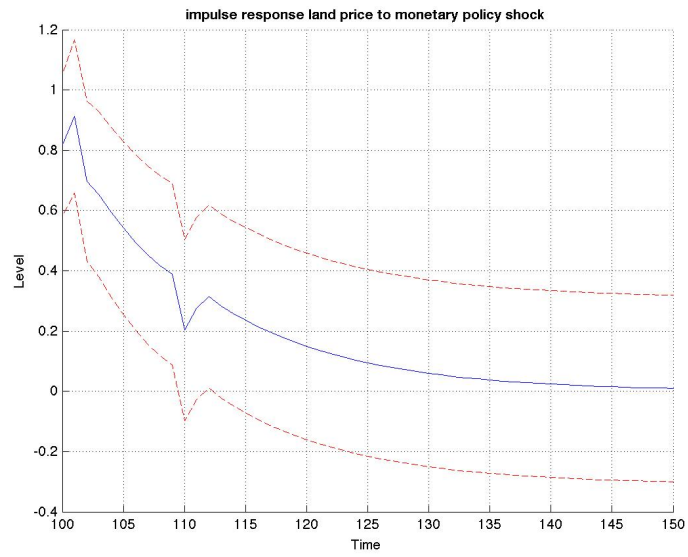


Figure II.10: Impulse responses to an expansionary monetary policy shock in  $t=100$

