Systems and Systemic Risk in Finance and Economics

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Systems and Systemic Risk in Finance and Economics *

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Neo: There are only two possible explanations: either no one told me, or no one knows.

The Architect: Precisely. As you are undoubtedly gathering, the anomaly’s systemic, creating fluctuations in even the most simplistic equations... an anomaly that if left unchecked might threaten the system itself.

Source: *The Matrix Reloaded*

1 Introduction

While the subject of systemic risk is enjoying a well-deserved surge of interest, many of the systemic risk researchers and practitioners would privately admit of yearning for a more fundamental definition and analysis of systemic risk to complement the operational and somehow ad-hoc definitions currently in use. For concreteness, let us mention the widely used and useful joint IMF, BIS and FSB (2009) definition proposed that defines a systemic event as

the disruption to the flow of financial services that is (i) caused by an impairment of all or parts of the financial system; and (ii) has the potential to have serious negative consequences for the real economy.

While in broad agreement with this definition, our aim is to micro found the concept and to approach the question from the more fundamental concept of an “economic system.” Indeed, we do not believe it satisfactory to analyse systemic risk before first having reflected on what a system is in the first place. If the term “system” has no meaning over and above “aggregate,” “systematic” or even “non-chaotic aggregate,” then it is not clear what systemic risk means over and above aggregate or systematic risk.

Within the rapidly expanding literature on systemic risk – mostly post 2007/8 – several authors stand out, for example the useful survey paper by Bisias

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1 It may be worthwhile pointing out that one of the most insightful and prescient books attempting to understand the foundations of the notion of systemic risk is Kambhu et al. (2007), written on the back of a conference in 2006 prior to the crisis starting in 2007.
et al. (2012) and the recent insightful books by Gai (2013) or Fouque and Langsam (2013). These authors use the term in a variety of ways, shedding promising light on different relevant aspects of systemic risk without a need for a definitive definition. That is common and commonsensical practice in science, of course. From these works it can be gleaned that the authors clearly do view systemic risk as a concept distinct from “aggregate risk” or “systematic risk,” even if a commonly accepted definition of systemic risk is lacking. Our aim is to base the idea of systemic risk on a more solid epistemological basis, thereby allowing for a clear distinction between systemic risk and other forms of risk.

Risk also often refers to the risk of the external exogenous shocks that play the role of inputs, as opposed to the risk that the outcomes of these shocks create once they work through the system. For instance, “idiosyncratic risk” stands for idiosyncratic shocks that hit individual exogenous factors such as endowments and preferences but wash out in the aggregate, while “aggregate risk” stands for the shocks that do not wash out, such as common shocks. In fact, aggregate risk is often simply the arithmetic sum of all idiosyncratic risks, and we shall argue later on that this concept would not in the philosophy of science count as a systemic, or emergent, property.

In our view, systemic risk is a fruitful concept if we interpret it in a systems context. It not only captures the exogenous risk that hits the system and may prevent the system from functioning properly, but it is also the endogenous risk that is generated by the system itself, even if the proper functioning of the system itself is not at risk. So according to the direct and indirect meanings given to these terms in everyday language,

systemic risk comprises the risk to the proper functioning of the system as well as the risk created by the system.

Of course, these two risks can overlap, and a shock within the system and then amplified by the system can lead to the auto destruction of large components of the system up to the entire the system itself, or indeed up to the real economy that embeds the system from which the shock emanates.

This concept of endogenous risk and its inherent positive feedback loops is closely related to systemic risk, see Danielsson and Shin (2003) and Danielsson and Zigrand (2008). The idea of amplification through interaction has a bit of the flavour described by Miller and Page (2007), “in systems char-
acterised by the central limit theorem, interactions cancel one another out and result in a smooth bell curve. In complex systems, interactions \textit{reinforce one another} and result in behaviour that is very different from the norm.” Similarly, a system is defined by its connective structure between elements. A network whose structure of interactions does not allow for diversification or averaging of individual effects can by itself be a significant propagator of shocks (see e.g. Jovanovic (1987) in a game-theoretic setting, Durlauf (1993) in a local-interaction setting and Acemoglu et al. (2012) in a network setting). In that sense, the interesting emergent properties in social systems, such as stock market crashes, banking crises, runs on the repo, riots etc., are primarily those that occur if large-number-type forces do not apply because of feedback loops.

In the rest of the paper we provide an intuitive account of the economic thought on systems and the development of the notion of systemic risk. The journey is illustrated by putting the ideas of system systemic risk and endogenous risk in a historical perspective.

2 What is a System?

The first task is to set out what we mean by a system. We believe that there is value in formalising the notion of a system somewhat and that the scientific content of the term “system” ought to be founded on the everyday use of this term. The following discussion is inspired by the textbook exposition in Mandy (1988) and a fuller epistemological discussion can be found in Appendix A.

In everyday language, certain realities qualify as forming a system while others do not. For instance:

| cluster of stars and planets          | vs | solar system           |
| collection of milestones              | vs | metric system          |
| lump of nerve cells                   | vs | nervous system         |
| miscellanea                           | vs | information system     |
| population of individuals as such     | vs | social system          |
| buses and trains                      | vs | public transportation system |
| lines of computer code                | vs | operating system       |
In the science of economics and finance,

<table>
<thead>
<tr>
<th>population of prices</th>
<th>vs</th>
<th>price system</th>
</tr>
</thead>
<tbody>
<tr>
<td>collection of banks</td>
<td>vs</td>
<td>banking system (Federal Reserve System, Eurosystem)</td>
</tr>
<tr>
<td>set of local monetary arrangements</td>
<td>vs</td>
<td>international monetary system</td>
</tr>
<tr>
<td>bilateral payment and settlement arrangements</td>
<td>vs</td>
<td>payment system</td>
</tr>
<tr>
<td>tax and spending rules</td>
<td>vs</td>
<td>fiscal system</td>
</tr>
</tbody>
</table>

With these examples in mind, the casual speaker would perhaps mention three characteristics of what constitutes a system – as opposed to merely a collection:

1. these expressions make reference to something, to a central concept;
2. their elements imply relationships;
3. in systems that evolve, which is the case especially of social and economic systems, they keep their identity.

In the scientific context of financial economics, the corresponding characteristics for economic systems are:

1. deductibility from the axiom of economicity, or the economising axiom, implying reference to a common principle and formal elements that are functions and serve an economic purpose, i.e. economic calculus\(^2\);
2. irreducibility, and therefore emergent properties due to interrelations;
3. continuity of identity.

To summarise, deductibility from the axiom of economicity bestows upon the elements the consistent and coherent shine and approach encapsulated in the effort to realise economic surplus. All elements being logically deduced

\(^2\)In the words of Allais (1981) “In their essence all economic operations, whatever they may be, can be thought of as boiling down to the pursuit, realisation and allocation of distributable surpluses.”
from this same axiom, they do not allow dislocations between the various elements. Irreducibility allows the system to distinguish itself from its parts by its *organisation*.

For social and economic systems, deductibility from the axiom of economicity further implies that the system is distinct from its parts due to the *function* the system plays. In the physical sciences this need not necessarily be the case. For instance, the beautiful organisation of the atoms of a crystal is an end product in itself, one not performing any function. A social system on the other hand must be *functioning*, or as Vining (1984) says, it must be a *working thing*, a machine functioning towards accomplishing an *aim*, an aim embedded in the principle of deductibility from the axiom of economicity. It may not be perfect, but it possesses some minimal requirements, not least consistency, coherence and an ongoing goal.

Systems that possess strong or novel emergent properties are often called *complex systems*. In analysing the price formation system for instance, Mills (1927) argued that the behaviour of the whole is not to be explained in terms of the behaviour of the individual elements that make it up, that the attributes of the whole are not to be arrived at by summing the attributes of the separate parts. Later on, Simon (1996, p. 184) writes that “in [complex systems] the whole is more than the sum of the parts in the weak but important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole.” As examples, a living cell is more than the sum of inert molecules that it is composed of, and human conscience is more than a sum of neurons. Below we see corresponding examples from the financial world – a price system, a payment system and a banking system – in detail.

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3 The discussion of what a “system” is is not unlike the long-standing discussion as to what a “machine” is. Adam Smith (1795) on page 60 already wrote that “Systems in many respects resemble machines. A machine is a little system, created to perform, as well as to connect together, in reality, those different movements and effects which the artist has occasion for. A system is an imaginary machine invented to connect together in the fancy those different movements and effects which are already in reality performed.” More recently, Ashby (1945, 1947, 1956) and Farmer (2007) have argued that a taxonomy of machines would be based on *function*, not on form, ancestry or material composition. For instance, Ashby’s definition of a machine is “that which behaves in the same way as does a closed single-valued transformation,” capturing the mechanical or “machine-like” working that is mapping its internal state and the state of its surroundings into a new state, and then adds that “it should be noticed that the definition refers to a way of behaving, not to a material thing.”
3 Economic and Financial Systems

In the context of systemic risk, the advantage of viewing a system as a working mechanism, and not as an outcome of the thing that is doing the working, is twofold. First, only a working mechanism can break down in the first place. Second, it allows researchers to focus on the mechanism that creates destructive systemic events, as opposed to the outcomes or the original input shocks that can lead to the displacement of the original equilibrium and to the unfolding of the destructive mechanism. Since it is unlikely that researchers can adequately forecast the next triggering shock, it would be valuable to understand how susceptible the system is to shocks – its systemic fragility – and how to redesign the system in a way to prevent the build-up of destructive positive feedbacks or cascades, and to introduce circuit breakers and other devices that can cut the feedbacks.

Walras and Pareto were both preoccupied with the idea of the economy as a complete system, with all phenomena interdependent on each other. The graphical construction of the misleadingly named “Edgeworth Box” by Pareto (Pareto, 1909, p. 191) can be viewed as the essence of this desire. Pareto in fact set himself the task of developing a system of laws capable of describing the behaviour of society as a whole. One of the first authors to take a further step back and to focus on the systemic aspects of the economy, and to empirically study the price system in particular, was Frederick C. Mills in his book on The Behavior of Prices (Mills, 1927). The fact that the concept of a system was not a well-defined mathematical object led to a fascinating and heated exchange with Jacob Viner to which we shall return in Section 3.5. Rutledge Vining (1984) summarises this exchange by saying that while for Viner the value of a thing and the theory of what determines its amount are what economics is about, to Mitchell and Mills the orienting fact is not the value of a thing but rather the working or performing of an economic system, of which one of the outcomes are those prices. In the same spirit, systemic risk is not only the risk that a price or a quantity are outliers, but rather it is risk associated directly with the machine or mechanism that does the working and generates prices, allocations and so forth.
3.1 Appraising Economic and Financial Systems

How often have we heard politicians complaining post 2008 that “the financial system is broken”? For instance, how should one even start to think about the question of how a given new policy rule would affect an observable statistical characteristic of systemic risk, or of volatility of prices, or of the fairness and trust in the financial system? In fact, herein lies another crucial distinction, already advanced by Knight (1947), between physical and social systems in that people agree what it means for a watch to work or for a human body to be healthy, but such a statement about the economy is much less clear in reality and a definition is not obvious.

The literature on systems has progressed since that exchange of opinions in the 1920s, and authors have expressed the intuitive meanings of the system concept in more formal forms. For our purposes of studying systemic risk, and in particular when viewed as a practical endeavour, the work by Vining (1984) is of direct relevance. He tried to see the economic system from the view of an economist in the role of practitioner, and more precisely in the role of a specialist advisor to legislators and citizens in a legislative frame of mind. Partly based on earlier work (Vining, 1969), Vining suggests the following concepts which we shall adopt also:

**Economic System.** The something that is said to be working or functioning and that can be deliberately modified is called economic system, denoted as $\mathcal{S} \equiv \{\theta, M\}$. It is the modifiable operating mechanism, or operating entity, that performs. $\theta$ is the set of constraining and prescriptive rules, the “rules of the game” as Frank Knight put it. This $\theta$ is what the legislators alter, not $M$. A change in $\theta$ can be minor in the sense of not altering the essence of the system (e.g. raising the capital ratios by a fraction), or a change in $\theta$ can alter the functioning of the

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4 "Individual medicine itself is “scientific” only to the extent that men agree on the meaning of health and disease... In this field, the degree of agreement which is practically requisite may be taken for granted. In “social medicine,” the case is distinctly to the contrary; the main problem of realizing social health is that of defining it, of agreeing as to what is to be striven for... The practical problem of achieving... a “healthy” society is not a scientific or technical one in the proper, instrumentalist sense. To begin with, it is the very different kind of problem involved in formulating, which means rationally agreeing upon, “rules of the game,” which is not a matter of means and ends.”

5 There are a number of similar conceptual frameworks for thinking about comparative economic systems, including Montias (1976) and Neuberger and Duffy (1976).
system (e.g. breaking up the banks). $M$ is the \textit{exogenous statistical mechanism} or \textit{generating mechanism} that cannot easily be amended by changing the rules.

While \{$\theta, M$\} as a conceptual experiment is in practice too vast to describe as a mathematical object, for concreteness let us say that

$$M = \{(\Omega, \mathcal{F}, \mathcal{T}, \mathcal{F}_X, \mathbb{P}), (I, \chi^I, \mathcal{N}), A\}$$

The elements in $M$ are exogenous and not under the control of the authorities (and are therefore not also in $\theta$):

$$(\Omega, \mathcal{F}, \mathcal{T}, \mathcal{F}_X, \mathbb{P})$$ represents the probability space with state space $\Omega$ and the informational structure(s) $\mathcal{F}_X$ generated over time $\mathcal{T}$ by the exogenous state processes $X$, such as the endowment processes, taste shocks, productivity shocks, etc. These information structures can differ between agents, including the policy makers. 

$$(I, \chi^I, \mathcal{N})$$ represents the set of a-priori agents and institutions $I$ with given characteristics $\chi^I$ (including types spaces, location and objective functions) and a-priori network structure of connections $\mathcal{N}$. This set also includes the given market mechanisms that enable trade, i.e. the set of trading venues and their market clearing mechanisms, be they batch auctions, limit order books, over-the-counter or dark, etc.

