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The art of brevity

Working paper

The Art of Brevity*

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Abstract

We analyze a class of sender-receiver games with quadratic payoffs, which includes the communication games in Alonso, Dessein and Matouschek (2008) and Rantakari (2008) as special cases, for which the receiver's maximum expected payoff when players have access to arbitrary, mediated communication protocols is attained in one-round of face-to-face, unmediated cheap talk. This result is based on the existence for these games of a communication equilibrium with an infinite number of partitions of the state space. We provide explicit expressions for the maximum expected payoff of the receiver, and illustrate its use by deriving new comparative statics of the quality of optimal communication. For instance, a shift in the underlying uncertainty that reduces expected conflict can worsen the quality of communication.

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1 Introduction

Conflicting interests often hinder communication between informed experts and uninformed decision makers. This is certainly the case in Crawford and Sobel's classic contribution where an informed sender strategically sends costless messages to an uninformed receiver, i.e. the sender engages in cheap talk (Crawford and Sobel, (CS) 1982). CS considers one-shot communication: the sender makes a single recommendation and the receiver immediately makes a decision based on that recommendation. The fact that the sender can foresee the effect of his influence, by anticipating how each recommendation will be interpreted and which decision it will induce, implies that perfect information transmission is not credible.

The literature has since studied several communication protocols that improve the efficiency of the one-shot equilibria in CS, for instance, by engaging in repeated rounds of communication (Krishna and Morgan, 2004), by using a noisy channel (Blume, Board and Kawamura, 2007), by appealing to a correlation device on which to base the encoding and decoding of messages (Blume, 2012) or, more generally, by relying on a trustworthy, or even strategic, mediator (Goltsman, Horner, Pavlov and Squintani (GHPS), 2009; Ivanov, 2009).[1] A driving force behind these efficiency-enhancing communication protocols is that they introduce noise in the sender's message: a communication protocol that induces noisier recommendations for decisions that on average favor the sender can discipline a risk-averse sender and thus enhance information transmission.

In this vein, a natural question is when general communication protocols, including mediated communication, can improve upon one-shot communication and to understand when these gains from mediation are likely to be either large or small. A limitation of this strand of literature to address this question is that efficiency bounds of communication equilibria exist only for the leading example in CS; the characterization of optimal mediation in Goltsman, Horner, Pavlov and Squintani (GHPS) (2009) is done for the case of quadratic preferences, constant bias between sender and receiver and a uniformly distributed state. We extend the analysis of GHPS to a broader class of sender-receiver games and study the structure of the

[1] The role of mediation with finite messages is considered in Ganguly and Ray (2012). Ambrus et al (2010) examine hierarchical cheap talk where the messages get passed through a sequence of agents and show that mixed-strategy equilibria can exist that dominate the direct communication game.

payoff set supported by communication equilibria.

Our main result is a characterization of a class of sender-receiver games for which there exists a one-shot, unmediated communication equilibrium for which the receiver's expected payoff cannot be improved upon by using arbitrary mediation rules (Proposition 2). That is, for these games the receiver obtains no efficiency gains from prolonged conversations, using noisy channels or arbitrary mediation; brief conversations are optimal. In particular, the cheap talk equilibrium that achieves the maximum of the receiver's payoff supports an infinite number of different decisions. That is, brief conversations that are optimal are also very detailed. Importantly, the class of sender-receiver games for which this is true includes the communication games in Alonso, Dessein and Matouschek (2008) and Rantakari (2008), where the bias between the sender and the receiver is linear and increasing and vanishes at some point of the state space.[2]

One lesson from GHPS is that the gains from mediation with respect to the most efficient equilibrium in CS are highest for intermediate levels of conflict. Indeed, as the conflict vanishes one-shot communication approaches full revelation while intensifying the conflict leads to no information transmission, even in the presence of a mediator. Our results show, however, that it is not only the magnitude of the conflict that determines the gains from mediation but also its shape. Indeed, Proposition 2 imposes no constraints on the average conflict (as measured by the expected bias between the sender and the receiver). Therefore, for any level of expected conflict one can find a sender-receiver game in our setup where the receiver does not gain from mediation.

Apart from the insights on the gains from mediation, this result has also a more practical appeal. The cheap talk model in CS, and specifically their leading example, has been a workhorse model in applications featuring costless communication, where the restriction to one-shot communication has often been defended on tractability grounds. A concern with this approach, however, is that the insights derived in such settings may not be robust to the parties agreeing to switch to welfare-improving communication protocols. However, for the class of games and the one-shot infinite equilibrium in those games identified in Proposition 2, the receiver cannot strictly gain from having access to a neutral mediator. This implies,

[2] See also Alonso (2007) and Gordon (2010). A single round of unmediated communication obtains the upper bound on the principal's payoff also in Rantakari (2013).

for instance, that the comparative statics in Alonso, Dessein and Matouschek (2008) and Rantakari (2008) are derived under optimal communication.

Finally, we explicitly compute the receiver's maximum expected payoff when brief conversations are indeed optimal (Proposition 3). This result is of separate applied interest, as it provides the loss due to strategic communication without explicitly solving for the equilibrium itself and for distributions other than the uniform distribution. We illustrate the use of this result by deriving new comparative statics on optimal communication. For instance, we show by example that a shift in the underlying distribution that reduces the expected conflict can actually worsen communication (Corollary 2).

The basic logic of the analysis is as follows. First, the revelation principle allows us to restrict attention to games where the sender privately and truthfully discloses the state to a mediator, who in turn issues a recommendation to the receiver which the latter is willing to follow. The need to ensure the sender's sincerity and the receiver's obedience implies that, for games with quadratic payoffs, interim and ex-ante payoffs can be expressed as linear functionals of state-contingent average decisions (Lemma 1). From this representation we identify a class of games for which a tight relationship exists between local and global properties of the communication equilibria: the interim payoff to the sender for either the lowest or the highest type is a linear function of the ex-ante payoff to the sender or the receiver (Proposition 1). Therefore globally optimal communication equilibria must also be locally optimal for either the highest or the lowest type of sender. We then show that if the sender and the receiver are perfectly aligned for either the highest or the lowest state, then there exists a one-shot infinite communication equilibrium such that communication is very detailed around the state of perfect alignment. In particular, an extreme type of sender is able to fully reveal his type, and thus to obtain his preferred decision with certainty. Since local properties of extreme types translate to global properties for these games, perfect communication for an extreme type implies that this communication equilibrium is also ex-ante optimal for the receiver. Moreover, this logic then naturally extends to settings where the point of alignment is interior, when we can split the communication game around this point of alignment

Our paper follows the recent literature that analyzes the gains from adopting more sophisticated communication protocols in sender-receiver games. The paper most related to ours is GHPS, who studies different conflict resolution procedures, among them optimal mediation. While we expand their methodology to a broader class of games, our focus is on characterizing the class of games where one-shot communication is optimal. Ivanov (2013) provides sufficient conditions for mediation to be valuable, that is, for the receiver to benefit at all from communicating with the sender. Finally, Blume (2012) is also concerned with optimal mediation involving one-shot communication. In particular, Blume (2012) shows that the efficiency bound reported by GHPS for the leading example in CS can be achieved in one-shot communication if parties can rely on a correlation device that sends private signals before the sender becomes informed.[3] In contrast, in our paper optimal communication is face-to-face and does not rely on the use of a correlation device.

