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Preferences over Equality in the Presence of Costly Income Sorting†

By Gilat Levy and Ronny Razin*

We analyze preferences over redistribution in societies with costly (positive) sorting according to income. We identify a new motivation for redistribution, where individuals support taxation in order to reduce the incentives to sort. We characterize a simple condition over income distributions which implies that even relatively rich voters—with income above the mean—will prefer full equality (and thus no sorting) to societies with costly sorting. We show that the condition is satisfied for relatively equal income distributions. We also relate the condition to several statistical properties which are satisfied by a large family of distribution functions. (*JEL* D31, D63, H23)

The presence of income sorting or stratification in society has received plenty of attention in the economics and sociology literature.[1] Relocating to a leafy suburb, sending your child to a private school, or engaging in conspicuous consumption of a sports car, jewelry, or designer clothes, have all been mentioned as ways in which people try to guarantee that they mix, interact, or match with those with the same or higher income than theirs.[2]

When individuals participate in such costly sorting, what are their preferences over redistribution? Beyond being a traditional tool for creating equality, income redistribution will potentially decrease the incentive to sort as it might decrease the benefit of mixing with other rich individuals. In this paper we explore how costly income sorting shapes individual and political preferences over redistribution.

To analyze this question, we introduce a simple model in which individuals differ in their income. We assume that the utility of an individual exhibits complementarities in his disposable income and that of those he interacts with. We consider incentive compatible partitions of society into "clubs," where all individuals in the same

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[1] See for example Bénabou (1996), Fernández and Rogerson (2001), Kremer (1997), and Wilson (1987).

[2] The literature on conspicuous consumption includes contributions by Liebenstein (1950), Bagwell and Bernheim (1996), Pesendorfer (1995), and Heffetz (2011). Glazer and Konrad (1996) consider signaling of wealth via charitable donation, which exhibits positive externalities. Moav and Neeman (2012) analyze the trade-off between conspicuous consumption and human capital as signals for unobserved income.

club pay the same costly signal and interact only with each other. This framework can be seen as a reduced form of several economic environments:

Example 1: The education market: The literature on sorting in children's education (see, for example, Epple and Romano 1998 and Fernández and Rogerson 2003) typically assumes a single crossing condition, i.e., that richer individuals care more about the education of their child. If there are peer effects, i.e., complementarities in the ability of pupils, or if education is financed locally with school quality determined by a majority vote in the community, then agents will sort into schools or neighborhoods according to income. Our model can be viewed as a reduced form of these models.[3] The costly signals can then be entry fees to private schools or land and house prices in a wealthy suburb (where children would attend the state school);[4] in both cases these costs imply that the child mixes with children of relatively rich individuals.

Example 2: The marriage market: Another example explored in the literature is that of the marriage market. Pesendorfer (1995) describes a "dating" market where individuals of different types, be it their education, entertainment skills, or human capital, are matched with one another. The utility from matching is supermodular, which induces high types to distinguish themselves by acquiring the newest fashion design. As typically human capital and education attainment are correlated with income, our model is a reduced form for this matching environment as well; the different signals would be the different fashion labels that would allow individuals to identify one another.

In environments such as the ones described above, would individuals prefer to live in an equal society—which will reduce the incentive to sort—or in an unequal society where one can mix with the rich but has to pay a cost for doing so? We focus on the income distribution as the main parameter determining such preferences. One intuition would be that income distributions characterized by high income inequality might push the middle classes to advocate more redistribution as it will soften the pressures to engage in costly signaling. On the other hand, another intuition is that it might induce the middle classes to be more concerned about the incomes of the wealthier groups they wish to mingle with, and therefore not support redistribution.[5]

In our main result we show that it is the latter intuition which holds. In particular, we identify a necessary and sufficient condition (Condition 1) over income distributions which implies that all individuals up to the mean (and possibly some above) prefer full equality to *any* incentive compatible partition of society and *any* linear tax level. We show that Condition 1 is satisfied for relatively equal societies. If a society is sufficiently unequal on the other hand, the condition will be violated and

[3] Within this literature several papers also consider the effect of redistributive policies. Fernández and Rogerson (2003) consider provision of quality of schooling and analyze different equalizing policies which target the finance of education. Epple and Romano (1998) model the supply side, i.e., the market for private schools, and show how more wealthy and able agents are screened into better quality schools.

[4] See Bradford and Kelejian (1973) who show empirically that the decision of the middle classes to live in the suburbs depend (negatively) on the share of the poor in the city.

[5] Naturally, for individuals with income below the mean, there is also the standard motivation to support redistribution to simply increase their own income.

there will be incentive compatible partitions of society for which some individuals below the mean would oppose redistribution. High income inequality implies that the middle class can, by sorting, avoid a large mass of very poor individuals, while keeping the cost of sorting relatively low.[6] In other words, the cost and benefits of sorting generate endogenous preferences over the income distribution; in distributions with high income inequality, individuals will be against redistribution, while sufficient income equality will imply that individuals will prefer to live in a fully equal society.

We show that familiar distribution functions satisfy Condition 1. For example, sufficiently equal Pareto and log-normal distributions satisfy Condition 1. Moreover, Condition 1 is satisfied by all functions which are *new better than used in expectations* (NBUE),[7] where NBUE is satisfied by functions with increasing hazard rate (and thus all log-concave density functions). Such functions for example are the uniform, exponential, normal, and sufficiently equal Gamma and Weibull functions.

We also consider the efficiency properties of sorting vis-à-vis full equality. We show that full redistribution is efficient (in a utilitarian sense) compared to any partition into clubs if and only if the distribution function is NBUE. This implies that whenever full redistribution is efficient, it is also supported by a large coalition. However, it may be supported by such a coalition even if it is not efficient, i.e., when Condition 1 is satisfied but the income distribution is not NBUE. In particular, for many plausible income distributions like the Pareto and lognormal distributions, exclusive sorting is efficient but garners very little political support.

In the classical work of Meltzer and Richards (1981), an individual favors taxation if (and only if) her income is below the mean income. While the empirical literature supports a positive relation between income and preferences over taxation, a puzzling observation is that many voters with income below the mean vote for parties on the right who traditionally oppose further taxation.[8] The opposite happens as well; De La O and Rodden (2008) use the Eurobarometers and World Values Survey data to show that on average well over 40 percent of the wealthiest individuals in Europe vote for parties of the left. Moreover, some evidence also indicates that voters in more equal societies are more positive towards further taxation and transfers, while voters in relatively unequal societies have less positive attitudes towards taxation.[9]

Our paper ties together these two empirical observations, as in the model it is in sufficiently equal (unequal) societies where one might find rich (poor) agents voting to the left (right). Our analysis contributes then to the political economy literature explaining one or both of the above empirical observations, by identifying

[6] Indeed India is one example of a society with a large fraction of the very poor, coupled with low income tax rates and a large degree of income sorting, as manifested for example in the marriage market. See Banerjee et al. (2013) who measure the effects of castes (often correlated with income) as well as costly signals (such as education) on the marriage market.

[7] In reliability theory and specifically the analysis of life distributions (Barlow and Proschan 1966), NBUE describes the stochastic life span of a device, which is less reliable with time.

[8] See for example Frank (2004). Gelman et al. (2007) show that the positive relation between income and voting right is strong in poor American states and weak in rich states.

[9] See Perotti (1996) and Kerr (2013).

an explanation that is based on the effects that redistribution has on the patterns of costly sorting in societies.[10]

Within this literature, the most related papers are by Bénabou (2000); Corneo (2002); and Corneo and Grüner (2002). In Corneo and Grüner (2002), individuals' consumption levels signal their wealth, and therefore redistribution reduces the information value of signaling. Our analysis complements this paper by focusing on equilibria in the sorting market.

In a model where redistribution has potential efficiency gains, Bénabou (2000) identifies a U-shaped relation between income inequality and political support for redistribution. His paper highlights a strong relation between political support for redistribution and efficiency. In his model redistribution has value by providing insurance or a safeguard against credit constraints. When the income distribution is sufficiently equal, there is little conflict about redistribution per se and the efficiency gains make redistribution more politically successful.[11] As in Bénabou (2000), in our model redistribution might have efficiency gains/costs in the form of its effect on the sorting market. Our paper complements Bénabou (2000), as in our model the gains/costs from sorting are distributed unevenly in the population, which implies that the connection between efficiency and support for redistribution is not as clear cut.

Corneo (2002) assumes that individuals care about their rank in society. In his model progressive taxation prevents the negative externalities that arise when agents bias their labor supply to increase their rank. These biases are strongest in an equal society and as a result a progressive tax is efficient when inequality is low.

Other papers explain the empirical observations on the misalignment of class and preferences for redistribution and how it is related to income inequality. Piketty (1995) and Bénabou and Tirole (2006) show how different beliefs, i.e., whether success is a function of luck or effort, could induce multiple equilibria, one with a large welfare state and low effort and one with a small government and high effort.[12] Benabou and Ok (2001) show how a future redistribution of a concave (convex) function of the current income distribution will induce those below (above) the mean to vote against (in favor of) redistribution. Galor and Zeira (1993) show how credit constraints and education externalities imply that middle income voters prefer a more equal society, as this will allow the poor to gain higher education levels.[13]

Our model is also related to a recent literature on the cost of signaling. Hoppe, Moldovanu, and Sela (2009) consider a model in which individuals signal their attributes. Their model is an incomplete information model with two-sided heterogeneity, finite types and perfect signaling, which implies that the condition they find for efficiency of signaling is stronger than ours.[14] Several other papers focus on coarse

[10] For a good summary of this literature, see Alesina and Giuliano (2011).

[11] This explains the decreasing part of the U-shaped relation between income inequality and political support for redistribution. See also Corneo (2002) for a similar relation.

[12] See also Alesina and Angeletos (2005).

[13] Other related papers explain these phenomena by assuming that agents have preferences over a multidimensional policy or identity space (Roemer 1998, Levy 2004, Alesina and La Ferrara 2005, and Shayo 2009).

[14] Specifically, they show that signaling is more (less) efficient compared with random matching (which in our model, in utilitarian terms, is equivalent to full equality) if the distribution over types satisfies decreasing (increasing) failure rate.

matching, for example Rege (2008); Hoppe, Moldovanu, and Ozdenoren (2011); and McAfee (2002), and show the conditions under which coarse matching provides sufficiently high surplus compared with random or perfect matching.