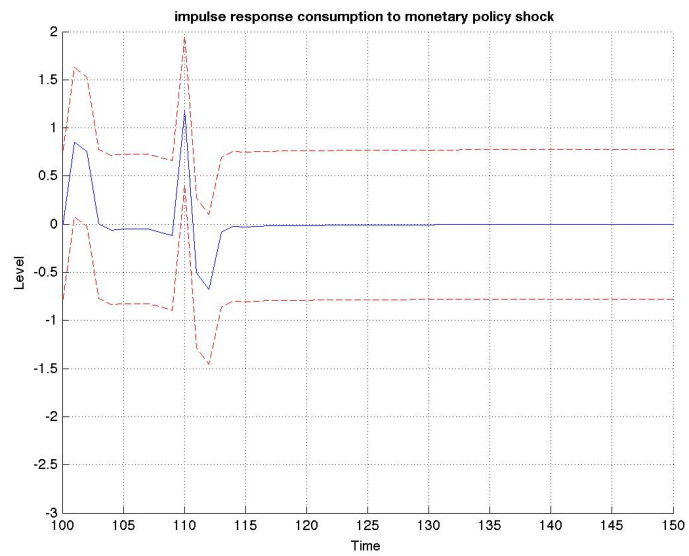
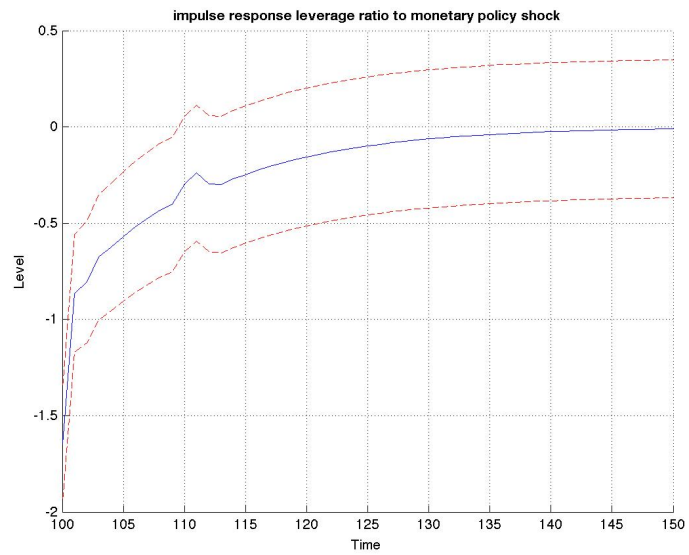
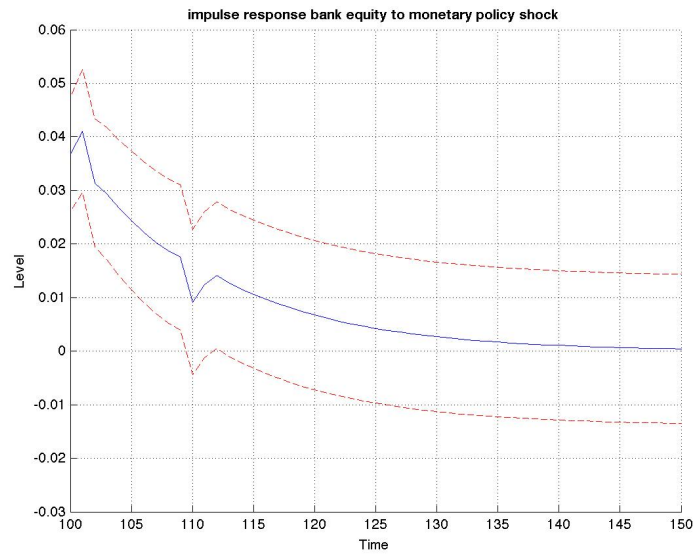


