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# *On the Confirmation of the Law of Demand*

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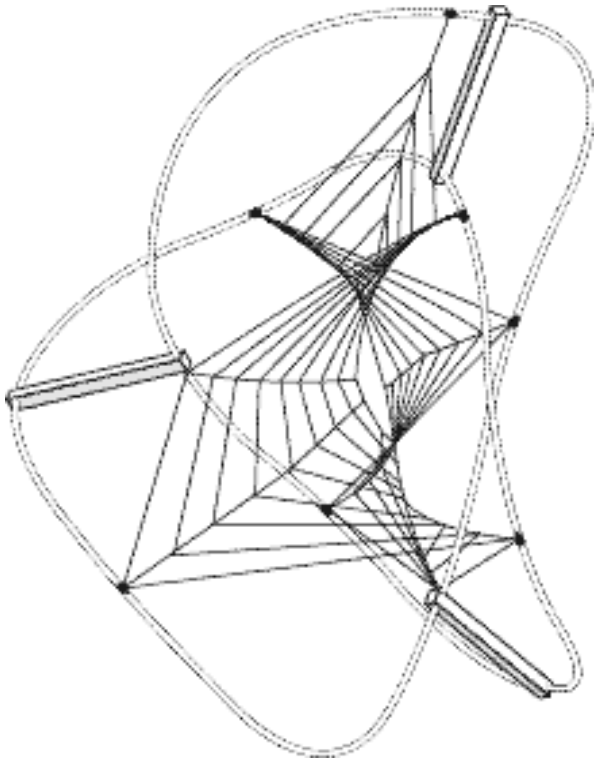

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# ON THE CONFIRMATION OF THE LAW OF DEMAND

Philippe Mongin

**Abstract**: The paper applies confirmation theory to a famous statement of economics, the law of demand, which says that ceteris paribus, prices and quantities demanded change in opposite directions. Today's economists do not accept the law unless definite restrictions hold, and have shown little interest in deciding whether or not these restrictions were satisfied empirically. However, Hildenbrand (1994) has provided a new derivation of the law of aggregate demand and used this theoretical advance to devise a test that may be the first rigorous one ever performed on the law. The paper accounts for Hildenbrand's and, in less detail, his predecessors' contributions within the philosophical framework of Hempel (1965) and Glymour (1980). Its salient result is that economists have accepted the "consequence condition", and rejected the "converse consequence condition", and thus implicitly adhered to a Hempelian-Glymourian view of confirmation and testability.

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# On the Confirmation of the Law of Demand

## 1. Introduction

The present paper deals with one of the famous topics of old and modern economics — the *law of demand*. Among the early statements, Marshall's (1890) is the most famous. It simply says that the consumer's demand for a good varies inversely with its price, given that every other price and the consumer's income remain fixed. After Hicks (1939), Samuelson (1947), and their post-war followers (Chipman et alii, 1971) rebuilt consumer theory, economists stopped claiming the inverse relation of price and demand in Marshall's straightforward way. As any microeconomics text explains nowadays, a price change triggers out two separate effects on demand which may go in opposite directions, with the consequence that the final effect is indeterminate. This explains why, despite its obviously strong practical importance and possibly high theoretical relevance, the law of demand has virtually disappeared from the scene of economics. But there is a recent major exception to this state of affairs. A mathematical economist well-known to the profession, Hildenbrand (1994) has managed to recover an aggregate version of the law from assumptions put on the income distribution. He uses this novel derivation in order to carry out an econometric test of the law against consumption data provided by national statistics. By and large, Hildenbrand claims that his empirical check comforts the time-honoured statement, a conclusion that is the more striking since Hildenbrand also argues that economists have never attempted a serious test before.