Tournaments have been analyzed as another form of sorting; Fernández and Galí (1997) show that with credit constraints, markets perform less well than tournaments at sorting individuals according to ability. Hopkins and Kornienko (2010) explore, in the context of a tournament, the effect of equality in the distribution of rewards vis-à-vis an equality in the distribution of income. They show that the latter induces effort whereas the former hampers it.

Our model can also shed light on recent empirical findings on inequality and happiness, which show that happiness can decrease even when everyone's income had increased, if inequality increases as well.[15] In the standard approach (e.g., Meltzer and Richards 1981) when utility is proportional to disposable income this cannot arise. One explanation that has been put forward in this literature is that agents have direct preferences over income inequality. In our model such preferences arise endogenously, and indeed, it is easy to find examples in our model in which the income of all individuals increases along with inequality, while the utility of a sizable fraction of the population decreases as a result of the changes in the cost and benefit of sorting. Consider for example a society with sufficient income equality, which initially has no sorting. As income increases for all, together with income inequality, sorting may then arise. The poorest individuals who will then be excluded, for example from private schools, will then be worse off (as long as income had not increased too much).

The remainder of this paper is organized as follows. Section I presents the model. In Section II we derive our main result in a simple environment in which we compare full redistribution to a society with at most two clubs and no taxation. We generalize these results to any incentive compatible partition and any linear tax in Section III, where we also discuss more general utility functions. In Section IV, we discuss the implications of our results to income distribution functions that have been used to fit the data. Section V concludes.

I. The Model

The population is composed of agents who differ in their income, x, which is distributed according to some distribution $F(x)$ and density $f(x)$, strictly positive on some $[0, \nu]$, $0 < \nu \leq \infty$. Let $\mu(m)$ denote the mean (median) of the distribution, with $m \leq \mu$.

We assume that when an individual with disposable income x interacts with an individual with disposable income y, as in the marriage market, or belongs to a club in which the average income is y, as in the case of peer or network effects in education, he receives a utility xy. The assumption of supermodularity is important as it creates the incentive to (positively) sort. Our results could be adjusted to other supermodular functions as we discuss in Section III.

[15] A recent example is Oishi, Kesebir, and Diener (2011). See also Alesina, Di Tella, and MacCulloch (2004).

We incorporate in this environment a set of costly signals (such as private schools with different fees) that will enable sorting. Thus, when some individuals use a costly signal they will interact randomly with, and only with, other individuals who use the same signal. When an agent with income x_i uses a signal that costs b, his utility will therefore be

$$x_i E[x_j | j \in X_b] - b, \tag{1}$$

where X_b is the set of other agents who use the same signal. The quasilinear nature of the utility function is simple to use but is not necessary for our results; our main result can be extended to the case in which the utility of an agent with income x_i who mixes in the same "club" with the population whose average income is x_j is $(x_i - b)(E(x_j) - b)$ instead.[16] Also, in some applications, the signal might provide an intrinsic utility on top of the sorting value, e.g., private schools might provide, aside from peer effects, better education. With some monotonicity condition, this can be accommodated in the model.

By single crossing, if some agent with x_i prefers to use a signal with cost $b > b'$, all agents with $x > x_i$ will prefer b over b'. We will therefore focus on monotone sorting, i.e., with connected intervals. We will abstract away from the supply side, i.e., how the signals or their costs are being determined.[17] But when agents choose optimally which signal to use, no matter how the supply side arises, the costs of the signals have to satisfy some incentive compatibility constraints:

DEFINITION 1: *An incentive compatible partition is a vector* $\mathbf{x} = (x_0, x_1, \ldots x_{n-1}, x_n)$ *with* $x_0 = 0, x_n = \nu$ *and* $x_i < x_{i+1}$, *such that all agents with type* $x \in [x_i, x_{i+1})$ *for* $i = 0, 1, \ldots, n-1$ *pay* b_i *and interact with agents in* $[x_i, x_{i+1})$ *only*,[18] *with* $b_0 = 0$ *and*

$$b_i - b_{i-1} = x_i(E[x_j | x_j \in [x_i, x_{i+1}]] - E[x_j | x_j \in [x_{i-1}, x_i]]) \geq 0. \tag{2}$$

In such an incentive compatible partition, the prices are such that for all i, the agent in x_i is indifferent between joining the club below her and the club above her. By single crossing, all other agents act optimally by joining the club they are assigned to in the partition. For simplicity and without loss of generality, we are restricting the price of joining the lowest club in the partition to zero. Henceforth, when we say a partition or a sorting environment, we mean an incentive compatible partition. For expositional purposes, we will present in Section II all the main results for the case of sorting with at most two clubs, i.e., where the incentive-compatibility partition is $\mathbf{x} = (0, \hat{x}, \nu)$. These results generalize to any incentive compatible partition as we show in Section III.

Our key assumption is that what matters for the utility from matching is (at least to some degree) the absolute, disposable, income. This will imply that when income

[16] Specifically, Condition 1 in Proposition 1 would be a sufficient condition for this utility function.
[17] For such analysis see Damiano and Li (2007) and Rayo (2013).
[18] For completeness, when $i = n - 1$, the last interval is closed from above as well, i.e., $[x_{n-1}, x_n]$.

inequality is reduced, so are the incentives to sort or the willingness to pay for sorting. In particular, with full redistribution, the income of all is the same, at μ, and sorting cannot arise.[19] Note that the utility from matching in such an equal society would be μ^2.

Our main analysis focuses on deriving a simple condition such that if satisfied, all agents up to the mean (and possibly some above) prefer full redistribution (henceforth FR) to *any* incentive compatible partition with sorting. Thus, our approach is to find conditions that will apply to all partitions, rather than focusing on a particular one. This allows us to pursue results in environments in which there are typically multiple equilibria, or in environments in which we, the modelers, do not have a precise grasp of the supply side of the sorting market.[20]

While we abstract away from a specific political model, we will show that preferences over FR are characterized by a cutoff and all voters with income up to that cutoff will support redistribution. The larger is the coalition supporting FR, the more likely the FR is to be politically implemented. Thus, the preferences we characterize—where a coalition of more than 50 percent of all agents up to at least the mean support FR—can be manifested as the political outcome of many political models. For example, it would arise in a two-candidate competition, any political model that supports the median voter results or some supermajority rules, as well as some environments which allow for lobbying or noise voters.[21]

II. Preferences over Redistribution

In this section, for expositional purposes, we focus on a comparison between a society with FR and a society with a simple incentive compatible partition of the form $\mathbf{x} = (0, \hat{x}, \nu)$, where $\hat{x} \in [0, \nu]$. In the next section we will generalize all the results below to any incentive compatible partition and also allow for linear taxes.

In Section IIA we will characterize a simple, necessary, and sufficient condition on the distribution function, which will imply that a coalition of all agents up to the mean (and some above) will support FR over any society with sorting. As the mean and those above him do not enjoy redistribution per se, this will allows us to identify a new motivation for redistribution which arises due to sorting only. We also show that this condition is more likely to be satisfied when $F(x)$ is more equal, and characterize a set of income distributions that satisfies it. In Section IIB we will derive the conditions under which FR is efficient in a utilitarian sense compared with all sorting environments, and in Section IIC we discuss the relation between efficiency and political support.

Preliminaries.—Note that the utility from FR is μ^2 for all agents; the income of each agent will be μ and thus the utility from a match is μ^2. We now construct the

[19] Formally, under full resdistribution, in any incentive compatible partition it has to be that $b_i = 0$ for all i.

[20] A different approach is taken by Moav and Neeman (2012) who focus on a refinement of equilibria.

[21] Note that we do not consider the preferences of the firms or organizations that provide signals; to maintain the political model one can assume that they compose a negligible part of the population.

utility from $\mathbf{x} = (0, \hat{x}, \nu)$ or in short the cutoff $\hat{x}$. Suppose that all agents above $\hat{x}$ pay $b(\hat{x})$ and all below pay nothing. The type at the cutoff $\hat{x}$ will be indifferent between paying the cost of sorting and gaining $\hat{x}E[x_j | x_j > \hat{x}]$, versus not paying and gaining a utility of $\hat{x}E[x_j | x_j < \hat{x}]$, where

$$(3) \qquad \underline{E}_{\hat{x}} \equiv E[x | x \leq \hat{x}] = \frac{\int_0^{\hat{x}} xf(x)\, dx}{F(\hat{x})},$$

$$\overline{E}_{\hat{x}} \equiv E[x | x \geq \hat{x}] = \frac{\int_{\hat{x}}^{\upsilon} xf(x)\, dx}{1 - F(\hat{x})}.$$

In an incentive compatible environment, the price of the signal must satisfy:

$$(4) \qquad b(\hat{x}) = \hat{x}(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}}).$$

The expected utility of an individual $x < \hat{x}$ is therefore $x\underline{E}_{\hat{x}}$ and the expected utility of an individual $x > \hat{x}$ can be written as $x\overline{E}_{\hat{x}} - b(\hat{x}) = x\overline{E}_{\hat{x}} - \hat{x}(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}})$ or:

$$(5) \qquad (x - \hat{x})\overline{E}_{\hat{x}} + \hat{x}\underline{E}_{\hat{x}}.$$

Expected utility from using the signal can be interpreted as the utility of the cutoff type, plus an information rent component that depends on the distance from the cutoff. This utility is increasing and convex in the income x; the slope for $x < \hat{x}$ is $\underline{E}_{\hat{x}}$ and the slope for $x > \hat{x}$, is $\overline{E}_{\hat{x}}$. This implies:

LEMMA 1: *The utility from sorting with any $\hat{x}$ is increasing and convex in x; as the utility from FR is equal to all, then whenever a voter with income x' prefers FR, then all voters with $x < x'$ do so as well.*

Note that if $\hat{x} = 0$, then $b(\hat{x}) = 0$, which is equivalent to having no club at all so that the whole population matches randomly, each gaining a utility of $x\mu$. This implies that preferences over redistribution would be standard: all agents up to the mean would support redistribution, and all agents above the mean would be against it. However, when $\hat{x} > 0$, both the cost of the club and the benefit of the club increase. It is obvious that if $\mu < \hat{x}$ then the mean and in fact all those with $x < \hat{x}$, prefer FR to sorting, as then both their own and their match's income would be higher. It is therefore left to consider clubs in which $\mu > \hat{x}$, which we now consider.

