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Asset Pricing with Index Investing^{*}

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Abstract

We provide a novel theoretical analysis of how index investing affects capital market equilibrium. We consider a dynamic exchange economy with heterogeneous investors and two Lucas trees and find that indexing can either increase or decrease the correlation between stock returns and in general increases (decreases) volatilities and betas of stocks with larger (smaller) market capitalizations. Indexing also decreases market volatility and interest rates, although those effects are weak. The impact of index investing is particularly strong when stocks have heterogeneous fundamentals. Our results highlight that indexing changes not only how investors can trade but also their incentives to trade.

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I. Introduction

Starting from the 1970's, passive index investing has been gaining popularity among institutional and individual investors. According to the 2013 Investment Company Fact Book (<http://www.icifactbook.org>), 33 percent of households that invested in mutual funds in 2012 owned at least one index mutual fund. The proportion of index funds in all equity mutual fund assets increased from 8.7 percent in 1998 to 17.4 percent in 2012. Moreover, the funds benchmarked to the S&P 500 index managed 33 percent of all assets invested in index mutual funds. Index investing was initially promoted by proponents of the efficient market hypothesis (e.g., Malkiel, 1973; Samuelson, 1974) and has an increasing number of supporters due to inability of money management industry as a whole to outperform the market (e.g., Malkiel, 1995; Fama and French, 2010; Lewellen, 2011) and high costs of active investment for society (e.g., French, 2008). It is blessed even by successful investors like Warren Buffett, who in his 2013 letter to Berkshire Hathaway shareholders argues that “the goal of the non-professional should not be to pick winners – neither he nor his ‘helpers’ can do that – but should rather be to own a cross-section of businesses that in aggregate are bound to do well. A low-cost S&P 500 index fund will achieve this goal.”

Despite the growing popularity of index investing, its impact on properties of capital market equilibrium is not well understood. The objective of our study is to fill this gap. We build a dynamic general equilibrium model of an exchange economy with two Lucas trees and two groups of investors dubbed type P investors (professional investors) and type I investors (index investors). We interpret the type P investors as professional market participants such as hedge funds, actively managed mutual funds, proprietary traders, etc., who can implement complex trading strategies that involve individual assets. The type I investors are unsophisticated market participants, like individuals who manage their savings and retirement accounts, and can trade only the market portfolio of Lucas trees (index). In practice, indexing can result from inability of ordinary investors to model stock returns, to keep track of a large number of open trading positions, to minimize transaction costs while trading individual stocks, etc. To maintain the generality of our analysis, we do not specify the reason why the type I investors are restricted to trade the index.

Consistent with our interpretation of the investors, we also assume that the type I investors are more risk averse than the type P investors, so even without indexing the investors in our model would trade stocks to share risk. Indexing changes the set of trading strategies that the type I investors can implement compared to an unconstrained economy (an economy in which fundamentals are the same but all investors can trade individual stocks) and, therefore,

affects the equilibrium variables. To identify the effect of indexing, we find the equilibria in the constrained and unconstrained economies and compare their characteristics including the risk-free rate, the volatilities and betas of stocks, and the correlation between stock returns.

Our analysis delivers several results. First, we find that in general indexing increases (decreases) volatilities and betas of stocks with relatively large (small) market capitalizations. Second, indexing can either increase or decrease the correlation between stock returns, and this conclusion challenges a wide-spread belief that indexing always increases the correlation. Third, indexing decreases market volatility and the risk-free rate, although those effects are relatively weak. Fourth, the effect of index investing is much stronger when stocks have heterogeneous fundamentals such as the expected growth rate and volatility of dividends.

To see economic intuition behind these effects, consider first an unconstrained economy in which all investors can trade all assets. When investors have heterogeneous risk preferences, they dynamically share risk and this affects statistical properties of stock returns. Assume, for example, that a positive cash flow shock hits one of the stocks and increases its price. Because less risk-averse investors in equilibrium hold more stocks than those who are more risk averse, this shock disproportionately increases their wealth. To maintain their optimal portfolio weights, less risk-averse investors buy more shares of the affected stock from those who are more risk averse and drive its price up even further. Thus, dynamic risk sharing tends to increase the volatility of returns. Moreover, in response to a wealth shock less risk-averse investors buy shares of all stocks and, as a result, the returns on the stocks become correlated even if their fundamentals evolve independently.¹

When some investors follow an indexing strategy, they hold an equal number of shares of each stock (the total number of shares of each stock in the model is normalized to one). Therefore, in response to cash flow shocks the investors can trade only the market portfolio. As a result, risk sharing is less effective than in the unconstrained economy and its impact on the equilibrium is subdued. In particular, indexing decreases market volatility inflated by risk sharing. Also, the risk-free rate decreases because more stocks are held by more risk-averse investors and less risk-averse investors borrow less from them. These effects are stronger when each tree produces a nontrivial part of the total dividend but the trees differ in size and portfolio distortions caused by the inability of agents to trade individual stocks are particularly pronounced.

The effect of indexing on individual stock returns depends on the relative size of the stock. On the one hand, due to indexing investors rebalance their portfolios less actively in response

¹Xiong (2001), Kyle and Xiong (2001), Cochrane, Longstaff, and Santa-Clara (2008), Bhamra and Uppal (2009), Ehling and Heyerdahl-Larsen (2012), and Longstaff and Wang (2012) discuss how risk sharing among investors affects the dynamics of stock returns.

to changes in dividends on a smaller tree, which are less aligned with returns on the market portfolio and cannot be hedged well when only index is tradable. As a result, the volatility of returns on a smaller tree and its beta are smaller than in the unconstrained economy. On the other hand, the price of a larger stock becomes more sensitive to changes in its dividend because investors respond to all shocks by trading only the index and effectively trade the larger stock more than in the unconstrained economy. Therefore, the volatility and beta of this stock tend to be higher than in the economy without indexing.

Indexing also changes the correlation of stock returns and can either increase or decrease it. When stocks have similar fundamentals, the market portfolio is almost optimal for the investors and they actively trade it to share risk. Buying and selling the portfolio as a whole, the investors effectively trade all stocks simultaneously, so the correlation between returns can be higher than in the unconstrained economy.² However, the market portfolio can substantially deviate from the unconstrained optimal portfolio when the stocks have different sizes. In this case, the investors trade stocks less aggressively, risk sharing is inhibited, and the correlation between stock returns produced by risk sharing is lower than in the unconstrained economy. This finding highlights that indexing changes not only how investors can trade but also their incentives to trade and challenges the perception of indexing as an unambiguous source of positive correlation between stock returns, which is shared by practitioners (e.g., Sullivan and Xiong, 2012) and appeared in popular press.³ Also note that there is no contradiction between the decrease in the correlation of stock returns produced by indexing in our model and numerous studies that document an increase in the correlation between a stock and an index when the stock is added to the index (e.g., Vijh, 1994; Barberis, Shleifer, and Wurgler, 2005; Greenwood and Sosner, 2007; Boyer, 2011). Indeed, our model describes the implications of passive indexing as a broad phenomenon that can inhibit risk sharing, whereas a migration of a single stock in or out of an index has a minuscule effect on the ability of investors to share risk.

The described effects of indexing exist even when dividends of all stocks have the same expected growth rate and the same volatility, so the heterogeneity in stocks is solely due to different realizations of their dividends. The difference in the dividend processes makes the impact of index investing much stronger. This result is explained by much larger portfolio distortions brought about by the inability of investors to trade individual stocks when stock dividends have different dynamics. For example, consider a case in which one stock has a low expected dividend growth rate and dividend volatility, whereas for the other stock both of these characteristics are relatively high. The risk-averse investors would hold relatively more shares of

²A similar effect arises in Barberis and Shleifer (2003), Basak and Pavlova (2013, 2014), and Grégoire (2014).

³“Simple Index Funds May Be Complicating the Markets”, *The Wall Street Journal*, February 18, 2012.

the first stock in the unconstrained economy but indexing forces them to hold the same number of the shares of each stock and, hence, makes the portfolio highly suboptimal. To ensure that the new portfolio satisfies the equilibrium conditions, the expected stock returns and return volatilities substantially deviate from their values in the unconstrained economy.

The analysis of dynamic economies with multiple trees, heterogeneous investors, and market frictions is a challenging task and our paper also makes a methodological contribution to the literature by demonstrating how to find an equilibrium in an economy with indexing. The idea of our approach is to characterize the equilibrium in terms of quasilinear differential equations for the price-dividend ratio of the index and the wealth-consumption ratio of the index investors, which can be solved by a fast and general numerical procedure. The approach works for arbitrary coefficients of risk aversion and allows us to take into account the effect of hedging demand by index investors on the equilibrium properties. This is particularly important in our setting because index investors are identified with individual investors, who tend to be more risk averse than unconstrained professional market participants.

Our paper belongs to the growing literature that uses a dynamic exchange economy framework with heterogeneous investors to study equilibrium effects of various economic frictions that make financial markets incomplete.⁴ Such frictions include restricted stock market participation (e.g., Basak and Cuoco, 1998), short-sale and borrowing constraints (e.g., Detemple and Murthy, 1997; Basak and Croitoru, 2000; Kogan, Makarov, and Uppal, 2007; Gallmeyer and Hollifield, 2008; Chabakauri, 2014), portfolio concentration constraints (e.g., Pavlova and Rigobon, 2008), margin constraints (e.g., Gromb and Vayanos, 2002; Gârleanu and Pedersen, 2011; Brumm, Grill, Kubler, and Schmedders, 2013; Chabakauri, 2013; Rytchikov, 2014), and transaction costs (e.g., Buss and Dumas, 2013; Buss, Uppal, and Vilkov, 2013). Gromb and Vayanos (2010) survey the literature on the frictions that are sources of limits to arbitrage. Dumas and Lyasoff (2012) develop a general approach for solving incomplete-market models with one Lucas tree.

The closest to our analysis is the paper by Shapiro (2002), who considers a general equilibrium model in which a fraction of investors can implement only particular trading strategies that are consistent with the investor recognition hypothesis (IRH), and the indexing strategy is one of them. In contrast to our paper, which explicitly characterizes the equilibrium and examines the volatility of returns and their correlation, Shapiro (2002) does not solve the model for the equilibrium characteristics and largely focuses on qualitative implications of portfolio

⁴Dynamic exchange economies with one Lucas tree, heterogeneous investors, and complete markets are studied by Wang (1996), Chan and Kogan (2002), Weinbaum (2009), Xiouros and Zapatero (2010), Longstaff and Wang (2012), Cvitanić, Jouini, Malamud, and Napp (2012), Bhamra and Uppal (2014), among others.

constraints for interest rates and risk premia. Moreover, Shapiro (2002) assumes that the constrained investors have logarithmic preferences, which makes his analysis more tractable but less realistic.

A dynamic model with logarithmic investors and indexing is also considered by Grégoire (2014), who uses perturbation analysis to approximate the solution to the model and demonstrates that indexing increases comovement of stock returns. In contrast to Grégoire (2014), the investors in our model have heterogeneous preferences with arbitrary coefficients of risk aversion and, therefore, trade to share risk. We show that indexing can hamper risk sharing and decrease the correlation of stock returns. The model in Grégoire (2014) cannot produce this effect because of the assumed homogeneity of investors' preferences.

Our paper is also related to the research on equilibrium effects of institutional investors whose compensation is benchmarked to a particular index and who can trade multiple risky assets (e.g., Cuoco and Kaniel, 2011; Basak and Pavlova, 2013; Basak and Pavlova, 2014; Buffa, Vayanos, and Woolley, 2014). One of the insights of this research is that in the presence of index-related incentives fund managers tilt their portfolios towards the index, so indexing arises endogenously and can be partially responsible for the identified effects of institutional investors on the equilibrium. In contrast to these papers, which study the implications of active money management by institutional investors on asset prices, we investigate the impact of pure passive indexing on the capital market equilibrium.

Finally, our paper builds upon the literature on dynamic equilibria in exchange economies with multiple Lucas trees and homogeneous investors (e.g., Menzly, Santos, and Veronesi, 2004; Cochrane, Longstaff, and Santa-Clara, 2008; Martin, 2013). As in those papers, the time variation in the dividend shares of individual trees in our model spills over into equilibrium characteristics. However, when investors are identical they hold the market portfolio and indexing is irrelevant. This does not happen in our model because we combine the multiple tree framework, which is necessary for studying the effects of indexing, with the heterogeneity in investors' preferences.

The rest of the paper is organized as follows. Section II presents our model and describes its equilibrium. Section III contains numerical analysis of the model and reports our main findings. Section IV summarizes the results of the paper and proposes directions for future research. Appendix contains all proofs.

II. Model

A. Assets

There are three assets in the economy: a risk-free short-term bond in zero net supply and two risky stocks. The supply of each stock is normalized to one share, which is a claim to a stream of dividends produced by a Lucas tree. The dividends D_{1t} and D_{2t} follow geometric Brownian motions

$$\frac{dD_{it}}{D_{it}} = \mu_{Di}dt + \Sigma_{Di}dB_t, \quad i = 1, 2, \quad (1)$$

where μ_{Di} are constant expected dividend growth rates, Σ_{Di} are constant 1×2 matrices of diffusions, and B_t is a 2×1 vector of independent Brownian motions. The rate of return on the bond r_t as well as the stock prices S_{1t} and S_{2t} are determined in the equilibrium. The excess return on each stock i is defined as

$$dQ_{it} = \frac{dS_{it} + D_{it}dt}{S_{it}} - r_tdt$$

and the vector $Q_t = [Q_{1t} \ Q_{2t}]'$ follows a diffusion process

$$dQ_t = \mu_{Qt}dt + \Sigma_{Qt}dB_t, \quad (2)$$

where the matrix of the risk premia $\mu_{Qt} = [\mu_{Q1t} \ \mu_{Q2t}]'$ and the matrix of the diffusions $\Sigma_{Qt} = [\Sigma'_{Q1t} \ \Sigma'_{Q2t}]'$ are also determined in the equilibrium.

Taken together, the stocks form a market portfolio (index), which pays the aggregate dividend $D_t = D_{1t} + D_{2t}$ and has the price $S_t = S_{1t} + S_{2t}$. Using Itô's lemma and equation (1), the dynamics of the dividend D_t can be written as

$$\frac{dD_t}{D_t} = \mu_{Dt}dt + \Sigma_{Dt}dB_t, \quad (3)$$

where $\mu_{Dt} = u_t\mu_{D1} + (1 - u_t)\mu_{D2}$, $\Sigma_{Dt} = u_t\Sigma_{D1} + (1 - u_t)\Sigma_{D2}$, and $u_t = D_{1t}/D_t$. The excess return on the index is defined as

$$dQ_{It} = \frac{dS_t + D_tdt}{S_t} - r_tdt$$

and using equation (2) its dynamics can be described as

$$dQ_{It} = \mu_{It}dt + \Sigma_{It}dB_t, \quad (4)$$

where $\mu_{It} = (\mu_{Q1t}S_{1t} + \mu_{Q2t}S_{2t})/S_t$ and $\Sigma_{It} = (\Sigma_{Q1t}S_{1t} + \Sigma_{Q2t}S_{2t})/S_t$. By construction, the index is value-weighted and its expected returns and diffusions are value-weighted averages of expected returns and diffusions of the individual stocks.