This paper has been motivated by Hildenbrand's findings, but given the paucity of philosophy-of-science discussions of the law of demand, I investigate it generally, and this leads me to explore consumer theory, one of the best structured parts of today's neoclassically inspired microeconomics. I have selected my case study despite two slight drawbacks - for one, consumer theory

is fairly mathematical, for another, Hildenbrand's programme is still ongoing - because it is exceptionally useful to understand how economists conceive of, and attempt to solve, the classic problem of the confirmation of scientific theories. I carry out the philosophical investigation within the framework of Hempel's (1965) tentative requirements for a suitable confirmation concept, prominent among which are the *consequence condition* and the *converse consequence condition*. As these requirements conflict with each other, it seems relevant both to confirmation theory and economic methodology to find out which of the two effectively prevails when it comes to confirming the law of demand. I argue that Hildenbrand puts this duality in sharp focus, and that he implicitly endorses the consequence condition and rejects the converse consequence condition, exactly as Hempel and his followers, like Glymour (1980), would recommend. I briefly extend this Hempelian interpretation to the earlier justifications of the law of the demand, and would push it more generally for economics if space permitted.

## 2. Consumer theory and the law of demand

Consumer theory relies on the virtually unique hypothesis that the individual consumer maximizes the utility of his basket of goods under the constraint set by his money income and the prevailing market prices. This hypothesis entails the following theorem due to Slutsky (1915) and developed by Hicks (1939): a small change in the price of a good *j* every other price and the consumer's income remaining constant, brings about a change in the demand for a good *k* that decomposes additively into a *substitution* and an *income effect*. The first effect measures the change in demand *while making allowance for the change in the consumer's real income induced by the price change*. The second effect measures the residual, i.e., the change in demand resulting from the change in real income. When changes in price and in demand relate to the same good, the Maximization Hypothesis entails that the substitution effect goes in the direction predicted by the law of demand, i.e., price and demand changes have opposite signs. By contrast, the hypothesis never restricts the direction of the income effect. In the case of an own price change, the latter may run counter to the former, and if large enough, defeat the

law. For concreteness, think of an increase in the price of bread, with other prices and income remaining constant. The increased cost of bread relative to other commodities would lead the consumer to substitute the latter for the former if his effective wealth - or real income, to be distinguished from his fixed money income - remained the same; however, the consumer is also impoverished by the price increase, and the consequence of this underlying change is prima facie unpredictable. After all, it may be best for the consumer to buy more bread, just because its increased cost makes meat and wine less affordable than they were before. This theoretical possibility is central to modern consumer theory.

I now restate these arguments formally, using differential calculus in the economists' way.[1] Denote by $x = (x_1,...,x_l) \in R^l_+$ the physically available baskets of $l$ goods, by $p = (p_1,...,p_l) \in R^l_{+*}$ the price vector, and by $I$ the consumer's budget.[2] Assuming that he maximizes a utility function $u(x)$ under the budget constraint $p \cdot x = I$, and that this maximization problem has a well-defined and unique solution, one derives a *demand function* $x_j(p, I)$ for each good $j$. Denote by $(\bar{p}, \bar{I})$ the initial values of prices and income, and consider the mapping: $p \mapsto s(p, \bar{x}) = x(p, p \cdot \bar{x}))$. The value of $s$ represents the basket of goods that the consumer would select at the new price vector $p$, if, for these prices, he could afford the initial basket $\bar{x} = x(\bar{p}, \bar{I}))$. The supposition is a precise rendering of the vague expression "the consumer's real income is unchanged". The mapping $s$ is called the *Slutsky compensated demand function* because Slutsky (1915) introduced this rendering; an alternative one, due to Hicks (1939), will be introduced shortly. To investigate the effect of a price change $p_k$ under the ceteris paribus clause that $p_l, l \neq k$, and $I$ are constant, we differentiate $s$ partially with respect to $p_k$, taking into account the budget constraint equation (the latter requires that all the budget be spent).

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[1] For a fuller exposition, the reader may consult Mas-Colell, Whinston and Green (1995).

[2] The notation $R^l_+$ and $R^l_{+*}$ refers to the nonnegative orthant of the *l*-dimensional Euclidean space, with and without $0$ included respectively.