$A$ represents the exogenous elements of the commodities space $X$ and the securities payoff structure (details of which can be amended through $\theta$).

Notice that this construction satisfies the three defining characteristics of a system outlined in Section 2. $S$ refers to a central concept by being a functioning mechanism designed to fulfil an economic purpose. Through $M$, $S$ also exhibits (possibly endogenous) interrelations between its components and is a dynamic system, evolving stochastically over time.

\textbf{Economy.} The physical outcome of the system’s working, which legislators react to, is the \textit{economic reality}, for simplicity called \textit{the economy}. This outcome is manifested in strictly physical phenomena, though perhaps not all observable. It is this immense mass of physical things and
events, populations and processes of events of real elements over space and time, of which there are observations. It is the physical reality. Let us denote the economy by $\pi_t$. In practice – absent lump-sum transfers and perfect information about agents' characteristics – the outcome $\pi_t$ cannot usually be directly modified by policy makers, only $\theta$ can.

**Performance Characteristics.** The third basic notion is that of a numerical specification of the stable (or stationary) properties of the outcome in terms of which the performance characteristics of an economic system are described. $\pi_t$ cannot be observed fully by anybody, instead the performance vector observable to policy makers is $\beta_t$. Its components numerically specify statistical properties of $\pi_t$ that have acquired meaning as operating characteristics. In other words, they convey to responsible persons a sense of how well or badly a currently operating economic system is working. The functional dependence of $\beta_t$ upon $\{\theta, M\}$ captures the essence of policy making.$^6$

One is rarely primarily interested in the state of any particular individual thing; the central focus is always the statistical properties of developing populations and processes. The names by which the features discussed are called are those of population phenomena: instability, concentrations, depression, underdevelopment, imbalance, blight, industry concentration. In our case, $\beta$ reflects observable aspects of systemic risk.

**Norms.** When deciding to change the rules, one has norms in mind against which the actuality is compared. I paraphrase, but what makes economics economics, a social science, is the problem of identifying and specifying what those norms or standards are against which responsible observers gauge the actuality that they in fact see. These norms reflect the axiom of economicity.

$^6$Goodhart’s Law (Goodhart, 1975) and Lucas Critique (Lucas, 1976) are accommodated within this framework. If, based on past analysis and observations, we found some statistical regularities in $\beta_t$ that may be exploited to move the economy in some desirable direction, once $\theta$ is modified to direct the system in the desired direction, it may well be that the new $\beta_t$ post change no longer satisfies this statistical regularity. This is taken into account when saying that $\beta_t$ is seen as a mapping from $\{\theta, M\}$.
3.2 On the Policy Maker’s Information System

It is worth emphasising a point made in Bonisch and Di Giammarino (2010) that in order to monitor and influence risk in the financial system \(\{\theta, M\}\) in actual practice, the policy maker and the supervisors need to have a sufficiently complete picture of the overall mechanism \(M\) as well as of the policy mapping \(\theta \mapsto \beta\). Since the relevant information about these systems that constitute \(M\) is disaggregated and held by dispersed financial participants who often have a strategic interest in keeping it private, with no participant having more than a small window on the whole \(M\), in practice this means that information flows need to be incentive-compatible, intelligently organised, using the right market variables to infer information about the mechanism (Mises, 1920; Hayek, 1945) and linked into the policy maker’s Information System.

Policy makers (for the sake of brevity, we include supervisors and regulators under that heading) currently do not have all the information and data sets they need. Furthermore, they receive vast amounts of data (“big data”) that are currently nor explored fully because the information is either simply gathered with no resources expended on properly analysing it, or it is siloed inside one of the regulatory bodies, or because the various regulatory bodies do not communicate properly with each other. The latter is often true at the national level and certainly at the international level. A shared infrastructure is required that allows data to be linked, accessed and analysed intelligibly across policy desks, institutions and jurisdictions. In short, a proper information system is required as a replacement for the decentralised collection of informational silos we currently have.\(^7\)

As a start, this system needs to be built upon agreed and meaningful universally accepted standards and sufficient statistics carefully chosen for the systemic risk platform, “encoded into a widely accepted, non-proprietary, searchable, computer-readable data format” (Bonisch and Di Giammarino,

\(^{7}\)Some authors are doubtful whether current and past data as currently gathered would be sufficient to understand how complex evolving systems behave in the future, partly because some relevant variables are not observable to observers and partly because the system is changing or the relevant variables cannot be guaranteed to be stationary and ergodic (or can be made so). For instance, Hayek (1989, p.93) writes that “the numerical measurements with which the majority of economists are still occupied today may be of interest as historical facts, but ...with the functions of the system these magnitudes have evidently very little to do.”
Incomparable data cannot be strung into an information system. Excessive and unstructured data is also of no use to the regulators. A high-frequency trading company can submit disks with terabytes of data and code to a regulator, but this information is unlikely to be of much use. Similarly, the 2.5 billion pages of documentation that Goldman Sachs provided to the Financial Crisis Inquiry Commission following their request for information on Goldman’s mortgage derivatives business has been perceived as wilfully obstructive. A clearer dialogue with investment firms that insures a firmer buy-in would be beneficial. Finally, it is doubtful that the policy community can fully comprehend and analyse the financial system on its own. In particular, workable and standardised confidentiality clauses need to be established that allow the harnessing of the academic community.

3.3 On Performance Characteristics $\beta_t$.

All particulars become meaningless if we lose sight of the pattern they jointly constitute.

Polanyi (1962, p.57)

In order to appreciate the meaning of $\beta$ in relation to $\pi$ and $\{\theta, M\}$, one can cite Mills (1935) who writes that ultimately it is the overall behaviour of the economy that is of most interest: “The data of economics (the price of potatoes on a given day, the rate of interest on a certain date) are the result of a great number of individual actions, and it is the mass, not the individual, which concern us. The mass must be studied as a whole, and its properties as a functioning unit ascertained.”

Similarly, one can cite the physicist Schrödinger (1944):

In the course of the last sixty or eighty years, statistical methods and the calculus of probability have entered one branch of science after another. (…) On its first appearance the new weapon was mostly accompanied by an excuse: it was only to remedy our shortcoming, our ignorance of details or our inability to cope
with vast observational material. (...) In textbooks on gas-
theory it has become a stock phrase, that statistical methods are
imposed on us by our ignorance of the initial coordinates and
velocities of single atoms – and by the insurmountable intricacy
of integrating $10^{23}$ simultaneous differential equations, even if we
knew the initial values. (...) But inadvertently, as it were, the
attitudes changed. It dawns upon us that the individual case is
entirely devoid of interest, whether detailed information about it
is obtainable or not, whether the mathematical problem it sets
can be coped with or not. (...) The working of the statistical
mechanism itself is what we are really interested in.

Herbert Simon, when discussing weak emergence, agrees with this when he
says (Simon, 1996, p.171): “‘Weak emergence’ shows up in a variety of ways.
In describing a complex system we often find it convenient to introduce the-
toretical terms, like inertial mass in mechanics, or voltage in the theory of
circuits, for quantities that are not directly observable but are defined by
relations among the observables. We often use such terms to avoid reference
to details of the component subsystems, referring only to their aggregate
properties.” Weather forecasting, for instance, does not attempt to forecast
the behaviour of each molecule but rather a distribution of pressure across
space and time and whether or not we can expect rain in London on Tuesday
afternoon. The answer to which is typically yes.

The same interest in the performance of the overall system exists in the social
sciences. Hayek (1976, chapter 10) writes that: “the efforts of the legislator
can thus be directed only towards increasing the chances for all, not in the
sense that the incidence of the diffused effects of his decision on the various
individuals will be known, but only in the sense that he can aim at increas-
ing the opportunities that will become available to some unknown persons.”
This should be clear to anybody observing modern computer based equities
markets for example (see Linton et al., 2013, for an in-depth analysis). It is
likely that no one entity has access to all the quotes and all the transactions
between all the players across all markets – lit and dark, OTC and organised
– across all securities. These outcomes of the price system are represented
by an element of $\pi_t$. Instead, when policy makers try to figure out whether a
larger minimum tick size would be a step in the right direction, they would
like to compare summary statistics, such as market price and limit order
book stability, overall transaction costs, the typical time it takes to offload
a large position within a given price impact bound, the degree of confidence of final investors in the markets, the typical perceived market quality, the evenness of the playing field, etc., of the current regime \( \theta \) with those of the proposed new regime \( \theta' \). In other words, they are not deliberating about the mapping from \( \{\theta, M\} \) to a \( \pi_t \) but to a \( \beta_t \).

The same applies to the central question in Allen and Gale (2000) on comparing financing systems. The authors did not intend to find out how each individual’s situation and actions would change at any moment in time if an economy moved from a market-based financing system \( \theta \) to a banking-based financing system \( \theta' \) as reflected in \( \pi_t \), but rather what the overall efficiency gains and losses are, the quality and extent of risk sharing, the ease with which firms can raise of raising financing for new projects etc., as reflected in \( \beta_t \). Neither did the many authors who analysed the transition from command-based economies to a free economy system attempt to predict the detailed outcome of the transition for everyone and everything (see for instance Mandy (1992) and Anderson and Kegels (1998)).

Herein also lies one of the more telling failures of much of mainstream financial thinking that underlies recent financial regulations, such as Basel II, III etc., namely the fallacy of composition. It is not true that the financial system is safe if each financial institution is safe (see Danielsson et al. (2001) for an early warning). One cannot easily deduct a property of an aggregate from the property of the elements that make up the aggregate, precisely because individual elements interact with each other and with the constraints of the problem in a way that aggregation can profoundly alter the characteristics. The recognition of this fact during the financial crisis that began in 2007 led to the design of “macroprudential regulation” to complement traditional “microprudential regulation.”

### 3.4 System of Systems

Vining says that once one empirically studies the thing that does the working and the statistical properties of some population process, it would then follow that the working part of this system consists of a collection of statistical mechanisms. In other words, the world might be best viewed as a system of systems. Nevertheless, one has a need to analyse parts of the system without analysing everything. For instance, when analysing the banking
system, and depending on the precise questions one is asking, one may be required to abstract first from the interactions with other systems, and then only interrelate the systems at a second stage.

This idea is also related to what Simon (1996) calls a *hierarchic nearly decomposable system*: “By a *hierarchic* system I mean a system that is composed of interrelated subsystems, each of the latter being in turn hierarchic in structure until we reach some lowest level of elementary subsystem. In most systems in nature it is somewhat arbitrary as to where we leave off the partitioning and what subsystems we take as elementary... At the moment we shall accept the fact that scientists do this all the time and that, if they are careful scientists, they usually get away with it... As a second approximation we may move to a theory of *nearly decomposable* systems, in which the interactions among the subsystems are weak but not negligible.” We now study a few such subsystems of the overall economic system in more detail.

### 3.5 Example: Price System

We have already seen in Section 3 that Mills and Viner were engaged in an intellectual battle over the meaning of a “price system,” as opposed to “prices.” Mills’ objective was:

> to secure a fuller understanding of the numerous relations, of varying degrees of intimacy, which tie all prices into a system – “a highly complex system of many parts connected with each other in diverse ways, a system infinitely flexible in detail yet stable in the essential balance of its interrelations, a system like a living organism in its ability to recover from the serious disorder into which it periodically falls.”

Yet, in spite of the numbers of factors and the complexity of the facts, there is hope of finding regularities in the field to be studied. It was one of the more fruitful scientific discoveries of modern times that regularities appear even when sheer chance rules. Though in the realm of chance the individual event may be unpredictable, definite principles of order prevail among groups of such events... The price system seems to furnish a striking and

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8Quoted by Mills from Mitchell (1913) (p. 31).
curious illustration of such a tendency in the realm of economics, a tendency of which there is, perhaps, a rational explanation.

... The isolated prices that were quoted in the dawn of the money economy must have been connected to other prices by ties that were but feeble and remote, and the circle within which the influence of a given price transaction was felt must have been a very narrow one. But as the money economy developed these ties increased in number and in strength. Small net-works of price relations expanded and established contacts with other such networks. The price system grew until the mesh of price relations included, as it does today, all industrialized communities. While this “mutual assimilation” was taking place, those regularities which, in Peirce’s view, constitute the “habits” of nature were coming into existence. Characteristic modes of behavior were being impressed upon various groups of prices by underlying natural forces, by the pressure of competition, by factors connected with the business cycle, and by other agencies the effects of which probably cannot be traced in detail. And the process by which price ties have been formed and by which characteristic modes of behavior have been acquired still goes on.

... If the price system could be fully explored, if all ties and connections could be traced and all fluctuations explained, then, perhaps, man could understand and control the economic system he has created. We can doubtless come much closer to that objective than we now are. It is doubtful, in a changing world, that it will ever be attained.

In this quote, we can see that Mills literally describes the price system as a complex dynamic system exhibiting emergent properties brought about by interrelations and networks. Since at the time of writing (1927) general equilibrium theory was still in its infancy and neither the tools of economic dynamics, nor of complexity theory, nor of network theory had been available, it is no wonder that his views were met by scepticism. In a critical article as a response to Mills on the definition of a system, Viner said (Viner, 1929),
and the discussion\(^9\) in (Bye, 1940, esp. pages 209 and 214):

For the price theorist, the points of outstanding interest in this volume are: the faith expressed by the author in the existence of a “price system;” his description of the characteristics of such a system; and his confidence that his method of analysis is adapted to, and that his investigation has contributed to, the demonstration of its existence. The outstanding contribution which the volume achieves is the reduction to precision and measurability of concepts that economists generally use vaguely and loosely. It is therefore unfortunate that the reader is given so little help in determining just what constitutes a “system” of prices and how one is to know when the “order,” the “regularities and uniformities,” which are apparently the criteria of a “system,” have been discovered in sufficient degree to warrant confidence in its existence.

... I think you speak of the “price system.” ... The word “system” is nowhere defined. I would like a discussion of the concept of “system” ... That is one of my stumbling blocks.

