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Credit Constraints and Growth in a Global Economy

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Abstract

We show that in an open-economy OLG model, the interaction between growth differentials and household credit constraints, more severe in fast-growing countries, can explain three prominent global trends: a divergence in private saving rates between advanced and emerging economies, large net capital outflows from the latter, and a sustained decline in the world interest rate. Micro-level evidence on the evolution of age-saving profiles in the U.S. and China corroborates our mechanism. Quantitatively, our model explains about 40 percent of the divergence in aggregate saving rates, and a significant portion of the variations in age-saving profiles across countries and over time.

JEL Classification: F21, F32, F41

Key Words: Household Credit Constraints, Age-Saving Profiles, International Capital Flows, Allocation Puzzle.

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1 Introduction

Two of the most important developments in the global economy of the recent decades are the integration of emerging markets into world capital markets and their rapid growth, particularly in certain parts of Asia. Alongside these events are three striking and unprecedented macroeconomic trends: (1) a large and persistent increase in the private saving rate in emerging Asia against a steady decline in the private saving rate in advanced economies; (2) the emergence of global imbalances, with developing countries running a large current account surplus and advanced economies a current account deficit; (3) a sustained fall in the world long-term interest rate.

The striking divergence in the saving rates of advanced economies vs. those of Emerging Asia is depicted in Figure 1.1. The gap in private saving rates across regions, interestingly, was rather small at the time of their integration around 1990 (top panel). The diverging pattern is even more obvious when it comes to household saving rates in countries such as the U.S. and China (bottom panel). In the late 1980s, China’s household saving rate was a mere 3.5 percentage points higher than that of the U.S.. By 2008, it had reached almost 30% while the U.S. saving rate had declined to about 2.5% — leading to the popular caricature of a ‘debt-ridden’ U.S. put into sharp contrast against a ‘thrifty’ Asia.

Observations on the macroeconomic level, however, can mask heterogenous behavior at the micro level. The microeconomic approach typically stresses heterogeneity in consumption and saving behavior over the lifecycle. A natural question to ask is: how did different age groups contribute to the staggering divergence in household saving rates? Estimating age-saving profiles for the U.S. and China, we document the following stylized facts: (i) the saving rate of young individuals fell significantly in the U.S. over 1988-2008—by about 13 percentage points—but remained approximately constant in China; (ii) the saving rate of the middle-aged rose in both countries, but by about 15 percentage points more in China than in the U.S.; (iii) there is a marked divergence of saving rates for the retirees—with China’s elderly seeing a sharp rise and the U.S.’ seeing a large drop. The elderly, however, contribute less to aggregate saving than the other age groups.
In this paper, we attempt to bridge the macroeconomic approach with the microeconomic approach in understanding the set of global facts (1)-(3). First, we provide an overlapping-generation theory of growth in an open economy where households face different levels of credit constraints across countries. We show that the interaction between growth differentials and heterogenous credit constraints — more severe in Emerging economies — can generate the set of aggregate facts, while yielding distinct predictions on age-specific saving behavior across countries and over time. Second, using consumption survey data, we provide new micro-level evidence on age-saving profiles in the U.S. and China and their evolution. These micro facts are important as they provide an additional empirical barometer against which the quantitative performance of our theory can be assessed. This leads to our third contribution, which is to develop a quantitative multi-period OLG model, calibrated to aggregate and micro data for the U.S. and China.

Explaining the global patterns (1)-(3) constitutes a challenge for standard open-economy growth models. Standard theories predict that a fast growing economy such as China should experience a fall in saving rate as agents borrow against their higher future income to augment consumption and investment, contributing to a rise in the world interest rate. In face of high domestic investment needs, China would become a net capital importer rather than a net exporter. This discrepancy between theory and evidence is forcefully pointed out by Gourinchas and Jeanne (2011), who refer to it as the “allocation puzzle”. For any theory to successfully capture the global phenomenon, the challenge is to explain why saving can outpace investment in a growing economy—but also in light of Figure 1.1, to account for the asymmetric evolution of saving rates across countries with the same underlying mechanism.

The pronounced divergence in household saving rates and the differential saving behavior across age groups motivate our theory. Our baseline theoretical framework, analyzed in Section 2, consists of large open economies populated with agents living for three periods. This structure provides scope for both international and intergenerational borrowing.¹ Young

¹Our baseline framework is an extension of Jappelli and Pagano’s (1994) closed-economy, three-period OLG model with household credit constraints. Our environment differs from theirs in several dimensions: (1) the multi-country, open-economy aspect of our setup; (2) the asymmetry in household credit constraints across countries; (3) more general preferences; (4) more general income profiles.
agents are all subject to borrowing constraints, but the tightness of the constraint is more severe in developing countries than in advanced economies. A fall in the world interest rate induces greater borrowing by the young—through a loosening of constraints, and greater savings of the middle-aged—through a dominant income effect.\(^2\) Asymmetric credit constraints imply that the young’s saving rate falls by more in less constrained economies, while the rise in the middle-aged’s saving rate is larger in more constrained ones. This leads to different responses of aggregate saving rates across countries which, combined with initial level differences, generate a divergence in saving rates in the long run.

In this framework, the decline in the world interest rate is brought about by the increasing size of Asia relative to the rest of the world. Faster growth in Emerging Asia results in a greater weight being put on its (lower) long-run autarkic interest rate in determining the dynamics of the world interest rate.\(^3\) The interaction of growth and heterogenous credit constraints is key. Without growth differentials, or with symmetric constraints across countries, the world interest rate would not permanently decline—critical for the saving divergence. Moreover in the transition, tighter credit constraints in Asia serve to limit the impact of the positive wealth effect caused by fast productivity growth for young consumers.

In Section 3, we dissect household survey data to provide new micro-evidence on saving behavior by age groups. We select two exemplary economies that arguably occupy opposite positions in the spectrum of credit constraint tightness, and are also the two most important contributors to global imbalances—the U.S. and China. The empirical challenge is to accurately measure age-saving profiles in the presence of potentially large biases inherent to household surveys in both countries—distinct problems yet equally taxing. The U.S. consumption survey data suffer from significant underreporting biases that can, in addition, be time-varying (Slesnick (1992)). The Chinese household survey suffers from limited data availability at the individual level. A common practice to circumvent this problem is to use the

\(^2\)In our baseline model, the income effect dominates if the elasticity of intertemporal substitution is smaller than one, as usually assumed and in line with most of the empirical evidence (see Campbell (2003)).

\(^3\)What matters for the long-run dynamics of the world interest rate is that Emerging Asia’s autarkic steady-state interest rate is lower. Note however that if Asia is capital scarce initially, its interest rate can be higher than that of Advanced economies at the time of opening.
age of the household head in constructing age-saving profiles. We demonstrate that two biases ensue in the presence of multi-generational households, which are prevalent in China: a selection bias which tends to overestimate the saving rate of the young and its change over time, and an aggregation bias which tends to underestimate those of the middle-aged (the Deaton and Paxson (2000) critique). We attempt to remove these biases to the best of our efforts and estimate age-saving behavior for both economies over two decades. The corrected age-saving profiles generally conform better with standard lifecycle hypotheses and lend broad support to the qualitative implications of our theory.

Equipped with both macroeconomic and microeconomic facts, we assess in Section 4 the quantitative relevance of our model. We turn to an extended, quantitative version of the model, calibrated to the experiences of the U.S. and China over the period 1968-2008, incorporating in particular the evolution of demographics and income profiles in both countries. We find that the model can explain more than 40 percent of the divergence in aggregate saving rates between the U.S. and China, and a significant portion of the evolution in the shape of the age-saving profile in both economies. The model however falls short of explaining the very large increase in household savings in China, especially for the elderly, pointing to the need for complementary mechanisms, potentially country-specific. Regarding current account imbalances, the model captures well the dynamics observed in the data, with China experiencing a small current account deficit at the time of opening, before building up a large current account surplus. Finally, the model predicts a significant drop in the world interest rate.

While the cross-country asymmetry in credit constraints is essential and the key driver of our results, our analysis indicates that the sharp aging of the population in China and differences in income profiles across countries, in their interaction with credit constraints, also contribute to the divergence in saving rates. We find in the data that the age-income profile in China reaches its peak at an earlier age than in the U.S. and falls more steeply in old age, especially in the more recent period. This particular feature reduces the strength of positive wealth effects on middle age consumption implied by faster growth and a falling interest rate—thus contributing to the large increase in the saving rate in China generated by the model
The role of demographics matters insofar as the rapid aging of the Chinese population, mostly a result of the one-child policy, implies an increase in the share of the middle-aged savers—a composition effect which also amplifies the increase in household savings in China.

To the best of our knowledge, combining the macro and micro-level approaches is a distinctive feature of this paper. Past papers on international capital flows between developed and developing economies have usually taken up the former. Among these, theories relying on market imperfections are most closely related to our work (see Gourinchas and Rey (2013) for a recent survey). Frictions that impact savings include asset scarcity in developing countries (Caballero, Farhi, and Gourinchas (2009)), incomplete financial markets and uninsurable risk in these economies (Mendoza, Quadrini and Rios-Rull (2009)), lack of firm’s access to liquidity to finance investment in periods of rapid growth (Benhima and Bacchetta (2011)), and international borrowing constraints (Benigno and Fornaro (2012)). Financial frictions on investment are analyzed in Song, Storesletten and Zilibotti (2011), Buera and Shin (2011), and Benhima (2012). Aguiar and Amador (2011) provide a political economy perspective with contracting frictions, where fast growing countries tend to experience net capital outflows.

There are three distinguishing elements that mark our theory from the aforementioned. The first is the emphasis on growth in emerging economies as a key driver of these aggregate phenomena—as opposed to capital market integration or shocks to financial markets in developing countries that are typically analyzed. The second aspect is the ability of our model to explain the saving rate divergence across countries (a time-series effect)—as opposed to mere differences in levels. Third, we emphasize household saving divergence as the main driver of global imbalances, in contrast with investment-based or corporate-saving-based explanations.

4Gourinchas and Rey (2013) also point out the role of the shape of income profiles in generating differences in savings and autarky interest rates across countries. Note that wealth effects on middle-aged consumers do not operate in the three-period model of Section 2 since agents receive zero labor income in old age.

5See also Carroll and Jeanne (2009), Sandri (2010), and Angeletos and Panouzi (2011).

6Exceptions are Caballero et al. (2009), Buera and Shin (2011), and Benhima and Bacchetta (2011) who also analyze the impact of faster growth in developing countries.

7Song, Storesletten and Zilibotti (2011), Buera and Shin (2011), and Benhima (2012) show that financial frictions can suppress firm investment demand, leading to net capital outflows from developing countries. From an empirical standpoint however, the period of pronounced global imbalances saw an increase in investment to GDP in Asia rather than a fall (Figure 1.4). Sandri (2010) and Benhima and Bacchetta (2011) focus on
There is compelling evidence supporting the view that the saving divergence was the main driver of global imbalances. The U.S. experience over the period 1970-2008, depicted in the top panel of Figure 1.2, shows a strong relationship between household saving and the current account, while there is hardly any relationship between investment and the current account. China echoes this experience (bottom panel). In the cross section, turning to a large group of countries over the period 1998-2007, it is also evident from Figure 1.3 that the dispersion in saving rates accounts for most of the dispersion in the current account. Gourinchas and Jeanne (2011) provide further support to this view, showing that saving wedges, rather than investment wedges, are necessary for the standard neoclassical model to replicate the patterns of international capital flows. This paper offers a theory of saving wedges, focusing specifically on household savings.\(^8\)

Our quantitative findings are also related to previous papers highlighting the role of demographics, combined with lifecycle saving behavior, in explaining international capital flows. These include empirical studies such as Lane and Milesi-Ferretti (2002), and quantitative analyses focusing on OECD countries such as Domeij and Flodén (2006) and Ferrero (2010).

The decline in the household saving rate in the U.S. and its rise in China have, independently, garnered a lot of attention. The particular stance we take in this paper is that global forces shaped these patterns simultaneously. That is not to say that there are no separate, country-specific, reasons why the U.S. saving rate may have declined and China’s saving rate may have risen. As our theory relies on one single global mechanism, unsurprisingly, it falls short of explaining the full divergence of saving rates across countries. We thus view the alternative explanations relevant to each of these economies as complementary to ours in accounting for the full dynamics of savings. Our work is therefore partly related to a series of papers attempting to explain the large decline in the U.S. household saving rate, summarized in Parker (2000) and Guidolin and La Jeunesse (2009),\(^9\) as well as to a large literature tackling corporate savings.

\(^8\)Empirically, Karabarbounis and Nieman (2012) find that corporate savings have risen uniformly in developing and advanced economies. Bayoumi, Tong, and Wei (2011), using firm-level data, show that Chinese firms’ saving rate is not significantly higher than the global average.

\(^9\)The decline in the U.S. saving rate has been attributed to positive wealth effects (Poterba (2000), Juster et al. (2006), Caroll et al. (2011)); financial innovation and relaxation of borrowing constraints (Parker (2000),
the “Chinese saving puzzle” (Modigliani and Cao (2004)), recently surveyed in Yang, Zhang and Zhou (2011), and Yang (2012). In a nutshell, our work provides a micro-founded explanation for the emergence of a ‘global saving glut’ (Bernanke (2005)) that induced a decline in the world interest rate and the subsequent saving divergence.

The paper proceeds as follows. Section 2 develops the theoretical framework and provides some key intuitions and analytical results, along with a numerical experiment illustrating the impact of fast growth and integration of emerging markets on the global economy. Section 3 investigates micro-level evidence on saving behavior by age groups in China and the U.S.. Section 4 examines the quantitative performance of a fully-calibrated model for these two economies. Section 5 concludes.

2 Theory

The world economy consists of large open economies, populated with overlapping generations of consumers who live for three periods. We let $\gamma \in \{y, m, o\}$ denote a generation. Consumers supply one unit of labor when young ($\gamma = y$) and when in middle age ($\gamma = m$), and retire when old ($\gamma = o$). In youth, consumers are credit-constrained, but the severity of that constraint differs across countries. In all other aspects our framework is standard. All countries use the same technology to produce one homogeneous good, which is used for consumption and investment, and is traded freely and costlessly. Preferences and production technologies have the same structure and parameter values across countries. Technologies only differ to the extent that labor input in each country consists of only domestic labor, and firms are subject to changes in country-specific productivity levels and labor force.

Boz and Mendoza (2012), and Ferrero (2012)); changes in social security and redistribution schemes (Gokhale, Kotlikoff and Sabelhaus (1996), Huggett and Ventura (2000)).

Some compelling explanations emphasize the role of precautionary savings (Blanchard and Giavazzi (2005), Chamon, Liu and Prasad (2010), and Chamon and Prasad (2010)); structural demographic changes (Curtis, Lugauer and Mark (2011), Ge, Yang and Zhang (2012), and Choukhmane, Coeurdacier and Jing (2013)); changes in life-income profiles and pension reforms (Song and Yang (2010), Guo and Perri (2012)); gender imbalances and competition in the marriage market (Wei and Zhang (2009)).
2.1 Production

Let $K^i_t$ denote the aggregate capital stock at the beginning of period $t$ in country $i$, and $e^i_t L^i_{y,t} + L^i_{m,t}$ the total labor input employed in period $t$, where $L^i_{y,t}$ denotes the size of generation $\gamma$ and $e^i_t$ the relative productivity of young workers ($e^i_t < 1$). The gross output in country $i$ is

$$Y^i_t = (K^i_t)^\alpha [A^i_t (e^i_t L^i_{y,t} + L^i_{m,t})]^{1-\alpha},$$

where $0 < \alpha < 1$, and $A^i_t$ is country-specific productivity. The capital stock in country $i$ depreciates at rate $\delta$ and is augmented by investment goods, $I^i_t$, with law of motion

$$K^i_{t+1} = (1 - \delta) K^i_t + I^i_t.$$  

Factor markets are competitive so that each factor, capital and labor, earns its marginal product. Thus, the wage rates per unit of labor in youth and middle age for country $i$ are

$$w^i_{y,t} = e^i_t (1 - \alpha) A^i_t (k^i_t)^\alpha, \quad w^i_{m,t} = (1 - \alpha) A^i_t (k^i_t)^\alpha,$$

where $k^i_t \equiv K^i_t/[A^i_t(e^i_t L^i_{y,t} + L^i_{m,t})]$ denotes the capital-effective-labor ratio. The rental rate earned by capital in production equals the marginal product of capital, $r^i_{K,t} = \alpha (k^i_t)^{\alpha-1}$. The gross rate of return earned between period $t-1$ and $t$ in country $i$ is therefore $R^i_t = 1 - \delta + r^i_{K,t}$.