2 The Model

There are two players, the informed sender (he) and the uninformed receiver (she). The payoffs to both players depend on the realized state of nature $\theta \in \Theta = [0, 1]$ and the chosen action $y \in Y \subset \mathbb{R}$. The state of nature is distributed according to the distribution $F(\theta)$ which admits a continuous density $f(\theta)$ with full support on Θ. The payoffs of the receiver and the sender are

$$\begin{aligned} u_R(y, \theta) &= -(y-\theta)^2, \\ u_S(y, \theta) &= -(y-y_S(\theta))^2, \end{aligned} \tag{1}$$

where $y_S(\theta)$ is a differentiable function of the state. In our specification, the sender's preferred action matches the realized state θ while the receiver's preferred action is $y_S(\theta)$, where the bias $b(\theta) = y_S(\theta) - \theta$ measures the distance between the sender's and the receiver's preferred choices. Apart from the differentiability requirement, we don't impose any additional assumptions on the shape of $y_S(\theta)$. In particular, for our representation in Lemma 1 we don't require $y_S(\theta)$ to be non-decreasing.

While the sender observes θ, the receiver has authority over the action y. Prior to the receiver selecting an action, the players exchange messages according to a fixed communication protocol. While communication protocols may involve complex communication

[3] For implementation in correlated equilibria, see also Vida and Forges (2013).

procedures with multiple rounds in which messages are exchanged, possibly with the help of a trustworthy mediator, the revelation principle applies: any equilibrium outcome of any sender-receiver game with communication can be replicated by a (canonical) communication equilibrium (Forges, 1986; Myerson, 1986). A communication equilibrium involves the use of a neutral mediator to which the informed party sends a single, private, costless message, which is a report of the state of nature, after which the mediator issues a recommendation to the receiver. Moreover, in a communication equilibrium the sender is sincere and the receiver obedient: reporting the true state is optimal for the sender, and abiding by the mediator's recommendation is optimal for the receiver. We can then restrict our attention to communication equilibria where the message space is the type space Θ and the space of mediator recommendations is the action space Y. Moreover, the receiver's preferred choice for any belief she may have about θ cannot fall outside $[0,1]$. Therefore, we can set $Y=[0,1]$.

Let $\mathcal{F}=\mathcal{B}(Y)$ be the Borel σ-algebra in Y. Formally, a mediation rule M is a family of probability measures on $(Y,\mathcal{F})$ indexed by Θ, $\{p\left(.|\theta\right)\}_{\theta\in\Theta}$, that completely describe the mediator's behavior: the mediator's recommendation is distributed according to the measure $p\left(y|\theta\right)$ following a report θ. Moreover, to ensure the sender's sincerity and the receiver's obedience, the family $\{p\left(.|\theta\right)\}_{\theta\in\Theta}$ must satisfy

$$\int_Y -\left(y-y_S(\theta)\right)^2\left(dp\left(y|\theta\right)-dp\left(y|\theta'\right)\right)\geq 0,\quad \forall\theta,\theta'\in\Theta. \qquad \text{(IC-S)}$$

$$y=E\left[\theta|y\right], \qquad \forall y\in Y. \qquad \text{(IC-R)}$$

The constraint (IC-S) is the sender's truthtelling constraint: the sender has no incentive to misrepresent the state when the mediator commits to randomizing its recommendation according to $p\left(y|\tilde{\theta}\right)$ following a report $\tilde{\theta}$. The constraint (IC-R) ensures the receiver's obedience: given the mediation rule $\{p\left(.|\theta\right)\}_{\theta\in\Theta}$, and given the sender's truthtelling behavior, the mediator's recommendation refines the receiver's belief about the realized state. Then (IC-R) simply states that whenever the mediator recommends action y, the receiver's optimal action given her updated beliefs over the state is indeed y. The particular form of (IC-R) follows from the fact that, with quadratic payoffs, the decision maker's optimal action given her beliefs equals the expected state. A mediation rule that simultaneously satisfies (IC-R) and (IC-S) is called incentive compatible.

We are interested in mediation rules that maximize the ex-ante welfare of the receiver.

We refer to a mediation rule as *unimprovable* for a player, if there is no incentive compatible mediation rule that yields a higher ex-ante expected payoff for that player. An optimal mediation rule is thus an incentive compatible mediation rule that is unimprovable for the receiver. Our quadratic setup allows also a simple informational interpretation of optimal mediation rules. Indeed, given the receiver's behavior (IC-R) and the functional form of the receiver's preferences (1), the receiver's payoff coincides with her residual variance after listening to the mediator's recommendation. Therefore optimal mediation rules also maximize the amount of information that is transmitted, when the informativeness of a signal is measured in terms of its expected residual variance.

We are particularly interested in a class of sender-receiver games where optimal mediation can be achieved through *brief conversations*. To be specific, we define a brief conversation as a Bayesian-Nash equilibrium of a game in which the sender sends a single, costless message observed by the receiver who then takes an action. That is, a brief conversation is the Nash equilibrium of a one-shot, face-to-face, unmediated cheap talk game. Formally, a brief conversation is characterized by (i.) the sender's communication rule $\mu(\theta) : \Theta \to \Delta M$ which specifies the probability of sending message $m \in M$ conditional on observing state θ, (ii.) the receiver's response $y(m) : M \to Y$ which maps messages into actions and (iii.) the receiver's belief function $g(\theta \mid m) : M \to \Delta\Theta$ which states the posterior probability of θ after observing message m. In a Bayesian-Nash Equilibrium, the communication rule is optimal for the sender given the receiver's response, the receiver's response is optimal for the receiver given the belief function and the belief function is derived from the communication rule using Bayes' rule whenever possible.

3 Analysis

We start by deriving an alternative representation of the equilibrium payoffs induced by a communication equilibrium. To this end, let $U_S(\theta)$ be the sender's interim payoff when the state is θ, and V_S and V_R the ex-ante expected payoffs of the sender and the receiver in a given communication equilibrium. As in GHPS, for any incentive compatible M, defined by $\{p(.|\theta)\}_{\theta\in\Theta}$, let $\bar{y}(\theta) = \int_Y y dp(y|\theta)$ and $\bar{\sigma}^2(\theta) = \int_Y (y - \bar{y}(\theta))^2 dp(y|\theta)$ be the equilibrium expected decision and variance of the decision. Quadratic payoffs are convenient as knowledge of $\bar{y}(\theta)$ and $\bar{\sigma}^2(\theta)$ suffices to obtain the state-dependent payoffs for any mediation rule. Indeed, one immediately has

$$\begin{aligned} U_S(\theta) &= -(\bar{y}(\theta) - y_S(\theta))^2 - \bar{\sigma}^2(\theta), \\ V_S &= -E\left[(\bar{y}(\theta) - y_S(\theta))^2\right] - E\left[\bar{\sigma}^2(\theta)\right], \\ V_R &= -E\left[(\bar{y}(\theta) - \theta)^2\right] - E\left[\bar{\sigma}^2(\theta)\right]. \end{aligned}$$

In principle, knowledge of both values $\bar{y}(\theta)$ and $\bar{\sigma}^2(\theta)$ would be required to obtain $U_S(\theta)$, $\theta \in [0,1]$, and knowledge of the functions $\bar{y}$ and $\bar{\sigma}^2$ would be necessary to deduce V_S and V_R. However, the restrictions imposed on the set of equilibrium payoffs by (IC-S) and (IC-R) imply that, for games with quadratic payoffs, interim and ex-ante payoffs can be obtained solely on the basis of the state-contingent average decision $\bar{y}(\theta)$, $\theta \in [0,1]$.