Vining explains why a price system is not a population of prices, or a price vector, but rather the thing that does the working that generates the price vector in the first place:

The thing whose properties are being described is a random process of events in geographic space and time. The thing upon which measurements are actually made for an estimation of these properties is a kind of population – but not a population of prices. A price, being a number, is a measured characteristic of an element of a quasi-population in question and is not itself an element... Mills’ data covered a period of some 36 years. During this period, billions upon billions of transactions occurred as events, each having a momentary existence at some time and place and

\(^9\)It is telling that in 1940, 13 years after Mills’ publication, the Social Science Research Council organised a conference and published a book (Bye, 1940) on the appraisal of Mills’ *The Behavior of Prices*. 

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each involving an exchange of goods for money. That which bears
the name \textit{price} is a variable defined upon this event and whose
value is given by the ratio of the number of units of money to the
number of units of goods exchanged in that transaction. . . . But
the primary reality is the transactions themselves, occurring as
events in time and geographic space. It is this process of events
that manifests the properties that Mills adapted his data to de-
scribe.

We view this as saying that what matters is not a collection of prices, or even
of prices over time, but that a price system, being the performing mechanism,
represents the entire \textit{price mapping}, as used in finance, intermediated by a
connecting tissue of demand and supply functions:

\[
P : \mathcal{X} \times \Pi \rightarrow \mathbb{R}^N
\]

where the space \( \mathcal{X} \) is the space of all transactions, i.e. a space labelling
the commodities (including time, trade counterparties, spatial and any other
properties), and \( \Pi \) regroups the given structure, including the probability
space \( (\Omega, \mathcal{F}, \mathcal{T}, \mathbb{F}_X, \mathbb{P}) \) with the (exogenous) driving state processes \( X \) de-
fined on it, the institutions, the policy environment, the set of agents and
their interrelations, their motivations and beliefs, the informational struc-
ture including expectations formation and learning, the type of auctioning
mechanism used (e.g. batch auction, limit order book), etc.

There is no reason to assume that there is one centralised market clearing
auctioneer, and the specification allows for a more realistic “economy of mar-
kets” in lieu of a “market economy” in the taxonomy of Allais (1967).10 A
quick glance at modern markets, both for Mars bars and for financial se-
curities, confirms the view that the same commodity or the same financial
security is traded at any moment in time at many different prices.

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10Allais writes in his Nobel speech “During the winter of 1966-1967, I was led to discard
the Walrasian general model of the market economy, characterized at any time, whether
there be equilibrium or not, by a single price system, the same for all the operators, - a
completely unrealistic hypothesis, - and to establish the theory of economic evolution and
general equilibrium, of maximum efficiency, and of the foundations of economic calculus,
on entirely new bases resting on the concept of distributable surplus which I had elaborated
and used in my 1943 book, and on a new model, the model of the economy of markets (in
the plural)."
So the price system is a $S := \{\theta, M\}$ where the mechanism $M$ is the entire price formation process, including those elements $\Pi_0$ of $\Pi$ that are exogenous and that cannot be readily affected by policy actions as well as those elements $\theta$ of $\Pi$ that can be amended via a policy choice.

Also,

$$\pi_t = \{(P(\mathcal{X}))(t), \text{ all transactions details}\},$$

and

$$\beta_t = \text{observable (incl. statistical) properties of } \pi_t$$

As argued above, observation is incomplete, first because much of the data is not observable by any one entity. Furthermore, what matters to the policy maker are statistical observations, such as price stability, financial stability, risk, fairness and transparency which are features of a price system, not of a price vector.

A price system is a working thing, it is functioning. Its functions are to clear markets, allowing uncountable decisions taken each day by consumers and businesses to interact and to become compatible, determining the allocation of scarce resources between competing uses. In other words, they provide a rationing function when there is a shortage (rationing of demand) or an excess (rationing of supply). They provide a signalling function to guide investors, producers and individuals in their decisions. They also help in refining one’s information. Nowadays these operations are made more explicit through the term price formation process.

In terms of emergence, the interrelations between commodities or transactions are obvious. There are substitution, income, wealth and informational effects, complementarities in production and consumption, common factors and the like. How do these interrelations substantiate themselves in terms of emergence as defined above? Observing in a Walrasian economy the (partial equilibrium) excess aggregate demand functions for each commodity $i$ separately as the mapping $p^i \mapsto Z^i(p^i; \cdot)$, being unaware of the variables represented as the $\cdot$, one knows the demand functions for each commodity separately as a function of its own price, but not as a function of all other prices and common factors. In other words, one can attempt to deduce, for each $i$ separately, $p^i : Z^i(p^i; \cdot) = 0$, giving us a price vector $(p^1, \ldots, p^n)$, but this price vector is not the same as the functioning mechanism that brings
parties together, discovers values by also considering substitutes, clears markets and reallocates resources to more valuable uses. As pointed out by Walras (1874) in his criticism of partial equilibrium analysis (and of the assumption of a constant marginal utility of money), what is missing is what joins the economy together, what makes it function by reallocating resources across commodities and agents. What is missing is the reference to the axiom of economicity, and therefore the mechanism as described above would not function properly since surplus goes to waste. At the same time, the reference attribute is missing, since commodities are not all compared simultaneously to a common yardstick of value, such as cash or a numéraire commodity.

Price systems and aggregate demand functions exhibit further emergent properties that are difficult to deduce from the behaviour of individual demand functions. For instance, the Law of Demand states that the aggregate demand for any bundle of commodities is monotonically decreasing in its price. Evidently the Law of Demand holds in aggregate if each individual’s demand satisfies monotonicity, but more interestingly there are conditions on the distribution of endowments across agents that guarantee that the aggregate demand is monotonic even though these restrictions do not guarantee that individual demand functions are monotonic (see for instance Hildenbrand (1983) and Chiappori (1985)). Alternatively, de Villemeur (2001) has shown that given a certain distribution of preferences, there are (restrictive) classes of economies for which none of the individual demand functions satisfy the Law of Demand while the aggregate does. Later on we shall see that systemic risk is to some extent related to the complementary situation whereby individual reactions to price changes do not cancel each other out and thereby do not guarantee the aggregate Law of Demand (since the Law of Demand implies uniqueness of equilibrium and stability), but instead reinforce each other through some institutional arrangements to lead to upward-sloping demand functions that correspond to some form of instability in that, in an appropriate dynamic sense, higher prices lead to a larger demand which in turn leads to higher prices and so forth.
3.6 Example: Payment and Clearing System

Payment systems such as the UK CHAPS (the Bank of England assumes the role as operator of the underlying RTGS processor) or the European TARGET 2 system (operated by the Eurosystem, of which CHAPS is a member) satisfy the definition of a system in that a collection of payers, receivers and transactions are weaved into a network of operations with a central mechanism operating in order to fulfil one of the most crucial functions in finance. The central role is taken up by the payment processor or clearing house. This role is often played by a central bank, but payment systems have existed prior to central banks and many clearing systems have at their centre a privately owned entity that settles in commercial bank money (as opposed to central bank base money).

The reason why the payment function calls for a system is that the payment functions are naturally best served in a network with a centre, a system in our definition. Jevons (1875) (see Figure 1) explains how a decentralised payment mechanism with multiple banks quickly becomes absurd if payments were to be settled by the carrying about of coin, and that it is sufficient to “appoint, as it were, a bankers’ bank to hold a portion of the cash of each bank, and then the mutual indebtedness may be balanced off.” The principle of economicity that calls for an economising of resources drives the desire for a central payment hub, and hence for a payment system.

Given this difficulty of moving coin, in the UK the payment business has found a solution with a central entity and became a proper “system” in about 1775 when “a few of the city bankers hired a room where their clerks could meet to exchange notes and bills, and settle their mutual debts. This society was of a nature of a strictly private and secret club. . . . Although it remains a private and voluntary association, unchartered, and in fact unknown to the law, the Clearing House has steadily grown in importance and in the publicity of its proceedings. . . . The Bank of England long remained entirely outside of the confederation, but more recently it has become a member, so far as regards the presentation of claims upon other banks.”

11A check system is a special sort of payment system based on paper-based debit instruments, called checks. Such payment systems are still widespread in many parts of the world, and they harbour extra difficulties, see BIS (2001). Knight (1956) for instance writes that “the object for which check system is the name is not a population of checks.”
Jevons (1875) describes on p. 261 the clearing system in the UK at the time. Bank customers are labelled by lower case letters and they bank with local banks, labelled \( P \) to \( W \). A depositor \( a \), client of bank \( P \), wishes to pay depositor \( r \), a client of bank \( U \). \( a \) sends a cheque to \( r \) who deposits it into his account with bank \( U \). \( U \) forwards it to its London-based correspondence bank \( Y \) who presents it through the Clearing House to \( X \), who debits it to \( P \). “Nothing can exceed the simplicity and perfection of this arrangement.”
Similarly in the US, before the Fed and the Fed Funds interbank lending market, banks created as a central mechanism a kind of primitive repo system to transfer funds around the banking system in an attempt to create a single big-bank approximation (see Mehrling, 2011).

Because a systemic event in the payment system may lead quickly to banking illiquidity and insolvency and therefore to a banking system event, the payments system is commonly viewed as a source of systemic risk to the entire financial and economic system, and beyond. A clear practical application of this risk to the case of the 1987 crash, with the corresponding flow network of margin calls and margin payments and ultimately the intervention of the Fed, can be found in Brimmer (1989). Similarly, when the Bank of New York computer system malfunctioned on 21 and 22 November 1985 and would not accept incoming payments for a prolonged period, the resultant illiquidity position soon ballooned to a point where no one counterparty bank could take on the risk of making a sufficiently large loan, and the Federal Reserve Bank of New York stepped in with a $22.6 billion discount window advance, see Volcker and Corrigan (1986). On a bigger scale still, on 11 September 2001 banks experienced difficulties in making their payments because of widespread damage to property and communications systems in Lower Manhattan. The Federal Reserve responded by supplying abundant liquidity to the banking system through discount window loans and open market operations – actions that helped restore payments coordination (see McAndrews and Potter, 2002). A paper by Afonso and Shin (2009) provides a formal model to think about systemic risk in payment systems.

3.7 Example: International Financial System?

If today the daily central interbank clearing and settlement process provides a centre for the clearing of the banking interrelationships within a given jurisdiction, can one speak of an international payment and banking system? Furthermore, with offshore credit and money creation, can one say that the set of national and supranational central banks – even if viewed as one – truly are able to control the global money supply?

To some extent, whether there is an effective central international clearing process is an empirical question. National CBs play a special role in the safety of payment systems given their role of providing settlement accounts
nearly free of default risk and their ability to inject currency in a crisis event. But even if the interbank settlements can be done locally in a network of such facilities, there is no World Central Bank that can inject intraday liquidity in an elastic fashion. Is this a role the IMF or the World Bank can effectively fulfil?

Even within Europe in 2013, the European banking landscape is still fragmented due to local regulatory, supervisory and resolution arrangements, due to multiple CBs and LOLRs, and due to disparate deposit guarantee schemes subjected to guarantees, some partly funded and most unfunded, by the national taxpayers in the absence of a European Banking Union. While banks operate cross-border, the segmentations create a banking sector that may not fully function like a system in the absence of both a banking union and a fiscal union (Jassaud, 2009), in which case it would indicate inefficient functioning and prone to more violent reinforcing feedback loops than otherwise.

\[
\begin{align*}
\{ \text{Banking Sector hit} \} & \Rightarrow \text{State injections} \\
\{ \text{Bank borrowing costs up} \} & \\
\uparrow & \\
\{ \text{value of state guarantees weakens} \} & \Leftarrow \{ \text{sovereign over extends} \} \\
\{ \text{gvt leans on FIs to buy more sovereigns} \} & \Rightarrow \{ \text{credit weakens} \}
\end{align*}
\]

Only a banking union makes the array of banks into a coherent system. But since banks hold sovereign debt of other countries, even a banking union cannot break the loop unless there is a fiscal union (e.g. ESM), for the failure of a sovereign still causes trouble to banks in other countries, which will put pressure on those other local sovereigns in the absence of a fiscal union. More details can be found in Danielsson (2013).

The question of the importance of a fiscal union in the functioning of an international financial system – as opposed to a collection of local banking systems or local payment systems – is not a recent one. For instance, Dupriez (1976) and Goodhart (1999) study whether international bodies, such as the IMF, can act as international LOLR, thereby giving the international banking sector a centre and coherence. Goodhart concludes that since international bodies are at a crucial disadvantage compared with national CBs (for instance neither the IMF nor the ECB have a single government standing behind it with international powers and taxing authority, and on top of that the IMF
cannot issue its own fiat liabilities): “For the time being the considerable (and even surprising) extent of segmentation in national financial systems within Europe will enable the present system of crisis resolution being centred on national institutions to continue (with the ECB playing a consultative, overseeing and advisory role). Once the European financial system becomes more integrated, the dysjunction between a centralised, federal monetary system and decentralised national fiscal powers will become more difficult to reconcile. It will be interesting to observe how this dysjunction will be resolved in future.”

Given that there is no universal supranational currency with its clearing settlement agency and lender of last resort having the power to create supranational money elastically, not much seems to have happened since the complaints by Keynes (1943), and it is hard to disagree as a result that the world financial system is not functioning as efficiently and safely as it could. The Asian crisis of 1998 provides a good example and the role of the IMF as an international lender of last resort during this episode is discussed further in Danielsson (2013).

4 Our Approach to Systemic Risk

What makes the subject of systemic risk so fascinating, and difficult, is the same thing that makes economics in general so fascinating and so difficult, and this something (which will return under the guise of the fallacy of composition in a subsequent section) is what Friedman (1980) had in mind when he said:

The great mistake everyone makes is to confuse what is true for the individual with what is true for society as a whole. This is the most fascinating thing about economics. In a way, economics is the most trivial subject in the world, and yet it is so hard for people to understand. Why? I believe a major reason is because almost any interesting economic problem has the following characteristic: what is true for the individual is the opposite of what is true for everybody together.