We let $g^i_{A,t}$ and $g^i_{L,t}$ denote the growth rate of productivity and of the size of consecutive cohorts, respectively, so that $A^i_t = (1 + g^i_{A,t}) A^i_{t-1}$ and $L^i_{y,t} = (1 + g^i_{L,t}) L^i_{y,t-1}$.

2.2 Households

A consumer born in period $t$ earns the competitive wage rate $w^i_{y,t}$ when young and $w^i_{m,t+1}$ in the following period. Let $c^i_{\gamma,t}$ denote the consumption of an agent in country $i$ belonging to generation $\gamma$. The lifetime utility of a consumer born in period $t$ in country $i$ is

$$U^i_t = u(c^i_{y,t}) + \beta u(c^i_{m,t+1}) + \beta^2 u(c^i_{o,t+2}),$$

where $u$ is the utility function.
with standard isoelastic preferences \( u(c) = (c^{1 - \frac{1}{\sigma}} - 1)/(1 - \frac{1}{\sigma}) \). The discount factor \( \beta \) satisfies \( 0 < \beta < 1 \) and the intertemporal elasticity of substitution coefficient satisfies \( \sigma \leq 1 \).\(^{11}\)

Let \( a_{\gamma,t+1}^i \) denote the net asset holdings at the end of period \( t \) of an agent belonging to generation \( \gamma \). An agent born in period \( t \) faces the following sequence of budget constraints:

\[
\begin{align*}
    c_{y,t}^i + a_{y,t+1}^i &= w_{y,t}^i, \quad (5) \\
    c_{m,t+1}^i + a_{m,t+2}^i &= w_{m,t+1}^i + R_{t+1}^i a_{y,t+1}^i, \quad (6) \\
    c_{o,t+2}^i &= R_{t+2}^i a_{m,t+2}^i. \quad (7)
\end{align*}
\]

When young, individuals can borrow in order to consume \( (a_{y,t+1}^i < 0) \). When middle-aged, they earn the competitive wage, repay their loans, consume and save for retirement. When old, they consume all resources available. A bequest motive is omitted for convenience but is introduced later in the quantitative analysis (Section 4).

We assume that young agents are subject to credit constraints: they can only borrow up to a fraction \( \theta^i \) of the present value of their future labor income,

\[
a_{y,t+1}^i \geq -\theta^i \frac{w_{m,t+1}^i}{R_{t+1}^i}. \quad (8)
\]

The tightness of credit conditions, captured by \( \theta^i \), can differ across countries. We are interested in the case in which (8) is binding for all countries.

**Assumption 1** *Credit constraints for the young are binding at all times in all countries.*

This assumption is satisfied if two conditions hold: (1) \( \theta^i \) is small enough—smaller than the fraction of intertemporal wealth that the young would consume in the absence of credit constraints; (2) the wage profile is steep enough—and steeper the higher the \( \theta^i \).\(^{12}\) When credit

\(^{11}\)Our analytical expressions are still valid when \( \sigma > 1 \), but some of our mechanisms rely on a sufficiently low e.i.s. coefficient. Most of the empirical literature since the seminal paper of Hall (1988) finds estimates of the elasticity of intertemporal substitution below 0.5 (see Ogaki and Reinhart (1998), Vissing-Jørgensen (2002), and Yogo (2004) among others). The macro and asset pricing literature (discussed in Guvenen (2006)) typically assumes higher values between 0.5 and 1.

\(^{12}\)The conditions are \( \theta^i < \eta_t^s \) and \( \frac{w_{m,t+1}^i}{R_{t+1}^i w_{y,t}^i} > \frac{1-\eta_t^s}{\beta^i} \), for all \( t \), where \( \eta_t^s \equiv \frac{\beta^{-2\sigma}(R_{t+1}^i R_{t+2}^i)^{1-\sigma}}{1+\beta^{-\sigma}(R_{t+2}^i)^{1-\sigma}}[1+\beta^{-\sigma}(R_{t+1}^i)^{1-\sigma}]^{-\sigma} \). In
constraints are binding, the net asset position of the young is

\[ a_{y,t+1}^i = -\theta^i \frac{w_{m,t+1}^i}{R_{t+1}^i}. \]  

(9)

The net asset position of a middle-aged agent at the end of period \( t \) is obtained from the Euler condition that links \( c_{m,t}^i \) and \( c_{o,t+1}^i \), yielding

\[ a_{m,t+1}^i = \frac{1}{1 + \beta^i (R_{t+1}^i)^{1-\sigma}} (1 - \theta^i) w_{m,t}^i. \]  

(10)

Changes in \( R_{t+1}^i \) affects middle-aged asset holdings through a substitution and income effect, the latter dominating when \( \sigma < 1 \).

### 2.3 Autarky Equilibrium

Under financial autarky, market clearing requires that the total capital stock accumulated at the end of period \( t \) is equal to aggregate country wealth:

\[ K_{t+1}^i = L_{y,t}^i a_{y,t+1}^i + L_{m,t}^i a_{m,t+1}^i. \]  

(11)

Along with (9) and (10), this gives the law of motion for \( k_i \), the capital-effective-labor ratio in country \( i \). In the full depreciation case \( (\delta = 1) \), the dynamic of \( k_i \) is given implicitly by\(^{13}\)

\[ (1 + g_{A,t+1}^i) (1 + g_{L,t}^i) \left[ 1 + e_{t+1}^i (1 + g_{L,t+1}^i) + \theta^i \frac{1 - \alpha}{\alpha} \right] k_{t+1}^i = \frac{(1 - \theta^i)(1 - \alpha)}{1 + \beta^i} \left\{ \alpha (k_{t+1}^i)^{\alpha-1} \right\}^{1-\sigma} (k_i)^\alpha. \]

Figure 2.1 depicts the autarkic law of motion for capital for two different values of the credit constraint parameter, \( \theta_L \) and \( \theta_H > \theta_L \). We can now characterize the impact of \( \theta^i \) on the steady state of the economy. To zero in on the effect of differences in credit constraints, we assume the case of log utility, these conditions amount to \( \theta^i < (1 + \beta + \beta^2)^{-1} \), and \( \frac{w_{m,t+1}^i}{R_{t+1}^i} w_{y,t}^i > \frac{\beta(1 + \beta)}{1 - \theta^i (1 + \beta + \beta^2)} \). Note that Assumption 1 is made for analytical convenience but our mechanism goes through as long as the credit constraint is binding in the more constrained economies.

\(^{13}\)Most of our theoretical results are derived for \( \delta = 1 \), but they hold more generally. The numerical illustration in Section 2.6 assumes \( \delta < 1 \).
Figure 2.1: Law of Motion and Steady State: Autarky and Integration.
Parameter values are \( \sigma = 0.5, \beta = 0.97 \) (annual), \( \alpha = 0.28, \delta = 10\% \) (annual), \( \theta_H = 0.2, \theta_L = 0.02, g_A = 1.5\% \) (annual), \( g_L = 1\%, e = 0.33 \). A period lasts 20 years.

constant and identical productivity and labor force growth rates \( g_A \) and \( g_L \) across countries, and a fixed relative productivity of young workers \( e \).

**Theorem 1** Suppose that \( \delta = 1 \). There exists a unique, stable, autarky steady state. All else equal, more constrained economies have a higher capital-to-efficient-labor ratio \( (dk^i/d\theta^i < 0) \) and a lower interest rate \( (dR^i/d\theta^i > 0) \).

The proof of Theorem 1 and all other proofs are relegated to Appendix A. More constrained economies accumulate more capital as a result of less dissaving of the young and lower debt repayment of the middle-aged, and hence feature a lower rate of return in the long run. In the case \( \sigma = 1 \), the autarky steady-state interest rate in country \( i \) is

\[
R^i = (1 + g_A)(1 + g_L)\frac{1 + \beta \alpha[1 + e(1 + g_L)] + \theta^i(1 - \alpha)}{(1 - \alpha)(1 - \theta^i)}.
\]

(12)

This expression shows that the rate of return is also increasing in productivity and labor growth rates, \( g_A \) and \( g_L \), and in the relative efficiency of young workers \( e \)—all of which raise the marginal productivity of capital.\(^{14}\) Demographics matter not only through its impact on

\(^{14}\)We analyze the impact of productivity growth differentials on the transition in our numerical experiment. Effects related to cross-country differences in demographics and income profiles are discussed in Section 4.
labor force growth, but also on the population composition: a higher proportion of young agents relative to middle-aged agents due to high $g_L$ increases the proportion of borrowers relative to savers and hence puts upward pressure on the rate of return to capital.

### 2.4 Integrated Equilibrium

Under financial integration, capital flows across borders until rates of return are equalized across countries. Financial integration in period $t$ implies that $R_{i,t+1} = R_{t+1}$ and $k_{i,t+1} = k_{t+1}$, for all $i$. The capital market equilibrium condition becomes

$$
\sum_i K_{i,t+1} = \sum_i \left( L_{y,ia_{y,t+1}} + L_{ma_{m,t+1}} \right),
$$

which, along with (9) and (10), gives the law of motion for $k_t$. Next, we characterize the integrated steady state where the growth rates of productivity and labor, as well as the relative efficiency of young workers, are identical across countries.

**Proposition 1** Suppose that $\delta = 1$. Let $\theta_L \equiv \min_i \{ \theta^i \}, \theta_H \equiv \max_i \{ \theta^i \}$, with $\theta_L \neq \theta_H$. The steady state world interest rate $R$ satisfies

$$
R(\theta_L) < R < R(\theta_H),
$$

where $R(\theta)$ denotes the autarky steady state interest rate for credit constraint parameter $\theta$.

Proposition 1 points to the first factor that can cause a fall in the rate of return faced by less constrained economies: financial integration with more constrained ones. Figure 2.1 illustrates this effect in a two-country case, assuming that the less constrained country starts at its autarkic steady state $k(\theta_H)$ whereas the more constrained one is initially capital scarce—so that the two economies have identical capital-effective-labor ratios at the time of opening.\footnote{This assumption is made for the ease of graphical representation. One way to think about it is that the more constrained economy experiences an episode of fast productivity growth before integration, which drives its capital-effective-labor ratio down at the time of opening.}

Upon integration, the transition path of capital is determined by the integrated law of motion,
which lies in between the autarkic ones. Effectively, the world economy behaves like a closed-economy with credit constraint parameter \( \bar{\theta} \equiv \sum_i \lambda^i \theta^i \), where \( \lambda^i \) denotes the relative size of country \( i \) measured by its share in world effective labor

\[
\lambda^i \equiv \frac{A_{i,t}(eL^i_{y,t} + L^i_{m,t})}{\sum_j A_{j,t}(eL^j_{y,t} + L^j_{m,t})}, \tag{15}
\]

Along the convergence to the integrated steady state \( k(\bar{\theta}) \) depicted in Figure 2.1, the world interest rate experiences a sustained decline.

The second factor that can lead to such decline is faster growth in more constrained economies. Indeed in the long run, the world interest rate is determined (up to a monotonously increasing transformation) as a weighted average of the autarky steady-state interest rates of all countries, with weight on country \( i \) increasing in \( \lambda^i \).\(^{16}\) Hence as the more constrained economies grow faster and account for a greater share of the world economy over time, the world interest rate falls.

**Proposition 2** A relative expansion of the more constrained economies (i.e., an increase in the share \( \lambda^i \) of a country with low \( \theta^i \)) causes a fall in the world interest rate. A relative expansion of less constrained economies has the opposite effect.

### 2.5 Saving and Investment

We now show that asymmetric credit constraints lead to heterogeneous responses of saving rates to a fall in the world interest rate across countries, both at the aggregate level and for each generation.\(^ {17}\) In the integrated steady state, the aggregate net saving to GDP ratio of country \( i \) is

\[
\frac{S^i}{Y^i} = -\frac{g}{1 + e(1 + g_L)} (1 - \alpha) \frac{\theta^i}{R} + \frac{g}{1 + g} \frac{1}{1 + e(1 + g_L)} (1 - \alpha) \frac{1}{1 + \beta^{-\sigma} R^{1-\sigma}}, \tag{16}
\]

\(^{16}\)This statement follows directly from the proof of Proposition 1. In the special case where \( \sigma = 1 \), an alternative representation of the long-run world interest rate is given by Equation (12), substituting the world average credit constraint parameter \( \bar{\theta} \) in place of \( \theta^i \).

\(^{17}\)Formal definitions of savings, at the aggregate level and for each generation, are given in Appendix B.
where $R$ is at its steady-state value, and $g \equiv (1 + g_A)(1 + g_L) - 1 > 0$. Equation (16) shows that more constrained economies (lower $\theta^i$) place a greater weight on the middle-aged savers and less weight on young borrowers, resulting in a higher saving rate. Moreover, it implies that in response to a fall in the world interest rate $R$, the saving rate increases by more in the more constrained economy, $\frac{\partial^2(S/Y)}{\partial \theta \partial R} > 0$. Combined with differences in levels, these slope differences imply that a fall in $R$ induces a divergence in saving rates across countries.

**Proposition 3** In an integrated global economy with heterogeneous credit constraints, a fall in the world rate of return induces a greater dispersion in saving rates across countries.

Away from the steady state, one can analyze the response of savings to a change in interest rate by first examining separately the response of each generation’s saving rate (expressed below as a share of GDP so that they add up to the aggregate net saving rate). We show in Appendix B that for $\delta = 1$,

$$
\frac{S_{y,t}^i}{Y_{t}^i} = -(1 + g_{A,t+1}^i) \frac{1 + g_{L,t}^i}{1 + c_{t}^i(1 + g_{L,t}^i)} (1 - \alpha) \alpha^{\frac{\sigma}{1 - \alpha}} \theta_t^i \left( \frac{1}{R_{t+1}} \right)^{\frac{1}{1 - \alpha}} ,
$$

$$
\frac{S_{m,t}^i}{Y_{t}^i} = \frac{1}{1 + c_{t}^i(1 + g_{L,t}^i)} \left[ \frac{1 - \theta_t^i}{1 + \beta^{-\sigma} R_{t+1}^{1 - \sigma}} + \frac{\theta_t^i}{R_{t}} \right] (1 - \alpha) ,
$$

$$
\frac{S_{o,t}^i}{Y_{t}^i} = \frac{1}{1 + g_{A,t}^i} \frac{1}{1 + c_{t}^i(1 + g_{L,t}^i)} \frac{1}{1 + \beta^{-\sigma} R_{t}^{1 - \sigma}} (1 - \theta_t^i)(1 - \alpha) \left( \frac{k_{t-1}}{k_{t}} \right)^{\alpha} .
$$

These equations demonstrate that the partial effect of a fall in the interest rate $R_{t+1}$ is more borrowing by the young—the combined effect of a lower discount rate and higher future wage—and also more saving by the middle-aged if $\sigma < 1$. The strength of these effects, however, varies across countries. Specifically, the increase in borrowing by the young is larger in the less constrained economy (high $\theta^i$), while the increase in saving of the middle-aged is larger in the more constrained economy (low $\theta^i$). The net response of the aggregate saving rate again depends on $\theta^i$: higher $\theta^i$ gives more importance to the young borrowers’ larger dissavings, whereas lower $\theta^i$ gives more importance to the middle-aged’s rising saving.