Evaluating the derivatives at $p = \bar{p}$, we obtain the *Slutsky equations*, which hold for all goods $j, k$, and all initial values $\bar{p}, \bar{I}$:

$$(*)\ \frac{\partial x_j(\bar{p},\bar{I})}{\partial p_k} = \frac{\partial s_j(\bar{p},\bar{I})}{\partial p_k} - \frac{\partial x_j(\bar{p},\bar{I})}{\partial I} \cdot x_k(\bar{p},\bar{I})).$$ [3]

Owing to (*), the effect on demand $\Delta x_j$ of a small change $\Delta p_k$ can be decomposed as the sum of two elementary effects:

$$\Delta x_j \approx \frac{\partial x_j(\bar{p},\bar{I})}{\partial p_k} \cdot \Delta p_k = \left[ \frac{\partial s_j(\bar{p},\bar{I}))}{\partial p_k} \cdot \Delta p_k \right] + \left[ -\frac{\partial x_j(\bar{p},\bar{I})}{\partial I} \cdot x_k(\bar{p},\bar{I}) \cdot \Delta p_k \right]$$

The first term is the *substitution effect*, and the second, the *income effect*; we thus have reached the decomposition sketched informally. Notice that the additive formula can only hold as an approximation for discrete - as against infinitesimal - changes $\Delta p_k$.

There is a more famous, but also more complex way of obtaining the decomposition. Denote by $\bar{v}$ the utility value reached initially, i.e., $\bar{v} = u(\bar{x}) = u(x(\bar{p},\bar{I}))$, and consider the mapping $p \mapsto h(p,\bar{v})$, which by definition associates to $p$ the demand vector $x(p,I)$ leading to the utility value $\bar{v}$ for an appropriately changed $I$. I.e., the value of $h$ is the basket of goods that the consumer would select at the new prices if his income were adjusted so as to keep him at his initial utility level. This supposition delivers an alternative rendering of the expression, "the consumer's real income remains unchanged", and $h$ is called the *Hicks compensated demand function*. Now, the Maximization Hypothesis entails that the following equations hold for all goods $j, k$, and all initial values $\bar{p}, \bar{I}$:

$$(**)\ \frac{\partial x_j(\bar{p},\bar{I})}{\partial p_k} = \frac{\partial h_j(\bar{p},\bar{v})}{\partial p_k} - \frac{\partial x_j(\bar{p},\bar{I})}{\partial I} \cdot x_k(\bar{p},\bar{I}).$$

Comparison with (*) establishes the remarkable result that the Slutsky and Hicks compensated demand functions have identical price derivatives. Accordingly, *for small enough changes $\Delta p_k$, it does not matter which way the*

[3] I made the innocuous change of notation from $s(\bar{p},\bar{x})$ to $s(\bar{p},\bar{I})$.

*substitution effect is defined*. Henceforth, I follow the standard practice of treating (*) and (**) as if they were the same "Slutsky equations".

These equations can be written in matrix term as $M^D = M^{SE} + M^{IE}$ , where (we may drop the bars):

$$M^D = \left[\frac{\partial x_j(p,I)}{\partial p_k}\right]_{j,k=1,\ldots,l}, M^{SE} = \left[\frac{\partial s_j(p,I)}{\partial p_k}\right]_{j,k=1,\ldots,l}, M^{IE} = \left[-\frac{\partial x_j(p,I)}{\partial I}\cdot x_k(p,I)\right]_{j,k=1,\ldots,l}.$$

The Maximization Hypothesis entails that the *substitution effect matrix* $M^D$ is negative definite, but does not restrict the *income effect matrix* $M^{IE}$.[4] If $j = k$, the approximation formula becomes:

$$\Delta x_j \approx \left[\frac{\partial s_j(p,I)}{\partial p_j}\cdot \Delta p_j\right] + \left[-\frac{\partial x_j(p,I)}{\partial I}\cdot x_j(p,I)\cdot \Delta p_j\right],$$

and the negative definiteness of $M^{SE}$ implies that the own substitution effect is *negative*, while the own income effect can be of any sign. If the latter is positive (i.e., if $\frac{\partial x_j(p,I)}{\partial I} < 0$) and its magnitude (which also depends on $x_j(p,I)\cdot\Delta p_j$) exceeds the magnitude of the substitution effect, the total effect will be positive. This formally explains why consumer theory does not recover the traditional law of demand, as Cournot, and more famously, Marshall expounded it:

"There is then one general law of demand: -The greater the amount to be sold, the smaller must be the price at which it is offered in order that it may find purchasers; or, in other words, the amount demanded increases with a fall in price, and diminishes with a rise in price" (1890-1920, p. 84) .[5]

The older economists were fully aware that their broad statement needed qualifying. Marshall (ibid., p. 84-85 and 94-95) lists a number of cases where the law fails, regrettably without clarifying the difference between genuine

[4] A $l \times l$ matrix $M$ is *negative definite* if for all $v \in R^l$, $v' \cdot M \cdot v < 0$, and *negative semi-definite*, if for all $v \in R_*^l$, $v' \cdot M \cdot v \leq 0$ (where $v'$ is $v$ in transposed form).