While aggregate constraints imposed upon a system are at the heart of the problem, and call for a general equilibrium approach, the mapping between
the individual and the aggregate itself is far from trivial because it is subject to nonlinear feedback loops and complex interrelationships. The explicit analysis of this mapping constitutes our approach to systemic risk.

4.1 Feedback Loops

We have mentioned the necessarily tight link between systemic risk and endogenous risk, especially endogenous risk of the positive feedback loop sort. The latter takes the economy outside of a damping or Central-Limit-Theorem-cancelling of effects, and into complex systems. The recognition of the importance of feedbacks goes back a long way to the work of von Bertalanffy (Bertalanffy, 1950, 1968) and especially of Wiener (1948) who formalised the notion of feedback and control, and who coined the term cybernetics. While the physical sciences latched on to negative feedback loops as regulating, self-regulating and equilibrating forces, largely cancelling out positive feedback loops in a display of homeostasis, less thought in the natural sciences was invested into large scale positive feedback loops. While positive local feedback loops have been part-and-parcel in the biological studies for a long time, overall stability prevailed.\(^{12}\)

In terms of global dynamics, the article by Maruyama (1963) has tried to refocus research on positive feedback and he even suggested to extend cybernetics into a second branch of cybernetics dealing with positive feedback systems. In the biological sciences, many destructive positive feedback loops were subsequently uncovered in the 1970s and 1980s. For instance, the circulatory system typically balances positive and negative feedback loops and manages to counteract small perturbations and restore normal functioning. However, when a larger shock perturbs, it can drive the circulatory system into a regime where homeostatic forces no longer operate and where the initial shock gets amplified, possibly until the system gets destroyed. Consider for instance the runaway condition of explosive heat death whereby an initial loss of water from the circulatory system causes a thickening of the blood, reducing the body’s cooling rate and accelerating the heat built-up. As another example, a massive loss of blood weakens the heart by decreasing its blood supply, slowing down the rate of pumping, and hence the circulation

\(^{12}\)The relationship between positive feedback loops and stability can be subtle and depends on the definition of positive feedback loops, see Ashby (1956).
of blood, which weakens the heart still further. The reader is referred to the fascinating book by DeAngelis et al. (1980) for many more examples.

Researchers such as Ashby (1947), (von Foerster, 1960, 1974), Prigogine and Varela et al. (1974), have argued that there is no need to limit one’s analysis to either negative or positive feedback loops, but that a system can exhibit both in a self-organised fashion. Such self-organised systems exhibit dynamics where a displacement does not either blindly revert to the original situation or explode and die, but where the displacement can lead to the creation of a new equilibrium order. The various steady-states a system can find itself in is often called an “attractor,” and Ashby (1962)’s principle of self-organisation states that any system will eventually end up in one of these attractors. Economists have long understood the dynamics of multiple basins of attractions in models with multiple equilibria due to external economies (Krugman, 1991). But one can go further and view the multiple basins not as multiple equilibria existing simultaneously and of which one is chosen either by history or by some rational expectations selection, but as situations that arise through purposeful higher-level actions and organisation. In that sense, one can define a super-system as the entire construct encompassing what one usually calls a system as well as the endogenous forces that act upon a displaced system and reorganise (or “self-organise”) it in a new fashion. This idea of a super-system is a natural one in the social sciences where higher-level negative feedback loops (representing policy interventions, legal amendments, wars, etc.) tend to ultimately constrain the lower-level positive feedbacks. Such a super-system would then itself never change,\textsuperscript{13} and the local behaviour of the super-system corresponds to the behaviour of the concrete system we denote by $S := \{\theta, M\}$.

4.2 Short Historical Background on Positive Feedback Loops in Economics

There is no doubt that negative feedback loops play an important role in finance and economics, preserving the structure of systems over long periods of time. Also, much of the empirical academic work on the economy relies

\textsuperscript{13}Consequently, there can never be a risk to the super-system if one considers it closed and all-encompassing, the risk merely reflects the likelihood that the super-system can find itself in a bad system state $\hat{S} := \{\theta, M\}$.
on stationarity induced by the stabilising forces of negative feedback loops, and so does much of theory through the assumption of mean-reverting driving processes (e.g. Cox et al., 1985). Endogenous crises and dysfunctional episodes, however, do not find an obvious place in this literature.

It is interesting that Maruyama (1963) writes that his work on the second cybernetics has been inspired by economists, in particular by Myrdal (1957). Myrdal described how in an underdeveloped laisser-faire economy “the few privileged people accumulate more wealth and power while the living standard of the poor tends to fall. Low standard of living, poor health, and low efficiency in work aggravate one another. Racial or social discrimination, and other social, psychological and cultural factors may be added in the ‘vicious circle.’”

In the study of economics, positive feedback loops appeared much earlier still, for instance in Quesnay’s tableau économique and in Smith (1776) while Ricardo and Malthus formalised positive feedback loops. Malthus (1798) describes that more production sustains a larger population, and that with a larger population, more production is possible, which ideally results in ever greater growth and productivity.

A monetary feedback loop was identified by Wicksell (1898), the so-called cumulative process. If the banking system (including the central bank) pushes the rate of interest below the natural market equilibrium rate, the quantity of loans demanded swells and is met by the creation of additional bank money.

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14 Quesnay (1766) analysed the circular flows of financial funds in an agricultural kingdom and the resulting feedbacks through a network of classes (producers, rentiers and a sterile class) of citizens. In his own words, “L’intérêt du cultivateur est le premier ressort de toutes les opérations économiques et de tous les succès de l’agriculture; plus les productions sont constamment à haut prix, plus le retour annuel des reprises des fermiers est assuré, plus la culture s’accroît, et plus les terres rapportent de revenu, tant par le bon prix des productions, que par l’augmentation de reproduction annuelle; plus la reproduction s’accroît, plus les richesses de la nation se multiplient, et plus la puissance de l’Etat augmente.”

15 Hayek (1989, p.93) writes that the division of labour is one of the major factors that generate self-generating orders due to the mutual adjustment of activities of people who do not know each other. “This foundation of modern civilization was first understood by Adam Smith in terms of the operation of feedback mechanism by which he anticipated what we now know as cybernetics.”

16 With finite resources, Malthus however discounts this possibility and argues for a negative feedback loop in that population would tend to grow geometrically while subsistence only increases arithmetically.
This creation leads to inflation, and reduces the real rate even further (since the nominal rate is held low by banks and central banks). This in turn further stimulates the demand for loanable funds.

The fiscal multiplier, devised by Kahn (1931) and later used by Keynes, can also be viewed as a reinforcing atemporal feedback loop. In fact, in his *General Theory*, Keynes (1936) not only considers the main point of his General Theory to lie in the destabilising forces that operate in financial markets (further emphasised in his rebuttal of Viner, Keynes (1937)), he defines intertemporal *cycles* in terms of feedback loops that represent endogenous rupturings of coherence often originating in financial usages:

By a cyclical movement we mean that as the system progresses in, e.g. the upward direction, the forces propelling it upwards at first gather force and have a cumulative effect on one another but gradually lose their strength until at a certain point they tend to be replaced by forces operating in the opposite direction; which in turn gather force for a time and accentuate one another, until they too, having reached their maximum development, wane and give place to their opposite.

More recently, while one of the great advocates of borrowing dynamic tools from physics into economics, Samuelson also warned that positive feedback loops may arise a fair bit more in economics than in physics (Samuelson, 1947, p.336):

Analytically the pure endogenous cycle is usually likened to the motion of a frictionless pendulum which satisfies a simple Newtonian second order differential equation. Upon closer examination difficulties appear with this notion. In the first place, all dampening must be ruled out or else the cycle will come to an end; similarly, in most theories anti-dampening or explosive behaviour is ruled out. Now in a physical system there are grand “conservation” laws of nature that guarantee that the system must fall on the thin line between dampening and anti-dampening, between stability and anti-stability. But there is nothing in the economic world corresponding to these laws, and so it would seem infinitely improbable that the coefficients and structural relations of the system be just such as to lead to zero dampening.
Economic systems are subjected to both compensating and cumulative processes. But reinforcing loops need not always be welfare deleterious. For instance, endogenous growth is one example of a cumulative process. Romer (1986), Lucas (1988b) and Arthur (1989) (and subsequently elaborated in Arthur (1990)) provided instances of reinforcing loops based upon instances of increasing returns: “Under increasing returns, by contrast many outcomes are possible. Insignificant circumstances become magnified by positive feedbacks to ‘tip’ the system into the actual outcome ‘selected.’ The small events of history become important.” This last point, that an economy kicked in the right direction with a small but sufficient push can lead to a resulting developments that is disproportionately large compared to the initial push, has been observed by Myrdal already.

The empirical assessment of feedback loops is a delicate econometric (and philosophical) matter pertaining to the concept of “causality” that is outside the scope of this note. Amongst the methods used in practice one finds nonrecursive simultaneous equation models (see e.g. Simon (1953) and Strotz and Wold (1960) for a survey of the issues, or Paxton et al. (2011) for a non-technical modern introduction) and tools based on spectral methods (Granger, 1969).

4.3 Interrelationships and Nonlinear Dynamics

Macro outcomes are hard to figure out, predict and forecast because of the manifold interrelationships and the (possibly resultant) nonlinearities that reinforcing feedback loops can induce. The two themes of interrelationships and feedback loops are closely knit together, for in the absence of interrelations, some effects cannot bounce back from further afield or are cancelled by offsetting effects. If that happens, individual effects cannot gather momentum to create amplifying movements and multiplier effects through macroeconomic complementary and cumulative processes. Progress is halted prematurely, see Mandy (1990). In that sense, one can view interrelationships as a mechanism that strengthens positive feedback loops. These interrelations can be purely economic, such as balance sheet intermediated, informational or market-based (e.g. via prices of commonly held asset), or they can be social. There is a growing literature on social interactions, both local and global, that studies how small differences in fundamentals can lead to large differences in

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aggregate social or economic variables through social multiplier effects. These multipliers can arise because of positive complementarities in agents’ utility functions, whereby the utility to an agent of undertaking an action increases with the number of agents undertaking the same action (e.g. Schelling, 1971, 1972, 1978; Ellison, 1993; Morris, 1997; Glaeser and Scheinkman, 2000; Brock and Durlauf, 2001). Froot et al. (1992) applied the idea to finance by allowing the asset demand by one investor to induce others to purchase shares.

Large changes in outcomes can also arise from small parameter changes because of thresholds and phase transitions resulting from the way the parameters affect the network formation. For instance, in the simple Poisson network with \( n \) nodes studied by Erdős and Rényi (1961), there are a number of thresholds for the probability \( p(n) \) of nodes forming a given link. If the probability crosses such a threshold, the network exhibits completely different emergent properties for large enough \( n \), such as having links at all, having at least one component with three nodes, cycles emerging, possessing a giant component, being connected, etc. In Poisson networks, the network formation process \( p(n) \) is exogenous and probabilities are independent, insuring that for large \( n \) the distributive properties become Poisson. In preferential attachment models (see Price, 1965) on the other hand, the network is growing through new nodes and links, with the probability of a node having a new link being larger the more links the node already has. The degree distribution in such networks, called “scale-free networks,” is approximately a power distribution, exhibiting fatter tails than Poisson networks. One of the avenues to be explored by financial economists is an attempt to microfound these complementarities and connections through more finance–based mechanisms and to combine the local with the market interactions.

When such interactions are viewed as occurring over time, nonlinear dynamics appear. Nonlinearities played an important role in modelling economic dynamics in the 1940s and 1950s until linear stochastic difference equations began to dominate the literature in the 1960s (see Scheinkman, 1990, for an insightful analysis). More recently, however, nonlinear difference and differential equations have taken the front stage again following the discovery of simple chaotic systems (e.g. the logistic map studied by May (1974) or the related tent map) and of non-market interaction models (e.g. random field models (e.g. Follmer, 1974; Glaeser and Scheinkman, 2000)). These approaches emphasise the sensitive dependence on initial conditions and on parameters as well as the emergence of macro phases. Complexity again ap-
pears, this time due to the interactions through time of the driving processes: one can easily describe how the system moves from today to tomorrow, but predicting what the system will look like in a few time steps is very hard even in the absence of any randomness. Nonlinear systems, possibly based on interactions,\footnote{In Bak et al. (1993) for instance, independent sectoral shocks create aggregate fluctuations that exhibit self-organised criticality, even if the number of sectors grows large (and one could therefore have guessed that the independent shocks wash out in aggregate).} therefore have the advantage of being able to capture sensitivity to initial conditions and amplification while remaining confined to a bounded region, an empirically desirable feat that linear systems cannot achieve.

4.4 Definition

From these observations that shocks can get around the normal homeostatic forces, the system runs the risk that the repercussions feed back into hugely destructive behaviour. In that light, one may define systemic risk\footnote{While the term “systemic fragility” goes back at least to Minsky (1977b) and the term “systemic vulnerability” to Cline (1982), it appears that the expression “systemic risk” in the academic literature was coined at the onset of the Latin American sovereign debt crisis of the early 1980s in Cline (1984) (see Ö zgöde (2011) for an account of this period), although the term has not been further defined or clarified in those works.} as (also see Hendricks (2009), IMF, BIS and FSB (2009) and Büsch (2011)):

**Definition 1 (Systemic Risk)** Systemic Risk is the risk of an event – labelled a systemic event – occurring in a given system \((\theta, M)\) that leads, at least temporarily, to an altered and damaged transitional “system” \((\theta, M')\) whose proper functioning is impeded. In the extreme, the structure of the system itself is damaged and the system no longer functioning.

Endogenous Systemic Risk refers to the risk of a systemic event where, in addition, the forces that drive the build-up of the systemic event or the forces that are responsible for the destructive transitions in the damaged system once the systemic event is realised, are positive feedback loops and/or cascades within the system that cannot be adequately kept in check.