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\textsuperscript{18}Normalizing by each generation’s factor income yields similar expressions, up to some multiplicative terms common across countries.
We also note that the presence of credit constraints moderates the negative impact of future growth $g_{A,t+1}$ on the saving rate: the dissavings of the young can only increase up to the extent permitted by the binding credit constraints. Thus the standard wealth effect of growth on saving is mitigated when growth is experienced by a country with tight credit constraints. Moreover, in the absence of a wage income for the elderly, the wealth effect of growth does not operate on middle-aged consumers. More generally, the strength of this effect is reduced when the income profile falls in old age.

Investment is governed by the same forces that underlie the neoclassical growth model. Under financial integration, differences in investment-output ratios across countries are largely determined by their relative growth prospects. With full depreciation ($\delta = 1$), investment to GDP ratios obey

$$\frac{I_i^t / Y_i^t}{I_j^t / Y_j^t} = \frac{1 + \tilde{g}_i^{t+1}}{1 + \tilde{g}_j^{t+1}},$$

where $1 + \tilde{g}_i^{t+1} \equiv (1 + g_{A,t+1}^{i})\frac{(1+e_{i,0}^{t+1}(1+g_{L,t+1}^{i}))}{e_{i,0}^{t+1}(1+g_{L,t}^{i})+1}$ denotes the combined growth rate in productivity and effective labor input in country $i$.

### 2.6 Numerical Illustration

We now conduct a numerical experiment to illustrate how financial integration of emerging markets and their faster growth impinge on the world economy in our framework. Each period lasts 20 years. We consider two economies, $H$ and $L$, with credit constraint parameters $\theta_H > \theta_L$. These represent Advanced economies and Emerging Asia, respectively. Both regions are in autarky in period $t = -1$, and financial opening occurs in period $t = 0$ (corresponding to 1970 and 1990, respectively). The Advanced economy starts at its own steady state in period $t = -1$, while Emerging Asia is capital-scarce.\footnote{Another experiment focusing on the impact of growth differentials, where economies are open throughout and start from their initial integrated steady state, is omitted for the sake of brevity but available upon request. The main findings are qualitatively similar.} Labor grows at the same constant growth rate of 1% (annually) in both regions. Productivity in Advanced economies grows at the steady-state growth rate of 1.5% throughout, while productivity in Emerging Asia
grows faster at 3% per year between $t = -1$ and $t = 1$ (i.e., the 1970-2010 period). The productivity growth path of Asia, along with the initial relative values of effective labor and capital-effective-labor ratios, $k^{-1}_L/k^{-1}_H$, are chosen to match Asia’s relative output share in 1970 and 2010, as well as the relative capital-effective-labor ratios of the two regions in 1990, as measured by Hall and Jones (1999). All growth rates are perfectly anticipated.

Preference and technology parameters are standard. The intertemporal elasticity of substitution is taken to be $\sigma = 0.5$, and the discount factor $\beta = 0.97$ on an annual basis. The depreciation rate is set at 10 percent per year. The capital share $\alpha$ is set at 0.28 and the relative productivity of young workers is fixed at $e = 0.33$. For illustrative purposes, we set $\theta_H = 0.20$ and $\theta_L = 0.02$. At this stage, we focus on the qualitative implications of the experiment. A quantitative evaluation of a richer version of the model is taken up in Section 4, where all parameters are calibrated to aggregate and micro data.

Figure 2.2 displays the results. Since Asia is capital-scarce initially, it features a higher rate of return than Advanced economies in period $t = -1$. Rates of return across the two regions are equalized when capital markets integration occurs in period $t = 0$. The rapid decline of Asia’s (shadow) autarky interest rate, combined with its increasing weight in the economy, leads the world interest rate to decline from the very outset. The saving rates across regions diverge between $t = 0$ and $t = 1$ (i.e., 1990-2010), consistent with the data. On the micro level, the rise in saving rate in Asia is mostly driven by the middle-aged while the fall in the Advanced economy is driven by the young. Finally, due to a spike in investment at the time of integration, Asia temporarily runs a small current account deficit at opening, before running a current account surplus of more than 3 percent of GDP in the subsequent

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20The parameter $\alpha$ is matched to the share of labor income in the U.S.. The relative productivity of young workers is chosen to match U.S. age-income profile data. See Section 4 for more details.

21Three factors determine the dynamics of interest rates. The first two factors pin down the paths of interest rates that would prevail if both regions remained in autarky throughout. The ‘growth effect’ tends to raise the interest rate in Asia due to higher marginal productivity of capital, while the ‘convergence effect’ tends to lower the interest rate in Asia as it rapidly accumulates capital from a capital-scarce starting point. After the opening of capital markets, the ‘integration effect’ determines the world interest rate according to the relative size of each economy. Here the convergence and integration effects dominate the Asian growth effect.

22The reason the young see a slight rise in saving rate in Emerging Asia, despite the fall in the world interest rate, is that growth halts after period 1, causing them to borrow less than in earlier periods where they anticipate rapid income growth. The convergence in aggregate saving rates in later periods is mostly driven by the behavior of the elderly.
Figure 2.2: Fast Growth in Asia and Integration: Qualitative Implications.

period. The dynamics of the current account in Asia resembles the one observed in the data, with small deficits in the early 1990’s and large surpluses in the 2000’s.

Comparisons with Alternative Models. In the absence of credit constraints, the aggregate saving rate would fall in the fast-growing economy as the young borrow more against their higher future income. Investment would rise and the country would run a large current account deficit. The fall in the world interest rate would be reduced due to a stronger growth effect, and the interest rate would not experience a prolonged decline in the long run. A simulation of the model where Emerging Asia has the same degree of credit constraints as Advanced economies generates saving and investment dynamics that are qualitatively similar.

23The interest rate could even rise temporarily if the growth effect dominates the convergence effect.
to those in a model without constraints. The saving rate of the fast growing economy falls at time of integration (the opposite for the other country) and then saving rates tend to converge across economies as agents respond similarly to changes in the interest rate in all countries. Thus, both the presence of credit constraints and their heterogeneity are vital for our results. Finally, the shape of the age-income profile, typical of an OLG model, is also important for the savings divergence. Credit constraints are binding for the young because they start with a lower labor income. Moreover, the positive wealth effect of growth and falling interest rates on middle-aged consumers is strongly mitigated when their income in old age is low. A flatter age-income profile would bring the model closer to a standard representative agent model without constraints.

3 Micro Evidence on Savings by Age Groups

Motivated by the predictions of our theory at the micro level, we now provide direct evidence on savings by age groups in Advanced economies and emerging Asia and their evolution over the last two decades. Because of limited data availability, we focus on two exemplary countries — the U.S. and China. These two economies are the most important contributors to global imbalances, and arguably occupy opposite positions in the spectrum of household credit constraint tightness. A number of complex issues arise when using household survey data to construct age-saving profiles. This section describes our careful treatment of these issues and the way we attempt to deal with potential biases. We use our micro findings later to calibrate our quantitative model and evaluate its performance. Readers only interested in the quantitative implications can proceed directly to Section 4.

3.1 Evidence for the U.S.

The Consumer Expenditure Survey (CEX) provides the most comprehensive data on disaggregated consumption, and is therefore our primary data source for the U.S.. Annual data from 1986 to 2008 are available for six age groups: under 25, 25-34, 35-44, 45-54, 55-64, and
above 65. Details of the data are provided in Appendix C.2.

**Underreporting Biases.** The main issue involved in using CEX data is their sharp discrepancy with the National Income and Product Account (NIPA) data. This discrepancy is well-documented in Slesnick (1992), Laitner and Silverman (2005), Heathcote, Perri and Violante (2010), and Aguiar and Hurst (2013), and arises from underreporting of both consumption and income in the CEX data. The degree of underreporting has become more severe over time for consumption but not for income, the consequence of which is a stark rise in the aggregate saving rate as computed from CEX data, compared to an actual decline as measured in NIPA data (Figure 3.1). Some important corrections of the CEX are therefore needed to estimate reasonable age-saving profiles for the U.S..

![Figure 3.1: U.S. Aggregate Saving Rate: NIPA vs. Unadjusted CEX.](image)

*Notes: CEX and BEA for the NIPA rate.*

**Correction Method.** Inspired by previous works (Parker et al. (2009) among others), we assume that NIPA data are well measured, and propose a correction method to bring consistency between CEX and NIPA data. Our correction method adjusts income uniformly across all age groups so as to match NIPA data. On the consumption side, we take into account the fact that the degree of underreporting may vary across goods, which becomes a concern if the composition of the consumption basket differs across age groups (see Aguiar and Hurst (2013) for recent evidence). While allowing the degree of underreporting in CEX to vary
over time and across consumption goods, our correction method relies on the identification assumption that it is constant across age groups.

In practice, to correct for underreporting in consumption, we use CEX and NIPA data on aggregate consumption for 15 sectors to construct time-varying, sector-specific adjustment factors $\chi_{kt} = C_{kt}^{NIPA}/C_{kt}^{CEX}$, where $C_{kt}^{D}$ denotes aggregate consumption of good $k$ in dataset $D$.\(^{24}\)

For all sectors, $\chi_{kt}$ is greater than 1, and rises over time as the underreporting bias in CEX consumption becomes more severe. We use the sector-specific factors to adjust CEX sectoral consumption data by age: given $c_{jkt}^{CEX}$ the average consumption of goods of sector $k$ by individuals of age $j$ as reported in CEX, we define $\hat{c}_{jkt} = \chi_{kt}c_{jkt}^{CEX}$. The adjusted consumption expenditure for age $j$ is then obtained as $\hat{c}_{j,t} = \sum_k \hat{c}_{jkt}^{CEX}$.\(^{25}\)

Similarly, our adjusted measure of income for age $j$ is $\hat{y}_{j,t} = \sum_k \chi_{kt}^{health}y_{jkt}^{CEX}$, where $y_{jkt}^{CEX}$ denotes the average income reported in CEX for age $j$ in year $t$, and $Y_{t}^{D}$ the aggregate income in dataset $D$. By construction, our corrected consumption and income measures match NIPA in the aggregate.\(^{26}\)

Finally, our estimated saving rate for age $j$ in period $t$ is $\hat{s}_{j,t} = (\hat{y}_{j,t} - \hat{c}_{j,t})/\hat{y}_{j,t}$.

**Corrected U.S. Age-Saving Profiles.** Figure 3.2 displays the estimated saving rates by age groups for the years 1988 and 2008 using our correction method. Age-saving profiles are in line with the lifecycle theory, and their shapes show some interesting evolution. In two decades, the group of young people (under 25) saw a decline of 12.7 percentage points in their saving rate, while those between 35-54 a small increase of about 2.3 percentage points, and the eldest group a large decline of about 19 percentage points.

\(^{24}\)The 15 sectors matched between NIPA and CEX are: Food and alcoholic beverages, Shelter, Utilities and public services, Household expenses, Clothing and apparel, Vehicles purchases, Gas and motor oil, Other vehicle expenses, Public transportation, Health, Entertainment, Education, Tobacco, Miscellaneous and cash contributions, Life/personal insurance.

\(^{25}\)Another issue is that health expenditures are treated differently in NIPA and CEX. Health expenditures in CEX are restricted to ‘out-of-pocket’ expenses, but NIPA also includes health contributions (Medicare and Medicaid), leading to very large adjustment factor $\chi_{health}$. This mostly affects our consumption estimates for the old, for whom ‘out-of-pocket’ health expenditures constitute a large share of their consumption basket in CEX. We address this concern by adjusting sectoral adjustment factors for mis-measurement in health expenditures while still matching NIPA consumption data in the aggregate. See details in Appendix D.1.

\(^{26}\)A small discrepancy remains for consumption since NIPA includes expenditure types (e.g., ‘Net foreign travel and expenditures abroad by U.S. residents’ and ‘Final consumption expenditures of nonprofit institutions serving households’) which cannot be matched with CEX categories.
Figure 3.2: Age-Saving Profile for the U.S. in 1988 (left panel) and 2008 (right panel).

Notes: CEX data, 1988-2008; estimates of saving rates are obtained using CEX adjusted data (sectoral-specific adjustment factors, correcting for health expenditures). Details of the correction techniques are given in Appendix D.1.

3.2 Evidence for China

The main data source for China is the Urban Household Survey (UHS) conducted by the National Bureau of Statistics, available for the year 1986 and annually over the period 1992-2009. We use the sample of urban households which covers 112 prefectures across 9 representative provinces, with an overall coverage of about 5,500 households in the 1992 to 2001 surveys and 16,000 households in the 2002 to 2009 surveys. The UHS data records detailed information on income, consumption expenditures, and demographic characteristics of households. It also provides employment, wages and other characteristics of individuals in the household. Further information about the data can be found in Appendix C.3.

The main issue that arises with UHS data is that, whereas the survey provides detailed individual information on income, consumption is only available at the household level. For this reason, previous studies analyzing age-specific saving behavior in China use household-level data. That is, the saving rate they impute to a certain age is the average household saving rate computed over all households whose head is of this age. Following this approach, Song et al. (2010), Chamon and Prasad (2010), and Chamon, Liu and Prasad (2010) find evidence against standard lifecycle motives of saving in China. In particular, they find that

27The 1986 survey covers a different sample of 12,185 households across 31 provinces.
the traditional hump-shaped age-saving profile is replaced by a U-shaped profile in recent years, with saving rates being highest for the young and close to retirement age, and lowest for the middle-aged. This would run counter to our prediction that the middle-aged savers in China should have contributed the most to the rise in household saving rate in the last two decades. In the next section however, we argue that the ‘household approach’ is subject to measurement error, for which we then attempt to correct.

**Aggregation and Selection Biases.** Deaton and Paxson (2000) have forcefully shown the problems associated with using the household approach to construct age-saving profiles in the presence of multi-generational households. If a large fraction of households comprise members that are at very different lifecycle stages, the age-saving profile obtained from household data will be obscured by an *aggregation bias*. For instance, suppose that middle-aged individuals have a high saving rate as they save for retirement, but middle-aged household heads live with younger adults or elderly members who have much lower saving rates. In this case, the household approach would lead to an under-estimation of the saving rate of the middle-aged. More generally, the aggregation bias tends to flatten the true age-saving profile. A second potential bias arises from the possibility that household headship is not random. If being a head at a certain age is correlated with certain characteristics (such as income) that affect saving behavior, the age-saving profile estimated by the household approach would suffer from a *selection bias*. Moreover, any time-variation in these two biases would affect the estimated change in age-specific saving behavior over time.

<table>
<thead>
<tr>
<th>Table 1: Frequency of Multi-Generational Households in China.</th>
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<td>2 generations</td>
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<td>3 generations</td>
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A multi-generational household is the norm in the case of China, thus making the aggregation bias a serious concern (Table 1). In urban households, more than 50 percent of households are multi-generational (defined as households in which the maximum age difference between two adults is above 18 years), and roughly one out of six includes three different
generations. Multi-generational households are observed when young adults (typically in their twenties) stay in their parents’ household or when older individuals (typically in their seventies) live with their children. A closer look at the data shows that, towards the end of the sample period, young adults tend to stay longer with their parents, while the elderly tend to join their children’s household at a later age as a result of an increase in life expectancy (see Appendix D.2 for further details). These evolutions are likely to introduce some bias in the estimates of changes in age-specific saving rates obtained from the household approach.

Figure 3.3 offers suggestive evidence of a potential bias arising from the fact that household heads are not selected randomly. The figure displays the income premium of household heads as a function age, with the average income of heads of a given age expressed as the log ratio of the average income of all individuals of that age. Both young and elderly household heads are significantly richer than their non-household head counterparts. This is no surprise — only the richer individuals can afford to live independently when young or in old age. If high individual income is correlated with high individual saving rate, the household approach would therefore tend to over-estimate the saving rates of the young and of the elderly. The evolution of the income premium over time, apparent in the figure, suggests that the selection bias is likely to be more severe for the elderly in 1992, and more severe for the young in 2009.