Figure II.11: Impulse responses 2 to an expansionary monetary policy shock in  $t=100$

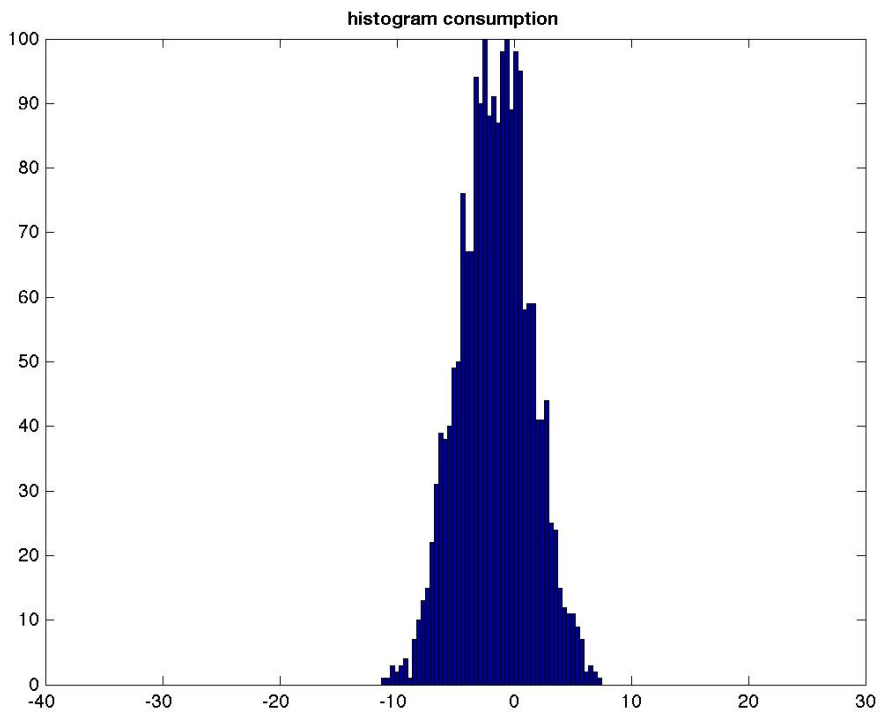
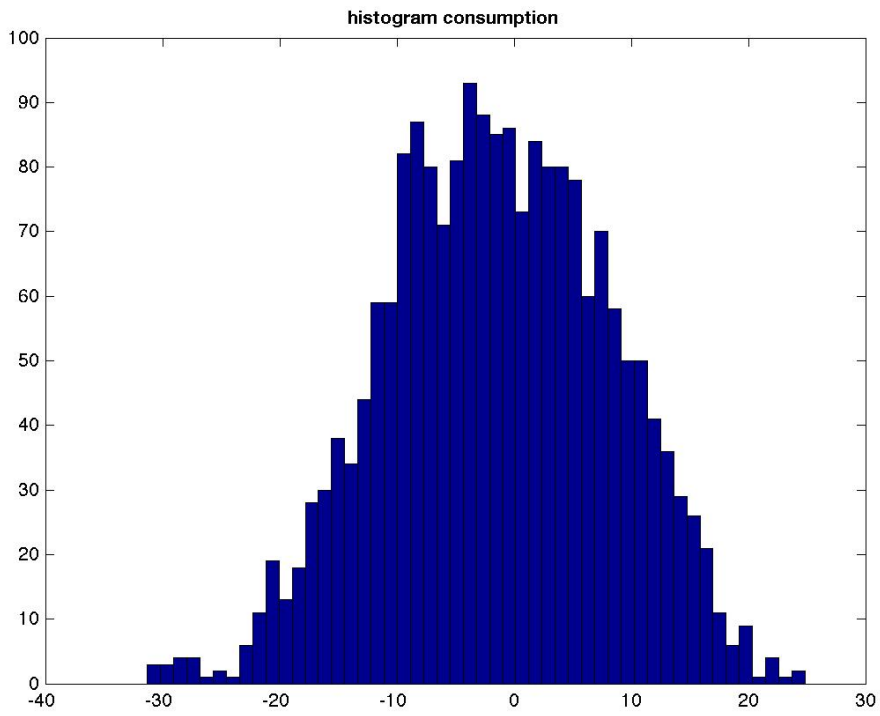