We would like to make the following clarifying remarks related to this definition.
**Remark 1.** We follow common usage and define the risk to be *downside risk*, i.e. *damage* or even *breakdown* to and by the system.

**Remark 2.** A system that is damaged and no longer functioning properly is strictly speaking not really a system as defined earlier, hence the quotation marks around “system.” The damage can occur during the transition back to the original equilibrium only, or the damage can occur during the transition to a new “damaged” order as well as so long as the system remains in this dysfunctional state, or possibly forever if the transition leads to the death of the system.

**Remark 3.** Contrary to common usage of the term in the economic or financial literature, any system can suffer from systemic risk, not just crucial systems such as the banking system. We follow standard usage of the term “systemic risk” by restricting it to serious risks to systems that matter in terms of social welfare, that is systems that society depends on in a meaningful way. In that sense, the ill-functioning system has repercussions for the *economic environment*, often coined the “real economy,” in which it is embedded.

**Remark 4.** In that sense, the definition requires judgment in a case-by-case manner as to whether any given event is “destructive enough.” This it shares with many other definitions in economics where thresholds are necessarily blurred. What is the critical threshold that makes an instrument money, a bank undercapitalised, a portfolio under-diversified, a budget deficit unsustainable, a portfolio illiquid, an economy in recession, a cancellation rate on a public limit order book excessive, an income distribution unjust, an industry too concentrated, a risk systemic? Still, this does not render systemic risk unmeasurable, provided a reasonable threshold has been chosen. Such a threshold ought to be neither too low (crying wolf too often and losing the required commitment) nor too high (leading agents to disregard the possibility of the event), and the ex-ante probabilities of a serious systemic event as defined here ought to reflect this threshold.

**Remark 5.** Those definitions inherited from the engineering and the critical infrastructure protection literatures naturally emphasise the “to” rather than the “by,” whereas the social sciences emphasise the fact that systemic risks do not simply exist, but that they are made “by” humans and can be shaped “by” human choices in the present (for a short survey, see Cavelty (2011)).
We have deliberately chosen to focus on endogenous systemic risk, the risk that a given shock leads to destructive feedback loops (and to a loss of diversity with an endogenously coordinated vanishing risk appetite) and therefore to a large shock to the system as a whole in an endogenous fashion, as opposed to one extreme exogenous shock destroying part of the system directly, along the lines summarised for instance in Barro and Ursúa (2012). See Danielsson et al. (2012) for a more in-depth analysis of exogenous versus endogenous extreme events, and for a discussion of atemporal (where the feedback loops are realised atemporally in one go) and intertemporal (where the effects occur sequentially over time, possibly with delays) feedback loops.

Remark 6. A feedback loop and a cascade can be seen as isomorphic since a cascade is a Markov chain and a feedback loop can be represented as an infinite Markov chain with the same state variables repeatedly appearing along the chain.

Remark 7. Given the complexity involved in defining systemic events, perhaps the hope of measuring the systemic event probabilistically is too much to ask for, and Hansen (2012) argues that instead one may wish to pursue a research agenda attempting to quantify systemic uncertainty instead, referring to the distinction by Knight (1921) and others.

Remark 8. The definition above defines a systemic event as the event where the destructive forces are unleashed. When fully specified, the description of the systemic event includes the description of the build-up of imbalances prior to the unleashing of the crash, the “systemic build-up event” so to speak. For instance, the systemic event $E$ not only specifies the event whereby a loss to a small sector, say the subprime sector, snowballs into a full-blown banking crisis, but the states in $E$ also correspond to states that witnessed the

---

19This distinction has already been made by Pareto (1909) in his discussion of economic crises (in particular p. 531) where he writes that “Crises have two main types of causes, namely: (α) Any objective change in the conditions of production, if extensive enough, can give rise to a crisis. The food shortages of former times were related to this cause. (β) Subjective synchronism of economic movements turn movements which otherwise would have given rise to smaller displacements of the economic equilibrium into intense crises.” To what can be described as Pareto’s version of an endogenous crisis, characterised by sudden synchronised actions that no longer cancel each others out but all synchronise, we add the further reinforcing characteristic of positive feedback loops and the concomitant non-linearities to constitute an endogenous systemic crisis. This difference also forms the basis for the distinction between systemic risk and aggregate, or systematic, risk in Section 6.
regulation-induced latent build-up of off-balance sheet structures that caused extended bank leveraging with serious maturity mismatches and liquidity vulnerabilities. Build-up events are further analysed in the next section.

Remark 9. Along the path driven by positive feedback loops, the final outcome can for simplicity be categorised into two groups, depending on the degree of alteration of the system. First, the hysteretic case\(^{20}\) is one where the system is hurled down by vicious loops and ends up in a situation (a so-called attractor) where the system infrastructure is roughly the same as before the unfolding, albeit with a local system behaviour that is quite distinct from the one before the event due to the nonlinearities involved. The formal equations in the model representations also remain the same. As an example, capital (human, financial or real) may be destroyed, but production exhibits different returns to scale, interest rates are very low, the division of labour and the level of employment have been affected with follow-on repercussions throughout the economy, counterparty exposures have changed, the political power has gradually shifted, or beliefs now anchor the economy at a new equilibrium.\(^{21}\) Second, in the structural case, the feedback loops can lead to a system structure that itself has been profoundly altered. In practice, the system of equations modelling the economy in the post systemic event situation is a different one.\(^{22}\) The ecology of economic participants can change, banks for instance can disappear and be replaced by pure mar-

\(^{20}\)Systems that are dependent not only on their current environment or state but on the past environment are often said to exhibit \textit{hysteresis} or \textit{path-dependence}.

\(^{21}\)Sah (2007) provides the interesting example of corruption induced hysteresis. Depending on minor shocks to the initial situation whereby by chance more citizens get matched with more corrupt officials, the updating of beliefs as to the extent of corruption on behalf of both officials and citizens makes them more likely to adopt the corrupt/bribing actions. The more corruption/bribing occurs, the larger the incentives to extend and accept bribes, and the positive feedback loop dynamics quickly lead to an equilibrium where the majority of officials is corrupt and where citizens expect to be facing a corrupt official, thereby self-perpetuating the corrupt state of affairs. The structure of the economy has remained the same along this path, but the system functions very differently at the end.

\(^{22}\)For instance, when markets become too volatile in Danielsson and Zigrand (2008), the market mechanism breaks down because this extreme risk cannot be absorbed by the financial sector in aggregate due to binding risk-sensitive regulatory constraints of the Basel type. Of course, and as mentioned earlier when describing self-organisation, ideally one would wish all changes, including the structural changes, to be endogenous within the formal model. In the example just given, one would expect the regulators to amend the regulatory envelope and to extend permissions to violate Basel rules temporarily for instance.
ket solutions, money consists of currency only, laws and regulations change, OTC transactions are forbidden, industries are nationalised, capital controls are introduced, and so forth. In the extreme, the new structure no longer corresponds to a system in light of inconsistencies, say in case the rule of law breaks down or the rules are contradictory and do not allow a functioning system. While in the hysteretic case the economy could – perhaps slowly or upon a solid push – return to its initial state without major structural changes, it could not in the structural case.

4.5 Mechanisms Creating Systemic Risk in Finance

The destructiveness of the nonlinear dynamics may depend on a variety of factors, for instance the network structure of exposures and market connections, the size of balance sheets, leverage, the reliance on short term funding, the liquidity profile of assets and the capitalisation profile of financial institutions. Risk is nonlinear, it often builds up slowly and under the radar screen as balance sheets expand during periods of moderation with low perceived risk, only to be violently revealed during a systemic event. This view of crises has been stated eloquently by Pareto (1909) and later revisited by Crockett (2000). Pareto writes that “during an upswing everyone is content and no one talks about a crisis. That period, however, is surely preparing the way for the downswing which distresses everyone and which alone is given the name crisis. The upswing normally lasts longer than the downswing. Things go up little by little, they are hurled down in a single stroke.”

Once the systemic event has obtained and things are hurled down, the functioning of the system is impeded. It may also raise doubts among market participants, not only about the markets themselves but about the wider system envelope. On page 13 of Kambhu et al. (2007), Ervin writes about the 1998 Russian default and the LTCM event: “Investor losses were estimated to be on the order of $100 billion. Every working assumption about the Russian market came into doubt: the rules, the participants, the prices, the functioning of markets, even the legal system. This was surely a systemic crisis for Russia. Moreover, it threatened to become a systemic crisis for the international financial system when the market turmoil affected a particular hedge fund, LTCM, and the liquidity of core markets in the financial system.”

Some of the precise theoretical feedback loops that have been identified,
or at least conceived of, have been covered in the historical section 4.2, to which one can add the following early descriptions applicable to finance. Hawtrey (1919) developed his “inherent instability of credit” around the idea of balance sheet interactions whereby if I spend more, then somewhere someone has more income, which is then mechanically available for more spending, and indirectly prices of the good purchased rise in a price-credit spiral:

\[
\text{more spending} \quad \Rightarrow \quad \text{more income for others, prices up} \\
\uparrow \\
\begin{align*}
\{ & \text{capital gains to inventory holders} \\
& \text{creditworthiness improved} \Rightarrow \text{more credit} \} & \Leftarrow \text{existing inventories revalued up}
\end{align*}
\]

In Hawtrey’s view, the job of the CB is to prevent a credit bubble from getting started, and furthermore such bubbles tend to end at the hands of the CB as the lender of last resort. Minsky (1977a), in what has become known as the “Minsky Instability Hypothesis,” renders the feedback more violent by expanding revaluation not only to inventories but also to collateral and portfolios:

\[
\text{more capital credit/spending} \quad \Rightarrow \quad \text{more income, more income capitalised} \\
\uparrow \\
\begin{align*}
\{ & \text{capital gains to securities holders} \\
& \text{creditworthiness up} \Rightarrow \text{more credit} \} & \Leftarrow \text{portfolio & collateral values up}
\end{align*}
\]

Let us add that listing possible destabilising loops such as those above does not mean that they are necessarily likely to occur or actually operating. That question remains an empirical and a theoretical exercise. It remains true, however, that the logical possibilities and scenarios can be helpful in establishing a systemically safe system.

In a future Systemic Risk Centre Special Paper we shall present a wide variety of current practical mechanisms that have the potential of creating, in today’s financial system, the sort of systemic risk we defined in this paper, a partial glimpse of which can be found in Danielsson et al. (2012). Those mechanisms can be economic, regulatory, legal, technological, political, social or financial.
We shall also attempt to provide tools to limit their ex-ante build-up and their ferocity, or to cut through the loops once unleashed.

5 Systemic Risk and Social Welfare

5.1 Distortions and Externalities

Not all systemic events are made equal. The size of the inefficiencies attached to systemic events depend on the extent to which society depends on the system under consideration. Also, some events that would be bunched together under the “systemic event” heading may not be inefficient. For instance, Allen and Gale (1998) construct a theoretical example where bank runs (when liquidation costs are either infinite or zero) enable the production of a first-best allocation by allowing efficient risk-sharing between early and late consumers and by allowing the banks to hold efficient portfolios.

Our notion of systemic risk, however, nearly by definition, hard-wires a wedge between private and social costs because any one agent contributing to overall systemic risk does not typically fully bear all the costs imposed upon all other agents and therefore does not rationally internalise the consequences of his actions. And symmetrically, if mitigating actions were found, specific macroprudential policies would be called due to externalities, because each individual contributor to systemic risk free-rides on the willingness of others to pay for financial stability and to provide liquidity. It follows that the total self-declared willingness to pay for financial stability and liquidity is less than the social value of this financial stability. Systemic risk policies have a public good component.

Those actions that contribute to systemic risk can themselves in turn arise from distortions, such as informational frictions, market power, too-big-to-fail, abuse or any one of a wide variety of constraints put upon the agent, including poorly designed rules and regulations. What makes things worse is that some of these actions that contribute to systemic risk may have externalities of the “fallacy of composition” sort whereby each agent’s individual action is well meaning and is perceived to reduce systemic risk, but where in fact all such actions taken together increase systemic risk. This type of
externality\textsuperscript{23} is of special interest because the failure of individual effects to aggregate at the system level is precisely the reason why a systems analysis is needed in the first place.

Now what can be done about these social costs? In some cases, pinpointed actions of the authorities can enforce a first-best allocation. Examples include the case of clear-cut externalities such as the single polluter problem where a market for pollution property rights or a (personalised) tax on pollution can improve matters by forcing the polluter to internalise the costs by effectively buying pollution rights at the right price, or the bank-run scenarios in Allen and Gale (1998) even with costly liquidation where the central bank can intervene optimally.

In general, however, it would appear that on a systems level the mapping from the “original polluter” to “pollution” is rarely clear and instead is mediated through a web of endogenous and largely unobservable interactions.\textsuperscript{24} Furthermore, a financial system may simply not have an original exogenous distorting action that can be addressed directly. And even if one could pinpoint to the original sin, it is at this stage not clear yet which instruments are available legally or acceptable morally that would reduce the cost without creating unintended consequences whose costs may well outweigh the initial reduction in social cost. In the spirit of the theory of second best (given the obvious constraints on policy tools and the multiplicity of distortions), alleviating one obvious distortion may well lead to an outcome that is Pareto inferior. What needs to be done is that the problems are studied until they become much better understood and that sufficient data are gathered and statistical and economic impact analyses performed.

5.2 A Simple Black Box Framework to Illustrate Systemic Externalities

It may be useful in this section to illustrate the ideas developed so far within a simplistic reduced-form model. Consider an economy consisting of \( I \) financial institutions (FI). The strictly concave, increasing and differentiable objective function \( u^i \) over consumption \( x^i_t \) of institution \( i \) is separable in time

\textsuperscript{23}This fallacy is discussed in Samuelson (1947) and goes back to Mill (1850).

