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Teaching resource

Canadian Provinces Voting Power under the 1971 Victoria Charter: Presentation of Detailed Calculations

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I. Introduction

In 1971 a conference was held in Victoria, the capital of British Columbia, between the federal government and the governments of the ten Canadian provinces. This conference produced what became known as the *Victoria Charter* which sought, *inter alia*, to establish a formula for amending the Canadian constitution without requiring the unanimous consent of the legislatures of all the ten Canadian provinces.

According to this charter the federal government, as well as any province having or having ever had 25 percent of the Canadian population, would be able to veto any proposed constitutional amendment – thereby awarding veto powers also to the provinces of Quebec and Ontario. In order for a proposed amendment to pass it would have to be ratified not only by the federal government and by the legislatures of Quebec (Q) and Ontario (O), but also by the legislatures of at least four additional provinces – two of which must belong to the Atlantic region, and the remaining two must belong to the West region and contain at least 50% of the population of the Western provinces.[1]

The Victoria Charter failed to pass because although it was ratified by all other Canadian provinces, the premier of Quebec at that time, Robert Bourassa, stalled and rejected it. Eleven more years elapsed until in 1982 the *Canada Act* passed specifying the conditions for amending the Canadian constitution – according to which no province has veto power. Nevertheless, the proposed rule for amending the Canadian constitution according to the 1971 Victoria Charter is of special interest to voting-power scholars for several reasons:

- It is the most well-known, perhaps so far the only, real-life decision rule demonstrating lack of co-monotonicity between the Shapley–Shubik (1954) and Banzhaf (1965) indices of a priori voting power – thereby proving that these indices cannot be considered as measuring the same thing. It is also a real-life example (which has not been used so far in the literature) for demonstrating the violation of the *added blocker postulate* (ABP) by the Shapley-Shubik index – thereby casting serious doubt on the reasonableness of this index. (ABP is discussed below in section III).

- The voting-power exact calculations associated with the Victoria Charter must be performed manually because, so far, there exists no online automatic program that can perform these calculations. Since, as far as I know, only the *final results* of these calculations have appeared in the literature,[2] and as these calculations are rather

[1] There are four provinces belonging to the Atlantic (A) region (New Brunswick, Prince Edward Island, Nova Scotia, Newfoundland), and four provinces belonging to the Western region (Manitoba, Saskatchewan, Alberta, British Columbia). The population of British Columbia and that of any one of the other three Western provinces (which are called in the sequel the Prairie [P] provinces) constituted at least 50% of the populations of all the four Western provinces; similarly, the populations of all the three prairie provinces also constituted at least 50% of the populations of all the four Western provinces. Thus although British Columbia was not awarded veto power, it was awarded, according to both the Banzhaf (Bz) and the Shapley–Shubik (S-S) indices, voting power which was equal to the sum of voting powers of all the other three prairie provinces.

[2] D.R. Miller (1973) published the voting-power results of the Canadian provinces under the Victoria Charter only according to the Shapley–Shubik index, while Straffin (1977) published these results also according to the Banzhaf index – and thus was able to point out that these two indices were not co-monotonic.

elaborate, it seems to me useful to outline them in detail – at least as a supplement to voting-power teaching modules.

II. The Detailed Calculations

The decision rule for amending the Canadian constitution according to the 1971 Victoria Charter can be represented either as a single unweighted voting game,[3] or as the meet of two weighted voting games,[4] but not as a single weighted voting game. In order to compare the relative regional voting powers according to the Victoria Charter, I shall first ignore the veto power of the Canadian federal government and consider only the votes of the ten Canadian provinces. Table 1 lists the 12 types of possible vulnerable coalitions (i.e., winning coalitions containing at least one critical member). This table lists, for each type of winning coalition, the number of ways it can be formed and the number of times each of the various provinces is critical.

Thus, for example, winning coalitions of Type #1 consist of six provinces: British Columbia (BC), one prairie (1P) province (i.e., Alberta, Saskatchewan, or Manitoba), Ontario (O), Quebec (Q), and two of the four Atlantic (2A) provinces (New Brunswick, Nova Scotia, Prince Edward Island, Newfoundland). Because there are six possibilities for choosing two out of the four Atlantic provinces and three possibilities for choosing one out of the three prairie provinces, there are altogether 18 (= 6 × 3) possible winning coalitions of Type #1. BC, O, and Q are critical in all of them, each of the prairie provinces is critical in six of them, and each of the Atlantic provinces is critical in nine of them.

Similarly, winning coalitions of Type #2 consist of seven provinces: British Columbia (BC), two prairie provinces (2P), Ontario (O), Quebec (Q), and two Atlantic provinces (2A). Because there are six possibilities for choosing two out of the four Atlantic provinces and three possibilities for choosing two out of the three prairie provinces, there are altogether 18 (= 6 × 3) possible winning coalitions of Type #2. BC, O, and Q are critical in all of them, none of the prairie provinces is critical in any of them, and each of the Atlantic provinces is critical in nine of them.