Figure II.12: Macroprudential policy effects

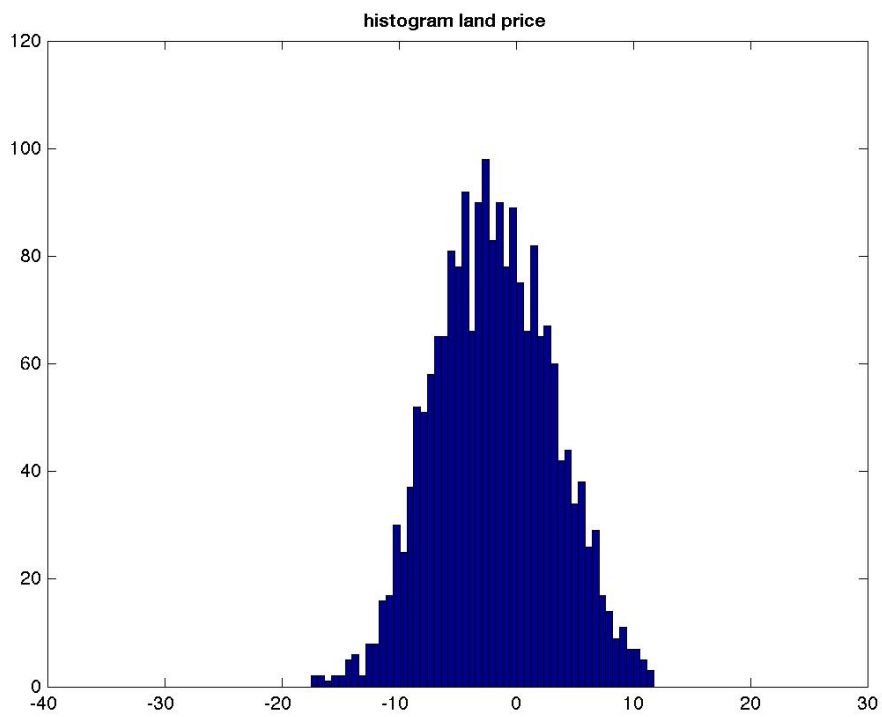
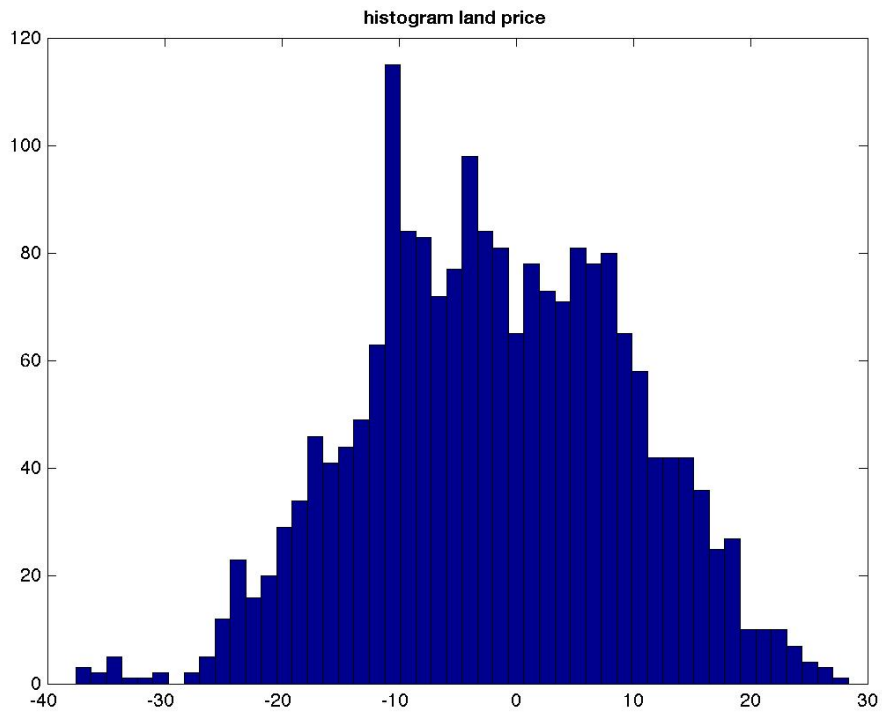