B. Agents

The economy is populated by two groups of competitive agents dubbed type P investors (professional investors) and type I investors (index investors). Each group consists of a unit mass of identical investors who have the standard constant relative risk aversion (CRRA) preferences. The investors differ across the groups in two respects. First, they have different coefficients of risk aversion, which are γ_P and γ_I for the type P and type I investors, respectively. Second, the trading strategies that the investors can implement depend on their type: the type P investors can trade all assets individually, whereas the type I investors are constrained and can trade only the risk-free bond and the index of the stocks. More specifically, the type P investors form an arbitrary portfolio of the stocks $\omega_{Pt} = [\omega_{P1t} \ \omega_{P2t}]'$, where ω_{P1t} and ω_{P2t} are the fractions of their wealth W_{Pt} allocated to stocks 1 and 2, respectively, and invest the rest of their wealth $\alpha_{Pt} = 1 - \omega_{P1t} - \omega_{P2t}$ in the bond. In contrast, the type I investors allocate their wealth W_{It} between the index and the bond, which receive the weights ω_{It} and $\alpha_{It} = 1 - \omega_{It}$, respectively.

The two types of investors admit a natural interpretation. The type P investors can be thought of as professional traders such as hedge funds, actively managed mutual funds, proprietary traders, etc., who are relatively risk tolerant and can implement sophisticated trading strategies that involve individual assets. The type I investors are unsophisticated market participants like individual investors who manage their savings and retirement accounts. They are more risk averse than professional investors and trade only the index, not individual stocks. In practice, indexing can be an optimal response of investors to various factors like information processing costs, organizational and management costs, transaction costs, etc. For example, investors with limited attention may allocate their learning capacity to market factors rather than firm-specific information (e.g., Peng and Xiong, 2006) and invest in a market as a whole. Investors may prefer to categorize assets in particular classes and invest in indexes because this simplifies the asset choice (e.g., Barberis and Shleifer, 2003). Even mutual fund and pension fund managers whose compensation is related to the index performance directly or indirectly through the response of the fund flows to the fund performance may find it optimal to partially allocate assets under management to index portfolios (e.g., Basak and Pavlova, 2013). We do not specify the reason why the type I investors can trade only the index because this preserves the generality of our analysis and allows us to study the implications of pure passive indexing

that is not contaminated by other economic frictions.

The optimization problem of the investors has the standard form: each investor $j = P, I$ chooses a consumption stream C_{jt} and portfolio weights ω_{jt} that maximize the expected CRRA utility

$$U_t = \mathbb{E}_t \left[\int_t^\infty e^{-\beta s} \frac{C_{js}^{1-\gamma_j}}{1-\gamma_j} ds \right] \quad (5)$$

subject to a budget constraint, which is

$$dW_{Pt} = (r_t W_{Pt} - C_{Pt})dt + W_{Pt} \omega'_{Pt} (\mu_{Qt} dt + \Sigma_{Qt} dB_t) \quad (6)$$

for the type P investors and

$$dW_{It} = (r_t W_{It} - C_{It})dt + W_{It} \omega'_{It} (\mu_{It} dt + \Sigma_{It} dB_t) \quad (7)$$

for the type I investors.

C. State variables

The model has two Lucas trees and two types of investors. Therefore, it is natural to assume that the state of the economy is described by two variables. The first one measures relative importance of each investor type and we choose it to be the consumption share of the type I investors $s_t = C_{It}/D_t$. In general, s_t follows a diffusion process

$$ds_t = \mu_{st} dt + \Sigma_{st} dB_t, \quad (8)$$

where the scalar μ_{st} and the 1×2 matrix Σ_{st} are determined by equilibrium conditions.⁵

The other state variable measures relative size of each tree and is chosen to be the share of the dividend on the first stock in the aggregate dividend: $u_t = D_{1t}/D_t$.⁶ The stochastic equation for u_t follows from applying Itô's lemma to the definition of u_t and using equations (1) and (3):

$$du_t = \mu_{ut} dt + \Sigma_{ut} dB_t, \quad (9)$$

where the drift μ_{ut} and the diffusion Σ_{ut} are determined by exogenous model parameters and

⁵The consumption share of one of the agents is often used as a state variable in economies with heterogeneous agents (e.g., Bhamra and Uppal, 2009, 2014; Longstaff and Wang, 2012; Chabakauri, 2013; Rytchikov, 2014).

⁶This state variable is standard in the models with multiple Lucas trees (e.g., Menzly, Santos, and Veronesi, 2004; Cochrane, Longstaff, and Santa-Clara, 2008; Martin, 2013).

given by

$$\mu_{ut} = u_t(1 - u_t)(\mu_{D1} - \mu_{D2} - (\Sigma_{D1} - \Sigma_{D2})(u_t\Sigma_{D1} + (1 - u_t)\Sigma_{D2}')), \quad (10)$$

$$\Sigma_{ut} = u_t(1 - u_t)(\Sigma_{D1} - \Sigma_{D2}). \quad (11)$$

As the state variable s_t , the variable u_t takes values in the range from 0 to 1. When $0.5 < u_t < 1$, the first tree contributes to the aggregate dividend more than the second tree, so we refer to the former as a larger tree and to the latter as a smaller tree. The terminology is opposite when $0 < u_t < 0.5$.

D. Equilibrium

We define an equilibrium in the model as a set of processes for the risk-free rate r_t , expected excess returns μ_{Qt} , diffusions of returns Σ_{Qt} , consumption streams C_{jt} , $j = P, I$, and portfolio strategies ω_{jt} , $j = P, I$, such that

1. C_{jt} and ω_{jt} solve the utility optimization problem of investor j ;
2. the aggregate consumption is equal to the aggregate dividend: $C_{It} + C_{Pt} = D_t$;
3. the markets for the stocks and bond clear:

$$\omega_{Pit}W_{Pt} + \omega_{Iit}W_{It} = S_{it}, \quad i = 1, 2, \quad (12)$$

$$\alpha_{Pt}W_{Pt} + \alpha_{It}W_{It} = 0, \quad (13)$$

where $\omega_{Iit} \equiv \omega_{It}S_{it}/(S_{1t} + S_{2t})$ is the fraction of the type I investors' wealth allocated to stock i through investing in the index.

Assuming that the state of the economy is fully described by the two variables s_t and u_t , we look for the equilibrium processes r_t , μ_{Qt} , Σ_{Qt} , μ_{It} , and Σ_{It} as functions of the state variables: $r_t = r(s_t, u_t)$, $\mu_{Qt} = \mu_Q(s_t, u_t)$, $\Sigma_{Qt} = \Sigma_Q(s_t, u_t)$, $\mu_{It} = \mu_I(s_t, u_t)$, and $\Sigma_{It} = \Sigma_I(s_t, u_t)$. The same representation should exist for the drift and diffusion of s_t : $\mu_{st} = \mu_s(s_t, u_t)$, $\Sigma_{st} = \Sigma_s(s_t, u_t)$. For the characterization of the equilibrium, it is convenient to introduce i) the price-dividend ratios of the index and individual stocks as functions of the state variables: $S_t/D_t = f(s_t, u_t)$, $S_{it}/D_{it} = f_i(s_t, u_t)$, $i = 1, 2$ and ii) the wealth-consumption ratios of the type I and type P investors as functions of the state variables: $W_{It}/C_{It} = h(s_t, u_t)$ and

$W_{Pt}/C_{Pt} = h_P(s_t, u_t)$. Finally, we introduce the risk aversion of a representative investor as

$$\Gamma_t = \left(\frac{s_t}{\gamma_I} + \frac{1-s_t}{\gamma_P} \right)^{-1}. \quad (14)$$

The following proposition characterizes the equilibrium in the model and indicates how to compute various equilibrium characteristics. To simplify notation, in the rest of the paper we omit the subscript t for all variables as well as the arguments s and u of all functions.

PROPOSITION 1 *The equilibrium in the model is characterized by the functions r , μ_s , Σ_s , Σ_I , f , and h that solve a system of algebraic and differential equations (A1) – (A6). The market price of risk η and the expected excess returns on the index μ_I are given by equation (A7). The price-dividend ratio f_i of stock $i = 1, 2$ solves equation (A8). The expected excess returns on individual stocks μ_{Qi} , $i = 1, 2$, and return diffusions Σ_{Qi} , $i = 1, 2$, are given by equation (A9). The optimal portfolio weights ω_I and ω_P and the numbers of the shares held by the type I and type P investors N_{Ii} and N_{Pi} , $i = 1, 2$, are given by equations (A10), (A11), and (A13).*

Proof. See Appendix.

To identify the effect of index investing, it is insightful to compare the equilibrium from Proposition 1 with the equilibrium in an identical unconstrained economy, that is, an economy in which the fundamentals are the same but all investors can trade all individual assets. The equilibrium in the unconstrained economy with heterogeneous agents and two Lucas trees is described by Proposition 2 in Chabakauri (2013), which shows that the differential equations for the price-dividend and wealth-consumption ratios as well as the expressions for the risk-free rate r , expected excess stock returns μ_{Qi} , diffusions Σ_{Qi} , and market prices of risk η appear to be identical in the economies with and without indexing. The difference between the economies comes from the diffusion of the consumption share s , which in the unconstrained economy has a closed form representation

$$\Sigma_s^{unc} = \frac{\gamma_P - \gamma_I}{\gamma_P \gamma_I} s(1-s) \Gamma \Sigma_D, \quad (15)$$

where Γ is defined in equation (14). In contrast, in the economy with indexing it is given by equation (A3), which can be written as

$$\Sigma_s = \Sigma_s^{unc} \Pi_I - \frac{s}{h + s h_s} (h \Sigma_D + h_u \Sigma_u) (I_2 - \Pi_I), \quad (16)$$

where $\Pi_I = (\Sigma_I' \Sigma_I) / (\Sigma_I \Sigma_I')$ is the projection operator on the space of index returns (on the vector of diffusions Σ_I) and I_2 is a 2×2 identity matrix.

Equation (16) deserves several comments. First, it highlights the role of market incompleteness for the type I investors, who face two-dimensional uncertainty associated with the shocks dB_1 and dB_2 but can trade only one risky asset. Equation (15) implies that without indexing the variation in the consumption share s is driven by the total dividend D (associated with the total risk in the economy) and this is the result of risk sharing between investors with different risk preferences. However, the composition of D in terms of the individual dividends D_1 and D_2 does not matter because all investors trade all risky assets and thereby perfectly hedge shocks to the relative dividend shares.⁷ In contrast, equation (16) shows that in the presence of indexing the diffusion Σ_s contains two terms. The first is the projection of Σ_s^{unc} on the index returns; it represents the variation in the total dividend that can be shared by investors using index as the only tradable asset. The second term in equation (16) contains the projector $I_2 - \Pi_I$, so it is orthogonal to the space of index returns and captures the variation in s produced by the variation of the total dividend that cannot be shared by investors. Effectively, the type I investors face additional exposure to the unhedgeable part of the fundamental shocks and absorb it by changing their consumption.⁸

Second, equation (16) shows how the magnitude of the variation in the state variable s is affected by indexing. On the one hand, because some changes in the total dividend are unspanned by index returns, indexing hampers risk sharing between investors and the volatility of s produced by it decreases (the projection of Σ_s^{unc} on the index in the first term of equation (16) is smaller than Σ_s^{unc}). On the other hand, the state variable is affected by the unspanned part of the fundamental shocks (as indicated by the second term in equation (16)) and the volatility of s increases. Which effect dominates depends on various factors including the fundamentals of the assets, the magnitude of portfolio distortions brought by indexing, etc. In Section III we consider the situations of both types.

Third, equations (15) and (16) show why it is more difficult to find the equilibrium in the economy with indexing than in the unconstrained economy. Because Σ_s^{unc} does not depend on the price-dividend and wealth-consumption ratios, the differential equations for these ratios in the unconstrained economy are linear, decoupled, and easy to solve. In contrast, equation (A4) implies that the projection operator Π_I in equation (16) is determined by the values of f and h , so in the presence of indexing the dynamics of the state variable s are entangled with the dynamics of the price-dividend and wealth-consumption ratios. As a result, the differential equations (A5) and (A6) are quasilinear, not linear, and do not have a closed-form solution.

⁷Note that the dividend share u is still a state variable because it affects the expected growth rate and volatility of the total dividend.

⁸Loosely speaking, the projection operator on the space of tradable assets Π_I becomes the identity operator in the unconstrained economy and equation (16) reduces to equation (15).

The formulas from Proposition 1 also reveal several technical tricks that help us simplify the description of the equilibrium. In general, the computation of an equilibrium in an economy with two trees and two types of investors involves the solution of three differential equations: two of them are for the price-dividend ratios of the stocks and the third is for the wealth-consumption ratio of one of the investors. In the unconstrained economy, those equations can be solved independently from each other but this is not the case in an economy with restrictions on portfolio weights in which the equations for the ratios typically become entangled and should be solved simultaneously (e.g., Chabakauri, 2013). Proposition 1 implies that in the presence of the indexing constraint the computation of the equilibrium can be simplified by sequentially solving two sets of differential equations: the first is a pair of quasilinear equations for the price-dividend ratio of the index and the wealth-consumption ratio of the type I investors; the second is a pair of linear equations for the price-dividend ratios of the individual stocks. This simplification occurs because the projection operator Π_I , which modifies the dynamics of the state variable s , depends only on the price-dividend ratio of the index, not individual stocks. The latter immediately follows from the definition of Π_I and equation (A4).

Indexing changes the equilibrium because it distorts portfolios of the type I investors. Therefore, the directions and magnitudes of the effects of indexing on the equilibrium variables can be interpreted by comparing the numbers of the shares of each stock held by each type of the investors in the benchmark economy and the economy with indexing. Equation (A13) shows that those numbers can be inferred from the investors' portfolio weights and wealth-consumption ratios. We use them in the next section to quantify portfolio distortions produced by indexing.