[5] Cournot (1838-1974, p. 85) antidates Marshall by far, but his statement is less detailed.

exceptions and failures of the appended ceteris paribus clause.[6] Economists have always known that vintage wines, fancy clothes, or luxurious carriages, might sell better at higher prices, and they have long realized that increasing wages do not always affect the demand for labour negatively. In due course, they observed that rallies on stock markets could attract buyers instead of repelling them. For a long time, they simply put aside the problematic classes of luxuries, factors of production, and speculative buying. Today's theorists take a more systematic attitude, if possible by handling exceptions in terms of strong positive income effects, more commonly however, by amending consumer theory itself. Thus, for *luxuries*, they make the "signalling" assumption that on some range of high values, prices enter the utility function directly. Within the same class, or taken as another group, fashion goods can be accommodated by the other non-standard assumption that preferences are ex post influenced by consumption. As to *speculative buying*, economists enrich the static framework of consumer theory with future periods in which buyers become sellers; only after this modelling change does the Slutsky decomposition become relevant again. That other prominent class, *factors of production*, is more simply discarded as being out of scope. According to the current categorization, consumer theory deals with the demand for final goods alone, and it is incumbent on the theory of the firm to investigate the demand for factors of production.

In sum, the more obvious exceptions to the old law are still exceptions for consumer theory. They call for special explanations in which the Slutsky equations play a subordinate rôle if they come in at all. One way of assessing the added value of consumer theory would be to find violations of the law where this theory applies without too much discrepancy, so that positive income effects would provide the bulk of the explanation. Here is a possible example - the so-called *Giffen goods*, which are defined by the following characteristics: (i) they are *inferior*, i.e., the demand for them varies inversely with income, every price remaining constant; (ii) the consumer has a low income and devotes a

[6] Here I take Marshall's ceteris paribus clause to be the same as that of today's theory, i.e., to refer to prices and income only. Alternative interpretations have been offered.

large amount of it to these goods; (iii) they have close substitutes. The name for these goods comes from the 19th century English civil servant whom Marshall (1890-1920, p. 109-110) credited for identifying them. Ever since Marshall, Giffen goods have been a matter of dispute: first of all, do they exist at all, or are they just a theoretical possibility? Second, if they exist, do they give rise to an actual failure of the law, or is this failure a theoretical possibility? Marshall seemed to have little doubt that Giffen goods existed but did not clearly say whether upward sloping demand curves had been observed for them. It is part of the folklore of economics that the poor Irish workers in 19th century Midlands would consume *more* bread when the price of bread increased. However, careful investigations shed a doubt on the available evidence; it is not even clear how Giffen would have had a chance of recording it. The few related examples in the literature are hardly more telling.[7] This explains why the founders of modern consumer theory, Hicks and Samuelson, take an ambiguous attitude towards the "Giffen case" or "Giffen paradox". Granting the real existence of Giffen goods, if they *had* an upward sloping demand curve, they *would* provide fascinating evidence for the theory; since the mathematical assumptions needed for a strong own positive income effect roughly correspond to (i) and (ii). Unfortunately, it is impossible to go beyond this conditional statement as long as the empirical issue is not sorted out, and given the constraints on consumption data and the heavy ceteris paribus clause, this may well never happen.

