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Efficiency in Decentralized Oligopolistic Markets

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Abstract: The paper analyses quantity competition in economies in which a network describes the set of feasible trades. A model is presented in which the identity of buyers, of sellers, and of intermediaries is endogenously determined by the trade flows in the economy. The analysis first considers small economies, and provides sufficient conditions for equilibrium existence, a characterization of prices and flows, and some negative results relating welfare to network structure. The second and central part of the analysis considers behavior in large markets, and presents necessary and sufficient conditions on the network structure for equilibria to be approximately efficient when the number of players is large.

Keywords: Decentralized Markets; Intermediation; Oligopoly; Efficiency; Market Power.

JEL Codes: L13, D6, D85, C7.

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1   Introduction

Classical models of competition rely on the anonymity of markets to explain prices and trade. According to this view, exchanges in an economy take place in centralized markets and the identity of players has no effect on prices and terms of trade. Recent models of decentralized competition depart from such a stark paradigm by considering markets in which exchanges take place in bilateral relationships. Prices and outcomes in such economies crucially depend on the set of feasible trades and on the implied market power. Results to date have mainly focused on economies in which the identity of buyers and sellers is exogenously determined, and in which only trade from sellers to buyers are feasible. This study analyzes decentralized oligopolistic markets in which the role of players in the economy is determined in equilibrium, and presents conditions on the structure of an economy for trade to be almost efficient when the number of players is large.

The paper introduces a static model of decentralized oligopolistic trade for economies in which a network describes the set of feasible trading relationships. In the model, individuals choose their supply to neighboring players, correctly anticipating that the equilibrium price for a trade will be given by a buyer's marginal value. Since trade affects the marginal rate of substitution of both players involved in a transaction, supply decisions influence both the price at which goods are purchased and the price at which they are sold. Traders maximize their private utility, taking into account how their supply decisions affect prices. The resulting flow of goods endogenously determines whether an individual buys, sells, or does both based on preferences, production possibilities and the position held in the network. That is, supply chains arise endogenously in this model. Intermediation and significant price dispersion are generic phenomena in small or poorly connected economies.

When the number of players is small, trade is necessarily inefficient because of the price distortions implied by quantity competition. However, when the number of players is large, simple conditions can be imposed on an economy to ensure that trade is approximately efficient. To study large markets with a fixed topology, the analysis introduces the notion of community structure of a trade network. Communities consist of subsets of players who share the same potential trade partners in the economy. For instance, when the network captures the geography of an economy, a community identifies the subset of players at a given geographical location. The analysis of large markets fixes the community structure and considers what happens when the number of players in some communities is large. Even when communities are large, trade between communities and intermediation may still be required to support an efficient allocation. Our main result establishes that trade is almost efficient if and only if it is possible to clear markets without recourse to intermediation in any large community. If so, direct competition among players belonging to neighboring large communities eliminates

resale and restores efficiency. In contrast, when intermediation is required to implement the efficient allocation, players mediating trade necessarily command a rent and distort trade, as their supply decisions affect feasible outcomes. If so, trade remains inefficient even when an economy is arbitrarily large.

The first part of the analysis develops baseline results for economies in which the number of players is small. It presents: sufficient conditions for pure strategy equilibrium existence; a characterization equilibrium prices, flows and markups; and some negative results on welfare. A key feature of the outflow model is that resale markups are strictly positive due to the double-marginalization problem faced by players acting as intermediaries. Thus, goods never cycle in equilibrium, and not all linked players with different marginal rates of substitution elect to trade. Individuals would never purchase units previously sold, because a higher price would have to be paid; and individuals with low willingness to pay might prefer not to sell their goods, as trade might increase the price paid on the units purchased.[1] Equilibrium behavior involves price discrimination across locations of the trade network. Intermediation arises both because of the scarcity of trading partners and because of the different prices that prevail throughout the economy in equilibrium.

Results on welfare first establish that trade is always inefficient in economies populated by finitely many players, and then present some negative conclusions relating welfare to network structure. Adding trading relationships does not necessarily improve social welfare in these markets, as more goods may flow to low value markets when sellers price discriminate locations in which the goods are most desired. More surprisingly, even though players have the option not to trade with any one of their neighbors, the welfare of an individual may decline when the number of players in his neighborhood increases. Since trading relationships are common knowledge, whenever new links raise the demand of an individual, price discrimination by his suppliers may decrease the amount of goods sold to him and consequently his welfare.

The second part of the analysis studies behavior in economies with a large number of players. It considers economies in which players are of finitely many types and are positioned at finitely many locations connected by a network. Necessary and sufficient conditions are presented for trade to be almost efficient when the number of players in every community is large. Any economy in which intermediation is required to clear markets, is inefficient independently of the size and structure of the market. Intermediaries necessarily command a rent whenever they are needed to distribute goods, and competition among them can reduce, but not eliminate resale markups. Efficient trade in such markets is equivalent to the existence of flows of goods that are both direct and efficient. Necessary and sufficient conditions for the existence of such flows are derived. These conditions are analogous to market clearing requirements for two-sided markets,

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[1]This implication differs from that of the Cournot model in which any two players with different marginal rates of substitution always elect to trade.

and require the aggregate demand of any subset of communities not to exceed the aggregate supply of communities with whom direct trade is feasible. Equilibrium outcomes can be fully characterized even when the conditions for efficiency fail. In such instances, intermediation and distortions persist even when the number of players at every location is arbitrarily large. Similar results are developed for economies in which only a subset of communities is large. Efficient trade in large communities is again equivalent to the existence of direct and efficient flows of goods between those communities. Efficiency in small communities further requires that players in those locations be able to directly trade in some large market. Results also establish that social welfare converges monotonically to efficiency when all communities grow large.

The analysis concludes by presenting an alternative quantity competition model in which individuals decide how much to buy, and in which units are sold at marginal value. Similar results hold, even though the distribution of rents differs. More rents flow to buyers, and social welfare is generally higher than when players choose how much to sell.

Literature Review: A vast recent literature has analyzed trade in buyer-seller networks. Such models usually take the identity of buyers and sellers in an economy as an exogenous characteristic of the market, and describe the set of feasible trades from sellers to buyers with a network. These papers differ mainly in how competition is modelled. Kranton and Minehart 2001 model competition among sellers as simultaneous ascending price auctions, and studies the formation of efficient link patterns. Corominas-Bosch 2004 models trade as a centralized non-cooperative bargaining game, and provides sufficient conditions on the network structure for the equilibrium of the bargaining game to coincide with the Walrasian outcome. Ilkilic 2010 discusses market power in the context of a linear-quadratic quantity competition model of two-sided markets. Lever 2010 analyses Bertrand competition between duopolists, and the relationship between network structure and welfare. Several papers on decentralized bargaining (Abreu and Manea 2012, Manea 2011, Polanski 2007, Polanski and Vega-Redondo 2013, Polanski and Winter 2010) also consider models of trade in two-sided markets, and analyze efficiency in such markets. All of these models however, rule out intermediation by assumption, and implicitly set the identity of buyers and sellers as a primitive of the problem.

Other papers have introduced some notion of intermediation in the context of a two-sided market. Blume, Easley, Kleinberg and Tardos 2007 study buyer-seller networks in which all trades have to be mediated by price-setting middlemen. Equilibria in this setting always implement an efficient allocation, and middlemen command a positive rent if and only if they possess an essential connection in the network structure. Siedlarek 2013 allows for more general structures of intermediation in the context of a model of coalitional bargaining, and shows that efficiency obtains when no intermediary is essential. The efficiency result however, relies partly on exogeneity of sellers and buyers and partly on the centralized nature of the bargain-

ing protocol considered. Manea 2014 has recently developed similar results in the context of a decentralized bargaining model, and identified frictions that might arise in such models. A related literature analyzes competition between owners of links on a network in which individuals selfishly route flows (Chawla and Roughgarden 2007, Acemoglu and Ozdaglar 2007, Choi Galeotti and Goyal 2014). This literature usually takes a Bertrand approach, and was developed to model competition between internet providers pricing information streams.

The quantity competition model presented here differs from the models discussed above, as the roles of individuals in a supply chain are endogenously determined in equilibrium. Kakade, Kerns and Orthiz 2004 characterizes the competitive equilibria of a general networked market in which the roles of players are endogenous. However, price taking behavior implies that network structure cannot directly affect market power. Condorelli and Galeotti 2012 analyzes sequential trade of a single unit in a general networked market in which there is some incomplete information about the value of the good. In the model prices decrease along the supply chain as trade reveals a low value for the good. This setting is closest in spirit to that of the current paper. But due to the complications arising from incomplete information and dynamics, the model remains stylized and does not deliver the results presented here.

The spirit of the analysis is close to the oligopoly and trade literature surveyed in Leahy and Neary 2010, which provides motivation for developing a tractable model of oligopolistic general equilibrium trade while highlighting the challenges posed by this exercise. Recent studies in the matching literature have also addressed the problem of intermediation in decentralized markets. Prime examples in this literature are Ostrovsky 2008, and Hatfield and Kominers 2012. Although these studies are motivated by similar questions, differences in the environment and in the notion of equilibrium remain significant. The framework analyzed is also evocative of classical cooperative game theory results which analyze properties of core allocations in similar environments, such as Kalai, Postlewaite & Roberts 1978.

Roadmap: Section 2 analyses outflow competition. It presents the model, a characterization of equilibrium prices and flows, and results for small economies. Section 3 discusses outflow competition in large economies, and presents conditions for efficiency. Section 4 discusses inflow competition. Section 5 concludes. All proofs can be found in appendix.

2   Outflow Competition

The section begins with a description of the economy and of the outflow competition model, and proceeds with results on equilibrium behavior and welfare in small economies.

The Economy and Efficiency

Consider an economy with a finite set of players, V, and two goods. For convenience, refer to

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the two goods as consumption q, and money m. Any player i in the economy can trade goods only with a subset of players $V_i \subseteq V \setminus \{i\}$, which is called the *neighborhood* of Player i. Assume that $j \in V_i$ if and only if $i \in V_j$. This structure of interaction defines an undirected graph $G = (V, E)$ in which $ij \in E$ if and only if $j \in V_i$. Refer to G as the *trade network*. Assume that trade network is connected, as any component would act as a separate economy otherwise.[2]

Denote by q_j^i the flow of consumption good from individual i to individual j. Since trade can occur only between players that know each other, $q_j^i = 0$ whenever $ji \notin E$. For any player i, define the *total purchases* and the *total sales* of consumption good respectively as

$$q_i^\circ = \sum_{k \in V_i} q_i^k \quad \text{and} \quad q_\circ^i = \sum_{k \in V_i} q_k^i.$$

Let the *net-trade* of Player i be defined as the difference between these two quantities, $q_i = q_i^\circ - q_\circ^i$, and let the *resale* of Player i be defined as the smallest among the two, $r_i = \min\{q_i^\circ, q_\circ^i\}$. When an individual purchases more (fewer) units than those he sells, his resale consists of all the units that he sells (buys). Bold letters are used to denote vectors of flows. In particular, $\mathbf{q}^i$ denotes the vector of consumption flows from i to his neighbors in V_i; $\mathbf{q}$ denotes the Cartesian product of all the $\mathbf{q}^i$'s; and $\mathbf{q}^{-i}$ denotes the Cartesian product of $\mathbf{q}^j$ for all $j \neq i$.

The utility of every individual in the economy is separable in the two goods, and linear in money. In particular, Player i's utility for a net-trade q_i and an amount of money m simply satisfies

$$u_i(q_i) + m.$$

The net-trade of any player i is bounded from below by a non-positive number $-Q_i$. Refer to Q_i as the capacity of Player i. Since $Q_i > 0$ is possible, players can sell more units, than they purchase. This setup can capture both endowment and production economies. In the production interpretation of the model, $-u_i(q_i)$ can be viewed as the cost of supplying $-q_i > 0$ units to the market. Non-negativity constraints on monetary holdings are neglected throughout the analysis. It is implicitly assumed that monetary endowments are sufficiently large for such constraints never to bind. The following standard assumptions on payoffs are maintained throughout.

Assumption A1 *For any player $i \in V$, u_i is three times continuously differentiable, strictly increasing and strictly concave on $[-Q_i, \infty)$.*

For convenience, denote an economy by $\mathcal{E} = \{V, G, \mathbf{Q}, \mathbf{u}\}$. For any profile of flows $\mathbf{q} \in \mathbb{R}_+^E$, social welfare is evaluated by the sum of payoffs. Since the payoffs are quasi-linear and monetary endowments are large, any interior Pareto optimum maximizes the sum of the utilities of the

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[2]A network is connected if for any $i, k \in V$ there exists an m tuple of players $(j_1, ..., j_m)$ such that $j_1 = i$, $j_m = k$, and $j_{k+1} \in V_{j_k}$ for all $k = 1, 2..., m-1$. Any maximal connected subgraph of G is a component of G (Bollobas 1998).

non-linear good. As all trades can be executed when the network is connected, the definition of efficiency abstracts from the network structure and only identifies welfare maximizing net-trades.[3]

Efficiency: *A profile of net-trades* $\bar{\mathbf{q}} \in R_+^V$ *is efficient for an economy* $\mathcal{E}$ *if it solves*

$$\bar{\mathbf{q}} \in \arg\max_{\mathbf{q} \in \mathbb{R}^V} \sum_{i \in V} u_i(q_i) \ \text{ s.t. } \ q_i \geq -Q_i \ \text{ for } \forall i \in V.$$

Assumption A1 implies that efficient net-trades exist, and that they would attain as a competitive equilibrium in the corresponding centralized market.

Outflow Competition

In the model of competition considered, the description of the economy is common knowledge.[4] Players can only trade with their neighbors, and simultaneously decide how many units of consumption to sell to each of them while being required not to sell more units than their capacity Q_i. As customary in quantity competition models, prices are determined so that buyers pay all of the units purchased at their marginal value. In particular, the price paid by Player i for units sold from a neighbor j is determined by i's inverse demand curve for consumption,

$$p_i^j(\mathbf{q}) = p_i(q_i) = u_i'(q_i) = u_i'(q_i^\circ - q_\circ^i). \tag{1}$$

The proposed pricing mechanism could be micro-founded in the context of a two-stage model in which suppliers first commit to sales of consumption to known buyers, and then compete on prices to supply these buyers. Indeed, if suppliers were able to commit to outflows, and if they were to compete on prices at each local market given their outflow decisions, equation (1) would still dictate pricing, since no supplier would benefit from a unilateral deviation in the price-setting game. Price reductions would not affect the quantity sold as all units supplied are sold, while price increases would reduce revenues because of falling sales. This observation was first made in Kreps and Scheinkman 1983 while studying Bertrand competition with quantity commitment. Their results extend immediately to the outflow framework, since no restrictions were imposed on the number of buyers.[5] Thus, pricing in the outflow model captures behavior in markets in which local supply decisions have to be made prior to competition.