III. Numerical results

A. Model parameters

In our numerical analysis, we consider two specifications for the dynamics of Lucas trees. In the first one, the dividend growth rates and volatilities of the trees are identical and set as $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \ 0]$, and $\Sigma_{D2} = [0 \ 0.045]$. We refer to this specification as a model with homogeneous trees and use it to identify the effects of indexing that exist only due to the difference in the relative size of the stocks. In the second specification, the growth rates and volatilities of the trees are different: $\mu_{D1} = 0.01$, $\mu_{D2} = 0.03$, $\Sigma_{D1} = [0.01 \ 0]$, and $\Sigma_{D2} = [0 \ 0.08]$. This is a model with heterogeneous trees and it allows us to explore the consequences of constructing an index from stocks with different fundamentals. We follow previous studies (e.g., Basak and Cuoco, 1998; Dumas and Lyasoff, 2012; Chabakauri, 2013) and

identify the aggregate dividend with the aggregate consumption and the chosen values of the parameters are in the ballpark of the estimated mean and volatility of the consumption growth rate in the United States. In both specifications the dividends of the trees are uncorrelated.

Because we interpret the type P investors as financial professionals and the type I investors as individual investors, we set $\gamma_P = 1$ and $\gamma_I = 5$, and this choice reflects that individual investors are more risk averse than professionals. In contrast to the vast majority of the papers that study equilibria in incomplete markets, we do not assume that constrained investors have logarithmic preferences. On the one hand, this complicates the analysis because hedging demand of such investors affects the properties of the equilibrium and should be taken into account. On the other hand, the choice of $\gamma_I > 1$ makes the analysis more realistic. The time preference parameter β is 0.03 for all investors.

B. Numerical technique

As follows from Proposition 1, all equilibrium processes in our model can be expressed in terms of the price-dividend ratio f and the wealth-consumption ratio h , which satisfy the system of quasilinear differential equations (A5) and (A6). To solve these equations, we use the standard finite-difference approach, which prescribes to approximate our infinite-horizon economy by an economy with a large finite horizon T , discretize the time interval $[0, T]$ and domains of state variables, and solve the discretized equations backward as a sequence of systems of linear algebraic equations (e.g., Lapidus and Pinder, 1999).

More specifically, we introduce a vector of functions $F = [f \ h]'$, denote the first and second partial derivatives of F with respect to the state variables s and u as F_s , F_u , F_{ss} , F_{uu} , and F_{us} , and write the system of equations (A5) and (A6) adjusted for a finite horizon economy as

$$A_{ss}(F, F_s, F_u, s, u)F_{ss} + A_{uu}(F, F_s, F_u, s, u)F_{uu} + A_{us}(F, F_s, F_u, s, u)F_{us} + A_s(F, F_s, F_u, s, u)F_s + A_u(F, F_s, F_u, s, u)F_u + A(F, F_s, F_u, s, u)F + 1 + \frac{\partial F}{\partial t} = 0, \quad (17)$$

where A_{ss} , A_{uu} , A_{us} , A_s , A_u , and A are diagonal matrices with elements that correspond to the coefficients of differential equations (A5) and (A6). Note that equation (17) includes the time derivative $\partial F / \partial t$, which appears as an additional term in Itô's lemma applied to the time-dependent price-dividend ratio and indirect utility function in the derivation of equations (A5) and (A6) presented in Appendix.

Next, we set $T = 500$ and using a backward recursion solve equation (17) at discrete moments $t = T, T - \Delta t, \dots, \Delta t, 0$ and in discrete states $s = 0, \Delta s, 2\Delta s, \dots, 1$, and

$u = 0, \Delta u, 2\Delta u, \dots, 1$, where $\Delta t = 0.1$, $\Delta s = 0.01$, and $\Delta u = 0.01$. In particular, the time t solution $F_{(t)}$ is found by solving discretized equation (17) in which all derivatives of $F_{(t)}$ are replaced with their finite-difference approximations and the equation coefficients are computed using the solution $F_{(t+\Delta t)}$ at time $t + \Delta t$ obtained in the previous step. Thus, the coefficients of the discretized equation do not depend on the time t solution and $F_{(t)}$ solves a system of linear algebraic equations. Because the time horizon T is large, the sequence $F_{(t)}$, $t = T, T - \Delta t, \dots, \Delta t, 0$, converges to a time-independent solution F , which describes an equilibrium in the infinite-horizon economy. We verify the convergence by observing that the discrete approximation of the derivative $\partial F / \partial t$ has the order of magnitude 10^{-7} at $t = 0$.

The iteration procedure starts from the terminal solution $F_{(T)} = [\Delta t \quad \Delta t]'$, which follows from the index price and the type I investors' wealth at the terminal date being equal to $S_T = D_T \Delta t$ and $W_{IT} = C_{IT} \Delta t$, respectively, so the price-dividend and wealth-consumption ratios at time T are $f_{(T)} = \Delta t$ and $h_{(T)} = \Delta t$. The spacial boundary conditions for the discretized version of equation (17) are obtained by taking the limits $s \rightarrow 0$, $u \rightarrow 0$, $s \rightarrow 1$, and $u \rightarrow 1$ in equation (17). The computation of the boundary conditions is incorporated directly into the numerical algorithm. Appendix B in Chabakauri (2013) provides further details.

Having solved equation (17) and obtained f and h , we find r , μ_s , Σ_s , and Σ_I as functions of the state variables using equations (A1) – (A4). Also, we compute η and μ_I from equation (A7). To find the price-dividend ratios f_i , we solve differential equations (A8). Note that those equations are linear because their coefficients are known functions of the state variables, so they are solved using the finite-difference approximation that no longer requires a backward recursion. The remaining equilibrium variables are obtained from equations (A9) – (A13).

To find the equilibrium in the benchmark economy without indexing, we also use the finite-difference approximation. However, in this case the differential equations for the price-dividend ratios and wealth-consumption ratios are linear and decoupled, so each of them is solved individually without a backward recursion. These computations closely follow Chabakauri (2013).

C. Benchmark: economy without index investing

Consider first an unconstrained economy in which all investors can trade all assets individually. An equilibrium in such an economy is characterized by Chabakauri (2013) and we use it as a benchmark for identifying and quantifying the impact of indexing. The equilibrium variables in the unconstrained economy with homogeneous trees are presented in Figure 1.

FIGURE 1 IS HERE

Figure 1 demonstrates that the stock volatilities and the volatility of the index tend to be higher than the volatilities of the dividends and this is an outcome of dynamic risk sharing between agents with different risk preferences (e.g., Bhamra and Uppal, 2009; Longstaff and Wang, 2012).⁹ Indeed, when a positive cash flow shock hits one of the stocks, it disproportionately increases wealth of the type P investors, who are less risk averse and invest a higher fraction of their wealth in stocks. To maintain their optimal portfolio weights, they buy more stocks from the more risk-averse type I investors and drive the price up even further. The effect is stronger for the larger stock (e.g., the first stock when $u > 1/2$) since this stock is traded more actively when the type P investors rebalance their portfolios. Also, the larger stock has a higher beta and shocks to its dividend have a higher price of risk. This is not surprising because the larger stock is a better proxy for the whole market and the risk associated with it has a bigger effect on the investors' consumption. Because the type P investors trade both stocks in response to a shock to one of them, the stock returns are positively correlated even though the correlation between dividends is zero (e.g., Cochrane, Longstaff, and Santa-Clara, 2008; Ehling and Heyerdahl-Larsen, 2012).¹⁰

Even though the total number of the shares of each stock in our economy is normalized to one and the stocks have identical dividend processes, their sizes as well as the statistical properties of their returns are different in all states except those with $u = 1/2$. As a result, the investors tend to hold more shares of one stock than of the other. In particular, the graph for the ratio of the number of shares N_{I2}/N_{I1} demonstrates that the type I investors prefer to hold more shares of the larger stock because it provides a better combination of risk and return.

FIGURE 2 IS HERE

Figure 2 presents the equilibrium in an economy with heterogeneous trees. It confirms many observations made in the case of homogeneous trees and reveals new effects. In particular, the volatilities of both individual stocks and the index tend to be higher when the economy is dominated by the more volatile second tree. This happens because both the fundamental volatility and the volatility produced by risk sharing are higher. The most interesting observation from Figure 2 is that stock returns can be negatively correlated even though the dividends are uncorrelated. To the best of our knowledge, this possibility has not been reported in the literature, which mainly considers economies with homogeneous trees and documents only positive excess correlation of stock returns produced by risk sharing (as in Figure 1).

⁹The volatilities of the stocks and index are computed as $\sigma_i = \sqrt{\Sigma_{Q_{i,1}}^2 + \Sigma_{Q_{i,2}}^2}$ and $\sigma_{ind} = \sqrt{\Sigma_{I,1}^2 + \Sigma_{I,2}^2}$.

¹⁰The correlation between stock returns is computed as $\rho = (\Sigma_{Q1,1}\Sigma_{Q2,1} + \Sigma_{Q1,2}\Sigma_{Q2,2})/(\sigma_1\sigma_2)$.

To better understand the sign of the correlation, we follow Cochrane, Longstaff, and Santa-Clara (2008) and decompose the covariance between stock returns as

$$\begin{aligned} \text{cov}(dQ_1, dQ_2) = & \text{cov}\left(\frac{dD_1}{D_1}, \frac{dD_2}{D_2}\right) + \text{cov}\left(\frac{df_1}{f_1}, \frac{df_2}{f_2}\right) \\ & + \text{cov}\left(\frac{dD_1}{D_1}, \frac{df_2}{f_2}\right) + \text{cov}\left(\frac{dD_2}{D_2}, \frac{df_1}{f_1}\right). \end{aligned} \quad (18)$$

Equation (18) demonstrates that the covariance depends not only on the covariances of dividends and changes in the price-dividend ratios but also on how dividends on one stock covary with changes in the price-dividend ratio of the other stock. In our economy, the first term in equation (18) is zero because dividends are uncorrelated. The second term is small, so, as in Cochrane, Longstaff, and Santa-Clara (2008), the covariance between stock returns is mainly determined by the last two terms. We find that the negative correlation between stock returns in the economy with heterogeneous trees arises because the last term in equation (18) is negative and large for a wide range of realizations of the state variables s and u . Indeed, a negative shock dD_2 increases the share of the first tree u . Figure 2 shows that the price-dividend ratio f_1 is an increasing function of u in many states of the economy (the pattern is particularly pronounced around $s = 0$ and $u = 1$), so in those states $\text{cov}(dD_2/D_2, df_1/f_1) < 0$. The absolute value of the covariance is large due to the large volatility σ_{D_2} , which substantially exceeds σ_{D_1} . This contrasts with the case of homogeneous trees in which f_1 is an increasing function of u in a smaller region and the volatility σ_{D_2} is the same as σ_{D_1} , so the last two terms in equation (18) have similar magnitudes, their sum is positive, and the correlation between stock returns is positive.

It remains to explain why the price-dividend ratios f_1 and f_2 increase with u . Figure 2 shows that the risk-free rate r is a decreasing function of u around $u = 1$ and the pattern is stronger than in the case of homogeneous trees because interest rates are lower in economies with lower expected dividend growth rates and $\mu_{D_1} < \mu_{D_2}$. As a result, the cash flows are discounted at a lower rate around $u = 1$ (where the first tree dominates the economy) and the price-dividend ratios tend to be higher. Thus, the ratios f_i are increasing functions of u around $u = 1$ and this gives rise to the negative correlation between stock returns. Note that the described effect crucially relies on the heterogeneity of both drifts and diffusions of the dividend processes: when either of them is homogeneous, the correlation is positive in all states of the economy.

D. Main analysis: economy with index investors

In this section, we study the equilibrium effects of indexing by comparing the equilibrium variables in the constrained and unconstrained economies. We separately discuss the cases with homogeneous and heterogeneous trees.

D.1. Homogeneous trees

Consider first the economy with homogeneous trees. Because the fundamentals of the trees follow the processes with identical parameters, indexing changes the equilibrium only because the trees have different sizes produced by different realizations of the dividends.

FIGURE 3 IS HERE

Figure 3 shows the changes in the equilibrium variables produced by indexing. For the majority of the variables we plot relative changes but for those variables that can be equal or close to zero we present absolute changes.

First of all, the change in the ratio N_{I2}/N_{I1} shows how indexing distorts investors' portfolios. Because only the market portfolio and the risk-free bond are held by both types of investors in the equilibrium with indexing and the market portfolio contains an equal number of the shares of each stock, indexing implies that $N_{I2}/N_{I1} = 1$ in all states of the economy. Since in the unconstrained economy the type I investors prefer to hold more shares of the larger stock (see the discussion of Figure 1), indexing increases (decreases) the relative number of the shares of the smaller (larger) stock in their portfolio. The graphs for the changes in the number of the shares N_{P1} and N_{P2} held by the type P investors further indicate that in total the type I investors hold more stocks in the economy with indexing than in the benchmark economy because the increase in the number of the shares of the smaller stock is not offset by only a slight decrease in the number of the shares of the larger stock.

An immediate consequence of a more uniform distribution of the shares across investors is the reduction in risk sharing among them. Indeed, because the type P investors hold fewer stocks than in the benchmark economy, their incentives to rebalance portfolios in response to cash flow shocks are subdued. As a result, indexing decreases the volatility of the index σ_{ind} , which is inflated by risk sharing in the unconstrained economy. A lower volatility of the index implies that the market portfolio is safer than in the unconstrained economy and this is why the more risk-averse type I investors hold more equity. The effect is particularly strong when the stocks have unequal sizes and the type P investors notably decrease their holdings of the

smaller stock but disappears as $u \rightarrow 0$ or $u \rightarrow 1$ because in these limits the market coincides with one of the stocks and indexing is irrelevant.

Figure 3 shows that the portfolio distortions brought about by indexing also affect the risk-free rate r , which is lower in the constrained economy. This happens because the stock holdings of the less risk-averse investors, who maintain leveraged positions in stocks in many states of the economy, decrease and they borrow less to finance their portfolio. As for the index volatility, the effect is particularly strong when the stocks differ in size.

The impact of indexing on betas and volatilities of individual stocks as well as on the market prices of risk is less straightforward. As follows from Figure 3, indexing decreases the beta and volatility of the smaller stock but the effect is opposite for the larger stock. Also, the market price of risk associated with the shock dB_i , $i = 1, 2$, is higher when stock i is larger. These observations also admit an intuitive explanation. Because the leveraged type P investors hold a smaller number of the shares of the smaller stock, they are more reluctant to rebalance their portfolios in response to its dividend shocks. As a result, returns on this stock become less volatile and less related to the returns on the market, that is, have a lower beta. Effectively, the smaller stock becomes safer due to indexing and, hence, the shocks associated with it have a lower price of risk. The effect is opposite for the larger stock but it is weaker because the type P investors only slightly increase their holdings of this stock compared to the unconstrained economy. Note that the beta, volatility, and price of risk are higher for the larger stock in the unconstrained economy, so our results imply that indexing increases the cross-sectional dispersion in these characteristics.

