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Mortgages and monetary policy

Discussion paper

Mortgages and Monetary Policy*

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Abstract

Mortgage loans are a striking example of a persistent nominal rigidity. As a result, under incomplete markets, monetary policy affects decisions through the cost of new mortgage borrowing and the value of payments on outstanding debt. Observed debt levels and payment to income ratios suggest the role of such loans in monetary transmission may be important. A general equilibrium model is developed to address this question. The transmission is found to be stronger under adjustable- than fixed-rate contracts. The source of impulse also matters: persistent inflation shocks have larger effects than cyclical fluctuations in inflation and nominal interest rates.

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1 Introduction

Most theories of how monetary policy affects the real economy rely on some form of nominal rigidity. Frequently made assumptions, supported by empirical evidence, are that prices and wages of individual firms or households are pre-set in nominal terms for a given period of time, with the result that nominal variables under the control of a monetary authority affect relative prices and real incomes. A specific form of nominal rigidity, but somewhat overlooked in the literature, characterizes also standard mortgage loans. In particular, fully-amortizing mortgages require the homeowner to make *nominal* instalments—regular interest and amortization payments—for the duration of the loan. The installments are calculated so as to guarantee that the principal is repaid in full by the end of the loan's life. A conventional fixed-rate mortgage (FRM) in the United States, for instance, carries a fixed nominal interest rate and prescribes constant nominal installments for the entire life of the loan, typically 30 years. An adjustable-rate mortgage (ARM), typical for the United Kingdom or Australia, also prescribes nominal installments, calculated each period so that, given the current short-term nominal interest rate, the loan is expected to be repaid in full by the end of its life.[1]

This paper studies the macroeconomic consequences of the nominal rigidity inherent in standard mortgage loans. In particular, our aim is to characterize the channels through which the rigidity facilitates the transmission of monetary policy into the real economy, especially into housing investment, and to investigate the strength of the transmission in general equilibrium. In order to isolate the effects of the rigidity, the paper abstracts from other nominal frictions.

[1]The majority of mortgage loans in advanced economies are fully-amortizing mortgages with a term of 15 to 30 years, either FRMs or ARMs. On average, over the period 1982-2006, FRMs accounted for 70% of mortgage originations in the United States (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10); before 1982, they were essentially the only mortgage type available. Other countries in which FRMs—with interest rates fixed for at least 10 years—have traditionally dominated the mortgage market include Belgium, Denmark, and France (in addition, the typical mortgage in Germany and the Netherlands has rates fixed for 5 to 10 years); in other advanced economies, ARMs (with an interest rate linked to a short-term market rate) or FRMs with interest rates fixed for less than 5 years prevail; see Scanlon and Whitehead (2004) and European Mortgage Federation (2012a). Such cross-country heterogeneity in mortgage markets appears to be due to different government regulations (e.g., Green and Wachter, 2005; Campbell, 2012). The structure of mortgage markets and mortgage contracts is taken here as given and we consider only the two extremes: FRMs with an interest rate fixed for the entire term and ARMs.

Recent monetary policies in a number of advanced economies have aimed at reducing long-term interest rates or have committed to low short-term interest rates for long periods of time. One of the goals of such policies is to encourage housing investment (e.g., Board of Governors, 2012). Concerns have also been expressed about the consequences of potential future rises in short-term interest rates for existing homeowners with ARMs (Bank of England, 2013). The model developed in this paper provides a step towards a framework allowing formal, general equilibrium, analysis of the effects of such policies on aggregate housing investment and income redistribution.

Mortgage payments (interest and amortization) as a fraction of income—the so called 'debt-servicing costs'—are nontrivial. Our estimates suggest that, on average over the past 30-40 years, they were equivalent to 15-22% of the pre-tax income of the 3rd and 4th quintiles of the U.S. wealth distribution, representing the typical 'homeowner' (Campbell and Cocco, 2003). A similar picture emerges also from scattered information for some other countries. Hancock and Wood (2004) report that in the United Kingdom mortgage debt servicing costs (for pre-tax income) fluctuated between 15% and 20% over the period 1991-2001. And in Germany, mortgage debt servicing costs are reported to be around 27% of disposable income (European Mortgage Federation, 2012b). Mortgage debt to (annual) GDP ratios in advanced economies are also considerable, reaching on average around 70% in 2009 (International Monetary Fund, 2011, Chapter 3).

The nominal rigidity in mortgages leads to two channels of monetary policy transmission. One channel works through new borrowing (a price effect), the other through outstanding mortgage debt (current and expected future wealth effects). As a preliminary step illustrating the real effects through the first channel, Figure 1 shows the quantitative impact of alternative paths of the short-term (i.e., one-period) nominal interest rate on a typical mortgage holder's expected debt-servicing costs over the life of a typical 30-year mortgage, either FRM or ARM (one period here, equals one quarter). In this example, a household considers buying a house by taking out a mortgage in period 1 worth four times its annual post-tax income.[2] Let us assume no uncertainty (for easier exposition) and that short- and long-term nominal interest rates, as well as mortgage rates, satisfy standard no-arbitrage conditions[3]. In addition, assume that the real interest rate and the household's real income are constant (the real rate is 1% per annum) and that the household's nominal income changes in line with inflation. The two real variables are purposefully held constant so that any real effects on the household's budget occur only due to nominal factors.

Panel A of Figure 1 considers the effect of a mean-reverting decline of the short rate. Its steady-state level is 4%, which is roughly the average for the period from 1990 onwards. As the right-hand side chart shows (lines labeled 'steady state'), at the steady-state interest rate, debt-servicing costs are front-loaded and decline monotonically over the life of the mortgage, here from 29% to 6.5%. This is the well-known 'tilting' effect, which occurs due to a positive inflation rate (3%).[4] Now instead suppose that in period 1 the short rate is equal to 1% ('monetary policy easing') and reverts back to the steady state with persistence of 0.95, the average autocorrelation in the data. Under this path, the tilting is weakened: at the front end of the mortgage debt-servicing costs decline, while at the back end they somewhat increase. For example, in period 1, they decline by 9 percentage points under ARM and by 4 percentage points under FRM. The decline under FRM is smaller than under ARM because the FRM interest rate, due to the mean-reverting nature of the short rate,

[2]This is based on the average ratio, 1975-2010, of the median price of a new home (assuming a loan-to-value ratio of 76%) to the median household income (assuming an income tax rate of 23.5%). The data on both house prices and incomes are from the U.S. Census Bureau. The loan-to-value ratio is the average ratio for single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10). The tax rate is an estimate discussed in Section 4.

[3]Specifically, (i) the expectations hypothesis—i.e., the interest rate on an n-period nominal zero-coupon bond is equal to the average of one-period nominal interest rates between periods 1 and n; (ii) the Fisher effect—i.e., the one-period nominal interest rate at time t is equal to the real interest rate plus the inflation rate between periods t and $t+1$; and (iii) mortgages are priced by arbitrage with the zero-coupon bonds (i.e., in the case of FRM, the mortgage interest rate is such that when the installments are evaluated at the prices of zero-coupon bonds—which are determined by the expectations hypothesis—the present value of a $1 loan is $1; in the case of ARM, the mortgage rate is equal to the one-period interest rate, implying again that the present value of a $1 loan is equal to $1). The principles of mortgage pricing and installment calculations are discussed by, e.g., Fabozzi, Modigliani, and Jones (2010); in the context of a two-period mortgage, they are explained in Section 3.

[4]Positive inflation deflates the real value of mortgage payments in later periods of the life of the loan, which has to be compensated by higher real payments at the beginning, for the present value of a $1 loan to equal to $1.

declines by less than the short rate itself. However, the impact of a decline of the short rate on debt-servicing costs is not always larger under ARM. Panel B of Figure 1 depicts a situation—a hump-shaped decline of the short rate—characterized by a stronger impact under FRM. This is because the FRM rate anticipates the future decline in the short rate, thus declining immediately in period 1.[5]

The two cases illustrate that changes in the path of the short rate, occurring due to purely nominal factors (the expected path of inflation), redistribute the expected debt burden over the life of the loan. Here, reducing real mortgage payments closer to the front end, where debt-servicing costs are the highest. This lowers the effective cost of the loan under a concave utility function and increases housing demand. A monetary policy 'tightening' has the opposite effect. An implicit assumption in this discussion, and a necessary condition for this effect to matter to the household, is that the household cannot fully offset the impact of the short rate on debt-servicing costs through other financial instruments.

In addition to the above (price) effect, which relates to new mortgage loans, in a world with uncertainty monetary policy also affects household decisions ex-post, through current and future debt-servicing costs on outstanding mortgage debt (wealth effects). In the case of FRM, only the inflation rate matters: a higher inflation rate reduces the real value of outstanding debt and thus the real value of the payments households have to make. The strength of this effect increases with inflation persistence. In the case of ARM, both the short-term nominal interest rate and the inflation rate are relevant. An equiproportionate (persistent) increase in the two rates, for instance, initially increases the real payments, as the impact of a higher nominal interest rate dominates the effect of higher inflation. Over time, however, the effect of persistently high inflation gains strength, reducing the real value

[5]If the steady-state short rate was equal to 1% (i.e., zero inflation rate), debt-servicing costs would be constant at 15%. The more persistent the 3 percentage point decline of the short rate is, the closer debt-servicing costs get to 15% and the smaller is the difference between the above effects under ARM and FRM. In the case of 8% steady-state short rate—the average for the period 1970-1989—a mean-reverting decline in the short rate by 3 percentage points (0.95 persistence) results in declines in debt-servicing costs in the first period by 11 percentage points under ARM and 6 percentage points under FRM. At the 8% steady-state short rate, the tilting is 42% in the first period and 3% in the final period, making a given reduction more valuable (under a concave utility function) than in the case of the 4% steady-state rate.

of the payments.

These channels are studied numerically in a general equilibrium model with incomplete asset markets and long-term mortgage loans. As in the above examples, mortgages are priced by arbitrage, but unlike in the examples, the short rate, the real interest rate, and the household's real income are endogenous. These variables are determined by a monetary policy rule, the marginal product of capital (owned by mortgage lenders), and labor supply decisions by homeowners in competitive factor markets. The two types of the short rate dynamics considered in the numerical example above arise endogenously in response to different shocks. The monetary policy rule consists of two parts: systematic responses of the central bank to movements in output and inflation and exogenous changes in an implicit inflation target. In equilibrium, the latter works like a level factor in models of the yield curve and allows the model to replicate the persistence and volatility of long-term nominal interest rates; the former affects the cyclical volatility of the long-short spread. Due to the long-term nature of the mortgage loan, the persistence of nominal interest rates affects the quantitative importance of the nominal rigidity.

The results can be summarized as follows. First, monetary policy has a larger effect on housing investment under ARM than under FRM. Broadly speaking, this is because the price and wealth effects reinforce each other under ARM, but tend to offset each other under FRM. Second, the effects of the stochastic part of the policy rule are larger than the effects of the systematic part. In the latter case, general equilibrium adjustments in the expected future path of the real interest rate tend to offset the real effects of the nominal rigidity in mortgages, whereas in the former case such offsetting forces are weaker. Third, higher inflation redistributes income from lenders to borrowers under FRM, but (at least initially) from borrowers to lenders under ARM.[6] An implication of our findings for the current policy debate is that, other things being equal, low nominal interest rates are likely to have larger real effects in ARM than FRM countries and the impact will be larger the longer is the time

[6]The result that monetary policy transmission is stronger under ARM than under FRM is consistent with cross-country empirical findings of Calza, Monacelli, and Stracca (2013).

horizon for which the rates are expected to stay low.

The paper proceeds as follows. Section 2 relates it to the literature. Section 3 uses a simple three-period problem to explain the nature of the nominal rigidity and the two channels of transmission. Section 4 describes the general equilibrium model and its equilibrium. Section 5 discusses the mapping between the model and the data and calibrates the model. Section 6 reports the findings and explains the general equilibrium adjustments. Section 7 concludes and offers suggestions for future research. A supplemental material contains a list of the model's equilibrium conditions, the computational method, a description of the data counterparts to the variables in the model, and estimates of mortgage debt servicing costs for the United States.

2 Related literature

The paper is related to distinct strands of the literature. First, a number of earlier studies recognize that inflation/nominal interest rates may affect housing demand and construction. The role of the tilting effect has been investigated in the context of mortgage contract design (Lessard and Modigliani, 1975), a supply-demand econometric model of the housing market (Kearl, 1979), and a consumer's problem under a constant inflation rate (Schwab, 1982; Alm and Follain, 1984).[7]

Second, following the seminal contribution of Iacoviello (2005), a number of dynamic stochastic general equilibrium (DSGE) models study the role of housing and housing finance in the monetary transmission mechanism (Iacoviello, 2010, contains various references). This literature, however, is concerned with a different channel than ours, focusing on the interaction between sticky prices, borrowing constraints, and the collateral value of housing. In ad-

[7]In addition, Poterba (1984) notes that, as the U.S. income tax brackets are set in nominal terms, mortgage finance and inflation also interact due to the tax deductibility of mortgage interest payments. This feature adds an additional layer of nominal rigidity into a mortgage contract, but is abstracted from in this paper. More recently, Brunnermeier and Julliard (2008) argue that the main channel through which inflation and mortgages affect housing decisions is money illusion, which makes households ignore the effects of inflation on the real value of future mortgage payments.

dition, housing finance in this literature takes the form of one-period loans, which (as shown in Section 3) eliminates from the transmission mechanism the nominal rigidity we focus on.[8] Iacoviello and Neri (2010) estimate a version of Iacoviello (2005) and find that housing demand shocks—modeled as shocks to the marginal utility of housing—are important drivers of housing investment and house prices over the business cycle. Shocks to the marginal utility of housing are also key in the land collateral mechanism of Liu, Wang, and Zha (2013). The price channel in our model may be viewed as a structural interpretation of such shocks, as it shows up in a similar way in the optimality condition for housing.

Housing and monetary policy have also been studied in the context of home production models (Edge, 2000; Aruoba, Davis, and Wright, 2012) and models with liquidity effects (Li and Chang, 2004; Dressler and Li, 2009; Ghent, 2012). Except for Ghent (2012), who works with FRMs specified in *real* terms, these studies abstract from mortgage loans.[9]

Third, mortgages (or long-term housing debt more generally) are considered by a number of studies focusing on issues unrelated to monetary policy: optimal mortgage choice (Campbell and Cocco, 2003), consumption smoothing (Hurst and Stafford, 2004; Li and Yao, 2007), equilibrium homeownership rates (Chambers, Garriga, and Schlagenhauf, 2009a,b), and equilibrium foreclosures (Garriga and Schlagenhauf, 2009; Chatterjee and Eyigungor, 2011; Corbae and Quintin, 2011). The objects of analysis of these studies are either a single household's decisions or steady-state equilibria in models without aggregate shocks. This allows the inclusion of various option-like features, such as refinancing or default, which our model with aggregate shocks abstracts from.[10]

[8] The interest rate on the one-period loan is in this literature specified as either the current short rate (Iacoviello, 2005, and many others), a weighted average of current and past short and long rates (Rubio, 2011), or evolving in a Calvo-style 'sticky' fashion (Graham and Wright, 2007). A staggered evolution of the interest rate introduces a form of nominal rigidity into the housing loan, but due to the one-period nature of the loan, households can undo its effects. Calza et al. (2013) distinguish between one- and two-period contracts, aimed at capturing ARM and FRM respectively. Their FRM thus contains the nominal rigidity studied here, but it lasts for only two periods; as a one-period loan, their ARM does not contain the rigidity.