Figure 3.3: Income Premium of Household Heads in China.
Notes: Income premium of household heads is the log difference between the average income of heads of a given age and the average income of individuals of the same age. Source UHS (1992-2009).

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Any household with one adult or several adults belonging to the same generation, possibly with a child, is considered as uni-generational. Children are defined as individuals aged less than 25 with no income.
**Projection Method.** To improve upon the household approach, the key challenge is to identify individual consumption. Our approach applies a projection method proposed by Chesher (1997, 1998) and Deaton and Paxton (2000) to disaggregate household consumption into individual consumption, from which we estimate new age-saving profiles. Essentially, the idea is to recover the consumption of each individual member of the household using cross-sectional variations in the composition of households as a source of identification. In practice, this is done by projecting household consumption on the number of household members belonging to various age groups, controlling for observable household characteristics. Following Chesher (1997), we conduct a non-linear least squares estimation of the following model for each year:

\[ C_h = \exp(\gamma \cdot Z_h) \left( \sum_{j=19}^{99} c_j N_{h,j} \right) + \epsilon_h, \]

where \( C_h \) is the aggregate consumption of household \( h \), \( N_{h,j} \) is the number of members of age \( j \) in household \( h \), and \( Z_h \) denotes a set of household-specific controls (income group, number of adults, number of children, uni-vs. multi-generational, etc.).\(^{29}\) The estimated consumption of an individual of age \( j \) living in a household with characteristics \( Z_h \) is then equal to \( \exp(\hat{\gamma} \cdot Z_h) \hat{c}_j \). Further details of the method are given in Appendix D.2, along with a number of robustness checks. In particular, we estimated individual age-saving profiles through an alternative method based on the restricted sample of uni-generational households (more than 40% of the entire sample). This alternative approach, which uses a different sample of households and a different identification strategy, yields very similar age-saving profiles. We also applied both methods to estimate individual age-saving profiles from the Chinese Household Income Project (CHIP) survey data, available for the years 1995 and 2002, finding consistent results across methods and across surveys.

**Estimated Age-Saving Profiles for China.** Figure 3.4 exhibits the estimated age-saving profiles, at the beginning and at the end of the sample period.\(^{30}\) In Appendix D.2, we show

\(^{29}\)This assumes that individual consumption can be written as multiplicatively separable functions of individual age and household characteristics. The identification therefore relies on the restriction that the effect of household characteristics on individual consumption is independent of age.

\(^{30}\)By symmetry with the U.S., we show the Chinese age-saving profile for the year 2008, but estimates from
that our estimates differ substantially from the ones produced by the household approach based on the age of the household head. Echoing the results of Deaton and Paxton (2002) for Taiwan and Thailand, we find that the age-saving profiles computed by the individual approach are more in accord with the lifecycle theory of saving. In particular, the young do save less than the middle-aged, especially so in the most recent period. Over time, we observe a large increase in the saving rate of the middle-aged, between 15-20 percentage points, while the saving rate of the youngest declines slightly. The striking increase in the saving rate of the elderly (> 65) is quite peculiar and seems at odds with standard lifecycle motives.

Figure 3.4: Estimated Age-Saving Profiles for China in 1992 (left panel, also showing the average for 1986, 1992 and 1993) and 2008 (right panel), Individual Method. 
Notes: UHS data. Saving rates are estimated using a projection method to identify individual consumption (Chesher (1997)), controlling for household characteristics as described in Appendix D.2.

### 3.3 Summary of Micro Evidence

Our baseline three-period model predicts that in the face of a fall in the world interest rate (caused by capital markets integration and fast growth in Asia), (i) the saving rate of the young falls by more in developed countries than in emerging markets; (ii) the saving rate of the middle-aged increases by more in emerging markets than in developed countries. As a result, age-saving profiles across countries become more distant from each other over time. In the data, the saving rate of the young did fall by about 10 percentage points more in the U.S. the 2009 survey are very similar. For the beginning of the sample period, due to the lack of observations in 1988, we show the estimated profile for 1992 along with averages over the first three years of observations to minimize issues related to the smaller size of our sample in early years.
than in China, whereas the saving rate of the middle-aged (35-54) in China rose by about 15 percentage points more in China than in the U.S.. Within countries, the increase in the saving rate of the middle-aged together with the fall in the saving rate of the young make the profiles more hump-shaped, both in theory and in the data. Overall, apart from the large increase in the saving rate of the elderly in China, our empirical findings appear to support the qualitative predictions of our theory.

4 Quantitative Analysis

Equipped with facts on the macro and micro level, we now assess the ability of the model to match the evolution of saving rates in the U.S. and China over the period 1988-2008, both on the aggregate and by age groups. The quantitative model enriches the baseline model of Section 2 along several dimensions, and is fully calibrated to the experiences of these two economies. First, we increase the number of periods/generations in order to yield more refined micro and aggregate predictions. Having more periods allows us to incorporate the exact shapes of age-income profiles across countries, and their variations over time. Second, we introduce a bequest motive to allow for a savings initiative by the old. The demographic evolution in each country is also calibrated to the data — thus incorporating the aging of population in both countries. Model parameters that are not directly observable are calibrated to micro and macro data for the U.S. and China at the beginning of the sample period.

4.1 A Multi-Period OLG Model with Asymmetric Constraints

A brief description of the quantitative model follows. Unless specified otherwise, the notations are retained from Section 2.

Preferences and Bequests. We consider agents whose economic life runs for $J + 1$ periods. Age is indexed by $j = 0, ..., J$. We let $c_{j,t}^i$ denote the consumption of an agent of age $j$ in period $t$ and country $i$. In order to obtain a more realistic saving behavior for the old, we augment our baseline model with a bequest motive along the lines of Abel (2001). The lifetime
utility of an agent born in period $t$ in country $i$ is

$$U_i^t = \sum_{j=0}^{J} \beta^j u(c_{j,t+j}^i) + \phi \beta^J u(R_{t+J+1}^i b_{t+J}^i),$$  

(18)

where $b_t^i$ denotes the amount of bequest left in period $t$ by an agent born in period $t - J$, and $\phi$ captures the strength of the bequest motive. Agents receive bequests at age $J - \Delta$, $0 < \Delta < J$, and bequests are shared equally among offsprings.\(^{31}\) Thus the amount of bequest $q_{t+j-\Delta}^i$ received by an agent born in period $t$ is related to $b_{t+j-\Delta}^i$ as follows

$$q_{t+j-\Delta}^i = \frac{L_{t+J-\Delta}^i}{L_t^i} b_{t+J-\Delta}^i,$$

(19)

where $L_t^i$ denotes the size of the generation born in period $t$, so that $L_t^i / L_{t-\Delta}^i$ captures the number of children (born in period $t$) per agent born in period $t - \Delta$.

**Production.** The production sector is analogous to the one in the qualitative model. Gross output in country $i$ is

$$Y_t^i = (K_t^i)^\alpha \left[ A_t^i \sum_{j=1}^{J} c_{j,t}^i L_{t-j}^i \right]^{1-\alpha} = A_t^i \tilde{L}_t^i (k_t^i)^\alpha,$$

(20)

where $\tilde{L}_t^i \equiv \sum_{j=0}^{J} c_{j,t}^i L_{t-j}^i$ denotes the total efficiency-weighted population, and $k_t^i = K_t^i / (A_t^i \tilde{L}_t^i)$ denotes the capital-effective-labor ratio. The efficiency weights $\{c_{j,t}^i\}_{j=0}^{J}$ capture the shape of the age-income profile in period $t$ and country $i$. Indeed, the competitive wage received by agent of age $j$ in country $i$ in period $t$ is $w_{j,t}^i = c_{j,t}^i (1 - \alpha) A_t^i (k_t^i)^\alpha$. The gross rate of return between $t - 1$ and $t$ is $R_t^i = 1 - \delta + \alpha (k_t^i)^{\alpha-1}$.

**Credit Constraints.** Consider the intertemporal problem of a consumer born in period $t$ and country $i$. This agent faces a sequence of gross rates of return $\{R_{t+j+1}^i\}_{j=0}^{J}$ and labor income $\{w_{j,t+j}^i\}_{j=0}^{J}$, and receives bequest $q_{t+j-\Delta}^i$. Let $a_{t+j}^i$ denote his end-of-period net asset

\(^{31}\)Under the assumption that (i) agents remain in childhood for $H$ periods before they start their economic life and (ii) that they procreate at age $P$, then $\Delta = H + P$. 

27
holdings at age $j$. Flow budget constraints are

$$
c_{j,t+j} + a_{j,t+j}^i = R_{t+j}^i a_{j-1,t+j-1}^i + w_{j,t}^i + 1_{(j=J-\Delta)} q_{t+J-\Delta}^i,
0 \leq j \leq J-1,
\tag{21}
$$

$$
c_{J,J,t} + b_{J,t}^i = R_{t+J}^i a_{J-1,J-1}^i + w_{J,t,J}^i,
\tag{22}
$$

with $a_{-1,t-1}^i = 0$. Define the discounted present value of current and future labor income

$$
H_{j,t}^i = w_{j,t}^i + \sum_{\tau=1}^{J-j} \frac{w_{j+\tau,t+\tau}^i}{\prod_{s=1}^\tau R_{t+s}^i},
0 \leq j \leq J-1,
\tag{23}
$$

and $H_{J,t}^i = w_{J,t}^i$. The credit constraint faced by the agent at age $j \leq J-1$ is

$$
a_{j,t+j}^i \geq -\theta^i \frac{H_{j+1,t+j+1}^i}{R_{t+j+1}^i}.
\tag{24}
$$

**Equilibrium.** As described in Appendix B, one can solve for the autarkic and integrated steady-states of the model, as well as its transitory dynamics for a given evolution of productivity, demographics and efficiency parameters. In autarky, the model equilibrium is given by a path for the capital-effective-labor ratio $k_t^i$ and bequests $(q_t^i, b_t^i)$ such that: (i) all agents maximize their intertemporal utility (Eq. 18) with respect to their consumption decisions, subject to their sequence of budget constraints (Eqs. 21-22) and credit constraints (Eq. 24); (ii) the consistency condition (Eq. 19) between bequests received and bequests left is satisfied; (iii) the market for capital clears at every date. Under financial integration, a similar definition of an equilibrium holds, with the market for capital clearing globally. When solving for equilibrium, the presence of bequests adds a layer of complexity since the paths of capital and bequests have to be determined together in a dynamic fixed point problem. A detailed description of the numerical solution method is provided in the appendix.
4.2 Calibration

Two economies are considered in the quantitative analysis, the U.S. and China, \( i \in \{US, CH\} \). Each period lasts 10 years and agents live for 6 periods \( (J = 5) \). Empirically, the six age groups map into the following age brackets: under 25, 25-34, 35-44, 45-54, 55-64, and above 65. We simulate the model under a scenario similar to the one analyzed in the experiment of Section 2.6. Specifically, China grows faster over four consecutive periods, from period \(-2\) to period 2 (corresponding to 1968-2008), and integrates with the U.S. in period 0 (i.e., 1988) after two periods of accelerated growth. The calibration methodology is the same as before—albeit more comprehensive, and applied to the U.S. and China rather than Advanced economies and Emerging Asia. Table 2 provides a complete summary of our calibration.

<table>
<thead>
<tr>
<th>Table 2: Calibration Summary.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Age-Income Profile</strong> ( (e^i_{j,t}) )</td>
</tr>
<tr>
<td>U.S.</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>steady state</td>
</tr>
<tr>
<td><strong>Demographic Growth</strong> ( (g^i_{L,t}) )</td>
</tr>
<tr>
<td>U.S.</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td>U.S.</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td><strong>Other Parameters</strong></td>
</tr>
<tr>
<td>Share of capital ( (\alpha) )</td>
</tr>
<tr>
<td>e.i.s. coefficient ( (\sigma) )</td>
</tr>
<tr>
<td>Bequest motive ( (\phi) )</td>
</tr>
</tbody>
</table>

**Demographics.** The age distribution for each country and its evolution over time are obtained from the World Population Prospects data, sampled every decade since 1970 (United Nations, 2010 revision).\(^{32}\) For each country, the demographic growth rate before 1970 and the sequence of growth rates \( g^i_{L,t} \) post-1970 are chosen to best fit the observed age distributions.

\(^{32}\)Data availability limits us to set the demographic structure in 1968 (resp. 1978 up to 2008) to the one measured in the data for the year 1970 (resp. 1980 up to 2010).
Table 3: Demographic Structure in the U.S. and China.
Model-implied demographic structure vs. data from World Population Prospects (United Nations, 2010).

<table>
<thead>
<tr>
<th>Year</th>
<th>U.S.</th>
<th>China</th>
<th>U.S.</th>
<th>China</th>
<th>U.S.</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968</td>
<td>41.6</td>
<td>52.5</td>
<td>39.5</td>
<td>52.1</td>
<td>32.8</td>
<td>38.1</td>
</tr>
<tr>
<td>1988</td>
<td>43.6</td>
<td>53.8</td>
<td>40.5</td>
<td>51.3</td>
<td>38.8</td>
<td>38.6</td>
</tr>
<tr>
<td>2008</td>
<td>32.4</td>
<td>29.8</td>
<td>34.1</td>
<td>31.3</td>
<td>33.2</td>
<td>37.9</td>
</tr>
</tbody>
</table>

Share of young (15-34) (% of population above 15)
Share of middle-aged (35-54) (% of population above 15)
Share of above 55 (% of population above 15)

from 1970 to 2010. The model does not have enough degrees of freedom to perfectly match the data, and we target more specifically the share of the middle age groups (35-44 and 45-54) as they contribute the most to aggregate saving. Nonetheless, our calibration produces a close match to the overall demographic structure, as shown in Table 3. Implied demographic growth rates are reported in Table 2. The main feature of the data is the large fall in population growth in China starting in 1990, largely a result of fertility controls (one-child policy), and the ensuing rapid aging of the population (see Table 3). Post 2008, the population growth rate is assumed to be 1% in both countries.

Age-Income Profiles. The relative efficiency parameters \(e_{i,j,t}^{U}\) are calibrated to the wage income profile (net of taxes) across age groups, as observed in the UHS data for China (starting in 1992) and in the CEX for the U.S. (starting in 1988). Figure 4.1 depicts wage income profiles measured for the initial year of observations in our data and for the year 2008, with the efficiency parameter for age group 45-54 normalized to unity. In the U.S., coefficients are remarkably stable over time. We therefore set U.S. efficiency parameters \(e_{i,j,t}^{US}\) equal to their average values over 1988-2008 in every period. Compared to the U.S., the Chinese

---

33 Our calibration matches fairly well the proportion of middle-aged in the U.S. at the expense of understating the aging of the country over the period 1970-2010, which is largely driven by individuals above 75 whose share has significantly increased. If one considers only individuals between 15 and 75, which is the relevant counterpart to our model, the match between model and data is very accurate.

34 This corresponds to the average population growth rate in the U.S. since 1970. We assume that the one-child policy in China will remain at least partially in place—leading to a low population growth, in line with the most recent years.

35 We only observe a slight flattening of the U.S. age-income profile after age 55 in the recent period. Since this change is quantitatively small, our results are not affected if we take into account time variation in the U.S. income profile. We set it constant in our benchmark calibration to eliminate one possible source of change.
profile reaches its peak earlier (at age 35-44) and falls more steeply in old age. This feature is particularly striking in the more recent period, due to a marked increase in relative wages for the 25-34 and 35-44 age brackets. For periods $t = 0, 1, 2$, we set $e^{CH}_{j,t}$ to the values observed in the data for the years 1992, 1998 and 2008, respectively. Owing to data limitations, relative efficiency parameters in earlier periods are set equal to their values in period 0. Going forward, we assume that relative efficiency parameters in China remain equal to their 2008 values for one period before converging to the steady state level of the U.S. gradually in four periods.