Figure 3 also shows that the price-dividend ratios f_i increase relative to the unconstrained economy when stock i is smaller and this is explained by the effects of indexing on r and η_i . Indeed, the approximate Gordon formula $f_i \approx 1/(r + \eta \Sigma'_{Di} - \mu_{Di})$ shows that the price-dividend ratio increases when both the risk-free rate and the market price of risk become lower but may decrease when the decrease in the risk-free rate is offset by an increase in the market price of risk. As follows from the discussion above, the former happens for the smaller stock and the latter may happen for the larger stock.

The effects of indexing on the volatilities can also be tracked down to the changes in the dynamics of the state variable s . Consider equation (A9), which decomposes the diffusions Σ_{Qi} into three components: one of them represents the fundamental diffusion Σ_{Di} and the two others are associated with the diffusions of the state variables s and u . The changes in $\Sigma_{Q1,1}$ (the first element in the matrix diffusion Σ_{Q1}) and its components produced by indexing are presented in the upper panels of Figure 4. Because the stocks have identical fundamental processes, we consider the volatility only of the first of them. Also, we focus only on the diffusions associated

with the innovation dB_1 because the changes in the diffusions associated with dB_i are the main determinants of the changes in the volatility of stock i . This follows from the approximation $\Delta\sigma_i^2 \approx 2(\Sigma_{Q_{i,1}}\Delta\Sigma_{Q_{i,1}} + \Sigma_{Q_{i,2}}\Delta\Sigma_{Q_{i,2}})$ and the inequalities $\Sigma_{Q_{1,1}} > \Sigma_{Q_{1,2}}$ and $\Sigma_{Q_{2,1}} < \Sigma_{Q_{2,2}}$, which hold due to the presence of large constant dividend diffusions in the components $\Sigma_{Q_{1,1}}$ and $\Sigma_{Q_{2,2}}$. The dominant role of $\Delta\Sigma_{Q_{1,1}}$ in shaping the change in the volatility of the first stock is evident from the comparison of its graph and the graph for $\Delta\sigma_1/\sigma_1^{unc}$ in Figure 3.

FIGURE 4 IS HERE

Figure 4 demonstrates that the effect of indexing on the volatility is primarily determined by the second component $(f_{1s}/f_1)\Sigma_{s,1}$ in equation (A9): the deviation from its unconstrained counterpart is an order of magnitude larger than the same deviation of the component $(f_{1u}/f_1)\Sigma_{u,1}$ and almost perfectly coincides with $\Delta\Sigma_{Q_{1,1}}$. The center right panel and the bottom right panel of Figure 4 further decompose the change in $(f_{1s}/f_1)\Sigma_{s,1}$ into two parts related to the changes in the ratio f_{1s}/f_1 and in $\Sigma_{s,1}$ using the approximation $\Delta((f_{1s}/f_1)\Sigma_{s,1}) \approx \Delta(f_{1s}/f_1)\Sigma_{s,1} + (f_{1s}/f_1)\Delta\Sigma_{s,1}$. Comparing these graphs with the graph for the total change in $(f_{1s}/f_1)\Sigma_{s,1}$ we conclude that the latter is largely determined by the change in $\Sigma_{s,1}$ but the effect is also shaped by the factor f_{1s}/f_1 .

Figure 4 also shows the graphs for f_{1s}/f_1 and $\Sigma_{s,1}$ in the benchmark economy and how they change in the economy with indexing. In particular, $\Sigma_{s,1}$ is negative and its absolute value increases with u . Indeed, because the less risk-averse type P investors hold more stocks, any positive shock dB_1 increases disproportionately their wealth and consumption, so the consumption share of the more risk-averse type I investors s goes down. Therefore, $\Sigma_{s,1}$ is negative. The magnitude of the effect grows with the contribution of the asset to the aggregate dividend volatility, and this explains why it is stronger for larger u . Indexing decreases the absolute value of $\Sigma_{s,1}$ for the smaller stock (the first stock when u is small) because the necessity to trade the whole index (both stocks) makes investors less responsive to changes in its dividend. The effect is opposite but weaker for the larger tree. The ratio f_{1s}/f_1 is also negative because the price-dividend ratio decreases with s : for higher s the proportion of the more risk-averse investors in the economy and the required risk premium are higher and prices are lower. As a result, the effect of indexing is stronger when the economy is dominated by the risk averse type I investors (s is large).

Indexing has an ambiguous effect on the correlation of stock returns. Figure 3 demonstrates that the correlation increases when the stocks have comparable sizes (around $u = 0.5$) but

decreases when the sizes are notably different. In general, the impact of indexing on the correlation is determined by the relative strength of the following two effects that work in opposite directions. On the one hand, indexing increases the correlation because each investor holds an equal number of the shares of each stock and trades both stocks in lockstep. On the other hand, indexing reduces the correlation because it hampers risk sharing between investors, which is the main source of the correlation when stocks can be traded individually and their dividends are uncorrelated (as explained in Section III.C). The latter effect is strong when the stocks have different sizes and it dominates, so the correlation becomes lower than in the benchmark economy. When the stocks have comparable sizes, risk sharing is almost unaffected, so the former effect dominates and the correlation between stocks appears to be higher than without indexing.

D.2. Heterogeneous trees

Although indexing changes various equilibrium characteristics, the magnitudes of the effects in an economy with homogeneous trees tend to be relatively small. Indeed, the inability of some investors to rebalance the portfolio of individual stocks matters only when returns on the stocks have different statistical properties. When the dividends follow stochastic processes with identical parameters, only different realizations of dividend shocks and ensuing heterogeneity in stock sizes contributes to the heterogeneity in stock returns, which appears to be limited. As a result, the impact of indexing is also relatively weak. The outcome can be substantially different when the heterogeneity in the stock sizes is accompanied by the heterogeneity in the dividend processes.

FIGURE 5 IS HERE

Figure 5 shows how the equilibrium characteristics change due to indexing in the model with heterogeneous trees. On the one hand, many effects are qualitatively similar to those observed in the economy with homogeneous trees. In particular, indexing reduces the risk-free rate r and the volatility of index returns σ_{ind} . On the other hand, the heterogeneity in the fundamentals brings about several new important effects. To understand why this happens, consider the graph for the change in the ratio N_{I2}/N_{I1} , which provides two observations. First, except for a small area around the point $u = 0, s = 1$ indexing forces the type I investors to hold relatively more shares of the second stock (the stock with more volatile dividends) than they would in the unconstrained economy. This contrasts with the homogeneous asset economy

in which investors are bound to hold more shares of a smaller stock. Second, the portfolio distortion is much larger than in the case with homogeneous trees. This explains why indexing has a stronger impact on the equilibrium as evidenced by other graphs in Figure 5. For example, the correlation between stock returns can decrease by almost 0.15, whereas the effect does not exceed 0.01 in the economy with homogeneous trees. Similarly, the changes in the volatilities, betas, and risk-free rate reported in Figures 3 and 5 differ by an order of magnitude.

Because index investors have to tilt the composition of their portfolios towards the more volatile stock, they respond to the distortion in the risk-return tradeoff by reducing their total exposure to stocks in many states of the economy. As follows from the graphs for $\Delta N_{P1}/N_{P1}^{unc}$ and $\Delta N_{P2}/N_{P2}^{unc}$, the type P investors substantially increase their holdings of the first stock but do not change the holdings of the second stock when the stocks are comparable in size or the first stock is slightly larger than the second one. In those states the type I investors effectively reduce their exposure to stocks by holding fewer shares of the first of them. This effect disappears only when one of the stocks is much larger than the other (when $u \rightarrow 0$ or $u \rightarrow 1$) and the type I investors hold more shares of the smaller stock as in the case with homogeneous trees.

The increase in the holdings of the first stock by the type P investors implies that they use more shares of this stock when they rebalance their portfolios in response to cash flow shocks. As a result, the first stock is more volatile, has a higher beta, and shocks to its dividends command a higher price of risk. The increase in the latter is not offset by the decrease in the risk-free rate and explains the decline in the price-dividend ratio of the first stock.

The effects are opposite for the second stock, which has the same allocation across investors as in the unconstrained economy when it is large and held more by the type I investors when it is small (this immediately follows from the graph for $\Delta N_{P2}/N_{P2}^{unc}$). In the latter case, it is used less for risk sharing by the type P investors, so its volatility, beta, and associated price of risk decrease compared to the unconstrained economy. Together with the lower risk-free rate, the lower price of risk explains why the price-dividend ratio of the second stock is higher in the economy with indexing.

Note that the effect of indexing on many variables is pronounced only when $s > 0.5$ and $u > 0.5$. Indeed, when s is large the index investors consume a substantial fraction of the aggregate dividend and have a strong impact on the properties of the equilibrium. The dependence on u is less straightforward. As follows from the graph for $\Delta(N_{I2}/N_{I1})$, the magnitude of portfolio distortions caused by indexing increases with u . As discussed above, the type I investors decrease their holdings of the index to partially offset the constraint to hold more shares of the second stock in many states of the economy and especially when $u > 0.5$. As a result, the type P investors hold more shares of the first stock and this causes the effects described above. When

u is low, there are two effects. On the one hand, as in the case with homogeneous trees indexing forces the type I investors to hold more shares of the smaller stock, which is the first stock when u is small. On the other hand, because the type I investors reduce their exposure to the index compared to the unconstrained economy they hold fewer shares of the first stock. Overall, these effects approximately offset each other, the impact of indexing is mitigated, and the equilibrium characteristics are almost the same as in the unconstrained economy.

As in Section III.D.1, the effect of indexing on the volatilities can be explained by changes in the dynamics of the state variable s . Again, the volatilities of the individual stocks are largely determined by the diffusions $\Sigma_{Q1,1}$ and $\Sigma_{Q2,2}$ and as before we decompose them according to equation (A9). The results are presented in Figures 6 and 7. Because now the dividend processes of the two stocks are different, we consider the decomposition of diffusions for both of them.

FIGURES 6 AND 7 ARE HERE

As in the model with homogeneous trees, the dominant role in the changes of $\Sigma_{Q1,1}$ and $\Sigma_{Q2,2}$ is played by the components $\Delta((f_{1s}/f_1)\Sigma_{s,1})$ and $\Delta((f_{2s}/f_2)\Sigma_{s,2})$, respectively. In their turn, these components are almost exactly equal to $(f_{1s}/f_1)\Delta\Sigma_{s,1}$ and $(f_{2s}/f_2)\Delta\Sigma_{s,2}$, so indexing affects the volatilities mostly through the changes in the diffusions $\Sigma_{s,1}$ and $\Sigma_{s,2}$. Also, as before the absolute values of the factors f_{1s}/f_1 and f_{2s}/f_2 increase with s , so the effect of indexing is particularly pronounced when the proportion of index investors is relatively large.

However, the effect of indexing on $\Sigma_{s,1}$ and $\Sigma_{s,2}$ is different from that in Figure 4. First, it is strong only when the first (less volatile) stock is larger. Second, indexing decreases the absolute value of $\Sigma_{s,2}$ but in most states increases the absolute value of $\Sigma_{s,1}$. This pattern is totally consistent with the logic discussed above. Due to indexing, in many states of the economy more shares of the first stock are held by the type P investors who become more exposed to the cash flow shocks dB_1 but cannot efficiently hedge them because the first stock can be traded only as a part of the index. As a result, the relative consumption ratio s becomes more sensitive to dB_1 . The decrease in the absolute value of $\Sigma_{s,2}$ has the same nature as the decrease in the diffusions of s in the case with homogeneous trees.

The effect of indexing on the correlation between stock returns is also qualitatively different from what we observe when the trees are homogeneous. In the vast majority of the states, the decrease in the correlation produced by hampered risk sharing dominates the lockstep trading effect responsible for the increase in the correlation (these effects are discussed in Section III.D.1), so in the economy with indexing the returns are less correlated than in the uncon-

strained economy. As almost all other effects, the decrease in the correlation is strong when the first stock is relatively large.

IV. Conclusion

In this paper we investigate the impact of index investing on various characteristics of capital market equilibrium. It is widely believed that the tendency of many market participants to trade indexes instead of individual securities makes returns more volatile and increases the correlations between them. Our analysis reveals that this logic is theoretically flawed because it does not take the equilibrium effects of index investing into account. We argue that indexing changes not only how investors can trade but also their investment opportunities, which determine the incentives to trade. In particular, we demonstrate that indexing can hamper risk sharing among investors, which is responsible for excessive volatility of returns and makes them correlated even when the asset fundamentals are independent. As a result, indexing can decrease the correlations between returns and their volatilities.

Our results also highlight the role of the heterogeneity in the assets' market capitalizations and dividend processes in shaping the impact of indexing. We show that in general indexing increases (decreases) volatilities and betas of stocks with relatively large (small) market capitalizations and its impact is especially strong when stocks differ in their expected dividends and dividend volatilities. The latter case is particularly realistic and empirically relevant.

Our analysis can be extended in several ways. In particular, our model can accommodate alternative types of indexes such as fundamental indexes, which were proposed in the literature and implemented in practice (e.g., Arnott, Hsu, and Moore, 2005). Also, it would be interesting to consider a setting with multiple trees in which only a subset of all trees is included in the index. Such a model could help to investigate how the choice of assets that are included in the index affects the equilibrium properties as well as to provide a fully-fledged general equilibrium analysis of the correlations between the assets included and excluded from the index. This extension is likely to be more technically complicated than our model due to a larger number of state variables. Finally, it may be interesting to endogenize the dividend processes using a production economy framework and examine the impact of indexing on the firms' behavior. The analysis of how portfolio constraints affect corporate policies could be a particularly fruitful direction for future research.

Appendix. Proof of Proposition 1.