The calculations pertaining to winning coalitions of Types #3-12 are conducted similarly.

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[3] The decision rule according to the Victoria Charter cannot be represented as a single weighted voting game because it is not *trade robust* (cf. Taylor and Zwicker, 1992; 1999, ch. 2). A simple voting game is not trade robust if it is possible to list two or more winning coalitions and then move players (voters) from one winning coalition to another until all of them become losing coalitions. Thus, for example, according to the Victoria Charter decision rule one winning coalition is U = {Ontario, Quebec, New Brunswick, Nova Scotia, Alberta, British Columbia} and another winning coalition is V = {Ontario, Quebec, Prince Edward Island, Newfoundland, Alberta, Saskatchewan, Manitoba}. However, if one moves Prince Edward Island and Newfoundland from coalition V to U and British Columbia from coalition U to V, then one obtains two new coalitions, U' and V', respectively, both of which are losing..

[4] If we assign weight 5 to each of Quebec and Ontario, weight 2 to British Columbia, and weight 1 to each of the remaining seven provinces, the Victoria Charter decision rule can be represented as the meet of the following two 5-player weighted (and proper) voting games: [7; 5,1,1,1,1] × [8; 5,2,1,1,1]. I thank Moshé Machover for this idea. Straffin (1988, p. 73) gives the meet of the following two 4-player weighted games [2; 1,1,1,1] × [3; 2,1,1,1] as an example where the Shapley–Shubik and Banzhaf indices are not co-monotonic. Except for the absence of the two veto-wielding provinces, this example is similar to the Victoria Charter example. However, note that the first weighted game in Straffin's example is improper.

TABLE 1: Possible Winning Coalitions According to the Victoria Charter

(1) Type	(2) Possible Winning Coalition	(3) No. of Provinces	(4) No. of Ways Can Be Formed	(5) No. Times BC is Critical	(6) No. Times each P is Critical	(7) No. Times O and Q are Critical	(8) No. Times each A is Critical
1	BC + 1P + O + Q + 2A	6	18	18	6	18	9
2	BC + 2P + O + Q + 2A	7	18	18	–	18	9
3	3P + O + Q + 2A	7	6	–	6	6	3
4	BC + 1P + O + Q + 3A	7	12	12	4	12	–
5	BC + 3P + O + Q + 2A	8	6	–	–	6	3
6	BC + 2P + O + Q + 3A	8	12	12	–	12	–
7	3P + O + Q + 3A	8	4	–	4	4	–
8	BC + 1P + O + Q + 4A	8	3	3	1	3	–
9	BC + 3P + O + Q + 3A	9	4	–	–	4	–
10	BC + 2P + O + Q + 4A	9	3	3	–	3	–
11	3P + O + Q + 4A	9	1	–	1	1	–
12	BC + 3P + O + Q + 4A	10	1	–	–	1	–
Total			88	66	22	88	24

Legend:
BC = British Columbia; P = prairie province, one of Alberta, Saskatchewan, or Manitoba; O = Ontario; Q = Quebec; A = Atlantic province, one of New Brunswick, Nova Scotia, Prince Edward Island, or Newfoundland.
Source: Columns (1)-(4) in this table are taken from D.R. Miller (1973, Table I, p. 141).

The relative Banzhaf (Bz) index of every province according to the Victoria Charter is equal to the number of times the province is critical divided by the sum of critical occasions of all the 10 provinces.[5] As can be seen from Table 1, this sum is equal to 404 = [66 + (22×3) + (88×2) + (24×4)]. Hence the Bz index of British Columbia is 66/404 = 0.1634, the Bz index of each of the three prairie provinces is 22/404 = 0.0545, the Bz index of Ontario and of Quebec is 88/404 = 0.2178, and the Bz index of each of the four Atlantic provinces is 24/404 = 0.0594.

To calculate the Shapley-Shubik (S-S) index of every province one proceeds as follows.[6] For every province one looks at each type of coalition in which the province is critical and then compute the following product: $C \times (M-1)! \times (10 - M)!$ where C is the number of times the province is critical and M is the number of

[5] This index appears in the 1965 article by John F. Banzhaf. The Bz index values of the 10 Canadian provinces according to the Victoria Charter calculated below agree with those reported by Straffin (1977, p. 110) and later by Kilgour and Levesque (1984, p. 468).

