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Working paper

On the Value of Persuasion by Experts *

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Abstract

A sender can influence the behavior of a receiver by controlling the informativeness of a public signal. We show that the sender cannot benefit from becoming an expert, that is, from privately learning some information about the state. We then show that in some instances an uninformed sender is ex-ante strictly better off than an expert sender.

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1 Introduction

When does an expert benefit from her private information? We consider a general class of Bayesian persuasion games where a sender can influence the behavior of a receiver by controlling the informativeness of a public signal. In the original model of Kamenica and Gentzkow (2011) (KG henceforth), the sender commits to the public signal prior to observing any private information about the state. We extend the model to consider senders who are experts, that is, senders who privately observe some information about the state prior to choosing the public signal. By becoming an expert, the sender could possibly adjust the public signal according to her new information. However, we show that the sender cannot benefit from becoming an expert. We establish this result by following a replication argument in much the same manner as the revelation principle: for any equilibrium of the informed sender's game, one can design a feasible signal of the uninformed sender's game that induces the same joint distribution over duples of action and state. We then show that in many instances an ignorant sender is ex-ante strictly better off than an expert sender.

Our paper is related to the recent literature that studies the strategic design of a public signal by an informed sender. Gill and Sgroi (2008, 2012) consider a privately-informed principal who can subject herself to a test designed to provide public information about her type, and can optimally choose the test's difficulty. Li and Li (2013) study a privately-informed candidate who can choose the accuracy of a costly public signal (campaign) about the qualifications of the politicians competing for office. Rayo and Segal (2010) study optimal advertising when a company can design how to reveal the attributes of its product, but it cannot distort this information. In those models the sender is constrained on the set of signals that she can choose from. We take a different approach and study the class of Bayesian persuasion models where the sender is unconstrained on her choice. As in KG, we allow the sender to choose any signal that is correlated with the state. We contrast the sender studied by KG, who can commit to a public signal prior to becoming privately informed, to a sender who is privately informed before she can choose a public signal. Perez-Richet (2014) considers an informed sender who might be constrained on her choice of a signal. In his model the receiver can only take two actions (validation or non-validation) and there are only two types of senders, who receive the same net payoff from validation. In contrast to that paper, we consider any finite set of types for the sender, any compact action space for the receiver, and allow the sender's utility to depend on the state. While Perez-Richet focus on describing the equilibrium of the game in the binary state-binary type setup, our main objective is to derived an upper bound on the sender's benefit from private information.

2 The Model

Our model features a game between a sender (she) and a receiver (he). The receiver chooses an action that affects the utility of both players. The sender has no authority over the receiver's actions, yet she can influence them through the design of a public signal that is correlated with the state. We contrast two cases. The first case is the one studied by KG: the sender has no private information about the state, or equivalently, she can commit to the public signal (or to a disclosure rule) before becoming privately informed. In the second case, the sender has private information about the state and cannot commit to the public signal before becoming informed.

Preferences and Prior Beliefs: All players are expected utility maximizers and process information according to Bayes rule. The receiver selects an action a from a compact set A. The sender and the receiver have preferences over actions characterized by continuous von Neumann-Morgenstern utility functions $u_S(a, \theta)$ and $u_R(a, \theta)$, with $\theta \in \Theta$ and Θ a finite state space. Players share a common prior belief p belonging to the interior of the simplex $\Delta(\Theta)$.

Private Information: The sender privately observes the realization of an exogenous signal π_e, with finite realization space. Let the sender's type $t \in \Delta(\Theta)$ represent her interim belief after observing the signal's realization, i.e., $\Pr[\theta|t] = t_\theta$, and $\beta(t)$ the probability of t. Let T be the (finite) set of possible beliefs induced by π_e. Bayes rules requires $E_\beta[t] = \sum_{t \in T} \beta(t)t = p$. The set T and the probabilities $\beta(t)$ are common knowledge.

Information Control: After observing her private signal but before the receiver chooses his action, the sender supplies a signal π, consisting of a finite realization space Z_π and a family of likelihood functions over Z_π, $\{\pi(\cdot|\theta)\}_{\theta \in \Theta}$, with $\pi(\cdot|\theta) \in \Delta(Z_\pi)$. Signal π is "commonly understood": π is observed by all players who agree on the likelihood functions $\pi(\cdot|\theta)$, $\theta \in \Theta$ (see Alonso and Câmara 2014 for a discussion of this assumption). We retain the assumptions of KG regarding the set of signals available to the controller: she can choose *any signal* that is correlated with the state, and signals are costless to the controller.

While the sender is free to design any signal π, she can only supply one signal. We allow the sender to use a mixed strategy σ over any finite set of signals Π. Formally, let Π_t be set of signals chosen with positive probability and let $\sigma(\pi|t)$ is the probability that a sender with type t chooses signal $\pi \in \Pi$. Let $\Pi = \cup_{t\in T}\Pi_t$ be the set of signals that are chosen by some sender's type.