The equilibrium functions r , μ_s , Σ_s , Σ_I , f , and h solve the following system of equations:

$$r = \beta + \Gamma \left(\mu_D - \frac{1}{2}(\gamma_I + 1)s \left(\Sigma_D + \frac{1}{s}\Sigma_s \right) \left(\Sigma_D + \frac{1}{s}\Sigma_s \right)' - \frac{1}{2}(\gamma_P + 1)(1-s) \left(\Sigma_D - \frac{1}{1-s}\Sigma_s \right) \left(\Sigma_D - \frac{1}{1-s}\Sigma_s \right)' \right), \quad (\text{A1})$$

$$\mu_s = -\Sigma_s \Sigma_D' + \frac{s(1-s)}{\gamma_I \gamma_P} \Gamma \left(\mu_D(\gamma_P - \gamma_I) + \frac{\gamma_I(\gamma_I + 1)}{2} \left(\Sigma_D + \frac{1}{s}\Sigma_s \right) \left(\Sigma_D + \frac{1}{s}\Sigma_s \right)' - \frac{\gamma_P(\gamma_P + 1)}{2} \left(\Sigma_D - \frac{1}{1-s}\Sigma_s \right) \left(\Sigma_D - \frac{1}{1-s}\Sigma_s \right)' \right), \quad (\text{A2})$$

$$\Sigma_s = \frac{\gamma_P - \gamma_I}{\gamma_I \gamma_P} s(1-s) \Gamma \Sigma_D \Pi_I - \frac{s}{h + s h_s} (h \Sigma_D + h_u \Sigma_u) (I_2 - \Pi_I), \quad (\text{A3})$$

$$\Sigma_I = \frac{\left(f \Sigma_D + f_u \Sigma_u + f_s \frac{\gamma_P - \gamma_I}{\gamma_I \gamma_P} s(1-s) \Gamma \Sigma_D \right) \left(f \Sigma_D + f_u \Sigma_u - \frac{s f_s}{h + s h_s} (h \Sigma_D + h_u \Sigma_u) \right)'}{\left(f \Sigma_D + f_u \Sigma_u - \frac{s f_s}{h + s h_s} (h \Sigma_D + h_u \Sigma_u) \right) \left(f \Sigma_D + f_u \Sigma_u - \frac{s f_s}{h + s h_s} (h \Sigma_D + h_u \Sigma_u) \right)'} \times \left(\Sigma_D + \frac{f_u}{f} \Sigma_u - \frac{f_s}{f} \frac{s}{h + s h_s} (h \Sigma_D + h_u \Sigma_u) \right), \quad (\text{A4})$$

$$\begin{aligned} & \frac{1}{2} f_{ss} \Sigma_s \Sigma_s' + \frac{1}{2} f_{uu} \Sigma_u \Sigma_u' + f_{su} \Sigma_s \Sigma_u' + f_s (\mu_s + (1 - \Gamma) \Sigma_D \Sigma_s') \\ & + f_u (\mu_u + (1 - \Gamma) \Sigma_D \Sigma_u') + (\mu_D - r - \Gamma \Sigma_D \Sigma_D') f + 1 = 0, \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} & \frac{1}{2} h_{ss} \Sigma_s \Sigma_s' + \frac{1}{2} h_{uu} \Sigma_u \Sigma_u' + h_{us} \Sigma_u \Sigma_s' \\ & + h_s \left(\mu_s - (\gamma_I - 1) \left(\Sigma_D + \frac{1}{s} \Sigma_s \right) \Sigma_s' \right) + h_u \left(\mu_u - (\gamma_I - 1) \left(\Sigma_D + \frac{1}{s} \Sigma_s \right) \Sigma_u' \right) \\ & - \left(\frac{\gamma_I - 1}{2} \left(\Sigma_D + \frac{1}{s} \Sigma_s \right) \left(\Sigma_D + \frac{1}{s} \Sigma_s \right)' + \frac{(\gamma_I - 1)r + \beta}{\gamma_I} \right) h + 1 = 0, \end{aligned} \quad (\text{A6})$$

where $\Pi_I = (\Sigma'_I \Sigma_I) / (\Sigma_I \Sigma'_I)$ and the subscripts s and u of the functions f and h indicate derivatives. The market price of risk η and the expected returns on the index μ_I are

$$\eta = \gamma_P \left(\Sigma_D - \frac{1}{1-s} \Sigma_s \right), \quad \mu_I = \Gamma \Sigma_I \Sigma'_D. \quad (\text{A7})$$

The price-dividend ratio f_i of stock $i = 1, 2$ solves the following differential equation:

$$\begin{aligned} \frac{1}{2} f_{iss} \Sigma_s \Sigma'_s + \frac{1}{2} f_{iuu} \Sigma_u \Sigma'_u + f_{isu} \Sigma_s \Sigma'_u + f_{is} (\mu_s + (\Sigma_{Di} - \eta) \Sigma'_s) \\ + f_{iu} (\mu_u + (\Sigma_{Di} - \eta) \Sigma'_u) + (\mu_{Di} - r - \eta \Sigma'_{Di}) f_i + 1 = 0. \end{aligned} \quad (\text{A8})$$

The expected excess returns on individual stocks μ_{Qi} and return diffusions Σ_{Qi} are

$$\mu_{Qi} = \gamma_P \Sigma_{Qi} \left(\Sigma_D - \frac{1}{1-s} \Sigma_s \right)', \quad \Sigma_{Qi} = \Sigma_{Di} + \frac{f_{is}}{f_i} \Sigma_s + \frac{f_{iu}}{f_i} \Sigma_u. \quad (\text{A9})$$

The optimal portfolio weights of the type I and type P investors are

$$\omega_I = \frac{1}{\Sigma_I \Sigma'_I} \left(\frac{\mu_I}{\gamma_I} + \frac{h_s}{h} \Sigma_I \Sigma'_s + \frac{h_u}{h} \Sigma_I \Sigma'_u \right), \quad (\text{A10})$$

$$\omega_P = (\Sigma_Q \Sigma'_Q)^{-1} \left(\frac{\mu_Q}{\gamma_P} + \frac{h_{Ps}}{h_P} \Sigma_Q \Sigma'_s + \frac{h_{Pu}}{h_P} \Sigma_Q \Sigma'_u \right), \quad (\text{A11})$$

where the wealth-consumption ratio of the type P investors h_P is

$$h_P = \frac{1}{1-s} (u f_1 + (1-u) f_2 - s h). \quad (\text{A12})$$

The numbers of the shares of each stock N_{Ii} and N_{Pi} held by the type I and type P investors are

$$N_{Ii} = \frac{s \omega_{Ii} h}{s \omega_{Ii} h + (1-s) \omega_{Pi} h_P}, \quad N_{Pi} = \frac{(1-s) \omega_{Pi} h_P}{s \omega_{Ii} h + (1-s) \omega_{Pi} h_P}, \quad (\text{A13})$$

where

$$\omega_{I1} = \frac{\omega_I u f_1}{u f_1 + (1-u) f_2}, \quad \omega_{I2} = \frac{\omega_I (1-u) f_2}{u f_1 + (1-u) f_2}. \quad (\text{A14})$$

We derive equations (A1) – (A14) in several steps.

A. Price-dividend ratios

First, we derive equations for the price-dividend ratios f_1 , f_2 , and f . By definition, $S_i = D_i f_i$.

Applying Itô's lemma to this equation, we get

$$\frac{dS_i}{S_i} = \frac{dD_i}{D_i} + \frac{df_i}{f_i} + \frac{dD_i}{D_i} \frac{df_i}{f_i},$$

where

$$df_i = f_{is}(\mu_s dt + \Sigma_s dB) + f_{iu}(\mu_u dt + \Sigma_u dB) + \frac{1}{2}f_{iss}\Sigma_s\Sigma'_s dt + \frac{1}{2}f_{iuu}\Sigma_u\Sigma'_u dt + f_{ius}\Sigma_u\Sigma'_s dt.$$

Using equation (1),

$$\begin{aligned} \frac{dS_i + D_i dt}{S_i} - r dt = & \left(\mu_{Di} - r + \frac{1}{2} \frac{f_{iss}}{f_i} \Sigma_s \Sigma'_s + \frac{1}{2} \frac{f_{iuu}}{f_i} \Sigma_u \Sigma'_u + \frac{f_{ius}}{f_i} \Sigma_u \Sigma'_s \right. \\ & \left. + (\mu_s + \Sigma_{Di} \Sigma'_s) \frac{f_{is}}{f_i} + (\mu_u + \Sigma_{Di} \Sigma'_u) \frac{f_{iu}}{f_i} + \frac{1}{f_i} \right) dt + \left(\Sigma_{Di} + \frac{f_{is}}{f_i} \Sigma_s + \frac{f_{iu}}{f_i} \Sigma_u \right) dB. \end{aligned}$$

This process should coincide with the process for excess returns from equation (2), so

$$\begin{aligned} \mu_{Qi} = \mu_{Di} - r + \frac{1}{2} \frac{f_{iss}}{f_i} \Sigma_s \Sigma'_s + \frac{1}{2} \frac{f_{iuu}}{f_i} \Sigma_u \Sigma'_u + \frac{f_{ius}}{f_i} \Sigma_u \Sigma'_s \\ + (\mu_s + \Sigma_{Di} \Sigma'_s) \frac{f_{is}}{f_i} + (\mu_u + \Sigma_{Di} \Sigma'_u) \frac{f_{iu}}{f_i} + \frac{1}{f_i}, \end{aligned} \quad (\text{A15})$$

$$\Sigma_{Qi} = \Sigma_{Di} + \frac{f_{is}}{f_i} \Sigma_s + \frac{f_{iu}}{f_i} \Sigma_u. \quad (\text{A16})$$

Equation (A15) is effectively a differential equation for f_i :

$$\begin{aligned} \frac{1}{2} f_{iss} \Sigma_s \Sigma'_s + \frac{1}{2} f_{iuu} \Sigma_u \Sigma'_u + f_{isu} \Sigma_s \Sigma'_u + f_{is} (\mu_s + \Sigma_{Di} \Sigma'_s) \\ + f_{iu} (\mu_u + \Sigma_{Di} \Sigma'_u) + (\mu_{Di} - r - \mu_{Qi}) f_i + 1 = 0. \end{aligned} \quad (\text{A17})$$

By definition of the market price of risk η , $\mu_{Qi} = \Sigma_{Qi} \eta'$. Plugging this representation for μ_{Qi} in equation (A17) and using equation (A16), we arrive at equation (A8). The same steps applied to the index yield the differential equation for the index price-dividend ratio f :

$$\begin{aligned} \frac{1}{2} f_{ss} \Sigma_s \Sigma'_s + \frac{1}{2} f_{uu} \Sigma_u \Sigma'_u + f_{su} \Sigma_s \Sigma'_u + f_s (\mu_s + \Sigma_D \Sigma'_s) \\ + f_u (\mu_u + \Sigma_D \Sigma'_u) + (\mu_D - r - \mu_I) f + 1 = 0. \end{aligned} \quad (\text{A18})$$

The index diffusion is related to the diffusions of the state variables as

$$\Sigma_I = \Sigma_D + \frac{f_s}{f} \Sigma_s + \frac{f_u}{f} \Sigma_u \quad (\text{A19})$$

and this equation is similar to equation (A16).

B. Utility maximization problem of the type P investors

Next, consider the consumption and portfolio problem of the type P investors. Recall that they can invest in any combination of stocks, so from their perspective the market is complete. The first order conditions of their optimization problem can be interpreted as pricing equations that relate the risk-free rate r and the expected excess returns μ_Q to their discount factor Λ (e.g., Cochrane, 2005):

$$r = -\frac{1}{dt} E \left(\frac{d\Lambda}{\Lambda} \right), \quad \mu_{Qi} = -\frac{1}{dt} E \left(\frac{d\Lambda}{\Lambda} \frac{dS_i}{S_i} \right), \quad i = 1, 2. \quad (\text{A20})$$

Since the investors have the CRRA preferences, their discount factor is $\Lambda = \exp(-\beta t)(C_P)^{-\gamma_P}$. Hence,

$$\frac{d\Lambda}{\Lambda} = -\beta dt - \gamma_P \frac{dC_P}{C_P} + \frac{\gamma_P(\gamma_P + 1)}{2} \left(\frac{dC_P}{C_P} \right)^2.$$

Using the definition of the consumption share s , the consumption of the type P investors is $C_P = (1 - s)D$. Itô's lemma applied to this equation together with equations (3) and (8) yields

$$\frac{dC_P}{C_P} = \left(\mu_D - \frac{\mu_s + \Sigma_D \Sigma'_s}{1 - s} \right) dt + \left(\Sigma_D - \frac{1}{1 - s} \Sigma_s \right) dB.$$

Therefore,

$$\begin{aligned} \frac{d\Lambda}{\Lambda} = & -\beta dt - \gamma_P \left(\mu_D - \frac{\mu_s + \Sigma_D \Sigma'_s}{1 - s} - \frac{\gamma_P + 1}{2} \left(\Sigma_D - \frac{1}{1 - s} \Sigma_s \right) \left(\Sigma_D - \frac{1}{1 - s} \Sigma_s \right)' \right) dt \\ & - \gamma_P \left(\Sigma_D - \frac{1}{1 - s} \Sigma_s \right) dB. \end{aligned}$$

Using equation (A20), we find the risk-free rate r and the expected excess returns μ_Q and μ_I :

$$r = \beta + \gamma_P \mu_D - \frac{\gamma_P}{1 - s} (\mu_s + \Sigma_s \Sigma'_D) - \frac{\gamma_P(\gamma_P + 1)}{2} \left(\Sigma_D - \frac{1}{1 - s} \Sigma_s \right) \left(\Sigma_D - \frac{1}{1 - s} \Sigma_s \right)', \quad (\text{A21})$$

$$\mu_Q = \gamma_P \Sigma_Q \left(\Sigma_D - \frac{1}{1 - s} \Sigma_s \right)', \quad \mu_I = \gamma_P \Sigma_I \left(\Sigma_D - \frac{1}{1 - s} \Sigma_s \right)'. \quad (\text{A22})$$

C. Utility maximization problem of the type I investors

The type I investors maximize the CRRA utility from equation (5) subject to the budget constraint from equation (7). Because they can trade only the index and the risk-free bond, from their perspective the market is incomplete and the utility maximization problem should be solved directly. In particular, their indirect utility function J satisfies the standard Hamilton-Jacobi-Bellman (HJB) equation

$$\max_{\{C_I, \omega_I\}} \left[e^{-\beta t} \frac{C_I^{1-\gamma_I}}{1-\gamma_I} + \mathcal{D}J \right] = 0, \quad (\text{A23})$$

where $\mathcal{D}J = E[dJ]/dt$ is given by

$$\begin{aligned} \mathcal{D}J = & J_W(rW_I - C_I + \omega_I W_I \mu_I) + \frac{1}{2} J_{WW} W_I^2 \omega_I^2 \Sigma_I \Sigma'_I + J_{Ws} \omega_I W_I \Sigma_I \Sigma'_s + J_{Wu} \omega_I W_I \Sigma_I \Sigma'_u \\ & + J_s \mu_s + J_u \mu_u + \frac{1}{2} J_{ss} \Sigma_s \Sigma'_s + \frac{1}{2} J_{uu} \Sigma_u \Sigma'_u + J_{us} \Sigma_s \Sigma'_u + J_t \end{aligned}$$

and the subscripts of J denote derivatives with respect to the corresponding variable. When investors have the CRRA preferences, it is standard to look for the indirect utility in the following form:

$$J = \frac{1}{1-\gamma_I} W_I^{1-\gamma_I} h^{\gamma_I} \exp(-\beta t), \quad (\text{A24})$$

where the function h depends on the state variables s and u . The maximization in equation (A23) with respect to C_I together with equation (A24) yields the optimal consumption:

$$C_I = W_I h^{-1}, \quad (\text{A25})$$

so h is the optimal wealth-consumption ratio. Similarly, the maximization in (A23) with respect to ω_I gives the optimal weight of the index:

$$\omega_I = \frac{1}{\Sigma_I \Sigma'_I} \left(\frac{\mu_I}{\gamma_I} + \frac{h_s}{h} \Sigma_I \Sigma'_s + \frac{h_u}{h} \Sigma_I \Sigma'_u \right). \quad (\text{A26})$$