There is an alternative way of relating consumer theory to the law of demand that, if consistently pursued, would do justice to the contribution of this theory. Instead of searching for exceptions outside the established list, economists could conceivably use the Slutsky equations as a theoretical warrant for the law within the limits set by the list. Hicks's late treatment (1956, ch. VII) is exemplary for this attitude. He stresses that a diversified consumer cannot spend much of his income on any inferior good $j$ in particular, which means that the positive income effect of such a good cannot be large either. For concreteness, Hicks supposes that the consumer spends a 5% fraction of $I$ on

[7] Stigler (1947) dismissed all existing historical evidence. The issue is still hotly debated.

sugar and the price of this commodity is halved. The income difference that compensates - in the sense (*) - for the real income increase is thus equal 2,5% of $I$, which means that the income effect cannot exceed the effect on the demand for sugar of a 2,5% rise in $I$. Such a small rise must be spread out on all goods, which leads Hicks to conclude: "we are justified in saying that the income-effect is normally very small" (1956, p. 65). This is how Hicks reaches the conclusion that, if the Slutsky equations hold, the law of demand is "in practice likely to hold" (1956, p. 66). Underlying the argument is an admixture of theoretical reasoning, computation, and sheer commonsense, that is typical of the way economists proceed when they lack detailed evidence.

Consistently with his attempted justification, Hicks argues that the level of aggregation *over goods* matters a great deal to the empirical confirmation of the law. Since income effects depend on the amount spent, they are stronger for goods grouped together than taken in isolation, as in the previous sugar example. Accordingly, observed violations of the law may be due to a measurement problem. Available consumption data rely on classifying goods into broad categories such as food, clothing, travel, and for these artificially constructed commodities the magnitude of the income effect can become quite large. If the construct has the property of an inferior good, this effect will be positive, with the consequence that a Giffen-like exception may emerge from the data.

It is not easy to assess the all-considered view of the law of demand among today's economists, but I would conclude that most of them take it to be *empirically acceptable within its established limits*. Rather than taking the Slutsky equations as a way of uncovering significant exceptions, they use them to argue that exceptions are bound to be rare. The law, it is sometimes said, holds true of "ordinary consumer goods" for "an ordinary consumer's budget". This ill-defined statement is meant to exclude all troublesome cases at once, i.e., (a) luxuries, fashions, and speculative goods, because they weigh little in the budget; (b) factors of production, because they are irrelevant to the consumer; (c) Giffen goods if they ever exist and have upward sloping demand

curves. Only specialists are aware of the problem created by aggregation over commodities.

Philosophers of science will probably be struck by the shallow basis on which the economists' conviction lies. This is how things often go in the field. Cournot and Marshall should be forgiven for discussing the law and its exceptions casually, but the cavalier attitude has persisted virtually up to now. Although suggestive, Hicks's argument is far from compelling. I did not invoke Samuelson because he refrained from taking sides, being content with the analytical claim that "it is not possible to deduce [from the Slustky equations] ... the ordinary expression for the law of demand" (1947, p. 115). Hicks and Samuelson were theorists, but the 20$^{th}$ century gap between theoretical and applied economics cannot serve as an excuse because *even applied economists have neglected the law of demand*. They concentrate on estimating and testing the Slutsky and associated relations; they also investigate straightforward empirical claims, e.g., that consumers spend on food proportionally less the richer they are. The authoritative summary of econometric evidence by Deaton and Mullbauer (1980) does not discuss the law. What these authors call the "law of demand" (in quotes, p. 44 and 51) is the statement that the own substitution effect is negative. Phlips's (1983), Blundell's (1988) and Chiappori's (1990) surveys similarly omit the topic. This is the more curious since some of the reported evidence could be used as a step towards the missing test. For instance, it helps to estimate the so-called *Engel curves*, which relate the consumers' demand to their income, prices being fixed, because this identifies inferior goods. Generally, before Hildenbrand's work, I do not know of any sustained attempt to substantiate the view that the law holds to a satisfactory degree of approximation.

I complete this overall discussion by resolving two semantic ambiguities. The first relates to the very meaning of the expression "law of demand" in today's writings. With typical nonchalance, economists employ this to mean three different things, i.e., (a) the traditional law (Hicks's and Samuelson's use), (b) the property that the own substitution effect is negative (Deaton and Muellbauer's use), and finally, another mathematical statement (c) I now

introduce. Suppose that prices change from $\bar{p}$ to $p$, the consumer's income being constant, so that $x(\bar{p},\bar{I})$ and $x(p,\bar{I})$ are the corresponding demand vectors. Then, the following inequality:

(***) $(p - \bar{p}) \cdot (x(p,\bar{I}) - x(\bar{p},\bar{I})) < 0$

relates the two vectors of changes in prices and quantities. Geometrically, it says that the vectors point in opposite directions of the *l*-dimensional commodity space.[8] If the change takes place in $p_j$ alone, the inequality provides a discrete version of the traditional law. As this comparison shows, to redefine the law by (***) alleviates Marshall's ceteris paribus clause; now, only the consumer's income has to be fixed. I need this *generalized* version of the law of demand because Hildenbrand specializes in it. This is unsurprising given his testing purposes. Beside the handier ceteris paribus clause, there may be an advantage related to the point that the law of demand fares badly when goods are grouped together; presumably, a test of the one-good law would involve a fair amount of aggregation of this kind. Hildenbrand also has theoretical reasons for preferring (***), but they do not belong to the present paper.[9]

Here is the second, more subtle semantic problem: does the law of "consumer" demand relate to the individual or some aggregate of individuals? Cournot envisaged only the market demand, in which Marshall also was primarily interested. This should not be taken to mean that these two economists disbelieved that there were *individual* demand laws. On the contrary, there is definite evidence that Marshall conceived of the law as being well-established — granted the exceptions — at the individual level. He certainly believed that collective demand functions smoothed out irregularities, but this is a different point — his statement only requires the function to be decreasing, not differentiable or even continuous. Similarly, nothing can be

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[8] Hicks (1956, p. 139) first introduced (***) as another form of the law of demand. An earlier version of his involved $h_j(p,\bar{v})$ instead of $x_j(p, I)$ (1939-1946, p. 52).

[9] The law in generalized form has implications for the stability of general equilibrium, while it has none in its one-good form.

concluded from Hicks's often-quoted claim that "a study of individual demand is only a means to the study of market demand" (1939-1946, p. 34). Most of the passage is in fact concerned with the individual consumer (e.g., p. 32). Even his later work is oriented the same way. From this brief survey, I conclude that the influential writers have never called into question that there was a sense, both theoretical and empirical, in placing the law of demand at the individual level.

They follow the strategy of establishing, first, a suitably qualified version of the law at the individual level, and then and only then, moving to the aggregate levels of the given market, or the economy, as a whole. The resulting aggregative law may call for qualifications — in terms of both ceteris paribus clauses and exceptions — different from those required by the individual law. The former may be easier to formulate and better supported empirically, but the latter would be no less of a putative law for that. Admittedly, economists often replace the individual unit by the *household*. As long as they do not analyze this entity, the shift remains verbal. Some have proposed to reconstruct the household's demand function as the collective (and in part unintended) result of prior interactions between its members. If the law of demand held true of the redefined functions, it would be genuinely collective; however, much remains to be done in order to extend consumer theory in this direction (see, e.g., Browning and Chiappori, 1998).

Following a less detailed, but more widespread, non-individualistic construction of the law, it emerges at the *market* level, while being false as a generality when taken at the individual level. This interpretation has been offered as a commentary on the existing work by Hausman: the law, he writes, is "a generalization about markets, not individuals" (1992, p. 28). This and related statements, like Becker's (1962), may point out the direction of future success, but they do not capture the present orthodoxy of consumer theory. They cannot be taken to *describe* what Marshall, Samuelson, and Hicks put into that theory. If Hildenbrand stresses that the law of demand should be reconstructed as an aggregate phenomenon, this is precisely because this line was never consistently pursued before.

### 3. Hildenbrand's programme and contribution

In *Market Demand* (1994) and related pieces, Hildenbrand distances himself from the standard treatment on two scores I have already touched on. First, he argues that consumer theory does not give a sufficient theoretical and empirical basis for accepting the law of demand. He states his complaint as follows: "I am afraid that all properties that have been formulated so far for individual demand functions, for example, the hypothesis of utility maximization or the Weak Axiom of Revealed Preference,[10] are entirely grounded on a priori reasoning" (1994, p. 12). When reading this, philosophers should bear in mind that economists never use "a priori" in the technical acceptation of "being true or false independently of all experience". Over and beyond propositions, such as linguistic truths, which satisfy this sense, economists mean a wide class of empirical propositions. The latter have the distinctive feature that only the *existing* stock of knowledge, whatever its origin, supports the claims made about their truth or falsehood. These propositions have not given rise to specially devised investigations, and in particular, were never tested. Hence, by criticizing "a priori" defences of the law of demand, Hildenbrand excludes both linguistically-based arguments for the law of demand (some were influential in the Austrian tradition of economics), and incomplete empirical defences (the main representative being Hicks's argument in last section).[11]