\textsuperscript{24}In contrast, standard models in public finance or in the economics of information assume that the structure of the economy is common knowledge.
and states of nature. Assets can be seen as financial or as real assets (in which case they are in positive net supply, like trees), and their exogenous and constant supply is $\delta$. Markets can be complete or incomplete. Assets pay real dividends at time $t+1$ of $\delta(t+1) = \delta(S_t, S_{t+1})$ units of consumption, where $S_{t+1}$ is an exogenous shock and $\sigma_t$ represents financial stability. Assets can be defined narrowly and made FI-dependent to capture OTC contracts and direct counterparty risks, as well as too-big-to-fail subsidies and other preferential treatments.

There is a production function $f$ for the driving state of systemic stability, $\sigma_t$. Much of the systemic drivers are summarised in the black box function $f$. We assume that the process $\sigma$ is observable to all, although interesting and realistic extensions would model $\sigma$ as a latent process subject to filtering. Individual FIs affect systemic stability, and therefore production $\delta$ next period, through their trading and portfolio choices $\phi^i$ as well as through the resources $z^i$ expended on financial stability. The term $z^i$ represents the total current cost of improving risk management systems, risk monitoring, education, more careful packaging and describing of structured products, cost of capital and liquidity buffers, extra incentives pay, etc., as well as the opportunity costs borne due to lowered risk. We assume that $f$ is concave in each variable separably, though not necessarily overall concave. To simplify matters to the extreme, assume that financial stability evolves according to the stochastic difference equation
\[ \sigma_{t+1} = \sigma_t + f\left(\{z^i_t\}_{i \in I}, \{\phi^i_t\}_{i \in I}; \sigma_t, q_t\right) + S_{t+1} \]
where $S$ is an exogenous iid shock process. This evolution rule is commonly known by all. Regulations on risk taking are captured through the additional constraint $z^i_t \geq z(\sigma_t)$. The mapping $z(\sigma_t)$ provides regulators with the freedom to choose pro- or counter-cyclical policies. One could further imagine extending the model by allowing $z$ to depend on FI-specific elements.

FIs affect systemic risk $\sigma_{t+1}$ via $f$ through their portfolio and risk choices. The initial portfolio distribution and cross-institution exposures map is given by $\{\phi^i_0\}_{i \in I}$. Together with the function $f$ they capture the initial as well as the evolving network structure of the FIs. For instance, larger positions of a given asset held by one given FI $i$ may increase global systemic risk depending on the initial distribution of risk. Similarly, larger investments

\[ \text{Some interesting non-linearities are ruled out in return for tractability.} \]
by one FI $i$ in risk reduction $z_i$ may reduce systemic risk. Furthermore, the
dependence of $f$ on $\sigma_t$ allows us to capture the fallacy of composition also
whereby the economy can find itself in a state $\sigma_t$ that leads to a *reduction*
in financial stability when FIs attempt to reduce risk individually, or when
*all* FIs attempt to reduce risk. Together, these actions feed into a new
endogenous distribution of future outputs $\delta$.\textsuperscript{26}

Financial stability is also affected by current asset prices $q_t$, not least due to
pecuniary externalities. Even if FIs would not normally put on a certain ac-
tion, in crises prices sometimes play a dual role, being affected by actions but
also forcing agents to take certain actions. Various value-at-risk constraints
for instance can have such an effect, forcing agents to sell more after a fall of
the price.

For instance, in the paper by Danielsson and Zigrand (2008), a free-riding
externality amongst financial institutions creates systemic risk by incentivis-
ing institutions not only to load up on more of the risky securities and skimp
on the liquid securities, but also by increasing the risk that the risky secur-
ities (which represent claims to productive processes) will not be able to
pay off because of a mis-match in the ownership of the productive securities
and a lack of liquid working capital. Stylistically, the externality amounts
to the choice of a low $z_i$ and/or a large holding of some risky assets $\phi_i$ and
consequently a low value of $f$. The regulator tries to counteract this exter-
nality by imposing a regulatory Value-at-Risk (VaR) constraint, represented
stylistically here by $z_i \geq z(\sigma_t)$, which has the effect of lowering the ex-ante
probability of this event. It is also shown in that paper that with additional

\textsuperscript{26}As an illustration for $I = 2$, one can set $f(z^1, z^2; \sigma) = (\alpha - z^2)(z^1)^2 + (\alpha - z^1)(z^2)^2 + \sigma(z^1 + z^2)^2$. For large enough $\sigma$, $f$ increases in $z^1$, in $z^2$ as well as jointly in $(z^1, z^2)$. In
other words, in an economy already relatively safe, any increase in expenditures on safety
increase safety further. For low enough $\sigma$, however, increased individual safety $z^i$ still
increases systemic safety, but if all FIs brusquely try to increase safety (say in a fire sale
when bank capital is already low), then systemic risk increases. If one furthermore pursued
the suggestion by Governor Kohn (p.16 in Kambhu et al. (2007)) that in a market-based
system, sound risk management by *all* (original emphasis) market participants is essential
to protect against the risk of a low-probability – or “tail” – event causing a financial crisis,
then this can for instance be achieved by further multiplying $f$ by an expression such as
$\min\{z^1, z^2\}$.

\textsuperscript{27}Since $\sigma$ enters into production and since the difference equation exhibits a unit root,
a change in $\sigma$ can affect the behaviour of the system in a non-negligible way over a very
long horizon, despite the fact that it does not affect the deeper structure of the system,
such as markets, motivations, power and the like.
noise trading, the imposition of the VaR gives rise to a fallacy of composition at the outset since the VaR constraint can lead to excessive and coordinated forced sales of assets in response to the noise trades. Once the economy is in a crisis (in their paper a low $\sigma$ represents an inadequate capitalisation level of banks), the individual safe action of selling risky securities creates less safety in aggregate as fire sales depreciate capital buffers and cause great uncertainty and risk of meltdown. Alternative justifications for the imposition of regulatory constraints on risk taking commonly given are moral hazard considerations or convex compensation arrangements that can lead agents to engage in overly risky trades. In order to mitigate this, individual traders within financial institutions are commonly subjected to capital and VaR restrictions, as in Adrian and Shin (2013).

The decentralised price-taking allocation problem is characterised as follows. Denote the market clearing (setting $\sum_i \phi^i_t = \bar{\phi}$) price vector by $q_t$ and the portfolio vector by $\phi^i_t$. Each period agent $i$ receives an endowment of $e^i_t$ units of the consumption good. The budget constraint yields a consumption of $x^i_{t+j} = \phi^i_{t+j-1}(q_{t+j} + \delta^i_{t+j}) + e^i_{t+j} - \phi^i_{t+j}q_{t+j} - z^i_{t+j}$ for given choices of $z^i_{t+j}$ and $\phi^i_{t+j}$. Each Fi $i$ solves

$$\max_{\{q_t, z_t\}} E_t \left[ \sum_{j=0}^{\infty} \beta^j u^i (\phi^i_{t+j-1}(q_{t+j} + \delta^i_{t+j}) + e^i_{t+j} - \phi^i_{t+j}q_{t+j} - z^i_{t+j}) \right.$$ 

$$+ \sum_{j=0}^{\infty} \lambda^i_{t+j}(z^i_{t+j} - z(\sigma^i_{t+j})) \right]$$

s.t. $\sigma^i_{t+1} = \sigma_t + f(\{z^i_t\}_i \in I), \{\phi^i_t\}_i \in I; \sigma_t, q_t) + S_{t+1}$

The first-order-conditions with respect to $\phi^i_t$ are

$$q_t = \beta E_t \left[ \frac{\partial u^i}{\partial x^i_{t+1}} (q_{t+1} + \delta_{t+1}) \right] + \beta \frac{\partial f}{\partial \phi^i_t} G^i_t \quad \text{where} \quad (2)$$

$$G^i_t := E_t \left[ \sum_{j=1}^{\infty} \left( \beta^j + \frac{\partial u^i}{\partial x^i_{t+j+1}} \phi^i_{t+j} \frac{\partial \delta^i_{t+j+1}}{\partial \sigma^i_{t+j}} - \frac{\partial u^i}{\partial \phi^i_{t+j}} \frac{\partial \lambda^i_{t+j}}{\partial \delta^i_{t+j+1}} - \frac{\partial u^i}{\partial \phi^i_{t+j}} \frac{\partial \lambda^i_{t+j}}{\partial \sigma^i_{t+j}} \right) \right] (3)$$

FI $i$ weighs the marginal rate of substitution between today’s consumption and one additional purchase of assets (the marginal cost $q_t/1$), on the left-hand side, with the discounted future net marginal benefits of that additional
unit purchased. The latter is composed of a direct portfolio benefit and of a systemic risk benefit, \( \beta \frac{\partial f}{\partial \phi} G^i_t \). The marginal portfolio benefit expressed in current numéraire is the value to FI \( i \) of the additional consumption at \( t + 1 \) since the portfolio pays a dividend and can then be sold. The marginal systemic risk benefits again can be split into two. The first term represents the marginal net benefit expressed in today’s numéraire (or cost if negative) of the present value of all future production increments made possible because of the effect of the current choice of portfolio holdings on systemic stability (including the future losses due to excessively risky portfolios). The second term is the present value of the gains arising from future reductions in the tightness of the regulatory constraint due to a more prudent (or less prudent if negative) choice of investments today.

The first-order-conditions with respect to \( z^i_t \) are

\[
\left(1 - \lambda^i_t \frac{\partial u^i}{\partial x^i_t} \right) \frac{1}{\frac{\partial f}{\partial z^i_t}} = G^i_t
\]

This amounts to setting the marginal rate of transformation between current consumption to FI \( i \) and financial stability, on the left-hand side, equal to the relevant sum across future periods of marginal rates of substitution between consumption to FI \( i \) today and financial stability, on the right hand side.

The FOCs can be combined to read

\[
q_t = \beta E_t \left[ \frac{\partial u^i}{\partial z^i_{t+1}} (q_{t+1} + \delta_{t+1}) \right] + \left(1 - \lambda^i_t \frac{\partial u^i}{\partial x^i_t} \right) \frac{\partial f}{\partial \phi} \frac{\partial f}{\partial z^i_t}
\]

Given the number of externalities present in this problem, not least through its network effects and its public good nature, the private benevolent contributions to the provision of financial stability are too low and socially suboptimal because FI \( i \) in its private cost-benefit analysis, when deciding upon how much to invest in \( z^i_t \) and in \( \phi^i_t \), ignores the spillover benefits or costs of stability arising from his own actions for all other FIs. Or in other words, when engaging in systemically unsafe behaviour, the FI ignores the welfare losses imposed on society in general. To see this concretely, we can derive the Bowen-Lindhal-Samuelson conditions from the social problem (with welfare weights proportional to the inverse of marginal utilities of consumption):
\[
q_t = \beta E_t \left[ \frac{\partial u^i}{\partial x^i_t} \right] (q_{t+1} + \delta_{t+1}) + \frac{\partial f}{\partial \phi^i_t} \sum_{h \in I} G^h_t
\]

(6)

\[
\left( 1 - \frac{\lambda^i_t}{\partial u^i / \partial x^i_t} \right) \frac{1}{\partial f / \partial z^i_t} = \sum_{h \in I} G^h_t
\]

(7)

The FOC can again be combined into (5).

In contrast to the decentralised outcome (2, 4), the social problem (while still assuming decentralised market clearing and price-taking behaviour) balances the MRT (and this term is independent of \(i\)) with the sum of all MRS of all FIs. The planner compares the welfare-weighted marginal cost to FI \(i\) of investing one more unit into stability with the marginal benefit accruing to all FIs, not just to the FI \(i\) that does the investing. Alternatively the planner balances the marginal benefit to FI \(i\) from engaging in a slightly more risky endeavour with the total societal marginal cost, including the MRT and all the deleterious effects on other agents’ utilities. For instance, assume that \(\lambda^i_t = 0\). Since total societal gains \(\sum G^h_t\) are larger than the private gains \(G^i_t\), the right-hand side in (7) is smaller in the case of free-riding than in the case where FIs are compelled to internalise the effects on systemic risk, and hence \(\partial f / \partial z^i_t\) is larger. In light of the assumed concavity of \(f\), and assuming the normal situation where \(f\) is (locally) increasing in \(z^i_t\), investments \(z^i_t\) in financial stability on behalf of FI \(i\) are lower than socially optimal. Systemic risk is suboptimally large.

One recovers the well-known frictionless Euler equations by one of three means. The first instance occurs if there is no external effect of individual portfolio choices on systemic stability, \(\partial f / \partial \phi^i_t = 0\). The second case obtains if the systemic risk monitoring technology is infinitely effective, \(\partial f / \partial z^i_t \to \infty\). The final case calls for a (possibly personalised and contingent) regulatory policy of \(\zeta^i_t\) chosen so as to align incentives through nudging \(\lambda^i_t = \partial u^i_t / \partial x^i_t\). Indeed, if regulators had full information on FIs and the means of micro-regulating them finely, and if FIs accepted, without gaming, the policy \(\zeta^i_t(\sigma_t; \bar{s}^i_t)\) as a given function of \(\sigma_t\) only with \(\bar{s}^i_t = (\phi^i_t, z^i_t, w^i_t)\) considered exogenous, then the regulatory policy that implements a frictionless constrained optimum (equalising the payoff-relevant intertemporal marginal rates of substitution.
across FIs) is given by a policy \( \{ z^i_t \} \) that satisfies at any time and in any state:

\[
\frac{\partial z^i_{t+j}}{\partial \sigma_{t+j}} = \beta^{j+1} E_{t+j} \left[ \frac{\partial u^i}{\partial x_{t+j}^i} \frac{\partial \delta_{t+j+1}}{\partial \sigma_{t+j}} \right] \phi^i_{t+j}
\]

This policy sets all \( \{ G^i_t \} \) to zero and confirms \( \lambda^i_t = \frac{\partial u^i}{\partial x^i_t} \) for all times and states. We do not believe that large FIs would consider such a regulatory policy to be exogenous, but rather that self-selection constraints would need to be imposed.

