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Understanding the space-time (in)consistency of the national accounts

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ABSTRACT

Space-time consistency means that an earlier PPP, updated by relative inflation rates, equals a more recent PPP. I show that in the absence of data errors Divisia price indices are space-time consistent provided that the consumer's utility function is homothetic.

JEL codes: C43; O47; I31
Key words: PPP; Divisia; price index; path-dependence; consistency

Equation Chapter 1 Section 1

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1. Introduction[2]

The economic gap between two countries today can be measured in two ways. Either we can employ today's purchasing power parities (PPPs) to translate the two countries' nominal GDP (or consumption) per head into a common currency. Or we can use an earlier PPP and earlier GDP per head figures, uprated to reflect the growth of real GDP per head between the earlier and the current period. Unfortunately, the two answers are unlikely to be the same, an example of what I call space-time inconsistency in the national accounts. The existence of space-time inconsistency has long been recognised. But there is disagreement over its cause or whether it is a problem or just a fact of life: see for example Deaton (2010), Deaton and Heston (2010), Feenstra *et al*. (2013), Johnson *et al*. (2013) and Deaton and Aten (2014). Is inconsistency due just to data errors or should we expect to observe it even with perfect data?

This paper proves a sufficient condition under which space-time consistency will prevail. Suppose we measure both domestic price indices and PPPs by Divisia price indices. And suppose that these Divisia indices possess the property of *path-independence*. Then in the absence of errors in the data, a multi-country set of national accounts using these price indices will be consistent across space and time. However, Divisia price indices are path-independent only if the underlying consumer's utility function is homothetic, a strong condition.

2. The inconsistency problem

The direct way to measure the economic gap between two countries, A and B, at time t ($Z_t^{B/A}$) is to make use of the PPP between them at time t:

$$\left[Z_t^{B/A}\right]_{direct} = \left(\frac{V_t^B}{V_t^A}\right)\left(\frac{1}{PPP_t^{B/A}}\right) \qquad (1)$$

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where $V_t^J = P_t^J Y_t^J$, $J = A, B$, is nominal GDP (or consumption) per capita at time *t* measured in local currency units. Here *Y* denotes real GDP per capita, *P* denotes the domestic price level (as measured typically by the GDP deflator), and *PPP* is the overall purchasing power parity, the PPP for GDP, measured in units of *B*'s currency per unit of *A*'s currency, e.g. U.K. pounds sterling per U.S. dollar. The indirect way makes us of the gap in an earlier period *r*, uprated by the relative increase in living standards between *r* and *t*:

$$\begin{aligned}\left[Z_t^{B/A}\right]_{indirecct} &= \frac{\left(Y_t^B / Y_r^B\right)}{\left(Y_t^A / Y_r^A\right)}\left(\frac{V_r^B}{V_r^A}\right)\left(\frac{1}{PPP_r^{B/A}}\right) \\ &= \left(\frac{V_t^B}{V_t^A}\right)\left(\frac{P_r^B / P_t^B}{P_r^A / P_t^A}\right)\left(\frac{1}{PPP_r^{B/A}}\right)\end{aligned} \tag{2}$$

The second line follows by applying the definition of nominal GDP per capita.

Consistency across space and time requires that the direct and indirect measures yield the same answer, which implies that:

$$PPP_t^{B/A} = PPP_r^{B/A}\left(\frac{P_t^B / P_r^B}{P_t^A / P_r^A}\right) \tag{3}$$

i.e. the more recent PPP must equal the older one after uprating the latter by inflation in the two countries. I shall refer to equation (3) as *the condition for space-time consistency in the national accounts*.[3] Though both the direct and indirect indices purport to measure the same thing, namely the gap between the two countries at time *t*, there is no guarantee that they will be equal in practice, i.e. that condition (3) will be satisfied.

In the latest version of the Penn World Table, Version 8, growth rates of GDP and its components are based on successive sets of PPPs, from 1955 to 2005 (Feenstra *et al.*, 2013). But a completely different set of time series for GDP and its components based on national accounts are also made available in PWT 8.0. These growth rates are often strikingly different from the PPP-based ones (see e.g. Figures 1 and 2 in Feenstra *et al*. 2013).

The degree of inconsistency seems large enough to cause concern. But how much inconsistency is consistent with economic theory? Divisia price indices can shed light on this.

[3] Both the OECD and the latest Penn World Table use the right hand side of (3) to estimate PPPs for years when no direct evidence exists.