Breaking away from "a priori" reasoning thus conceived, Hildenbrand aims at providing the law with the genuine test it never had. However, in order to reach this target, *Market Demand* follows a heavily theoretical strategy which comes as a surprise, given the straightforward empiricism of his critique. Hildenbrand argues for his roundabout approach on the ground that the law of demand, even in the generalized form, cannot be tested directly: "The Law of Demand ... does not refer to the actual evolution of prices, but to hypothetical changes within the same period" (1994, p. 5). I understand this terse statement

[10] After Samuelson, economists sometimes take one of the revealed preference axioms as alternatives to the Maximization Hypothesis; see below.

[11] The analysis of the economic a priori in this paragraph draws on Mongin (2005).

as follows. It takes time for any real consumer to express his demand vector as a reaction to prices and income, which poses a first obstacle to the test, since many things — not only price-income variables, but even the consumer's utility function — may change from one period to another. Now, one may take the periods to be so short that they exclude any disturbing change. In other words, with a suitably small time unit, each period's *empirical* price-demand schedule can meaningfully be compared with the *theoretical* mapping $p_j \mapsto x_j(p, \bar{I})$. But this only leads to a second obstacle: the smaller the time unit, the fewer the available observations. In the limit, there will be a *single* price-demand pair observed for any given period; any other price-demand pair is "hypothetical" in the sense of not belonging to the empirical scheme of *that* period. Hildenbrand's view amounts to a dilemma: if the time unit chosen for statistical measurement is too long, the test is impossible, and if it is too short, the test is also impossible.

What I have described as a *dilemma*, others would describe as a *trade-off*. There may be a value of the time-unit that reduces both horns to an acceptable level of inconvenience; applied economists may sensibly conclude that three or six months approximate the optimal trade-off, and these are the time units effective in many consumption studies. I do not want to push this objection too far because, if pressed, Hildenbrand could call important pragmatic reasons to the rescue. The evidence on consumption is roughly divided between *time series*, which are data relative to different time periods, and *cross-sectional data*, which relate to different households within the same time period. The most revealing time series, called *panel data*, follow the same households' consumption at different times. Although panel data are becoming increasingly available, and are now specially favoured by consumption theorists, the bulk of evidence is still made out of either cross-sectional data or time-series relative to changing populations; both are indeed common in national statistics. The latter involve, or may involve, changes in too many of those variables — the households' incomes, utility functions, and identities — which a test of the law of demand requires one to keep fixed. It makes things worse that time-series do not normally exhibit sufficient variation in the price

and quantity variables which are the objects of the statistical investigation (Hildenbrand, 1994, p. 9; see also Deaton and Mullbauer, 1980). There remain cross-sections, which have the single defect of letting the households' identities change. All of Hildenbrand's tests are based on these data. Instead of claiming that the law of demand is by itself untestable, he could argue that *it is pragmatically better to test it indirectly* because (i) cross-sectional data are the only ones that are both widespread and usable, and (ii) a test based on these data is of necessity indirect (since it involves considering different individuals, contrary to the law).

I move to Hildenbrand's second disagreement with today's conception of the law of demand. He believes that if it ever approximates an empirical truth, this can only be *at the market level*. "The Law of Demand does not refer to the demand of an individual household, but to market demand, that is to say, to the mean demand of a large population of households, for example to all private households in Germany or the United Kingdom" (1994, p. 3-4). In view of the last section, this claim clashes with consumer theory. So it comes as a surprise that Hildenbrand finds support in Hicks: "The market [income effect] is the sum of the individual *I*'s... For [this effect] to be negative, there must be a balance of negativeness among the individual *I*'s that compose it... The probability of exceptional cases is diminished when we take a large group of heterogeneous consumers together" (1956, p. 136). As Hildenbrand (1989, p. 258-259) himself notes, this passage means a gesture without serious consequence. It is typical of the way economists often toy with theoretical possibilities that they do not really entertain.[12]