[9] In addition to these quantitative-theoretical studies, a number of authors investigate the relationship between monetary policy and housing empirically, in various regression models (see Kearl, Rosen, and Swan, 1975; Kearl, 1979) and structural VARs (e.g., Bernanke and Gertler, 1995; Iacoviello and Minetti, 2008; Calza et al., 2013). Using data for a number of developed economies, Calza et al. (2013) find stronger monetary transmission in ARM than FRM countries.

[10] An exception in this regard is Koijen, Van Hemert, and Van Nieuwerburgh (2009), who study a mortgage

Fourth, the paper is related to studies investigating the redistributive effects of monetary policy when debt contracts are specified in nominal terms (Doepke and Schneider, 2006; Meh, Rios-Rull, and Terajima, 2010; Sheedy, 2013). We show that in the case of mortgages the distributional consequences depend, even qualitatively, on whether the loan is ARM or FRM.

Finally, for our numerical analysis we use an approximation of mortgage loans proposed by Kydland, Rupert, and Sustek (2012), which makes mortgages easy to handle in DSGE models. The focus of their paper is the lead-lag cyclical pattern of residential investment, rather than monetary policy transmission. To that end, they take the mortgage and inflation rates as exogenous, following an estimated VAR process with total factor productivity. As such, the interest and inflation rates process fed into their model reflects shocks and frictions our model abstracts from.

3 The nominal rigidity and channels of transmission

In a deterministic three-period problem of a single household, this section explains the nature of the nominal rigidity and the resulting two channels of monetary policy transmission. Using, at this stage, a deterministic three-period example allows us to describe the rigidity in a transparent way. An extension to an infinite horizon and uncertainty is straightforward but at the cost of extra notation (probabilities and histories of events) and cumbersome expressions.[11]

Time is denoted by $t = 1, 2, 3$. Each period the household is endowed with constant real income w and in period 1 has no outstanding mortgage debt (we introduce outstanding debt

choice problem with some option-like features in a model with aggregate shocks. Their agents and mortgages, however, live for only two periods.

[11] The issues discussed here apply equally to other long-term loans with nominal installments, such as car loans. The focus of the paper is on mortgages as they have much longer term than car loans and housing makes up a bigger chunk of household investment than automobiles. As shown below, a particular nominal rigidity characterizes also long-term coupon bonds, typically issued by corporations. This paper abstracts from corporate debt for the reason that, in contrast to single-family housing, long-term corporate assets are predominantly (more than 75%) financed through retained earnings and other forms of equity (Rajan and Zingales, 1995).

later in this section). In period 1, the household makes a once-and-for-all housing investment decision, financing a fraction θ of the investment with a loan and a fraction $1-\theta$ with income. The loan can be used only for the housing investment and the house lasts for periods 2 and 3. The life-time utility function of the household is $V = \sum_{t=1}^{3}\beta^{t-1}u(c_t)+\sum_{t=2}^{3}\beta^{t-1}g(h)$, where β is a discount factor, c_t is consumption of a nonhousing good in period t, h is housing, and $u(.)$ and $g(.)$ have the standard properties. The household maximizes the utility function with respect to c_1, c_2, c_3, and h, subject to three per-period budget constraints: $c_1+h = w+l/p_1$, $c_2 = w - m_2/p_2$, and $c_3 = w - m_3/p_3$, where $l = \theta p_1 h$ is the nominal value of the loan, m_2 and m_3 are nominal loan installments (to be specified below), and p_t is the aggregate price level in period t (the price of goods in terms of an abstract unit of account). Assume there is a financial market that prices assets by arbitrage but in which the household does not participate due to, for instance, high entry costs (in the actual model this assumption will be partially relaxed). Assume also that monetary policy controls a one-period nominal interest rate i_t. The absence of arbitrage restricts i_t to satisfy $1 + r = (1 + i_t)/(1 + \pi_{t+1})$, where $1+r$ is a gross rate of return on real assets, assumed to be constant and given by some pricing kernel $\mu^* = (1 + r)^{-1}$, and $\pi_{t+1} \equiv p_{t+1}/p_t - 1$ is the inflation rate between periods t and $t + 1$.

3.1 Mortgages

Mortgage installments satisfy $m_2 \equiv (i_2^M + \gamma)l$ and $m_3 \equiv (i_3^M + 1)(1 - \gamma)l$. Here, i_t^M denotes the mortgage interest rate (henceforth referred to as the 'mortgage rate'). Under FRM, $i_2^M = i_3^M = i^F$; under ARM, i_2^M and i_3^M may be different. Further, γ is the amortization rate in the first period of the life of the mortgage, when the outstanding nominal debt is l. In the second period, the outstanding nominal debt is $(1 - \gamma)l$ and the amortization rate is equal to one (i.e., the mortgage is repaid in full). FRM prescribes constant nominal installments: $m_2 = m_3$. The amortization rate therefore solves $i^F + \gamma = (i^F + 1)(1 - \gamma)$, which yields $\gamma = 1/(2 + i^F) \in (0, 0.5)$, for $i^F > 0$. Note that $d\gamma/di^F = -1/(2 + i^F)^2 \in (-0.25, 0)$. For a given l, m_2 and m_3 therefore increase when i^F increases. Under ARM, $\gamma = 1/(2 + i_2^M) \in (0, 0.5)$, for $i_2^M > 0$. If $i_3^M > i_2^M$ then $m_3 > m_2$ and vice versa. It is also the case that $d\gamma/di_2^M \in (-0.25, 0)$ and therefore that m_2 increases when i_2^M increases.

3.1.1 Mortgage pricing and housing investment under FRM

In the absence of arbitrage, i^F has to satisfy

$$1 = Q_1^{(1)}(i^F + \gamma) + Q_1^{(2)}(1 - \gamma)(i^F + 1), \tag{1}$$

where $Q_1^{(1)} = (1+i_1)^{-1}$ and $Q_1^{(2)} = [(1+i_1)(1+i_2)]^{-1}$ are the period-1 prices of one- and two-period zero-coupon bonds, determined according to the expectations hypothesis. Condition (1) states that the present value of installments for a mortgage of size one is equal to one. Notice that if $\gamma = 1$, the mortgage becomes a one-period bond and if $\gamma = 0$, the mortgage becomes a coupon bond. It is straightforward to show that, for $\gamma \in [0, 1)$, $i_1 < i_2$ implies $i_1 < i^F < i_2$ and vice versa.

The household's only first-order condition is $u'(c_1)(1 + \tau_H) = \beta(1 + \beta)g'(h)$, where

$$\tau_H = -\theta \left\{ 1 - \left[\mu_{12} \frac{i^F + \gamma}{1 + \pi_2} + \mu_{12}\mu_{23} \frac{(1 + i^F)(1 - \gamma)}{(1 + \pi_2)(1 + \pi_3)} \right] \right\} \tag{2}$$

is a wedge between the marginal utility of period-1 nonhousing consumption and the marginal lifetime utility of housing, and where $\mu_{t,t+1} \equiv \beta u'(c_{t+1})/u'(c_t)$ is the household's 'stochastic' discount factor. Notice that the wedge works like an ad-valorem tax/subsidy on housing investment and that the expression within the square brackets is the present value of the marginal real installments from the household's perspective (i.e., evaluated at its stochastic discount factor rather than the pricing kernel of the financial market, μ^*). The present value represents the cost of the mortgage to the household. Because the household does not trade in the financial markets, in general, $\mu_{t,t+1} \neq \mu^*$ and the present value is different from one. When it is less (greater) than one, the wedge is negative (positive).

Equation (2) shows that the wedge depends on nominal variables i^F, π_2, π_3; i.e., it is not possible to rewrite the wedge in terms of real variables alone. By controlling i_1 and i_2—and thus, through the no-arbitrage conditions, i^F, π_2, and π_3—monetary policy affects τ_H and the household's optimal choice of h. This channel of transmission will be referred to as the *price effect*, as it affects the cost of new borrowing and thus the effective price of housing investment paid by the household. Notice that r also affects τ_H: for a given i_t, it affects π_{t+1} through the Fisher equation. But because of the long-term and nominal nature of the loan, r alone is not a sufficient statistic for the cost of the loan to the household. In contrast, in standard models used for monetary policy analysis (e.g., the New-Keynesian models), r is such a summary statistic.[12]

When $\mu_{t,t+1} = \mu^*$, $\tau_H = 0$ and monetary policy is neutral. When $\mu_{t,t+1} \neq \mu^*$, the wedge is nonzero for any $\gamma \in [0,1)$, not just the FRM γ which makes $m_2 = m_3$. The value of γ, however, controls the form of the nominal rigidity. In the extreme case, $\gamma = 0$ (a coupon bond), the nominal payments are concentrated in period 3 and monetary policy works primarily through changing the real value of the repayment of the principal; in the case of FRM, the nominal payments are distributed evenly across the two periods, producing the tilting effect as m_3 gets more deflated, in real terms, than m_2. When $\gamma = 1$ (one-period loan), monetary policy is neutral: $\tau_H = -\theta\{1 - \mu_{12}[(1+i_1)/(1+\pi_2)]\}$, where $(1+i_1)/(1+\pi_2) = 1 + r = (\mu^*)^{-1}$, and μ_{12} is evaluated at $c_2 = w - \theta(1+r)h$.[13]

3.1.2 Mortgage pricing and housing investment under ARM

Under ARM, $i_2^M = i_1$ and $i_3^M = i_2$ ensures the absence of arbitrage:

$$Q_1^{(1)}(i_2^M + \gamma) + Q_1^{(2)}(1-\gamma)(i_3^M + 1) = \frac{i_1 + \gamma}{1 + i_1} + \frac{(1-\gamma)}{(1+i_1)}\left[\frac{(i_2+1)}{(1+i_2)}\right] = 1.$$

[12] Monetary policy transmission in that class of models works through sticky prices, resulting in sluggish π_{t+1}, which allows i_t to directly affect r_{t+1}.

[13] Neutrality also results when the housing loan takes the form of a 2-period *zero-coupon* bond; i.e., $m_2 = 0$ and $m_3 = (1+i_1)(1+i_2)l$. It is straightforward to show that in this case the wedge depends only on the ratio of $(\mu_{12}\mu_{23})$ and $(\mu^*)^2$, where μ_{23} is evaluated at $w - (1+r)^2\theta h$. Neutrality also results under index-linked mortgages (see Section 3.2.1) and in the trivial case of $\theta = 0$.

The household's first-order condition takes the same form as under FRM, but with a wedge

$$\tau_H = -\theta \left\{ 1 - \left[\mu_2 \frac{i_1 + \gamma}{1 + \pi_2} + \mu_2 \left(\frac{\mu_3}{\mu^*} \right) \frac{1 - \gamma}{1 + \pi_2} \right] \right\}, \tag{3}$$

where we have substituted $(\mu^*)^{-1}$ for $(1 + i_2)/(1 + \pi_3)$. Again, for $\gamma \in [0, 1)$, τ_H depends on nominal variables and monetary policy affects the household's optimal choice of h. For instance, a decline in i_1 reduces the marginal real installments in the first period of the life of the mortgage: through the Fisher effect (holding r constant), π_2 declines one for one with i_1 but—as $\gamma \in (0, 0.5)$ and $d\gamma/di_2^M \in (-0.25, 0)$—the effect on the numerator is stronger than the effect on the denominator.

3.1.3 Outstanding mortgage debt

Let us now abstract from the housing investment decision and focus instead on how monetary policy affects the real value of payments on outstanding mortgage debt. Suppose that in period 1 the household has some outstanding mortgage debt l_0, taken out in period 0 and maturing in period 2. The household's budget constraint in period 1 is $c_1 = w - \widetilde{m}_1$, where $\widetilde{m}_1 \equiv m_1/p_1 = [(i_1^M + \gamma)/(1 + \pi_1)]\widetilde{l}_0$, with $\widetilde{l}_0 \equiv l_0/p_0$. The mortgage rate i_1^M is predetermined in period 1; it is equal to some i_0^F under FRM and to i_0, the period-0 short rate, under ARM. Clearly, a higher π_1 generates a positive *current wealth effect* in period 1. This is the standard wealth effect present also in the case of one-period loans ($\gamma = 1$).

In period 2, the real payments on this 2-period loan are, respectively under FRM and ARM,

$$\widetilde{m}_2 = \frac{i_0^F + 1}{(1 + \pi_1)(1 + \pi_2)}(1 - \gamma)\widetilde{l}_0 \quad \text{and} \quad \widetilde{m}_2 = \frac{1 + r}{1 + \pi_1}(1 - \gamma)\widetilde{l}_0,$$

where in the second equation we have substituted $1 + r$ for $(i_1 + 1)/(1 + \pi_2)$. Thus, for $\gamma \in [0, 1)$, a higher π_1 generates not only positive wealth effects in period 1, but also positive *expected future wealth effects*, as it reduces the real payments in period 2. If the increase in the inflation rate is persistent, under FRM the expected future wealth effects occur also due to expectations of a higher π_2. In the case of ARM, the absence of the nominal interest rate and period-2 inflation rate in $\widetilde{m}_2$ is due to the 2-period term of the loan considered here. Suppose, instead, that the loan has a 3-period term, maturing in period 3. In period 2, the real mortgage payments are then

$$\widetilde{m}_2 = \frac{i_1 + \gamma_2}{(1+\pi_1)(1+\pi_2)}(1-\gamma_1)\widetilde{l}_0,$$

where γ_2 is a period-2 amortization rate. In this case, an expected increase in π_2 which, by the Fisher equation, leads to an equiproportionate increase in i_1, does not reduce the expected period-2 real payments, as in the case of FRM, but increases them. It is straightforward to check that, as $\gamma_2 \in (0, 0.5)$ and $d\gamma_2/di_1 \in (-0.25, 0)$, an increase in i_1, accompanied by an equiproportionate increase in π_2, increases the real installments.[14] In period 3, the ARM payments are

$$\widetilde{m}_3 = \frac{1+r}{(1+\pi_1)(1+\pi_2)}(1-\gamma_2)(1-\gamma_1)\widetilde{l}_0,$$

where we have substituted $1 + r$ for $(i_2 + 1)/(1 + \pi_3)$. A higher π_2, while increasing the real payments in period 2, leads to their reduction in period 3.