**Initial Conditions and Productivity Growth.** Given the calibrated population growth rates and income profiles, initial relative productivity levels and subsequent productivity growth rates are set to match the output of China relative to the U.S. over the period 1968-2008, and to allow the capital-effective-labor ratio in China to reach about 70% of that of the U.S. in 1988, per Hall and Jones (1999). The resulting annual productivity growth rate for China is 3.6% between 1968-78, and 4.5% between 1978-2008. We assume that U.S. productivity grows at an annual rate of 1.5% throughout, and that China grows at the same rate after 2008. Such differences in productivity growth across countries may seem small compared to observed real GDP growth differentials between the U.S. and China (5% on average over 1978-2008), but a significant part of Chinese growth in our experiment is driven by increasing in age-saving profiles and facilitate the interpretation of our results.
inputs, i.e. labor (increasing share of middle-aged workers, who are the most productive) and
capital, and by increased efficiency of workers in the 25-34 and 35-44 age groups.

Other Calibrated Parameters. We use $\alpha = 0.28$ for the share of labor in value added,
corresponding to the average share of labor income in the U.S. over the period 1988-2008.\textsuperscript{36}
The depreciation rate is set to 10\% on an annual basis. The remaining parameters are the
elasticity of intertemporal substitution $\sigma$, the discount factor $\beta$, the bequest parameter $\phi$,
and the credit constraint parameters $\theta^i$. These parameters are chosen so as to minimize the
discrepancy between empirical and model-implied saving behavior, at both the macro and
micro levels, in the integration period. Specifically, let $s^i_0$ and $s^i_{j,0}$ denote the model-implied
aggregate saving rate and the saving rate of agents of age $j$ in country $i$ in the integration
period, and let $s^{i,d}_0$ and $s^{i,d}_{j,0}$ denote their counterparts in the data. We search over a large grid
the vector of parameter values $\psi \equiv [\sigma, \beta, \phi, \theta^{US}, \theta^{CH}]$ that minimizes the distance

$$\sum_i \left| s^i_0(\psi) - s^{i,d}_0 \right| + \sum_i \sum_{j=0}^J \omega^i_j (s^i_{j,0}(\psi) - s^{i,d}_{j,0})^2,$$

where the weights $\omega^i_j$ on different age groups in country $i$ satisfy $\sum_{j=1}^J \omega^i_j = 1$ and reflect their
shares in the total effective population $\bar{L}^i$.\textsuperscript{37} The optimal parameter values are described in
Table 2. Values of 0.5 for $\sigma$ and 0.955 for $\beta$ are in the range of empirical estimates. Other
parameter values are more difficult to gauge. Credit constraint parameters are found to be
very different across countries ($\theta^{CH} = 0, \theta^{US} = 0.24$). In the data, the household debt-to-GDP
ratio in China is about 1/9 of that in the U.S. in 2007, suggesting indeed very large differences
in household access to credit. Although our calibration procedure yields $\theta^{CH} = 0$, the results
are not very sensitive to the value of $\theta^{CH}$ provided that it is an order of magnitude smaller
than $\theta^{US}$.

\textsuperscript{36}We use OECD Quarterly National Accounts data, correcting for mixed income as in Gollin (2002).
\textsuperscript{37}We adopt absolute deviations for the macro variables instead of squared differences. Otherwise the opti-
mization would only weight micro outcomes since micro discrepancies are on average much larger than macro
ones. The grid of parameter values is described in Appendix B.
4.3 Results

We now present the results for our benchmark calibration. When evaluating the performance of the model, it is important to keep in mind that some key parameters are calibrated to match savings data at the date of integration, without using post-1990 data.

On the aggregate level, the qualitative implications of the three-period model are broadly preserved (Figure 4.2). Quantitatively, the aggregate saving rate increases sharply in China (+6.3 percentage points between $t = 0$ and $t = 2$, i.e., 1988-2008), while falling significantly in the U.S. (−2.9 percentage points over the same period). The model explains more than 40% of the savings divergence observed in the data. At the time of opening, China runs a small current account deficit, due to a growth-driven investment boom, before turning into a large surplus: the current account moves from −2.2% of GDP to +6.5% between 1988 and 2008. In the data, China indeed ran current account deficits in the 1990s (and so did other Asian countries, to an even greater extent) before moving into a surplus. Our model predicts such a pattern as the standard neoclassical forces (capital flowing towards the capital-scarce and fast growing economy) dominate initially when China is still relatively small. But as its relative size in the world economy rapidly increases, the world interest rate drops significantly, and the contrasting responses of saving rates across countries lead to a reversal in current account positions.

Turning to micro-level predictions, Figure 4.3 juxtaposes the model-implied age-saving profiles in 1988 ($t = 0$) and 2008 ($t = 2$) with those estimated from the data. For the U.S., the model matches the increasing spread in the saving rates of the young (under 25) and middle-aged (35-54) observed in the data over two decades. The model nevertheless slightly overpredicts the fall in the young’s saving rate and the rise in the middle-aged saving rate. It also fails to account for the saving rate of the old in 1988. For China, the model provides a reasonably good fit to the relatively flat age-saving profile observed at the beginning of the period. Over the subsequent two decades, the model-implied saving rates for individuals between 25-54 rise substantially, although by less than in the data. Interestingly, the model implies a large increase in saving rates for those of age 25-34 and 35-44 (by about 10 percentage
Figure 4.2: Quantitative Results: Aggregate Dynamics.

Notes: Benchmark calibration displayed in Table 2.

Figure 4.3: Quantitative Results: Age-Saving Profiles.

Notes: Benchmark calibration displayed in Table 2. Details on the construction of empirical profiles are provided in Section 3. Averages over the years 1986, 1992 and 1993 are used for China in 1988.

points), capturing about 50 percent of the particularly pronounced rise observed in the data for these age groups. As for young individuals (under 25), the model predicts a constant saving rate as the credit constraint is binding in every period — in line with the data and
in contrast to the U.S.. As a result, the model captures the increasing discrepancy between the saving rates of the very young and the middle-aged over time. At the other end of the age-saving profile however, the model is unable to explain the large increase in the saving rate of the elderly. To summarize, our model can with one mechanism explain a significant portion of the rise in saving rates for most age groups in China, and the simultaneous increase in borrowing of the young in the U.S.. The shapes of the profiles generated by the model are broadly in line with the data, with the exception of the oldest age group — particularly in China.

4.4 Alternative Calibrations and Sensitivity Analysis

To provide further intuition on the channels driving the dynamics of savings across countries and age groups, we now examine the output of the model under alternative calibrations. In particular, we investigate the role played by the asymmetry of credit constraints and the shape of income profiles. We also assess the quantitative contribution of changes in the Chinese income profile and of fast aging in China. For the sake of brevity, we display graphically only variables that exhibit significant changes relative to our benchmark calibration.

Symmetric Credit Constraints. We first investigate the quantitative role of credit constraint heterogeneity by setting all credit constraint parameters to the U.S. level, while keeping all other parameters constant. Results for the variables of interest are shown in Figure 4.4. Upon integration, the aggregate saving rate falls substantially in China while increasing in the U.S.. China experiences a very large current account deficit (-10.5%) at the time of opening, before turning into a surplus. On the micro level, age-saving profiles for China are also markedly different from our benchmark calibration. In particular, the 1988 profile exhibits a much more pronounced inverted-U shape, given the massive borrowing of the young Chinese households in anticipation of faster growth.38 Simulations of the model in the absence of any credit constraints yield very similar, counterfactual results, thus affirming the importance of both the presence of credit constraints and their being tighter in China for our findings.

38Similarly in 2008, the saving rates of individuals below 45 are significantly below those in our benchmark.
Flat Age-Income Profiles. We next demonstrate the importance of the shape of the income profile. The experiment sets relative efficiency parameters to unity at all ages in both countries, while keeping all other parameters at their benchmark values. As in the previous experiment, fast-growing China sees a large fall in aggregate savings and a massive current account deficit (Figure 4.5). There are two aspects of the shape of the calibrated age-income profiles that matter for our results. The downward-sloping part of the profile gives stronger saving motives to the middle-aged, and at the same time limits the wealth effect of growth. The upward-sloping part of the profile is even more crucial as credit constraints only matter to the extent that younger individuals have a desire to borrow. With flat age-income profiles, credit constraints are not binding in any country in the steady state (despite aggregate productivity growth), thus bringing the model dynamics very close to a frictionless neoclassical representative agent model. More generally, this experiment illustrates the dynamics induced by a growth shock in a model where most agents have a desire to save in the steady state.\textsuperscript{39}
Contribution of Changes in Income Profile in China. As noted in Section 4.2, the age-income profile in China changed significantly over time. Towards the end of the sample period, the income profile reaches a higher peak at a younger age and falls more steeply in old age (see Figure 4.1). By providing further incentives to save for retirement and by reducing wealth effects for middle-aged consumers, this evolution contributes to the rise in Chinese savings. We assess the quantitative importance of this channel by keeping the relative efficiency parameters in China \(e_{j,t}^{CH}\) equal to their initial values until 2018. All other parameters remain at their benchmark values. Figure 4.6 depicts the evolution of macro variables of interest, along with the 2008 Chinese age-savings profile. Compared to our benchmark, aggregate savings and current account surplus in China rise less over the period 1988-2008 (by 1.2% and 60bps, respectively), due to a smaller increase in the middle-aged’s saving rates over the period. As a consequence, the fall in interest rate is reduced.

Contribution of Fast Aging in Asia. Starting from the early seventies, China experienced a very fast demographic transition due to the implementation of the one-child policy. Our benchmark experiment takes this evolution into account. We investigate its quantitative role by running an experiment where demographic growth in China remains at 3% per year until
2018 (an extreme scenario where the fertility rate stays identical to its 1968 value) before converging to its steady-state value of 1%. All other parameters values are kept to their benchmark values. Results for the evolution of macro variables, displayed in Figure 4.7, are not very different from our benchmark simulation.\textsuperscript{40} Aggregate savings in China increase at a slower pace in the two decades following integration, while the interest rate decreases by less over that period. Indeed in the absence of a demographic transition, the share of middle-aged savers does not increase. As a result of this composition effect, the extent of the rise in savings in China (and the fall in the world interest rate) is therefore smaller compared to our benchmark simulation. Higher demographic growth also limits the fall in interest rates by raising the marginal productivity of capital. These effects tend to dominate in the short run — but in the long run, as China reaches an even greater weight in the world economy, the world interest rate falls further, causing a larger divergence in savings.

![Figure 4.7: Quantitative Results with Delayed Demographic Transition in China.](image)

\textit{Notes}: In the ‘alternative’ calibration, demographic growth in China is set to its initial value of 3% until 2018. All other parameters are set to their benchmark values displayed in Table 2.

## 5 Conclusion

This paper develops a lifecycle theory of savings in large open economies with heterogeneous levels of household credit constraints. We show that faster growth in (more constrained) emerging markets can lead to a divergence in household saving rates across developed and emerging economies, as well as a persistent decline in the world interest rate. The theory provides, with a single mechanism, micro-foundations to the global saving glut (Bernanke\textsuperscript{38})

\textsuperscript{40}The change in demography has little impact on age-savings profiles.
(2005)) and a potential answer to the “allocation puzzle” (Gourichas and Jeanne (2011)). The age-saving profiles estimated from U.S. and China’s survey data are broadly consistent with the lifecycle hypothesis, and at the same time lend empirical support to our theoretical predictions on the contrasting evolution of saving profiles between these two economies. A quantitative version of the model calibrated to macro and micro data for the U.S. and China can explain more than 40% of the divergence in their aggregate household saving rates and a substantial share of the evolution of saving rates across age-groups in both countries. Our model however falls short of explaining the full extent of the “Chinese saving puzzle” (Modigliani and Cao (2004)).

In examining micro-level evidence for China, we point out the biases that may arise from employing household-level data to estimate age-specific saving behavior. Our endeavors to correct for these biases allow us to establish new empirical facts. The novel evidence we provide, along with remaining discrepancies between data and theory, can potentially form the basis for future research. In particular, the saving behavior of the old in China warrants further study. Plausible explanations for their puzzling behavior, not considered in this model, include the evolution of pension systems and health insurance or changes in life expectancy. Finally, our theory can be easily applied to a larger cross-section of countries, thus providing an additional dimension for assessing its performance in accounting for savings and current account patterns across countries.

References


Figures for the Introduction. See Appendix C.1 for data description

Figure 1.1: Private and Household Saving Rates.
Figure 1.2: Current Account, Savings and Investment: Evidence for the U.S. and China.
Figure 1.3: Current Account and Savings in a Large Cross-Section of Countries.

Figure 1.4: Investment: Post-Crisis Recovery in Asia.
A Proofs

Proof of Theorem 1: Consider a country $i$ characterized by $\theta^i$. Note that for $\delta = 1$, we have $R^i_t = \alpha (k^i_t)^{1-\alpha}$. The law of motion for $k^i_t$ is implicitly given by:

$$k^i_{t+1} + \beta^{-\sigma} \alpha^{1-\sigma} (k^i_t)^{\alpha (1-\sigma)+\sigma} = \frac{(1-\theta^i)(1-\alpha)}{(1+g^i_{A,t+1})(1+g^i_{L,t}) \{1+e^i_{t+1}(1+g^i_{L,t+1})+\theta^i(1-\alpha)\}} (k^i_t)^{\alpha}.$$

If a steady-state level of capital $k^i$ exists, it therefore satisfies

$$k^i + \beta^{-\sigma} \alpha^{1-\sigma} (k^i)^{\alpha (1-\sigma)+\sigma} = \frac{(1-\theta^i)(1-\alpha)}{(1+g^i_{A})(1+g^i_{L}) \{1+e(1+g_L)+\theta^i(1-\alpha)\}} (k^i)^{\alpha}.$$

Substituting the steady-state gross rate of return $R^i = \alpha (k^i)^{1-\alpha}$, we can write

$$1 + \beta^{-\sigma} (R^i)^{1-\sigma} = C (\theta^i) R^i,$$

with $C (\theta) = \frac{(1-\alpha)(1-\theta)}{(1+g^i_{A})(1+g^i_{L})\{\alpha[1+e(1+g_L)]+\theta(1-\alpha)\}}$. Note in particular that $\partial C/\partial \theta < 0$. If $\sigma = 1$, the steady-state exists, is unique, and satisfies

$$R^i = \frac{1+\beta}{\beta C (\theta^i)} = (1+g^i_{A})(1+g^i_{L}) \frac{1+\beta \alpha[1+e(1+g_L)]+\theta^i(1-\alpha)}{(1-\alpha)(1-\theta^i)}.$$

For $\sigma < 1$, $R^i$ is such that $v_{\theta^i}(R^i) = 0$, where $v_{\theta}(R) \equiv 1 + \beta^{-\sigma} R^{1-\sigma} - C (\theta) R$ for $R > 0$. We now show that $v_{\theta}(R) = 0$ has a unique solution. Differentiating $v_{\theta}$ with respect to $R$, we get

$$\frac{\partial v_{\theta}}{\partial R} = \beta^{-\sigma} (1-\sigma) R^{-\sigma} - C (\theta),$$

which implies the following equivalence:

$$\frac{\partial v_{\theta}}{\partial R} \geq 0 \iff R \leq \frac{1}{\beta} (1-\sigma) \frac{1}{\sigma} C (\theta)^{-\frac{1}{\sigma}}.$$

Hence $v_{\theta}$ is increasing for $R \leq \frac{1}{\beta} (1-\sigma) \frac{1}{\sigma} C (\theta)^{-\frac{1}{\sigma}}$ and decreasing for $R \geq \frac{1}{\beta} (1-\sigma) \frac{1}{\sigma} C (\theta)^{-\frac{1}{\sigma}}$. We also have $\lim_{\theta} v_{\theta}(R) = 1 > 0$ and $\lim_{\theta} v_{\theta}(R) = -\infty$. Since $v_{\theta}$ is a continuous function, it
follows that $v_\theta(R) = 0$ has a *unique* solution, $R(\theta)$. This is our first result. We also note in passing that our characterization of $v_\theta$ implies

$$R < R(\theta) \iff v_\theta(R) > 0.$$  \hspace{1cm} (A-1)

We now show that countries with a higher $\theta$ have a higher rate of return in autarky steady state. Consider $\theta^i < \theta^j$ and let $R^i = R(\theta^i)$ (resp. $R^j = R(\theta^j)$) denote the well-defined solution to $v_{\theta^i}(R^i) = 0$ (resp. $v_{\theta^j}(R^j) = 0$). For any $R > 0$, we can write

$$v_{\theta^j}(R) - v_{\theta^i}(R) = (C(\theta^j) - C(\theta^i)) R > 0,$$

where the first equality follows from the definition of $v_\theta$, and the inequality follows from $\partial C/\partial \theta < 0$. In particular, for $R = R^i$, we have $v_{\theta^j}(R^i) - v_{\theta^i}(R^i) = v_{\theta^i}(R^i) > 0$, which by remark (A-1) above, is equivalent to $R^i < R^j$. We therefore have shown that

$$\theta^i < \theta^j \iff R^i < R^j.$$  

This establishes our second result, $\partial R^i / \partial \theta^i > 0$, and the fact that $dk^i / d\theta^i < 0$ follows immediately. It is worthwhile to note that the theorem also holds for $\sigma > 1$. Our proof naturally extends to that case.