Lemma 1 *Let M be an incentive compatible mediation rule that induces in equilibrium $\bar{y}(\theta)$ and $\sigma^2(\theta)$, $\theta \in [0,1]$. Then*

$$U_S\left(\hat{\theta}\right) = E\left[\bar{y}(\theta)K_{S(\hat{\theta})}(\theta)\right] - y_S^2(\hat{\theta}),\ \hat{\theta} \in [0,1], \tag{2}$$

$$V_S = E\left[\bar{y}(\theta)K_S(\theta)\right] - E\left[y_S^2(\theta)\right], \tag{3}$$

$$V_R = E\left[\bar{y}(\theta)K_R(\theta)\right] - E\left[\theta^2\right], \tag{4}$$

where, letting $I_{[0,\hat{\theta}]}$ be the characteristic function of the set $[0,\hat{\theta}]$, we have[4]

$$\begin{aligned} K_{S(\hat{\theta})}(\theta) &= 2y_S(\theta) - \theta - 2\frac{y_S'(\theta)}{f(\theta)}\left(1 - F(\theta) - I_{[0,\hat{\theta}]}(\theta)\right), \\ K_S(\theta) &= 2y_S(\theta) - \theta, \\ K_R(\theta) &= \theta. \end{aligned} \tag{5}$$

Lemma 1 provides expressions for $U_S(\theta)$, V_S and V_R as affine functionals of the average decision $\bar{y}$ without explicit recourse to either $\bar{\sigma}^2$ or to any additional information of the mechanism M. Lemma 1 is analogous to well known results in mechanism design with quasilinear utility and convex type spaces: if $\bar{y}(\theta)$ plays the role of an "allocation" and $\bar{\sigma}^2(\theta)$ (which enters additively in $U_S(\theta)$) plays the role of a type-dependent transfer, then an application of the envelope theorem to (IC-S) implies that $U_S(\theta)$ can be obtained from

[4] The characteristic function of the set A, I_A, is such that $I_A(x) = 1$ if $x \in A$ and $I_A(x) = 0$ otherwise.

knowledge of the interim payoff of one type and the entire allocation $\bar{y}(\theta), \theta \in [0,1]$. Under mediation, however, we must also ensure that the receiver is obedient, i.e. (IC-R) must also hold. Lemma 1 shows that this additional constraint eliminates the degree of freedom in specifying the interim payoff to a fixed sender's type. That is, knowledge of the "allocation" $\bar{y}$ suffices to compute both interim and ex-ante expected payoffs. Note, however, that Lemma 1 remains silent on the set of implementable $\bar{y}$. For example, the conflict of interest may be so severe that only a babbling equilibrium can be sustained (and thus the set of implementable $\bar{y}$ is a singleton), which nevertheless would still satisfy (2), (3) and (4).

3.1 Interim and ex-ante payoffs under mediation.

In principle, if the space of implementable $\bar{y}$ is sufficiently rich, one may conjecture that knowledge of $U_S(\theta)$ for some type θ is not enough to derive $U_S(\theta')$ for some other type θ', and would also be insufficient to infer the ex-ante welfare of the sender and the receiver. In other words, if the set of mediation rules is sufficiently rich one would expect that mediation rules that exhibit the same local behavior, by inducing the same $U_S(\theta)$ for some type θ, may have widely different global properties and thus generate different ex-ante payoffs. By contrast, the following proposition characterizes a class of sender-receiver games in which local behavior univocally determines the global welfare properties of any incentive compatible mediation rule. For the remainder of this paper let $h(\theta) = f(\theta)/(1-F(\theta))$ and $r(\theta) = f(\theta)/F(\theta)$ be the hazard rate and reversed hazard rate of the distribution $F(\theta)$.

Proposition 1 *Suppose that, for some $\alpha, \beta \in \mathbb{R}$, $y_S(\theta)$ takes one of the following forms*

$$y_S(\theta) = \alpha E\left[\tilde{\theta}|\tilde{\theta} \geq \theta\right] + \beta, \tag{6}$$

$$y_S(\theta) = \alpha E\left[\tilde{\theta}|\tilde{\theta} \leq \theta\right] + \beta, \tag{7}$$

$$y_S(\theta) = \alpha \int_0^\theta h(\theta')d\theta' + \beta, \tag{8}$$

$$y_S(\theta) = \alpha \int_0^\theta r(\theta')d\theta' + \beta. \tag{9}$$

(i) If either (6) or (7) holds, then for any two mediation rules M and M' we have

$$U_S^{M'}(i) - U_S^{M}(i) = (2\alpha - 1)\left(V_R^{M'} - V_R^{M}\right), \tag{10}$$

where $i = 0$ if $y_S(\theta)$ satisfies (6), while $i = 1$ if $y_S(\theta)$ satisfies (7).

(ii) If either (8) or (9) holds, then for any two mediation rules M and M' we have

$$U_S^{M'}(i) - U_S^{M}(i) = V_S^{M'} - V_S^{M}, \tag{11}$$

where $i = 0$ if $y_S(\theta)$ satisfies (8), while $i = 1$ if $y_S(\theta)$ satisfies (9).

We can interpret Proposition 1 as a refinement of Lemma 1. Lemma 1 shows that knowledge of the function $\bar{y}$ is sufficient to compute ex-ante and interim payoffs for any incentive compatible M. Nevertheless, the functional difference in the linear functionals defining (2), (3) and (4) implies that two average decisions $\bar{y}$ and $\bar{y}'$ that induce the same $U_S(\theta)$ for some type θ may very well yield different ex-ante payoffs. Adding more structure to our model, however, can lead to an equivalence between local interim payoffs and global ex-ante payoffs. For instance, if $y_S(\theta)$ can be written as (6) or (7), then a linear relation exists between the expected payoff to the receiver and the payoff to the sender at an extreme type for any incentive compatible mediation rule, while if either (8) or (9) holds then the change in the ex-ante payoff to the sender when switching from M to M' equals the change in the payoff to an extreme type.

The intuition behind Proposition 1-i and 1-ii is based on the representation (2),(3) and (4) in Lemma 1 coupled with the obedience constraint by the receiver (IC-R). Suppose that (6) holds. Then, the function $y_S(\theta) - y_S'(\theta)/h(\theta)$ is linear in the state, implying that $K_{S(0)}$, obtained by setting $\hat{\theta} = 0$ in (5), can be expressed as an affine function of K_R, which defines the expected utility of the receiver. The final step is to observe that the law of the iterated expectations applied to (IC-R) implies that $E[\bar{y}(\theta)]$ is constant across all mediation mechanisms (and equal to $E[\theta]$). Then (10) follows immediately from Lemma 1 as $E\left[\bar{y}(\theta)K_{S(0)}(\theta)\right]$ is a linear transformation of $E[\bar{y}(\theta)K_R(\theta)]$ in the set of implementable $\bar{y}$. Similar reasoning shows that $E\left[\bar{y}(\theta)K_{S(1)}(\theta)\right]$ and $E[\bar{y}(\theta)K_R(\theta)]$ are linearly related if (7) holds. Conversely, if (8) holds then $K_{S(0)}(\theta) - K_S(\theta)$ is a constant, while $K_{S(1)}(\theta) - K_S(\theta)$ is constant if (9) holds. Again, as $E[\bar{y}(\theta)]$ is constant over the space of mediation rules then either case would imply (11).