To summarize, current inflation produces standard wealth effects under both FRM and ARM, as well as under one-period loans. In addition, with mortgages there are expected future wealth effects. When a higher current short rate transmits one for one into a higher inflation rate next period, as the Fisher effect dictates, it unambiguously reduces future real payments on outstanding mortgage debt under FRM; under ARM, it increases the payments in the immediate periods, but reduces them in later periods of the life of the mortgage. The more persistent the increase in the inflation rate is, the larger is the expected future reduction in the real value of mortgage payments.[15]

[14] The properties of γ_2 listed here are derived from the equation $(i_1+\gamma_2)(1-\gamma_1) = (i_1+1)(1-\gamma_2)(1-\gamma_1)$, which states that the installments in periods 2 and 3 have to be the same, conditional on i_1. This yields $\gamma_2 \approx (1-\gamma_1)/(2+i_1-\gamma_1)$, which, for some $\gamma_1 \in (0,1)$, is in the interval (0,0.5). Taking the derivative with respect to i_1 then confirms that $d\gamma_2/di_1 \in (-0.25, 0)$, for $\gamma_1 \in (0,1)$.

[15] In a model with both the outstanding and new debt, the wealth effects interact with the price effect, as they affect consumption of the nonhousing good and thus $\mu_{t,t+1}$, the valuation of the marginal real

3.2 Alternative housing finance arrangements

For comparison, we now discuss alternative housing finance arrangements.

3.2.1 Index-linked mortgage

An index-linked mortgage, also known as a price-level adjusted mortgage, is a mortgage (here with a 2-period term) that adjusts the principal for changes in the price level. Under this mortgage, the nominal installments are $m_2 = (i_2^M + \gamma)[(1+\pi_2)l]$ and $m_3 = (i_3^M + 1)(1-\gamma)[(1+\pi_2)(1+\pi_3)l]$. Arbitrage imposes $i_2^M = i_3^M = r$. As a result, real installments, m_2/p_2 and m_3/p_3, do not depend on nominal variables, rendering monetary policy neutral. The wedge in this case is $\tau_H = -\theta\left\{1 - [\mu_{12}(\gamma + r) + \mu_{12}\mu_{23}(r+1)(1-\gamma)]\right\}$. Notice that the same wedge results under FRM or ARM if $\pi_t = 0$ for $t = 2, 3$.

3.2.2 Sequence of one-period loans

Suppose we let the household adjust h and l in period 2. That is, the household chooses $l_t = \theta p_t h_t$ in periods $t = 1, 2$ and pays back $(1 + i_{t-1})l_{t-1}$ in periods $t = 2, 3$. This is a common assumption in the DSGE models noted in Section 2.[16] Such arrangement is similar to period-by-period refinancing: each period, an existing mortgage is fully prepaid (with the one-period interest paid) and a new mortgage—of a possibly different size and with a different interest rate—is taken out. The sequence of loans results in wedges in periods $t = 1, 2$ given by $\tau_{Ht} = -\theta[1 - \mu_{t,t+1}(1+r)]$, which are nonzero for $\mu_{t,t+1} \neq \mu^*$, but do not depend on nominal variables. Clearly, both period-by-period refinancing and keeping the mortgage until maturity—an implicit assumption in our set up—are extreme cases. In reality, refinancing is an option, which the household may occasionally exercise. This paper abstracts from optimal refinancing. The nominal rigidity in mortgages is thus

installments.

[16]The constraint in these models is slightly different from our version of it. Usually it takes the form $[(1+i_t)/(1+\pi_{t+1})](l_t/p_t) \leq \theta p_{t+1}^H h_t$. That is, repayment of the one-period loan with interest, in real terms, must be less or equal to a fraction of the value of the house next period, where p^H is the relative price of the house in terms of the nonhousing good. Additional assumptions guarantee that the constraint always holds with equality. These details are unimportant for the point being made here.

at its extremum and the results are best viewed as an upper bound on the strength of the transmission mechanism under investigation.

4 The model

The model embeds a version of the household's problem of Section 3.1 in a general equilibrium framework. As in that section, and in the examples in the Introduction, mortgages are priced by arbitrage and the expectations hypothesis and the Fisher equation hold. The model differs from the three-period example in six respects: i) the time horizon is infinite and the same types of decision are made every period; ii) there are aggregate shocks; iii) houses consist of land and structures; iv) mortgages resemble standard 30-year mortgage loans, rather than maturing in just two or three periods; v) households have some ability to smooth the impact of mortgage payments through financial assets; and vi) the household's income, the short-term nominal interest rate, and the real interest rate are endogenous. The model also includes various taxes, transfers, and government expenditures. They are parameters and their role is to facilitate a sensible mapping of the model into data. The presence of land in the model is unimportant for the main results, but it allows us to derive the model's implications for house prices, as opposed to only prices of structures.

4.1 Environment

The economy's population is split into two groups, 'homeowners' and 'capital owners', with measures Ψ and $(1-\Psi)$, respectively. Within each group, agents are identical. An aggregate production function combines capital and labor to produce a single good. Capital owners own the economy's capital stock, whereas homeowners supply labor and own the economy's housing stock. Such abstraction is motivated by cross-sectional observations.[17] The two

[17]Capital owners and homeowners in the model correspond to, respectively, the 5th and the sum of the 3rd and 4th quintiles of the U.S. distribution of wealth: in the data, the 3rd and 4th quintiles hold most of their assets in housing, while the 5th quintile hold almost the entire corporate equity in the economy (the 5th quintile also own housing, but it is a less important component of their asset structure; the 1st and 2nd quintiles are essentially renters with no assets); see Campbell and Cocco (2003), Figure 1. In addition,

types of agents trade a one-period nominal bond and capital owners provide mortgage loans to homeowners, pricing them by arbitrage. Where applicable, the notation is the same as in Section 3. Only new variables and functions are therefore defined. When a variable's notation is the same for both agent types, an asterix ($*$) denotes the variable pertaining to capital owners.

4.1.1 Capital owners

A representative capital owner maximizes expected life-time utility

$$E_t \sum_{t=0}^{\infty} \beta^t u(c_t^*), \quad \beta \in (0,1),$$

where $u(.)$ has standard properties, subject to a sequence of budget constraints

$$c_t^* + x_{Kt} + \frac{b_{t+1}^*}{p_t} + \frac{l_t^*}{p_t} = \left[(1-\tau_K)r_t + \tau_K \delta_K\right] k_t + (1+i_{t-1})\frac{b_t^*}{p_t} + \frac{m_t^*}{p_t} + \tau_t^* + \frac{p_{Lt}}{1-\Psi}. \tag{4}$$

Here, x_{Kt} is investment in capital, b_{t+1}^* is holdings of the one-period nominal bond between periods t and $t+1$, τ_K is a capital income tax rate, $\delta_K \in (0,1)$ is a depreciation rate, k_t is capital, and τ_t^* is a lump-sum transfer. In addition, $1/(1-\Psi)$ is new residential land, which the capital owner receives each period as an endowment, and p_{Lt} denotes its price in terms of consumption. The capital stock evolves as

$$k_{t+1} = (1-\delta_K)k_t + x_{Kt} \tag{5}$$

and the depreciation is tax deductible in order to make the capital income tax rate in the model comparable with its estimates in the literature.

All mortgages in the economy are either FRM or ARM and are approximated using the formulation of Kydland et al. (2012), which is convenient both analytically and computa-

the 3rd and 4th quintiles derive almost all of their income from labor, whereas labor income is much less important for the 5th quintile (Survey of Consumer Finances; see also Section 5.2 for details).

tionally, while being reasonably accurate (see their paper for details). Three state variables track the outstanding nominal mortgage debt and its effective amortization and interest rates. Denoting by d_t^* the outstanding debt owed to the capital owner, the nominal mortgage payments received by the capital owner in period t are

$$m_t^* = (R_t^* + \gamma_t^*) d_t^*, \tag{6}$$

where R_t^* and γ_t^* are, respectively, the effective interest and amortization rates. The state variables evolve as

$$d_{t+1}^* = (1 - \gamma_t^*) d_t^* + l_t^*, \tag{7}$$

$$\gamma_{t+1}^* = (1 - \phi_t^*) (\gamma_t^*)^\alpha + \phi_t^* \kappa, \tag{8}$$

$$R_{t+1}^* = \begin{cases} (1 - \phi_t^*) R_t^* + \phi_t^* i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \tag{9}$$

where $\phi_t^* \equiv l_t^* / d_{t+1}^*$ is the fraction of new loans in the outstanding debt next period and $\kappa, \alpha \in (0, 1)$ are parameters controlling the evolution of the amortization rate.

Under FRM, the first-order condition for l_t^* ensures that i_t^F is such that the capital owner is indifferent between new mortgages and rolling over the one-period bond from period t on. The first-order condition is an infinite-horizon counterpart to equation (1); see Appendix A. Under ARM, the current one-period interest rate i_t is applied to both new and outstanding mortgage loans, making the capital owner again indifferent between mortgages and rolling over the bond. Notice that, even though new loans are extended every period, each new loan (both FRM and ARM) is a long-term loan, starting with an amortization rate κ. A one-period loan would result as a special case of this formulation if we set $\alpha = 0$ and $\kappa = 1$, which implies $\gamma_t = 1 \; \forall t$.

4.1.2 Homeowners

A representative homeowner maximizes expected life-time utility

$$E_t \sum_{t=0}^{\infty} \beta^t v(c_t, 1-n_t, h_t),$$

where n_t is labor and $v(.,.,.)$ has the standard properties. This maximization is subject to

$$c_t + p_{Ht}x_{Ht} - \frac{l_t}{p_t} + \frac{b_{t+1}}{p_t} = (1-\tau_N)(w_t n_t - \tau) + (1 + i_{t-1} + \Upsilon_{t-1})\frac{b_t}{p_t} - \frac{m_t}{p_t} + \Omega_t, \tag{10}$$

$$\frac{l_t}{p_t} = \theta p_{Ht} x_{Ht}. \tag{11}$$

Here, x_{Ht} is newly constructed houses, p_{Ht} is their relative price, τ_N is a labor income tax rate, and τ is a pre-tax labor income deduction.[18] Further, Υ_{t-1} is a bond market participation cost, governed by a function $\Upsilon(-\widetilde{B}_t)$, where $\widetilde{B}_t \equiv B_t/p_{t-1}$ is homeowners' real aggregate holdings of the bond. The function $\Upsilon(.)$ is assumed to be bounded below by minus one, increasing, and convex. In addition, $\Upsilon(.) = 0$ when $\widetilde{B}_t = 0$, $\Upsilon(.) > 0$ when $\widetilde{B}_t < 0$, and $\Upsilon(.) < 0$ when $\widetilde{B}_t > 0$. We think of $\Upsilon(.) > 0$ as capturing a premium for unsecured consumer credit, which is increasing in aggregate borrowing[19]; $\Upsilon(.) < 0$ is meant to capture some intermediation costs on household savings, which reduce the interest rate on savings below the market interest rate i_t. In order to avoid the participation cost affecting the definition of aggregate output, it is rebated to the homeowner as a lump-sum transfer $\Omega_t = B_t\Upsilon_{t-1}/p_t$. In a nonstochastic steady state, $\widetilde{B} = 0$ and the first-order conditions for b_{t+1} and b^*_{t+1} imply $\mu = \mu^*$ and hence $\tau_H = 0$.

[18] As in the three-period example, θ is treated as a parameter. Similar assumption is made also by Chambers et al. (2009a) and has empirical support: over the period 1973-2006, there has been very little variation in the cross-sectional average of the loan-to-value ratio for single family *newly-built home* mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10).

[19] This can be though of as capturing the notion that as aggregate unsecured credit grows, the creditworthiness of borrowers declines.

The housing stock evolves as

$$h_{t+1} = (1-\delta_H)h_t + x_{Ht}, \tag{12}$$

where $\delta_H \in (0,1)$. Mortgage payments are again given as

$$m_t = (R_t + \gamma_t)d_t, \tag{13}$$

where

$$d_{t+1} = (1-\gamma_t)d_t + l_t, \tag{14}$$

$$\gamma_{t+1} = (1-\phi_t)\left(\gamma_t\right)^{\alpha} + \phi_t \kappa, \tag{15}$$

$$R_{t+1} = \begin{cases} (1-\phi_t)R_t + \phi_t i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM.} \end{cases} \tag{16}$$

4.1.3 Technology

An aggregate production function, operated by perfectly competitive producers, is given by $Y_t = A_t f(K_t, N_t)$, where K_t is the aggregate capital stock, N_t is aggregate labor, and $f(.,.)$ has the standard neoclassical properties. Total factor productivity (TFP) evolves as $\log A_{t+1} = (1-\rho_A)\log A + \rho_A \log A_t + \epsilon_{A,t+1}$, where $\rho_A \in (0,1)$, A is the unconditional mean, and $\epsilon_{At} \sim iidN(0, \sigma_A)$. The real rate of return on capital, r_t, and the real wage rate, w_t, are determined by the marginal products of capital and labor, respectively. The resource constraint of the economy is $C_t + X_{Kt} + q_t X_{St} + G = Y_t$, where C_t is aggregate consumption, X_{Kt} is aggregate investment in capital, X_{St} is aggregate investment in housing structures, and G is (constant) government expenditures. Here, q_t is the marginal rate of transformation between housing structures and the other uses of output, and hence the relative price of structures. It is given by a strictly increasing convex function $q(X_{St})$, which makes the economy's production possibilities frontier concave in the space of $(C_t + X_{Kt} + G)$ and (X_{St})—a specification akin to that of Huffman and Wynne (1999). Its sole purpose is to ensure realistic volatility of housing investment in response to shocks; if the production possibilities frontier was linear, given the calibration of the shocks, the volatility would be too high.

As in Davis and Heathcote (2005), new houses consist of structures and land and are produced by perfectly competitive homebuilders according to a production function $X_{Ht} = g(X_{St}, X_{Lt})$. Here, X_{Ht} is the aggregate number of new homes produced in period t, X_{Lt} is the amount of new residential land used, and g has the standard neoclassical properties. Homebuilders choose X_{Ht}, X_{St}, and X_{Lt} to maximize profits $p_{Ht}X_{Ht} - q_t X_{St} - p_{Lt}X_{Lt}$, subject to the above production function.