3. The inconsistency issue: insights from Divisia index numbers

Given prices p_i of N commodities and corresponding budget shares w_i, the growth rate of a Divisia price index (P^D) at time t is defined as:

$$\frac{d \ln P^D(r,t)}{dt} := \sum_{i=1}^{N} w_i(t) \frac{d \ln p_i(t)}{dt} \tag{4}$$

The level of the Divisia index depends both on the current time t and on the reference time r. We can also interpret t and r as indexing countries, in which case r is the reference country. Just as time was assumed to be continuous so we can now think of a continuum of countries as is common elsewhere in economics, e.g. a consumer, firm or product is often assumed to be an infinitesimal part of a continuum. The level of the Divisia index in period (country) t is found by integration:

$$\ln P^D(r,t) = \sum_{i=1}^{N} \left[\int_r^t w_i(\tau) \frac{d \ln p_i(\tau)}{d\tau} d\tau \right] \tag{5}$$

Here I have normalised the index to equal 1 in the reference period (country): $P^D(r,r) = 1$.

Divisia indices have many desirable properties: first, by definition the value index is the product of the price and quantity indices and second, the indices are consistent in aggregation. But they also suffer from two drawbacks. The first is that they are defined in continuous time so in practice they cannot be calculated exactly. However they can be approximated by chain indices. For example, a discrete approximation to the Divisia price index is given by the chained Törnqvist price index (P_t^{ChT}) whose discrete growth rate is (compare equation (4)):

$$\Delta \ln P_t^{ChT} = \sum_{i=1}^{N} \left(\frac{w_{it} + w_{it-1}}{2} \right) \Delta \ln p_{it} \tag{6}$$

The second drawback is path dependence. This means that a Divisia index may fail the circularity test.

As is well known (Hulten, 1973; Balk, 2005), there is one condition which is necessary and sufficient for a Divisia index not to be path-dependent, i.e. for it to be path-*independent*. This is (in the case of consumption) that observed behaviour results from consumers maximising a utility function which is homothetic. This boils down to the requirement that the expenditure shares must depend only on the prices and not on some other factors which may change over the path. These other factors include real income (Engel's Law) but in a cross-country context budget shares have been shown to depend to an important extent on background factors like climate, demography and culture (Oulton 2012a and 2012b) and these are another cause of non-homotheticity.

We can now state the main theoretical result of this paper:

Proposition If the Divisia price index is path-independent, then (in the absence of data errors) comparisons made using Divisia price indices are consistent across space and time. That is, the direct and indirect methods of equations (1) and (2) yield the same answer and the space-time consistency condition, equation (3), is satisfied.

Proof We can write the Divisia price index (5) in non-parametric form by eliminating the variable indexing countries or time (τ):

$$\ln P^D(r,t) = \sum_{i=1}^{N}\left[\int_{\ln p_i(r)}^{\ln p_i(t)} w_i d \ln p_i\right]$$

We can then re-write this as a line integral using vector form and dot product notation:

$$\ln P^D(r,t) = \int_G \mathbf{w}(\mathbf{p},\mathbf{X}) \cdot d\ln\mathbf{p}$$

where $\ln\mathbf{p}$ is the vector of log prices and $\mathbf{w}(\mathbf{p},\mathbf{X})$ is the vector of expenditure shares; the latter are shown as functions of prices and possibly other variables (e.g. income or scale) represented by the vector **X**. The integral is taken over a path *G* which commences with the share and price vectors **w** and ln **p** having the values of country or period *r* and finishes with these same vectors having the values of country or period *t*.

Now take logs in the condition for space-time consistency, equation (3):

$$\ln PPP_t^{B/A} = \ln PPP_r^{B/A} + \ln\left(P_t^B / P_r^B\right) - \ln\left(P_t^A / P_r^A\right)$$

Translate this condition into Divisia price indices:

$$\ln P_t^D(A,B) = \ln P_r^D(A,B) + \ln P_B^D(r,t) - \ln P_A^D(r,t) \qquad (7)$$

Here I have added a time subscript (*r* or *t*) to indicate the date to which a cross-country index applies and a country subscript (*A* or *B*) to indicate the country to which a time-series index applies. In line integral terms equation (7) can be written as:

$$\int_G \mathbf{w}(\mathbf{p},\mathbf{X}) \cdot d\ln\mathbf{p} = \int_H \mathbf{w}(\mathbf{p},\mathbf{X}) \cdot d\ln\mathbf{p} + \int_I \mathbf{w}(\mathbf{p},\mathbf{X}) \cdot d\ln\mathbf{p} - \int_J \mathbf{w}(\mathbf{p},\mathbf{X}) \cdot d\ln\mathbf{p} \qquad (8)$$

Let $\ln\mathbf{p}(X,s)$ be the log price vector for country *X* at time *s*. Then the paths have the following characteristics:

Path	*Type*	*Endpoints*
G	Cross-country, time *t*	$\ln \mathbf{p}(A,t), \ln \mathbf{p}(B,t)$
H	Cross-country time *r*	$\ln \mathbf{p}(A,r), \ln \mathbf{p}(B,r)$
I	Time series, country *B*	$\ln \mathbf{p}(B,r), \ln \mathbf{p}(B,t)$
J	Time series, country *A*	$\ln \mathbf{p}(A,r), \ln \mathbf{p}(A,t)$

So taking into account the minus sign we see that the right hand side of (8) describes a path (*JHI*) whose endpoints are $\ln \mathbf{p}(A,t)$ and $\ln \mathbf{p}(B,t)$, the same as those of path *G* on the left hand side. Hence by path-independence the two sides of (8) are equal. In other words space-time consistency holds for Divisia price indices when the indices are path-independent. Figure 1 illustrates for the two-good case

Remark In Figure 1 the overall path has sharp corners when it switches from time series to cross section variation. This doesn't affect the argument as the main theorems on line integrals continue go through. All that is required is that the overall path be continuous, but not necessarily differentiable, at every point.

The Proposition shows that there are three reasons why national accounts might not exhibit space-time consistency in practice:

1. The Divisia price indices may not be path-independent.
2. Even if they are path-independent they have to be approximated, e.g. by chain indices, and this leads to error (approximation error).
3. There are errors in the underlying price data.

4. Conclusions

The conditions for path-independence are very strong: they require that budget shares be determined wholly by relative prices and not at all by income or by background factors like demography or climate. If to the contrary the utility function is not homothetic, then we need to employ a true cost-of-living or Konüs index in place of a Divisia one. Since a Konüs price index holds utility constant it is path-independent and so space-time consistency will prevail. Fortunately there is a close relationship between Divisia and Konüs indices (Balk, 2005;

Oulton, 2008). In Oulton (2012b) I presented an empirically practical method of estimating a Konüs index across countries from data available in the 2005 ICP. If this work were extended then we could see how much of the observed inconsistency is due to non-homotheticity and how much to other causes such as data errors.

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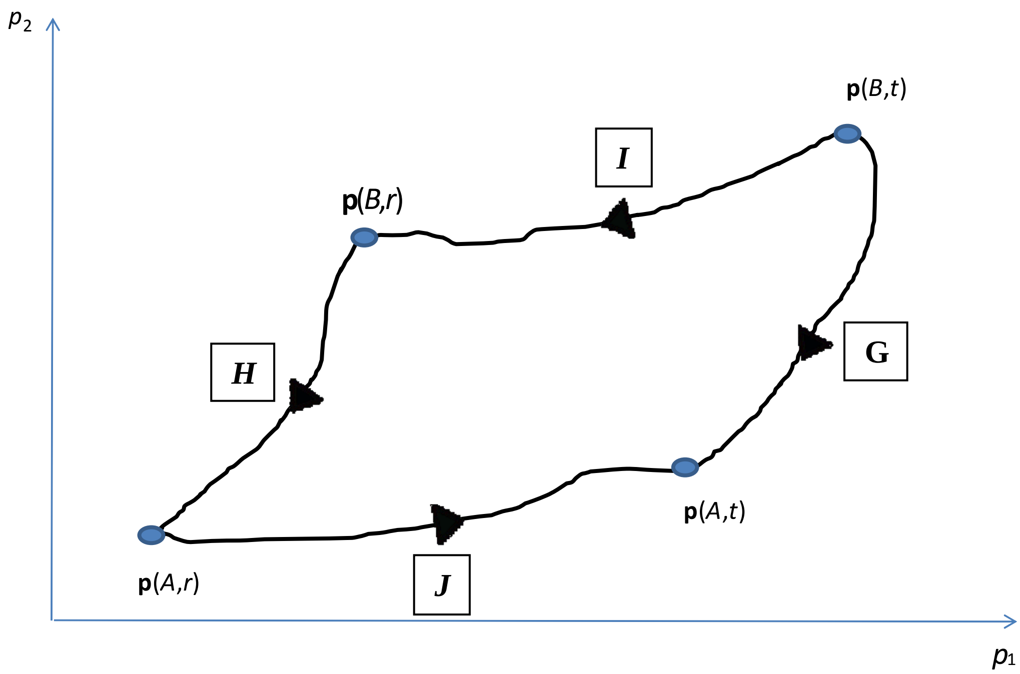

Figure 1 Price variation over time and across countries: the two-good case

Note: $\mathbf{p}(X,s)$ is the price vector for country X $(A \le X \le B)$ at time s $(r \le s \le t)$.