Figure II.13: Macroprudential policy effects 2

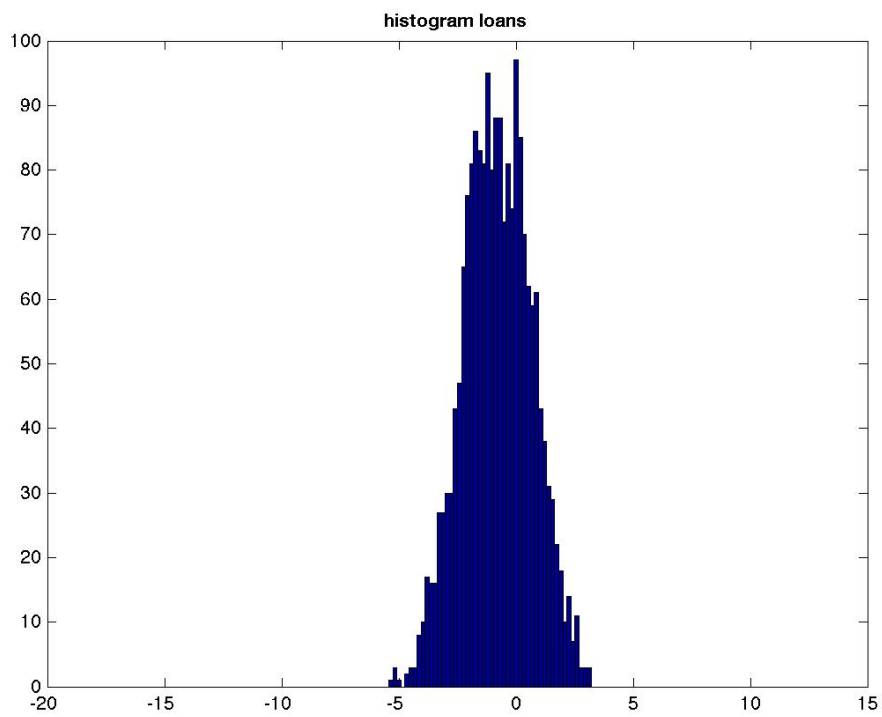
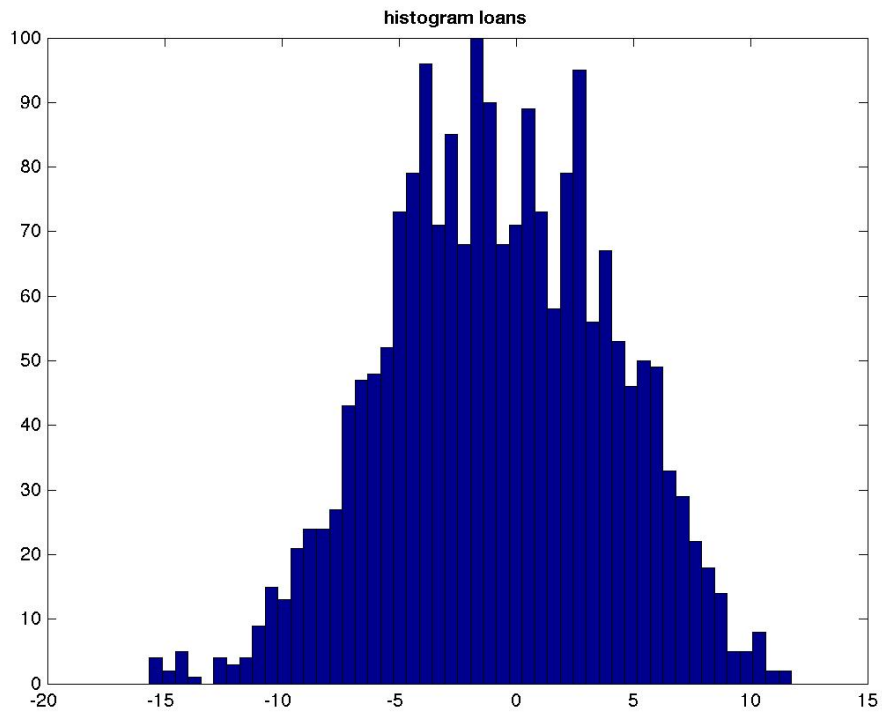