This is equation (A10). The substitution of equations (A25) and (A26) back into equation

(A23) yields a differential equation for h :

$$\begin{aligned} & \frac{1}{2}h_{ss}\Sigma_s\Sigma'_s + \frac{1}{2}h_{uu}\Sigma_u\Sigma'_u + h_{us}\Sigma_u\Sigma'_s + h_s\mu_s + h_u\mu_u \\ & + \frac{\gamma_I - 1}{2} \left(\left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u \right) \left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u \right)' - \frac{1}{\Sigma_I\Sigma'_I} \left(\frac{\mu_I}{\gamma_I} + \frac{h_s}{h}\Sigma_I\Sigma'_s + \frac{h_u}{h}\Sigma_I\Sigma'_u \right)^2 \right) h \\ & + \frac{1}{\gamma_I}((1 - \gamma_I)r - \beta)h + 1 = 0. \quad (\text{A27}) \end{aligned}$$

D. Dynamics of the state variable s and returns on the index

Next, we find expressions for μ_s , Σ_s , μ_I , and Σ_I . The definition of the consumption share s implies that $C_I = sD$, so using Itô's lemma

$$\frac{dC_I}{C_I} = \mu_C dt + \Sigma_C dB, \quad \frac{dC_I^{-\gamma_I}}{C_I^{-\gamma_I}} = \left(-\gamma_I \mu_C + \frac{1}{2} \gamma_I (\gamma_I + 1) \Sigma_C \Sigma'_C \right) dt - \gamma_I \Sigma_C dB, \quad (\text{A28})$$

where

$$\mu_C = \mu_D + \frac{\mu_s + \Sigma_s \Sigma'_D}{s}, \quad \Sigma_C = \Sigma_D + \frac{1}{s} \Sigma_s. \quad (\text{A29})$$

Note that using $C_I = W_I h^{-1}$, the indirect utility function from equation (A24) can be rewritten as

$$J = \frac{1}{1 - \gamma_I} C_I^{-\gamma_I} W_I \exp(-\beta t).$$

Applying Itô's lemma to this equation and taking into account equations (7) and (A28), we get

$$\frac{dJ}{J} = \left(-\beta - \gamma_I \mu_C + \frac{1}{2} \gamma_I (\gamma_I + 1) \Sigma_C \Sigma'_C + r - h^{-1} + \omega_I (\mu_I - \gamma_I \Sigma_I \Sigma'_C) \right) dt + (\omega_I \Sigma_I - \gamma_I \Sigma_C) dB. \quad (\text{A30})$$

Alternatively, Itô's lemma applied to equation (A24) yields

$$\frac{dJ}{J} = \frac{\mathcal{D}J}{J} dt + \left((1 - \gamma_I) \omega_I \Sigma_I + \gamma_I \frac{h_s}{h} \Sigma_s + \gamma_I \frac{h_u}{h} \Sigma_u \right) dB. \quad (\text{A31})$$

Noting that equations (A24) and (A25) imply that

$$e^{-\beta t} \frac{C_I^{1-\gamma_I}}{1 - \gamma_I} = J h^{-1}$$

and using the HJB equation (A23), we get $\mathcal{D}J = -Jh^{-1}$ and rewrite equation (A31) as

$$\frac{dJ}{J} = -h^{-1}dt + \left((1 - \gamma_I)\omega_I\Sigma_I + \gamma_I\frac{h_s}{h}\Sigma_s + \gamma_I\frac{h_u}{h}\Sigma_u \right) dB. \quad (\text{A32})$$

Matching the drifts and diffusions in equations (A30) and (A32) and using μ_C and Σ_C from equation (A29), we get

$$\begin{aligned} \frac{1 + \gamma_I}{2} \left(\Sigma_D + \frac{1}{s}\Sigma_s \right) \left(\Sigma_D + \frac{1}{s}\Sigma_s \right)' + \frac{r - \beta}{\gamma_I} + \omega_I \left(\frac{\mu_I}{\gamma_I} - \left(\Sigma_D + \frac{1}{s}\Sigma_s \right) \Sigma_I' \right) \\ = \mu_D + \frac{1}{s}(\mu_s + \Sigma_s \Sigma_D'), \end{aligned} \quad (\text{A33})$$

$$\omega_I \Sigma_I - \frac{h_s}{h}\Sigma_s - \frac{h_u}{h}\Sigma_u = \Sigma_D + \frac{1}{s}\Sigma_s. \quad (\text{A34})$$

Equation (A34) helps to derive a system of equations for Σ_I and Σ_s . Plugging the optimal portfolio weight ω_I from equation (A26) into equation (A34) yields

$$\frac{\mu_I \Sigma_I}{\gamma_I(\Sigma_I \Sigma_I')} - \left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u \right) \left(I_2 - \frac{\Sigma_I' \Sigma_I}{\Sigma_I \Sigma_I'} \right) = \Sigma_D + \frac{1}{s}\Sigma_s, \quad (\text{A35})$$

where I_2 is a 2×2 unit matrix. Multiplying this equation by Σ_I' , we get

$$\mu_I = \gamma_I \left(\Sigma_D + \frac{1}{s}\Sigma_s \right) \Sigma_I', \quad (\text{A36})$$

which together with the expression for μ_I from (A22) gives

$$\Sigma_s \Sigma_I' = \left(\frac{\gamma_I}{s} + \frac{\gamma_P}{1 - s} \right)^{-1} (\gamma_P - \gamma_I) \Sigma_D \Sigma_I'. \quad (\text{A37})$$

The substitution of this equation in equation (A36) yields $\mu_I = \Gamma \Sigma_D \Sigma_I'$, where Γ is defined in equation (14). This expression for μ_I is a part of equation (A7). Plugging it into equation (A35), introducing the matrix $\Pi_I = (\Sigma_I' \Sigma_I) / (\Sigma_I \Sigma_I')$, which is a projector operator on the vector Σ_I , and rearranging the terms, we get

$$\Sigma_s = (\gamma_P - \gamma_I) \left(\frac{\gamma_I}{s} + \frac{\gamma_P}{1 - s} \right)^{-1} \Sigma_D - s \left(\frac{h_s}{h}\Sigma_s + \frac{h_u}{h}\Sigma_u + \frac{1}{\gamma_I} \Gamma \Sigma_D \right) (I_2 - \Pi_I). \quad (\text{A38})$$

The resolution of this equation for Σ_s yields

$$\Sigma_s = (\gamma_P - \gamma_I) \left(\frac{\gamma_I}{s} + \frac{\gamma_P}{1-s} \right)^{-1} \Sigma_D \Pi_I - \frac{s}{h + sh_s} (h\Sigma_D + h_u \Sigma_u) (I_2 - \Pi_I). \quad (\text{A39})$$

Using the definition of Γ , we obtain equation (A3).

Equations (A19) and (A39) jointly determine Σ_s and Σ_I . The substitution of Σ_s from (A19) in (A39) yields an equation for Σ_I :

$$\begin{aligned} \Sigma_I = \frac{f_s}{f} \left[(\gamma_P - \gamma_I) \left(\frac{\gamma_I}{s} + \frac{\gamma_P}{1-s} \right)^{-1} \Sigma_D + \frac{s}{h + sh_s} (h\Sigma_D + h_u \Sigma_u) \right] \Pi_I \\ + \left[\Sigma_D + \frac{f_u}{f} \Sigma_u - \frac{f_s}{f} \frac{s}{h + sh_s} (h\Sigma_D + h_u \Sigma_u) \right]. \end{aligned} \quad (\text{A40})$$

To solve this equation, we use the following lemma.

LEMMA. *Consider a linear space with a scalar product (\cdot, \cdot) and denote by Π_x the orthogonal projection on vector x . Also, let a and b be two vectors and assume that $(b, b) > 0$. Then, the equation for x*

$$x = \Pi_x a + b \quad (\text{A41})$$

has the unique solution

$$x = \frac{(a + b, b)}{(b, b)} b.$$

Proof of Lemma. The application of the operator Π_x to both sides of equation (A41) gives $x = \Pi_x a + \Pi_x b$, which together with the initial equation (A41) implies that $\Pi_x b = b$. Hence, the vector b belongs to the subspace spanned by the vector x , so $x = \lambda b$, $\lambda \in \mathbb{R}$. The substitution of this expression in equation (A41) yields $\lambda b = \Pi_b a + b$, which implies $\lambda = (\Pi_b a + b, b) / (b, b)$. Finally, $(\Pi_b a, b) = (a - (I - \Pi_b)a, b) = (a, b)$, where I is the identity operator, and this completes the proof. Q.E.D.

Equation (A40) has exactly the form of equation (A41) with Σ_I corresponding to x . Hence,

$$\begin{aligned} \Sigma_I = \frac{\left(f\Sigma_D + f_u \Sigma_u + f_s (\gamma_P - \gamma_I) \left(\frac{\gamma_I}{s} + \frac{\gamma_P}{1-s} \right)^{-1} \Sigma_D \right) \left(f\Sigma_D + f_u \Sigma_u - \frac{sf_s}{h+sh_s} (h\Sigma_D + h_u \Sigma_u) \right)'}{\left(f\Sigma_D + f_u \Sigma_u - \frac{sf_s}{h+sh_s} (h\Sigma_D + h_u \Sigma_u) \right) \left(f\Sigma_D + f_u \Sigma_u - \frac{sf_s}{h+sh_s} (h\Sigma_D + h_u \Sigma_u) \right)'} \times \\ \times \left(\Sigma_D + \frac{f_u}{f} \Sigma_u - \frac{f_s}{f} \frac{s}{h + sh_s} (h\Sigma_D + h_u \Sigma_u) \right). \end{aligned}$$

This is equation (A4). To derive the expression for μ_s , we use equation (A33), which together with equation (A36) yields

$$\frac{1 + \gamma_I}{2} \left(\Sigma_D + \frac{1}{s} \Sigma_s \right) \left(\Sigma_D + \frac{1}{s} \Sigma_s \right)' + \frac{r - \beta}{\gamma_I} = \mu_D + \frac{1}{s} (\mu_s + \Sigma_s \Sigma_D'). \quad (\text{A42})$$

Equations (A21) and (A42) can be viewed as a system of linear equations for r and μ_s . Its solution is given by equations (A1) and (A2).

E. Differential equations for f and h

Equation (A5) for the price-dividend ratio f follows from (A18) after noting that $\mu_I = \Gamma \Sigma_D \Sigma_I'$ and Σ_I is given by equation (A19). Equation (A6) for the wealth-consumption ratio h is derived from equation (A27). Using the expression for μ_I from (A36) and noting that (A38) implies that

$$\left(\frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u + \Sigma_D + \frac{1}{s} \Sigma_s \right) (I_2 - \Pi_I) = 0,$$

we get

$$\begin{aligned} & \left(\frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right) \left(\frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right)' - \frac{1}{\Sigma_I \Sigma_I'} \left(\frac{\mu_I}{\gamma_I} + \frac{h_s}{h} \Sigma_I \Sigma_s' + \frac{h_u}{h} \Sigma_I \Sigma_u' \right)^2 \\ &= \left(\frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right) \left(\frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right)' \\ &- \left(\Sigma_D + \frac{1}{s} \Sigma_s + \frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right) \Pi_I \left(\Sigma_D + \frac{1}{s} \Sigma_s + \frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right)' \\ &= \left(\frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right) \left(\frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right)' \\ &- \left(\Sigma_D + \frac{1}{s} \Sigma_s + \frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right) \left(\Sigma_D + \frac{1}{s} \Sigma_s + \frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right)' \\ &= -2 \left(\Sigma_D + \frac{1}{s} \Sigma_s \right) \left(\frac{h_s}{h} \Sigma_s + \frac{h_u}{h} \Sigma_u \right)' - \left(\Sigma_D + \frac{1}{s} \Sigma_s \right) \left(\Sigma_D + \frac{1}{s} \Sigma_s \right)'. \end{aligned}$$

This transformation allows us to eliminate the quadratic terms with h_s and h_u from equation (A27) and get equation (A6).

F. Optimal portfolios and numbers of shares

The optimal portfolio policy of the type I investors is given by equation (A26). The optimal portfolio of the type P investors stated in (A11) is derived from their utility optimization problem following exactly the same steps that are used to derive equation (A26). To find the wealth-consumption ratio function h_P , we exploit the market clearing conditions. Indeed, summing

up equations (12) and (13), we get that $W_P + W_I = S_1 + S_2$. Using the definitions of the price-dividend ratios $S_i = f_i D_i$ and the wealth-consumption ratios $W_I = h C_I$, $W_P = h_P C_P$, this equation can be rewritten as $(1 - s)h_P + sh = uf_1 + (1 - u)f_2$. After resolving it for h_P , we get (A12).

Finally, we derive the numbers of the shares N_{Ii} and N_{Pi} . The definition of the portfolio weights implies that $N_{Ii}S_i = \omega_{Ii}W_I$ and $N_{Pi}S_i = \omega_{Pi}W_P$, so

$$\frac{N_{Ii}}{N_{Pi}} = \frac{\omega_{Ii}W_I}{\omega_{Pi}W_P} = \frac{\omega_{Ii}C_I h}{\omega_{Pi}C_P h_P} = \frac{s\omega_{Ii}h}{(1 - s)\omega_{Pi}h_P}, \quad (\text{A43})$$

where the second equality uses the definition of the wealth-consumption ratio and the last equality uses the definition of the state variable s . Together with the market clearing condition $N_{Ii} + N_{Pi} = 1$, equation (A43) yields the expressions from (A13). The portfolio weights of the individual stocks in the index are proportional to $S_i/(S_1 + S_2)$, $i = 1, 2$, which in terms of the price-dividend ratios f_1 and f_2 are

$$\frac{S_1}{S_1 + S_2} = \frac{uf_1}{uf_1 + (1 - u)f_2}, \quad \frac{S_2}{S_1 + S_2} = \frac{(1 - u)f_2}{uf_1 + (1 - u)f_2}.$$

Taking into account that the type I investors allocate ω_I to the index, we get the expressions from (A14). Q.E.D.