The concavity of the utility function implies that the price paid by any player i decreases

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[3]If the network had more than one component, efficiency would also have to account for the infeasibility of trade across components.

[4]The assumption is strong, but considerably simplifies the analysis. It is plausible for environments in which the network captures geographical or legal trade costs. It would be interesting to study a setup in which players have incomplete information about the global network structure. The analysis presented here would apply only to setups in which players hold correct beliefs about equilibrium net-trades at neighboring locations.

[5]The proposed two-stage model would always possess Subgame Perfect equilibria in which prices and flows coincide with the Nash equilibria of the outflow competition model.

when his inflows increase, increases when his outflows increase, and is not directly affected by other flows in the economy. That is, $\partial p_i(q_i)/\partial q_i^j < 0$ and $\partial p_i(q_i)/\partial q_j^i > 0$ for any neighbor $j \in V_i$. When choosing their outflows, sellers account for the distortions that their supply decisions might induce both on the prices they receive for each unit sold and on the price they pay for each unit bought. The *welfare* of an individual i given a profile of flows $\mathbf{q}$ is therefore determined by the map,

$$w_i(\mathbf{q}) = u_i(q_i) + \sum_{k \in V_i} \left[p_k(q_k)q_k^i - p_i(q_i)q_i^k \right],$$

where prices are pinned down by equation (1), and where the summation denotes the trade surplus of Player i. In what follows the expression *outflow equilibrium* will be used to refer to a pure strategy Nash equilibrium of the outflow competition model.

Outflow Equilibrium *Flows $\mathbf{q} \in R_+^E$ constitute an outflow equilibrium, if for any $i \in V$,*

$$\mathbf{q}^i \in \arg\max\nolimits_{\mathbf{y}^i \in \mathbb{R}_+^{V_i}} w_i(\mathbf{y}^i, \mathbf{q}^{-i}) \quad \text{s.t.} \quad y_o^i \leq Q_i.$$

The outflow constraint $q_o^i \leq Q_i$ requires total sales not exceed capacity, and is more demanding than requiring net-trades to be bounded by capacity. The constraint implies that players cannot resell units unless these can potentially be produced in house. Although the stronger restriction is far from ideal in many applications, it has no effect on the results and it is only imposed for clarity, as it guarantees that action sets do not depend on the supply decisions of other players. All of the conclusions presented (including those on existence) would also hold under the weaker outflow constraint $q_i \geq -Q_i$. However, the model would cease to be a game as the set of feasible actions would be determined in equilibrium by means of a fixed point argument.

Outflow Equilibrium Existence

The first result presents sufficient conditions for outflow equilibrium existence, and a characterization of equilibrium flows of consumption. Bounds are imposed on the slope and the curvature of every demand function to guarantee that the payoff of every player remains well-behaved (a standard assumption in imperfect competition models). Denote the elasticity of the inverse demand curve of Player i with respect to quantity by $\eta_i(q) = -(Q_i + q)u_i''(q)/u_i'(q)$. Also, denote Player i's *total cost* of supplying outflows and Player i's *revenue* from suppling units to market $j \in V_i$ respectively by

$$\begin{aligned} C_i(\mathbf{q}^i, \mathbf{q}^{-i}) &= -u_i(q_i) + u_i'(q_i)q_i^\circ, \\ R_j^i(q_j^i, \mathbf{q}^{-i}) &= u_j'(q_j)q_j^i. \end{aligned}$$

The total cost of supplying outflows is determined by adding the cost of forgone net-trades

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$-u_i(q_i)$ to the expenditure on inflows $u_i'(q_i)q_i^\circ$. The welfare of an individual can thus, be expressed as the sum of the revenue made in each neighboring market, minus the total cost of supplying such outflows.

Consider the following constraints on the elasticity of the slope of the inverse demand curve with respect to quantity.

Assumption A2 *For any player $i \in V$, the utility u_i satisfies at least one of the following two conditions for any $q > -Q_i$:*
[B1] $-(Q_i + q)u_i'''(q)/u_i''(q) \in [-1, 2]$;
[B2] $-(Q_i + q)u_i'''(q)/u_i''(q) \in [-\eta_i(q)/V_i, 2\eta_i(q)]$.

Assumption A2 is evocative of the sufficient conditions for the existence of a solution to the monopoly problem, as it bounds the elasticity of demand from above.[6] Conditions in A2 are stronger however, as the elasticity must also be bounded from below when players are allowed both to buy and to sell units.

The next result establishes that assumption A2 guarantees the existence of an outflow equilibrium, and characterizes the conditions identifying any outflow equilibrium.

Proposition 1 *If A1 and A2 hold, outflow equilibria exist and coincide with the solutions $(\mathbf{q}, \boldsymbol{\mu}) \in \mathbb{R}_+^E \times \mathbb{R}_+^V$ to the complementarity problem:*

$$f_j^i(\mathbf{q}, \boldsymbol{\mu})q_j^i = 0 \quad and \quad f_j^i(\mathbf{q}, \boldsymbol{\mu}) = u_i'(q_i) - u_j'(q_j) - u_j''(q_j)q_j^i - u_i''(q_i)q_i^\circ + \mu_i \geq 0 \quad for \quad ij \in E$$
$$f_i(\mathbf{q}, \boldsymbol{\mu})\mu_i = 0 \quad and \quad f_i(\mathbf{q}, \boldsymbol{\mu}) = Q_i - q_\circ^i \geq 0 \qquad\qquad\qquad\qquad for \quad i \in V.$$

Existence obtains because the set of feasible outflows of every player is non-empty, convex and compact, and because assumption A2 guarantees that best responses are continuous and single-valued (and that Brouwer's fixed point theorem thus applies). In particular, condition B1 requires revenues to be concave and total costs to be convex in any market; while condition B2 requires total costs to be convex and revenues to be concave only when revenues increase in a market. Either condition implies that best responses are single-valued, as the payoff of every player is concave whenever increasing. Any combination of the bounds would also grant existence, as the lowerbounds discipline only total costs, while the upperbounds only revenues. It can be readily verified that common families of preferences meet the proposed conditions for outflow equilibrium existence.

Remark 2 *An outflow equilibrium exists if one of the following two conditions holds:*
(a) $u_i(q) = \beta_i(Q_i + q)^{\alpha_i}$ *for* $\alpha_i \in (0, 1)$ $\beta_i \in \mathbb{R}_{++}$ *any $i \in V$;*
(b) $u_i(q) = -\beta_i e^{-\alpha_i(Q_i + q)}$ *for* $\alpha_i \in \mathbb{R}_{++}$ $\beta_i \in \mathbb{R}_{++}$ *any $i \in V$.*

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[6]Classical monopoly requires the elasticity of inverse demand to be bounded above by 2.

The second part of Proposition 1 characterizes outflow equilibria as solutions to the system of best responses (where μ_i denotes the multiplier on the capacity constraint of Player $i \in V$). When the outflow constraint $q_\circ^i \leq Q_i$ does not bind, the optimality of an interior outflow $q_j^i > 0$ simply requires that

$$p_j(q_j) - p_i(q_i) = -p_j'(q_j)q_j^i - p_i'(q_i)q_i^\circ.$$

If so, the markup on the flow q_j^i (the difference between price received and the marginal cost of forgone consumption) is completely determined by two wedges: one distorts of the price paid by Player j, while the other distorts the price paid by i for all inflows purchased. The first wedge is due to the fact that i is a Cournot supplier of j, while the second wedge is due to the fact i is a monopsonistic buyer at his location. Pricing behavior in the outflow model favors suppliers as the demand curve is used to clear each local market. Section 4 explores the consequences of the alternative setup in which sellers own the trading location and in which buyers commit to inflows.

Four Player Examples

Before the formal discussion of outflow equilibrium properties, consider a simple economy with four players, labeled $\{a, b, c, d\}$. Let $Q_a = 5$, $Q_b = 2$, and $Q_c = Q_d = 1/2$, and assume that preferences of every Player i satisfy $u_i(q) = (Q_i + q)^{1/2}$. Throughout the examples, for convenience, interpret Q_i and $Q_i + q_i$ respectively as the endowment and the equilibrium consumption of Player i. If no trade takes place in the economy, social welfare is worth 5.06. Efficiency instead, requires players to split the consumption good equally. Social welfare at this allocation is maximal and equal to 5.66. Equal sharing however, is not an outflow equilibrium even when all trades are feasible. When the trade network is complete, in the unique outflow equilibrium Player a sells to all of his neighbors, and Player b resells some of the goods purchased from a to c and d. Players c and d do not trade with each other since they are identical and in a symmetric position.[7] Equilibrium flows do not equalize marginal rates of substitutions. The price paid by consumers c and d for each unit of consumption purchased is 0.41. This price exceeds the price charged by consumer a to b on the units traded, 0.34. Even though a has the option not to sell to his competitor, b, he prefers to do so, because it is profitable, and because it is impossible to prevent b from supplying the final consumers, c and d. Thus, Player b is able to impose a 21% markup on all the units that he resells. Equilibrium flows for this economy are reported in the first network of figure 1. Consumption, prices and welfare can be found in the first matrix of table 1. In equilibrium, consumers a and b curtail their supply to c and d in order to maximize their gains from trade. The allocation is inefficient and social welfare is equal to 5.61.

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[7]If the link cd were removed from the trade network, equilibrium flows and prices would not be affected, since no trade takes place between c and d.
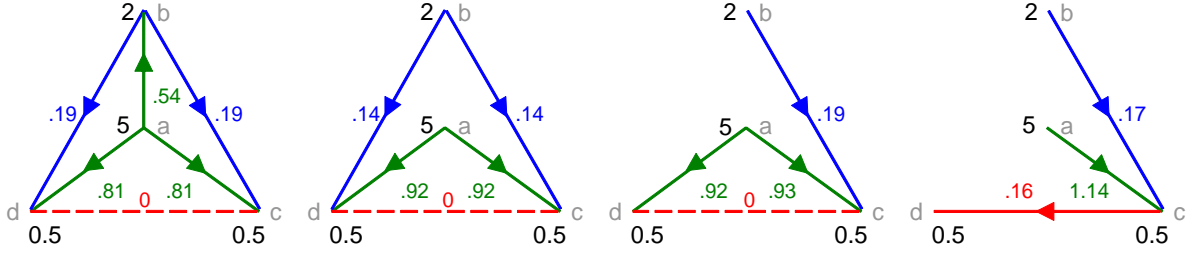
FIGURE 1: Flows in the four examples: on the vertices are endowments and identities and on the edges are equilibrium flows and their direction.

Severing the link between players a and b surprisingly does not favor a by giving him the opportunity to commit not to sell to b. When the link ab is removed from the trade network, consumption of every player, except b, increases. The final consumers, c and d, purchase more goods at a lower price and are better off. But consumers a and b are worse off. The equilibrium remains inefficient, and social welfare decreases further to 5.59. The price paid by consumers c and d for each unit of consumption purchased decreases to 0.4, and coincides with the Cournot equilibrium price for the economy without a network. The second network in figure 1 and the second matrix in table 1 describe the unique equilibrium of this economy.

	p	x	w		p	x	w		p	x	w		p	x	w
a	-	2.84	2.53	a	-	3.16	2.51	a	-	3.15	2.53	a	-	3.86	2.41
b	0.34	2.16	1.44	b	-	1.72	1.42	b	-	1.81	1.42	b	-	1.83	1.42
c	0.41	1.50	0.82	c	0.40	1.56	0.83	c	0.39	1.62	0.83	c	0.39	1.65	0.87
d	0.41	1.50	0.82	d	0.40	1.56	0.83	d	0.42	1.42	0.80	d	0.62	0.66	0.71
+	-	8.00	5.61	+	-	8.00	5.59	+	-	8.00	5.58	+	-	8.00	5.41

TABLE 1: Equilibrium prices paid, consumption $x = Q + q$, and welfare for each player in the four economies.

If the link between players b and d is also removed from the network, consumer d remains with only a and c as potential suppliers, while consumer c can still purchase from both a and b. In equilibrium, a and b still supply all of their neighbors. But, consumer c opts not to resell to d despite having more consumption good than d, since selling to d would increase the price paid on all the units purchased. In the outflow model, linked players with different marginal rates of substitution occasionally choose not to trade, as a commitment not to resell can significantly reduce the price paid on all the units purchased. The third network in figure 1 and the third matrix in table 1 characterize the unique equilibrium of this economy. Since Player c has two suppliers, while Player d has only one that is active, Player c pays a lower price for consumption than d. Player a sells more units in the competitive market than in the one in which he is a monopolist. Social welfare decreases further to 5.58. Finally, consider the economy in which

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all individuals can only trade with c. In such a market, players a and b sell to c who in turn supplies d with some of the units purchased. The final network in figure 1 and the final matrix in table 1 describe the unique equilibrium of this economy. Player c's markup on the units sold to d amounts to 58%. Resale takes place despite the large intermediation rent. However, price distortions constrain sales from Player c to Player d, as resale sharply increases the price paid by Player c to his suppliers. Social welfare drops to 5.41. Consumers a and d are worse off, while consumer c is better off as he mediates any trade with d.

Outflow Equilibrium Properties

Two sets of results are presented about outflow equilibria in economies with a small number of players. The first addresses the properties of equilibrium flows and pricing, while the latter presents several negative conclusions on welfare.

In the outflow model, consumption flows from players with low marginal value to players with high marginal value, as assumption A1 implies that $q_i^j > 0$ only if $u_j'(q_j) > u_i'(q_i)$. The worst possible use of the goods owned is therefore consumption and not trade; and buyers are never willing to pay more than their marginal value for the last unit purchased. Consumption flows only in one direction on every link, and at most $|E|/2$ flows are positive in equilibrium. Individuals sell, or resell, goods to their neighbors only if the gains from trade compensate them both for the monopsony price distortion on inflows and for the Cournot distortion on outflows. A positive difference in marginal rates of substitution is therefore necessary, but not sufficient for trade to take place among pairs of linked individuals. Small differences in marginal rates of substitution may not suffice for trade to take place, as the monopsony distortion may prevent trade between players who value consumption similarly. Equilibrium retail markups are always strictly positive, as $q_i^\circ \geq r_i > 0$ implies that $p_j(q_j) > p_i(q_i)$ even when q_j^i is small. Intermediation however, remains a common phenomenon because of the limited number of trading relationships that can be used to transfer goods, and because of the sellers' incentives to price discriminate neighboring buyers. The latter motive explains why intermediation can take place in equilibrium even when the trade network is complete. The next proposition summarizes several useful properties of outflow equilibria. For convenience, refer to an individual as a *source* (*sink*) if he does not buy (sell) consumption.