A. *Sorting versus Equality*

Note that the mean prefers FR to any club $\hat{x} < \mu$ if and only if

$$(6) \qquad (\mu - \hat{x})\overline{E}_{\hat{x}} + \hat{x}\underline{E}_{\hat{x}} \leq \mu^2.$$

Divide by μ to get

$$\left(1 - \frac{\hat{x}}{\mu}\right)\overline{E}_{\hat{x}} + \frac{\hat{x}}{\mu}\underline{E}_{\hat{x}} \leq \mu. \tag{7}$$

As

$$\mu = (1 - F(\hat{x}))\overline{E}_{\hat{x}} + F(\hat{x})\underline{E}_{\hat{x}}, \tag{8}$$

then FR is preferred to coarse sorting for any cutoff $\hat{x}$ if and only if:

(Condition 1) $$x/\mu \geq F(x) \quad \text{for all} \quad x < \mu.$$

Recall that when $\hat{x} > \mu$, the mean trivially prefers FR. We then have:

PROPOSITION 1: *The mean (and all below) prefers FR to any cutoff $\hat{x}$ if and only if $F(x)$ satisfies $x/\mu \geq F(x)$ for all $x < \mu$.*

Note that when $x/\mu > F(\hat{x})$ for some $\hat{x} < \mu$, then the mean will strictly prefer FR to the environment with $\hat{x}$ and by continuity some agents with $x > \mu$ will do so as well. Thus, a coalition which is larger than all those up to the mean, including relatively rich agents, will support FR. On the other hand, when this condition fails, this implies that there exists a cutoff $\hat{x}$ for which the mean and some individuals poorer than the mean prefer sorting to FR.

The condition is simple and intuitive, as it encodes the benefit and costs of belonging to a club $[\hat{x}, \nu)$ for the mean relative to FR. The left hand side, $\frac{\hat{x}}{\mu}$, measures the relative cost of joining the club. The information rent of the mean is proportional to $(1 - \hat{x}/\mu)$; specifically, a high $\hat{x}/\mu$ implies a low information rent. The right hand side, $F(\hat{x})$, denotes the relative benefit of the club. A higher $F(\hat{x})$ implies a large fraction of the poor is excluded from the club. Belonging to this club allows the mean to stay away from a large constituency of poor individuals (and thus match with a high probability with the higher incomes). Condition 1 goes over all the possible club configuration which include the mean, and demands that he prefers full redistribution to these clubs.

To see this interplay between cost and benefits, consider first a value of $\hat{x}$, which is close to the mean. For a high $\hat{x}/\mu$, the mean is almost indifferent between joining the club or not as the information rent is minimal. In this case, all the rent is extracted from him and so the club is too costly. Indeed, when $\hat{x} = \mu$ we have $\hat{x}/\mu = 1 > F(\mu) = F(\hat{x})$ and so the condition is satisfied in a neighborhood of $\hat{x} = \mu$. Thus, for a sufficiently high $\hat{x}$, there are no benefits that can convince the mean to prefer the club to FR.

On the other hand, when $\hat{x}$ is small, the mean has a high information rent and therefore perceives the club as relatively less costly. This means that it might be hard to satisfy Condition 1. Specifically, if $F(\hat{x})$ is high for such a small $\hat{x}$, the benefit of the club is also high as then there are many, very poor, individuals who are excluded from the club. This arises when F is sufficiently concave, as we discuss below. Condition 1 insures that this will not happen by keeping $\hat{x}$ sufficiently larger than $F(\hat{x})$. In other words, insuring that the benefit from sorting relative to its cost cannot be too high.

In Section V we generalize the analysis and show that Condition 1 is necessary and sufficient also when considering any incentive compatible partition, as well as such a partition with an interior (linear) tax and redistribution scheme. Generalizing Condition 1 to allow for some taxation under sorting is trivial. Generalizing it to any partition with more than one signal does not follow immediately however. In particular, whenever $x/F(x)$ is increasing, adding more signals below some cutoff $\hat{x} < \mu$ reduces the signaling cost for all types $x > \hat{x}$ and thus improves the utility from sorting. Still, we are able to show that Condition 1 is necessary and sufficient for all partitions; the intuition is that Condition 1 insures that the mean prefers FR both to a partition $[0, x_1, v]$ and to a partition $[0, x_2, v]$, for $\mu > x_2 > x_1$, which together imply that the mean would also prefer it to a partition $[0, x_1, x_2, v]$.

We next discuss the relation of Condition 1 to inequality and then some familiar properties of distribution functions, which ensure that Condition 1 is satisfied.

Condition 1 and Inequality.—From the intuition above, we can see that Condition 1 can be violated when there is a large share of very poor agents (a large $F(x)$ for a small x), which is typically associated with a high level of inequality. A sufficiently concave function with a high $f(x)$ for small x will therefore violate Condition 1. However, a more equal income distribution, with sufficiently low $f(x)$ for small x, would render Condition 1 viable. To see this, consider Figure 1. The figure illustrates that when $F(x)$ is too concave, it will be above x/μ for small levels of x, whereas if this is not the case (specifically whenever $f(0) < 1/\mu$ or in other words when $F(x)$ is not too concave), then for all x, $F(x)$ lies below x/μ:

> The straight line coresponds to x/μ. If $F(x)$ is sufficiently equal, then it is completely below x/μ, whereas if it is too concave, or in other words there is a large share of the very poor, it is above x/μ for small values of x.

As another illustration, consider the almost fully equal distribution, with almost all weight on μ. In that case, for any club, the benefit from being in the club is associating with a type of average income close to μ (as in FR) while the cost is strictly positive as it is in the order of, for a cutoff $\hat{x}$, $\hat{x}(\mu - \underline{E}_{\hat{x}}) > 0$.

We now make this relation between Condition 1 and equality more precise. In particular, we show that whenever Condition 1 is satisfied by some $F(x)$, then it is also satisfied by $G(x)$, if G belongs to a set of mean-preserving contractions of F.

We say that G is a *monotone* mean-preserving contraction of F if to obtain G, for all values smaller than μ, weight always shifts upwards to higher values, still below μ (naturally some weight shifting must occur also above μ to preserve the mean and the second-order stochastic dominance of G, but we can be agnostic about their exact nature). Formally, G has to be a mean-preserving contraction of F satisfying: (i) $F(\mu) = G(\mu)$, (ii) for any interval $Y = [y_1, y_2] \subset [0, \mu]$ for which $\int_Y g(x)\,dx < \int_Y f(x)\,dx$, then there exists an interval $Y' = \left[y_1', y_2'\right] \subset [0, \mu]$ such

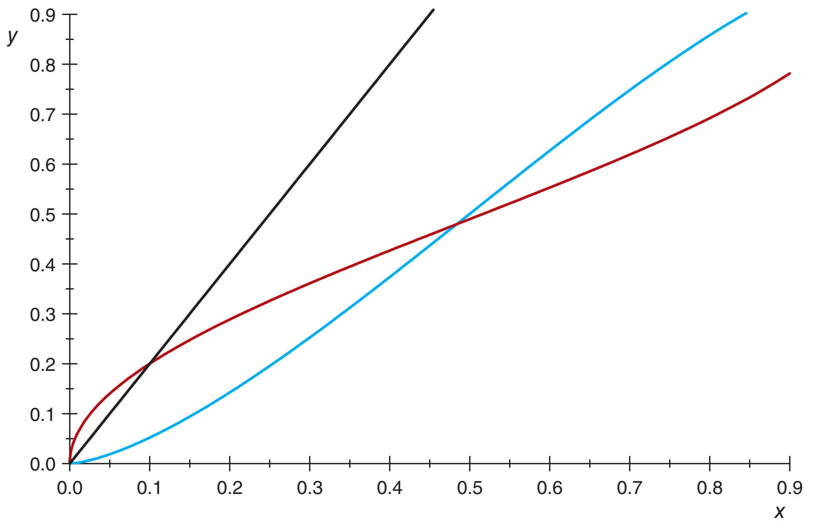

FIGURE 1

that $y_1' \geq y_1$ and $y_2' \geq y_2$ and $\int_{Y'} (g(x) - f(x))\, dx = \int_Y (f(x) - g(x))\, dx$.[22] We then have:

PROPOSITION 2:

(i) *Suppose that $F(x)$ satisfies Condition 1. Then all $G(x)$ obtained from $F(x)$ by some monotone mean-preserving contraction also satisfy Condition 1.*[23]

(ii) *Suppose that $F(x)$ does not satisfy Condition 1. Then there exists a monotone mean-preserving contraction of $F(x)$ that would satisfy Condition 1.*

PROOF:

(i) Note that $G(\mu) = F(\mu)$ and that $G(x) \leq F(x)$ for any $x < \mu$ by the definition of a monotone mean-preserving contraction. Thus, $G(x) \leq F(x) \leq x/\mu$ and $G(x)$ satisfies Condition 1.

(ii) One way to do so would be to shift (almost) all weight from $[0, \zeta\mu]$ to $[\zeta\mu, \mu]$ (and a corresponding change above μ), where $\zeta = F(\mu)$. Condition 1 is satisfied then for all $x > \zeta\mu$ (as then $x/\mu > 1 > F(x)$), as well as for $x < \zeta\mu$ for which $F(x) \rightarrow 0$. ■

Condition 1 is then more likely to be satisfied when $F(x)$ is more equal in the monotone mean-preserving contraction sense. Note that with more general utility functions, such as $h(x)g(y)$, a condition similar to Condition 1 can be constructed and a similar relation between the Condition and inequality can be derived, as in Proposition 2 (see Section III).

[22] It is easy to find such a mean-preserving contraction.

[23] It is possible to construct nonmonotone mean preserving contractions (for example, with smaller weight on low and high values below the mean, and higher on intermediate values below the mean) in a way that would violate Condition 1.