Figure II.14: Macroprudential policy effects 3



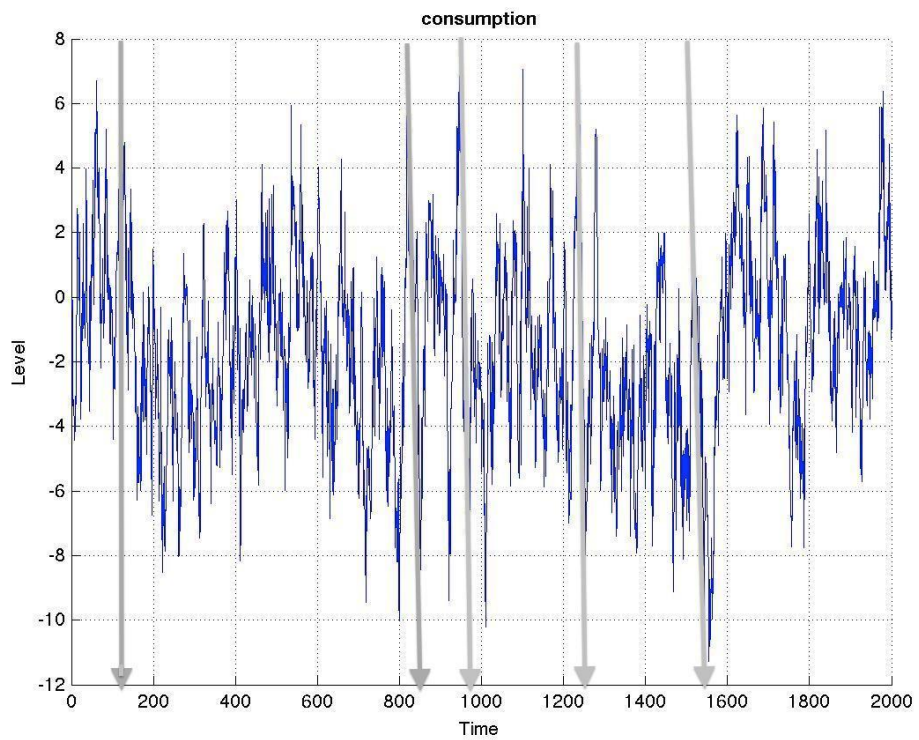
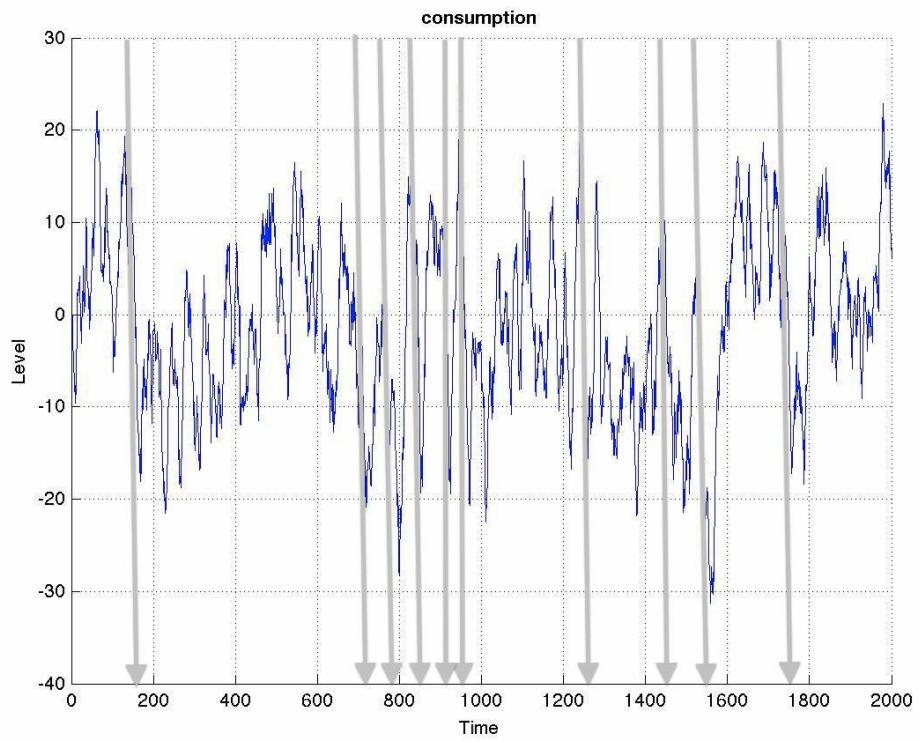