4.1.4 Monetary policy and government

Monetary policy is modeled as an interest rate feedback rule with a stochastic inflation target (e.g., Ireland, 2007)

$$i_t = (i - \overline{\pi} + \overline{\pi}_t) + \nu_\pi(\pi_t - \overline{\pi}_t) + \nu_y(y_t - y). \tag{17}$$

Here, $\nu_\pi > 1$, $\nu_y \geq 0$, i is the nonstochastic steady-state nominal interest rate, $\overline{\pi}_t$ is the inflation target, $y_t \equiv \log Y_t - \log Y_{t-1}$ is the output growth rate, and y is its nonstochastic steady-state value (equal to zero). The inflation target follows an AR(1) process $\overline{\pi}_{t+1} = (1-\rho_\pi)\overline{\pi} + \rho_\pi \overline{\pi}_t + \epsilon_{\pi,t+1}$, where ρ_π is less than but close to one, $\overline{\pi}$ is the nonstochastic steady-state inflation rate, and $\epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi)$. As shown in Section 4.2.2., in equilibrium, the inflation target shock works like a 'level factor', moving short and long rates equally, and allows the model to reproduce the observed volatility and persistence of the 30-year mortgage rate. A number of studies document that the level factor accounts for over 90% of the volatility of yields across maturities (see, e.g., Piazzesi, 2006) and shocks to the inflation target are often invoked as its structural interpretation (e.g., Atkeson and Kehoe, 2008). The model is closed by the government budget constraint: $G + T_t^* = \tau_K(r_t - \delta_K)K_t + \tau_N(w_t N_t - \tau\Psi) + \tau\Psi$, where T_t^* is a transfer to capital owners.

4.2 Equilibrium

The equilibrium concept is the recursive competitive equilibrium (e.g., Hansen and Prescott, 1995). First, let $z_t \equiv [\log A_t, \overline{\pi}_t, p_{t-1}, Y_{t-1}]$ be the vector of exogenous state variables and lagged endogenous variables p_{t-1} and Y_{t-1}, $s_t^* \equiv [k_t, b_t^*, d_t^*, \gamma_t^*, R_t^*]$ the vector of the capital owner's state variables, $s_t \equiv [h_t, b_t, d_t, \gamma_t, R_t]$ the vector of the homeowner's state variables, and $S_t \equiv [K_t, H_t, B_t, D_t, \Gamma_t, \Re_t]$ the vector of aggregate endogenous state variables, where the elements are, respectively, the aggregate capital, housing, bonds, mortgage debt, and its effective amortization and interest rates. Next, write the capital owner's optimization problem as

$$U(z, S, s^*) = \max_{[x_K, (b^*)', l^*]} \left\{ u(c^*) + \beta E[U(z', S', (s^*)')|z] \right\}, \tag{18}$$

where a prime denotes a value next period and the constraints (4)-(9) are thought to have been substituted in the utility and value functions. Similarly, write the homeowner's problem as

$$V(z, S, s) = \max_{[x_H, b', n]} \left\{ v(c, 1-n, h) + \beta E[V(z', S', s')|z] \right\}, \tag{19}$$

where the constraints (10)-(16) are thought to have been substituted in the utility and value functions. Let $W_t \equiv [X_{Kt}, p_t, i_t^M, X_{Ht}, B_{t+1}, N_t]$ be the vector of aggregate decision variables and prices, where $i_t^M = i_t^F$ under FRM and $i_t^M = i_t$ under ARM. Define a function $W_t = W(z_t, S_t)$.

A recursive competitive equilibrium consists of the functions U, V, and W such that: (i) U and V solve (18) and (19), respectively; (ii) r_t and w_t are given by the respective marginal products of capital and labor, p_{Ht} and p_{Lt} are given by the homebuilder's first-order conditions for structures and land, and $q_t = q(X_{St})$; (iii) i_t is given by the monetary policy rule (17) and the government budget constraint is satisfied; (iv) the bond, mortgage, housing, and land markets clear: $(1-\Psi)b_{t+1}^* + \Psi b_{t+1} = 0$, $(1-\Psi)(l_t^*/p_t) = \Psi\theta p_{Ht} x_{Ht}$, $\Psi x_{Ht} = g(X_{St}, X_{Lt})$, and $X_{Lt} = 1$; (v) aggregate consistency is ensured: $K_t = (1-\Psi)k_t$, $X_{Kt} = (1-\Psi)x_{Kt}$, $T_t^* = (1-\Psi)\tau_t^*$, $X_{Ht} = \Psi x_{Ht}$, $N_t = \Psi n_t$, $B_t = \Psi b_t$, $H_t = \Psi h_t$,

$(1-\Psi)m_t^* = \Psi m_t$, $(1-\Psi)d_t^* = \Psi d_t = D_t$, $\gamma_t^* = \gamma_t = \Gamma_t$, and $R_t^* = R_t = \Re_t$; (vi) the exogenous state variables follow their respective stochastic processes and the endogenous aggregate state variables evolve according to aggregate counterparts to the laws of motion for the respective individual state variables; and (vii) the individual optimal decision rules of the capital owner (for x_K, $(b^*)^{'}$, and l^*) and the homeowner (for x_H, $b^{'}$, and n) are consistent with $W(z, S)$, once the market clearing conditions (iv) and the aggregate consistency conditions (v) are imposed.

It is straightforward to check that the goods market clears by Walras' Law: $C_t + X_{Kt} + q_t X_{St} + G = Y_t$, where $C_t = (1-\Psi)c_t^* + \Psi c_t$. Equations characterizing the equilibrium are contained in Appendix A; a computational procedure resulting in log-linear approximation of $W(z, S)$ around the model's non-stochastic steady state is described in Appendix B. The first-order conditions of the capital owner for x_{Kt}, b_{t+1}^*, and l_t^* result in no-arbitrage conditions for capital, bonds, and new mortgages. As a result, the capital owner is indifferent between the three assets and the allocation of his period-t savings is determined by the homeowners's demand for bonds and new mortgages.[20]

4.2.1 Capital owner and homeowner blocks

It will be convenient to view the economy as consisting of two blocks. Given a set of decision rules for X_{Ht}, B_{t+1}, and N_t, the 'capital owner block' determines an aggregate decision rule for X_{Kt} and pricing functions for p_t and i_t^M. Similarly, given a set of decision rules and pricing functions for X_{Kt}, p_t and i_t^M, the 'homeowner block' determines aggregate decision rules for X_{Ht}, B_{t+1}, and N_t. In equilibrium, the two sets of decision rules and pricing functions have to be mutually consistent at each point in the state space (z, S). Working with these two blocks in partial equilibrium—i.e., taking the other block's decision rules as given—facilitates understanding of the general equilibrium results.[21]

[20] In the case of ARM, $i_t^M = i_t$ makes the capital owner indifferent between new mortgages and bonds and the first-order condition for l_t^* can be dropped from the description of the equilibrium. In the case of FRM, the first-order condition determines i_t^F.

[21] In terms of equations, the homeowner block consists of the optimality conditions for the homeowner's Bellman equation, while the capital owner block consists of the optimality conditions for the capital owner's

4.2.2 The equilibrium short rate

The capital owner's first-order conditions for b^*_{t+1} and x_{Kt} yield the Fisher equation. In a linearized form: $i_t = E_t\pi_{t+1} + E_t r_{t+1}$, where (abusing notation) the variables are in percentage point deviations from steady state. Given a stochastic process for r_t, by successive forward substitution the Fisher equation and the monetary policy rule (17) determine i_t. Excluding explosive paths for inflation (a common assumption) and given ρ_π close to one, the resulting expression for i_t is

$$i_t \approx \sum_{j=0}^{\infty} \left(\frac{1}{\nu_\pi}\right)^j E_t r_{t+1+j} + \overline{\pi}_t, \tag{20}$$

where, anticipating calibration described in the next section, $\nu_y = 0$ has been imposed. Due to its high persistence, $\overline{\pi}_t$ generates highly persistent movements in i_t and thus moves i_t and i^F_t approximately one for one. In this sense, it works like a level factor, moving all yields approximately equally. In contrast, the first term in equation (20) is much less persistent than $\overline{\pi}_t$, mainly due to a lower persistence of the A_t shock. It produces only temporary movements in i_t and thus smaller movements in i^F_t than in i_t. As a result, it moves the long-short spread, $i^F_t - i_t$. In this sense it works like a slope factor. The equilibrium inflation rate is determined from the monetary policy rule as $\pi_t = (\overline{\pi} - i)/\nu_\pi + \overline{\pi}_t + (i_t - \overline{\pi}_t)/\nu_\pi$, where i_t is given by (20). Notice that a higher ν_π reduces the volatility of i_t and π_t in response to movements in the expected future path of r_t.

The real interest rate r_t is pinned down by the marginal product of capital. In a log-linearized form, $r_t = A_t + (\varsigma - 1)K_t + (1 - \varsigma)N_t$. The equilibrium i_t thus depends on the stochastic paths of four variables: the exogenous state variables A_t and $\overline{\pi}_t$ and the endogenous variables K_t and N_t. Any general equilibrium adjustments of i_t thus occur through expected future paths of K_t and N_t. Recall that in equilibrium the capital owner is indifferent between saving in mortgages, bonds, or capital. An increase in the demand for mortgages, other things being equal, thus reduces K_{t+1}. This increases r_{t+1} and hence i_t.

Bellman equation; both blocks also contain the producers' conditions determining r_t, w_t, q_t, p_{Ht}, p_{Lt}, and X_{St}, the government budget constraint, and the monetary policy rule, so that the prices and transfers relevant to each block can be pinned down (given decisions/prices of the other block).

5 Calibration

A closed-form solution to the model's equilibrium conditions does not exist and the model's properties can be studied only numerically for specific functional forms and parameter values. The choice of parameter values is based on calibration. The model is quarterly and most parameter values are obtained by requiring the model to reproduce long-run averages of the data in nonstochastic steady state. Some second moments are also used. As most of the required historical data are readily available for the United States, the calibration is based on U.S. data.

An extra layer of complication, relative to most DSGE models, arises due to the need to match debt-servicing costs of homeowners. For this reason the model is required to be consistent with both the cross-sectional distribution of income, as well as the key aggregate ratios: $X_K/Y = 0.156$, $X_S/Y = 0.054$, $G/Y = 0.138$, $K/Y = 7.06$, $H/Y = 5.28$, averages for 1958-2006 (see Appendix C for the description of the data), and $N = 0.255$ (American Time-Use Survey, 2003, population 16+). Official data for mortgage debt servicing costs are not available for the United States. Estimates, however, can be obtained from different data sources (see Appendix D), resulting in long-run averages (1972-2006) in the ballpark of 18.5% of homeowners' pre-tax income. The model's steady-state counterpart to this ratio is $\widetilde{M}/(wN - \tau\Psi)$, where $\widetilde{M} = (R+\gamma)\widetilde{D}/(1+\pi)$ and $\widetilde{D}$ is real mortgage debt.

Consistency with the cross-sectional distribution of income is achieved through the transfer τ. Recall that homeowners in the model are an abstraction for the 3rd and 4th quintiles of the U.S. wealth distribution, while capital owners are an abstraction for the 5th quintile. In the data, the 5th quintile derives 40% of income from capital and 53% from labor; in the case of the 3rd and 4th quintiles, 81% comes from labor (SCF, 1998). As a result, if the only source of income of capital owners in the model was capital, and given the observed average capital share of output ς, they would account for too small fraction of aggregate income (28.3% in the model vs 48% in the data), while homeowners' share would be too large (71.7% vs 34%). As a result, the steady-state debt-servicing costs implied by the observed

θ and H/Y ratio (and steady-state amortization and interest rates) would be too low. The parameter τ adjusts for this discrepancy by transferring, in a lump-sum way, some of the labor income from homeowners to capital owners.[22]

5.1 Functional forms

The capital owner's per-period utility function is $u(c^*) = \log c^*$; the homeowner's utility function is $v(\overline{c}, n) = \omega \log \overline{c} + (1-\omega)\log(1-n)$, where $\overline{c}$ is the composite consumption good $\overline{c}(c, h) = c^{\xi} h^{1-\xi}$. The additive separability of the homeowner's utility function facilitates a transparent interpretation of the results as marginal utilities are independent of the consumption of other goods. Further, the goods production function is $f(K, N) = K^{\varsigma} N^{1-\varsigma}$ and the housing production function is $g(X_S, X_L) = X_S^{1-\varphi} X_L^{\varphi}$. As in Kydland et al. (2012), $q(X_{St}) = \exp(\zeta(X_{St} - X_S))$, where $\zeta > 0$ and X_S is the steady-state structures to output ratio (Y is normalized to be equal to one in steady state). A similar functional form is used also for the bond market participation cost: $\Upsilon(-\widetilde{B}) = \exp(-\vartheta \widetilde{B}_t) - 1$, where $\vartheta > 0$ and $\widetilde{B}_t = 0$ in steady state. It is straightforward to check that this function satisfies the properties set out in Section 4.1.2.

5.2 Parameter values

The model's parameters are summarized as follows: Ψ (population); δ_K, δ_H, ς, A, ρ_A, σ_A, ζ, φ (technology); τ_K, τ_N, G, τ (fiscal); θ, α, κ (mortgages); ϑ (bond market); $\overline{\pi}$, ν_π, ν_y, ρ_π, σ_π (monetary policy); and β, ω, ξ (preferences). The parameter values are listed in Table 1 and are discussed in detail in what follows. Most parameters can be assigned values without solving a system of steady-state equations. Four parameters (ω, ξ, τ_K, τ) have to be obtained jointly. A third set of parameters (ζ, ρ_π, σ_π) is assigned values by matching second moments of the data. Table 2 lists the steady-state values of the model's endogenous variables implied by the calibration and, where possible, the values of their data counterparts. As can be seen,

[22] The lump sum transfer can be interpreted as labor income of capital owners obtained by inelastic labor supply and a constant wage rate.

despite the highly stylized nature of the model, the steady state is broadly consistent with a number of moments not targeted in calibration.