We note that both (6) and (7) hold if the sender's preferred decision is an affine function of the state and the state is uniformly distributed.[5] This includes the leading example in CS,

[5] We leave the analysis of other cases that satisfy (6) to Section 4.1.

which has been the workhorse model in applications involving cheap talk communication. Indeed, the main studies of optimal mediation in CS-type of games all consider this "uniform-quadratic" example (see, Krishna and Morgan, 2004; Blume et al, 2007; GHPS; Ivanov, 2010; Blume, 2012). In fact, for this case we have the following corollary.

Corollary 1 *Suppose that θ is uniformly distributed and $y_S(\theta) = a\theta + b$. Then, if $a > 1/2$ the following statements are equivalent: (i) an optimal mediation rule maximizes V_S, (ii) an optimal mediation rule maximizes $U_S(0)$ and (iii) an optimal mediation rule maximizes $U_S(1)$.*

The observation that optimal mediation maximizes $U_S(0)$ is exploited by GHPS to characterize the optimal mediation rule for the constant bias case (i.e. when $a = 1$, $b \neq 0$). Interestingly, as long as $a > 1/2$ a sender's optimal mediation rule must also lead to the maximum payoff for the sender at the extremes of the type space, as well as when computing the sender's payoff at an ex-ante stage.

We end this section with two caveats regarding Proposition 1. First, the equivalence (10) establishes a bijection in terms of payoffs, not decisions or even average decisions. That is, if either (6) or (7) holds, then different mediation rules that yield the same expected utility for the receiver (and thus for either the lowest or highest type of sender) may induce totally different decisions. This is simple to see by noting that in the constant bias-uniform specification and for the range of biases in which a three partition equilibrium is feasible, one can construct a mediation rule that induces either the babbling equilibrium or the three partition equilibrium with fixed probabilities and such that the expected payoff to the sender is the same at the extreme types as the two partition cheap talk equilibrium. Clearly, however, average decisions cannot coincide under both communication rules. Second, either (6) or (7) are only sufficient for the existence of a bijection between $U_S(0)$ or $U_S(1)$ and V_R, as we don't incorporate information about the set of implementable $\bar{y}$. For instance, a bijection would trivially follow when the conflict of preferences between sender and receiver is so severe that any incentive compatible mediation rule implements a single decision.

4 Art of Brevity

We now turn our attention to optimal mediation rules, and study when they involve brief conversations. For the cases that satisfy (6) or (7) with $\alpha > 1/2$, Proposition 1 establishes that M is optimal if and only if the sender's payoff in state $\hat{\theta} = i \in \{0, 1\}$ cannot be improved by any other mediation rule. Clearly, $U_S(i) \leq 0$ and $U_S(i) = 0$ if and only if the receiver selects the sender's preferred decision when his type is i. The next proposition describes a class of games that satisfy (6) or (7) and admit a brief conversation where the receiver selects the sender's preferred decision at an extreme type, and this brief conversation must then be optimal.

Proposition 2 *Suppose that either*

$$y_S(\theta) = \alpha \left(E\left[\tilde{\theta}|\tilde{\theta} \geq \theta\right] - E\left[\tilde{\theta}\right]\right), \; \textit{with } \alpha > \max_{\theta \in [0,1]} \frac{\theta}{E\left[\tilde{\theta}|\tilde{\theta} \geq \theta\right] - E\left[\tilde{\theta}\right]}, \tag{12}$$

or

$$y_S(\theta) = \alpha \left(E\left[\tilde{\theta}|\tilde{\theta} \leq \theta\right] - E\left[\tilde{\theta}\right]\right) + 1, \; \textit{with } \alpha > \max_{\theta \in [0,1]} \frac{1-\theta}{E\left[\tilde{\theta}\right] - E\left[\tilde{\theta}|\tilde{\theta} \leq \theta\right]}. \tag{13}$$

Then, there exists a brief conversation that is unimprovable for the receiver. Importantly, this equilibrium induces an infinite number of different decisions, where $y = 0$ is an accumulation point if (12) holds, and $y = 1$ is an accumulation point if (13) holds.

A notable feature of (12) and (13) is that they require full alignment between sender and receiver at some extreme type.[6] Thus Proposition 2 rules out the sender-receiver games studied in CS where preferred decisions of sender and receiver never coincide. These types of games, with an infinite type- and action-space and where the bias $b(\theta)$ may vanish or even change sign, have been studied by Gordon (2010) who characterizes communication equilibria and provides conditions for the existence of infinite equilibria.

The logic behind Proposition 2 can be seen in two steps. First, the existence of a state of full alignment implies in our case that an infinite equilibrium exists. This is not immediate as Alonso (2007) and Gordon (2010) show that even if the bias vanishes at some state only finite

[6] Alternatively, the point of alignment may be some interior θ', where (12) holds for $\theta \geq \theta'$, conditional on $\theta \in [\theta', 1]$ while (13) holds for $\theta \leq \theta'$, conditional on $\theta \in [0, \theta']$.

equilibria maybe possible. However, either (12) or (13) imply that the range of preferred decisions of the sender contains the range of preferred decisions of the receiver (i.e. the sender is more *reactive* than the receiver) and thus Theorem 2 in Gordon (2010) guarantees the existence of an infinite equilibrium. Second, an infinite equilibrium on a bounded state space must necessarily have an accumulation point at a state at which the bias disappears. This implies that an infinite equilibrium guarantees that the receiver selects the sender's (and receiver's) preferred decision at some of point of alignment. Then (12) or (13) guarantee that there is a unique point of alignment, which occurs at an extreme type. For instance, (12) implies that $y_S(\theta) > \theta$ for $\theta > 1$, thus the only point of congruence is at $\theta = 0$ implying that the infinite equilibrium must necessarily have the sender of type $\theta = 0$ inducing decision $y = 0$. As this equilibrium maximizes the sender' payoff at $\theta = 0$, it must also be optimal for the receiver. Therefore, when (12) or (13) holds brief conversations that are optimal are very detailed around the point of full alignment.

An important implication of Proposition 2 is that the one-shot communication equilibrium characterized in many applied papers cannot be improved upon by having more rounds of communication, communicating through a noisy channel, or, more generally, by employing a neutral mediator. For example, Melumad and Shibano (1991), Stein (2002), Alonso, Dessein and Matouschek (2008) and Rantakari (2008) study communication games which are equivalent to sender-receiver games with preferences over actions given by (1) with $y_S(\theta) = a\theta$, $a > 1$, and $\theta \sim U[0, 1]$. Note that this case satisfies (12) with $\alpha = 2a$. Proposition 2 then establishes that the infinite equilibrium studied in those papers is necessarily the optimal communication protocol for the receiver. The relevance of this observation is that it addresses a typical concern regarding applied models with cheap talk communication, namely, that the results and insights may not be robust to the receiver adopting a more informative communication protocol. For instance, Proposition 2 implies that the findings in Alonso, Dessein and Matouschek (2008) and Rantakari (2008) regarding the impact of internal communication on organizational structure are obtain under the optimal communication protocol.