Figure II.15: Macroprudential policy effects 4

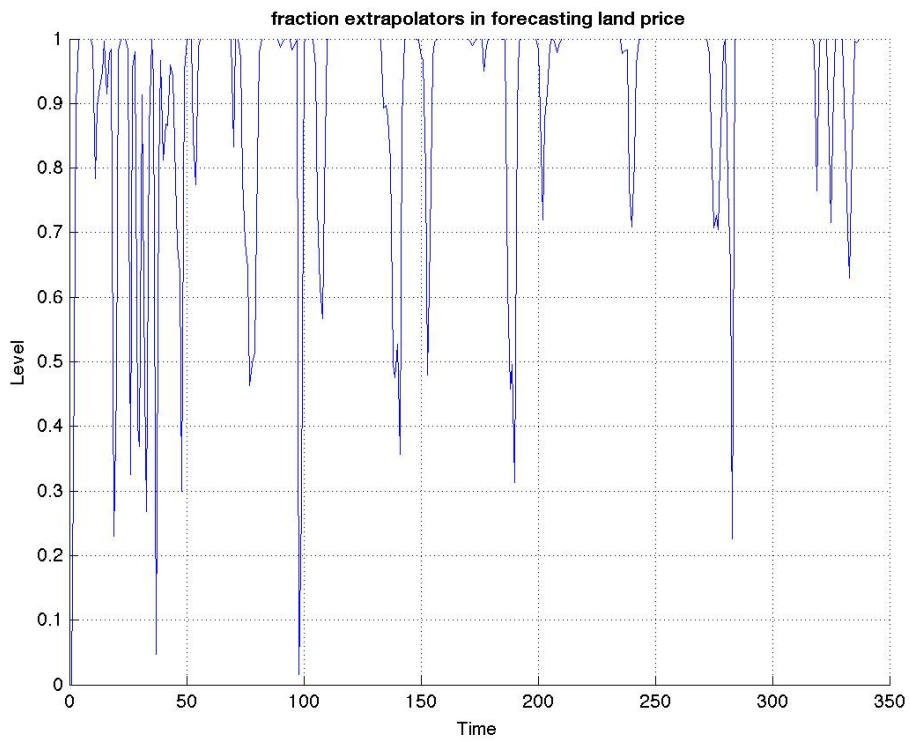
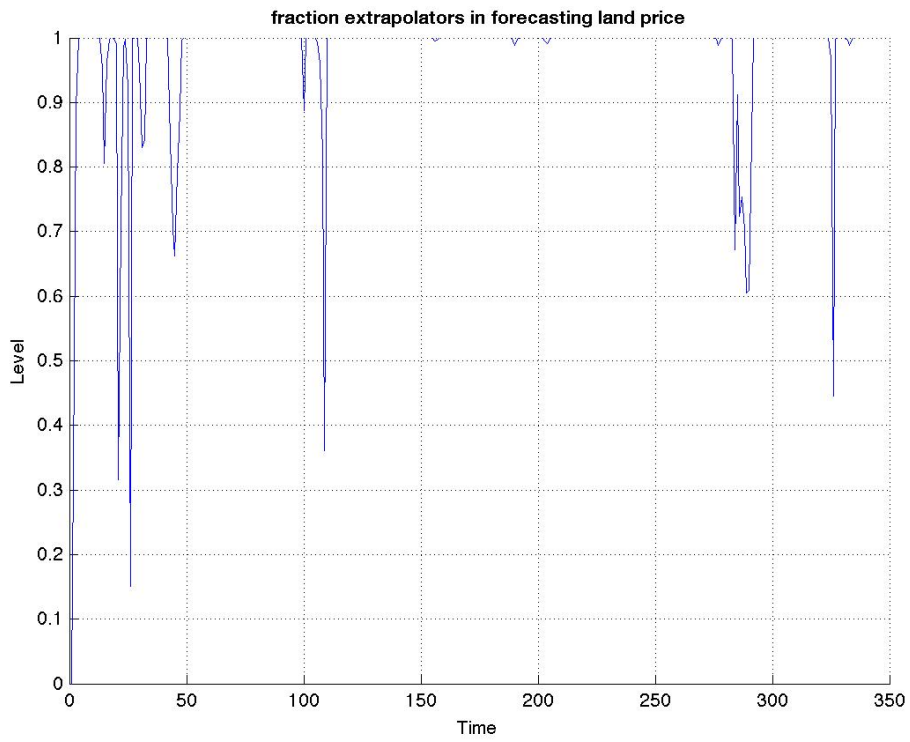


Figure II.16: Macroprudential policy effects 5