Condition 1 and Statistical Properties of Distributions.—We now continue to explore Condition 1. Below we show that functions with the familiar property of increasing failure or hazard rate (IFR) satisfy the condition. In fact, Condition 1 will be satisfied with a strict inequality for any IFR distribution. Intuitively, these distributions do not provide sufficient benefits from matching with the rich as the tail on high income falls too quickly.

However, we can also relate Condition 1 to a weaker property, called NBUE. In reliability theory, in the analysis of life distributions (see Barlow and Proschan 1966), a distribution F is said to be *new better than used in expectations*, in short NBUE, if it describes the stochastic life span of a device which is less reliable with time. Formally,

DEFINITION 2: *A distribution function satisfies NBUE if and only if* $\overline{E}_x - x \leq \mu$ *for all* x.

PROPOSITION 3: *Any NBUE function satisfies condition* 1.

PROOF:
Assume that $\overline{E}_x - x \leq \mu$ for any x. Using (8) we have that

$$\text{(9)} \qquad \mu = F(x)\underline{E}_x + (1 - F(x))\overline{E}_x \leq F(x)\underline{E}_x + (1 - F(x))(x + \mu)$$

$$\Leftrightarrow F(x)\mu \leq F(x)\underline{E}_x + (1 - F(x))x \Leftrightarrow \mu \leq \frac{x}{F(x)} + (\underline{E}_x - x)$$

$$\Rightarrow \mu < \frac{x}{F(x)} \quad \text{for any} \quad x > 0 \text{ as } \underline{E}_x < x. ||$$

Note that NBUE is a weaker condition than Condition 1, and thus there will be functions satisfying Condition 1 which are not NBUE. It is easy to establish that a function with IFR, which implies that the survival rate $1 - F$ is log-concave, also satisfies NBUE. The proposition below (essentially a corollary to Proposition 3 and results from the statistical literature) lists properties of distribution functions, which are stronger than NBUE, and hence functions with these properties satisfy Condition 1:

PROPOSITION 4:

(*i*) *Suppose that* f *satisfies decreasing mean residual life, i.e.,* $\overline{E}_{\hat{x}}[x] - \hat{x}$ *decreases in* $\hat{x}$ *or in other words* $\int_x^\infty (1 - F(v))\, dv$ *is log-concave; it then satisfies Condition* 1.

(*ii*) *Suppose that* f *has increasing failure rates or in other words that* $1 - F$ *is log-concave; it then satisfies Condition* 1.

(*iii*) *Suppose that* f *is log-concave; it then satisfies Condition* 1.

PROOF:
To see (i), note that $\overline{E}_{\hat{x}}[x] - \hat{x} = \mu$ for $\hat{x} = 0$. Decreasing mean residual life (DMRL) implies then that $\overline{E}_{\hat{x}}[x] - \hat{x} < \mu$, and thus NBUE is satisfied, which by

Proposition 3 implies that Condition 1 is satisfied. It is then easy to see (ii) and (iii) (this is based on Bagnoli and Bergstrom 2005 and Barlow and Proschan 1966): (ii) If f has IFR then $1 - F$ is log-concave, which also implies that $\int_x^\infty (1 - F(v))\, dv$ is log-concave, which is identical to DMRL and thus by (i) it satisfies Condition 1. (iii) If f is log-concave then also $1 - F$ is log-concave (and hence IFR), which then implies (ii) and thus Condition 1 is satisfied. ∎

The following flow chart is a graphical illustration of Proposition 4:

(10) $f(x)$ is log-concave

$$\Rightarrow 1 - F(x) \text{ is log-concave}$$

$$\Leftrightarrow F(x) \text{ has } \mathit{IFR}\ (f(x)/(1 - F(x)) \text{ increases})$$

$$\Rightarrow \int_x^\infty (1 - F(v))\, dv \text{ is log-concave}$$

$$\Leftrightarrow F(x) \text{ has } \mathit{DMRL}\ (\overline{E}_x - x \text{ decreases in } x)$$

$$\Rightarrow F(x) \text{ satisfies } \mathit{NBUE}\ (\overline{E}_x - x \leq \mu).$$

Log-concavity, of either f, its survival/reliability function $1 - F$, or the integral of the reliability $\int_x^\infty (1 - F(v))\, dv$, are all stronger properties than Condition 1 and are easy to verify. They are satisfied for example by the uniform, normal, logistic, and exponential functions, as well as for the Power, Weibull, Gamma, and Beta functions with shape parameters greater than one. Intuitively, it implies that the density does not increase too fast and thus prevents $x/F(x)$ from being too low, or that the density on the tails is not too "heavy," implying also a relatively equal distribution as discussed above.

B. *Sorting versus Equality: Efficiency*

In our analysis so far, we have considered political support for FR compared with sorting. As we have shown, sorting generally entails benefits for the rich and losses for the poor, and we have analyzed when the positive distributional effects of sorting that accrue to the rich (compared with FR) will also spread to the mean (for whom FR has no other redistributional effects).

We now explore for which distribution functions it is efficient—in a utilitarian sense—for society to have FR compared with sorting. While sorting always entails benefits when the utility from a match is supermodular, it is also costly,[24] which implies that in some environments it might be inefficient.

[24] We perceive the costs of sorting to be either deadweight loss, or benefit only a negligible proportion of society, which is the case in which the suppliers of the signals are highly concentrated.

For the next result, note that F is *new worse than used in expectations* (NWUE) if and only if $\mu + \hat{x} \leq \overline{E}_{\hat{x}}$ for any $\hat{x}$. We then have:

PROPOSITION 5: *FR is more* (*less*) *efficient than any club* $\hat{x}$ *if and only if F is NBUE* (*NWUE*).

PROOF:

Average utility from sorting for some $\hat{x}$ can be written as:

$$(11)\quad \int_0^{\nu} U_x(\hat{x})\, dF = \int_0^{\hat{x}} x\underline{E}_{\hat{x}} f(x)\, dx + \int_{\hat{x}}^{\nu} \big(\hat{x}\underline{E}_{\hat{x}} + x\overline{E}_{\hat{x}} - \hat{x}\underline{E}_{\hat{x}}\big) f(x)\, dx$$

$$= F(\hat{x})\underline{E}_{\hat{x}}^2 + (1 - F(\hat{x}))\hat{x}\underline{E}_{\hat{x}} - (1 - F(\hat{x}))\hat{x}\overline{E}_{\hat{x}} + (1 - F(\hat{x}))\overline{E}_{\hat{x}}^2$$

$$= \big(\underline{E}_{\hat{x}} - \overline{E}_{\hat{x}}\big)\big(F(\hat{x})(\underline{E}_{\hat{x}} + \overline{E}_{\hat{x}} - \hat{x}) + \hat{x}\big) + \overline{E}_{\hat{x}}^2.$$

The average utility from FR is:

$$(12)\quad \int_0^{\nu} U_x(FR)\, dF = \mu^2$$

$$= \mu\big(F(\hat{x})\underline{E}_{\hat{x}} + (1 - F(\hat{x}))\overline{E}_{\hat{x}}\big) = \mu\big(F(\hat{x})(\underline{E}_{\hat{x}} - \overline{E}_{\hat{x}}) + \overline{E}_{\hat{x}}\big).$$

Let $\Delta = \int_0^{\nu} U_x(\hat{x})\, dF - \int_0^{\nu} U_x(FR)\, dF$. Then:

$$(13)\quad \Delta = (\underline{E}_{\hat{x}} - \overline{E}_{\hat{x}})\big(F(\hat{x})(\underline{E}_{\hat{x}} + \overline{E}_{\hat{x}} - \hat{x}) + \hat{x}\big) + \overline{E}_{\hat{x}}^2 - \mu\big(F(\hat{x})(\underline{E}_{\hat{x}} - \overline{E}_{\hat{x}}) + \overline{E}_{\hat{x}}\big)$$

$$= (\underline{E}_{\hat{x}} - \overline{E}_{\hat{x}})\big(F(\hat{x})(\underline{E}_{\hat{x}} + \overline{E}_{\hat{x}} - \hat{x} - \mu) + \hat{x}\big) + \overline{E}_{\hat{x}}(\overline{E}_{\hat{x}} - \mu)$$

$$= (\underline{E}_{\hat{x}} - \overline{E}_{\hat{x}})\big(F(\hat{x})(\underline{E}_{\hat{x}} + \overline{E}_{\hat{x}} - \hat{x} - \mu) + \hat{x}\big) + \overline{E}_{\hat{x}} F(\hat{x})(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}})$$

$$= (\underline{E}_{\hat{x}} - \overline{E}_{\hat{x}})(F(\hat{x})(\underline{E}_{\hat{x}} - \hat{x} - \mu) + \hat{x})$$

$$= (\underline{E}_{\hat{x}} - \overline{E}_{\hat{x}})(1 - F(\hat{x}))(\mu + \hat{x} - \overline{E}_{\hat{x}}),$$

and thus $\Delta < 0$ ($\Delta > 0$) for any $\hat{x}$ if and only if $\mu + \hat{x} - \overline{E}_{\hat{x}} > 0$ ($\mu + \hat{x} - \overline{E}_{\hat{x}} < 0$) for any $\hat{x}$ which is the NBUE (NWUE) property. ∎

To see the intuition for the efficiency result, note that positive assortative matching outweighs the cost when variability in the distribution is sufficiently high (in which cases random matching results in significant losses). Hall and Wellner (1981) showed that any NBUE function has a coefficient of variation $CV(x) = \sqrt{\mathrm{Var}(x)}/E(x) \leq 1$, whereas for any NWUE, $CV(x) \geq 1$. Thus, under NBUE, the variability of the income distribution is too small and sorting is inefficient.[25]

C. *Efficiency and Political Outcomes*

Note that by Lemma 1, the utility from signaling is strictly convex in the income x. This implies that

$$U_{\mu}(\hat{x}) \; < \; \int_0^{\nu} U_x(\hat{x}) \; dF. \tag{14}$$

This is simply an implication of Jensen's inequality given the convexity of the utility of signaling as a function of x; the LHS is the utility of the mean μ from a club $\hat{x}$, and the RHS is the average utility in the population from a club $\hat{x}$.