Proposition 3 *If A1 and A2 hold, in any outflow equilibrium* $\mathbf{q}$:
(a) $q_j^i > 0$ *implies* $p_j(q_j) > p_i(q_i)$, *and the converse may not hold;*
(b) goods do not cycle and prices strictly increase along any supply chain;
(c) players with marginal utility lower (higher) than all their neighbors are sources (sinks);
(d) if unconstrained, sources sell to all their neighbors with strictly higher marginal utility;
(e) if $i, j \in V_k$ *and* $p_j(q_j) > p_i(q_i)$, *then* i *buys from* k *only if* j *buys from* k.

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Part (a) follows from the previous discussion, and (b) is an immediate consequence of goods being resold at strictly positive markups. In fact, because the marginal utility of consumption strictly increases along any supply chain, it can never be that an individual buys units he previously sold. Since goods do not cycle, flows of goods move from sources to sinks. Flows however, may have more than one source and/or sink in equilibrium. Part (c) follows because individuals with lower marginal utility than their neighbors would never buy, as only players with lower marginal utility could supply them, and must therefore be sources. Similarly, individuals with higher marginal utility than their neighbors would never sell, and must therefore be sinks. Part (d) shows why sources sell to every neighbor with higher marginal utility if the outflow constraint does not bind. If so, a positive difference in marginal rates of substitution is not only necessary, but also sufficient for trade to take place, because sources have no inflows and because outflow price distortions vanish with outflows. Part (e) finally, establishes that if two players have a neighbor in common, that neighbor sells to the low marginal utility player only if he sells to the high marginal utility player. Results in the web-appendix show that when the network is complete, the ranking of marginal utilities coincides with the ranking of supply costs.[8]

The next result exploits some of the properties of outflow equilibria to derive several negative conclusions on welfare. As before, evaluate *social welfare* by summing the welfare of each player in the economy,

$$\sum_{i \in V} w_i(\mathbf{q}) = \sum_{i \in V} u_i(q_i).$$

Results establish that inefficiencies are a common feature of the outflow equilibrium model, and show why adding links might have unexpected consequences on individual and social welfare.

Proposition 4 *If A1 holds, the following conclusions apply for $i, j \in V$:*
(a) an efficient outflow equilibrium $\mathbf{q}$ exists if and only if $\mathbf{q} = \mathbf{0}$ is efficient;
(b) equilibrium social welfare in a network (V, E) can be higher than in a network $(V, E \cup ij)$;
(c) equilibrium welfare of i in a network (V, E) can be higher than in a network $(V, E \cup ij)$.

When trade is required to support the efficient allocation, any outflow equilibrium is necessarily inefficient, as price distortions curtail trade in any local market. If so, social welfare may decline when new trading relationships are added to the network.[9] A new trading link can further distort the allocation of consumption, as profit maximization by sellers may reallocate consumption from high to low value buyers. The payoff of a player can also decline when a new trading partner is added to his neighborhood. If a new trading partner increases the marginal

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[8]Complete networks guarantee that low marginal utility players sell more. However, without further discipline on preferences it is impossible to guarantee that players with low marginal utilities also buy less from their neighbors. Section 2 of the web-appendix presents sufficient conditions for this to be the case.

[9]A negative relationship between social welfare and network density obtains in numerous other studies, and is often referred to as Braess's Paradox in the context of the networks literature.

value of consumption of Player i (due to the option value of reselling units), players selling to i might curtail their supply in order to extract some of the surplus generated in the new relationship. If this effect is sufficiently pronounced, Player i's payoff declines in any outflow equilibrium when a new trading partner is added to his neighborhood.[10]

Welfare conclusions rely on the market power frictions implicit in any quantity competition model. The main aim of the rest of the analysis is to provide conditions on the network structure for such frictions to vanish when the number of players in the economy is large.

Comments on Outflow Competition and Market Power: In the model presented, nodes on a network were interpreted as separate local markets. Competitors used their access to different locations to price discriminate their customers. As discrimination within a local market was ruled out by linear pricing, discriminating across markets was welfare maximizing for suppliers. Preferences and access to markets jointly determined prices, welfare and market power. Goods were exchanged at local prices that differed from the competitive equilibrium price. Resale at positive markups was common even in well connected economies, and was driven by the arbitrage opportunities that the different prices in the economy offered to traders. The monopsony wedges were the main force limiting intermediation in the model, as the cost of supplying units was shown to increase along any supply chain. Although an explicit characterization of market power remains desirable, it was not possible to deliver a tractable and general result mainly due to the non-linearities in the complementarity problem characterizing the outflow equilibria.

3   Large Markets and Efficiency

This section analyzes pricing behavior in economies with a large number of traders, and presents necessary and sufficient conditions on the network topology for outflow equilibrium trades to be approximately efficient. To impose some discipline on the network structure as the number of players diverges to infinity, the analysis introduces the notion of community structure of a trade network. A community will be defined as a complete subgraph in which all players share the same neighbors. Any trade network will be represented by a corresponding network among communities. The analysis fixes the topology among communities and studies outflow equilibrium behavior when communities are large. The aim of this exercise is to provide a tractable model of market power distortions in large markets. Communities here will be interpreted as cities to capture a distinctive feature of cities, namely that individuals of a given city are

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[10]Despite the negative conclusions obtained in proposition 4, it would be interesting to argue that it is always possible to add a link to any incomplete trade network that weakly increases either social or individual welfare. If so, the complete network would be both welfare maximizing and pairwise stable. The proof of this conjecture however, remains an open question.

located in the same geographical position (and can thus freely trade with other inhabitants of their city and with the same set of neighboring cities).

The first result presents necessary and sufficient conditions for the existence of trade flows among communities which are efficient and direct (without resale). Such conditions follow from an adaptation of Hall's marriage theorem to our more complex environment.[11] The analysis proceeds to show that these conditions are both necessary and sufficient for the existence of an efficient outflow equilibrium when the number of players in every community is large. When all communities have comparable magnitudes, these conditions imply that intermediation must be negligible for trade to be efficient in every community. When communities have different magnitudes approximate efficiency only requires the absence of intermediation between any two large communities. In such economies, trade in smaller communities may remain inefficient unless large communities can execute all of their trades. The section concludes with some examples and by discussing the relationship between social welfare and market size.

Community Structure and Market Clearing

The notion of community structure of a network G is now introduced. A subset of players $C \subseteq V$ is said to be a *community* in G if: (i) $V_i \cup \{i\} = V_j \cup \{j\}$ for any two players $i, j \in C$; and (ii) $V_i \cup \{i\} \neq V_k \cup \{k\}$ for any player $k \in V \backslash C$. Thus, a community consists of a completely connected subset of players who share the same players as neighbors.[12] A community differs from a clique (a maximal complete subgraph) in that players share the same neighbors if they belong to the same community, but not necessarily if they belong to the same clique. Denote by $\mathbf{C}$ the set of communities of a network G. The set $\mathbf{C}$ uniquely partitions the vertices of the original trade network into disjoint subsets of players. The *community structure* of a network G is network $\mathbf{G} = (\mathbf{C}, \mathbf{E})$ with communities as vertices $\mathbf{C}$ and with edges between any two communities $C, K \in \mathbf{C}$ defined so that $CK \in \mathbf{E}$ if $ij \in E$ for some $i \in C$ and $j \in K$. The definition of community implies that, whenever two communities are linked, all of their inhabitants can trade with each other. The rest of the analysis presents results in terms of the community structure. The approach is without loss of generality, as there is a one-to-one mapping between community structure and network structure.

To understand when intermediation is needed to clear markets at the efficient allocation, it is useful to understand some properties of the efficient net-trade $\bar{\mathbf{q}}$. Let $D = \{i \in V | \bar{q}_i > 0\}$ denote the set of players demanding consumption at the efficient profile $\bar{\mathbf{q}}$, and let $S = \{i \in V | \bar{q}_i < 0\}$ denote the set of players supplying consumption at $\bar{\mathbf{q}}$. Refer to players in D as *buyers,* and to players in S as *sellers*. For any community $C \in \mathbf{C}$, denote by $q_C^+ = \sum_{i \in C \cap D} \bar{q}_i$ the aggregate demand of that community, and denote by $q_C^- = \sum_{i \in C \cap S} \bar{q}_i$ its aggregate supply. The efficient

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[11] Hall's marriage theorem provides conditions on a bipartite graph for the existence of a match that clears the short side of the market (Bollobas 1998).

[12] The definition of community implies that $V_i \cup \{i\} \supseteq C$ for any $i \in C$.

net-trade of any community can thus be defined as the difference between these two quantities, $\bar{q}_C = q_C^+ + q_C^-$. For any subset of communities $T \subseteq \mathbf{C}$, let $\mathbf{V}_T$ denote the set of communities that can trade with at least one community in T,

$$\mathbf{V}_T = T \cup \{K \in \mathbf{C} \mid CK \in \mathbf{E} \text{ for some } C \in T\},$$

and define the *excess-supply* and the *excess-demand* of group T respectively as

$$\begin{aligned}
\sigma(T, \bar{\mathbf{q}}) &= -\sum_{C \in \mathbf{V}_T} q_C^- - \sum_{C \in T} q_C^+ \text{ and} \\
\delta(T, \bar{\mathbf{q}}) &= \sum_{C \in \mathbf{V}_T} q_C^+ + \sum_{C \in T} q_C^-.
\end{aligned}$$

The excess-supply (excess-demand) amounts to the difference between the aggregate supply (demand) of communities who can directly sell to (buy from) communities in T and the aggregate demand (supply) of communities in T. These definitions depend on the network structure and on the notion of efficiency, but not on any element of the outflow competition setup. For convenience, say that an economy meets condition MC (or *market clearing*) if any group of communities faces a non-negative excess-supply.

Condition MC: *Economy $\mathcal{E}$ satisfies MC if $\sigma(T, \bar{\mathbf{q}}) \geq 0$ for any $T \subseteq \mathbf{C}$.*

A simple economy satisfying MC is one in which every seller is linked to every buyer. If so, MC holds trivially as the aggregate excess supply equals zero by construction, $\sigma(\mathbf{C}, \bar{\mathbf{q}}) = 0$.[13] The next result generalizes Hall's marriage theorem to our environment. It establishes that condition MC is both necessary and sufficient for the existence of direct flows of consumption from sellers to buyers that support an efficient allocation $\bar{\mathbf{q}}$. The proposition also shows that MC is equivalent to requiring any group of communities to face a non-negative excess-demand. The result is related to Gale 1957, who studies the existence of market clearing flows in environments in which intermediation is possible, and in which the capacity of every link is bounded.

Proposition 5 *For any economy $\mathcal{E}$ the following three statements are equivalent:*
(a) the economy satisfies MC;
(b) $\delta(T, \bar{\mathbf{q}}) \geq 0$ for any $T \subseteq \mathbf{C}$;
(c) there exists $\mathbf{q} \in \mathbb{R}_+^E$ such that:

$$\begin{aligned}
\bar{q}_i &= +\sum_{j \in S \cap V_i} q_i^j \quad \text{for any } i \in D, &\text{(i)} \\
\bar{q}_i &= -\sum_{j \in D \cap V_i} q_j^i \quad \text{for any } i \in S. &\text{(ii)}
\end{aligned}$$

The third statement in the proposition amounts to the existence of consumption flows that clear

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[13] As MC hinges on the definitions of D and S and in turn on the definition of efficiency, some information about preferences and technologies may be required to test whether MC holds when the network is incomplete.

markets in environments in which intermediation is not feasible. Efficient and direct flows of consumption exist if and only if any subset of communities can have its efficient net-trade met by those communities to which it is linked to.[14] As in the marriage theorem, the more surprising part of the result is that MC is sufficient for the existence of such flows of consumption, since necessity obtains trivially. The existence of direct and efficient consumption flows plays a central role in the analysis of large markets, as results establish that intermediation necessarily distorts trade.

Large Markets and Outflow Competition Efficiency

The next results present sufficient conditions on community structure for the existence of an approximately efficient outflow equilibrium when some communities are large. Large markets are introduced by fixing community structure and increasing the number of players in some communities. The approach is convenient, as it affects the extent of the competition in each community without changing the overall topology among communities. When communities are interpreted as cities and the community structure as the network of the feasible trades among cities (where limitations arise either because of geography or because of trade barriers), the analysis provides conditions on the economy for trade to be efficient when some cities are large.

It is convenient to introduce the notion of a replica economy in the context of markets in which a subset of communities is large and comparable in magnitude (cities), while the remaining communities are small (villages). This approach is almost without loss of generality, since conclusions on approximate efficiency rely only on behavior in communities with the largest magnitude. Fix a baseline economy $\mathcal{E} = \{\mathbf{C}, \mathbf{E}, \mathbf{Q}, \mathbf{u}\}$, a number $z \in N_+$, and a subset of communities $\hat{\mathbf{C}} \subseteq \mathbf{C}$.

Replica Economy: $\mathcal{E}(\hat{\mathbf{C}})^z = \{\mathbf{C}^z, \mathbf{E}^z, \mathbf{Q}^z, \mathbf{u}^z\}$ *is a* $(\hat{\mathbf{C}}, z)$*-replica economy of* $\mathcal{E}$ *if:*
[R0] for any $C \in \mathbf{C}\backslash\hat{\mathbf{C}}$ there exists $C^z = C$;
[R1] for any $C \in \hat{\mathbf{C}}$ there exists $C^z = \{i.s | i \in C \ \& \ s \in \{1, ..., z\}\}$;
[R2] $\mathbf{C}^z = \{C^z | C \in \mathbf{C}\} \ \& \ \mathbf{E}^z = \{C^z K^z | CK \in \mathbf{E}\}$;
[R3] $Q_{i.s}^z = Q_i \ \& \ u_{i.s}^z = u_i$ for any $i \in V \ \& \ s \in \{1, ..., z\}$.