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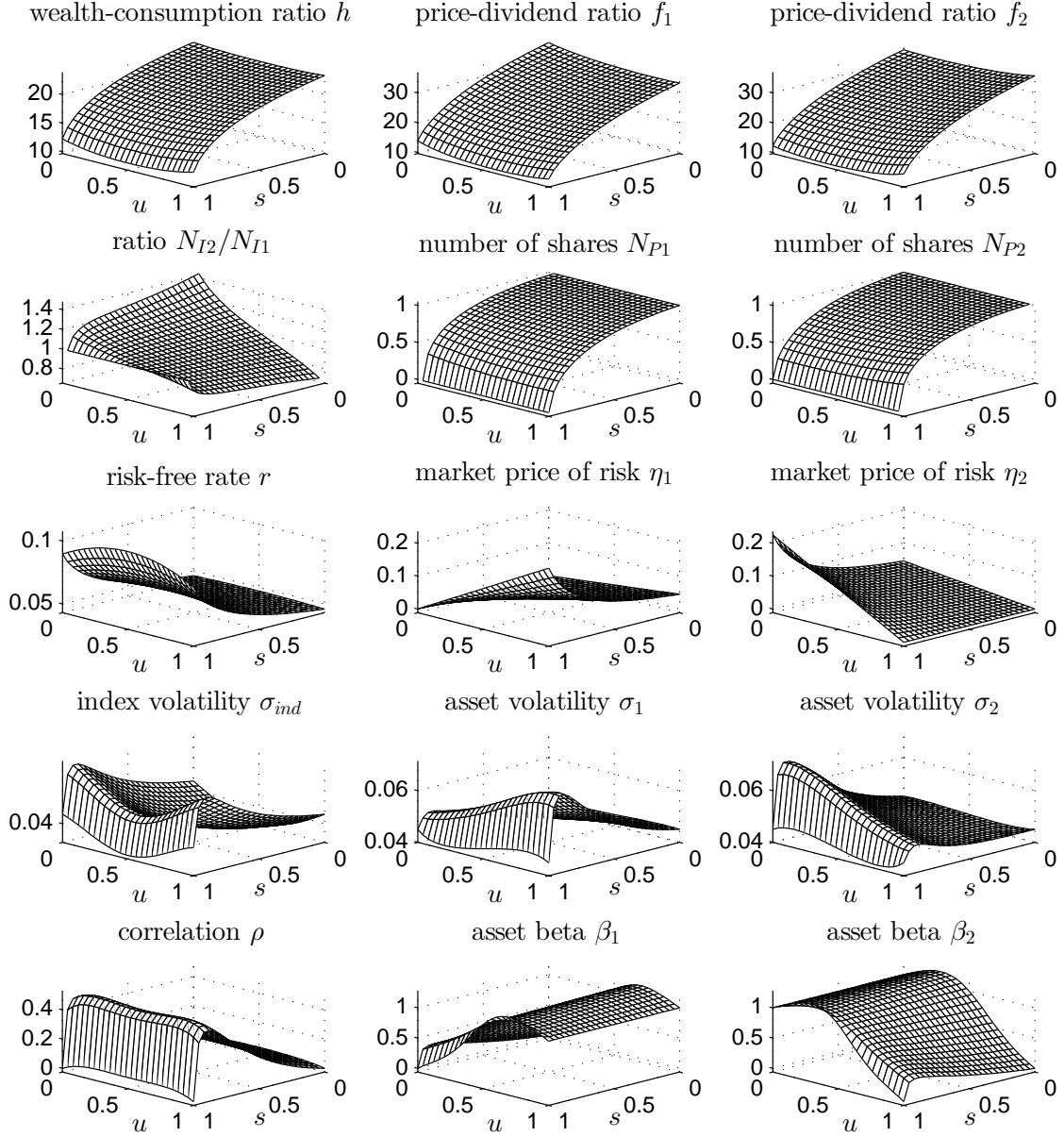


Figure 1. The equilibrium in the model with homogeneous trees (no indexing). This figure presents equilibrium variables in the model without indexing as functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D . The model parameters are as follows: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \ 0]$, $\Sigma_{D2} = [0 \ 0.045]$, $\beta = 0.03$, $\gamma_I = 5$, $\gamma_P = 1$.

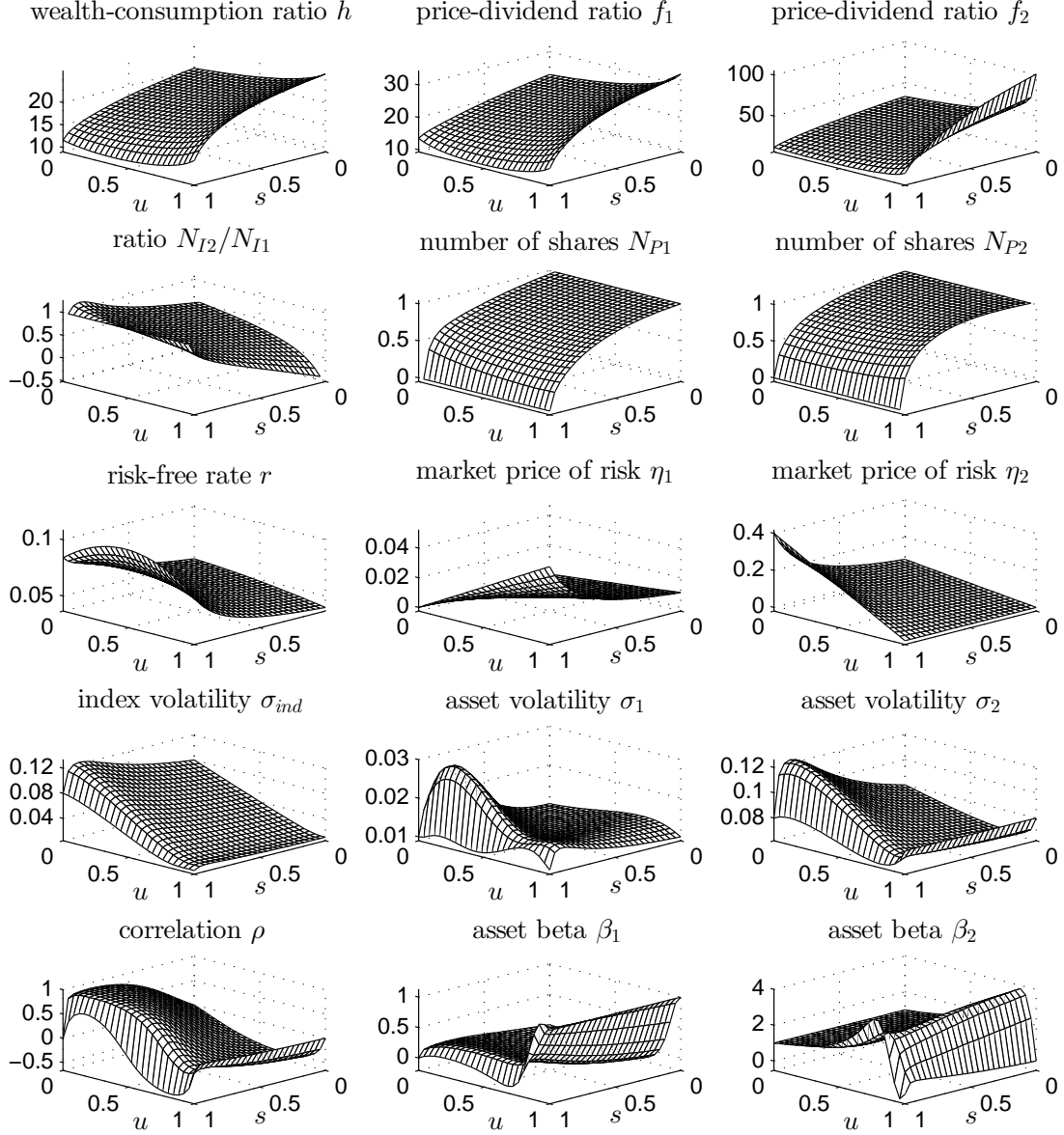


Figure 2. The equilibrium in the model with heterogeneous trees (no indexing).

This figure presents equilibrium variables in the model without indexing as functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D . The model parameters are as follows: $\mu_{D1} = 0.01$, $\mu_{D2} = 0.03$, $\Sigma_{D1} = [0.01 \ 0]$, $\Sigma_{D2} = [0 \ 0.08]$, $\beta = 0.03$, $\gamma_I = 5$, $\gamma_P = 1$.

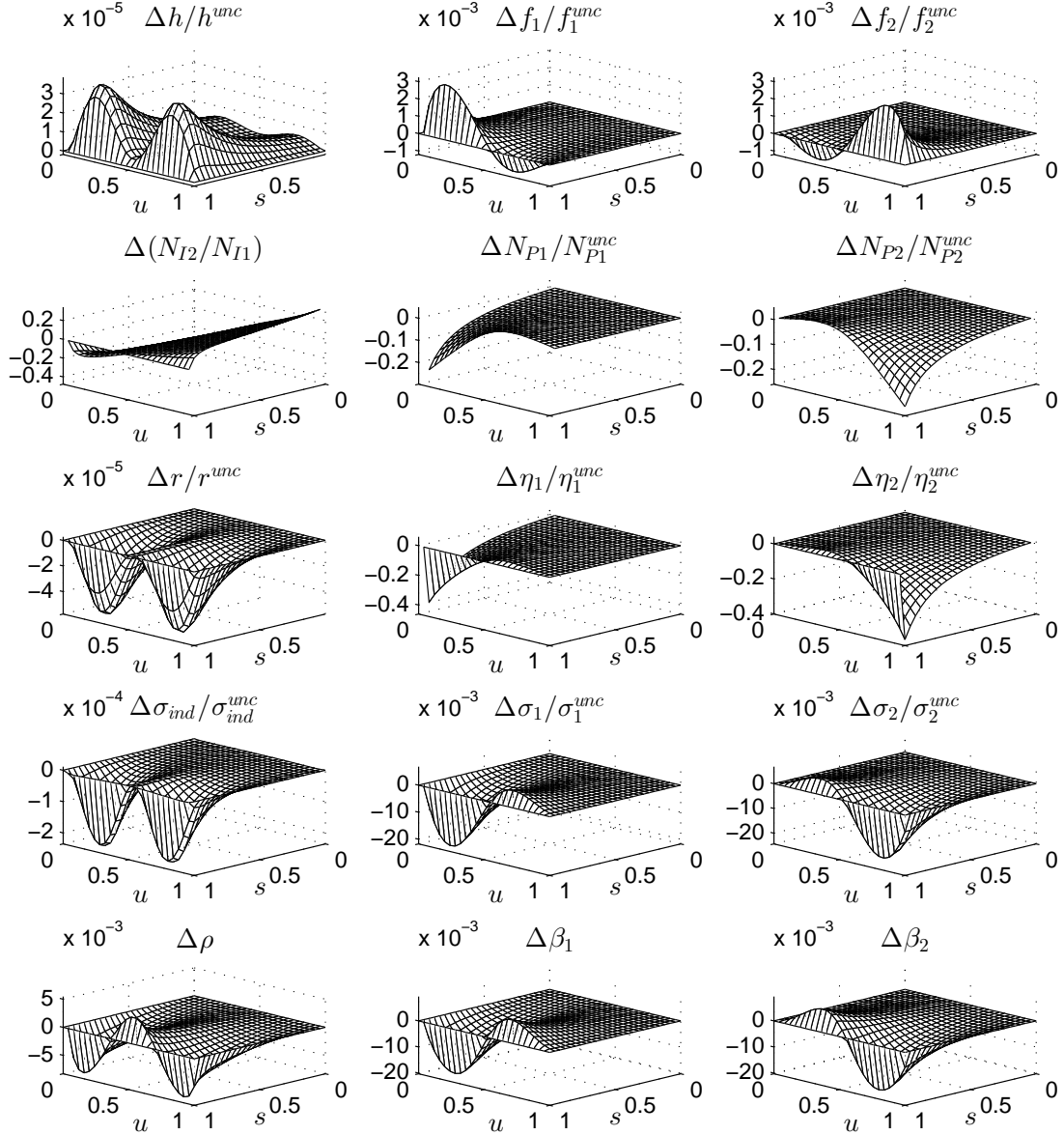


Figure 3. The effect of indexing on equilibrium characteristics in the model with homogeneous trees. This figure shows how equilibrium changes due to indexing in an economy with homogeneous trees. All variables are functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D . The model parameters are as follows: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \ 0]$, $\Sigma_{D2} = [0 \ 0.045]$, $\beta = 0.03$, $\gamma_I = 5$, $\gamma_P = 1$.