Thus, if FR is efficient, so that for any $\hat{x}$,

$$\int_0^{\nu} U_x(\hat{x}) \; dF \; < \; \int_0^{\nu} U_x(FR) \; dF, \tag{15}$$

it implies that

$$U_{\mu}(\hat{x}) \; < \; \int_0^{\nu} U_x(FR) \; dF \; = \; U_{\mu}(FR), \tag{16}$$

and all up to at least the mean will support FR. However, given the slackness in (15), it can also be the case that FR is not efficient (so that some sorting environment yields a higher average utility for society), while the mean and all below prefer FR:

COROLLARY 1: *If $F(x)$ is such that FR is more efficient relative to any club $\hat{x}$, then a coalition of all up to at least the mean will support FR. However, for some $F(x)$ and $\hat{x}$, the mean and all below will support FR even when the club $\hat{x}$ is more efficient than FR.*

Our analysis in Propositions 1, 3, and 5 allows us to identify for which income distributions FR has political support even among relatively rich voters (as in (16)) and for which income distributions FR is efficient (as in (15)).

Moreover, we identify that the wedge between efficiency and political support would arise for all functions that do not satisfy NBUE but satisfy Condition 1. The set of such functions contains a subset of long-tailed distribution functions, such as the Pareto or the log-normal distributions. For these functions (for all parameters), for a sufficiently high $\hat{x}$, sorting of the form $\{[0, \hat{x}), [\hat{x}, \infty)\}$ is efficient compared

[25] For the case of perfect continuous signaling, Hoppe, Moldovanu, and Sela (2009) show that $CV(x) \geq (\leq) 1$ is a sufficient and necessary condition for sorting to be efficient (not efficient) compared with random matching. For their discrete model which has incomplete information on a discrete set of types but perfect signaling, a necessary and sufficient condition for efficiency (inefficiency) of signaling is for the function to have decreasing (increasing) failure rate.

with FR. From a utilitarian point of view, it is better to separate the very rich from the rest. Intuitively, the costs of doing so are relatively low due to the small mass of the rich, while the benefits for the rich population are high due to the complementarities.

Formally, for distribution functions with a long tail, for a high enough $\hat{x}$, we have that $E_{\hat{x}} - \hat{x} > \mu$. This holds even if the income distribution is very close to being equal. The more equal it is though, the higher $\hat{x}$ has to be for sorting to be efficient. Moreover, when such income distributions are sufficiently equal, Condition 1 is satisfied: in the Pareto case it is satisfied when α, the shape parameter, is sufficiently large and in the case of the log-normal when σ, the shape parameter, is sufficiently small (see also Section IV).

Example: *Consider the Pareto distribution,* $F(x) = 1 - x_m^\alpha / x^\alpha$, *with* $x_m = 1$. *Suppose for example that* $\alpha = 1.5$, *which implies that* $\mu = \alpha/(\alpha - 1) = 3$. *In this case it is easy to verify that Condition* 1 *is satisfied. However, all* $\hat{x} \in (1.5, 3)$ *also satisfy* $\overline{E}_{\hat{x}} - \hat{x} > \mu$. *Thus, such clubs, which the mean belongs to, are efficient (compared with FR) but do not have the political support of the mean and all those below. This would hold for all* $\alpha \in [1.5, 2]$ and respectively, $\hat{x} \in (\alpha, \mu)$. *For values of* $\alpha > 2$ *(associated with greater equality), Condition* 1 *is still satisfied, with the club being efficient for all* $\hat{x} > \alpha > \mu$. *In this case the mean and all below strictly prefer FR simply as they are excluded from the club.*

The intuition behind this is that for such clubs to be efficient, $\hat{x}$ has to be sufficiently high (to push $E_{\hat{x}}$ to be high enough using the long tail). But this implies that the information rent enjoyed by the mean is too small (or nonexistent). Therefore, although efficient from the point of view of aggregate welfare, the distributional effects of sorting imply that it will have little political support in some cases.

As a political outcome is typically deemed to be more successful when a larger coalition supports it, our next question is whether political behavior is locally aligned with efficiency. That is, if $\hat{x}$ changes and as a result FR becomes more efficient relative to the club, is FR supported by a larger coalition? We can then show (the proof is in the Appendix):

PROPOSITION 6:

(*i*) *There exist* $F(x)$ *for which an increase in* $\hat{x}(< \mu)$ *increases the efficiency of the club (relative to FR) but decreases its political support (relative to FR).*

(*ii*) *If* $\overline{E}_{\hat{x}} - \hat{x} < (>)0$ for $\hat{x} \to 0$, *then for a small enough* $\hat{x}$, *an increase in* $\hat{x}$ *decreases (increases) the efficiency of the club (relative to FR) and decreases (increases) the size of the coalition supporting the club (relative to FR).*

The first part illustrates that efficiency and political support do not always go hand in hand. For example, for the Pareto distribution, there exist clubs to which the mean belongs to, where an increase in the exclusiveness of the club also increases the efficiency of the club. However, this implies that the information rent enjoyed by some of the members of the club is lowered (specifically those who are not too rich), which therefore decreases the political support of the club relative to FR. Our proof in the Appendix makes use of environments in which the local NBUE property is not satisfied for $\hat{x} < \mu$ but Condition 1 is satisfied.

For a small enough $\hat{x}$, the localized requirement for NBUE and Condition 1 is the same. This turns out to be sufficient to show that an increase in the efficiency of the club goes hand in hand with an increase in its political support. To see why, note that at $\hat{x} = 0$, from a utilitarian point of view, the average utility from sorting (essentially random matching with a zero price), is the same as the utility from FR. But this is also true for the mean himself: the utility from $\hat{x} = 0$ and the utility from FR are equivalent. Finally note that the joint requirement is a local DMRL ($\overline{E}_{\hat{x}} - \hat{x}$ decreasing) which intuitively implies that the increase in the benefit from a club, manifested through $\overline{E}_{\hat{x}}$, is too small relative to the increase in cost (in the order of $\hat{x}$).

D. *Sorting versus Equality: The Preferences of Poorer Agents*

We now look at smaller, majoritarian coalitions, which includes agents only up to the median. When we had considered the preferences of the mean, this had allowed us to identify the nonstandard incentives for redistribution, as from the point of view of their own income, those from the mean and up lose from redistribution. Moreover, it had allowed us to see when relatively rich voters support redistribution and when a large coalition can arise to support such policy.

Assuming that the median is poorer than the mean, as is typically the case, whenever the mean supports FR, so does the median. But for the median voter (or all below the mean), there are also income incentives for redistribution, which Condition 1 does not take into account. We now focus on the median voter to combine sorting and standard income motivations for redistribution. We show that these additional income incentives imply that whenever the income distribution is sufficiently *unequal*, the median (and all those below) would favor FR:

PROPOSITION 7: *The median* (*and all below*) *prefer FR to any partition if* $m < 0.5\mu$.

PROOF:

Note that $m\overline{E}_m$ is the highest utility the median can get in all clubs as $\overline{E}_m$ is the highest expected type he could match within a club he belongs to, and we are excluding the cost of the match. Note though that when $m < 0.5\mu$, we have that:

$$m\overline{E}_m = m\frac{\int_m^\infty xf(x)\,dx}{1-F(m)} = 2m\int_m^\infty xf(x)\,dx < \mu^2 \tag{17}$$

as $\mu = \int_0^\infty xf(x)\,dx > \int_m^\infty xf(x)\,dx$. ∎

The condition holds for sufficiently unequal distributions with half the population concentrated on relatively low incomes compared with the mean; the proposition adds then a counterpart to Condition 1. As sorting benefits arise through income complementarities, if the distribution is too unequal and the income of the median is simply too low, no sorting benefits will allow the median to prefer sorting to FR. Moreover, if $m < 0.5\mu$ is satisfied for some $F(x)$, it will also be satisfied for any $G(x)$ which is a monotone mean-preserving spread of F, i.e., when weight is transferred from high values in $[0, \mu]$ to low values in $[0, \mu]$.[26] Thus, the more unequal the distribution is in this second-order stochastic sense, the more likely is the condition to hold. Together, Propositions 1 and 7 imply that from the point of view of the median, either relatively equal or relatively unequal distributions would yield preferences for FR (as Proposition 1 provides a sufficient condition for the median).

Remark 1: Note that for all distributions with decreasing failure rates (DFR), then $m \leq \mu \ln 2 \approx 0.69\mu$, and thus redistribution will be favored in a large family of DFR distributions by the median and those below (and in particular those that are relatively more concave or more unequal). This arises as all DFR's with the same mean as some exponential, are more variable, i.e., stochastically dominated in a second-order sense, than the exponential one, which satisfies $m = \mu \ln 2$.[27] Thus, together with Proposition 4, both IFR functions and a large family of DFR functions will imply support for FR.

E. *Sorting: An "Ends against the Middle" Coalition*

We conclude this section with a very different political economy question. So far, we have considered government intervention only in the form of redistribution. This has led to monotone coalitions, characterized by a cutoff, where all voters below this cutoff advocate redistribution.

One other possible intervention for the government is to introduce taxes or subsidies in the housing or education markets; these will not only generate revenues from sorting, but will also affect the price and composition of sorting. For example, a tax on luxury goods or private schools might increase the exclusiveness of sorting.

To shed some light on this, we ask whether agents will prefer their club to be more or less inclusive. In other words, conditional on sorting, what form would voters prefer it to be.

For the poor voters who are not in the club, the higher is $\hat{x}$ the higher is the average income of those left to interact with them. For those in the club, the derivative of the utility from sorting (for some type x) is:

$$(18) \qquad (\overline{E}_{\hat{x}} - \hat{x})\left((x - \hat{x})\,\frac{f(\hat{x})}{1 - F(\hat{x})} - 1\right) + (\hat{x} - \underline{E}_{\hat{x}})\left(\hat{x}\,\frac{f(\hat{x})}{F(\hat{x})} - 1\right).$$

An increase in $\hat{x}$ directly increases $\overline{E}_{\hat{x}}$, $\underline{E}_{\hat{x}}$, and the price. What is clear from (18) is that once some x prefers an increase in $\hat{x}$, then all those above prefer an increase

[26] And similarly, weight shifts on values above μ to maintain the mean and the second order stochastic dominance of F.