**Proof of Proposition 1:** For $\delta = 1$ and any $\sigma \leq 1$, one can easily show that the steady state world interest rate $R$ satisfies

$$F(R) = \sum_i \lambda^i \frac{(1 - \theta^i)}{\sum_j \lambda^j (1 - \theta^j)} F(R^i),$$  \hspace{1cm} (A-2)

where $F(x) \equiv x / (1 + \beta^{-\sigma} x^{1-\sigma})$ and $R^i$ denotes the autarky steady state interest rate in country $i$. The bounds on $R$ in (14) follow from $F'(.) > 0$. Note that the proposition also holds for $\sigma > 1$.

**Proof of Proposition 2:** The result follows immediately from Equation (A-2).
**B Technical Appendix**

This appendix provides further details on the setup, solution method, and variable definitions for the quantitative model of Section 4, which embeds the three-period version of Section 2. The definitions given in Section B.6 apply straightforwardly to the three-period case. In that subsection, we derive the formulas for savings by age displayed in Section 2.5.

We consider agents whose economic life runs for $J+1$ periods. Age is indexed by $j = 0, \ldots, J$. We let $c^j_{i,t}$ denote the consumption of an agent of age $j$ in period $t$ and country $i$. In order to obtain a more realistic savings behavior for the old, we introduce a bequest motive along the lines of Abel (2001). The lifetime utility of an agent born in period $t$ in country $i$ is

$$U^i_t = \sum_{j=0}^{J} \beta^j u(c^j_{i,t+j}) + \phi \beta^J u(R^i_{t+J+1} b^i_{t+J}), \quad (B-1)$$

where $b^i_t$ denotes the amount of bequest left in period $t$ by an agent born in period $t - J$, and $\phi$ captures the strength of the bequest motive. Agents receive bequests at age $J - \Delta$, $0 < \Delta < J$, and bequests are shared equally among offsprings.\footnote{Under the assumption that (i) agents remain in childhood for $H$ periods before they start their economic life and (ii) that they procreate at age $P$, then $\Delta = H + P$.} Thus the amount of bequest $q^i_{t+J-\Delta}$ received by an agent born in period $t$ is related to $b^i_{t+J-\Delta}$ as follows

$$q^i_{t+J-\Delta} = \frac{L^i_t}{L^i_{t+J-\Delta}} b^i_{t+J-\Delta} \equiv \frac{b^i_{t+J-\Delta}}{n^i_t}, \quad (B-2)$$

where $L^i_t$ denotes the size of the generation born in period $t$ and $n^i_t$ denotes the number of children (born in period $t$) per agent born in period $t - \Delta$. Gross output in country $i$ is

$$Y^i_t = (K^i_t)^{\alpha} \left[ A^i_t \sum_{j=1}^{J} e^i_{j,t} L^i_{t-j} \right]^{1-\alpha} = A^i_t \bar{L}_t (k^i_t)^{\alpha}, \quad (B-3)$$

where $\bar{L}_t \equiv \sum_{j=0}^{J} e^i_{j,t} L^i_{t-j}$ denotes the total efficiency-weighted population, and $k^i_t = K^i_t / (A^i_t \bar{L}_t)$ denotes the capital-effective-labor ratio. The set of efficiency weights $\{e^i_{j,t}\}_{j=0}^{J}$ captures the shape of the age-income profile in period $t$ and country $i$. Indeed, the labor income received
by agent of age \( j \) in country \( i \) in period \( t \) is \( w_{j,t}^i = e_{j,t}^i (1 - \alpha) A_t^i (k_t^i)^\alpha \equiv e_{j,t}^i w_t^i \). Finally the gross rate of return between \( t - 1 \) and \( t \) is

\[
R_t^i = 1 - \delta + r_{K,t}^i = 1 - \delta + \alpha (k_t^i)^{\alpha - 1}.
\] (B-4)

### B.1 Individual Optimization

Consider the consumption-saving problem of an agent born in period \( t \) and country \( i \). This agent faces a sequence of gross rates of return \( \{R_{t+1}^i\}_{j=0}^J \) and labor income \( \{w_{j,t+1}^i\}_{j=0}^J \), and receives bequest \( q_{t+J-\Delta}^i \). Let \( a_{j,t+1}^i \) denote his end-of-period net asset holdings at age \( j \). Flow budget constraints are

\[
c_{j,t+1}^i + a_{j,t+1}^i = R_{t+1}^i a_{j-1,t+1}^i + w_{j,t+1}^i + 1_{(j=J-\Delta)} q_{t+J-\Delta}^i, \quad 0 \leq j \leq J - 1, \quad (B-5)
\]

\[
c_{j,t+1}^i + b_{j,t+1}^i = R_{t+1}^i a_{j-1,t+1}^i + w_{j,t+1}^i, \quad (B-6)
\]

with \( a_{-1,t-1}^i = 0 \). Define the discounted present value of current and future labor income

\[
H_{j,t}^i \equiv w_{j,t}^i + \sum_{\tau=1}^{J-j} \frac{w_{j+\tau,t+\tau}^i}{\prod_{s=1}^{\tau} R_{t+s}^i}, \quad 0 \leq j \leq J - 1, \quad (B-7)
\]

\[
H_{J,t}^i \equiv w_{J,t}^i. \quad (B-8)
\]

The credit constraint faced by the agent at age \( j \leq J - 1 \) is

\[
a_{j,t+1}^i \geq -\theta^i \frac{H_{j+1,t+j+1}^i}{R_{t+j+1}^i}. \quad (B-9)
\]

### B.2 Autarky Steady State

Consider a steady state for country \( i \) where \( e_{j,t}^i = e_j^i \) for all \( t \), productivity grows at constant rate \( g_A^i \), and \( L_{t+1}^i = (1 + g_L^i) L_t^i \), implying \( n_t^i = (1 + g_L^i)^\Delta \) for all \( t \). Let \( k^i \) denote the autarky
steady state level of \( k \) in country \( i \),

\[
\frac{K^i_t}{A^i_t L^i_t} = k^i.
\]

The age-income profile is given by \( w^i_{j,t} = e^i_j w^i_t \), where \( w^i_t = (1 - \alpha)A^i_t(k^i)^\alpha \). Thus an agent born in period \( t \) faces the wage sequence \( \{w^i_{j,t+j}\}_{j=0}^J \) with

\[
w^i_{j,t+j} = e^i_j w^i_{t+j} = e^i_j (1 + g^i_A)^j w^i_t = \tilde{w}^i_j w^i_t,
\]

where \( \tilde{w}^i_j = e^i_j (1 + g^i_A)^j \). Taking the steady state value of the gross rate of return \( R^i \) as given, consider the stationary individual optimization problem with normalized labor income sequence \( \{\tilde{w}^i_j\}_{j=0}^J \). For a given value of received bequest \( q \), let \( \{\tilde{a}^i_j(q)\}_{j=0}^J \) denote the optimal path of wealth for an agent in country \( i \), and \( \tilde{b}^i(q) \) the amount of bequest left by this agent. Define \( \bar{q}^i(R^i) \) the value of \( q \) such that

\[
q = \frac{1}{[(1 + g^i_L)(1 + g^i_A)]^\Delta \tilde{b}^i(q)},
\]

and let \( \bar{a}^i_j \equiv \tilde{a}^i_j(\bar{q}^i) \). Stationarity and homogeneity imply that, at steady state in country \( i \), the wealth at age \( j \) of an agent born in period \( t \) is \( a^i_{j,t+j} = \tilde{a}^i_j w^i_t \).

The market clearing condition at the end of period \( t \) is

\[
K^i_{t+1} = A^i_{t+1} \bar{L}^i_{t+1} k^i = \sum_{j=0}^{J-1} L^i_{t-j} a^i_{j,t}.
\]

Using the fact that \( a^i_{j,t} = \tilde{a}^i_j w^i_t / (1 + g^i_A)^j \) along with \( L^i_{t+1} = (1 + g^i_L)L^i_t \), the market clearing condition can be rewritten as

\[
\left( \sum_{j=0}^{J} \frac{e^i_j}{(1 + g^i_L)^j} \right) (k^i)^{1-\alpha} = (1 - \alpha) \sum_{j=0}^{J-1} \frac{\tilde{a}^i_j(k^i)}{[(1 + g^i_L)(1 + g^i_A)]^j+1},
\]

where the notation \( \tilde{a}^i_j(k^i) \) makes explicit the dependence of the path of net asset positions on
the steady state rate of return. Equation (B-13) implicitly defines the steady-state level of the capital-effective-labor ratio.

**B.3 Integrated Steady State**

Consider an integrated steady state where \( e_{j,t}^i = e_j \) for all \( i \) and \( t \), productivity grows at constant rate \( g_A \), and \( L_{t+1}^i = (1 + g_L) L_t^i \), implying \( n_t^i = (1 + g_L)^\Delta \) for all \( i \) and \( t \). At steady state,

\[
\frac{K_t^i}{A_t^i L_t^i} = k.
\]

Now the income profile by age is given by \( w_{j,t}^i = e_j w_t^i \), where \( w_t^i = (1 - \alpha) A_t^j k^\alpha \). Hence an agent born in period \( t \) faces the wage sequence \( \{w_{j,t+j}^i\}_{j=0}^{J} \) with

\[
w_{j,t+j}^i = e_j w_{t+j}^i = e_j (1 + g_A)^j w_t^i \equiv \bar{w}_j w_t^i. \tag{B-14}
\]

The integrated steady state can be determined along the same logic as for the autarky steady state. First, taking the steady state value of the gross rate of return \( R \) as given, we consider the stationary individual optimization problem with normalized labor income sequence \( \{\bar{w}_j\}_{j=0}^{J} \). For a given value of received bequest \( q \), let \( \{\bar{a}_j^i(q)\}_{j=0}^{J} \) denote the optimal path of wealth for an agent in country \( i \), and \( \bar{b}_i(q) \) the amount of bequest left by this agent. Define \( \bar{q}_i(R) \) the value of \( q \) such that

\[
q = \frac{1}{[(1 + g_L)(1 + g_A)]^\Delta \bar{b}_i(q)}, \tag{B-15}
\]

and let \( \bar{a}_j^i \equiv \bar{a}_j^i(q^i) \). Stationarity and homogeneity imply that, at the integrated steady state, the wealth at age \( j \) of an agent born in period \( t \) in country \( i \) is \( a_{j,t+j}^i = \bar{a}_j^i w_t^i \).

The market clearing condition at the end of period \( t \) is

\[
\sum_i K_{t+1}^i = k \sum_i A_{t+1}^i \bar{L}_{t+1}^i = \sum_i \sum_{j=0}^{J-1} L_{t-j}^i a_{j,t}^i. \tag{B-16}
\]
Let $\lambda^i \equiv A^i_t L^i_t / (\sum_h A^h_t L^h_t)$ denote the constant share of country $i$ in world effective labor. The market clearing condition can be rewritten as

$$\left( \sum_{j=0}^{J} \frac{e^j_j}{(1 + g_L)^j} \right) k_{t+1}^{1-\alpha} = (1 - \alpha) \sum_{j=0}^{J-1} \sum_i \lambda^i a^j_j(k) \left[ (1 + g_L) (1 + g_A) \right]^{j+1}. \quad (B-17)$$

### B.4 Dynamics

The law of motion for $k^i_t \equiv (k^i_t)^N_{t=1}$ depends on whether countries are financially integrated or in financial autarky. If countries are closed financially in period $t$, the market clearing condition in country $i$ is

$$A^i_{t+1} L^i_{t+1} k^i_{t+1} = \sum_{j=0}^{J-1} L^i_{t-j} a^i_{j,t}. \quad (B-18)$$

The generations who matter in period $t$ are those born in periods $t - J + 1$ to $t$. It is then immediate to show that market clearing in period $t$ pins down $k^i_{t+1}$ given

- lagged values $K^i_{L,t+1} \equiv \{k^i_{t-J+1}, \ldots, k^i_t\}$ and future values $K^i_{F,t+1} \equiv \{k^i_{t+2}, \ldots, k^i_{t+J+1}\}$,

- bequests $Q^i_{t+1} \equiv \{q^i_{\tau}\}_{\tau=t+1-\Delta}^t$,

- productivity sequence $\{A^i_{\tau}\}_{\tau=t-J+1}^{t+J}$,

- evolution of demographics, i.e., $\{L^i_{\tau}\}_{\tau=t-J+1}^t$,

- evolution of age-income profile, i.e., $\{e^j_{i,j+\tau}\}_{\tau=0}^J$ for $\tau = t - J + 1, \ldots, t$.

Note that bequests $\{q^i_{\tau}\}_{\tau=t+1-\Delta}^{t+J}$ are determined along with $k^i_{t+1}$.

If instead countries are financially integrated in period $t$, then rates of return are equalized across countries

$$R^i_{t+1} = R_{t+1}, \quad \text{for all } i,$$

and so are their capital-effective-labor ratios

$$k^i_{t+1} \equiv K^i_{t+1} / (A^i_{t+1} L^i_{t+1}) = k_{t+1}, \quad \text{for all } i.$$
The market clearing condition in period $t$ is

$$\sum_i K_{t+1}^i \equiv k_{t+1} \sum_i A_{t+1}^i \bar{L}_{t+1}^i = \sum_i \sum_{j=0}^{J-1} L_{t-j}^i a_{j,t}^i. \quad (B-19)$$

It is immediate to show that market clearing in period $t$ pins down $k_{t+1}$ given

- lagged and future values, $K_{L,t+1} \equiv \{k_i^{t-J+1}\}_{t=0}^\tau$ and $K_{F,t+1} \equiv \{k_i^{t+1}\}_{t=0}^{t+J}$,
- bequests $Q_{t+1} \equiv \{q_I^T\}_{t=0}^{t+1}$, where $q_I = (q_i^j)_{i=1}^N$,
- productivity sequence $\{A_i^T\}_{t=0}^{t+1}$ for $i = 1, ..., N$,
- evolution of demographics, i.e., $\{L_i^T\}_{t=0}^{t+1}$ for $i = 1, ..., N$,
- evolution of age-income profile, i.e., $\{e_{i,j}^T\}_{j=0}^{t-J}$ for $\tau = t - J + 1, ..., t$ and $i = 1, ..., N$.

### B.5 Simulations

In our main experiment, we consider a situation where countries start in financial autarky and integrate in period $X$ (we set $X = 0$ in the paper). Hence for $t \geq X + 1$, $k_i^t = k_i$ for all $i$. Around the integration period, we also feed the model with “shocks” to productivity, demography, and age-income profiles. We allow for shocks over the window $[X - \tau, X + \tau]$. All shocks are perfectly anticipated. In order to determine how the global economy responds to financial integration and other contemporaneous shocks, we use the following algorithm.