The first two conditions state that for any community in the baseline economy there is a corresponding replicated community in its $(\hat{\mathbf{C}}, z)$-replica. Replicated communities in $\hat{\mathbf{C}}$ consist of z copies of the players in the baseline community, while replicated communities not in $\hat{\mathbf{C}}$ coincide with those in the baseline economy. The last two conditions in the definition instead

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[14]In the context of Hall's marriage theorem, condition MC simplifies to having any group of players on one side of the market linked to a group of players on the other side of the market which has at least its size. Even in that environment the definition of MC relies both on the network structure and on the notion of efficiency (as a match between players on different sides of the market has value, whereas one between players on the same side has none).

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require that community structure is not affected by replication, and that all copies of a player have the same capacity and preferences. While increasing competition within each community, the notion of replica preserves the community structure in an economy and the composition of players within each community (only increasing the number of players in some communities). The concept of replica is introduced for sake of tractability, but any large market with a finite number of communities populated by finitely many types of players can be approximated by a replica economy.

A $(\hat{\mathbf{C}}, z)$-replica is said to be *balanced* if $\hat{\mathbf{C}} = \mathbf{C}$. Balanced replicas keep the relative size of communities constant, and guarantee that all communities remain comparable in magnitude. A convenient feature of balanced replication is that the efficient net-trades of players coincide in any replica of a baseline economy, as preferences are concave. Therefore, buyers (sellers) in an economy remain buyers (sellers) in anyone of its replicas. In contrast, when replication is not balanced $\hat{\mathbf{C}} \neq \mathbf{C}$, efficient net-trades may differ in any two replicas, since the composition of players in the economy might be affected by the replication process.

A sequence of replica economies $\left\{ \mathcal{E}(\hat{\mathbf{C}})^z \right\}_{z=1}^{\infty}$ is said to *converge to efficiency*, if there exists a sequence of outflow equilibria in which net-trades converge pointwise to the efficient net-trades for every player in every community. Similarly, a sequence of replica economies is said to *converge to approximate efficiency*, if there exists a sequence of outflow equilibria in which net-trades converge pointwise to the efficient net-trades for every player in any community belonging to $\hat{\mathbf{C}}$. The notion of approximate efficiency is introduced as the surplus of any small community is negligible compared to aggregate surplus in any large replica economy.

Begin by considering balanced replication. Proposition 6 is central to the analysis, and shows that condition MC is both necessary and sufficient for outflow equilibrium net-trades to converge to efficiency when all communities have comparable magnitudes. Furthermore, trade converges to efficiency only if no player resells consumption in the limiting economy.

Proposition 6 *If A1 holds, a sequence of balanced replica economies converges to efficiency:*
(a) only if no individual resells consumption in the limiting economy;
(b) if and only if MC holds in the baseline economy $\mathcal{E}$.

Intermediaries command a rent that distorts trade in markets of any size. When each local market becomes more competitive, players become price takers as sellers, but never as buyers since they retain local monopsony power when purchasing goods. The wedge on inflow prices cannot disappear for intermediaries mediating a non-negligible flow of consumption, as their trading decisions affect the allocation of consumption in the economy and consequently welfare. Thus, large economies failing condition MC never converge to efficiency, as players acting as intermediaries would necessarily command a rent if they were required to mediate trade. In the outflow model, intermediaries retain market power whenever their aggregate supply decisions

affect feasible outcomes. If instead MC holds, competition among sellers in large communities eliminates rents on all trades and thus any motive for resale. If so, intermediation vanishes, outflow equilibrium net-trades converge to efficiency, and a unique price reigns in the limiting economy. The result views anonymous centralized Walrasian markets as approximations of non-anonymous decentralized markets in which a large number of buyers and sellers can directly trade with each other (as would be the case in an economy in which every community with a non-negligible aggregate demand is able trade with every other community with a non-negligible aggregate supply). A convenient feature of the proposed balanced replication process is that MC can be imposed directly on the baseline economy rather than on the entire sequence of replicas. A testable implication of the outflow model is that resale markups are strictly positive even in large markets. In contrast, most price-competition models predict that intermediaries never command rent when more than one supply chain exists (Choi, Galeotti and Goyal 2014).

Next consider unbalanced replication. To guarantee that the efficient net-trades remain bounded when an unbalanced replica grows large, an additional technical assumption has to be imposed on preferences.

Assumption A3 *For any player $i \in V$, the utility u_i satisfies $\lim_{q \to \infty} u_i'(q) = 0$.*

Denote by $\hat{V} = \cup_{C \in \hat{\mathbf{C}}} C$ the set of players located in one of the large communities, and by $V \backslash \hat{V}$ the set of players located in one of the small communities of a $(\hat{\mathbf{C}}, \mathbf{z})$-replica. Let $\bar{\mathbf{q}}^z \in \mathbb{R}_+^V$ denote the efficient net-trades of any type of player in a $(\hat{\mathbf{C}}, z)$-replica, and let $\bar{\mathbf{q}}^\infty = \lim_{z \to \infty} \bar{\mathbf{q}}^z$. Such net-trades coincide for all copies of a player-type due to concavity of preferences. The results on unbalanced replication require further discipline on the community structure to guarantee that net-trades converge to efficiency in small communities. Consider the following additional requirement.

Condition FC: *A sequence of $(\hat{\mathbf{C}}, z)$-replica economies satisfies FC if*

(a) for any $i \in V \backslash \hat{V}$ such that $\bar{q}_i^\infty < 0$, there exist $j \in \hat{V} \cap V_i$;

(b) for any $i \in V \backslash \hat{V}$ such that $\bar{q}_i^\infty > 0$, there exist $j \in \hat{V} \cap V_i$ such that $\bar{q}_j^\infty < 0$.

The two conditions together imply that any small community requiring trade is linked to large community; while the second further implies that sellers from some large community can supply buyers living in a small community. Condition FC (or *full clearing*) may hold even if the baseline economy violates MC.

For the economy $\mathcal{E}$ and the subset of communities $\hat{\mathbf{C}}$, consider the *large economy* $\mathcal{E}^+ = \{\hat{\mathbf{C}}, \hat{\mathbf{E}}, \mathbf{Q}, \mathbf{u}\}$ obtained by deleting communities that do not belong to $\hat{\mathbf{C}}$,

$$\hat{\mathbf{E}} = \{CK \in \mathbf{E} | C, K \in \hat{\mathbf{C}}\}.$$

The next result generalizes Proposition 6 to unbalanced replication. It establishes that the existence of efficient and direct flows of consumption in $\mathcal{E}^+$ is necessary and sufficient for a sequence unbalanced replicas to converge to approximate efficiency. It also establishes that net-trades converge to efficiency in every communities when FC holds, as players in small communities can access a large pool of buyers and sellers to meet their efficient net-trades.

Proposition 7 *If A1 and A3 hold, a sequence of replica economies converges to:*
(a) approximate efficiency if and only if MC holds in the large economy $\mathcal{E}^+$;
(b) efficiency if and only if MC holds in $\mathcal{E}^+$ and FC holds for the sequence of replicas.

Whenever intermediation is not required to clear markets in large communities, equilibria that converge to approximate efficiency exist. In these equilibria, all goods are traded at a unique price in every large community, and there are no distortions to pricing, as a large number of sellers competes to supply any group of buyers belonging to $\hat{\mathbf{C}}$.[15] Prices, however, may differ in small communities as market power and resale rents still distort trade in such locations. An anonymous centralized market again approximates trading behavior in the large communities of non-anonymous decentralized markets when intermediation is superfluous. Condition FC further implies that trade from the large communities can clear every local market and restore full efficiency. If so, intermediation and distortions vanish even in small markets as competition from the larger communities disciplines prices by reducing rents on every trade. Convergence to efficiency would fail in smaller markets if FC were violated, as price distortions would necessarily curtail trade in some small and poorly connected communities.

Other studies on two-sided networks and matching have exploited variants of condition MC to clear markets and achieve efficiency in decentralized markets without intermediation. Within the outflow competition framework, MC was proven to be necessary and sufficient for convergence to efficiency even in environments in which resale was feasible. Our observation differs from most other studies exploiting Hall theorem type arguments as necessity of MC is not built in the trading environment by exogenous assumptions preventing resale. Results would extend to more general replication processes in which all communities grow at possibly heterogeneous rates. Convergence to approximate efficiency in those environments would still require MC holding among the largest communities. Conditions for convergence to full efficiency would, however, differ slightly, as communities that are neither large nor small could occasionally mediate trade between larger and smaller communities without creating frictions. The main aim of the section was to provide simple conditions on the economy for trading behavior in large decentralized oligopolistic markets to emulate behavior in large centralized competitive mar-

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[15]A convenient feature of the proposed unbalanced replication process is that condition MC can imposed directly in the large economy rather than on the entire sequence of replicas.

kets. Sequences of replicas were only invoked here, as a parsimonious method to model large decentralized markets in which locations are populated by finitely many types of traders.

Examples: Large Economies and Replication

Before proceeding to the final results, consider three examples of replica economy. In each example, the economy consists of three communities C_a, C_b, C_c. The preferences of all the individuals in community C_i satisfy $u_i(q) = (Q_i + q)^{1/2}$ for $i \in \{a, b, c\}$, where $Q_a = 2$, $Q_b = 1$, and $Q_c = 0$. Community C_a is populated only by sellers, community C_c only by buyers, while players in community C_b are neither sellers nor buyers. The first two examples differ only in the community structure, while the last two differ only in the replication process. Begin by considering the economy in which the three communities are linked, and form a grand community (depicted in the left plot of figure 2). The economy trivially satisfies condition MC, and thus converges to efficiency. As the economy becomes large, consumption in the sellers' community decreases monotonically, while consumption in the buyers' community increases monotonically. In the limit every player consumes one unit. The price paid by intermediaries in community C_b converges from below to the competitive equilibrium price, $1/2$; while the price paid by buyers in the import community C_c monotonically decreases to the same value. Intermediation by players in community C_b vanishes, and such players do not trade in the limit economy. Per-capita social welfare increases monotonically as the economy grows large. The two plots on the left in figure 3 depict consumption and prices in the unique equilibrium of this sequence of replicas.
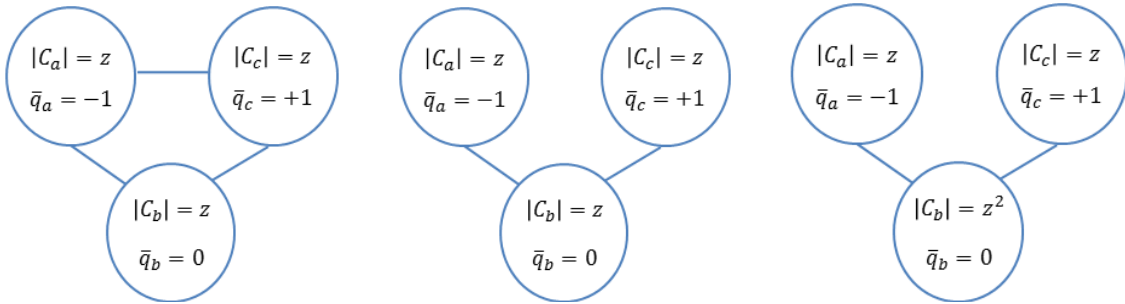

FIGURE 2: Community structure. Communities appear as linked circles.

Next consider the same economy, but suppose that sellers in community C_a cannot trade directly with buyers in community C_c (depicted in the central plot of figure 2). If so, players in community C_b act as middlemen buying from sellers in community C_a to supply buyers in community C_c. The economy cannot satisfy condition MC, since no direct trade between sellers and buyers is feasible. Thus, no sequence of outflow equilibria can ever converge to efficiency. In fact, outflow equilibrium consumption in the three communities does not converge. In the limit economy, players in communities C_a and C_b consume more than players in community C_c. The

price paid by middlemen in community C_b first grows and then declines converging to a value below the competitive equilibrium price. The price paid by buyers in community C_c instead, monotonically decreases, but always remains above the competitive price. The limit markup made by middlemen is approximately 30%. Per-capita social welfare increases monotonically as the economy grows large, but remains inefficient in the limit economy. The central plots in figure 3 depict consumption and prices in the unique equilibrium of this sequence of replicas. The outflow model recognizes that the second community structure cannot attain efficiency while mimicking an anonymous Walrasian market, as some players in community C_b must necessarily act as intermediaries while transferring a non-negligible amount of consumption from sellers in C_a to buyers in C_c.
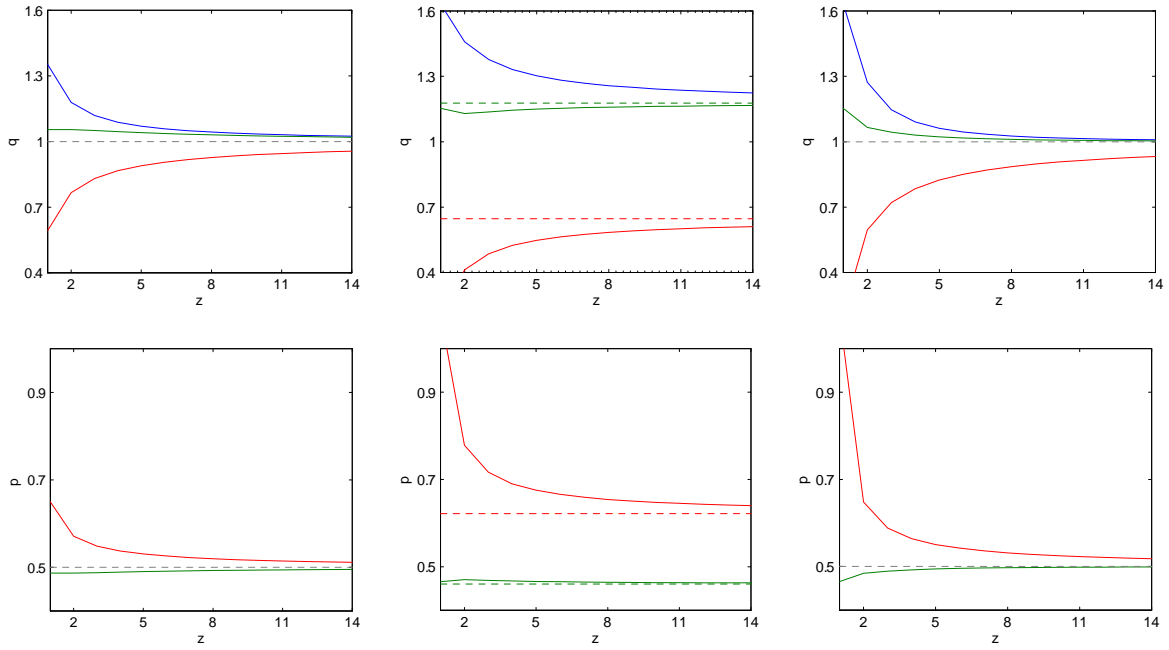

FIGURE 3: In the top row consumption in communities a (top), b (middle), and c (bottom); and in the bottom row prices in communities b (bottom), and c (top).