[6] This index appears in the 1954 article by Lloyd S. Shapley and Martin Shubik. The S-S index values of the 10 Canadian provinces according to the Victoria Charter calculated below agree with those reported by D.R. Miller (1973, p. 142) and later by Straffin (1977, p. 110).

coalition members in the given type. Thereafter one sums the results of these products over all types in which the province is critical and divide the outcome by 10!.

Thus the S-S index of BC is: [18(5!4!) + 18(6!3!) + 12(6!3!) + 12(7!2!) + 3(7!2!) + 3(8!1!)] / 10! = 453,600 / 10! = 0.125.

The S-S index of each of the three prairie provinces is: [6(5!4!) + 6(6!3!) + 4(6!3!) + 4(7!2!) + (7!2!) + (8!1!)] / 10! = 151,200 / 10! = 0.0417.

The S-S index of Ontario and of Quebec is: [18(5!4!) + 18(6!3!) + 6(6!3!) + 12((6!3!) + 6(7!2!) + 12(7!2!) + 4(7!2!) + 3(7!2!) +4(8!1!) +3(8!1!) +(8!1!) +(9!0!)] /10! = 1,144,800 / 10! = 0.3155.

And the S-S index of each of the four Atlantic provinces is: [9(5!4!) + 9(6!3!) + 3(6!3!) + 3(7!2!)] / 10! = 108,000 / 10! = 0.0298.

Note that according to the Bz index the a priori (relative) voting power of each of the Atlantic provinces is (slightly) larger than that of each of the prairie provinces (0.0594 > 0.0545), whereas according to the S-S index the a priori (relative) voting power of each prairie province is (considerably) larger than that of each of the Atlantic provinces (0.0417 > 0.0298) – thus demonstrating that in unweighted simple voting games the Bz and S-S indices are not necessarily co-monotonic.[7]

III. Violation of the Added Blocker Postulate (ABP) by the S-S Index

Now let us examine what would be the *ratios* between the (relative) voting powers of the various provinces if one takes into consideration the fact that the Canadian federal government (F), too, was awarded the right to veto any proposed constitutional amendment according to the Victoria Charter. In this case one must add F to each of the 12 possible types of winning coalitions listed in column (2) of Table 1, increase by 1 each of the entries in column (3) and add F also to the heading of column (7) in this table.

As a result of this change the denominator of the Bz index (i.e., the total number of critical occasions over all provinces) would increase by 88 (from 404 to 492), but as the number of times each province is critical remains the same, the Bz index of all provinces would decrease but the *ratios* between the Bz indices of the various provinces would remain unchanged.

However, ratios between the S-S indices of the various provinces would change as a result of adding the federal government as a veto player (or blocker). This is so because, for each type of winning coalition, the total number of coalition members (M) increases by 1, and the total number of players in this voting game increases from 10 to 11. So the ratios between the new factorial products changes and one obtains that:

The S-S index of BC is: [18(6!4!) + 18(7!3!) + 12(7!3!) + 12(8!2!) + 3(8!2!) + 3(9!1!)] / 11! = 3,516,480 / 11! = 0.0881.

The S-S index of each of the three prairie provinces is: [6(6!4!) + 6(7!3!) + 4(7!3!) +

[7] The Bz and S-S indices are always co-monotonic in weighted simple voting games.

4(8!2!) + (8!2!) + (9!1!)] / 11! = 1,172,160 / 11! = 0.0294

The S-S index of Ontario, Quebec and the Canadian federal government (F) is: [18(6!4!) + 18(7!3!) + 6(7!3!) + 12((7!3!) + 6(8!2!) + 12(8!2!) + 4(8!2!) + 3(8!2!) +4(9!1!) +3(9!1!) +(9!1!) +(10!0!)] /11! = 9,947,520 / 11! = 0.2492.

And the S-S index of each of the four Atlantic provinces is: [9(6!4!) + 9(7!3!) + 3(7!3!) + 3(8!2!)] / 11! = 760,320 / 11! = 0.0190.

So, for example, we obtain that according to the S-S index the voting power of each prairie province is 1.3993 (= 0.0417 / 0.0298) larger than that of each Atlantic province when one ignores the Canadian federal government as a veto player, but the voting power of each prairie province becomes 1.5474 (= 0.0294 / 0.0190) larger than that of every Atlantic province when, *ceteris paribus*, one adds the Canadian federal government as a veto player. Felsenthal and Machover (1998, pp. 266-267) viewed this phenomenon as a serious violation by the S-S index of a compelling postulate they called the *added blocker postulate* (ABP),[8] which does not only lead to different conclusions than those arrived at according to the Bz index due to their lack of co-monotonicity, but also casts serious doubts as to the reasonableness of the S-S index as a voting-power measure.

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[8] According to this postulate the distribution of voting powers among existing players should not change as a result of adding a new veto player. As we have explained, the Bz index satisfies this postulate but the S-S index almost always violates it in almost any simple voting game.