1. We assume each country starts at its autarkic steady state, and that the economy does not react to future shocks before period $X - T$, for $T > \tau$ large. That is, $k_i^t = k_i^* = k_i^{ts}$ for $t \leq X - T - 1$. The initial steady state for country $i$ is determined by country specific parameters $\theta_i^i$, along with $(g_L^i, g_A^i)$ and $\{e_{i,j}^j\}_{j=0}^\tau$, as described in Section B.2. From the initial steady state, we obtain bequests $q_i^t$ for $t \leq X - T - 1$, which gives us $Q_{X-T}^i$.

2. We assume the global economy has converged to its integrated steady state $k^*$ in period $X + T + 1$. That is, $k_t = k^*$ for $t \geq X + T + 1$. The final steady state is determined
3. We then determine the transition path \( \{k_t^i\}_{t=X-T}^{X+T} \) iteratively as follows.

- We start with a guess \( \{k_t^{i(0)}\}_{t=X-T}^{X+T} \). We set \( k_t^{i(0)} = k^*_i \) for \( t \leq X \) and \( k_t^{i(0)} = k^* \) for all \( i \) for \( t \geq X + 1 \).

- For \( n \geq 0 \), given the path \( \{k_t^{i(n)}\}_{t=X-T}^{X+T} \), the updated path \( \{k_t^{i(n+1)}\}_{t=X-T}^{X+T} \) is obtained as follows. First, we obtain \( \{k_t^{i(n+1)}\}_{t=X-T}^{X+T} \) by iterating on the autarkic forward-backward difference equation (FBDE) in each country (see Section B.4).

Specifically, for each country \( i \),

- We compute \( k_t^{i(n+1)} \) as the solution to the autarkic FBDE, given \( K^{i(n)}_{L,X-T} \), \( K^{i(n)}_{F,X-T} \) and \( Q^{i}_{X-T} \). Along with \( k_t^{i(n+1)} \), we get \( q^{i(n+1)}_{X-T} \).

- We compute \( k_t^{i(n+1)} \) as the solution to the autarkic FBDE, given \( K^{i(n)}_{L,X-T+1} \), \( K^{i(n)}_{F,X-T+1} \) and given past bequests \( Q^{i}_{X-T+1} = (q^{i(n)}_{X-T+2}, q^{i(n)}_{X-T+1}, q^{i(n+1)}_{X-T}) \). Along with \( k_t^{i(n+1)} \), we get \( q^{i(n+1)}_{X-T+1} \) which is later used for computing \( k_t^{i(n+1)} \).

- We repeat the previous steps until we have determined \( k_t^{i(n+1)} \).

Then, we determine the common path \( \{k_t^{i(n+1)}\}_{t=X+1}^{X+T} \) by iterating on the integrated FBDE. Specifically:

- We compute \( k_t^{i(n+1)} \) as the solution to the integrated FBDE given \( K^{i(n)}_{L,X+1} \), \( K^{i(n)}_{F,X+1} \) and past bequests \( (q^{i(n+1)}_{X-2}, q^{i(n+1)}_{X-1}, q^{i(n+1)}_{X}) \). Along with \( k_t^{i(n+1)} \), we get \( q^{i(n+1)}_{X+1} \) which is used for computing \( k_t^{i(n+1)} \).

- We proceed until we have determined \( k_t^{i(n+1)} \).

- We iterate on \( n \) until convergence, based on the distance between two consecutive paths \( \{k_t^{i(n)}\}_{t=X-T}^{X+T} \) and \( \{k_t^{i(n+1)}\}_{t=X-T}^{X+T} \).

4. We set \( T \) large enough for the distances \( |k_t^{i(\infty)} - k^*_i| \) and \( |k_t^{i(\infty)} - k^*| \) to fall below some convergence threshold.
B.6 Definitions

Investment in country $i$ in period $t$ is

$$I^i_t \equiv K^i_{t+1} - (1 - \delta)K^i_t = A^i_{t+1}L^i_{t+1}k^i_t - (1 - \delta)A^i_t\bar{L}^i_t k^i_t. \quad (B-20)$$

Let $W^i_{t-1}$ denote aggregate wealth in country $i$ at the end of period $t - 1$:

$$W^i_{t-1} \equiv \sum_{j=0}^{J-1} L^i_{t-j}a^i_{j,t-j}. \quad (B-21)$$

The net foreign asset position of country $i$ at the end of period $t - 1$ is defined as

$$NFA^i_{t-1} \equiv W^i_{t-1} - K^i_t. \quad (B-22)$$

Aggregate savings are defined as GNP minus aggregate consumption, i.e.,

$$S^i_t \equiv Y^i_t + (R_t - 1)NFA^i_{t-1} - C^i_t, \quad (B-23)$$

where $C^i_t = \sum_{j=0}^{J} L^i_{t-j}c^i_{j,t-j}$. One can easily show that:

$$S^i_t = \Delta W^i_t + \delta K^i_t.$$  

Savings net of capital depreciation correspond to the change in country wealth $\Delta W^i_t$.

The current account position of country $i$ in period $t$ is

$$CA^i_t \equiv NFA^i_t - NFA^i_{t-1} = \Delta NFA^i_t. \quad (B-24)$$

From the definitions above, it follows that $CA^i_t = S^i_t - I^i_t$. Finally, let $S^i_{j,t}$ denote the individual level of savings for an agent of age $j$ in country $i$ and period $t$, defined as

$$S^i_{j,t} \equiv NDI^i_{j,t} - c^i_{j,t}, \quad (B-25)$$

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where the first term denotes the agent’s net disposable income

\[ NDI_{j,t}^i \equiv (R_t^i - 1) a_{j-1,t-1}^i + w_{j,t}^i + 1_{(j=j-\Delta)} q_t^i. \] (B-26)

The saving rate for age \( j \) is computed as

\[ s_{j,t}^i = \frac{S_{j,t}^i}{NDI_{j,t}^i} = 1 - \frac{c_{j,t}^i}{NDI_{j,t}^i}. \] (B-27)

**Savings by Age in the Three-Period Model.** The general definitions (B-25)-(B-27) apply in particular to the three-period version of the model without bequests. Using the notations of Section 2, the level of saving of the young in country \( i \) and period \( t \) is

\[ S_{y,t}^i = L_{y,t}^i (w_{y,t}^i - c_{y,t}^i) = L_{y,t}^i a_{y,t+1}^i = -L_{y,t}^i \frac{\theta^i}{R_{t+1}} w_{m,t+1}^i, \]

where the last equality follows from Assumption 1. Normalizing by GDP, we get

\[ \frac{S_{y,t}^i}{Y_t^i} = -(1 + g_{L,t+1}^i) \frac{1 + g_{L,t}^i}{1 + e_t^i(1 + g_{L,t}^i)} \frac{\theta^i}{R_{t+1}} (1 - \alpha) \left( \frac{k_{t+1}}{k_t} \right)^\alpha. \]

The expression given in the text in Section 2.5 follows from the relationship that holds between \( k_{t+1} \) and \( R_{t+1} \) when \( \delta = 1 \). The level of saving of the middle-aged in country \( i \) and period \( t \) is

\[ S_{m,t}^i = L_{m,t}^i \{ w_{m,t}^i + (R_t - 1) a_{y,t}^i - c_{m,t}^i \} \]
\[ = L_{m,t}^i \{ (w_{m,t}^i + R_t a_{y,t}^i - c_{m,t}^i) - a_{y,t}^i \} \]
\[ = L_{m,t}^i \{ a_{m,t+1}^i - a_{y,t}^i \} \]
\[ = L_{m,t}^i \left\{ \frac{1}{1 + \beta - \sigma R_{t+1}^{1-\sigma}} (1 - \theta^i) + \frac{\theta^i}{R_{t}} \right\} w_{m,t}^i. \]

Normalizing by GDP, we get

\[ \frac{S_{m,t}^i}{Y_t^i} = \frac{1}{1 + e_t^i(1 + g_{L,t}^i)} \left[ \frac{1 - \theta^i}{1 + \beta - \sigma R_{t+1}^{1-\sigma}} + \frac{\theta^i}{R_t} \right] (1 - \alpha). \]
Finally, the level of saving of the old in country \( i \) and period \( t \) is

\[
S^i_{o,t} = r_{K,t}K^i_t + (R_t - 1)[L^i_{m,t-1}a^i_{m,t} - K^i_t] - L^i_{o,t}c^i_{o,t}
\]
\[
= (R_t - 1 + \delta)K^i_t + (R_t - 1)L^i_{o,t}a^i_{m,t} - (R_t - 1)K^i_t - L^i_{o,t}R_t a^i_{m,t}
\]
\[
= -L^i_{o,t}a^i_{m,t} + \delta K^i_t
\]
\[
= -L^i_{o,t} \frac{1}{1 + \beta - \sigma R_t^{1-\sigma}}(1 - \theta^i)w^i_{m,t-1} + \delta K^i_t,
\]

where the last term is dropped when net savings are considered. Normalizing by GDP, we get

\[
\frac{S^i_{o,t}}{Y^i_t} = -\frac{1}{1 + g^i_{A,t}} \frac{1}{1 + g^i_{L,t-1}} \frac{1}{1 + e^i_t(1 + g^i_{L,t})} \frac{1}{1 + \beta - \sigma R_t^{1-\sigma}}(1 - \theta^i)(1 - \alpha) \left( \frac{k_t-1}{k_t} \right)^\alpha.
\]

### B.7 Calibration

Section 4.2 summarizes the calibration of the quantitative model. The vector of unobservable parameters \( \psi \equiv [\sigma, \beta, \phi, \theta^{US}, \theta^{CH}] \) is determined as follows. Let \( s^i_0 \) and \( s^i_{j,0} \) denote the model-implied aggregate saving rate and the saving rate of agents of age \( j \) in country \( i \) in the integration period, and let \( s^i_{j,0} \) and \( s^i_{j,0} \) denote their counterparts in the data. We search over a large grid \( \Psi \) the vector \( \psi^* \) such that

\[
\psi^* = \arg\min_{\psi \in \Psi} \sum_i \left| s^i_0(\psi) - s^i_{0,d} \right| + \sum_i \sum_{j=0}^J \omega^i_j \left( s^i_{j,0}(\psi) - s^i_{j,0} \right)^2,
\]

where \( \omega^i_j = e^i_{j,0}L^i_{-j}/L^i_0 \), so that the weights on different age groups in each country add up to one and reflect their shares in the total effective population at the time of opening. We start the search with a coarse grid covering a wide range to identify the region of the parameter space where the solution lies and then refine the grid gradually. The final grid involves values for \( 1/\sigma \) in \([1.8 : 0.1 : 2.6]\), values for \( \beta \) (annualized) in \([0.945 : 0.005 : 0.985]\), values for \( \phi \) in \([0 : 0.01 : 0.06]\), values for \( \theta^{US} \) in \([0.18 : 0.01 : 0.26]\), and values for \( \theta^{CH} \) in \([0, 0.01, 0.02]\).
C Data

C.1 Aggregate Data (for the figures shown in the Introduction)

List of Countries

*Developed Countries:* Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States;

*Asian Countries:* Bangladesh, Cambodia, China, Fiji, Hong Kong SAR, China, India, Indonesia, Kiribati, Korea, Lao P.D.R., Malaysia, Maldives, Nepal, Pakistan, Papua New Guinea, Philippines, Singapore, Solomon Islands, Sri Lanka, Thailand, Tonga, Vanuatu, Vietnam.

Data on Savings, Private Savings, Investment and Current Account (% of GDP). Data for Emerging Asia and Developed Countries are from World Development Indicators (World bank), Penn World Tables and Asian Development Bank (ADB). Private savings are computed as the difference between Aggregate saving and Primary Government Surplus. Data for Primary Government Surplus in Asian countries are only available starting 1988 for a large sample of Asian countries.

Data on Household Saving Rates. Data for Developed Countries are from OECD (NIPA personal saving rate for the U.S.). Data for India are from the Central Bank of India. Data for China are from Song and Yang (2010) and authors’ calculations from Urban Household Survey (UHS).

Cross-Section of Saving and Current Accounts (Figure 1.3). Data for the period 1998-2007 are from World Development Indicators (World Bank) and Penn World Tables and cover a sample of 89 countries. The list of countries is available on request.

C.2 Data for the U.S.

Definitions

*Household disposable income:* sum of individual income net of taxes (in USD).
Household expenditure: household consumption expenditures (in USD).

Household saving: difference between household disposable income and consumption expenditure (in USD).

Household saving rate: Household saving divided by disposable income.

Consumer Expenditures Survey Data (CEX)
Annual data over the period 1986-2008 for consumption expenditures and income. Disaggregated by age groups (6 age groups): under 25, 25-34, 35-44, 45-54, 55-64, and above 65. Disaggregated by sectors of expenditures. The sectors covered in the CEX data are: Food and alcoholic beverages, Shelter Utilities and public services, Household expenses, Clothing and apparel, Vehicles purchases, Gas and motor oil, Other vehicle expenses, Public transportation, Health Entertainment, Education, Tobacco, Miscellaneous and cash contributions, Life/personal insurance.

NIPA Data from U.S. Bureau of Economic Analysis (BEA)
Consumption and income data for 1986-2008. Consumption expenditures data are disaggregated by sectors of expenditures. We match sectors in NIPA with the corresponding sectors in CEX. Only two categories in NIPA consumption expenditures (accounting for about 1% of total expenditures) do not appear in CEX data (Net foreign travel and expenditures abroad by U.S. residents, and Final consumption expenditures of nonprofit institutions serving households). Aggregate consumption expenditures from CEX data do not match aggregate NIPA data, as a result of underreporting of consumption in CEX—a bias which has increased over time. Income displays a similar bias but without trend.

C.3 Data for China

Definitions
Household disposable income: sum of individual disposable income net of taxes within a household.

Household consumption expenditures: sum of consumption expenditures within household.

Household savings: difference between household disposable income and household consump-
tion expenditure. Rates are computed by dividing by household disposable income.

*Individual savings:* difference between individual disposable income and individual consumption expenditure (estimated). Rates are computed by dividing by individual disposable income.

**Urban Household Survey Data (UHS)**

Annual data for the year 1986 and over the period 1992-2009 for consumption expenditures, income and household characteristics (number of household members, age of household members, employment status of household members...), for a large sample of urban households in China. Starting from 1992, households are chosen randomly — based on several stratifications at the provincial, city, country, township, and neighborhood levels — and are expected to stay in the survey for 3 years. The 1986 survey covers 47,221 individuals in 12,185 households across 31 provinces. The 1992-2009 surveys cover 112 prefectures across 9 representative provinces (Beijing, Liaoning, Zhejiang, Anhui, Hubei, Guangdong, Sichuan, Shaanxi and Gansu). The sample size has been extended over time from roughly 5,500 households in the 1992-2001 surveys to nearly 16,000 households in the 2002-2009 surveys. Disposable income is provided at the individual for the years 1992-2009 (but not for 1986) and at the household level for all years. Data for consumption expenditures are given at the household level. When estimating individual consumption expenditures and savings, we restrict our attention to individuals above 25 and income earners aged between 19-24. Individuals under 25 without income and all those below 18 are considered as children, whose consumption is imputed to other household members (typically their parents).42

**Chinese Households Income Project Data (CHIP)**

CHIP survey data are available for the years 1995 and 2002. Income and consumption by age for these two years are consistent across the UHS and CHIP datasets.

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42In our final specification, old dependents (i.e., individuals above 65 earning zero income and living with their offsprings) are treated in the same way as children, but the treatment of old dependents does not affect our results.
D  Data Treatment

D.1 Correction Methods for the U.S.