The final example considers an unbalanced replica of the second community structure, in which players in community C_b grow at a faster rate than other players in the economy (depicted in the right plot of figure 2). In particular, the zth element of the unbalanced replica considered here possesses z players in communities C_a and C_c, and z^2 in community C_b. Any economy in the sequence still violates condition MC, as no direct trade is feasible between the sellers' community and buyers' community. However, the unique symmetric equilibrium of this sequence of unbalanced replicas also converges to efficiency. Approximate efficiency obtains, as only one community is large in the limit and thus MC trivially holds in the large economy. Full efficiency also obtains, since large communities can clear the aggregate demand and the aggregate supply of any smaller community when the market is sufficiently large. The example highlights

why results on unbalanced replicas also apply to replication processes in which communities grow at different rates, and why any intermediation has to take place in the large communities. Consumption and prices for this sequence of replicas are shown in the right plots in figure 3.

Concluding Remarks on Balanced Replication

The analysis concludes with two additional observations on balanced replication: the first presents sufficient condition for symmetric equilibrium existence, while the latter relates welfare to market size. For any sequence of symmetric equilibria of a replica, let $\breve{q}^i_j = \lim_{z\to\infty} z q^{i.t}_{j.s}(z)$ denote the consumption sold in the limit economy by an individual of type i to all individuals of type j. In a limiting symmetric equilibrium, optimality of flows requires

$$\breve{q}^i_j (u'_j(\breve{q}_j) - u'_i(\breve{q}_i) + u''_i(\breve{q}_i)\breve{q}^\circ_i - \mu_i) = 0,$$

where μ_i denotes the non-negative multiplier on the outflow constraint, $\breve{q}^i_\circ \leq Q_i$. Outflow price distortions vanish in any symmetric equilibrium when a large number of individuals competes to supply each neighbor. The price distortions on inflows, instead, persist for those individuals who resell consumption. However, since the outflow wedges were the complicating factor in the proof of equilibrium existence, stronger results obtain.

Proposition 8 *If A1 holds, the following three results follow:*
(a) if MC holds, an efficient outflow equilibrium exists in the limit economy;
(b) if $u'''_i \geq 0$ for any $i \in V$, a symmetric outflow equilibrium exists in the limit economy;
(c) if $V_i \supseteq S$ for any $i \in D$, a unique outflow equilibrium exists in the limit economy.

In all the three cases, limiting revenues in each local market are concave. The result in (b) holds since the costs of supplying outflows are convex in the limit by the restriction on the third derivative. Results in (a) and (c) follow because condition MC directly implies the existence of an efficient outflow equilibrium in the limit economy. The strong conditions on the market structure imposed in (c) further imply that all equilibria converge to efficiency when all sellers and buyers can directly trade.

Finally, Proposition 9 relates per-capita social welfare to market size and shows that welfare increases monotonically as an economy gets replicated. Intuitively, the result obtains because, by definition, a balanced replica increases competition uniformly at every location of the trade network. Even economies failing MC become more competitive (though not perfectly competitive) as the number of players grows large.

Proposition 9 *If A1 holds and if any balanced replica possesses a unique symmetric equilibrium, then per-capita social welfare increases every time the economy is replicated.*

The proof of the proposition exploits the definition of balanced replica to establish a link between social welfare and network structure by studying how changes in the number of players at each location affect the Jacobian matrix of the complementarity problem characterizing the symmetric equilibria of a replica. Although the result offers limited testable implications as it holds only for balanced replication, it establishes an interesting property of such a process.

The aim of the section was to present conditions under which competition in decentralized oligopolistic markets could mimic perfect competition. For this to be the case, trade had to be direct in large communities and small communities had to be well connected. Economies, in which intermediation (by buyers or by sellers) was necessary, would never approximate perfect competition and efficiency, as market power distortions would inevitably persist.

4   Inflow Competition

This section shows why our conclusions are not specific to the outflow model, by outlining an alternative quantity competition model and comparing it to the outflow model. The web-appendix presents a more detailed discussion and examples. For sake of brevity, the analysis assumes that players are constrained in the amount of units that they can purchase. Alternative specifications in which the supply side is constrained yield similar results.

Inflow Competition: In the inflow competition model, players simultaneously decide how many units of consumption to buy from each of their neighbors (rather than deciding on how many units to sell to their neighbors). Any player i is constrained not to buy more than Q_i units of consumption. Prices are determined at each location so that all units are sold at their marginal cost. Thus, the price paid by Player i for units sold from a neighbor j is determined by the inverse supply curve at node j,

$$p_i^j(\mathbf{q}) = p_j(q_j) = u_j'(q_j).$$

Players sell all of their goods at unique price which coincides with their marginal value of consumption. Buyers expect such prices when choosing their demands. Assumption A1 again implies that $\partial p_j(q_j)/\partial q_i^j > 0$ and $\partial p_j(q_j)/\partial q_j^i < 0$ for any $i \in V_j$. The price earned by Player j decreases when his inflows increase, and increases when his outflows increase. Again an argument à la Kreps and Scheinkman could show how price competition among buyers would lead to such prices if individuals had to commit to their inflows.[16] The problem of an individual

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[16]Individuals offering a lower price would be worse off since part of their demand would not be met. Individuals offering a higher price would be worse off since their demand could be met at a lower price.

$i \in V$ reduces to

$$\max\nolimits_{\mathbf{q}_i \in \mathbb{R}_+^{V_i}} u_i(q_i) + \sum\nolimits_{j \in V_i}[p_i(q_i)q_j^i - p_j(q_j)q_i^j] \quad \text{s.t.} \quad q_i^\circ \leq Q_i.$$

If $q_i^j > 0$ and $q_i^\circ < Q_i$, optimality of the flow q_i^j requires that

$$p_i(q_i) - p_j(q_j) = -p_j'(q_j)q_i^j - p_i'(q_i)q_\circ^i.$$

The markup on the flow q_i^j (the difference between the buyer's marginal value and the price paid) is completely determined by two wedges: the monopoly price distortion on all units sold, and the Cournot distortion on the units purchased from seller j. Optimality in the inflow model differs from the outflow model, as different distortions affect equilibrium pricing. Whereas suppliers were able to commit to their sales in the outflow model, buyers are able to commit to their purchases in the inflow model. The ability to commit to trade flows benefits the players executing trades by allowing them to appropriate more gains from trade. Thus, an inflow economy is generally more efficient than an outflow economy, as more units flow to individuals with a higher marginal value. The expression *inflow equilibrium* is used to refer to a pure strategy Nash equilibrium of the inflow competition model.

Result Survey: Almost all of the results developed in the context of the outflow model also, apply to the inflow model. Sufficient conditions for inflow equilibrium existence differ slightly from conditions imposed in the outflow model, and are reported in the web-appendix. In the inflow model sellers supply all their customers at a single price. Buyers, however, often purchase goods from different suppliers at different prices, as price distortions can increase their expenditure when they concentrate their demand in a single market. As in the outflow model, resale is a common feature of equilibrium behavior, and linked individuals with different marginal rates of substitution do not necessarily trade. Sufficient conditions for trade to take place between pairs of linked individuals require gains from trade to exceed the outflow price distortion of the buyer. Examples in the web-appendix establish that adding links may still reduce welfare. Results on large markets are not affected by the change in the pricing paradigm. Again, economies in which intermediation cannot vanish never attain efficiency, whereas economies satisfying the condition MC do.

5   Comments

When can behavior in a decentralized oligopolistic market approximate perfect competition? Providing a simple answer to this question was the main aim of the analysis. To this end, a tractable model of oligopolistic competition in networked markets was introduced. Distinguish-

ing features of the model were the option to resell goods and the endogenous identity of buyers and sellers in the economy. Unsurprisingly, behavior in small markets could never approximate perfect competition, as local market power would inevitably distort trade. However, behavior in large decentralized markets was shown to approximate perfect competition whenever resale was not required to clear large markets. In such scenarios, efficient trades obtained in large communities directly without recourse to intermediation; all units were sold at one price in any large community; and intermediation persisted only to supply communities of negligible size. Behavior in these smaller communities was shown to approach perfect competition only when all of their trades could be executed in some large communities. The main implication of the model is that perfectly competitive resale markets do not exist when local market power constrains trade.

Strong assumptions on trade costs were implicit in the two quantity competition models presented. Trade was assumed to be costless between pairs of linked individuals, but extremely costly between any other pair of traders. Such restrictions were only imposed for the sake of clarity. In fact, the model and its conclusions would easily extend to environments in which a weighted network captures the heterogeneous trade costs between pairs of players. Assumptions also required the marginal utility of consumption to be positive for any player in the economy. The setup, however, can approximate environments in which intermediaries do not value consumption, when the marginal utility of such players is sufficiently low. Other limitations of the analysis were the omission of an explicit network formation model and the impossibility of migration. Indeed, it would be interesting to know if migration would always lead to efficient community structures. But this question lies beyond the scope of this manuscript.

6   Acknowledgements

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7   Appendix

Proof of Proposition 1.   Begin by establishing existence. For every player $i \in V$, the set of feasible outflows $X_i = \left\{ \mathbf{q}^i \in \mathbb{R}^{V_i} | q_\circ^i \leq Q_i \ \cap \ q_j^i \geq 0 \right\}$ is clearly non-empty, compact, convex. Sufficient conditions for the best reply maps to be single-valued require: that for every player i revenues from the sales to each neighbor $j \in V_i$ be concave in q_j^i; that his costs of supplying

units be convex in outflows $\mathbf{q}^i$; and that one of the two conditions be strict. Revenues are concave in each market, if for any $ij \in E$

$$\partial^2 R_j^i(q_j^i, \mathbf{q}^{-i})/(\partial q_j^i)^2 = 2u_j''(q_j) + q_j^i u_j'''(q_j) \leq 0. \tag{E1}$$

Since q_i is a linear function of every outflow q_j^i and since outflows affect costs only through consumption q_i, costs $C_i(\mathbf{q}^i, \mathbf{q}^{-i})$ are a convex in the vector $\mathbf{q}^i$ whenever $C_i(\mathbf{q}^i, \mathbf{q}^{-i})$ is convex in q_i

$$\partial^2 C_i(\mathbf{q}^i, \mathbf{q}^{-i})/(\partial q_i)^2 = -u_i''(q_i) + q_i^\circ u_i'''(q_i) \geq 0. \tag{E2}$$

Assumptions A1 and B1 imply that E1 and E2 hold, with at least one of the two holding strictly. In particular, since by feasibility $Q_j + q_j \geq q_j^\circ \geq q_j^i$, A1 and the upperbound in B1 imply that revenues are concave since

$$2u_j''(q_j) + q_j^i u_j'''(q_j) \leq (2u_j''(q_j) + (Q_j + q_j)u_j'''(q_j))\mathbb{I}(u_j'''(q_j) > 0) \leq 0,$$

where $\mathbb{I}(\cdot)$ denotes the indicator function ($\mathbb{I}(A) = 1$ if A is true, $\mathbb{I}(A) = 0$ otherwise). Moreover, since $Q_i + q_i \geq q_i^\circ$, A1 and the lowerbound in B1 imply that costs are convex since

$$-u_i''(q_i) + q_i^\circ u_i'''(q_i) \geq (-u_i''(q_i) + (Q_i + q_i)u_i'''(q_i))\mathbb{I}(u_i'''(q_i) < 0) \geq 0.$$

Since both indicator maps cannot hold at once either revenues are strictly concave, or costs are strictly convex. Thus, A1 and B1 imply that payoffs are strictly concave and continuous for each player. Strict concavity of payoffs and the compactness and convexity of choice set X_i require the best-response correspondences to be single-valued. Continuity of the payoffs implies (by Berge's theorem of the maximum) that best responses are continuous. Thus, the existence of outflow equilibrium is guaranteed by Brouwer's fixed point theorem.

Next observe that sufficient conditions for best reply maps to be single-valued do not need to discipline payoffs when the revenues from selling units are decreasing. In fact, such outflows could never be a best reply for the player selling the units, as marginal costs are positive by assumption A1. Thus, to grant existence it suffices to show A1 and B2 imply that E1 and E2 hold whenever revenues increase. The rest of the argument shows that A1 and B2 imply that the revenue of Player i from sales to every neighbor $j \in V_i$ is concave in q_j^i and that his costs of supplying units are convex in $\mathbf{q}^i$, whenever the revenue from selling units to i increases. Revenues in market i increase, if $u_i'(q_i) + q_i^j u_i''(q_i) \geq 0$. If so,

$$V_i u_i'(q_i) + q_i^\circ u_i''(q_i) \geq 0.$$

where the implication holds by summing over all neighbors j. Thus, A1 and the lowerbound in

B2 imply that costs are convex when revenues increase, since

$$-u_i''(q_i) + q_i^\circ u_i'''(q_i) \geq V_i u_i'(q_i) \left(-\frac{u_i''(q_i)}{u_i'(q_i)V_i} - \frac{u_i'''(q_i)}{u_i''(q_i)} \right) \mathbb{I}(u_i'''(q_i) < 0) \geq 0.$$

Similarly, observe that A1 and the upperbound in B2 imply that revenues are concave when revenues increase, since

$$2u_j''(q_j) + q_j^i u_j'''(q_j) \leq -u_i'(q_i) \left(\frac{u_i'''(q_i)}{u_i''(q_i)} - \frac{2u_i''(q_i)}{u_i'(q_i)} \right) \mathbb{I}(u_j'''(q_j) > 0) \leq 0.$$

As one of the two conditions on revenues and costs holds strictly, assumptionsA1 and B2 imply that the payoff of each player is strictly concave and continuous whenever increasing. The strict concavity of payoffs and the compactness and convexity of the choice set correspondence imply that the best-responses are single-valued. Again Brouwer's fixed point theorem applies and implies existence. Also observe that any combination of the two assumptions B1 and B2 would similarly grant existence.