Timing: The sender privately learns her type t and then choses signal π. The receiver observes π and its realization $z_\pi \in Z_\pi$, and chooses an action $a(z_\pi, \pi) \in A$. Payoffs are then realized.

Sender's Equilibrium Payoff: We consider Perfect Bayesian equilibria. If signal π_e is uninformative ($t = p$ for all $t \in T$), then the sender is uninformed and the game is equivalent to the model of KG. In this case, an equilibrium exists and we can use the results from KG to compute the sender's maximum expected equilibrium payoff V_U. If signal π_e is informative and an equilibrium exists, then let $V_I(t)$ be the equilibrium payoff of a sender with type t, and $\sum_{t\in T} \beta(t)V_I(t)$ the weighted sum of payoffs across all the sender's types.

3 Unimprovability of Expected Payoffs under Ignorance

We first show that privately obseving an informative signal prior to designing the public signal does not confer any advantage to the sender. Indeed, we show that, in any Perfect Bayesian equilibrium of the informed sender's game, averaging the payoffs of each type of sender cannot improve upon the sender's equilibrium payoff in a game in which she has no informational advantage over the receiver. We establish this result by following a replication argument in much the same manner as the revelation principle: For any equilibrium of the informed sender's game, one can design a feasible signal of the uninformed sender's game that induces the same joint distribution over duples of (action, state) in $A \times \Theta$.

Proposition 1 *For any equilibrium of the informed sender game, with type-dependent payoffs $V_I(t)$, the sender cannot benefit from her expertise,*

$$V_U \geq \sum_{t \in T} \beta(t) V_I(t).$$

Proof: In equilibrium, the expected utility of type t sender from signal π is

$$W(\pi, t) = \sum\nolimits_{Z_\pi \times \Theta} u_S(a(\pi, z_\pi), \theta) \Psi \left((z_\pi, \theta) \,|\pi, t \right),$$

where

$$\Psi \left((z_\pi, \theta) \,|\pi, t \right) = \pi \left(z_\pi | \theta \right) t_\theta,$$

$$a(z_\pi, \pi) \in \arg\max \mathrm{E}_{\mu(z_\pi, \pi)} \left[u_R(a, \theta) \right],$$

and $\mu \left(z_\pi, \pi \right)$ is derived from Bayes rule according to

$$\mu_\theta \left(z_\pi, \pi \right) = \frac{\pi \left(z_\pi | \theta \right) q_\theta^R(\pi)}{\sum_{\theta' \in \Theta} \pi \left(z_\pi | \theta' \right) q_{\theta'}^R(\pi)}, \text{ with } q^R(\pi) = \frac{\sum_{t' \in T} \sigma(\pi | t') \beta(t') t'}{\sum_{t' \in T} \sigma(\pi | t') \beta(t')}.$$

Therefore, the equilibrium expected payoff obtained by type t is

$$V_I(t) = \sum\nolimits_{\pi \in \Pi} W(\pi, t) \sigma(\pi | t) = \sum\nolimits_{\pi \in \Pi} \sigma(\pi | t) \sum\nolimits_{Z_\pi \times \Theta} u_S(a(\pi, z_\pi), \theta) \Psi \left((z_\pi, \theta) \,|\pi, t \right),$$

and the ex-ante expected utility over all types $\mathrm{E}\left[V_I(t) \right]$ would be

$$\sum\nolimits_{t \in T} V_I(t) \beta(t) = \sum\nolimits_{t \in T} \beta(t) \sum\nolimits_{\pi \in \Pi} \sigma(\pi | t) \sum\nolimits_{Z_\pi \times \Theta} u_S(a(\pi, z_\pi), \theta) \Psi \left((z_\pi, \theta) \,|\pi, t \right).$$

Define a new signal $\pi_u = \left\{ Z_u, \left\{ \Lambda \left(\cdot | \theta \right) \right\}_{\theta \in \Theta} \right\}$ with signal realization space

$$Z_u = \left\{ (\pi, z_\pi) \in \Pi \times \cup_{\pi \in \Pi} Z_\pi : z_\pi \in Z_\pi \right\},$$

and satisfying

$$\Lambda \left(\pi, z_\pi | \theta \right) p_\theta = \sum\nolimits_{t \in T} \Psi \left(z_\pi, \theta | \pi, t \right) \sigma(\pi | t) \beta(t)$$

for each $(\pi, z_\pi) \in Z_u$. Interchangeability of Bayesian updating implies that