[27] Specifically, by Theorems 4.4 and 4.7 in Barlow and Proschan (1965), $F(x) \geq 1 - e^{\frac{-x}{\mu}}$ for all $x < \mu$ if $F(x)$ is DFR, which implies that the median is lower in the DFR distribution.

in $\hat{x}$ as well. This reveals a possible "ends against the middle" coalition for small local changes.

PROPOSITION 8: *A coalition to increase $\hat{x}$ will always consist of agents below $\hat{x}$ and sometimes consists of all agents from some $x > \hat{x}$ and above.*

Moreover, it is also easy to find parameters for income distributions for which an "ends against the middle" majority coalition can arise to successfully increase the exclusiveness of the club.

III. Generalizing the Results

In this section we generalize our results. First we show that Condition 1 is sufficient for a coalition of all agents up to the mean to prefer FR compared with any incentive compatible partition, and any linear tax. We then generalize our efficiency results and finally we provide some results on general utility functions.

A. *General Incentive Compatibility Partitions and Linear Taxes*

We first extend Proposition 1 to any incentive compatible partition $\mathbf{x} = (x_0, x_1, \ldots x_{n-1}, x_n)$ and to any linear tax $t > 0$, i.e., when the disposable income of an agent of type x is $x^t = (1 - t)x + t\mu$. We can then show:

PROPOSITION 9:

(*i*) *The mean and all below prefer FR to any incentive compatible partition* $\mathbf{x}$ *and any $t \in [0, 1]$ if and only if $F(x)$ satisfies Condition* 1.

(*ii*) *When t is sufficiently large, the mean and all below prefer FR to any incentive compatible partition* $\mathbf{x}$.[28]

Note that when a tax $t > 0$ is in place, then the relevant income distribution becomes $F^t(x) = F\left(\frac{x - t\mu}{1 - t}\right)$, with $F^0(x) = F(x)$. In the proof we show that for any t, the mean prefers FR to any incentive compatible partition x if and only if $F^t(x)$ satisfies Condition 1. Note also that $F^t(x)$ is essentially a monotone mean-preserving contraction of F,[29] and thus if F satisfies Condition 1, so does $F^t(x)$. Thus, the necessary condition for $t = 0$ is also sufficient for any $t > 0$, implying that the mean prefers FR to any partition and any t if and only if $F(x)$ satisfies Condition 1. Moreover, when t is sufficiently high, Condition 1 is satisfied by F^t, as then income equality is high enough. Similarly, Condition 1 is necessary for a partition with one cutoff and turns out to be sufficient for any other partition with more than one cutoff.

[28] The proof is in the Appendix.

[29] In our main model we have considered only densities with full support while F^t is not full support, but this is not important for the gist of our analysis.

Next we turn to generalizing Proposition 5, which contrasts the efficiency of sorting and FR. We have a similar generalization to the above, and in particular, when t is high enough, FR is always efficient:

PROPOSITION 10:
(i) *FR is more efficient than any incentive compatible partition and any $t \in [0, 1]$ if and only if F is NBUE, whereas for any t, it is less efficient than any incentive compatible partition if and only if F^t is NWUE.*
(ii) *When t is large enough, FR is more efficient than any incentive compatible partition.*

PROOF:
Define

$$(19) \qquad \overline{E}_i^t \equiv E[x_j^t | x_j \geq x_i] = (1 - t)\overline{E}_i + t\mu.$$

Note that

$$(20) \qquad \mu + x_i^t - \overline{E}_i^t > 0 \Leftrightarrow \overline{E}_i < \frac{\mu}{1 - t} + x_i,$$

and so if F is NBUE, then also F^t is NBUE for any $t > 0$. In the Appendix we show that $\Delta = U(\mathbf{x}) - U(FR)$ can be written as:

$$(21) \qquad \Delta = \sum_{i=1}^{n-1} \big(E_{i-1}^t - E_i^t\big)(1 - F(x_i))\big(\mu + x_i^t - \overline{E}_i^t\big).$$

Given the above, NBUE (NWUE) of F^t is therefore necessary and sufficient for FR to be efficient (inefficient) compared with any partition, given some t. However, as $\mu + x_i^t - \overline{E}_i^t > 0 \Leftrightarrow \overline{E}_i < \frac{\mu}{1 - t} + x_i$, if F is NBUE then it also holds for F^t and in other words FR is more efficient ($\Delta < 0$) than any partition and any $t \in [0, 1]$ if and only if F is NBUE. Note that for a high enough t, $\mu + x_i^t - \overline{E}_i^t > 0$ for all x_i^t, implying that high enough equality is associated with the efficiency of FR compared with any partition and the (high enough) tax rate. ∎

B. *More General Utility Functions*

We now generalize our results to a larger set of utility functions. Let $\Phi(x, y)$ be the benefit of an individual with income x from membership in a club composed of other individuals with average income y. We assume that $\Phi_1, \Phi_2 > 0$ and for assortative matching that $\Phi_{12} > 0$.

In what follows we restrict attention to simple partitions with the cutoff $\hat{x}$. As we do above, we are interested in a condition under which the individual with average income prefers FR to sorting for any $\hat{x}$ (we focus on the interesting case in which $\hat{x} < \mu$, as otherwise $\Phi(\mu, \underline{E}_{\hat{x}}) < \Phi(\mu, \mu)$). We therefore need:

$$(22) \qquad \Phi(\mu, \overline{E}_{\hat{x}}) - \Phi(\hat{x}, \overline{E}_{\hat{x}}) + \Phi(\hat{x}, \underline{E}_{\hat{x}}) < \Phi(\mu, \mu) \quad \text{for all} \quad \hat{x} < \mu,$$

which is satisfied if:

$$\text{(Condition 2)}\quad \Phi(\mu, \overline{E}_{\hat{x}}) - \Phi(\mu, \mu) < \Phi(\hat{x}, \overline{E}_{\hat{x}}) - \Phi(\hat{x}, \underline{E}_{\hat{x}}) \quad \text{for all} \quad \hat{x} < \mu.$$

We first illustrate that also in this level of generality there is a sense in which more equality implies that Condition 2 is easier to satisfy. Consider an income distribution $F(\cdot)$. For any $\alpha \in (0, 1)$ define F^{α} as the income distribution with $F^{\alpha}(x) = (1 - \alpha)F(x) + \alpha\delta_{\mu}(x)$, where $\delta_{\mu}(x)$ is the degenerate distribution that has all mass on η, i.e., $\delta_{\mu}(x) = 0$ if $x < \mu$ and equals 1 otherwise. The property of supermodularity, $\Phi_{12} > 0$, and $\Phi_2 > 0$ will then be sufficient to guarantee that:

LEMMA 2: *If F satisfies Condition* 2 *then for any* $\alpha \in (0, 1)$, F^{α} *satisfies condition* 2.

PROOF OF LEMMA 2:

Let $\underline{E}_{\hat{x}}^{G}$ and $\overline{E}_{\hat{x}}^{G}$ denote the relevant expressions $\underline{E}_{\hat{x}}$ and $\overline{E}_{\hat{x}}$ under distribution G. Note that for any $\hat{x} < \mu$, $\underline{E}_{\hat{x}}^{F^{\alpha}} = \underline{E}_{\hat{x}}^{F}$ as the density, conditional on $[0, \hat{x}]$ is the same under both distributions. Note further that the expectation of F^{α} is μ. Finally note that $\overline{E}_{\hat{x}}^{F^{\alpha}} \leq \overline{E}_{\hat{x}}^{F}$ as all we did is convexify the conditional distribution with one that has a lower expectation. By $\Phi_{12} > 0$ and $\Phi_2 > 0$ and for any $\hat{x}$, the LHS of Condition 2 has decreased more than the RHS. ■

We now further explore Condition 2. In particular we want to analyze its relation to Condition 1. In the next two results, we show that a sufficient degree of concavity and a relatively weak supermodularity, imply that Condition 1 is sufficient for Condition 2:[30]

LEMMA 3: *Suppose that* $\Phi_{22}(x, y) \leq 0$ *and that* $\Phi_2(x, y)/\Phi_{12}(x, y) \geq x$ *for all x and y. Then Condition* 1 *implies condition* 2.

Example 1: Suppose that $\Phi(x, y) = (xy + 1)^{\beta}$, in this case,

$$\text{(23)}\qquad \frac{\Phi_2(x, y)}{\Phi_{12}(x, y)} = \frac{x(xy + 1)}{(xy + 1) + y(\beta - 1)x} \geq x \Leftrightarrow \beta \leq 1; \quad \text{and}$$

$$\Phi_{22}(x, y) \leq 0 \Leftrightarrow \beta \leq 1.$$

As a further illustration of the sufficiency of Condition 1 when Φ satisfies some concavity, let us consider a more specific form of complementarities, namely that,

$$\text{(24)}\qquad \Phi(x, y) = h(x)g(y) + f(x) + l(y).$$

[30] The proof of Lemma 3 is in the Appendix.

Note that to guarantee incentive to sort, $h' > 0$ and $g' > 0$ implying that Condition 2 becomes,

$$(25)\ \Phi(\mu, \overline{E}_{\hat{x}}) - \Phi(\mu, \mu) < \Phi(\hat{x}, \overline{E}_{\hat{x}}) - \Phi(\hat{x}, \underline{E}_{\hat{x}}) \Leftrightarrow \frac{h(\hat{x})}{h(\mu)} > \frac{g(\overline{E}_{\hat{x}}) - g(\mu)}{g(\overline{E}_{\hat{x}}) - g(\underline{E}_{\hat{x}})},$$

where by the mean value theorem, this is equivalent to:

$$(26) \qquad \frac{h(\hat{x})}{h(\mu)} > F(\hat{x})\left(\frac{g'(y' \in (\mu, \overline{E}_{\hat{x}}))}{g'(y'' \in (\underline{E}_{\hat{x}}, \overline{E}_{\hat{x}}))}\right).$$

LEMMA 4: *Suppose that* $h(0) = 0$. (i) *If* h *and* g *are concave then Condition* 1 *implies Condition* 2. (ii) *If* h *and* g *are convex then Condition* 2 *implies Condition* 1.