To prove the characterization finally observe that the first part of the proof implies that solutions can be found by Kuhn-Tucker first order conditions. The optimality of a flow q_j^i then implies that

$$q_j^i = 0 \ \ \text{if} \ \ u_i'(q_i) - u_i''(q_i)q_i^\circ - u_j'(q_j) + \mu_i > 0$$
$$q_j^i > 0 \ \ \text{if} \ \ u_i'(q_i) - u_i''(q_i)q_i^\circ - u_j'(q_j) - u_j''(q_j)q_j^i + \mu_i = 0$$
$$\mu_i \geq 0, \ \ Q_i - q_\circ^i \geq 0, \ \ \text{and} \ \ \mu_i(Q_i - q_\circ^i) = 0$$

which simply amounts to the complementarity problem stated in the result. ∎

Proof of Proposition 3. (a) First order necessary conditions immediately establish the result since $q_i^j > 0$ implies

$$u_j'(q_j) - u_i'(q_i) = \mu_i - u_j''(q_j)q_j^i - u_i''(q_i)q_i^\circ > 0,$$

where μ_i denotes the non-negative multiplier on the outflow constraint, $q_\circ^i \leq Q_i$, and where the latter terms are positive by assumption A1. Moreover, the converse clearly fails due to the positive price distortions inherent to the model.

(b) Let $T(q) = \{ij \in E | q_j^i > 0\}$ be the set of active trading links. If $ij \in T(\mathbf{q})$, by first order optimality $u_i'(q_i) < u_j'(q_j)$. Thus if a cycle $c = \{ij, jk, ..., li\} \in T(\mathbf{q})$, a contradiction arises since $u_i'(q_i) < u_j'(q_j) < u_k'(q_k) < ... < u_l'(q_l) < u_i'(q_i)$.

(c) If for $i \in V$ and for any $j \in V_i$ equilibrium dictates that $u_i'(q_i) \leq u_j'(q_j)$, then i cannot buy from any neighbor, since $u_i'(q_i) > u_j'(q_j)$ is necessary for $q_i^j > 0$. Similarly if $u_i'(q_i) \geq u_j'(q_j)$ for

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any $j \in V_i$, Player i cannot be selling to any neighbor, since $u'_i(q_i) < u'_j(q_j)$ is necessary for $q^i_j > 0$.

(d) By part (c) if i is a source $q^o_i = 0$. Which in turn implies that, if A1 holds, Player i sells to any $j \in V_i$ with $u'_i(q_i) < u'_j(q_j)$, provided that $q^i_o < Q_i$, since there exists $q^i_j > 0$ for which

$$-u'_i(q_i) + u'_j(q_j) + q^i_j u''_j(q_j) = 0.$$

(e) If A1 holds, optimality of the trade from k to i requires

$$u'_k(q_k) - u''_k(q_k) \sum_{l \in V_k} q^l_k + \mu_k = u'_i(q_i) + q^k_i u''_i(q_i) < u'_i(q_i) < u'_j(q_j)$$

which is both necessary and sufficient for a trade from k to j to occur. ∎

Proof of Proposition 4. (a) A profile of flows $\mathbf{q}$ is efficient if for any $ij \in E$ the flow q^i_j satisfies

$$[u'_j(q_j) - u'_i(q_i) - \lambda_i]q^i_j = 0 \ \& \ [q_i + Q_i]\lambda_i = 0, \tag{2}$$

where λ_i denotes the multiplier of the capacity constraint of Player i. Optimality conditions for an outflow equilibrium, instead, require that for any $ij \in E$ the flow q^i_j satisfies

$$[u'_j(q_j) - u'_i(q_i) - \mu_i + u''_j(q_j)q^i_j + u''_i(q_i)q^o_i]q^i_j = 0 \ \& \ [q_i + Q_i]\mu_i = 0, \tag{3}$$

where μ_i denotes the multiplier of the capacity constraint. If $\mathbf{q} = \mathbf{0}$ is efficient, it satisfies condition (2). But then $\mathbf{q} = \mathbf{0}$ is an outflow equilibrium, as it immediately satisfies condition (3). If, however, $\mathbf{q} \neq \mathbf{0}$ is efficient, then $\mathbf{q}$ cannot be an outflow equilibrium, as condition (3) and (2) coincide only if $\mathbf{q} = \mathbf{0}$, since utility is concave by assumption A1.

(b) Consider a market with three consumers $\{a, b, c\}$. Let $Q_a = 2$, $Q_b = 1$, and $Q_c = 0$, and assume that the preferences of any player i satisfy $u_i(q) = (Q_i + q)^{1/2}$. Increasing the set of trading relationships from $\{ac\}$ to $\{ac, ab\}$ reduces social welfare. When only players a and c are allowed to trade, Player a sells 0.4 units to c at a price of 0.8, and social welfare stands at 2.9. When instead consumer a is allowed to trade with b as well as c, he elects to supply both neighbors: b with 0.2 units and c with 0.36 units at different prices. Individual a's price discrimination of players b and c decreases sales to c. Player a curtails his supply to c in order to extract higher marginal rents from c, since he is able to recoup the loss in revenue by selling to b. Social welfare declines to 2.89. Flows, prices and quantities for the two economies are reported in figure 4 (left and center) and table 2 (left and center).
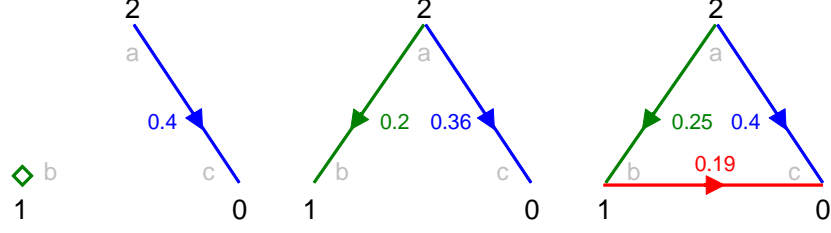
FIGURE 4: On the vertices endowments and identities and on the edges flows.

E1	p	x	w
a	-	1.60	1.58
b	-	1.00	1.00
c	0.79	0.40	0.32
+	-	3.00	2.90

E2	p	x	w
a	-	1.44	1.59
b	0.46	1.20	1.00
c	0.83	0.36	0.30
+	-	3.00	2.89

E3	p	x	w
a	-	1.35	1.54
b	0.49	1.06	1.03
c	0.65	0.59	0.39
+	-	3.00	2.96

TABLE 2: Prices, consumption & welfare: left $\{ac\}$, center $\{ac, ab\}$, right $\{ac, ab, bc\}$.

(c) Consider an economy with four players $\{a, b, c, d\}$, with initial endowments $\{2.97, 1, 0, 0.03\}$, and in which the preferences of any player satisfy $u_i(q) = (Q_i + q)^{1/2}$. When the set of feasible trades increases from $\{ad, bc\}$ to $\{ad, bc, dc\}$, Player d's welfare decreases. If only trades in $\{ad, bc\}$ are feasible, in the unique equilibrium of this economy players a and b supply their respective customers as monopolies.
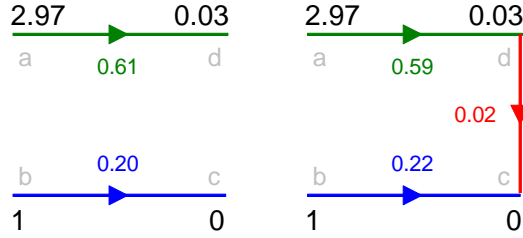

FIGURE 5: On the vertices endowments and identities and on the edges flows.

E1	p	q	w
a	-	2.36	1.92
b	-	0.80	1.12
c	1.12	0.20	0.22
d	0.63	0.64	0.42
+	-	4.00	3.68

E2	p	q	w
a	-	2.38	1.93
b	-	0.78	1.11
c	1.02	0.24	0.24
d	0.65	0.60	0.41
+	-	4.00	3.69

TABLE 3: Prices, consumption & welfare: left $\{ad, bc\}$, right $\{ad, dc, bc\}$.

But when the link cd is added to the network, Player d competes with b to supply c. In the unique equilibrium consumer d is worse off than when he cannot sell to c, since his payoff decreases from 0.42 to 0.41. Even though Player d chooses to supply c, having the option to sell affects the quantity sold to him from a, and thus reduces his welfare. All gains from trade on the newly created link are either kept by c or transferred to a. Player a being the monopoly supplier of d is able to extract more rents, as he faces a steeper demand schedule whenever d has the option to resell. Flows, allocations and prices for the two economies are reported in table 3 and in figure 5. $\blacksquare$

Proof of Proposition 5. The first step of the argument shows that (a) and (b) are equivalent. If $\sigma(H, \bar{\mathbf{q}}) \geq 0$ for $\forall H \subseteq \mathbf{C}$, it must be that for any $T \subseteq \mathbf{C}$

$$
\begin{aligned}
\delta(T, \bar{\mathbf{q}}) &= \delta(T, \bar{\mathbf{q}}) - \delta(\mathbf{C}, \bar{\mathbf{q}}) = -\sum_{C \in \mathbf{C} \backslash \mathbf{V}_T} q_C^+ - \sum_{C \in \mathbf{C} \backslash T} q_C^- = \\
&= \sigma(\mathbf{C} \backslash \mathbf{V}_T, \bar{\mathbf{q}}) - \sum_{C \in (\mathbf{C} \backslash T) \backslash \mathbf{V}_{\mathbf{C} \backslash \mathbf{V}_T}} q_C^- \geq 0,
\end{aligned}
$$

where the first equality holds since the efficient net trades add to zero $\delta(\mathbf{C}, \bar{\mathbf{q}}) = 0$, and where the second equality holds since $\mathbf{C} \backslash T \supseteq \mathbf{V}_{\mathbf{C} \backslash \mathbf{V}_T}$ for $\forall T \subseteq \mathbf{C}$. Similarly if $\delta(H, \bar{\mathbf{q}}) \geq 0$ for $\forall H \subseteq \mathbf{C}$, then for any $T \subseteq \mathbf{C}$

$$
\sigma(T, \bar{\mathbf{q}}) = \delta(\mathbf{C} \backslash \mathbf{V}_T, \bar{\mathbf{q}}) + \sum_{C \in (\mathbf{C} \backslash T) \backslash \mathbf{V}_{\mathbf{C} \backslash \mathbf{V}_T}} q_C^+ \geq 0.
$$

To prove the final step it is convenient to map condition MC to the original network structure. Let $D_i = D \cap V_i$, $S_i = S \cap V_i$, and $S_H = \cup_{i \in H} S_i$. First establish that MC holds if and only if for any $H \subseteq D$

$$
\tilde{\sigma}(H, \bar{\mathbf{q}}) \equiv -\sum_{j \in S_H} \bar{q}_j - \sum_{i \in H} \bar{q}_i \geq 0.
$$

For any $H \subseteq D$, let $T(H) = \{C \in \mathbf{C} \mid C \cap H \neq \emptyset\}$. If MC holds, observe that for any $H \subseteq D$

$$
\tilde{\sigma}(H, \bar{\mathbf{q}}) \geq -\sum_{C \in \mathbf{V}_{T(H)}} \sum_{i \in C \cap S} \bar{q}_i - \sum_{C \in T(H)} \sum_{i \in C \cap D} \bar{q}_i = \sigma(T(H), \bar{\mathbf{q}}) \geq 0,
$$

where the first inequality holds as $T(H)$ may include more buyers than H. Similarly, for any $T \subseteq \mathbf{C}$, let $H(T) = \{i \in D \mid i \in C \text{ for } C \in T\}$. If $\tilde{\sigma}(H, \bar{\mathbf{q}}) \geq 0$ for any $H \subseteq D$, MC holds as for any $T \subseteq \mathbf{C}$,

$$
\sigma(T, \bar{\mathbf{q}}) = -\sum_{C \in \mathbf{V}_T} \sum_{i \in C \cap S} \bar{q}_i - \sum_{i \in H(T)} \bar{q}_i = \tilde{\sigma}(H(T), \bar{\mathbf{q}}) \geq 0,
$$

where the first inequality holds as $i \in S_{H(T)}$ if and only if $i \in C \cap S$ for some $C \in \mathbf{V}_T$.

The next step exploits the previous simplification to establish that (c) implies (a) by contradiction. Observe that whenever (ii) holds for any $H \subset D$

$$- \sum_{i \in S_H} \bar{q}_i = \sum_{i \in S_H} \sum_{j \in D_i} q_j^i \geq \sum_{i \in S_H} \sum_{j \in H \cap D_i} q_j^i = \sum_{j \in H} \sum_{i \in S_j} q_j^i,$$

where first equality holds by (ii) and the last holds since any pair of players i, j that satisfies $j \in H$ and $i \in S_j$, also satisfies $i \in S_H$ and $j \in H \cap D_i$. If MC were violated, $\tilde{\sigma}(H, \bar{\mathbf{q}}) < 0$ for some $H \subset D$. But if so, the previous observation would imply that (i) would also be violated for some player $j \in H$ since

$$- \sum_{j \in H} \bar{q}_j + \sum_{j \in H} \sum_{i \in S_j} q_j^i \leq - \sum_{j \in H} \bar{q}_j - \sum_{i \in S_H} \bar{q}_i = \tilde{\sigma}(H, \bar{\mathbf{q}}) < 0.$$

The final step proves that $\tilde{\sigma}(H, \bar{\mathbf{q}}) \geq 0$ for any $H \subseteq D$ implies (c) by induction on D. First establish that the result holds for $|D| = 1$. Let i denote the only buyer in the economy. By assumption we have that $\tilde{\sigma}(D, \bar{\mathbf{q}}) \geq 0$, which in turn implies that $\tilde{\sigma}(D, \bar{\mathbf{q}}) = 0$ as supply cannot exceed aggregate demand by construction. Thus flows satisfying both (i) and (ii) can be found by setting $q_i^j = -\bar{q}_j$ for any $j \in S = S_i$.