There are other well known cases where brief conversations are optimal. First, when the conflict of interest between the sender and the receiver is extreme, either because the difference between preferred actions is large for states that are very likely[7] or because the sender's preferred decision decreases with the state, then the receiver cannot do better than simply choosing her preferred uninformed decision. In this case, optimal conversations are necessarily brief as nothing can be credibly communicated. The equilibria described in Proposition 2, however, always involve influential communication and the receiver strictly benefits from the sender's recommendation. Second, a more subtle example is presented in GHPS were they show that in the leading example of CS, when the constant bias b satisfies $b = 1/(2N^2)$ for some integer N, the most informative C-S equilibrium (which involves N different decisions) is unimprovable through mediation. That is, the constant bias case admits a non-generic set of cases where brief conversations are optimal.[8] In contrast, applying Proposition 2 one can construct sender-receiver games, as we do in Section 4.1, for which all games have a brief conversation that is optimal.

It is instructive to contrast the findings in the literature on optimal mediation in CS-type of games to Proposition 2. For the leading example in CS one has that: (i) For $b \geq 1/2$ no information transmission is possible, (ii) for $b < 1/2$, one-shot communication is generically not optimal, and (iii) if $b < 1/8$ multiple rounds of unmediated communication can achieve the maximum payoff (see GHPS for details).[9] In short, employing a mediator is most valuable when the conflict of interest is intermediate, since for small biases several rounds of cheap talk is unimprovable through mediation, while if the conflict is extreme no meaningful communication is possible, even with a neutral mediator. Further, one-shot communication can, generically, be improved upon either through long conversations or mediation.

We find however that for a state-dependent conflict of interest, the gains from mediation depend not only on the magnitude of the conflict of interest (as given by the expected bias),

[7] For instance, it is well known that for the leading example in CS, no influential communication is possible for $b > 1/2$. Moreover, as a simple corollary of Proposition 2, if $y_S(\theta) = \alpha \left(E\left[\tilde{\theta}|\tilde{\theta} \geq \theta\right] - E\left[\tilde{\theta}\right] \right) + E\left[\tilde{\theta}\right]$ then $y_S(0) = E[\theta]$ and an optimal mediation rule implements a constant decision $y = E[\theta]$. Note that this condition is compatible with a unifomly small bias in $[0, 1]$.

[8] As we noted in Corollary 1, the leading example in CS satisfies (10). The cases with $b = 1/(2N^2)$ are optimal as the sender at $\theta = 0$ obtains his preferred decision.

[9] More generally, Ivanov (2013) provides a simple sufficient condition for information transmission to be possible in sender-receiver games.

but also on the shape of the bias $b(\theta)$. Indeed, (12) imposes no upper limit on the value of α while the expected conflict increases without bound as α increases. The key difference, however, with the constant bias case is that in spite of an increased average conflict, full alignment at an extreme type persists and influential communication remains feasible.

4.1 Brief conversations, Uncertainty and the Quality of Communication

While the cheap talk setting of CS has found wide acceptance as a model of communication under conflicting preferences, applications have generally restricted attention to the "uniform-quadratic" example as expressions for the payoffs in models beyond that case have proven difficult to come by. However, for the class of sender-receiver games characterized in Proposition 2 we can explicitly compute the ex-ante payoff to the receiver under an optimal communication equilibrium.

Proposition 3 *For every $y_{i-1}, y_i \in Y$, $y_{i-1} < y_i$, define $g(y_{i-1}, y_i)$ by*

$$g(y_{i-1}, y_i) = E[\theta | \theta \in [y_{i-1}, y_i]]. \tag{14}$$

If $y_S(\theta)$ satisfies (12), then the maximum expected payoff of the receiver is

$$V_R^* = -E[\theta^2] + (1 - F(y^*)) g^2(0,1) \frac{g^2(y^*,1) - g^2(0,y^*)}{g^2(0,1) - g^2(0,y^*)} \tag{15}$$

with $y^* \in (0,1)$ the unique solution to

$$2\alpha (g(y^*,1) - g(0,1)) = g(0,y^*) + g(y^*,1). \tag{16}$$

To derive (15), the proof of the proposition constructs two incentive compatible mechanisms that give the sender the same interim utility at $\theta = 0$. The first mechanism is equivalent to a two partition equilibrium where the sender only reports whether his type exceeds a threshold y^*. Truthtelling by the sender and obedience by the receiver requires this threshold to satisfy the "arbitrage" condition (16). To define the second mechanism, M^λ, let M^* be the mechanism that implements the infinite equilbrium described in Proposition *2-i,* and let $M^\varnothing$ be the totally uninformative mediation rule (i.e. the babbling equilibrium). Then after the sender's report, with probability λ M^λ issues a recommendation according to

$M^{\varnothing}$, while with probability $1-\lambda$ it follows M^{*}. The probability λ is chosen such that the two partition equilibrium and M^{λ} yield the same $U_S(0)$. Then the linear relation (10) for games where $y_S(\theta)$ satisfies (6) implies that these two mechanisms must generate the same ex-ante payoffs for the receiver, from which we deduce that V_R^* satisfies (15).

We now use (15) to compare V_R^* for different distributions of the state. To ensure that comparative statics follow from changes in the distribution rather than the bias, in the next corollary we study three examples where the application of (12) leads to a sender's linear preferred decision $y_S(\theta)=a\theta$.

Example 1 (exponential). For each truncated exponential $f(\theta,\bar{\theta},\lambda)=\lambda e^{-\lambda\theta}/(1-e^{-\lambda\bar{\theta}}),\theta\in[0,\bar{\theta}]$, let $y_S(\theta,\bar{\theta},\lambda)$ be given by (12). Then $f(\theta,\bar{\theta},\lambda)$ converges pointwise to $\lambda e^{-\lambda\theta}$ and $y_S(\theta,\bar{\theta},\lambda)$ converges pointwise to $a\theta$ as $\bar{\theta}\to\infty$, where $a=\alpha$.[10] Finally, the limit variance of the truncated exponentials is the variance of an exponential $1/\lambda^2$.

Example 2 (linear) Consider a linear pdf that vanishes at the upper bound of the support, $f(\theta)=\frac{2}{\bar{\theta}_l}(1-\frac{\theta}{\bar{\theta}_l}),\theta\in[0,\bar{\theta}_l]$, with $Var\left[\theta\right]=\bar{\theta}_l^2/18$. Then, applying (12), we obtain $y_S(\theta)=a\theta$ with $a=2\alpha/3$.

Example 3 (uniform) Finally, applying (12) to a uniform distribution $f(\theta)=\frac{1}{\bar{\theta}_u},\theta\in[0,\bar{\theta}_u]$, with $Var\left[\theta\right]=\bar{\theta}_u^2/12$, we have $y_S(\theta)=a\theta$ with $a=\alpha/2$.

Corollary 2 *(i) For $a\geq 1$, the sender's maximum expected payoff is*

$$V_R^*=-\frac{x(a-1)}{xa-1}Var\left[\theta\right], \tag{17}$$

where $x=2$ for the limit of truncated exponentials, $x=3$ for the linear case, and $x=4$ for the uniform case.[11] *(ii) Suppose that*

$$\frac{2}{3}<\frac{\bar{\theta}_u}{\bar{\theta}_l}<\sqrt{\frac{4a-1}{2(3a-1)}}\ \textit{and}\ 3<\lambda\bar{\theta}_l<\sqrt{\frac{12\left(3a-1\right)}{2a-1}}. \tag{18}$$

[10] See proof of Corollary 2.