PROOF:

If g is concave (convex), then $\frac{g'(y' \in (\mu, \overline{E}_x))}{g''(y' \in (E_x, \overline{E}_x))} < (>)1$. If h is concave (convex), $\frac{h(x)}{h(\mu)} > (<) \frac{x}{y}$. ■

Example 2: Assume that $\Phi(x, y) = x^{\alpha} y^{\beta}$. (i) Whenever $\alpha, \beta \leq 1$, Condition 1 implies Condition 2, and thus when Condition 1 is satisfied, the mean and all below prefer FR to any partition with one club. (ii) Whenever $\alpha, \beta \geq 1$, Condition 2 implies Condition 1. Therefore, in this case, the set of distributions for which the mean and all below prefer FR to any partition with one club shrinks.

IV. Discussion: Some Empirically Estimated Income Distributions

Our analysis had identified a simple necessary and sufficient condition for at least a majoritarian coalition to prefer FR. We now discuss whether this condition is satisfied for income distributions, which are often used in the literature.

For the United States in the 1960s, Salem and Mount (1974) have advocated a version of the Gamma distribution that is IFR, i.e., with a shape parameter estimated to be around two.[31] For these distributions the higher the shape parameter, the lower the Gini coefficient is, and hence Condition 1 is satisfied for the sufficiently equal Gamma and Weibull distributions.

Other distributions that are typically considered in the literature are Pareto (which is DFR, i.e., decreasing failure rates), and the lognormal (which is first IFR and then DFR). Singh and Maddala (1976) claim that income distributions should be DFR

[31] The distribution is $f(x) = \frac{\lambda^{\alpha}}{A(\alpha)} x^{\alpha-1} e^{-\lambda x}$ on $[0, \infty]$ for $A(\alpha) = \int_0^{\infty} e^{-u} u^{\alpha-1}\, du$. For this distribution the median is $\frac{3\alpha - 1}{3\lambda}$, $\frac{1}{\sqrt{\alpha}}$ is the parameter of skewness, and the mean is $\frac{\alpha}{\lambda}$. For the decades of the 60s, their estimate of α is around 2 and λ is around $\frac{3}{10^4}$.

at least for high enough income, as the ability to make more money should increase with one's income, once some threshold is reached.[32]

It is easy to compute Condition 1 for Pareto distributions on $[1, \infty)$ and to see that it is satisfied for all such distribution with a sufficiently high shape parameter α, $\alpha \geq 1.5$. The higher the shape parameter α, the lower the Gini coefficient is (which equals $1/(2\alpha - 1)$), and thus we find that Condition 1 is satisfied for the more equal Pareto distributions.[33] For lower shape parameters, $\alpha \in (1, 2]$, when the Gini coefficient is high, the Pareto distribution satisfies $m < 0.5\mu$, and thus the condition identified in Proposition 7 is satisfied for the more unequal Pareto distributions.

The lognormal distribution is characterized by two parameters, $\tilde{\mu}$ (log-scale) and σ (the shape). The Gini coefficient is $2\Phi(\sigma/\sqrt{2}) - 1$ where $\Phi(x)$ is the standard normal distribution, and thus a lower σ is associated with a lower Gini. In this family of distributions we can show that Condition 1 is satisfied as long as σ is sufficiently low, $\sigma \leq 1.1$, and irrespective of $\tilde{\mu}$,[34] whereas the condition specified in Proposition 7 holds for all, more unequal, lognormal functions with σ sufficiently high, $\sigma > 1.174$. If one assumes in addition that $\tilde{\mu} > 0$ (as in typical income distributions), then the condition in Proposition 7 holds also for all $\sigma \in [1.1, 1.1774]$.

V. Conclusion

In this paper we have considered the implications of positive assortative (and costly) sorting on preferences for redistribution. We have identified a simple condition on income distributions, which is necessary and sufficient for all agents up to the mean to prefer full equality compared with any sorting environment. We have illustrated that this condition is associated with income equality, and that a large degree of income inequality implies that the middle classes prefer to match with the rich, even when it's costly, rather than live in an equal society. From a dynamic perspective, this indicates that both extreme environments may be stable. That is, once the distribution is sufficiently equal, agents will rather push for more equality, and when the distribution is sufficiently unequal, sorting will be stable and might breed more inequality.

Another avenue for future research is to consider government policies that combine redistribution with taxes or subsidies over the cost of signaling. We have identified that preferences over the exclusiveness of the club, as opposed to preferences over redistribution, do not satisfy single-crossing. Analysis that incorporates policies that both affect individuals' income via redistribution and the exclusiveness of sorting via such taxes is therefore promising and nontrivial.

[32] Singh and Maddala (1976) fit the data to some mixture of Pareto and Weibull, with an increasing *proportional* hazard rate $\left(x \frac{f(x)}{1 - F(x)}\right)$, which then converges to become constant. We note that Cramer (1978) advocates caution with respect to interpreting failure rates properties with regard to static distributions of income (where such properties should relate to time or age).

[33] Atkinson, Piketty, and Saez (2011) and Diamond and Saez (2011) provide evidence that the top tail of income distributions follows a Pareto distribution. See also Cowell (2011).

[34] One example for the estimation of σ is the estimation of the distribution of earnings of United Kingdom full time male manual workers (see Cowell 2011) with an estimated $\sigma^2 = 0.13$ well below the cutoff above. See also Pinkovskiy and Sala-i-martin (2009).

Mathematical Appendix

PROOF OF PROPOSITION 6:

Recall that the average utility from sorting minus that from FR, Δ, is:

$$\Delta = \int_0^{\nu} U_x(\hat{x})\, dF - \int_0^{\nu} U_x(FR)\, dF \tag{27}$$

$$= -\left(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}}\right)\left(\hat{x} + \mu - \overline{E}_{\hat{x}}\right),$$

and thus

$$d\Delta = -\left(d\overline{E}_{\hat{x}} - d\underline{E}_{\hat{x}}\right)\left(\hat{x} + \mu - \overline{E}_{\hat{x}}\right) - \left(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}}\right)\left(1 - d\overline{E}_{\hat{x}}\right). \tag{28}$$

Now consider the voter who is indifferent between FR and some club $\hat{x}$, some $z(\hat{x}) > \mu$. The voter $z(\hat{x})$ satisfies:

$$z(\hat{x})\overline{E}_{\hat{x}} - \hat{x}\left(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}}\right) = \mu^2 \tag{29}$$

$$z(\hat{x}) = \frac{\mu^2 + \hat{x}\left(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}}\right)}{\overline{E}_{\hat{x}}}$$

$$\frac{dz(\hat{x})}{d\hat{x}} = \frac{\left(\left(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}}\right) + \hat{x}\left(d\overline{E}_{\hat{x}} - d\underline{E}_{\hat{x}}\right)\right)\overline{E}_{\hat{x}} - d\overline{E}_{\hat{x}}\left(\mu^2 + \hat{x}\left(\overline{E}_{\hat{x}} - \underline{E}_{\hat{x}}\right)\right)}{\overline{E}_{\hat{x}}^2}.$$

Note that $d\overline{E}_{\hat{x}} = (\overline{E}_{\hat{x}} - \hat{x})\frac{f(\hat{x})}{1 - F(\hat{x})}$, $d\underline{E}_{\hat{x}} = \frac{f(x)}{F(x)}(x - \underline{E}_{\hat{x}})$.

For (i), consider the example of the Pareto distribution with $x_m = 1$, $\alpha = 2$ and $\hat{x} = 1.5$. Note that $\hat{x} < \mu = 3$. We find that $d\Delta > 0$ and $\frac{dz(\hat{x})}{d\hat{x}} > 0$. This implies that the club becomes more efficient relative to FR but also that the support of FR increases. On the other hand when $\alpha = 8$ and $\hat{x} = 1.1 < \mu \approx 1.14$, we have that both $d\Delta < 0$ and $\frac{dz(\hat{x})}{d\hat{x}} < 0$. It is then the case that greater efficiency of FR results in lower political support for it.

For (ii), note that as $\hat{x} \to 0$, $d\Delta$ converges to $-\mu(1 - d\overline{E}_{\hat{x}})$. Moreover, $\frac{dz(\hat{x})}{d\hat{x}}$ converges to $\mu^2(1 - d\overline{E}_{\hat{x}})$. Thus, if $d\overline{E}_{\hat{x}} < 1$ when $\hat{x} \to 0$, we have that $d\Delta < 0$ and $\frac{dz(\hat{x})}{d\hat{x}} > 0$, which implies that a higher $\hat{x}$ implies both a greater efficiency of FR and a greater political support of FR. If $d\overline{E}_{\hat{x}} > 1$ when $\hat{x} \to 0$, then a higher $\hat{x}$ implies a greater efficiency of the club and lower support of FR. Finally note that $\frac{d(\overline{E}_{\hat{x}} - \hat{x})}{d\hat{x}} = d\overline{E}_{\hat{x}} - 1$ and thus the relevant property is the local MRL for $\hat{x} \to 0$. ∎

PROOF OF PROPOSITION 9:

As we deal with general partitions, define

$$E_i \equiv E\big[x_j | x_j \in [x_i, x_{i+1}]\big], \tag{30}$$

and analogously

$$E_i^t \equiv E\big[x_j^t | x_j \in [x_{i-1}, x_i]\big] = (1-t)E_i + t\mu. \tag{31}$$

(i) We provide a direct proof for all partitions and t. We have already considered the case of one signal or a partition with $n = 2$. Note that if we start from some positive level of taxation t, the condition becomes $\frac{x^t}{\mu} \geq F(x)$ for all $x < \mu$, for which Condition 1 is sufficient. The necessary part of the proposition follows then from this case for $t = 0$.

We now show sufficiency using an induction on the number of elements in the partition. Suppose that the proposition is true for any partition with $j = k - 1$. Consider all partitions with $j = k$.