Next suppose that $\tilde{\sigma}(H, \bar{\mathbf{q}}) \geq 0$ for any $H \subseteq D$ is sufficient whenever $|D| \leq m - 1$ to prove that it is sufficient for $|D| = m$. Initially assume that $H \subset D$ exists such that $\tilde{\sigma}(H, \bar{\mathbf{q}}) = 0$. Consider the subgraph (V', E') with vertices $V' = S' \cup D'$ with $D' = H$ and $S' = S_H$, and with edges restricted to $E' = E \cap \{ij | i \in S' \cap j \in D'\}$. This subgraph satisfies $\tilde{\sigma}(K, \bar{\mathbf{q}}) \geq 0$ for any $K \subseteq D'$ trivially, since no condition was altered. Thus, since $|H| < m$, by the induction assumption it is possible to find flows $\mathbf{q} \in \mathbb{R}_+^{E'}$ such that (i) and (ii) hold in the subgraph,

$$
\begin{aligned}
\bar{q}_j &= \sum_{i \in S_j} q_j^i && \text{for } \forall j \in H, \\
\bar{q}_i &= - \sum_{j \in D_i \cap H} q_j^i && \text{for } \forall i \in S_H.
\end{aligned}
$$

To conclude the proof it suffices to show that, given such flows, the remaining players of the original graph still satisfy MC. Denote by $\hat{\mathbf{q}} \in \mathbb{R}^V$ the efficient net-trades $\bar{\mathbf{q}}$ shifted by such flows $\mathbf{q}$. That is for any $i \in V$, let

$$\hat{q}_i = \bar{q}_i - q_i^\circ + q_\circ^i.$$

Consider the subgraph (V'', E'') with vertices $V'' = S'' \cup D''$ with $D'' = D \backslash H$ and $S'' = S \backslash S_H$, and with edges restricted to $E'' = E \cap \{ij | i \in S'' \cap j \in D''\}$. For any $K \subset D \backslash H$ it must be

that

$$
\begin{aligned}
\tilde{\sigma}''(K, \hat{\mathbf{q}}) &= \tilde{\sigma}''(K, \hat{\mathbf{q}}) + \tilde{\sigma}''(H, \hat{\mathbf{q}}) + \textstyle\sum_{i \in S_H \cap S_K} \hat{q}_i = \\
&= \tilde{\sigma}(K, \bar{\mathbf{q}}) - \textstyle\sum_{i \in S_K} \sum_{j \in D_i \cap H} q^i_j + \tilde{\sigma}(H, \bar{\mathbf{q}}) + \sum_{i \in S_H \cap S_K} (\bar{q}_i + \sum_{j \in D_i \cap H} q^i_j) = \\
&= \tilde{\sigma}(K, \bar{\mathbf{q}}) + \tilde{\sigma}(H, \bar{\mathbf{q}}) + \textstyle\sum_{i \in S_H \cap S_K} \bar{q}_i = \tilde{\sigma}(K \cup H, \bar{\mathbf{q}}) \geq 0,
\end{aligned}
$$

since $\tilde{\sigma}''(H, \hat{\mathbf{q}}) = \tilde{\sigma}(H, \bar{\mathbf{q}}) = 0$ and $\sum_{i \in S_H \cap S_K} \hat{q}_i = 0$. Which in turn implies by induction that flows $\mathbf{q}'' \in \mathbb{R}^{E''}_+$ exist that satisfy condition (i) and (ii), since $|D \backslash H| < m$.

Finally if $\tilde{\sigma}(L, \bar{\mathbf{q}}) > 0$ for any $L \subset D$, consider $H \in \arg\min_{L \subset D} \tilde{\sigma}(L, \bar{\mathbf{q}})$ and choose any profile of flows $\dot{\mathbf{q}}$ from S_H to $D \backslash H$ such that

$$
\textstyle\sum_{j \in D \backslash H} \sum_{i \in S_H \cap S_j} \dot{q}^i_j = \tilde{\sigma}(H, \bar{\mathbf{q}}).
$$

Let $\ddot{\mathbf{q}} \in \mathbb{R}^V$ denote the efficient net-trades $\bar{\mathbf{q}}$ adjusted for such flows $\dot{\mathbf{q}}$. After such transfers, $\tilde{\sigma}(H, \ddot{\mathbf{q}}) = 0$ and $\tilde{\sigma}(L, \ddot{\mathbf{q}}) \geq 0$ for any $L \subseteq D$, since

$$
\tilde{\sigma}(L, \ddot{\mathbf{q}}) \geq \tilde{\sigma}(L, \bar{\mathbf{q}}) - \tilde{\sigma}(H, \bar{\mathbf{q}}) \geq 0.
$$

Thus, the $\ddot{\mathbf{q}}$ economy satisfies all the conditions required in the previous step of the proof and MC is sufficient. ∎

Proof of Proposition 6. (a) Let $(V(z), E(z))$ denote the trade network associated to community structure $(\mathbf{C}^z, \mathbf{E}^z)$. For convenience, occasionally denote $u_i(q_i(z))$ by $u_i(z)$. Whenever the equilibrium of the replicas converges to efficiency, it must be that $\lim_{z \to \infty}(u'_j(z) - u'_i(z)) = 0$ for any two players $i, j \in V(z)$ for which $\lim_{z \to \infty} q_i(z) > -Q_i$ and $\lim_{z \to \infty} q_j(z) > -Q_j$. Suppose by contradiction that some player $i \in V(z)$ resells units in the limit economy,

$$
\lim_{z \to \infty} r_i(z) = \lim_{z \to \infty} \min\{q^\circ_i(z), q^i_\circ(z)\} > 0.
$$

If so, optimality of flows from i to his neighbors $j \in V_i(z)$ requires that, when $q^i_j(z) > 0$,

$$
\lim_{z \to \infty}(u'_j(z) - u'_i(z)) = \lim_{z \to \infty}\left(\mu_i(z) - u''_i(z)q^\circ_i(z) - u''_j(z)q^i_j(z)\right) > 0,
$$

where $\mu_i(z)$ denotes the non-negative multiplier on the outflow constraint, $q^i_\circ(z) \leq Q_i$. Which contradicts the assumption that the economy becomes competitive.

(b) First the necessity of MC is proven. MC holds in economy $\mathcal{E}$ if and only if MC holds in any of its replicas $\mathcal{E}^z$. If MC did not hold, by Proposition 5, no direct flows would exist from seller to buyers that support the efficient net-trades in the original economy. Resale among players would thus be necessary for the efficient net-trades to be an outcome of such economy.

Define the *minimal resale* in the $(\mathbf{C}, z)$-replica economy as

$$r(z) = \min_{\mathbf{q} \in \mathbb{R}_+^{E(z)}} \max_{i \in V(z)} r_i(z) \quad \text{s.t. (i) and (ii) on page 17.}$$

MC fails if and only if $r(1) > 0$. The definition of replica implies $r(1) = r(z)$, because minimizing the maximum resale requires all players of the same type to buy and sell the identical amounts. This is the case, since the average flows across any two player types in a replica (that is $\sum_{s=1}^{z} \sum_{t=1}^{z} q_{j.t}^{i.s}/z^2$) define flows in the original economy in which resale exceeds $r(1)$ (by efficiency), which in turn implies that $r(1) \leq r(z)$, as the average resale of a player of type i cannot exceed the maximal resale of a player of type i. Thus, if $r(1) > 0$, any profile of flows leading to the efficient allocation requires at least one player to resell a positive amount of goods in the limit economy. But, by part (a) no such outcome can be supported in an efficient limiting outflow equilibrium as there is no resale in any such equilibrium.

The next part of the proof establishes that MC is sufficient for the existence of an efficient symmetric outflow equilibrium in the limit economy. First observe that the solution of the complementarity problem defining the symmetric equilibrium flows is lower hemi-continuous in the replica counter z, as each optimality condition defining the problem is continuous and differentiable in $1/z$, (see problem 4 in the proof of Proposition 9). Therefore, consider flows in the original economy $\mathbf{q} \in \mathbb{R}_+^E$ satisfying (i) and (ii) in Proposition 5. Such flow exist because MC holds. If so, $q_i = \bar{q}_i$ for any player $i \in V$. Define the sequence of flows $\mathbf{q}(z) \in \mathbb{R}_+^{E(z)}$ as follows: $q_{j.t}^{i.s}(z) = q_j^i/z$ for any $(i.s)(j.t) \in E(z)$, and $q_{j.t}^{i.s}(z) = 0$ otherwise. Such flows are direct and satisfy conditions (i) and (ii) in the replica. Moreover, $\lim_{z \to \infty} q_{j.t}^{i.s}(z) = 0$. Thus, $\lim_{z \to \infty} \mathbf{q}(z)$ satisfies all the outflow equilibrium requirements in the limit economy. In particular, if such flows were chosen by others, no player would be able to profitably affect the prices of the goods sold in the limit, as deviations on his behalf could only reduce prices since $\lim_{z \to \infty} q_{j.t}^{i.s}(z) = 0$. As gains from deviating from $\mathbf{q}(z)$ decrease along the sequence of replicas and vanish in the limit, the limit of $\mathbf{q}(z)$ is efficient, and belongs to the limit of the symmetric outflow equilibrium correspondence. Lower hemi-continuity of the equilibrium correspondence then guarantees the existence of a selection of equilibrium correspondence that converges to such a limit point. A direct proof of the final argument is possible, but more involved. ∎

Proof of Proposition 7. (a) The proof of the result emulates part (b) of Proposition 6. Let $\left(\hat{V}, \hat{E}\right)$ denote the trade network associated to community structure $(\hat{\mathbf{C}}, \hat{\mathbf{E}})$. Let $\hat{V}(z) = \cup_{C \in \hat{\mathbf{C}}} C^z$ denote the set of players located in a large community of the $(\hat{\mathbf{C}}, z)$-replica. To establish the necessity of MC holding in the economy $\mathcal{E}^+$ observe that if MC were not hold, by Proposition 5, no direct flows would exist from seller to buyers that support the efficient net-trades in $\mathcal{E}^+$. If so, resale among players in $\hat{V}$ would required for the efficient net-trades to be an outcome of the economy. Let $\hat{\mathbf{q}} \in \mathbb{R}^{\hat{V}}$ denote the profile of efficient net-trades in

the economy $\mathcal{E}^+$, and let $\bar{\mathbf{q}}^z \in \mathbb{R}^{V(z)}$ denote the profile of efficient net-trades in the economy $\mathcal{E}(\hat{\mathbf{C}})^z$. Such profiles exist by assumption A1. Observe that $\lim_{z\to\infty} \bar{q}^z_{i.s} = \hat{q}_i$ for any $i \in \hat{V}$ and any $s \in \{1, ..., z\}$, as net-trades of players in $V\backslash\hat{V} = V(z)\backslash\hat{V}(z)$ become negligible when the economy $\mathcal{E}(\hat{\mathbf{C}})^z$ is large. This is the case since the efficient allocation of consumption is independent of $\sum_{i\in V\backslash\hat{V}} \bar{q}^z_i$ when z is sufficiently large given that: assumption A3 implies that $\lim_{z\to\infty} \bar{q}^z_i < \infty$ for any $i \in V\backslash\hat{V}$; the constraint on sales implies $\lim_{z\to\infty} \bar{q}^z_i \geq -Q_i$; and the set $V(z)\backslash\hat{V}(z)$ contains a finite number of players. Moreover, since the total number of units sold by players in $V(z)\backslash\hat{V}(z)$ cannot exceed $\sum_{i\in V\backslash\hat{V}} Q_i$, resale among players in $\hat{V}(z)$ is always required to achieve the efficient net-trades when condition MC fails in the economy $\mathcal{E}^+$. If so, the argument developed in Proposition 6 applies, and establishes that any outflow equilibrium must be inefficient since at least a player in $\hat{V}(z)$ resells non-negligible amount of consumption in any given profile of flows that gives rise to the efficient net-trades.

The next part of the argument establishes why MC is sufficient for convergence to approximate efficiency. Observe that the solution of the complementarity problem defining the symmetric equilibrium flows is lower hemi-continuous in the replica counter z, as each optimality condition defining the problem is continuous and differentiable in z. In particular, optimality of a flow $q^{i.s}_{j.t}(z) = q^i_j > 0$ in a symmetric equilibrium of a $(\hat{\mathbf{C}}, z)$-replica requires that

$$u'_i(q_i) - u'_j(q_j) - u''_j(q_j)q^i_j - u''_i(q_i)q^\circ_i + \mu_i = 0,$$

where μ_i denotes the multiplier on the capacity constraint and where

$$\begin{aligned} q_i &= z\sum_{k\in V_i\cap\hat{V}} \left(q^k_i - q^i_k\right) + \sum_{k\in V_i\backslash\hat{V}} \left(q^k_i - q^i_k\right), \\ q^\circ_i &= z\sum_{k\in V_i\cap\hat{V}} q^k_i + \sum_{k\in V_i\backslash\hat{V}} q^k_i. \end{aligned}$$

This establishes the lower hemi-continuity in z of the complementarity problem defining the symmetric outflow equilibria, as each optimality condition defining the problem is continuous and differentiable in z.

Now construct candidate flows that converge to approximate efficiency. For the economy $\mathcal{E}^+$, let $\tilde{\mathbf{q}} \in \mathbb{R}^{\hat{E}}_+$ denote a profile of flows satisfying (i) and (ii) in Proposition 5. Such flows exist because MC holds, and satisfy $\sum_{j\in\hat{V}_i} (\tilde{q}^j_i - \tilde{q}^i_j) = \hat{q}_i$ for any player $i \in \hat{V}$. Consider the sequence of flows $\mathbf{q}(z) \in \mathbb{R}^{E(z)}_+$ obtained by setting $q^{i.s}_{j.t}(z) = \tilde{q}^i_j/z$ for any $(i.s), (j.t) \in \hat{V}(z)$, while setting the remaining flows $q^i_j(z)$ according to their respective symmetric equilibrium optimality conditions (defined above). Observe that by construction for any player in $i \in \hat{V}(z)$,

$$q_i(z) = \hat{q}_i + \sum_{k\in V_i\backslash\hat{V}} \left(q^k_i(z) - q^i_k(z)\right).$$

Consider any player $k \in V_i(z) \backslash \hat{V}(z)$, observe that $i \in V_k(z) \cap \hat{V}(z)$, if so $\lim_{z \to \infty} q_i^k(z) = 0$ or else the capacity constraint of Player k would be violated. Similarly, $\lim_{z \to \infty} q_k^i(z) = 0$, or else $q_k(z)$ would diverge to infinity, which is impossible as Player $i \in V_k(z) \cap \hat{V}(z)$ would not be choosing his flow to k optimally because $q_k^i(z) > 0$ is equivalent to $u_i'(q_i(z)) < u_k'(q_k(z))$ (which cannot hold in the limit as A1 implies $u_i'(q_i(z)) > 0$, while A3 that $\lim_{z \to \infty} u_k'(q_k(z)) = 0$). This establishes that flows $\mathbf{q}(z)$ converge to approximate efficiency as $\lim_{z \to \infty} q_i(z) = \hat{q}_i = \bar{q}_i^\infty$.