[11] This expression for the uniform case already appears in Alonso, Dessein and Matouschek (2008) and Rantakari (2008).

Then the expected bias is highest for the uniform case while it is lowest for the limit of truncated exponentials. However, the maximum expected payoff to the sender is highest for the uniform case but lowest for the exponential case.

As expected, (17) shows that the quality of communication improves when the conflict between sender and receiver decreases (i.e. for lower a). More interestingly, the corollary also shows that a shift in the distribution that lowers the expected bias $E\left[y_S(\theta)-\theta\right]$ can actually worsen communication and lower the receiver's expected payoff under an optimal communication protocol. Indeed, Corollary 2-ii provides a range of parameter values such that the distribution that leads to the the highest expected payoff for the receiver is also the one with the highest expected conflict.

Another way of stating this result is in terms of the receiver's benefit from communicating with the sender relative to making an uninformed decision. To this end, define the communication gain $G=(Var\left[\theta\right]-|V_R^*|)/Var\left[\theta\right]$ as the increase in the receiver's knowledge of the state due to communicating with the sender. From (17) the communication gain in our three examples is $G=\left(x-1\right)/\left(ax-1\right)$. As this expression increases in x, the communication gain is highest for a uniform distribution and lowest for the exponential distribution.

The intuition relies on the two separate roles that uncertainty plays in determining the gains from communication. All equilibria are partitional equilibria where intervals become smaller as one approaches the point of congruence at $\theta=0$. Therefore, holding constant the partition of the state space, a shift in the distribution that puts more mass on the states where communication is more detailed can only improve the receiver's payoff. However, the change in the distribution also changes the "arbitrage condition" determining the equilibrium partition. Suppose that the principal knows that the state lies in $[y,y+\Delta]$ so that her optimal choice exceeds y by $\psi(y,\Delta)=g\left(y,y+\Delta\right)-y$. That is, $\psi(y,\Delta)$ measures the responsiveness of the receiver when she knows that the state lies in an interval of length Δ. Then for a uniform $\psi(y,\Delta)$ does not vary in y while it decreases in y for the linear case. That is, the receiver becomes less responsive under a linear distribution than a uniform. To preserve incentive compatibility by the sender, the size of the intervals must be larger for a linear distribution so that the partitions are coarser under a linear distribution. Then Corollary 2-ii indicates that this second effect dominates for the range of parameters in (18) and a lower expected conflict actually leads to a lower expected payoff for the receiver.

5 Conclusion

The literature has emphasized the beneficial role of mediation in sender-receiver games where conflicting preferences hinder information transmission. We have identified a class of games, however, for which neither lengthy conversations nor mediation enhances the amount of information exchanged in equilibrium. In short, brief conversations are optimal in these cases. Importantly, the optimality of brief conversations persists even if the average conflict between the sender and the receiver is arbitrarily large. This shows that the value of mediation not only depends on the magnitude of the conflict between the sender and the receiver but also on how this conflict varies over the state space. In our case, as long as the conflict vanishes at one of the extreme points of the state space, brief conversations remain optimal.

Our proof of optimality of brief conversations (Proposition 2) relies on the existence of a one-to-one relation between the sender's interim payoffs at extreme types and the ex-ante expected payoffs of the players. This bijection also implies that optimal mediation rules are locally optimal for some sender's type. A natural question is the extent to which this assertion holds true in general. In other words, does an optimal mediation rule necessarily maximize the interim utility of some sender's type? Furthermore, our proofs made no use of the characteristics of the set of implementable average decisions, as we rely instead on properties of the payoff functions. Better understanding implementability can further our understanding of the benefits of mediation. We leave these two observations for future work.

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A Proofs

Proof of Lemma 1: Let M be an arbitrary incentive compatible mediation mechanism. We will derive the relations (2), (3) and (4) in three steps. First, we have that

$$\int_{Y\times\Theta} y\theta dp(y|\theta)dF(\theta) = \int_Y y\left(\int_\Theta \theta dg(\theta|y)\right)dH(y) = \int_Y y^2 dH(y), \tag{19}$$

where in the first equality we apply the Fubini-Tonelli Theorem and the second equality follows from (IC-R). Therefore, V_R can be written as

$$\begin{aligned} V_R &= -\int_{Y\times\Theta}(y-\theta)^2 dp(y|\theta)dF(\theta) = -\int_{Y\times\Theta}\left(y^2-2y\theta-\theta^2\right)dp(y|\theta)dF(\theta) \\ &= \int_{Y\times\Theta}\left(y\theta-\theta^2\right)dp(y|\theta)dF(\theta) = E\left[\bar{y}(\theta)\theta\right]-E\left[\theta^2\right]. \end{aligned}$$

where we applied (19) to the third equality and the law of iterated expectations to the last equality. This establishes (4) with $K_R(\theta)=\theta$.

Second, as utilities are quadratic and given (19) we immediately have

$$\begin{aligned} V_S &= -E\left[(y-y_S(\theta))^2\right] = -E\left[y^2-2yy_S(\theta)+y_S^2(\theta)\right] = \\ &= -E\left[y\theta-2yy_S(\theta)+y_S^2(\theta)\right] = -E\left[\bar{y}(\theta)\left(\theta-2y_S(\theta)\right)+y_S^2(\theta)\right], \end{aligned} \tag{20}$$

where we have again applied (19) to the third equality. This establishes (3) with $K_S(\theta) = 2y_S(\theta)-\theta$.

Third, fixing a probability measure $p(\cdot|\hat{\theta})$ from the mechanism M, the function $-\int_Y (y - y_S(\theta))^2 dp(y|\hat{\theta})$ has the same smoothness properties as $y_S(\theta)$. Our assumption that $y_S(\theta)$ is differentiable and Theorem 2 of Milgrom and Segal (2002) then imply that $U_S(\theta)$ is absolutely continuous, and for any two states θ and θ' satisfies the integral representation

$$\begin{aligned} U_S(\theta')-U_S(\theta) &= \int_\theta^{\theta'}\left(\int_Y 2(y-y_S(\tau))y_S'(\tau)dp(y|\tau)\right)d\tau \\ &= 2\int_\theta^{\theta'}\left(\bar{y}(\tau)-y_S(\tau)\right)y_S'(\tau)d\tau. \end{aligned}$$

Fixing a reference state $\tilde{\theta}$, integrating by parts, and rearranging we have

$$\begin{aligned} V_S &= U_S\left(\tilde{\theta}\right)-2\int_0^{\tilde{\theta}}\left(y(\theta)-y_S(\theta)\right)y_S'(\theta)d\theta+2\int_0^1\left(y(\theta)-y_S(\theta)\right)y_S'(\theta)\left(1-F(\theta)\right)d\theta \\ &= U_S\left(\tilde{\theta}\right)+E\left[y(\theta)\left(2y_S'(\theta)\frac{1-F(\theta)}{f(\theta)}-2\frac{y_S'(\theta)}{f(\theta)}I_{[0,\tilde{\theta}]}\right)\right]-E\left[y_S^2(\theta)\right]+y_S^2(\tilde{\theta}) \end{aligned}$$

where $I_{[0,\tilde{\theta}]}$ is the characteristic function of the set $[0,\tilde{\theta}]$ (i.e. $I_{[0,\tilde{\theta}]}(x)=1$ if $x\in[0,\tilde{\theta}]$ and $I_{[0,\tilde{\theta}]}(x)=0$ otherwise). Using the expression for V_S given in (20) and substituting above we obtain (2) with $K_{S(\hat{\theta})}$ given by (5). ■