In order to be consistent with the notion of homeowners and capital owners in the data, Ψ is set equal to 2/3. The parameter ς corresponds to the share of capital income in output and is set equal to 0.283, an estimate obtained by Gomme and Rupert (2007) from National Income and Product Accounts (NIPA) for aggregate output close to our measure of output (see Appendix C). The share of residential land in new housing φ is set equal to 0.1, an estimate reported by Davis and Heathcote (2005). The depreciation rates δ_K and δ_H are set equal to 0.02225 and 0.01021, respectively, to be consistent with the average flow-stock ratios for capital and housing investment, respectively. The level of TFP, A, is set equal to 1.5321, so that steady-state output is equal to one. The stochastic process for TFP has $\rho_A = 0.9641$ and $\sigma_A = 0.0082$, estimates obtained by Gomme and Rupert (2007) for the Solow residual of a production function with the same ς and measurements of capital and labor inputs used here (see Appendix C). The labor income tax rate is derived from NIPA using a procedure of Mendoza, Razin, and Tesar (1994), yielding $\tau_N = 23.5\%$. As noted above, $G = 0.138$. The mortgage parameter θ is set equal to 0.76, the average (1973-2006) of the cross-sectional mean of the loan-to-value ratio for single family newly-built home mortgages (Federal Housing Finance Agency, Monthly Interest Rate Survey, Table 10). Using the average (1972-2006) 30-year FRM interest rate of 9.31% per annum, Kydland et al. (2012) show that $\kappa = 0.00162$ and $\alpha = 0.9946$ provide a close approximation to the installments of a conventional 30-year mortgage. In a baseline case, the weight on inflation in the monetary policy rule, ν_π, is set equal to 1.35, which falls in the middle of the range of estimates reported by Woodford (2003), Chapter 1. This parameter will be treated as a free parameter in monetary policy experiments. The weight on output, ν_y, is set equal to zero.[23] The steady-state inflation rate, $\overline{\pi}$, is set equal to 0.0113, the average (1972-2006) quarterly inflation rate. In steady state, the first-order condition for l_t^* constrains i^F to equal to i. Given the values of i and $\overline{\pi}$,

[23]Experimentation with alternative values of ν_y did not significantly change the dynamic properties of the model. This is because output in the model responds to shocks in the typical mean-reverting fashion, producing only small growth rates after the impact period.

the first-order condition for b_t^* implies $\beta = 0.9883$. For the participation cost function $\Upsilon(.)$, the choice of ϑ is guided by available studies on prices of unsecured credit. Namely, setting ϑ equal to 0.035 gives approximately the same premium at $\widetilde{B} = -0.5$ as that predicted by the unsecured credit pricing function of Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), Figure 6, white-collar workers.[24]

Given the above parameter values, ω, ξ, τ_K, and τ are chosen jointly to match the values of K/Y, H/Y, debt-servicing costs, and N. The relationship between the parameters and the targets is given by the steady-state versions of the first-order conditions for x_{Kt}, x_{Ht}, and n_t, and the expression for steady-state debt-servicing costs noted above (the first-order conditions are contained in Appendix A). These restrictions yield $\omega = 0.2478$, $\xi = 0.6009$, $\tau_K = 0.3362$, and $\tau = 0.5886$.[25]

Conditional on all of the above values, the parameters ζ, ρ_π, and σ_π are calibrated by simulation under FRM. The parameters of the process for $\overline{\pi}_t$ are calibrated by matching the standard deviation (2.4%) and the first-order autocorrelation (0.97) of the 30-year FRM mortgage rate (annualized rate, unfiltered data).[26] The resulting parameter values are $\rho_\pi = 0.994$ and $\sigma_\pi = 0.0015$. The parameter ζ controls the volatility of the expenditure components of output and is used to match the volatility of aggregate consumption, relative to the volatility of output. Targeting the volatility of consumption has the advantage that approximately the same parameter value is obtained regardless of whether the FRM or the ARM economy is used. The resulting value is $\zeta = 0.35$.

[24] We thank Eric Young for this suggestion on how to calibrate the cost parameter.

[25] In principle, τ_K can be measured from NIPA in the same way as τ_N. Such alternative parameterization, however, is inconsistent with the observed capital to output ratio. This is because β is already pinned down by the first-order condition for bonds. Nevertheless, τ_K implied by the model is not far from the NIPA tax rate obtained by Gomme, Ravikumar, and Rupert (2011): 33.62% in the model vs 40.39% in NIPA.

[26] The 10-year government bond yield is actually used as a proxy for the 30-year mortgage rate. The two rates co-move closely for the period for which both series are available (from 1972), but the data for the 10-year yield are longer (1958-2007), thus providing a better estimate of the stochastic properties of the inflation target shock.

6 Findings

We start by presenting results for a version of the economy in which homeowners are completely excluded from the bond market (i.e., $\vartheta = \infty$ and, in equilibrium, $b_t^* = 0$). This is done in Subsection 6.1. The main results of the paper are qualitatively unaffected by this simplification but the general equilibrium mechanism is easier to explain. The explanation is provided in Subsection 6.2. Subsection 6.3 then presents the results for the case with homeowners' access to the bond market. In light of how the simplified economy works, these results are quite straightforward.

6.1 No access of homeowners to the bond market

Figure 2 plots the general equilibrium responses of selected aggregate variables to a 1 percentage point (annualized) increase in the inflation target in period 1. Recall that the only rigidity that allows the transmission of this shock to real variables is the structure of FRM and ARM contracts. The first two left-hand side charts show that, in line with equation (20), the short-term nominal interest rate, the FRM interest rate, and the inflation rate all increase approximately by 1 percentage point in period 1 and revert back to the steady state very slowly, more or less replicating the autocorrelation of the shock, 0.994. (The inflation rate and the short rate under ARM increase by a little more than 1 percentage point due to an increase in labor supply, discussed below, which increases the marginal product of capital and thus the first term in equation (20).) Next, under both FRM and ARM, the increase in the inflation rate reduces in period 1 the real value of payments on outstanding debt. This is the standard wealth effect present also in the case of one-period loans. However, it is quite small and is dwarfed by the effects of inflation in the subsequent periods. Under FRM, the cumulative effect of persistently high inflation gradually reduces the real value of mortgage payments. In contrast, under ARM, mortgage payments increase sharply in period 2, then start to decline over time as the inflation effect starts to slowly dominate the nominal interest rate effect.[27]

As discussed in Section 3, under both FRM and ARM, an equiproportional increase in the nominal interest and inflation rates increases the real mortgage installments of a new loan at the front end of the loan's life. This, other things being equal, increases τ_H. But given the relative sizes of the outstanding and new debt, housing demand is mainly driven by the wealth effects, rather than the price effect. Under FRM, housing investment gradually increases as the real value of mortgage payments on outstanding debt declines. The increase, however, is modest, reaching a peak of only 1.6%. In contrast, in the case of ARM, housing investment drops sharply in period 2, by 6.3%, as mortgage payments on existing debt increase.[28] As for investment in capital, it increases under both FRM and ARM. Under FRM, this is due to an incentive of the capital owner to save more in order to make up for the expected future decline in income from outstanding mortgages (a part of the increased saving goes into the new mortgage borrowing by homeowners). Under ARM, this is due to an incentive to smooth the effect of the temporary windfall of higher real mortgage payments from period 2 on. Finally, under FRM, output gradually declines as homeowners reduce labor supply in response to the positive wealth effects. Under ARM, output increases as homeowners increase labor supply in response to the negative wealth effects.

Figure 3 shows general equilibrium responses to a 1% increase in TFP. Two cases are considered: loose policy ($\nu_\pi = 1.05$) and tight policy ($\nu_\pi = 2.5$).[29] The top charts show

[27]Under both FRM and ARM, the path of real mortgage payments from period 2 on reflects both, payments on debt outstanding in period 1 as well as payments on new loans taken out from period 1 (inclusive) on. The outstanding stock in period 1, however, is almost 40 times larger than the quarterly flow of new loans, dominating thus the responses of $\widetilde{m}_t$, at least in the first 30-40 periods.

[28]In period 1, housing investment under ARM drops a little due to the price effect. The price effect is small because of a change in the valuation of the installments on a new loan: as consumption in period 2 drops due to the increase in mortgage payments on outstanding debt (and is subsequently expected to return back to steady state), consumption growth from period 2 on is positive, reducing $\mu_{t,t+1}$ from period 2 on. For a similar reason, in period 1 housing investment under FRM increases: future consumption is expected to increase with the expected future decline in real mortgage payments on outstanding debt, reducing the valuation of the mortgage payments on a new loan sufficiently enough to even reduce τ_H and increase housing investment in period 1.

[29]As, to a large extent, output is driven by the shock, for space constraints, the responses of output are not included in the figure. For space constraints the figure also does not include the responses of nominal interest rates. Under both FRM and ARM, the path of the short rate approximately copies the the path of the inflation rate, being above the inflation rate due to an increase in the marginal product of capital.

that loose policy lets inflation deviate from target much more than tight policy. Under both FRM and ARM, the inflation rate increases in response to the shock, as expected from our discussion in Section 4.2.2. Under loose policy, the initial increase is about 0.6 percentage points (annualized) under both contracts but the inflation rate is more persistent under FRM than under ARM (we will come back to this in the subsection below); under tight policy, the inflation rate increases only a little under both contracts, thus effectively producing the same allocations as under an index-linked mortgage. Any significant differences in the dynamics of housing investment across the two types of loan will thus be visible only under loose policy and the real effects of the nominal rigidity under each contract can be judged against the responses under tight policy.

The key observation to make from Figure 3 is that, in contrast to the case of the inflation target shock, the responses of housing investment are very similar across both mortgage types and policies. Especially in period 1 the responses are almost identical. From period 2 on, some differences exist between FRM and ARM, as the increase in the inflation rate increases the real payments on outstanding debt under ARM, while it reduces them under FRM. In contrast to housing investment, the responses of capital investment (under loose policy) differ significantly across the loan types. The next subsection explains these results.

6.2 Explaining the general equilibrium mechanism

We use partial equilibrium analysis of the capital owner and homeowner blocks to explain the general equilibrium mechanism in the model. Figure 4, panel A, shows the responses of the capital owner block (i.e., treating X_{Ht} and N_t as exogenous and constant) to 1% increase in A_t. The first chart shows a response of X_{Kt} familiar from the neoclassical growth model. The response is a little higher after period 2 under ARM than under FRM because of a (relatively small) additional increase in income due to higher real mortgage payments from outstanding debt under ARM, occurring due to an increase in the nominal interest and inflation rates plotted in the next two charts. These two nominal variables increase due to an increase in the first term in equation (20); the baseline value $\nu_\pi = 1.35$ is used. The FRM mortgage rate i_t^F also increases, but substantially less than i_t, as i_t is expected to mean revert relatively fast; the implied autocorrelation is about 0.95. Notice for future reference that about 2/3 of the increase in i_t transmit into π_{t+1}.

Panel B of Figure 4 shows the responses of the homeowner block (i.e., treating X_{Kt}, π_t, and i_t^M as exogenous) to 1 percentage point (annualized) mean reverting increase in i_t, assuming autocorrelation of 0.95. It is further assumed that 2/3 of i_t transmit into π_{t+1} and that i_t^F is related to i_t as in panel A. The first chart in panel B shows that in response to the shock, X_{Ht} declines by more under ARM than under FRM. As the next two figures in the panel show, under ARM the price and wealth effects work in the same direction (both τ_{Ht} and real mortgage payments on outstanding debt increase), whereas under FRM they work in opposite directions (τ_{Ht} increases but real mortgage payments on outstanding debt decline over time).[30] In addition, τ_{Ht} increases by less under FRM than under ARM. The first chart in the panel complements the responses of X_{Ht} with its response to 1% increase in A_t. This response is the same under both contracts, as i_t^M and π_t are, in this case, held constant. Taken the responses of X_{Ht} to the interest rate and TFP shocks together, we would expect a positive TFP shock, triggering movements of the nominal interest and inflation rates as in panel A, to increase X_{Ht} substantially more under FRM than under ARM. This, however, does not occur in general equilibrium, as Figure 3 showed.[31]

Panel C completes the picture. It shows the responses of the capital owner block to 10% increase in X_{Ht} (10% is used so that the order of magnitude of the shock is in line with the responses of X_{Ht} in panel B). As the capital owner supplies any amount of new mortgages demanded by the homeowner, such shock crowds out X_{Kt}. As a result, K_t starts to gradually decline and r_t to gradually increase, at least until the capital owner (induced by a higher r_t) sufficiently increases his overall saving. r_t thus follows a hump-shaped path, which produces

[30] Of course, in period 1, only the price effect is present. The positive wealth effects from period 2 on under FRM show up in the response of X_{ht} in the relatively fast recovery of X_{Ht} (faster than the decline of τ_{Ht}), whereas under ARM the negative wealth effect shows up as further decline in X_{Ht} in period 2.

[31] Note that i_t increases by only 0.3 percentage points in panel A, whereas panel B shows responses to an increase by 1 percentage point (a normalization).

hump-shaped responses of i_t and π_t, plotted in the charts in panel C. Anticipating future increases in i_t, i_t^F jumps immediately. In combination with the sluggish increase in π_t, this implies higher initial real mortgage installments on a new loan under FRM than under ARM.

In sum, while the partial equilibrium effect of the nominal rigidity on housing investment in the presence of a TFP shock is stronger under ARM than under FRM, the general equilibrium effect is stronger under FRM than under ARM. In combination, these two effects result in similar responses of X_{Ht} to the TFP shock regardless of the mortgage type. The working of this mechanism is apparent in Figure 3 in the more persistent response of inflation under FRM than under ARM—reflecting the hump-shaped component—and in the hump-shaped response of capital investment under FRM.[32] As shown below, in the full model, the general equilibrium adjustment is weakened and the responses are closer to what would be expected from partial equilibrium analysis.

Why, in contrast to the TFP shock, in the case of the inflation target shock the general equilibrium responses of housing investment differ across contracts? (Refer back to Figure 2.) Roughly speaking, this is because the responses of the two agent types are mutually consistent at, more or less, a constant real interest rate. In the case of FRM, while the homeowner demands more mortgage borrowing, the capital owner wants save more. (The real interest rate declines a little due to a small increase in capital accumulation, as not all desired saving gets absorbed by new mortgage borrowing.) In the case of ARM, the homeowner reduces demand for mortgages, whereas the capital owner increases his desired saving. However, the downward effect on the real interest rate from the resulting faster capital accumulation is neutralized by higher labor supply by the homeowner, which he uses to smooth the increase in real mortgage payments on outstanding debt. (The real interest rate increases a little as the effect of higher labor supply is somewhat stronger than the effect of higher capital stock, as was noted in our discussion of Figure 2 above.)

[32]The same general equilibrium adjustment also produces the similar responses of X_{Ht} under the two alternative values of ν_π, for a given contract.

6.3 The full model

With access to the bond market, homeowners have an additional margin with which to smooth the impact of the two shocks. In particular, they use the bond to borrow when either income declines or real mortgage payments increase and to lend when either income increases or real mortgage payments decline. The left-hand side chart of Figure 5, panel A, shows the responses of housing investment to the inflation target shock for the baseline autocorrelation of the shock of 0.994. Under both FRM and ARM, the responses are now smoother and somewhat muted, relative to Figure 2, but the main result that the effect of the shock is larger under ARM than FRM still holds (a maximum decline of −3.7% under ARM vs a maximum increase of 1.4% under FRM).

The right-hand side chart in panel A shows the responses for a lower autocorrelation of the shock, 0.75. In this case, while the responses are qualitatively similar to those in the left-hand side chart, quantitatively they are much smaller, especially in the ARM case: −0.4% vs −3.7% in period 1. High persistence of the shock is thus crucial for the quantitative importance of the nominal rigidity. Historically, through the lenses of the model, the baseline autocorrelation of 0.994 is the more relevant one, as it reproduces the observed autocorrelation of the long-term nominal interest rate.[33] An implication of this property of the model for policy—especially for ARM countries—is that, in order to have sizable effect on housing investment, changes in the short-term nominal interest rate have to be persistent.