The extent of underreporting of income and consumption in CEX, and their variations over time, are depicted in Figure D.1. We describe two alternative methods to deal with underreporting in CEX. The first one makes adjustments using only aggregate data, while the second one makes adjustments to consumption at the sector level. In the main text, we only refer to the second method, which is in principle more accurate.

![Figure D.1: CEX Underreporting of Income and Consumption (CEX/NIPA ratios).](image)

D.1.1 Method 1: Corrections Using Aggregate Data

Let $c_{j,t}^{CEX}$ and $y_{j,t}^{CEX}$ denote average consumption and income reported in CEX for age $j$ in year $t$, and let $C_t^D$ and $Y_t^D$ denote aggregate consumption and income in dataset $D$. We adjust consumption and income for all ages according to

$$
\tilde{c}_{j,t} = \frac{C_t^{NIPA}}{C_t^{CEX}} c_{j,t}^{CEX}, \quad \tilde{y}_{j,t} = \frac{Y_t^{NIPA}}{Y_t^{CEX}} y_{j,t}^{CEX}.
$$
By construction, consumption expenditures and income match NIPA in the aggregate. The corrected saving rate for age \( j \) in period \( t \) is \( \hat{s}_{j,t} = (\hat{y}_{j,t} - \hat{c}_{j,t})/\hat{y}_{j,t} \).

**D.1.2 Method 2: Corrections Using Sectoral Expenditure Data**

Since the degree of underreporting is likely to differ across types of goods, and since different age groups potentially have different consumption baskets, we implement sector-specific adjustments. Let \( C_{kt}^D \) be the aggregate consumption expenditures of goods in sector \( k \) at date \( t \) from dataset \( D \). Define the following sector-specific weight:

\[
\chi_{kt} = \frac{C_{kt}^{NIPA}}{C_{kt}^{CEX}}.
\] (B-28)

For all goods, the weights are greater than one due to underreporting in CEX, and they increase over time as the bias gets larger. Consider consumption of good \( k \) by age-group \( j \) in CEX, denoted by \( c_{jkt}^{CEX} \). Our corrected measure of consumption expenditures in sector \( k \) for group \( j \) is (up to the additional adjustment described below):

\[
\hat{c}_{jkt} = \chi_{kt} c_{jkt}^{CEX}.
\]

Total consumption expenditures of group \( j \) is then \( \hat{c}_{jt} = \sum_k \hat{c}_{jkt} \). The corrected income net of taxes \( \hat{y}_{jt} \) of group \( j \) is, as before: \( \hat{y}_{jt} = \frac{Y_{NIPA}^t}{Y_{CEX}^t} y_{jkt}^{CEX} \). Finally, the corrected saving rate of group \( j \) is \( \hat{s}_{j,t} = (\hat{y}_{j,t} - \hat{c}_{j,t})/\hat{y}_{j,t} \).

The sector-specific factors need to be slightly modified to account for the fact that health expenditures are treated differently in CEX vs. NIPA. Indeed health expenditures in CEX are restricted to ‘out-of-pocket’ expenses, but NIPA also includes health contributions (Medicare and Medicaid), leading to very large adjustment factor \( \chi_{\text{health}} \approx 5 \) — which primarily affects our consumption estimates for the old, for whom ‘out-of-pocket’ health expenditures constitute a large share of their consumption basket in CEX (\( \approx 12\% \)). Without additional correction, we

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\( ^{43} \)A small discrepancy remains since NIPA includes some expenditures (e.g., ‘Net foreign travel and expenditures abroad by U.S. residents’ and ‘Final consumption expenditures of nonprofit institutions serving households’) which cannot be matched with CEX categories.
would tend to under-estimate the saving rate of the old. To address this concern, we amend the computation of sectoral adjustment factors as follows. We set

$$
\chi_{\text{health},t} = \frac{\sum_{k \neq \text{health}} C_{NIPA}^{kt}}{\sum_{j \neq \text{health}} C_{CEX}^{jt}},
$$

and for other sectors $z \neq \text{health},$

$$
\chi_{z,t} = \frac{C_{NIPA}^{zt}}{C_{CEX}^{zt}} \left[ 1 + \frac{C_{NIPA}^{health,t}}{\sum_{k \neq \text{health}} C_{NIPA}^{kt}} - \frac{C_{CEX}^{health,t}}{\sum_{k \neq \text{health}} C_{CEX}^{kt}} \right].
$$

Compared to the set of simple sector-specific weights given in (B-28), this amendment reduces the adjustment factor for health to its average across other sectors while slightly increasing the adjustment factor of other goods. Doing so, we still match NIPA aggregate consumption. Age-saving profiles with or without adjustment for health expenditures are very similar, except for the group of individuals above 65. We also find that Method 1 (using only aggregate data) and Method 2 (using disaggregated expenditures data) produce results that are very similar.

### D.2 Correction Methods for China

In the text, we argue that in the presence of multi-generational households, age-saving profiles obtained from the ‘household approach’ are subject to an aggregation bias. Multigenerational households are very prevalent in China (see Table 1). Figure D.2, which mimics Figure 4 in Deaton and Paxson (2000), provides further evidence on Chinese household composition and its evolution over time. For the years 1992 and 2009, the figure plots, as a function of the age of individuals, the average age of the head in the households they live in. If everyone were a household head or lived with persons of the same age (i.e., in the absence of multi-generational households), the plot would be the 45-degree line. Instead, the plot lies above the 45-degree line for young people (many of whom live with their parents), then more or less runs along the line for those aged between 40-60, and then falls below the line for the elderly—many of whom live with their children. Comparing across years, the figure suggests that young individuals
are leaving their parents’ household on average later in 2009 than in 1992. Similarly, most likely as a result of an increase in life expectancy, the elderly join their children’s households at a later age in 2009 than in 1992. The fact that the degree of disconnect at various ages changes over time suggests that the household approach could lead to biases when estimating changes in saving rates across age groups.

Figure D.2: Average Age of Household Head By Age of Individual.

To correct for the biases inherent to the household approach, we provide two alternative ‘individual’ methods — one based on the sub-sample of uni-generational households, and another based on a projection technique proposed by Chesher (1997). In the main text, we describe only the second method as we believe it is more accurate for the early years of our sample, for which we have fewer observations. However we show below that the two methods yield similar age-saving profiles (even more so towards the end of the sample period), which is quite noteworthy as they rely on different sub-samples and different identification strategies.
D.2.1 Individual Method 1: Re-Sampling of Uni-Generational Households

Our first approach to deal with the aggregation bias consists in restricting our attention to the sub-sample of uni-generational households, which constitute more than 40% of the entire sample.\textsuperscript{44} Individual consumption is inferred from household consumption by applying an equal-sharing rule among members of the households.\textsuperscript{45} The main issue that arises with this approach is that individuals of a certain age who live in a uni-generational household may differ systematically, along a number of attributes, from individuals of the same age living in multi-generational households. We find that individuals in uni-generational households indeed differ from the whole sample in terms of income, gender, and marital status.\textsuperscript{46} In particular, (i) individuals who live in uni-generational households tend to be richer than average, and (ii) women tend to be over-represented among the young and under-represented among the old.\textsuperscript{47}

To address potential selection biases, we re-weight the observations to match the distribution of these attributes in the whole sample for each age, as described below. Given the limited number of uni-generational household observations for the youngest and oldest age groups, it is difficult to re-sample the data to match the distribution of all three attributes simultaneously for these groups. Since income and gender appear to be the variables having the greater impact on saving rates, we focus on these two variables to control for selection issues.\textsuperscript{48}

\textit{Re-sampling of the restricted sample to match the income distribution}

Young and old individuals who live alone tend to be richer than average. To address this issue,

\textsuperscript{44}Any household with one adult or several adults belonging to the same generation (i.e., with a maximum age difference less than 18 years), possibly with a child, is treated as uni-generational. Another benefit of restricting the analysis to uni-generational households is to minimize concerns related to intrahousehold transfers, which could potentially obscure actual saving behavior.

\textsuperscript{45}Some aggregation bias remains if the equal-consumption rule does not apply to husband and wife, for example, but it is reasonable to believe that consumption sharing is more equal within a generation than across generations.

\textsuperscript{46}We find no difference between the two samples along other characteristics (e.g., the number of children).

\textsuperscript{47}In terms of marital status, young and old individuals who live in uni-generational households are more likely to be married, the reason being that young people tend to move out of their parents’ household when they get married, and the elderly are more likely to move back to their offsprings’ household when they lose their spouse. The observed gender bias may come from the fact that young women marry and leave their parents at an earlier age than men, and that widows are more likely to live with their children than widowers.

\textsuperscript{48}Re-weighting observations to match the income distribution only, the income & gender distribution, or the income & marital status distribution yields similar age-saving profiles. The only notable difference is that estimated saving rates for the youngest individuals are lower when gender is not taken into account.
observations are re-weighted so that the distribution of individual income for each age in the restricted sample matches the aggregate income distribution. We first group individuals into 2-year age bins, and then assign weights to match the income decile distribution for each of the 2-year bins. When the number of observations is insufficient (especially at the ends of the age distribution), we use income quintiles. One potential problem with the approach is that very high weights are assigned to individuals in the lowest income quantile for the young, and that these young individuals may not be representative of the low-income youth who live with their parents. Another potential concern comes from the fact that the elderly living alone are more likely to receive monetary transfers from their children than those living in their children’s household. Hence by focusing on uni-generational households, the income of the old could be overestimated.\footnote{The information available in UHS data does not allow us to identify the component of individual income coming from inter-household transfers.}

Using CHIP survey data for the year 2002, for which more detailed information on inter-household transfers is available, we find that this bias exists but is small.

**Re-sampling of the restricted sample to match the income & gender distribution**

To correct for the gender bias among uni-generational households, we re-sample observations to match the income distribution separately for men and women, and then combine the two distributions with weights reflecting the gender composition in the whole sample.\footnote{We proceed in the same way when controlling for income & marital status.}

**D.2.2 Individual Method 2: Projection Method (Chesher (1997))**

In order to identify individual consumption from household consumption, we estimate the following model on the cross-section of households for every year

\[
C_h = \exp(\gamma . Z_h) \left( \sum_{j=1}^{99} c_j N_{h,j} \right) + \epsilon_h,
\]

where \(C_h\) is the aggregate consumption of household \(h\), \(N_{h,j}\) is the number of members of age \(j\) in household \(h\), and \(Z_h\) denotes a set of household-specific controls. Following Chesher
(1997), multiplicative separability is assumed to limit the number of degrees of freedom, and control variables enter in an exponential term. The control variables include:

- Household composition: number of children aged 0-10, number of children 10-18, number of adults, and depending on the specification, the number of old and young dependents. The coefficient associated with the number of children is positive, as children-related expenses are attributed to the parents.

- Household income group: households are grouped into income quintiles. The sign of the control variable (a discrete variable 1-5) is positive: individuals living in richer households consume more.

In the estimation, a roughness penalization term is introduced to guarantee smoothness of the estimated function $c_j = c(j)$. This term is of the form:

$$P = \kappa^2 \int [c''(j)]^2 \, dj,$$

where $\kappa$ is a constant that controls the amount of smoothing (no smoothing when $\kappa = 0$ and forced linearity as $\kappa \to \infty$). The discretized version of $P$, given that $j$ is an integer in $[19; 99]$, can be written $\kappa^2 (Jc_j)'(Jc_j)$, where the matrix $J$ is the $79 \times 81$ band matrix

$$J = \begin{bmatrix}
1 & -2 & 1 & 0 & \ldots & 0 & 0 & 0 \\
0 & 1 & -2 & 1 & \ldots & 0 & 0 & 0 \\
0 & 0 & 1 & -2 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \ldots & -2 & 1 & 0 \\
0 & 0 & 0 & 0 & \ldots & 1 & -2 & 1
\end{bmatrix},$$

and $c_j = [c_j]_{j=19,\ldots,99}$ is an $81 \times 1$ vector. Pre-multiplying $c_j$ by $J$ produces a vector of second differences. We set $\kappa = 10$. 

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As a robustness check, we use the projection method to estimate individual income distributions by age from household income data, and then confront the estimated distributions to the actual ones—which we observe for the period 1992-2009. The estimated income distributions are indeed very close to the observed ones.51

D.2.3 Estimated Profiles: Individual Methods vs. Household Approach

Figure D.3 shows the age-saving profiles estimated by the two individual methods for the years 1992 and 2009. Although the two methods use different samples of households and different identification strategies, they yield very similar age-saving profiles.52 The discrepancy is larger at the beginning of the sample period. For the 1992 survey, there are only about 5,000 households in our sample (compared to about 16,000 after 2001), 44% of which are uni-generational households. This makes it difficult to re-sample observations to match the aggregate distributions of attributes as for some combinations of age and income level, there are very few observations. The larger size of the sample in more recent years makes the problem less severe and the profiles produced by the two methods become even more similar. Figure D.4 displays the age-saving profiles obtained by applying the commonly used household approach based on the age of the household head. As expected, this approach generates flatter profiles (aggregation bias) and much higher saving rates for the youngest individuals (largely driven by the selection bias).

D.2.4 Robustness Checks

Treatment of transfers. Intra-household transfers are not directly observable but we believe they should affect our estimates to a lesser extent when we consider the sub-sample of uni-generational households. Regarding inter-household transfers, the UHS data on individual

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51For the year 1986, information on income is available only at the household level. For that year, we therefore use the projection method to estimate both individual income and individual consumption. The estimated age-saving profile for 1986 is then used to construct the average profile over the first three years of observations (1986, 1992, and 1993).

52This suggests that our re-sampling procedure to control for income and gender characteristics in the first method (using the sub-sample of uni-generational) takes care of selection issues quite well. The first method does give slightly lower saving rates, indicating that some unobservable characteristics correlated with the household composition are also correlated with saving behavior.
Figure D.3: Estimated Age-Saving Profile for China in 1992 and 2009, Individual Methods. Notes: The uni-generational method resamples the data to match gender and income distributions by age group in the full sample. The youngest cohort are taken to be those < 26 under this method due to lack of observation for individuals younger than 24 in the sample of uni-generational households. Chesher method controls for household characteristics as described in Appendix C.3. UHS data (1992 and 2009).

Figure D.4: Estimated Age-Saving Profile for China in 1992 and 2009, Household Method. Notes: The saving rate for a given age is obtained as the average household saving rate for households whose head is of that age. UHS data (1992 and 2009).
income include information on received transfers (gifts from relatives, alimony, pensions and grants), but without detailed information on the source of the transfer (e.g., other households or government). Our measure of individual income includes all received transfers. The UHS survey also gives information on household transfer expenditures, i.e., transfer payments to other households (gifts to relatives, alimony, family support), but only at the household level. In the aggregate, we find that transfer expenditures are an order of magnitude larger than (received) income transfers, possibly due to underreporting of the latter. Furthermore, including transfer expenditures implies an estimate of the aggregate household saving rate in China that is significantly lower than estimates typically reported in the literature. As a result transfer expenditures are ignored in the estimated saving rates that we report. When implementing individual methods 1 and 2 on a measure of expenditures obtained as the sum of household consumption and transfer expenditures, we find age-saving profiles for the years 1992 and 2009 similar to those depicted in Figure D.3, but shifted downward by about 3%. However, estimates of changes in saving rates across age groups over the sample period are very similar whether transfers are included or not.

Other robustness checks. In non-reported robustness checks, we investigate alternative sets of controls in the Chesher and uni-generational methods, and alternative treatments of zero-income observations in the Chesher method. We also try dropping the top 1% and top 5% income earners from the sample. Estimated age-saving profiles are similar across all procedures — with the exception of the saving rate of individuals under 25 in the early years of the sample, for which our estimates vary between $-5\%$ and $+5\%$ depending on the method and the controls. This is due to the small number of observations for this age group at the beginning of the sample, especially when focussing on uni-generational households. Finally, we use the two individual methods to estimate age-saving profiles with an alternative Chinese survey (Chinese Household Income Project, CHIP) for the two years where these data are available (1995 and 2002). Results are very similar both across methods and across surveys.