The proof concludes by establishing that flows $\mathbf{q}(z)$ must be arbitrarily close to symmetric equilibrium flows in the limit as z diverges. To verify that flows $q_{j.t}^{i.s}(z) = \tilde{q}_j^i/z$ for any $(i.s), (j.t) \in \hat{V}(z)$ are arbitrarily close to equilibrium flows as z diverges, observe that the conjectured flows satisfy $\lim_{z \to \infty} q_{j.t}^{i.s}(z) = 0$. Therefore, $\lim_{z \to \infty} \mathbf{q}(z)$ satisfies all the outflow equilibrium requirements for trades on links $ij \in \hat{E}$ in the limit economy. In particular, if such flows were chosen by others, no player would be able to profitably affect the prices of the goods sold in the limit, as deviations on his behalf could only reduce prices since $\lim_{z \to \infty} q_{j.t}^{i.s}(z) = 0$. Since gains from deviating from $\mathbf{q}(z)$ decrease along the sequence of replicas and vanish in the limit, the limit of $\mathbf{q}(z)$ is approximately efficient and belongs to the limit of the symmetric outflow equilibrium correspondence. Lower hemi-continuity of the equilibrium correspondence then guarantees the existence of a selection of equilibrium correspondence that converges to such a limit point.

(b) Necessity of FC is immediate from part (a) and the following considerations. If FC were violated, convergence to efficiency in small communities would require trade across and/or within small communities, or trade between buyers in small communities and buyers in large communities. Either scenario would necessarily result in distortions. In the first scenario distortions would be a trivial consequence of Proposition 3, while the second distortions would appear as inflow price distortions would never vanish for players purchasing units in the limit economy.

The next part of the proof establishes why MC and FC are sufficient for convergence to efficiency. Recall that $\mathbb{I}(\cdot)$ denotes the indicator function. Consider the sequence of flows $\mathbf{q}(z) \in \mathbb{R}_+^{E(z)}$ obtained by setting for any $ij \in E(z)$

$$
q_{j.t}^{i.s}(z) = \begin{cases} \frac{\tilde{q}_j^i}{z} & \text{if} \quad (i.s), (j.t) \in \hat{V}(z) \\ -\frac{\bar{q}_i^\infty \mathbb{I}(\bar{q}_i^\infty < 0)}{z |V_i \cap \hat{V}|} & \text{if} \quad i.s \in V(z) \backslash \hat{V}(z) \text{ and } j.t \in \hat{V}(z) \\ \frac{\bar{q}_j^\infty \mathbb{I}(\bar{q}_i^\infty > 0)}{z |V_j \cap \hat{V} \cap S|} & \text{if} \quad j.t \in V(z) \backslash \hat{V}(z) \text{ and } i.s \in \hat{V}(z) \end{cases}
$$

while setting the remaining flows $q_{j.t}^{i.s}(z)$ to zero. The proposed flows converge to efficiency since for any player in $i \in \hat{V}(z)$ by construction it must be that

$$
\lim_{z \to \infty} q_i(z) = \hat{q}_i + \lim_{z \to \infty} \sum_{k \in V_i \backslash \hat{V}} \left(q_i^k(z) - q_k^i(z) \right) = \bar{q}_i^\infty,
$$

while $q_i(z) = \bar{q}_i^\infty$ for players $i \in V(z)\backslash \hat{V}(z)$. The proposed flows $\mathbf{q}(z)$ must be arbitrarily close to symmetric equilibrium flows in the limit as z diverges. This is the case because conjectured flows $\mathbf{q}(z)$ satisfy $\lim_{z\to\infty} q_{j.t}^{i.s}(z) = 0$. Again $\lim_{z\to\infty} \mathbf{q}(z)$ satisfies all the outflow equilibrium requirements for trades on all links in the limit economy. If such flows were chosen by others, no player would be able to profitably affect the prices of the goods sold in the limit, as deviations on his behalf could only reduce prices by $\lim_{z\to\infty} q_{j.t}^{i.s}(z) = 0$, and because no player resells a non-negligible amount of consumption. As gains from deviating from $\mathbf{q}(z)$ again decrease along the sequence of replicas and vanish in the limit, the limit of $\mathbf{q}(z)$ is approximately efficient and belongs to the limit of the symmetric equilibrium correspondence. Lower hemi-continuity again guarantees the existence of a selection of equilibrium correspondence that converges to such a limit point. ∎

Proof of Proposition 8.   (a) This is a consequence of vanishing price distortions in any efficient limiting economy (which requires concave revenues and convex costs) and of Proposition 6 (which shows that MC implies that a limiting outcomes can be efficient).

(b) Since in any symmetric equilibrium of the limiting economy the outflow wedges vanish, revenues in each market are concave. Since the third derivative is positive, costs of supplying units are convex. Thus existence of a symmetric equilibrium in the limit economy follows as in Proposition 1.

(c) Let $i.s^* \in \arg\min_{j.t\in V(z)} u'_{j.t}(q_{j.t}(z))$. If $\bar{\mathbf{q}} \neq \mathbf{0}$, for any sequence of outflow equilibrium flows $\mathbf{q}(z) \in \mathbb{R}_+^{E(z)}$ it must be that $i.s^* \in S(z)$, because such a player does not purchase consumption by part (b) of Proposition 3, and because by definition of competitive equilibrium $0 \geq q_{i.s^*}(z) \geq \bar{q}_i$. By contradiction suppose that there exists a sequence of outflow equilibria that does not converge to efficiency. If so, the set of players linked to $i.s^*$ and with marginal utility strictly higher than $i.s^*$ diverges, since $V_j(z) \supseteq S(z)$ for $\forall j \in D(z)$ implies $V_j(z) \supseteq D(z)$ for $\forall j \in S(z)$, and since $\lim_{z\to\infty} |D(z)| = \lim_{z\to\infty} z|D| = \infty$. This immediately yields a contradiction if $\lim_{z\to\infty} (q_{i.s^*}(z)) > -Q_i$, because $Q_i < \infty$ and because by part (c) of Proposition 3 Player $i.s^*$ would sell a strictly positive amount of consumption in the limit to all his neighbors with strictly higher marginal utility.

If, instead, $\lim_{z\to\infty} (q_{i.s^*}(z)) = -Q_i$, let $V_+(z) = \{k \in V(z) | q_k(z) > -Q_k\}$ and let $i.s_* \in \arg\min_{j.t\in V_+(z)} u'_{j.t}(q_{j.t}(z))$. First notice that $\lim_{z\to\infty} |V_+(z)| = \infty$, since $\lim_{z\to\infty} z\sum_{i\in V} Q_i = \infty$ and since $u'' < 0$. Thus, no player in $V\backslash V_+(z)$ sells to $i.s_*$ for z large enough, since a large and diverging number players have strictly higher marginal utility than $i.s_*$, if $\bar{\mathbf{q}} \neq \mathbf{0}$. Hence, in the limit $i.s_*$ does not buy. If $i.s_* \in S(z)$ for z large, assuming that the sequence of outflow equilibria does not converge efficiency again yields a contradiction. In fact, part (c) of Proposition 3 would imply that Player $i.s_*$ sells a strictly positive amount of goods in the limit to all his neighbors with strictly higher marginal utility which is impossible since $\lim_{z\to\infty} (q_{i.s_*}(z)) > -Q_k$, since $Q_i < \infty$, and because $i.s_*$ has infinitely many neighbors with higher marginal utility in the

limit economy. A contradiction arises even if $i.s_* \in D(z)$ for z large and if the sequence of outflow equilibria does not converge to efficiency. In particular if $i.s_* \in D(z)$ for z large enough, it must be that $\bar{q}_i > 0 \geq \lim_{z \to \infty} q_{i.s_*}(z)$, since $i.s_*$ only sells for z large enough. Moreover, by definition of $i.s_*$ it must be that, for any $j.t \in V_+(z)$,

$$u'_{i.s_*}(\bar{q}_i) < u'_{i.s_*}(q_{i.s_*}(z)) \leq u'_{j.t}(q_{j.t}(z)),$$

which in turn by concavity implies that $\bar{q}_j > q_{j.t}(z)$ for any $j.t \in V_+(z)$. Also, notice that $\bar{q}_j \geq q_{j.t}(z) = 0$ for any $j.t \in V \backslash V_+(z)$. Hence, provided that $\bar{\mathbf{q}} \neq \mathbf{0}$, contradiction arises, since

$$0 = \sum_{j.t \in V(z)} \bar{q}_{j.t} > \sum_{j.t \in V(z)} q_{j.t}(z).$$

Thus the limit outflow equilibrium must be efficient. ∎

Proof of Proposition 9. Define the total quantity sold from an individual of type i to all individuals of type j in the unique symmetric equilibrium of a $(\mathbf{C}, z)$-replica by $\mathring{q}^i_j = z q^i_j(z)$. The inequalities defining the symmetric equilibrium of a replica (a complementarity problem) can be written in terms of such quantities as follows

$$
\begin{aligned}
f^i_j(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z) &= u'_i(\mathring{q}_i) - u'_j(\mathring{q}_j) - u''_j(\mathring{q}_j)(\mathring{q}^i_j/z) - u''_i(\mathring{q}_i) \sum_{k \in V_i} \mathring{q}^k_i + \mu_i \geq 0, \quad &(4)\\
f_i(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z) &= Q_i - \mathring{q}^i_\circ \geq 0,
\end{aligned}
$$

where $f^i_j \mathring{q}^i_j = 0$ and $f_i \mu_i = 0$. Let $\mathbf{f}(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)$ denote such complementarity problem. Define the set active constraints at the equilibrium of the replica by

$$T(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z) = \left\{ ij \in E | \mathring{q}^i_j > 0 \right\} \cup \left\{ i \in V | \mu_i > 0 \right\}.$$

Let $\mathbf{f}_T(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)$ denote the complementarity problem obtained by restricting attention to the active constraints. By assumption any replica economy possesses a unique equilibrium and conditions for existence are met. Thus, the Jacobian of the problem must be positive definite at the unique solution (Kolstad and Mathiensen 1987),

$$J_T(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z) = \nabla \mathbf{f}_T(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z) > 0,$$

where only the principal minor of Jacobian associated the active constraints has to be considered. The implicit function theorem then implies that at the unique equilibrium of the problem

$$\frac{\partial \mathbf{f}_T}{\partial \mathbf{q}} \frac{\partial \mathbf{q}}{\partial z} + \frac{\partial \mathbf{f}_T}{\partial \boldsymbol{\mu}} \frac{\partial \boldsymbol{\mu}}{\partial z} + \frac{\partial \mathbf{f}_T}{\partial z} = 0 \quad \Rightarrow \quad \frac{\partial (\mathbf{q}, \boldsymbol{\mu})_T}{\partial z} = -J_T(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)^{-1} \frac{\partial \mathbf{f}_T}{\partial z},$$

where the definition of the complementarity problem requires that

$$\frac{\partial f_j^i(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)}{\partial z} = \frac{u_j''(\mathring{q}_j)\mathring{q}_j^i}{z^2} \quad \text{and} \quad \frac{\partial f_i(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)}{\partial z} = 0.$$

Let $L(\mathring{\mathbf{q}}|z) = \{ij \in E|\mathring{q}_j^i > 0\}$ denote the set of active flows. Define: $J_L(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)^{-1}$ to be the leading minor of $J_T(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)^{-1}$ associated with indexes in $L(\mathring{\mathbf{q}}|z)$; $\mathbf{x} = \{u_j''(\mathring{q}_j)\mathring{q}_j^i\}_{ij \in L}$; and $\mathbf{Z} = \{z_{kl}^{ij}\}_{ij,kl \in L}$ as follows

$$z_{kl}^{ij} = \begin{cases} 1/z & \text{if} & ij = kl \\ 1 & \text{if} & j = k \ \cap \mathring{q}_j^i > 0 \\ 0 & \text{if} & \text{otherwise} \end{cases}.$$

For such notation, one gets that

$$\mathbf{Zx} = \{u_j''(\mathring{q}_j)(\mathring{q}_j^i/z) + u_i''(\mathring{q}_i)\sum_{k \in V_i} \mathring{q}_i^k\}_{ij \in L},$$

$$\frac{\partial \mathbf{q}}{\partial z} = -(1/z^2)J_L(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)^{-1}\mathbf{x},$$

where the second equality obtains, as the the replica counter never appears in an outflow constraint. The matrix $\mathbf{Z}$ is positive definite because, for an appropriately chosen order of links, it is lower triangular (as goods do not cycle), and because all elements on the main diagonal are positive. Differentiating per-capita social welfare with respect to z one gets that

$$\frac{\partial}{\partial z}\left(\frac{1}{V}\sum_{i \in V} u_i(\mathring{q}_i)\right) = \frac{1}{V}\sum_{ij \in E}(u_j'(\mathring{q}_j) - u_i'(\mathring{q}_i))(\partial q_j^i/\partial z) =$$

$$= -\frac{1}{V}\sum_{ij \in E}\left(u_j''(\mathring{q}_j)(\mathring{q}_j^i/z) + u_i''(\mathring{q}_i)\sum_{k \in V_i} \mathring{q}_i^k\right)(\partial q_j^i/\partial z) =$$

$$= -\frac{1}{V}\mathbf{x}'\mathbf{Z}'\frac{\partial \mathbf{q}}{\partial z} = \frac{1}{Vz^2}\mathbf{x}'\mathbf{Z}'J_L(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)^{-1}\mathbf{x} \geq 0.$$

The last expression is positive since it is a bilinear form and because both $\mathbf{Z}'$ and $J_L(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)$ are positive definite. In fact, because both are positive definite, consider the positive definite square root H of $J_L(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)^{-1}$ (i.e. $J_L(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)HH = I$). Then $\mathbf{Z}'J_L(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)^{-1} = H^{-1}(H\mathbf{Z}'H)H$. Therefore $\mathbf{Z}'J_L(\mathring{\mathbf{q}}, \boldsymbol{\mu}|z)^{-1}$ and $H\mathbf{Z}'H$ have the same eigenvalues. Since $H\mathbf{Z}'H = H'\mathbf{Z}'H$, such matrix is positive definite and thus has only non-negative eigenvalues. The third equality uses the observation that $\partial q_j^i/\partial z \neq 0$ implies that the first order condition must hold with equality. In fact, if $\partial q_j^i/\partial z < 0$, then $\mathring{q}_\circ^i < Q_i$ clearly holds and $\mathring{q}_j^i > 0$ or else $\mathring{q}_j^i$ could not decrease in equilibrium. If $\partial q_j^i/\partial z > 0$ instead, then $\mathring{q}_j^i > 0$ clearly holds and $\mathring{q}_\circ^i < Q_i$ or else $\mathring{q}_j^i$ could not increase in equilibrium. $\blacksquare$