Homeowners' access to the one-period bond market also weakens the general equilibrium adjustments in response to TFP shocks described in the previous subsection. Now homeowners respond to a positive TFP shock by increasing both housing investment and bond holdings. The resulting flow of funds from homeowners to capital owners and the lower demand for mortgages, relative to the case of no access to the bond market, mean that the crowding out of capital investment, and its implication for interest rates, does not need to occur as much as before in order for equilibrium to be reached. Panel B of Figure 5 shows

[33] High persistence of long rates is historically observed across developed economies, not just in the United States.

that, in response to the 1% positive TFP shock, housing investment now increases by more under FRM than under ARM. This is consistent with Figure 4, panel B, which shows that, in the extreme case of no crowding out, the increase in interest rates in response to the TFP shock dampens the response of housing investment more under ARM than under FRM.

A final set of results is contained in Table 3, which reports standard deviations and correlations with output of the model's variables and their counterparts in U.S. data (see Appendix C for a description of the data counterparts to the variables in the model). Given that the model has only two shocks, and purposefully abstracts from a number of empirically relevant frictions, these statistics serve the purpose of only gauging the model's general plausibility, rather than as a formal test of the theory. As is customary in the business cycle literature, the statistics are for HP-filtered series, both in the model and in the U.S. economy. Even though the inflation target shock has real effects, the TFP shock is the dominant shock and the model's business cycle moments are mainly determined by this shock. As Table 3 shows, for most statistics, the model is broadly in line with the data, but some discrepancies are worth noting. First, the correlation in the model between Y_t and N_t is negative, under both FRM and ARM. This is because, with limited means to smooth consumption over time, the intertemporal elasticity of labor—responsible for the positive comovement between hours and output in real business cycle models—is relatively small here and is dominated by wealth effects. Second, because the only shock driving the comovement between the slope factor and output in the model is the TFP shock, the negative correlation of the long-short spread with output in the model is stronger than in the data. The long-short spread is also only half as volatile, relative to output, as in the data. This likely reflects the absence of time-varying term premia in our model. Third, the model underpredicts the volatility of house prices, relative to that of output, accounting for about two thirds of the observed relative standard deviation, and produces too high comovement between house prices and the business cycle. This is because q_t, determined in the model by housing demand, is much less volatile than in the data and too strongly positively correlated with output. In order to reproduce the observed volatility of house prices and their correlation with output, shocks to q_t are needed.[34]

7 Concluding remarks

A parsimonious model containing either FRM or ARM loans was constructed in order to investigate the equilibrium effects on the real economy, and on aggregate housing investment especially, of the nominal rigidity inherent in fully-amortizing mortgages. The model economy has a population of homeowners and capital owners with selected key characteristics of each group observed in the data. Due to the nominal rigidity and incomplete asset markets, monetary policy transmits through the effective price of new housing and current and expected future wealth effects of payments on outstanding mortgage debt. The key finding is that monetary policy affects housing investment more under ARM than under FRM. In addition, shocks to long-run inflation have larger effects than cyclical fluctuations in inflation and nominal interest rates, occurring due to TFP shocks. Finally, the distributional consequences of monetary policy depend on the type of the mortgage loan. A persistent increase in the inflation rate redistributes real income from lenders to borrowers under FRM, but from borrowers to lenders under ARM, at least in the initial periods after the shock. An implication of our findings for the current policy debate is that, other things being equal, low nominal interest rates are likely to have a larger effect on the housing market in ARM than FRM countries and the effect of such a policy will be larger the longer is the time horizon for which the rates are expected to stay low.

Our aim was to make a step towards a better understanding of the aggregate and redistributive consequences of monetary policy in the presence of standard mortgage loans. In order to isolate the channels under investigation, and to describe their effects in a transparent way, the model has intentionally abstracted from other nominal frictions. Shocks were also

[34] In the multisectoral model of Davis and Heathcote (2005), TFP shocks in the construction sector play such a role.

limited to only two types, traditional business cycle shocks to TFP and shocks to long-run inflation. The long-run inflation shocks work like the level factor in models of the yield curve, moving short and long rates approximately equally, whereas TFP shocks resemble a slope factor, moving the long-short spread over the business cycle.

A natural next step is to incorporate other relevant shocks, margins of adjustment, or frictions to align the model more closely with the data and investigate how the quantitatively important elements of these richer environments impact on the basic conclusion of the paper. Based on our partial equilibrium results, we conjecture that mechanisms that weaken the general equilibrium adjustments in the path of the real interest rate will increase the importance of the nominal rigidity in the transmission mechanism.

The focus of the paper was only on conditional first moments in agents' decisions. That is, we have abstracted from the role of risk. Indeed, ARMs have different risk characteristics then FRMs. Furthermore, long-term interest rates contain risk premia that vary with the state of the economy. Incorporating these elements would be another fruitful extension of the model.

A third relevant extension, conditional on successfully achieving the second one, would be to include optimal refinancing and/or the choice between FRM and ARM. As we discussed, introducing such margins is going to weaken the effects of the nominal rigidity. In that sense, our results are best interpreted as providing an upper bound. The challenge of such extensions, however, is to avoid a bang-bang solution. Doing so necessarily involves the complication of homeowners' heterogeneity. The same difficulty also applies to the introduction of the option to default. In our model homeowners are not allowed to default. When real mortgage payments on outstanding debt increase, homeowners respond by cutting consumption and investment and increasing hours worked. These adjustments are especially relevant in the case of ARM. With default, depending on its costs, homeowners may instead choose to default on mortgage debt, especially in response to very large increases in real mortgage payments.

Finally, an interesting normative question regards optimal monetary policy. Under incomplete markets, the nominal rigidity in mortgage loans generates both a distortion in the optimality condition for housing and ex-post redistribution of income between homeowners and capital owners. With our mapping between these two groups of agents in the model and in the data, the latter group have better means of smoothing consumption over time and states of the world. In addition, mortgage income makes up only a small fraction of their total income. An optimal monetary policy may therefore essentially face a trade off between eliminating the distortion in the optimality condition for housing and providing insurance to homeowners against ex-post fluctuations in real mortgage payments. As the real effects of monetary policy differ depending on whether loans are FRM or ARM, optimal monetary policy is likely to depend on the type of the loan contract. Additional complexity in the design of optimal monetary policy is likely to arise when default is allowed. All these questions and issues are, however, beyond the scope of this paper and are left for future research.

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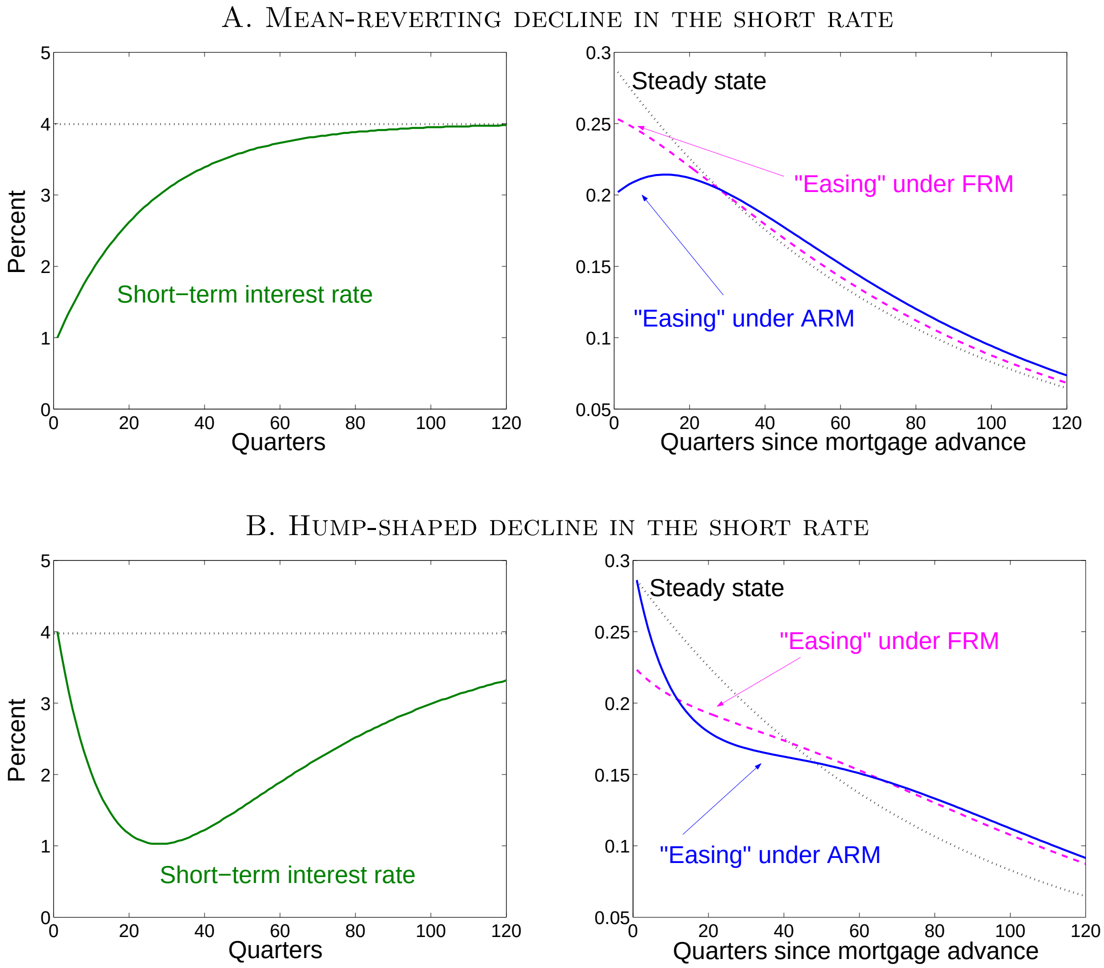


Figure 1: Monetary policy and debt-servicing costs. The left-hand side panels show alternative paths of the short-term nominal interest rate. The right-hand side panels show the corresponding mortgage payments as a fraction of post-tax income for FRM and ARM; the label 'steady-state' refers to the case when the short rate is at its steady-state level of 4%. The mortgage loan is equal to four times the household's post-tax income. In all cases, the real interest rate (1%) and real income are held constant; nominal income changes in line with inflation.

Table 1: Calibration

Symbol	Value	Description
Population		
Ψ	2/3	Share of homeowners
Technology		
A	1.5321	Steady-state level of TFP
ς	0.283	Capital share of output
δ_K	0.02225	Depreciation rate of capital
δ_H	0.01021	Depreciation rate of housing
ζ	0.35	Curvature of PPF
φ	0.1	Land share of new housing
Fiscal		
G	0.138	Government expenditures
τ_N	0.235	Labor income tax rate
τ_K	0.3362	Capital income tax rate
τ	0.5886	Labor income transfer
Preferences		
β	0.9883	Discount factor
ω	0.2478	Cons. composite's share in utility
ξ	0.6009	Share of market cons. in composite
Mortgages		
θ	0.76	Loan-to-value ratio
κ	0.00162	Initial amortization rate
α	0.9946	Amortization adjustment factor
Bond market		
ϑ	0.035	Participation cost function
Monetary policy		
ν_π	1.35	Weight on inflation
ν_y	0	Weight on output growth
$\overline{\pi}$	0.0113	Steady-state inflation rate
Exogenous processes		
ρ_A	0.9641	Persistence of TFP shocks
σ_A	0.0082	Std. of TFP innovations
ρ_π	0.994	Persistence of infl. target shocks
σ_π	0.0015	Std. of infl. target innovations

Table 2: Nonstochastic steady state and long-run averages of data

Symbol	Model	Data	Description
Normalized:			
Y	1.0	N/A	Output
Targeted in calibration:			
K	7.06	7.06	Capital stock
H	5.28	5.28	Housing stock
X_K	0.156	0.156	Capital investment
X_S	0.054	0.054	Housing structures
N	0.255	0.255	Hours worked
$\widetilde{m}/(wn-\tau)$	0.185	0.185	Debt-servicing costs (pre-tax)
i^M	0.0233	0.0233	Mortgage rate
Not targeted:			
Aggregate mortgage variables			
$\widetilde{D}$	1.61	2.35†	Mortgage debt
γ	0.0144	0.0118‡	Amortization rate
Capital owner's variables			
$(1-\tau_K)(r-\delta_K)$	0.012	0.013§	Net rate of return on capital
$[(r-\delta)k+\widetilde{m}^*]/[(r-\delta)k+\widetilde{m}^*+\tau^*]$	0.31	0.39¶,§§	Income from assets to total income
$\widetilde{m}^*/[(1-\tau_K)(r-\delta)k+\widetilde{m}^*+\tau^*]$	0.089	N/A	Mortg. payments to total (net) income
Homeowner's variables			
τ_H	0	N/A	Housing wedge
$\widetilde{m}/[(1-\tau_N)(wn-\tau)]$	0.24	N/A	Debt-servicing costs (post-tax)
$(wn-\tau)/(wn-\tau)$	1.00	0.81¶	Income from labor to total income
Distribution of wealth			
$(K+\widetilde{D})/(K+H)$	0.71	0.82¶	Capital owners
$(H-\widetilde{D})/(K+H)$	0.29	0.18¶	Homeowners

Note: Rates of return and interest and amortization rates are expressed at quarterly rates; capital owners = the 5th quintile of the SCF wealth distribution; homeowners = the 3rd and 4th quintiles of the SCF wealth distribution.

† Upper bound for the mortgage debt in the model due to the presence in the data of equity loans, second mortgages, and mortgages for purchases of existing homes.

‡ For a conventional 30-year mortgage.

§ NIPA estimate by Gomme et al. (2011).

¶ 1998 SCF; the model counterpart is defined so as to be consistent with SCF.

§§ The sum of capital and business income.