This leads on to the next question, namely the optimal regulatory rules given the informational restrictions under which regulators operate where decisions need to be based on a few observable statistics \( \beta \) in lieu of the full observation \( \pi \), including situations where this informational discrepancy is strategic due to private information issues such as moral hazard and adverse selection? How can the mappings \((f, \delta)\) serve to define what a systemically important financial institution is, both at the margin and overall? This assessment for FI \( i \) depends on the interactions with all other FIs through \((f, \delta)\). In real life, information extracted from \((f, \delta)\) can act as a variable entering into macro-prudential rules such as countercyclical capital buffers, leverage constraints or direct bank levies, all subsumed under \( z \) in this black-box representation. The regulator also needs to keep in mind that depending on \( \sigma \), the decentralised provision of financial stability can either be under– or overprovided for. To what extent do the empirical systemic risk measures currently proposed or in use, as summarised for instance by Bisias et al. (2012), capture these systemic externalities as laid out in the previous equations? These questions indicate the direction into which future research needs to move to. Making the endogenous risk mechanisms that are hidden in the black box above visible is one of the top priorities in academic research on systemic risk.

5.3 Should all Systemic Events be Prevented?

If one could, should crises always be prevented? In this regard it is difficult to improve upon the passage by Pareto (1897, p.297) that might have inspired Schumpeter’s own concept of creative destruction (Schumpeter, 1943):

If it was possible to completely avoid all crises, would it be useful
to do so? One may be tempted to respond in the affirmative, but a slightly deeper analysis of this question would lead to new doubts creeping in. It is not at all certain that the rhythmical movement is not one of the conditions of economic progress. (...). Of course, one needs to prevent making crises sharper still through imprudent measures taken, and one needs to attempt to reduce its ill-effects. (...). Completely suppressing a certain movement, or attempting to reduce its violence in exceptional cases, are two essentially different things.

Similarly, Friedman (1959) writes

After all, uncertainty and instability are unavoidable concomitants of progress and change. They are one face of a coin of which the other is freedom.

Other than for the fact that one could not prevent all systemic events in finance, short of shutting the entire financial system down, a cost-benefit analysis needs to be undertaken both in terms of ex-ante measures aimed at reducing build-ups of systemic risk and in terms of ex-post measures aimed at ironing out the painful consequences of a systemic event, or at least aimed at not making things even worse (as some pro-cyclical policies do).

Within the black-box framework presented in Section 5.2, stricter rules aimed at eliminating systemic events can be seen to have the following effects. First, there is a marginal cost today of imposing stricter rules in terms of marginal utility of foregone current consumption. Second, the level of \( \bar{z} \) needs to recognise the efficiency of the production function of systemic stability \( f \). Not only can the production function have low productivity, but short of raising \( \bar{z} \) to exorbitant levels one runs the risk that stricter regulations may run into a negative marginal productivity region, as happens during an episode of the fallacy of composition. Second, the effect of stricter rules on the public good \( \sigma \) further leads to a change in the investment opportunity set through \( \delta(\sigma_t, S_{t+1}) \). This can capture for instance the effect that a more regulated environment can stifle innovation and reduce the outliers in the future, perhaps reducing some very bad ones but also preventing some very favourable ones. In a dynamic economy with production, innovation and human capital, such a straight jacket on risky but innovative investments today can potentially lead to a reduction in longer-term growth rates, as well
as to a genuinely changed system. This is the essence of the ideas exposited above that link interrelationships to nonlinearities which lead to progress and cycles, including the crises and recessions embedded in cycles.

In a nutshell, a balance needs to be found between those interventions that can at low cost reduce externalities that would have otherwise led to easily avoidable losses (for example the provision of liquidity in a liquidity panic, or the nudging to efficiency in a prisoner’s dilemma situation) and those interventions that are excessively costly in terms of direct costs or indirect costs (such as stifled innovation or poor incentives).

### 5.4 Policy Interventions

It is our hope that by studying systemic risk further, some clarity can be gained that would allow policy makers to find the right policies that point in a social-welfare-improving direction, at least when viewed through the lens $\beta$ rather than $\pi$. This direction need not point to the immediate intended outcome. In other words, since policy makers are unlikely ever to be able to know and keep track of all the characteristics and all the actions of all financial players, policy makers’ objective functions are in practice directly over some macro characteristics of $\beta$. For instance, central bankers may follow a Taylor rule, fiscal policy makers look at metrics of income distribution and fiscal deficits, and financial policy makers may look at overall market quality metrics such as overall effective spreads or market resilience, metrics capturing the extent to which gains from trade can be reaped in financial markets, or in fact metrics capturing the build-up of certain fragilities that can lead to a systemic event.

In addition to these more ex-ante policies trying to control the distortions that can create systemic risk, a difficult undertaking given the lack of information and data on individual actors, we believe that following Pareto’s quote there is a lot of value in mitigating the negative ex-post effects of an event once the seed has been sown. This is because contrary to the huge variety of initial situations and shocks that can create the first losses, the accelerating mechanisms that aggravate the initial shocks and that are responsible for much of the social welfare losses are often the same. It is now

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28 One can interpret the Taylor Rule (Taylor, 1993) as counteracting the Wicksellian cumulative process loop by raising real rates when inflation picks up.
well understood that some rules and regulations can in such ex-post circumstances become aggravators in lieu of dampers. For instance, in Danielsson and Zigrand (2008) a free-riding externality amongst financial institutions creates systemic risk, and a regulatory VaR constraint has the effect of lowering the ex-ante probability of this event. However, once the systemic event has obtained anyway, the crisis is much deepened by the pro-cyclical regulatory VaR constraints.

One avenue suggested by Morris and Shin (2008) and by Keating and Marshall (2010) is to view the $f$ mapping as representing liquidity and to move from a risk-based regulation more towards a liquidity-based macroprudential regulation. As with stability, liquidity has a public good aspect and is therefore inadequately provided for in a laissez-faire setting. The central bank at the core of the monetary system plays the role of liquidity provider through its repo and haircut policies. Together with the regulator (say, through leverage constraints and the like that act as a tax), the central bank can therefore nudge the size and the composition of the balance sheets of financial institutions towards an internalisation of these externalities.

In that sense, what is needed are two sorts of measures, measures that prevent build-up of systemic fragility (including the feedback loops involved in the build-up), and measures that cut through the propagating feedback loops once the systemic event has realised. Amongst the former one can count leverage constraints, the removal of various rules that encourage pro-cyclical behaviour, or the various sources of moral hazard emphasised by Dow (2000). Well-known examples of the latter are lender of last resort and market-maker of last resort interventions, living wills, rules facilitating the quick unwinding of positions, smart circuit breakers on markets limit-order books, the temporary suspension of rules that would otherwise reinforce downturns, and so on.

This leads to the currently open issue of not only the composition but also the number of policy tools. As pointed out by Tinbergen (1952), in order to achieve optimality the number of instruments in general needs to match the number of objectives. In cybernetic systems more generally, the Law of Requisite Variety says that the variety in the outcomes, if minimal, can be decreased further only by a corresponding increase in that of the actions available. Only variety in actions can force down the variety in outcomes: “variety can destroy variety” (Ashby, 1956, p.207). Macroprudential authorities have currently at their disposal a very limited number of rather blunt
tools. More tools will need to be devised in order to achieve a preferrable outcome.

6 On the relationship between Systemic Risk and related concepts

Given the proposed definitions of systemic risk and complexity (namely, that a system is complex if its emergent properties are novel and sufficiently strong and that systemic risk is the risk of unbalanced and powerfully destructive positive feedback loops generated within the system that alter and damage, at least temporarily, the structure of the system), systemic risk can only occur in complex systems.

Like systematic risk, systemic risk as defined herein cannot be diversified either. However, the nonlinearities involved in the feedback loops suggest clearly that systemic risk is separate from what is usually called systematic risk, or non-diversifiable risk, in the finance literature, that represents the risk of an aggregate event (not purely a redistributive event). Typical examples of systematic risk would be aggregate technology shocks, aggregate endowment, output, preference or monetary shocks etc. These systematic events may correspond to very large shocks, such as for instance the power-law shocks in Gabaix (2009) and Barro and Ursúa (2012), but the system is expected to continue functioning normally and properly. This distinction has also been emphasised by Hansen (2012).

Formally, a simple multi-factor affine dynamic general equilibrium model that is hit by an extreme shock drawn from a fat-tailed distribution exhibits large losses and a jump in the state variables to new levels, but the equations that govern the evolutions are unaffected by the shock. The economy locally works as before, only from lower levels of wealth, etc. This we call an aggregate shock. A systemic shock implies (i) either that the structure of the economy changes (say, with some of the institutions vanishing) and therefore that the system of equations that governs the evolution itself has radically changed, or (ii) that the institutions and the equations governing their behaviour are still present, but that these equations are sufficiently non-linear or reflect sufficient hysteresis and path-dependencies that the local characteristics of the economy exhibit quite distinct properties. In both cases, the equations
reflect an ex-post more dysfunctional system.
A Appendix: System

This appendix has been inspired by Mandy (1988) and borrows freely from many authors and philosophers.

A.1 Everyday Use of the Term “System”

We said that the casual speaker would perhaps mention three characteristics of what constitutes a system, as opposed to being merely a collection (for a collection of quotes and illustrations, http://www.muellerscience.com/spezialitaeten/System/System_Definitionen.htm and http://en.wikiquote.org/wiki/System):

1. these expressions make reference to something;
2. their elements imply relationships;
3. and many systems change and evolve over time but keep their identity.

A.1.1 Reference

Even in cases where the reference is not explicitly spelled out, it would be hard to deny its existence. Each one of the expressions refers to

- a centre that is easily identifiable (sun, central nervous system (e.g. brain), federal state, central bank, etc.); or

- a fundamental idea or value, irrespective of the discipline studying it (e.g. “liberty” in the liberal system analysed by different disciplines); or

- alternatively to a unique “base” or “standard” of measure (e.g. the metric system, the standard of value of a price system); or

- a law or fundamental rule the elements of the system depend on (e.g. family links in a kinship system, resolution of equations in an equation system, data analysis in an information system).
Each system thereby implies reference to a principle recognised as the primary or original cause or fundamental rule or method all the elements in the system depend on, or as Kant said in 1786: “ein nach Prinzipien geordnetes Ganzes der Erkenntnis.” This commonality serves as the centre and the common reference, and creates the unity that sets a system apart from an aggregate. The concepts of “system” and “reference” are tightly linked, and some thinkers choose the concept of a “reference system” as the fundamental concept of their theoretical approach to systems (e.g. Duquesne de la Vinelle, 1969).

It is evident that such a reference is totally absent when talking of a bunch of stars, a pile of sand, lines of code (as opposed to an operating system) or miscellaneous facts; the casual speaker would not qualify them as “system.”

In the human and social sciences, this unity of a man-made system creates an aim or common interest for the system, “By Systems I understand any numbers of men joined in one Interest, or one Businesse” (Hobbes, 1651). In that sense social systems have been argued to be of a higher order of complexity than physical or biological systems. From a physics point of view, a plane advances from origin to destination due to the propulsive forces generated by the engines and the plane body and wings – a mechanical explanation. From an economics point of view, the plane flies to destination due to the mutually beneficial terms found by the airline as provider of the service and its customers as to the desirability and profitability of servicing this route – a finalist explanation.

While smaller social subsystems are created expressly with an aim in mind, the overall market system can have an aim also even though no one agent (and no social planner) has created it that way. As argued by Hayek (1989), the reason that the market system acquired the aim of implementing economicity

29 The Swiss mathematician J.H. Lambert wrote (Lambert, 1761): “We have only to attend to the solar system, and we shall perceive the utility of a central body on which the whole depends. In virtue of this body, it rarely happens that the planets and comets disturb each other, and these extraordinary instances form but trifling exceptions. But were we to retrench the central body, the general law would be destroyed, and the exceptions alone would remain.”

30 To quote Halliday (2005): “A physical system is just that: a physical system. What is systematized is matter itself, and the processes in which the system is realized are also material. But a biological system is more complex: it is both biological and physical – it is matter with the added component of life; and a social system is more complex still: it is physical, and biological, with the added component of social order, or value.
lies in the fact that the market system has itself emerged as a result of evolutionary forces operating via natural selection not only on individuals’ behaviour but also on rules and social structure, the \( \theta \) in our definition of a system \( S := (\theta, M) \).

Let us consider some examples from finance and economics.

**Banking System.** A set of deposit-taking banks becomes a banking system in the modern era only if the banks have a fundamental reference to a central bank. For one, banks rely on the central bank for liquidity. Since banks are required to convert deposits and credits into central bank money at the simple request of depositors or creditors, there is a permanent reference to central bank money and therefore the the central bank (CB). Banks also rely on a central payment and clearing system.

**Price System.** That the expression *price system* is commonly used in everyday speech stems from the fact that the exchange value of *all* commodities is expressed with reference to a basis or common denominator that then becomes the yardstick of value, the numéraire. It is this reference to a *unit* – often central bank currency – that allows all prices to enter into a system, also called the “numeration system.” It is thus possible to express the value of any commodity by one number, as opposed to specifying the full set of combinations of the quantities of the other commodities that the given commodity can be exchanged for, which is Ricardo’s definition of value (Ricardo, 1817).

A.1.2 Interrelationships

All due respect, you got no f*****g idea what it’s like to be Number One. Every decision you make affects every other f*****g thing. It’s too much to deal with almost.

Source: *Tony Soprano*

“The coelacanth? The prehistoric fish? But how could one possibly affect the other?”
“Ah. Now there you’re asking. The complexities of cause and effect defy analysis. Not only is the continuum like a human body, it is also very like a piece of badly put up wallpaper. Push down a bubble somewhere, another one pops up somewhere else.”

Source: Professor Urban “Reg” Chronotis in Douglas Adam’s
Dirk Gently’s Holistic Detective Agency

A second characteristic of a system are the relations that link the elements of a system to each other, forming a network of interrelations.

In a pile or cluster of unstructured individuals, elements are only juxtaposed, without any links between them. This is not the case in a system, where independence and dependence are replaced by mutual interdependence. It is this interdependence that can give rise to complex outcomes (see e.g. May, 1973).