Note that if $\mu < x_1$, then the utility of the mean is like in a partition with $j = 2$ and the same x_1, and so Condition 1 applies. If $x_1 < \mu < x_2$, consider his utility from a partition with $j = 3$ and the same x_1, x_2, which is the same again. Thus, if $x_{i-3} < \mu < x_{i-2}$ for $i \leq k$, his utility from the partition is the same as the utility from a partition with $j = i$ and the same $x_0, x_1, \ldots, x_{i-2}$, which by the induction hypothesis proves the result. Now assume that $x_{k-2} < \mu < x_{k-1}$. The mean's expected utility can be written as:

$$x_1^t E_0^t + (x_2 - x_1)(1-t)E_1^t + \cdots + (x_{k-2} - x_{k-3})(1-t)E_{k-3}^t$$
$$+ (\mu - x_{k-2})(1-t)E_{k-2}^t), \tag{32}$$

which is strictly lower than the utility from a partition with $j = k - 1$ and the same $x_0, x_1, \ldots, x_{k-2}$ in which case the last expectations are replaced by $\overline{E}_{x_{k-2}}$ and the rest is the same.

Finally consider the case of $\mu > x_{k-1}$. We first divide both sides by μ and then use Condition 1 repetitively:

$$\frac{x_1^t}{\mu} E_0^t + \frac{x_2^t - x_1^t}{\mu} E_1^t + \cdots + \frac{x_{k-1}^t - x_{k-2}^t}{\mu} E_{k-2}^t + \left(1 - \frac{x_{k-1}^t}{\mu}\right) E_{k-1}^t \tag{33}$$
$$= \frac{x_1^t}{\mu}(E_0^t - E_1^t) + \cdots + \frac{x_{k-1}^t}{\mu}(E_{k-2}^t - E_{k-1}^t) + E_{k-1}^t$$
$$\leq F(x_1)(E_0^t - E_1^t) + \cdots + F(x_{k-1})(E_{k-2}^t - E_{k-1}^t) + E_{k-1}^t$$
$$= F(x_1)E_0^t + \big(F(x_2) - F(x_1)\big)E_1^t + \cdots\big(F(x_{k-1}) - F(x_{k-2})\big)E_{k-2}^t$$
$$+ \big(1 - F(x_{k-1})\big)E_{k-1}^t = \mu,$$

where the inequalities follow from Condition 1 as the difference in the expectations terms is negative.

(ii) As we illustrate in the proof above, fixing t, the necessary and sufficient condition for FR to be preferred by the mean to any partition is Condition 1(t) which states that $\frac{x^t}{\mu} \geq \mu$ for any $x < \mu$, for which Condition 1 is sufficient. But for a high enough t, Condition 1 (t) would hold for any $F(x)$ (as it becomes sufficiently equal). For example, for all $t > F(\mu)$, $\frac{x^t}{\mu} \geq \frac{F(\mu)\mu}{\mu} > F(x)$ for any $x < \mu$. ■

PROOF OF PROPOSITION 10:

The utility of an individual with after tax income x^t from sorting is $x^t E_0^t$ if $x \in [0, x_1]$ and $x^t E_k^t - \sum_{i=1}^{k} x_i^t (E_i^t - E_{i-1}^t)$ if $x \in [x_k, x_{k+1}]$ for $k = 1, \ldots, n$. Integrating over all types x, we get:

(34)

$$U(\mathbf{x}) = F(x_1)(E_0^t)^2 + \sum_{i=1}^{n-1}(F(x_{i+1}) - F(x_i))(E_i^t)^2 - \sum_{i=1}^{n-1}(1 - F(x_i))x_i^t(E_i^t - E_{i-1}^t)$$

$$= \sum_{i=1}^{n-1} F(x_i)\left((E_{i-1}^t)^2 - (E_i^t)^2\right) + (E_{n-1}^t)^2 - \sum_{i=1}^{n-1}(1 - F(x_i))x_i^t(E_i^t - E_{i-1}^t)$$

$$= \sum_{i=1}^{n-1} F(x_i)(E_{i-1}^t - E_i^t)(E_{i-1}^t + E_i^t) + (E_{n-1}^t)^2 - \sum_{i=1}^{n-1}(1 - F(x_i))x_i^t(E_i^t - E_{i-1}^t)$$

$$= \sum_{i=1}^{n-1}(E_{i-1}^t - E_i^t)\left[F(x_i)(E_{i-1}^t + E_i^t) + (1 - F(x_i))x_i^t\right] + (E_{n-1}^t)^2$$

whereas the average utility from FR is:

$$(35)\quad \mu\left(F(x_1)E_0^t + \sum_{i=1}^{n-1}(F(x_{i+1}) - F(x_i))E_i^t\right) = \mu\left(\sum_{i=1}^{n-1}F(x_i)(E_{i-1}^t - E_i^t) + E_{n-1}^t\right).$$

The difference $\Delta = U(\mathbf{x}) - U(FR) =$

$$(36)\quad \sum_{i=1}^{n-1}(E_{i-1}^t - E_i^t)\left[F(x_i)(E_{i-1}^t + E_i^t) + (1 - F(x_i))x_i^t\right] + (E_{n-1}^t)^2$$

$$- \mu\left(\sum_{i=1}^{n-1}F(x_i)(E_i^t - E_{i+1}^t) + E_{n-1}^t\right)$$

$$= \sum_{i=1}^{n-1}(E_{i-1}^t - E_i^t)\left[F(x_i)(E_{i-1}^t + E_i^t - \mu) + (1 - F(x_i))x_i^t\right] + E_{n-1}^t(E_{n-1}^t - \mu).$$

Note that

$$(37)\quad E_{n-1}^t - \mu = E_{n-1}^t - \sum_{i=1}^{n-1}F(x_i)(E_{i-1}^t - E_i^t) - E_{n-1}^t = -\sum_{i=1}^{n-1}F(x_i)(E_{i-1}^t - E_i^t).$$

Therefore:

$$\Delta = \sum_{i=1}^{n-1} \left(E_{i-1}^t - E_i^t\right)\left[F(x_i)(E_{i-1}^t + E_i^t - \mu) + (1 - F(x_i))x_i^t\right] - E_{n-1}^t \sum_{i=1}^{n-1} F(x_i)\left(E_{i-1}^t - E_i^t\right) = \sum_{i=1}^{n-1} \left(E_{i-1}^t - E_i^t\right)\left[F(x_i)(E_{i-1}^t + E_i^t - E_{n-1}^t - \mu) + (1 - F(x_i))x_i^t\right] \tag{38}$$

We now add and subtract $\sum_{j=i+1}^{n-1} E_j^t$ in the summation, with the convention that if $i + 1 > n - 1$ these expressions are zero,

(39)

$$\Delta = \sum_{i=1}^{n-1} \left(E_{i-1}^t - E_i^t\right)\left[F(x_i)\left(E_{i-1}^t + E_i^t - E_{n-1}^t + \sum_{j=i+1}^{n-1} E_j^t - \sum_{j=i+2}^{n-1} E_j^t - \mu\right) + (1 - F(x_i))x_i^t\right] = \sum_{i=1}^{n-1} \left(E_{i-1}^t - E_i^t\right)\left[F(x_i)\left(E_{i-1}^t + \sum_{j=i}^{n-2} (E_j^t - E_{j+1}^t) - \mu\right) + (1 - F(x_i))x_i^t\right].$$

We now move the expressions $(E_j^t - E_{j+1}^t)F(x_i)(E_{i-1}^t - E_i^t)$ for any i up to their relevant position, $j + 1$, in the summation,

$$\Delta = \sum_{i=1}^{n-1} \left(E_{i-1}^t - E_i^t\right) \times \left[F(x_1)E_0^t + \sum_{j=1}^{i} (F(x_{j+1}) - F(x_j))E_j^t - F(x_i)\mu + (1 - F(x_i))x_i^t\right]. \tag{40}$$

Note that, $F(x_1)E_0^t + \sum_{j=1}^{i}(F(x_{j+1}) - F(x_j))E_j^t - F(x_i)\mu = (1 - F(x_i))(\mu - \overline{E}_i^t)$ and so we have,

$$\Delta = \sum_{i=1}^{n-1} \left(E_{i-1}^t - E_i^t\right)(1 - F(x_i))\left(\mu + x_i^t - \overline{E}_i^t\right), \tag{41}$$

and the rest is shown in the main text. ∎

PROOF OF LEMMA 3:

By the mean value theorem,

$$\Phi\left(\mu, \overline{E}_x\right) - \Phi(\mu, \mu) < \Phi\left(x, \overline{E}_x\right) + \Phi(x, \underline{E}_x) \tag{42}$$

$$\Leftrightarrow \frac{\Phi_2\left(x, y' \in \left(\underline{E}_x, \overline{E}_x\right)\right)}{\Phi_2\left(\mu, y'' \in \left(\mu, \overline{E}_x\right)\right)} > \frac{\overline{E}_x - \mu}{\overline{E}_x - \underline{E}_x} = F(x) \quad \text{where}$$

$$\Phi_2\left(x, y' \in \left(\underline{E}_x, \overline{E}_x\right)\right)\left(\overline{E}_x - \underline{E}_x\right) = \Phi\left(x, \overline{E}_x\right) - \Phi(x, \underline{E}_x)$$

$$\Phi_2\left(\mu, y'' \in \left(\mu, \overline{E}_x\right)\right)\left(\overline{E}_x - \mu\right) = \Phi\left(\mu, \overline{E}_x\right) - \Phi(\mu, \mu).$$

We show that under the two conditions $\frac{\Phi_2(x, y' \in (\underline{E}_x, \overline{E}_x))}{\Phi_2(\mu, y'' \in (\mu, \overline{E}_x))} \geq \frac{x}{\mu}$, or equivalently that:

$$\frac{\Phi_2\left(x, y' \in \left(\underline{E}_x, \overline{E}_x\right)\right)}{x} \geq \frac{\Phi_2\left(\mu, y'' \in \left(\mu, \overline{E}_x\right)\right)}{\mu}. \tag{43}$$

Note however that

$$\frac{\Phi_2(x, y' \in (\underline{E}_x, \overline{E}_x))}{x} \geq \frac{\Phi_2(\mu, y' \in (\underline{E}_x, \overline{E}_x))}{\mu} \geq \frac{\Phi_2(\mu, y'' \in (\mu, \overline{E}_x))}{\mu}, \tag{44}$$

where concavity in second element will imply the second inequality, and for the first inequality, a sufficient condition is that $\frac{\Phi_2\left(x, y' \in \left(\underline{E}_x, \overline{E}_x\right)\right)}{x}$ is decreasing in x, i.e., if $\frac{\Phi_2(x, y)}{\Phi_{12}(x, y)} \geq x$. ■

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