Proof of Proposition 1: (i) Suppose that (6) holds so that

$$y_S'(\theta)=\alpha h(\theta)\left(E\left[\theta'|\theta'\geq\theta\right]-\theta\right),$$

implying

$$\begin{aligned}
K_{S(0)}(\theta) &= 2y_S(\theta) - \theta - 2y_S'(\theta)\frac{1}{h(\theta)} \\
&= 2\alpha E[\theta'|\theta' \geq \theta] + 2\beta - \theta - 2\alpha(-\theta + E[\theta'|\theta' \geq \theta]) \\
&= (2\alpha - 1)\theta + 2\beta = (2\alpha - 1)K_R + 2\beta.
\end{aligned}$$

Conversely, if (7) holds then

$$y_S'(\theta) = \alpha r(\theta)(\theta - E[\theta'|\theta' \leq \theta]),$$

so that

$$\begin{aligned}
K_{S(1)}(\theta) &= 2y_S(\theta) - \theta + 2y_S'(\theta)\frac{1}{r(\theta)} \\
&= 2\alpha E[\theta'|\theta' \leq \theta] + 2\beta - \theta + 2\alpha(\theta - E[\theta'|\theta' \leq \theta]) \\
&= (2\alpha - 1)\theta + 2\beta = (2\alpha - 1)K_R + 2\beta
\end{aligned}$$

Let $i = 0$ if (6) holds, and $i = 1$ if (7) holds. Then,

$$\begin{aligned}
U_S(i) &= E\left[\bar{y}(\theta)K_{S(i)}(\theta)\right] - y_S^2(i) = E[\bar{y}(\theta)((2\alpha - 1)K_R + 2\beta)] - y_S^2(i) = \\
&= (2\alpha - 1)E[\bar{y}(\theta)K_R] + 2E[\bar{y}(\theta)\beta] - y_S^2(i) \\
&= (2\alpha - 1)V_R + (2\alpha - 1)E\left[\theta^2\right] + 2\beta E[\bar{y}(\theta)] - y_S^2(i).
\end{aligned}$$

Applying the law of iterated expectations to (IC-R) one readily obtains

$$E[\bar{y}(\theta)] = E[y] = E[E[\theta|y]] = E[\theta]. \tag{21}$$

Therefore, for any incentive compatible mediation rule we have

$$U_S(i) = (2\alpha - 1)V_R + C_R, \tag{22}$$

with $C_R = (2\alpha - 1)E\left[\theta^2\right] + 2\beta E[\theta] - y_S^2(i)$ finite and independent of the mediation mechanism. This establishes (10). Finally, if $\alpha > 1/2$ then (22) implies that there is a linear and increasing relation between $U_S(i)$ and the ex-ante payoff to the receiver V_R. Therefore, a mediation rule achieves the maximum $U_S(i)$ if and only if it maximizes V_R.

(ii) If $y_S'(\theta) = \alpha h(\theta)$, then $K_{S(0)}$ can be written as

$$K_{S(0)}(\theta) = 2y_S(\theta) - \theta - 2\alpha = K_S(\theta) - 2\alpha,$$

while if $y'_S(\theta) = \alpha r(\theta)$, then $K_{S(1)}$ can be written as

$$K_{S(1)}(\theta) = 2y_S(\theta) - \theta + 2\alpha = K_S(\theta) + 2\alpha$$

As average decisions must equal the state (as shown in (21)), then letting $i = 0$ if (8) is satisfied, and $i = 1$ if (9) is satisfied, we can write

$$\begin{aligned} U_S(i) &= E\left[\bar{y}(\theta)K_{S(i)}(\theta)\right] - y_S^2(i) = E\left[\bar{y}(\theta)K_S(\theta)\right] - 2(1-2i)E[\theta] - y_S^2(i) = \\ &= V_S + C_S, \end{aligned}$$

with $C_S = -2(1-2i)E[\theta] - y_S^2(i)$ finite and independent of the mediation mechanism, from which (11) follows. ■

Proof of Corollary 1: If $y_S(\theta) = a\theta + b$ then clearly (6) and (7) are both satisfied with $\alpha = 2a$. Then (10) implies that if $a > 1/4$ a mediation rule is optimal if and only if it maximizes $U_S(i)$, $i \in \{0,1\}$. Moreover, as preferred decisions are linear, and applying both (19) and (21) we have

$$\begin{aligned} V_S &= -E\left[\left(y-(a\theta+b)^2\right)\right] = -E\left[y^2 - 2ay\theta - 2by + (a\theta+b)^2\right] = \\ &= (2a-1)E[y\theta] - E\left[-2by + (a\theta+b)^2\right] = (2a-1)V_R + \tilde{C} \end{aligned}$$

with $\tilde{C} = E\left[(2a-1)\theta^2 + 2b\theta - (a\theta+b)^2\right]$. Thus for $a > 1/2$, V_S is increasing in V_R. Overall, if $a > 1/2$ we have that optimal mediation rules maximize the interim payoff of extreme types, and also the sender's expected payoff and, conversely, any mechanism that maximizes the interim payoff at an extreme state must necessarily be ex-ante optimal for both the sender and the receiver. ■

Proof of Proposition 2: Let $Y_S = \{y : y_S(\theta) = y\}$ which is a connected set given the continuity of $y_S(\theta)$ as implied by either (12) or (13). Then, if either (12) or (13) holds then $[0,1] \subset Y_S$. This implies that the sender is reactive and an infinite equilibrium exists (Gordon 2010, Theorem 2). Furthermore if (12) holds then $y_S(\theta) > \theta$ for $\theta \in (0,1]$. This means that the unique point of alignment is $\theta = 0$ and the equilibrium with an infinite number of actions must necessarily have an accumulation point at $y = y_S(0) = 0$, and thus $U_S(0) = 0$. As (12) satisfies (6) and the bound condition on α guarantees $\alpha > 1$ then (10) holds and the infinite equilibrium must be optimal. Conversely, if (13) holds then $y_S(\theta) < \theta$ for $\theta \in [0,1)$. This means that the unique point of alignment is $\theta = 1$ and the equilibrium with an infinite number of actions must necessarily have an accumulation point at $y = y_S(1) = 1$ and $U_S(1) = 1$. As (13) satisfies (7) and the bound condition on α guarantees $\alpha > 1$, then (10) holds and the infinite equilibrium must be optimal. ■

Proof of Proposition 3: To obtain (15) we will use the relation (10) and the fact that the set of implementable $\bar{y}$ is convex. To see this last point note that for any two incentive compatible M' and M'', that induce $\bar{y}'$ and $\bar{y}''$, a mediation rule that with probability λ issues recommendations according to M' and with probability $1-\lambda$ according to M'',where, importantly, the probability λ does not vary with the report of the sender, is incentive compatible and induces an average decision $\lambda\bar{y}' + (1-\lambda)\bar{y}''$. Let M^* replicate the infinite equilibrium described in Proposition 2 and let $M^{\varnothing}$ replicate the babbling equilibrium (i.e. under $M^{\varnothing}$ the receiver selects a single decision $E[\theta]$ is induced).