$$\begin{aligned}
\mu_\theta \left(z_\pi, \pi \right) &= \frac{\pi \left(z_\pi | \theta \right) \sum_{t' \in T} \sigma(\pi | t') \beta(t') t'_\theta}{\sum_{\theta' \in \Theta} \pi \left(z_\pi | \theta' \right) \sum_{t' \in T} \sigma(\pi | t') \beta(t') t'_{\theta'}} \\
&= \frac{\sum_{t' \in T} \Psi \left((z_\pi, \theta) \,|\pi, t \right) \sigma(\pi | t') \beta(t')}{\sum_{\theta' \in \Theta} \sum_{t' \in T} \Psi \left((z_\pi, \theta) \,|\pi, t \right) \sigma(\pi | t') \beta(t') t'_{\theta'}} \\
&= \frac{\Lambda \left(\pi, z_\pi | \theta \right) p_\theta}{\sum_{\theta' \in \Theta} \Lambda \left(\pi, z_\pi | \theta' \right) p_{\theta'}} \\
&= \mu_\theta^R \left(z_\pi, \pi \right).
\end{aligned}$$

Therefore, $a(\pi, z_\pi) \in \arg\max \mathrm{E}_{\mu^R(z_\pi,\pi)}\left[u_R(a,\theta)\right]$ with $\mu^R_\theta(z_\pi,\pi) = \frac{\Lambda(\pi,z_\pi|\theta)p_\theta}{\sum_{\theta'\in\Theta}\Lambda(\pi,z_\pi|\theta')p_{\theta'}}$, so $a(\pi, z_\pi)$ is an optimal response by the receiver under π_u. The expected utility of an uninformed sender when offering signal π_u is

$$\mathrm{E}_S\left[u_S|\pi_u\right] = \sum\nolimits_{Z_u\times\Theta} u_S(a(\pi,z_\pi),\theta)\Lambda\left(\pi, z_\pi|\theta\right)p_\theta.$$

By interchanging the order of summation we have

$$\begin{aligned}
\mathrm{E}_S\left[u_S|\pi_u\right] &= \sum\nolimits_{Z_u\times\Theta} u_S(a(\pi,z_\pi),\theta)\Lambda\left(\pi, z_\pi|\theta\right)p_\theta \\
&= \sum\nolimits_{t\in T}\beta(t)\sum\nolimits_{(Z_u)\times\Theta} u_S(a(\pi,z_\pi),\theta)\Psi\left(z_\pi,\theta|\pi,t\right)\sigma(\pi|t) \\
&= \sum\nolimits_{t\in T}\beta(t)\sum\nolimits_{\pi\in\Pi}\sigma(\pi|t)\sum\nolimits_{Z_\pi\times\Theta} u_S(a(\pi,z_\pi),\theta)\Psi\left(z_\pi,\theta|\pi,t\right) \\
&= \sum\nolimits_{t\in T}\beta_t V_I(t) \leq V_U.
\end{aligned}$$

■

By gathering information about the state prior to designing a signal, the sender can choose a signal that induces actions in the receiver that have a different correlation with the underlying state. However, the receiver's posterior belief is pinned down by the sender's behavior and the observed signal realization for the chosen signal. Therefore, an uninformed sender can replicate this behavior by designing a signal that, from an ex-ante perspective, induces the same joint-distribution between receiver's beliefs and the state. As any weighted average of type-dependent payoffs can be replicated by an ignorant sender, having access to a private signal of the state cannot allow the sender to improve her payoff.

Proposition 1 established that the sender cannot benefit from observing π_e. Can the sender be made strictly worse off because of her expertise? The following example shows that this is indeed possible. Let $\Theta = \{\underline{\theta}, \overline{\theta}\} \subset \mathbb{R}$, where $\underline{\theta} < \overline{\theta}$. Let $A = [\underline{\theta}, \overline{\theta}]$ and $u_R(a,\theta) = -(a-\theta)^2$, so that a receiver with interim belief q chooses action $a^*(q) = \sum_{\theta\in\Theta} q_\theta\theta$. Consider a sender who strictly benefits from higher actions, $u_S(a,\theta) = f(a)$ where $f'(a) > 0$.

Suppose that it is not optimal for the uninformed sender to choose a fully informative public signal. In this case, if the private signal π_e induces at least one sender's type to be sufficiently confident about $\overline{\theta}$, then the sender is on average hurt by her expertise. This is so because there is no equilibrium in the informed sender game that induces the same probability distribution over receiver's actions as in the uninformed sender game. To see this, by contradiction, suppose that there is an equilibrium in the informed sender game that induces the same probability over receiver's actions as in the equilibrium of the uninformed sender game. Since the uninformed sender does not choose a fully revealing signal, there exists at least one signal realization that is induced with strictly positive probability by both states. Therefore, the receiver takes with strictly positive probability at least one action strictly lower than $\overline{\theta}$. This implies that conditional on the state being $\overline{\theta}$, the receiver on average takes an action strictly lower than $\overline{\theta}$. Consequently, a sender's type who is sufficiently confident about $\overline{\theta}$ is strictly better by deviating to a fully informative signal, a contradiction to the equilibrium assumption.

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