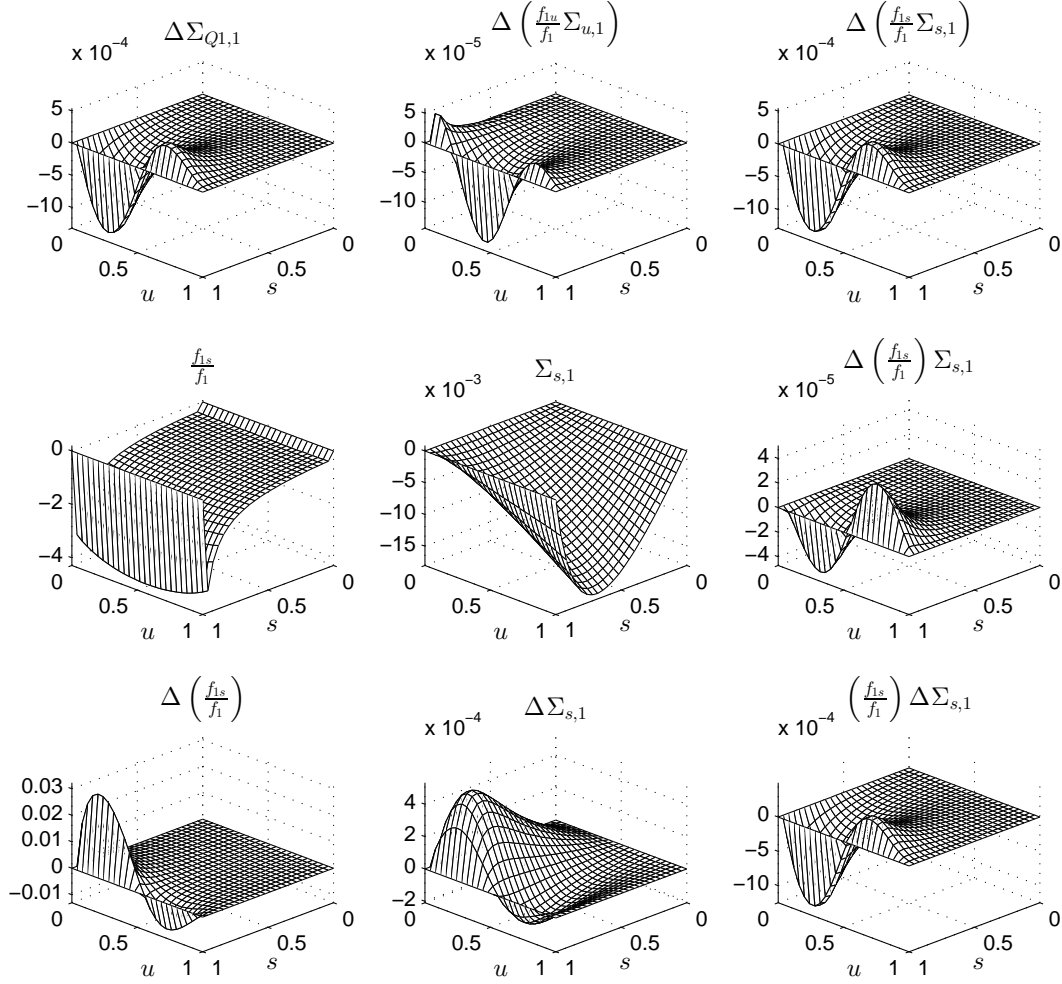


Figure 4. The effect of indexing on $\Sigma_{Q1,1}$ in the model with homogeneous trees. This figure presents the impact of indexing on the first component of the diffusion $\Sigma_{Q1} = \Sigma_{D1} + (f_{1s}/f_1)\Sigma_s + (f_{1u}/f_1)\Sigma_u$. All variables are functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D . The model parameters are as follows: $\mu_{D1} = \mu_{D2} = 0.018$, $\Sigma_{D1} = [0.045 \ 0]$, $\Sigma_{D2} = [0 \ 0.045]$, $\beta = 0.03$, $\gamma_I = 5$, $\gamma_P = 1$.

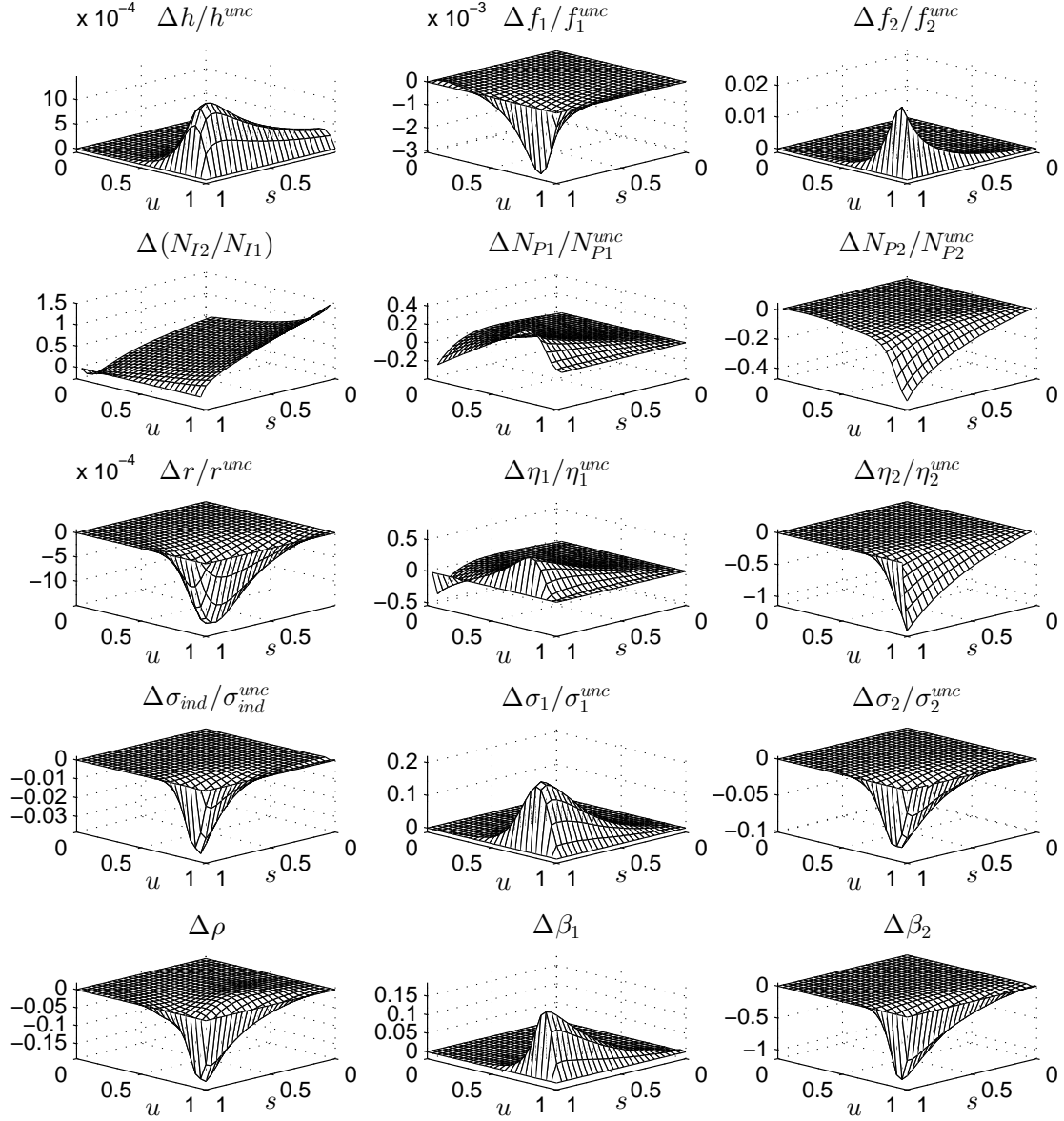


Figure 5. The effect of indexing on equilibrium characteristics in the model with heterogeneous trees. This figure shows how equilibrium changes due to indexing in an economy with heterogeneous trees. All variables are functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D . The model parameters are as follows: $\mu_{D1} = 0.01$, $\mu_{D2} = 0.03$, $\Sigma_{D1} = [0.01 \ 0]$, $\Sigma_{D2} = [0 \ 0.08]$, $\beta = 0.03$, $\gamma_I = 5$, $\gamma_P = 1$.

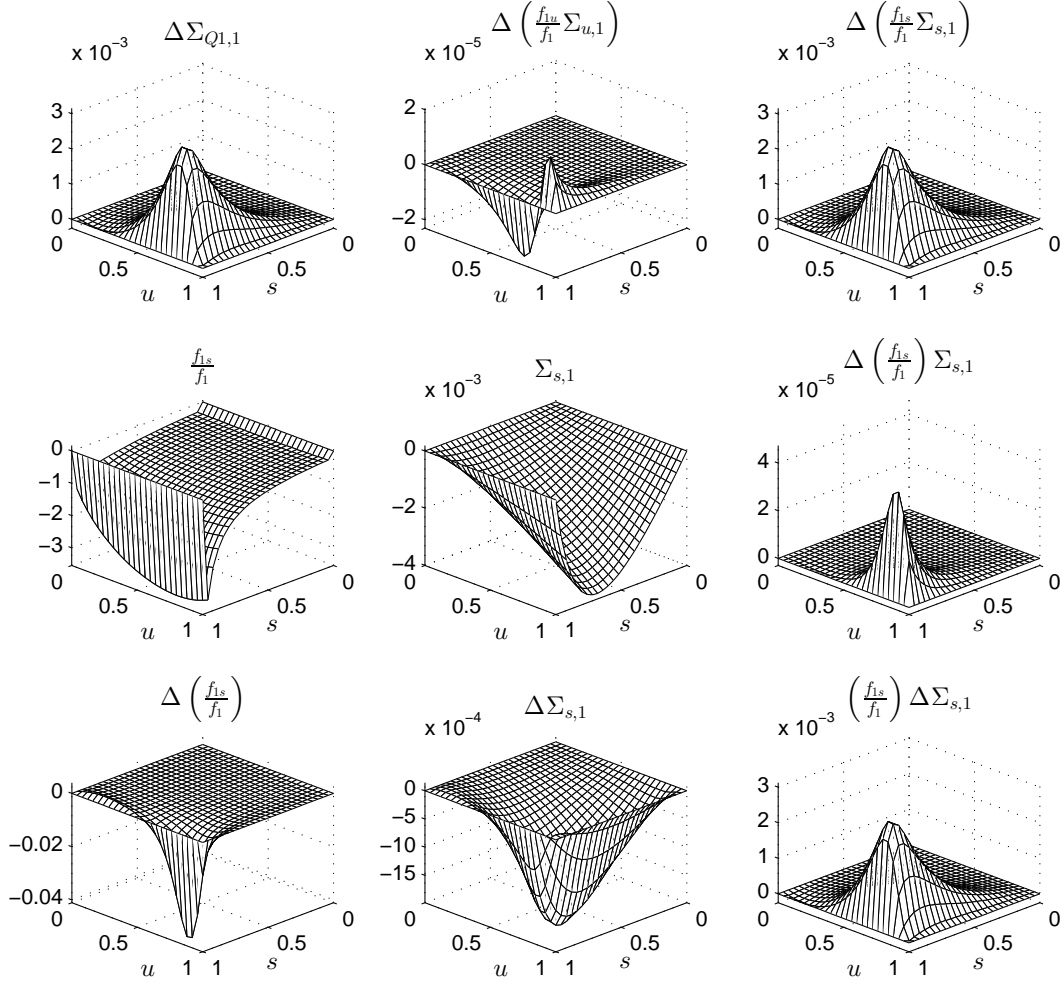


Figure 6. The effect of indexing on $\Sigma_{Q1,1}$ in the model with heterogeneous trees. This figure presents the impact of indexing on the first component of the diffusion $\Sigma_{Q1} = \Sigma_{D1} + (f_{1s}/f_1)\Sigma_s + (f_{1u}/f_1)\Sigma_u$. All variables are functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D . The model parameters are as follows: $\mu_{D1} = 0.01$, $\mu_{D2} = 0.03$, $\Sigma_{D1} = [0.01 \ 0]$, $\Sigma_{D2} = [0 \ 0.08]$, $\beta = 0.03$, $\gamma_I = 5$, $\gamma_P = 1$.

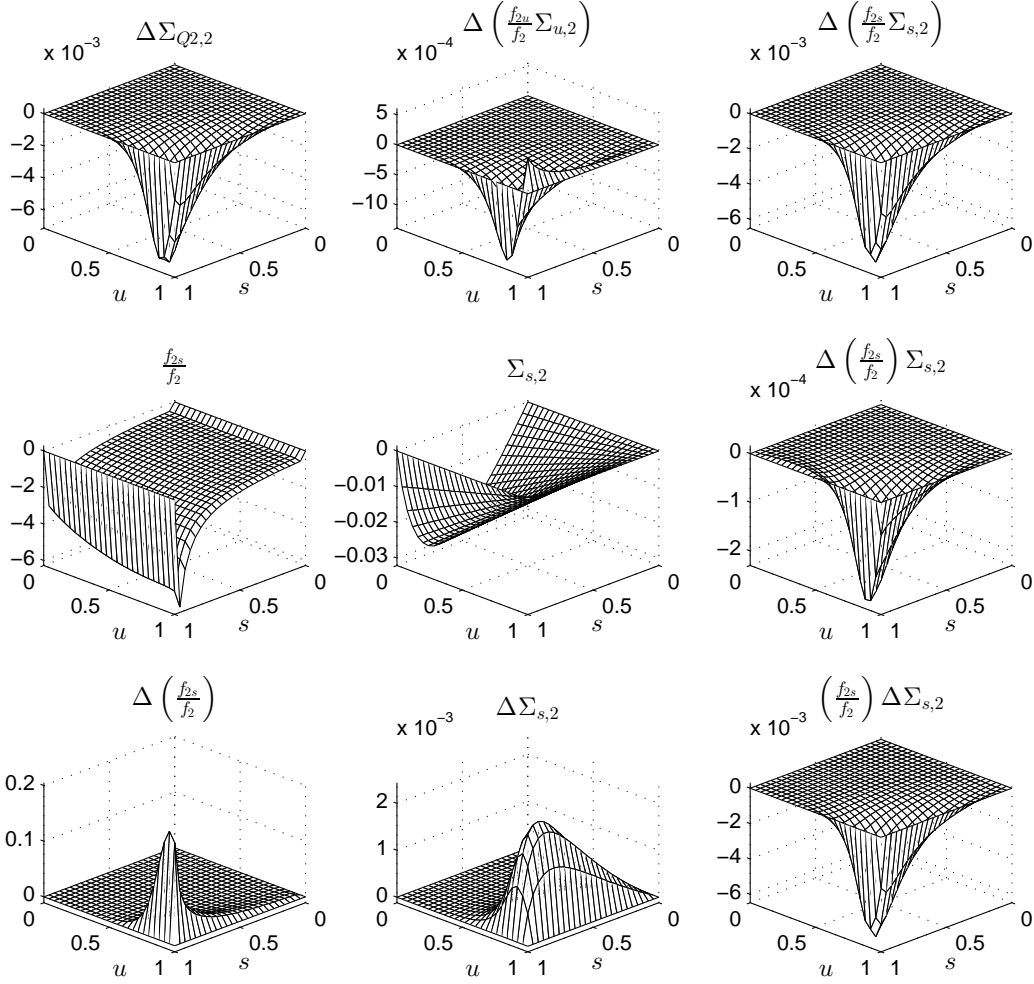


Figure 7. The effect of indexing on $\Sigma_{Q2,2}$ in the model with heterogeneous trees. This figure presents the impact of indexing on the second component of the diffusion $\Sigma_{Q2} = \Sigma_{D2} + (f_{2s}/f_2)\Sigma_s + (f_{2u}/f_2)\Sigma_u$. All variables are functions of the consumption share s of the type I investors and the share u of the first dividend D_1 in the aggregate dividend D . The model parameters are as follows: $\mu_{D1} = 0.01$, $\mu_{D2} = 0.03$, $\Sigma_{D1} = [0.01 \ 0]$, $\Sigma_{D2} = [0 \ 0.08]$, $\beta = 0.03$, $\gamma_I = 5$, $\gamma_P = 1$.