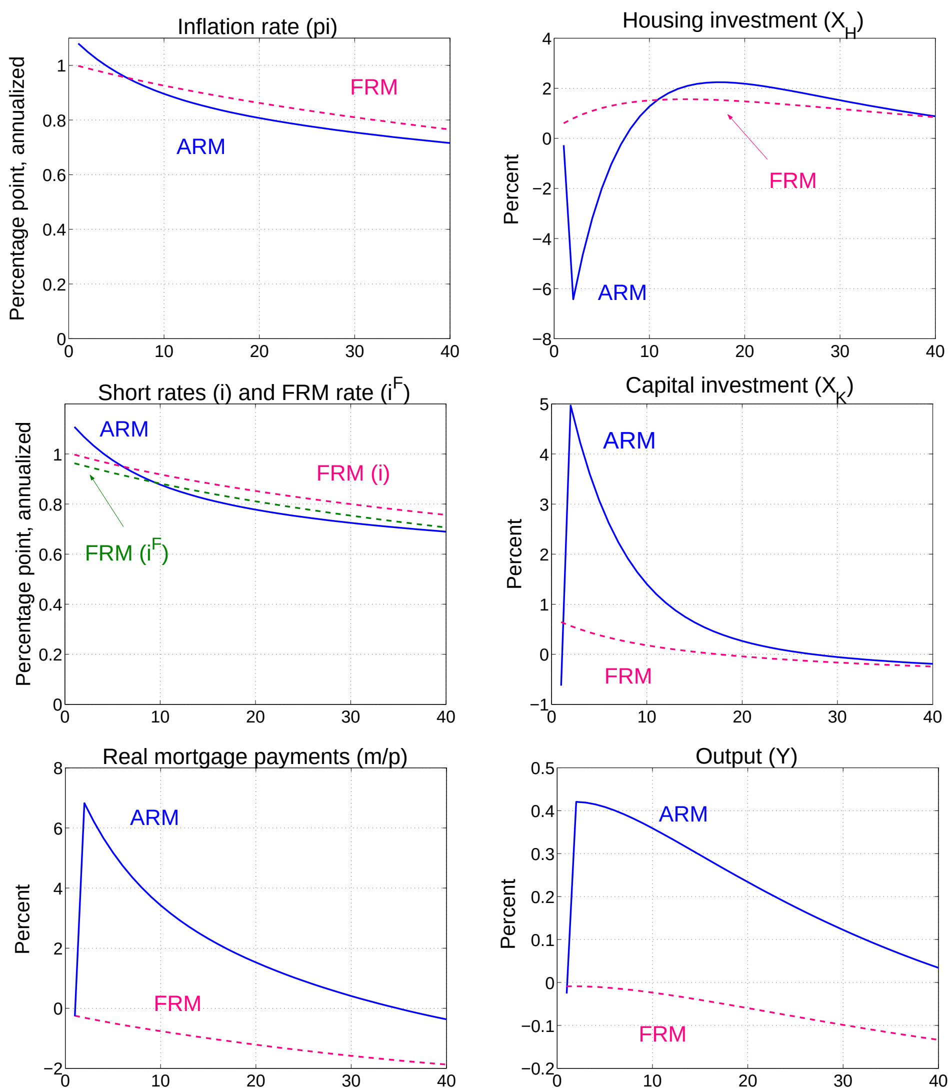


Figure 2: General equilibrium responses to 1 percentage point (annualized) increase in $\overline{\pi}_t$ in period 1; version without access of homeowners to the bond market.

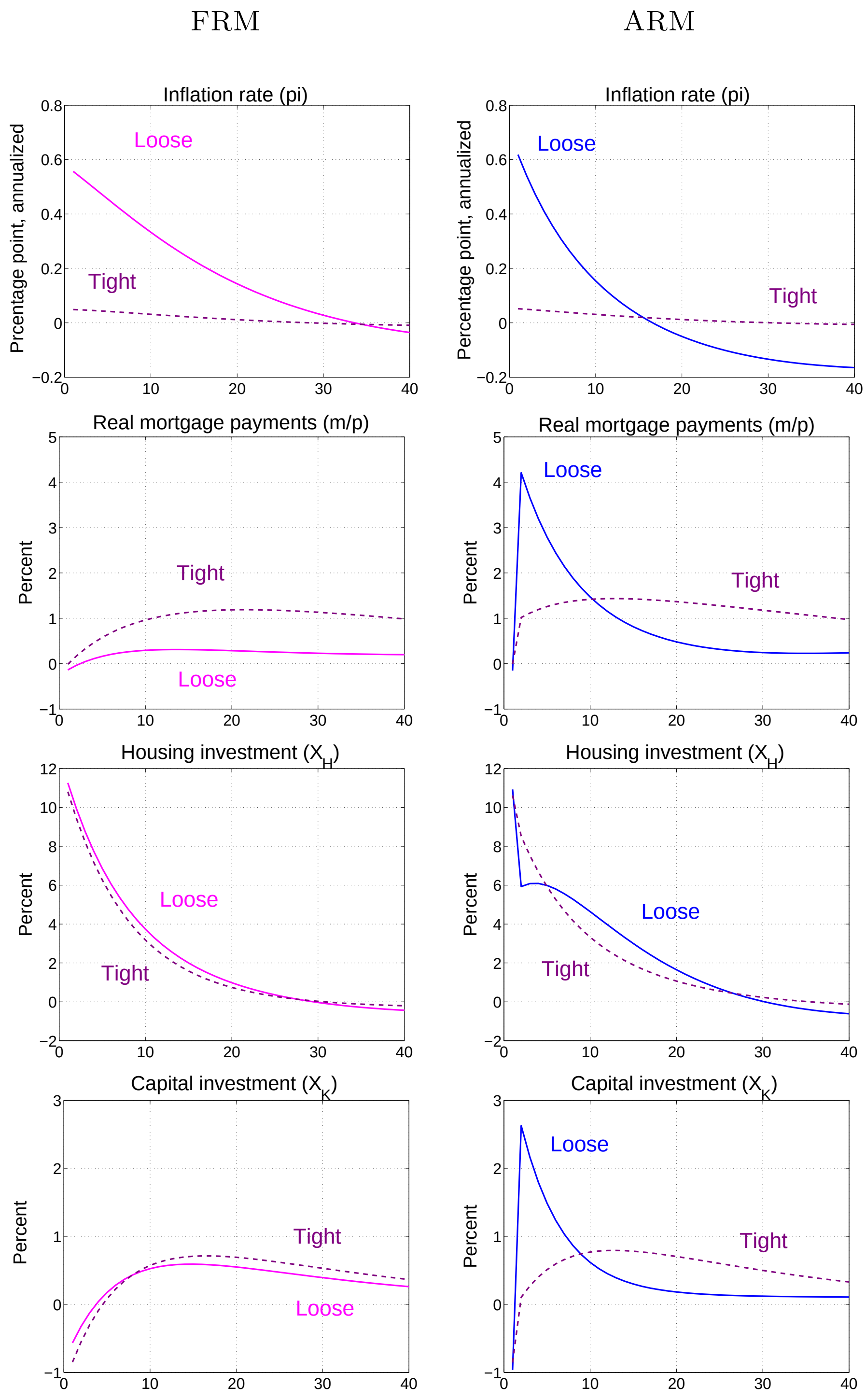


Figure 3: General equilibrium responses to 1% increase in A_t in period 1; version without access of homeowners to the bond market. Loose policy: $\nu_\pi = 1.05$; tight policy: $\nu_\pi = 2.5$.

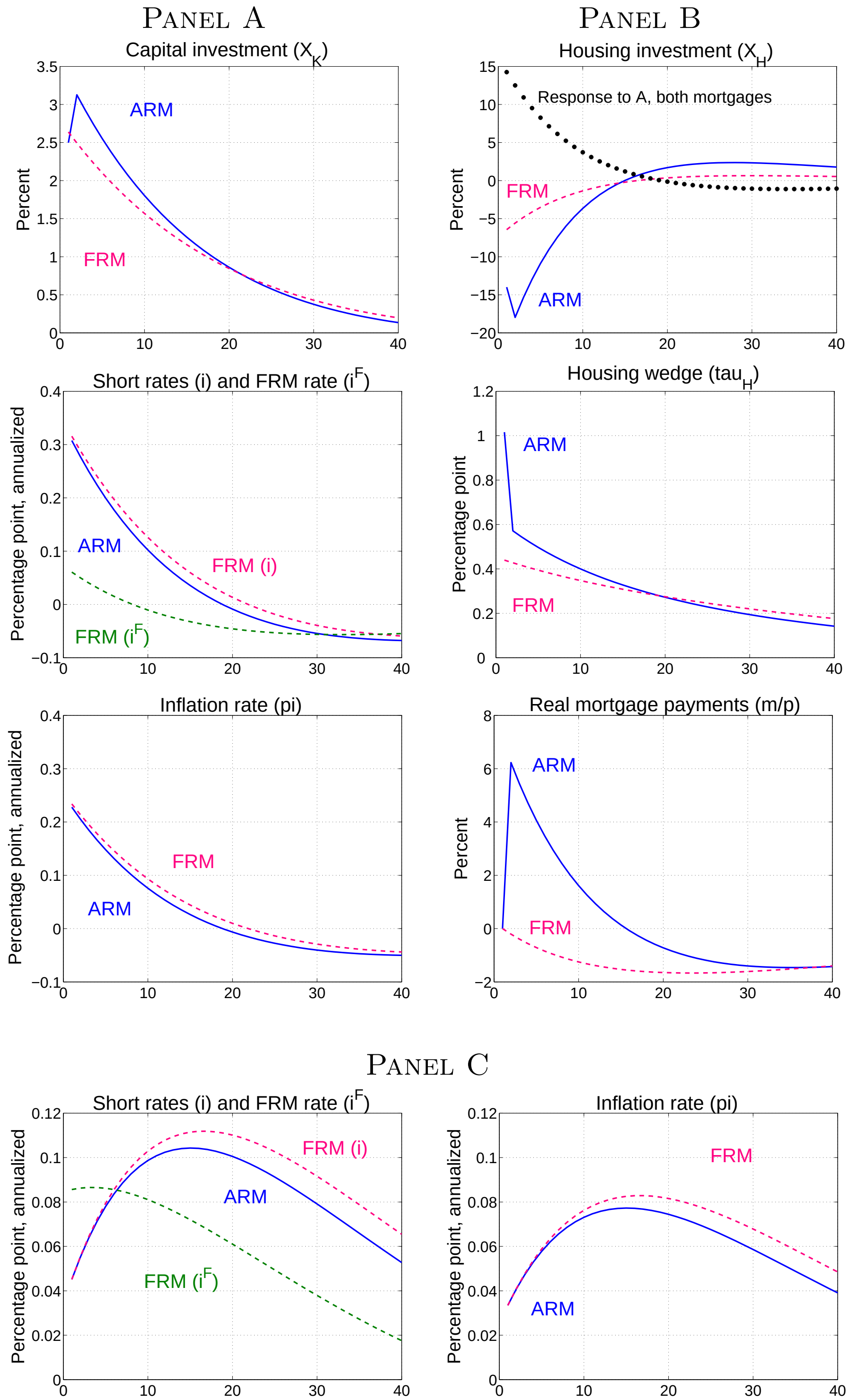
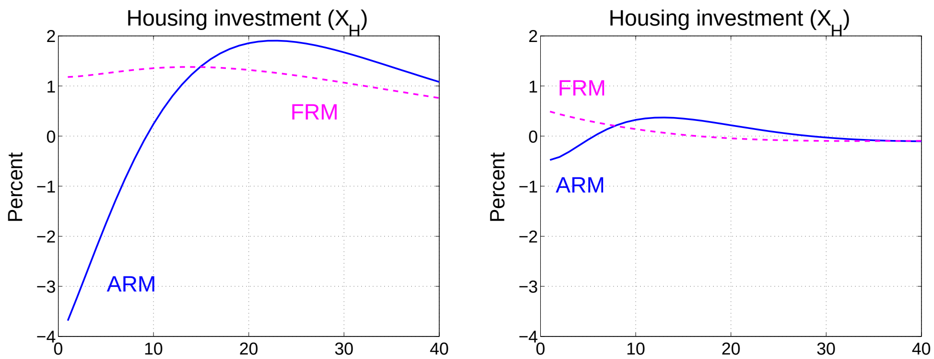


Figure 4: Explaining the general equilibrium adjustments to a positive A_t shock using partial equilibrium responses of the capital owner and homeowner blocks. Panel A: capital owner block—responses to 1% increase in A_t. Panel B: homeowner block—responses to 1 percentage point (annualized) increase in i_t (with 2/3 pass-through to π_{t+1}); in the first chart complemented with the response of X_{Ht} to 1% increase in A_t. Panel C: capital owner block—responses to 10% increase in X_{Ht}.

A. 1 perc. point (annualized) increase in $\overline{\pi}_t$

$\rho_\pi = 0.994$ $\rho_\pi = 0.75$



B. 1% increase in A_t

FRM ARM

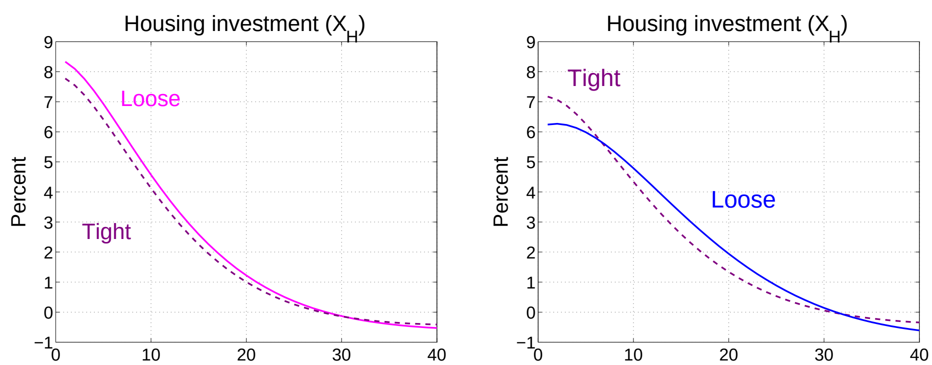


Figure 5: General equilibrium responses of housing investment in the version with homeowners' access to the bond market. Panel A: effects of varying the degree of persistence of the inflation target shock; panel B: responses to a TFP shock under loose policy ($\nu_\pi = 1.05$) and tight policy ($\nu_\pi = 2.5$).

Table 3: Business cycle properties

	US data	Model FRM	Model ARM
Std			
Y	1.92	0.94	1.04
Rel. std			
Y	1.00	1.00	1.00
C	0.42	0.42	0.35
X_S	6.94	9.48	8.20
X_K	2.45	1.76	3.01
N	0.92	0.24	0.30
π	0.58	0.85	0.81
i	0.58	0.85	0.85
i^F	0.35	0.77	N/A
$i^F - i$	0.42	0.21	N/A
q	0.58	0.18	0.15
p_H	1.57	1.13	0.97
Corr			
(C_t, Y_t)	0.79	0.88	0.94
(X_{St}, Y_t)	0.60	0.99	0.85
(X_{Kt}, Y_t)	0.73	0.92	0.83
(N_t, Y_t)	0.84	-0.67	-0.05
(π_t, Y_t)	0.14	0.23	0.41
(i_t, Y_t)	0.36	0.32	0.48
(i_t^F, Y_t)	0.01	0.09	N/A
$(i_t^F - i_t, Y_t)$	-0.49	-0.98	N/A
(q_t, Y_t)	0.41	0.99	0.85
(p_{Ht}, Y_t)	0.55	0.99	0.85

Note: All U.S. moments are for HP-filtered series, post-Korean war data. Interest and inflation rates are annualized. The 10-year government bond yield is used as a proxy for i_t^F due to its longer time availability; the inflation rate of the GDP deflator is used for π_t; the 3-month T-bill yield is used for i_t; the ratio of the residential investment deflator to the GDP deflator is used for q_t; the ratio of the average price of new homes sold (Census Bureau) and the GDP deflator is used for p_{Ht} (1975-2006). The model moments are averages of moments for 150 runs of the model; the artificial series of each run have the same length as the data series and are HP filtered.