We now construct a two partition equilibrium of the cheap talk game when y_S is given by (12). In such equilibrium the sender only discloses whether the state is above or below y^*, the receiver selects $g(0, y^*) = E[\theta|\theta \in [0, y^*]]$ if $\theta \leq y^*$ and $g(y^*, 1) = E[\theta|\theta \in [y^*, 1]]$ otherwise, and y^* must satisfy the arbitrage condition

$$y_S(y^*) - g(0, y^*) = g(y^*, 1) - y_S(y^*). \tag{23}$$

As $y_S(\theta)$ that satisfies (12) can be expressed as $y_S(\theta) = \alpha(g(\theta, 1) - g(0, 1))$, the existence of a solution $y^* \in [0, 1]$ to (23) then requires the existence of a solution to

$$2\alpha = \frac{g(0, y^*) + g(y^*, 1)}{g(y^*, 1) - g(0, 1)} \tag{24}$$

The right hand side of (24) is decreasing in y^* and achieves a minimum $(1 + E[\theta]) / (1 - E[\theta])$ when $y^* = 1$. From (12) then we have

$$2\alpha > \max \frac{\theta}{E\left[\tilde{\theta}|\tilde{\theta} \geq \theta\right] - E\left[\tilde{\theta}\right]} \geq \frac{2}{1 - E[\theta]} \geq \frac{1 + E[\theta]}{1 - E[\theta]}$$

This implies that we can always find an y^* that solves (24) and a two partition equilibrium exists. Denote by M^2 the mediation rule that induces this two partition equilibrium. The receiver's expected utility under M^2 is

$$\begin{aligned} V_R^{M^2} &= -\int_0^{y^*} (g(0, y^*) - \theta)^2 \, dF(\theta) - \int_{y^*}^1 (g(y^*, 1) - \theta)^2 \, dF(\theta) = \\ &= -E\left[\theta^2\right] + F(y^*) \, g^2(0, y^*) + (1 - F(y^*)) \, g^2(y^*, 1). \end{aligned} \tag{25}$$

Next consider the mediation mechanism M^{λ} that is a convex combination of M^* and $M^{\varnothing}$, that is with probability λ the mechanism M^{λ} issues the same recommendation as $M^{\varnothing}$ while with probability $1-\lambda$ it issues the same recommendation as M^*. We then have

$$V_R^{M^{\lambda}} = -\lambda Var\theta + (1-\lambda) V_R^*. \tag{26}$$

From Proposition 2, under the optimal one-shot equilibrium we have $U_S^{M^*}(0) = 0$. Moreover, $U_S^{M^2}(0) = -g^2(0, y^*)$ and $U_S^{M^{\varnothing}} = -g^2(0,1) < U_S^{M^2}(0)$. Therefore, there exists $\bar{\lambda}$ such that $U_S^{M^{\lambda}}(0) = U_S^{M^2}(0)$, which is then given by

$$\begin{aligned} -g^2(0, y^*) &= -\bar{\lambda} g^2(0,1) + (1-\bar{\lambda}) U_S^{M^*}(0), \\ \bar{\lambda} &= \frac{g^2(0, y^*)}{g^2(0,1)}, \end{aligned} \tag{27}$$

and (26) leads to

$$(1-\bar{\lambda}) V_R^{M^*} = V_R^{M^2} + \bar{\lambda} Var\theta$$

substituting the value $\bar{\lambda}$ given by (27) and $V_R^{M^2}$ given by (25) into this expression one obtains (15). ■

Proof of Corollary 2: First consider a truncated exponential of parameter λ, $f(\theta) = \frac{\lambda e^{-\lambda\theta}}{1-e^{-\lambda\bar{\theta}}}, \theta \in [0, \bar{\theta}]$. Then (12) translates to

$$y_S(\theta) = \frac{\alpha}{1-e^{-\lambda(\bar{\theta}-\theta)}} \left(\theta - \bar{\theta} e^{-\lambda(\bar{\theta}-\theta)} \frac{\left(1-e^{-\lambda\theta}\right)}{\left(1-e^{-\lambda\bar{\theta}}\right)} \right),$$

and pointwise we have $y_S(\theta) \to \alpha\theta$ as $\bar{\theta} \to \infty$. Taking the limit as $\bar{\theta} \to \infty$ to the arbitrage condition (16) gives

$$2\left(1 - \lambda y^* (a-1)\right) = \frac{\lambda y^*}{1-e^{-\lambda y^*}},$$

and (15) gives

$$\begin{aligned} V_R^* &= -\frac{2}{\lambda^2} + \frac{1}{\lambda^2} \left[1 + \frac{y^* \lambda \left(1-e^{-\lambda y^*}\right)}{2\left(1-e^{-\lambda y^*}\right) - y^* \lambda e^{-\lambda y^*}} \right] \\ &= -\frac{2(a-1)}{2a-1} \frac{1}{\lambda^2} = -\frac{2(a-1)}{2a-1} Var[\theta]. \end{aligned}$$

Now consider the linear case where (12) translates to $y_S(\theta) = \frac{2}{3}\alpha\theta$ with $\frac{2}{3}\alpha > 1$, implying that $\alpha = \frac{3}{2}a$ with $a > 1$. The solution to (16) is

$$y^* = \left(1 - \frac{\sqrt{36a^2 - 48a + 17} - 1}{6a - 4} \right) \bar{\theta}_l,$$

which, substituted in (15) leads to

$$\begin{aligned} V_R^* &= -\frac{1}{18}\bar{\theta}_l^2 \frac{-3\bar{\theta}_l y^* + \bar{\theta}_l^2 + (y^*)^2}{\bar{\theta}_l y^* + \bar{\theta}_l^2 - (y^*)^2} \\ &= -\frac{(a-1)}{3a-1}\frac{\bar{\theta}_l^2}{6} = -\frac{3(a-1)}{3a-1} Var[\theta]. \end{aligned}$$

Finally, consider the uniform case where (12) translates to $y_S(\theta) = \frac{\alpha}{2}\theta$ with $\frac{\alpha}{2} > 1$, so that $\alpha = 2a$ with $a > 1$. The solution to (16) is

$$y^* = \frac{\bar{\theta}_u}{2(2a-1)},$$

which, substituted in (15) leads to

$$\begin{aligned} V_R^* &= -\frac{\bar{\theta}_u - 2y^*}{12(\bar{\theta}_u + y^*)}\bar{\theta}_u^2 = \\ &= -\frac{a-1}{3(4a-1)}\bar{\theta}_u^2 = -\frac{4(a-1)}{4a-1} Var[\theta]. \end{aligned}$$

Part ii, follows from the fact that expected bias for the exponential, linear and uniform case are $(a-1)/\lambda$, $(a-1)/3\bar{\theta}_l$ and $(a-1)/2\bar{\theta}_u$ while the maximum expected payoff to the receiver in each case is $2(a-1)/\lambda^2(2a-1)$, $(a-1)\bar{\theta}_l^2/(18a-6)$ and $(a-1)\bar{\theta}_u^2/(12a-3)$.
■

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