Let us consider some examples from finance and economics.

Banking System. Banks in a deposit taking-banking system exhibit a highly dense network of links. A web of mutual exposures and credits is created across banks. The daily settlement process across banks settles these obligations each day by transferring corresponding settlement assets. Systemically important clearing and settlement systems have the CB as a settlement agent for systemic stability reasons (as the government supports the CB), so the settlement asset is CB money, i.e. the balances held in accounts at the CB. In the UK, this role is played by the Bank of England’s Real-Time Gross Settlement infrastructure.31

Price System. A price system relates the value of each commodity to a common yardstick, a common unit of account, and relative values can then be deduced between any two commodities with a non-zero price. Relative prices express interrelations between any set of commodities, allowing a direct comparison.

As an example of an economy that does not form a system, assume two separate islands with no links of any sort. There will be two price vectors, a

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31 For details, refer to Dent and Dison (2012), and for a description of the systemic importance of intraday liquidity provision (intraday needs arising due to the timing mismatches between incoming and outgoing payments) and the central bank provision of intraday liquidity, see Ball et al. (2011).
dichotomy. While each island has a price system, the overall pricing mechanism is not *a* price system.

As we shall see below, once the economy is formed of an array of interrelationships, its aggregate outcome may become hard to predict (so-called *irreducibility*), as Tony Soprano, the head of a New Jersey mobster family in the HBO TV series “The Sopranos”, has realised early on.

### A.1.3 Continuity of identity

A system contains within itself the seeds of some more complete evolution, but it does not admit of arbitrary alterations.

Elias Magnus Fries (1825)

Not all systems are dynamic. But most real life systems evolve and are adaptable to – as well as shape – their environments. In such a case, the system must not be an ephemeral accident at some time period.

Or in the words of Klir (1991), to qualify for the name “system,” *continuity of identity* needs to be present. This is true whether the system is atemporal and fixed or dynamic and changing. Something that is not able to preserve its structure amid change –if there is change– is never recognised as a system. As argued for instance by Allais (1967), Williamson\(^32\) and Hayek (1989), dynamic, adaptive and evolutionary forces over time can select \(\theta\) in a way to ensure the system tends to continue to function properly. This tendency to try to improve the usefulness of the economic system is exactly the definition of a self-organising system given by Ashby (1947). Modelling this phenomenon is difficult, so much so that some economists (e.g. Lucas, 1988a) go so far as to state that what matters to economists is the steady state of this adaptive process: “technically, I think of economics as studying decision rules that are steady states of some adaptive process, decision rules that are found to work over a range of situations and hence are no longer revised

\(^{32}\)E.g. refer to Williamson (1977): “A strategy which I find attractive is to adopt the viewpoint that economic systems knowingly or demonstrably sacrifice efficiency only with reluctance, whence a showing that a rule is to reduce efficiency will predictably give rise to adaptive [original emphasis] efforts to mitigate these losses.”
appreciably as more experience accumulates.” Similarly, it appears that the recent literature on complexity theory and on complex evolving systems has perhaps excessively abstracted away from goal directedness. In order to reintroduce it, we believe that the disciplines of law and political science would be brought fruitfully into economic and financial research precisely so as to model the dynamic evolution of the financial system more accurately and more purposefully.

Among financial examples we can pick:

**Banking System.** The banking system has evolved greatly over the last 100 years, to and from universal banking, oscillating back and forth between investment banks and financial holding companies. The system adapts to the realities of the world, in particular the regulatory cycles.

**Price System.** The price system evolves both in terms of absolute prices, i.e. inflation, and in terms of relative prices, say the relative prices of energy, fossil fuels or food stuffs. The location dimension has also changed dramatically, many world markets have become more integrated, while others (e.g. financial exchanges and other trading venues) have become more fragmented as monopolies have been broken. Still, the forces that underpin them remain the same because the functions remain the same. For instance, high frequency algorithms now arbitrage across trading venues, pocketing pennies and weaving liquidity pools together.

In other words, this property of a system is simply the fact that as a system evolves and changes, it remains recognisable as such: “A system is recognized as such by remaining recognizable as ‘itself’ in spite of changes in its detailed appearance” (Rapoport, 1986).

### A.2 Scientific use of the term “system”

La science est un système de relations.

Henri Poincaré (1927)

We now try to describe the major elements of what constitutes a system in scientific terms. The main ideas had started to crystallise in Kant (1781), and it is worth quoting him more fully:
By the term architectonic I mean the art of constructing a system. *Without systematic unity, our knowledge cannot become science; it will be an aggregate, and not a system.* Thus architectonic is the doctrine of the scientific in cognition, and therefore necessarily forms part of our methodology. Reason cannot permit our knowledge to remain in an unconnected and rhapsodistic state, but requires that the sum of our cognitions should constitute a system. It is thus alone that they can advance the ends of reason. By a system I mean the unity of various cognitions under one idea. (...) This idea is the conception—given by reason—of the form of a whole, in so far as the conception determines a-priori not only the limits of its content, but the place which each of its parts is to occupy. The scientific idea contains, therefore, the end and the form of the whole which is in accordance with that end. The unity of the end, to which all the parts of the system relate, and through which all have a relation to each other, communicates unity to the whole system, so that the absence of any part can be immediately detected from our knowledge of the rest; and it determines a-priori the limits of the system, thus excluding all contingent or arbitrary additions. *The whole is thus an organism (articulatio), and not an aggregate (coacervatio); it may grow from within (per intussusceptionem), but it cannot increase by external additions (per appositionem).* (...) *We require, for the execution of the idea of a system, a schema, that is, a content and an arrangement of parts determined a-priori by the principle which the aim of the system prescribes.*

### A.2.1 Deductibility

A system is a multitude of phenomena which, being related to one another as cause and effects, all spring from a first law.

Abbé Etienne Bonnot de Condillac

A scientific endeavour strives for logical consistency, trying to prevent contradictions and dichotomies. This is the aim of an axiomatic formalisation. The consistency and validity achieved through deductibility from first principles can be viewed as the scientific expression of the reference attribute of
A scientific theory in economics is grounded on the axiomatic principle of economics, let us call this the *axiom of economicity* or the *economising principle*. For Menger (1871), *economising* is the principle, the “aim” in Kant’s words, at the core of the theory of value because economising allows the satisfaction of many needs by scarce resources. Since the epistemological work of Robbins (1932), it has become generally accepted that the axiom of economicity is nothing but the economising and non-wastage of rare resources with multiple ends. This sets economics apart from other disciplines studying the same topic but from a different disciplinary angle due to the different starting principle that reflects the approach taken by various disciplines. The endogenous elements of a system follow from the logical organisation deduced from this principle of economising.

The exact form of the expressions of the axiom of economicity can be manifold, depending on the question at hand, be they the optimal allocation of scarce resources, or social welfare, or the full employment of factors of production, social choice or optimal taxation, etc.

Since formal elements (such as marginal utility and marginal productivity functions, demand and supply functions, production functions, monetary functions etc.) are deducted on the basis of a common desire to economise, these elements are coherent and serve a purpose, they become functions in an application of the calculus of economics. This coherence is reflected in a logical structure common to many economics problems, as pointed out by Koopmans (1957), due to the idea of maximisation subject to various sets of constraints.

This functional role of economic variables reveals their typically economic role without which they could not fulfil their *economic function* within the economic system, which in turn would mean that the system itself could not function efficiently, could not have hope that its aim is being fulfilled. This goal directness in economic and social systems may not be present in physical systems, though some have argued it is present in biological systems in light of the finality and functionality of certain organic characteristics through the

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33 We follow general usage (and Kant) and require definitively a “system” to be coherent, though some authors use the term slightly more generally, e.g. Stebbing (1930, p.198): “A system is said to be coherent if every fact in the system is related to every other fact in the system by relations that are not merely conjunctive. A deductive system affords a good example of a coherent system.”
action of natural selection. Goal directedness (Allais, 1981; Hayek, 1989; Klir, 1991) is simply the existence of an aim and a function. In turn, goal directedness itself is a reflection of the desire to organise in an efficient way to abide by the axiom of economicity, to economise. This desire may in turn be the result of a process of selection.34

**Price System.** Money fulfils a variety of monetary functions, including the *numeration function*, acting as the reference term of a price function. This function of the monetary price system follows from the necessity to compare the values of multiple commodities, requiring a common denominator, a common reference base for the economic actors. Its role is truly to allow the real world economy to function in a way that an economy based on pure barter could not. This function allows a set of values to become a system of prices.

At the same time, prices are determined by a pricing function reflecting the marginal valuations of the many actors in the economy. At equilibrium, subject to the various frictions of the economy, resources are allocated according to a common driver that attempts to equalise the MRS and MRT across the system, the invisible hand of the central auctioneer guided by competitive economising forces.

**Banking System.** Underlying the banking sector is the function of monetary liquidity. This function must be assumed in a modern financial system deriving from the *Banking Principle* by the existence of a lender of last resort, a function fulfilled by the central banks.

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34The coherence and well functioning of social systems allows them be analysed further in terms of the internal order they possess. *Order* is defined (Stebbing, 1930) as a “state of affairs in which a multiplicity of elements of various kinds are so related to each other that we may learn from our acquaintance with some spatial or temporal part of the whole to form correct expectations concerning the rest, or at least expectations which have a good chance of proving correct.” Hayek (1989) distinguishes between two sources of order, grown, or spontaneous, order on one hand and made, or deliberate, *order* on the other hand. These two orders distinguish themselves in terms of the distinct complexities they may achieve. Hayek argues that while both serve functions, made systems have more of a deliberate purpose compared to grown systems that are better called purposive.
A.2.2 Irreducibility

As mentioned before, a system is characterised by the interrelationships between the various components. The main idea is that a system possesses a degree of complexity that is larger than the one of its components, or in other words, that the system possesses properties that cannot be reduced to those of its components, the emergence basis. This *irreducibility* must be attributed to the presence of relations that unite the components.

A property possessed by the system but not by its constituent elements, and which cannot obviously be deduced from the properties of its constituent elements, is often called an *emergent* property. Emergence has the dictum that the whole is more than the sum of its parts. Classically this concept is attributed to the British thinkers on emergence, the so-called British Emergentists, John Stuart Mill\(^\text{35}\) and George Henry Lewes, although certain sources trace its origins back to the Greek antique medical doctor Galien as well as to Aristotle\(^\text{36}\) (Sartenaer (2013) provides a useful historical and philosophical background). Another term for “irreducibility” has been coined by Fuller (1963): “Synergy is the only word in our language that means behavior of whole systems unpredicted by the separately observed behaviors of any of the system’s separate parts or any subassembly of the system’s parts. There is nothing in the chemistry of a toenail that predicts the existence of a human being.” In a literal sense, the injection of medicines is an instance of emergence in the sense of the total being more than the sums since oftentimes the injection of a double dose has more than double the effect of two isolated doses. Or to give an economics example, the merger of two companies may lead, for any given level of inputs, to an output larger than the sum of the elements.

\(^{35}\)Mill (John Stuart Mill outlined his version of emergentism in System of Logic (1843)) called *heteropathic* and Lewes (1875) called *emergent* the case where the effect of the complex cause is not equivalent to the mathematical sum of the effects provoked by the isolated partial causes. Broad (1925) moves away from the narrow summation straightjacket and proposes a third interpretation of the classical maxim of emergence around the concept of *non-deductibility*, meaning the properties of the whole cannot be *deduced solely* by the properties of the elements. The contemporaneous version thereof is *irreducibility*. This is an active research topic in modern philosophy and researchers are proposing a variety of extensions to irreducibility that shall not concern us here. Please see Sartenaer (2010) for a useful background discussion and further details.

\(^{36}\)Aristotle writes in book 8 on Metaphysics, Section 1045a, that “In all things which have a plurality of parts, and which are not a total aggregate but a whole of some sort distinct from the parts, there is some cause of unity […].”
two isolated outputs through the operation of economies of scale and scope. This illustrates one of the relationships of emergence to nonlinearities. Systems that possess strong or novel emergent properties are often called complex systems. Not every system with emergent properties is necessarily complex. As Simon (1996) says on p. 184,

...in such systems [complex systems] the whole is more than the sum of the parts in the weak but important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole.

As examples, a living cell is more than the sum of inert molecules that it is composed of since it can reproduce by mitosis while the inert molecules it is composed of cannot. Human conscience is more than a sum of neurons. In the main text, we have seen the example of a price system in greater detail, but in a nutshell, the equilibrium pricing function that sets the excess demand vectors to zero possesses properties that cannot be found in the behaviour of prices if one considers the excess demand schedules for each commodity or for each agent separately, as in partial equilibrium analysis. For instance, aggregate rationality can be seen as an emergent property in household choice. Becker (1962) has shown that even irrational and impulsive agents tend to have demand functions that are downward-sloping due to the power of the budget constraint. Even though there will be many exceptions on individual demands, “the market demand curve in markets with many irrational households would, however, be negatively inclined, and the market’s revealed preferences system could be said to be rational (consistent and transitive) ... Hence the market would act as if “it” were rational not only when households were rational, but also when they were inert, impulsive, or otherwise irrational.”

A different approach to the emergence of aggregate rationality exploits natural selection. As we are reminded by Kirman (2006), Johnson (1968) argued that the system can have properties that are not dependent on the specific behaviour of individuals, and in particular that the aggregate structure may...
result from natural selection and therefore exhibits as a whole a certain rationality even if the individuals that make up the system do not. Under some circumstances, rationality emerges by natural selection. As a result the system as a whole behaves like a large optimising individual.\textsuperscript{38}

\textsuperscript{38}To quote Johnson, “it has been shown...that whether firms consciously seek to maximize profits and minimize costs or not, competition will eliminate the inefficient firms; and that whether consumer behaviour is rational or purely random, the demand curves for a product will tend to slope downwards as in the Marshallian analysis. In consequence, it is possible for economists to treat the economy as an interdependent system responding to change according to certain general principles of a rational kind, with considerably more confidence than appeared justifiable thirty years ago.”
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