Now to the programme explicitly stated. It preserves the conventional distinction between a negatively signed substitution effect and a nondescript income effect. Pursuing Hicks's glimpse in his way, Hildenbrand finds out that *the income distribution might have the favourable consequence of wiping out positive income effects*. "Aggregating individual demand over a large group can

[12] Becker's (1962) sketch of an argument for the aggregate law of demand is another example to the point.

lead to properties of the market demand function which, in general, individual demand functions do not possess. There is a qualitative difference in market and individual demand functions" (1983, p. 998). Hildenbrand's belief is based on a simple calculation that might have provided the heuristic for his whole research programme. If all consumers have the same utility function and the income density is uniform over $[0,1]$, it follows that the average income effect is non-positive.[13] Of course, in any relevant community, utility functions will differ across individuals, and the statistical distribution of income is unlikely to conform with the uniform assumption. But the unrealistic case disposes of the *theoretical* preconception that to move the law from the individual up to the market level can only mean further trouble. The challenge becomes to explore the class of statistical distributions that deliver a negative average income effect. This is a purely theoretical move, but depending on whether acceptably realistic distributions are found in the class, it may lead to an empirical grounding of the law of market demand. A distinctive feature of the resulting programme is that income effects become the sole object of attention. Hicks's hint was compatible with the different, actually much more Hicksian, strategy of establishing that aggregate income effects are small *in magnitude* relative to corresponding substitution effects. At the individual level, Hicks is prepared to strike a balance between effects of opposite signs; similarly when he considers the collective level.

Compared with the 1983 paper, which exemplifies Hildenbrand's heuristic, his 1994 book reveals a shift in the way he captures the statistical element of the law. Instead of putting his asssumptions on the income distribution directly, he now makes them on *the functional dependence between variations in income and the statistical diversity of consumption*. This more sophisticated strategy leaves him within the confines of the programme, because he still aims at showing that aggregate income effects are non-positive. There are two versions of the new approach, which differ in how they

[13] Here is the calculation:

$$-\int_0^1 x_j(p,I).\frac{\partial x_j(p,I)}{\partial I}.dI = -\frac{1}{2}\int_0^1 \frac{\partial x_j^2(p,I)}{\partial I}.dI = -\frac{1}{2}\left[x_j^2(p,1) - x_j^2(p,m)\right] = -\frac{1}{2}x_j^2(p,1) \le 0.$$

define the functional dependence just stated. Hildenbrand's condition of Increasing Spread of Household Demand is a less transparent measure of statistical variation than his alternative condition of Increasing Dispersion of Household Demand, but it leads to simpler mathematics, which is why I selected it here. (When I say, following Hildenbrand, "household" instead of "individual", this is just a nominal change motivated by the fact that statistical data concern households, not individuals.)

Hildenbrand's target of analysis is the generalized law of demand (***), and the prices changes considered in this statement are discrete, which blocks direct comparison with the Slutsky equations (*) or (**). However, (***) can be shown to be equivalent to the property that the matrix $M^D$ be negative definite, a property which implies the Marshallian law in differential form:

$$\frac{\partial x_j(p,I)}{\partial p_j} < 0,\ j = 1,...,l\ .$$

In view of this restatement, the Slutsky equations deliver a sufficient condition for (***) to hold. Returning to the matrix form of these equations, we know that $M^{SE}$ is negative definite. Hence, it is sufficient for the generalized law that the other term $M^{IE}$ be negative definite.

Conveniently, both the generalized law and the Slutsky equations keep the same linear form whether they bear on household demand functions or the *average* or *sum* of these functions. Hildenbrand's analysis goes in terms of average uncompensated and compensated demand functions:

$$X_j(p) = \frac{1}{|P|}\sum_{i\in P} x_j{}^i(p, I^i)\,,\ S_j(p) = \frac{1}{|P|}\sum_{i\in P} s_j{}^i(p, I^i)\,.$$

Here, $x^i$ and $s_j^i$ stand for household *i*'s uncompensated and compensated demand functions, $I^i$ for *i*'s fixed income, $P$ for the set of households, and $|P|$ for their number. It is routine to reformulate (*), (**), an

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