Supplemental material—appendices

Appendix A: Equilibrium conditions

This appendix lists the conditions characterizing the equilibrium defined in Section 4.2. Throughout, the notation employed is that, for instance, u_{ct} denotes the first derivative of the function u with respect to c, evaluated in period t. Alternatively, v_{2t}, for instance, denotes the first derivative of the function v with respect to the second argument, evaluated in period t.

Capital owner's optimality

The first-order conditions with respect to, respectively, x_{Kt}, b^*_{t+1}, and l^*_t:

$$1 = E_t\left\{\beta\frac{u_{c,t+1}}{u_{ct}}\left[1+(1-\tau_K)(r_{t+1}-\delta_K)\right]\right\},$$

$$1 = E_t\left[\beta\frac{u_{c,t+1}}{u_{ct}}\left(\frac{1+i_t}{1+\pi_{t+1}}\right)\right],$$

$$1 = E_t\left\{\beta\frac{\widehat{U}_{d,t+1}}{u_{ct}}+\beta\frac{U_{\gamma,t+1}}{u_{ct}}\zeta^*_{Dt}\left[\kappa-(\gamma^*_t)^\alpha\right]+\beta\frac{U_{R,t+1}}{u_{ct}}\zeta^*_{Dt}(i^F_t-R^*_t)\right\}.$$

In the last equation, which—as discussed in the text—applies only in the FRM case, $\widehat{U}_{dt}\equiv p_{t-1}U_{dt}$ is a normalization to ensure stationarity in the presence of positive steady-state inflation and U_{dt}, $U_{\gamma t}$, and U_{Rt} are the derivatives of the capital owner's value function with respect to d^*_t, γ^*_t, and R^*_t, respectively. These derivatives are given by the Benveniste-Scheinkman (BS) conditions:

$$\widetilde{U}_{dt} = u_{ct}\frac{R^*_t+\gamma^*_t}{1+\pi_t}+\beta\frac{1-\gamma^*_t}{1+\pi_t}E_t\left\{\widetilde{U}_{d,t+1}+\zeta^*_{lt}\left[(\gamma^*_t)^\alpha-\kappa\right]U_{\gamma,t+1}+\zeta^*_{lt}(R^*_t-i^F_t)U_{R,t+1}\right\},$$

$$\begin{aligned}U_{\gamma t} = {} & u_{ct}\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right)-\beta\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right)E_t\widetilde{U}_{d,t+1}\\ & +\beta\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right)\left\{\zeta^*_{lt}[\kappa-(\gamma^*_t)^\alpha]+\frac{(1-\gamma^*_t)\alpha(\gamma^*_t)^{\alpha-1}}{\frac{1-\gamma^*_t}{1+\pi_t}\widetilde{d}^*_t+\widetilde{l}^*_t}\right\}E_tU_{\gamma,t+1}\\ & +\beta\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right)\zeta^*_{lt}(i^F_t-R^*_t)E_tU_{R,t+1},\end{aligned}$$

$$U_{Rt} = u_{ct}\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right)+\beta\left(\frac{\frac{1-\gamma^*_t}{1+\pi_t}\widetilde{d}^*_t}{\frac{1-\gamma^*_t}{1+\pi_t}\widetilde{d}^*_t+\widetilde{l}^*_t}\right)E_tU_{R,t+1}.$$

In these expressions, $\widetilde{d}_t^* \equiv d_t^*/p_{t-1}$, $\widetilde{l}_t^* \equiv l_t^*/p_t$,

$$\zeta_{lt}^* \equiv \frac{\widetilde{l}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*\right)^2} \in (0,1),$$

and

$$\zeta_{Dt}^* \equiv \frac{\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*\right)^2} \in (0,1).$$

Notice that for a once-and-for-all mortgage loan ($l_t^* = l^*$ in period t and $l_t^* = 0$ thereafter) and no outstanding mortgage debt ($d_t^* = 0$ in period t), $\zeta_{Dt}^* = 0$ and $\zeta_{l,t+j}^* = 0$, for $j = 1, 2, \ldots$. In this case, the first-order condition for l_t^* and the BS condition for $\widetilde{U}_{dt}$ simplify. Once combined, the resulting equation is just an infinite-horizon extension of the mortgage-pricing equation (1) in the two-period mortgage example of Section 3. The complications in the general case arise because the mortgage payment m_t^* entering the budget constraint of the capital owner pertains to payments on the entire outstanding mortgage debt, not just the new loan. The simplified form also arises when $R_t = i_{t-1}$ (i.e., ARM) *and* $\gamma_t^* = \kappa$ (i.e., the amortization rate is constant through out the life of the mortgage, which is the case for $\alpha = 1$). This is because in that case the interest and amortization rates of m_t^* are the same as those of the new (marginal) mortgage payment.

The capital owner's constraints:

$$c_t^* + k_{t+1} + \widetilde{b}_{t+1}^* + \widetilde{l}_t^* = \left[1 + (1-\tau_K)(r_t - \delta_K)\right] k_t + (1 + i_{t-1})\frac{\widetilde{b}_t^*}{1+\pi_t} + \widetilde{m}_t^* + \tau_t^* + \frac{p_{Lt}}{1-\Psi},$$

$$\widetilde{m}_t^* = (R_t^* + \gamma_t^*)\frac{\widetilde{d}_t^*}{1+\pi_t},$$

$$\widetilde{d}_{t+1}^* = \frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*,$$

$$\gamma_{t+1}^* = (1-\phi_t^*)\left(\gamma_t^*\right)^\alpha + \phi_t^*\kappa,$$

$$R_{t+1}^* = \begin{cases} (1-\phi_t^*)R_t^* + \phi_t^* i_t^F, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases}$$

where $\phi_t^* \equiv \widetilde{l}_t^*/\widetilde{d}_{t+1}^*$ and $\widetilde{b}_t^* \equiv b_t^*/p_{t-1}$.

Homeowner's optimality

The first-order conditions with respect to, respectively, n_t, x_{Ht}, and b_{t+1}:

$$v_{ct}(1-\tau_N)w_t = v_{2t},$$

$$v_{ct}(1-\theta)p_{Ht} = \beta E_t\left\{V_{h,t+1} + p_{Ht}\theta\left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa - \gamma_t^\alpha)V_{\gamma,t+1} + \zeta_{Dt}(i_t^M - R_t)V_{R,t+1}\right]\right\},$$

$$1 = E_t \left[\beta \frac{v_{c,t+1}}{v_{ct}} \left(\frac{1 + i_t + \Upsilon_t}{1 + \pi_{t+1}} \right) \right],$$

where $\widetilde{V}_{dt} \equiv p_{t-1} V_{dt}$ and V_{ht}, V_{dt}, $V_{\gamma t}$, and V_{Rt} are the derivatives of the homeowner's value function. Further, ζ_{Dt} is the homeowner's analog to ζ^*_{Dt} and $i^M_t = i^F_t$ in the FRM case and $i^M_t = i_t$ in the ARM case. Rearranging the second equation yields

$$v_{ct} p_{Ht} (1 + \tau_{Ht}) = \beta E_t V_{h,t+1},$$

where the wedge τ_{Ht} is given by

$$\tau_{Ht} \equiv -\theta E_t \left[1 + \beta \frac{\widetilde{V}_{d,t+1}}{v_{ct}} + \zeta_{Dt} (\kappa - \gamma^\alpha_t) \beta \frac{V_{\gamma,t+1}}{v_{ct}} + \zeta_{Dt} (i^M_t - R_t) \beta \frac{V_{R,t+1}}{v_{ct}} \right].$$

For the same reasons as in the case of the mortgage-pricing equation of the capital owner, the wedge is more complicated than in the case of the two-period mortgage. Again, it becomes a straightforward infinite-horizon extension of either equation (2) or (3) when the housing investment decision is once-and-for-all and there is no outstanding mortgage debt. The derivatives of the value function with respect to d_t, γ_t, and R_t are given by BS conditions, which take similar forms to those of the capital owner:

$$\widetilde{V}_{dt} = -v_{ct} \frac{R_t + \gamma_t}{1 + \pi_t} + \beta \frac{1 - \gamma_t}{1 + \pi_t} E_t \left[\widetilde{V}_{d,t+1} + \zeta_{lt} \left(\gamma^\alpha_t - \kappa \right) V_{\gamma,t+1} + \zeta_{lt} (R_t - i^M_t) V_{R,t+1} \right],$$

$$\begin{aligned} V_{\gamma t} = & -v_{ct} \left(\frac{\widetilde{d}_t}{1 + \pi_t} \right) - \beta \left(\frac{\widetilde{d}_t}{1 + \pi_t} \right) E_t \widetilde{V}_{d,t+1} \\ & + \beta \left(\frac{\widetilde{d}_t}{1 + \pi_t} \right) \left[\zeta_{lt} (\kappa - \gamma^\alpha_t) + \frac{(1 - \gamma_t) \alpha \gamma^{\alpha - 1}_t}{\frac{1 - \gamma_t}{1 + \pi_t} \widetilde{d}_t + \widetilde{l}_t} \right] E_t V_{\gamma,t+1} \\ & + \beta \left(\frac{\widetilde{d}_t}{1 + \pi_t} \right) \zeta_{lt} (i^M_t - R_t) E_t V_{R,t+1}, \end{aligned}$$

$$V_{Rt} = -v_{ct} \left(\frac{\widetilde{d}_t}{1 + \pi_t} \right) + \beta \left(\frac{\frac{1 - \gamma_t}{1 + \pi_t} \widetilde{d}_t}{\frac{1 - \gamma_t}{1 + \pi_t} \widetilde{d}_t + \widetilde{l}_t} \right) E_t V_{R,t+1}.$$

In addition, there is a BS condition for the derivative with respect to h_t:

$$V_{ht} = v_{ht} + \beta (1 - \delta_H) E_t V_{h,t+1}.$$

Due to the aggregate consistency conditions $(1 - \Psi) \widetilde{d}^*_t = \Psi \widetilde{d}_t$, $\gamma^*_t = \gamma_t$, and $R^*_t = R_t$, it is not necessary to include the homeowners laws of motion for the mortgage variables among the equations characterizing the equilibrium. The constraints pertaining to the homeowner are:

$$c_t + p_{Ht}x_{Ht} - \widetilde{l}_t + \widetilde{b}_{t+1} = (1-\tau_N)(w_t n_t - \tau) + (1 + i_{t-1} + \Upsilon_{t-1})\frac{\widetilde{b}_t}{1+\pi_t} - \widetilde{m}_t + \Omega_t,$$

where $\Omega_t = [\widetilde{b}_t/(1+\pi_t)]\Upsilon_{t-1}$, and

$$\widetilde{m}_t = (R_t + \gamma_t)\frac{\widetilde{d}_t}{1+\pi_t},$$

$$\widetilde{l}_t = \theta p_{Ht} x_{Ht},$$
$$x_{Ht} = h_{t+1} - (1-\delta_H)h_t.$$

Production

The producer's first-order conditions:

$$r_t = A_t f_1\left((1-\Psi)k_t, \Psi n_t\right),$$

$$w_t = A_t f_2\left((1-\Psi)k_t, \Psi n_t\right).$$

Output:

$$Y_t = A_t f\left((1-\Psi)k_t, \Psi n_t\right).$$

The relative price of structures (i.e., the curvature of the production possibilities frontier):

$$q_t = q(\Psi x_{St}).$$

Homebuilding

Using the equilibrium condition

$$X_{Lt} = 1,$$

the production function and the first-order conditions of homebuilders (for the Cobb-Douglas production function):

$$x_{St} = \frac{1}{\Psi}(\Psi x_{Ht})^{\frac{1}{1-\varphi}},$$

$$p_{Ht} = q_t\frac{(\Psi x_{St})^{\varphi}}{1-\varphi},$$

$$p_{Lt} = p_{Ht}\varphi(\Psi x_{St})^{1-\varphi}.$$

For a given x_{Ht}, the first equation determines x_{St}, the second p_{Ht}, and the third p_{Lt}. Notice that when $\varphi = 0$, $x_{Ht} = x_{St}$ and $p_{Ht} = q_t$.

Monetary policy and the government

The monetary policy rule:

$$i_t = (i - \overline{\pi} + \overline{\pi}_t) + \nu_\pi(\pi_t - \overline{\pi}_t) + \nu_y(\log Y_t - \log Y_{t-1} - y).$$

The government budget constraint:

$$G + (1-\Psi)\tau_t^* = \tau_K(r_t - \delta_K)(1-\Psi)k_t + \tau_N(w_t\Psi n_t - \tau\Psi) + \tau\Psi.$$

Market clearing

The land and structures market clearing conditions have already been imposed in the home-building sector. The remaining market clearing conditions are for the bond market:

$$(1-\Psi)\widetilde{b}_t^* + \Psi\widetilde{b}_t = 0;$$

and mortgage market:

$$(1-\Psi)\widetilde{l}_t^* = \Psi\widetilde{l}_t.$$

It is straightforward to verify that the Walras' law holds (i.e., the goods market clears and national accounts hold):

$$(1-\Psi)c_t^* + \Psi c_t + (1-\Psi)x_{Kt} + q_t\Psi x_{St} + G = Y_t = r_t(1-\Psi)k_t + w_t\Psi n_t.$$

Stochastic processes

TFP:

$$\log A_{t+1} = (1-\rho_A)\log A + \rho_A \log A_t + \epsilon_{A,t+1}, \quad \text{where} \quad \epsilon_{A,t+1} \sim iidN(0, \sigma_A).$$

Inflation target:

$$\overline{\pi}_{t+1} = (1-\rho_\pi)\overline{\pi} + \rho_\pi\overline{\pi}_t + \epsilon_{\pi,t+1}, \quad \text{where} \quad \epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi).$$

Appendix B: Computation

The recursive competitive equilibrium (RCE) is computed using a linear-quadratic (LQ) approximation method for distorted economies with exogenously heterogenous agents (see Hansen and Prescott, 1995, for details). In a nutshell, the Bellman equation of each agent type (equations (18) and (19)) is LQ approximated. Following the split-up of the economy in Section 4.2 into the capital owner and homeowner blocks, the maximization problem of each block is solved in isolation, given a guess for the decision rules of the other block. The RCE of the entire economy is a fixed point in which the guesses coincide with the outcomes of each respective block's problem. The centering point of the approximation is the nonstochastic steady state and the approximation of the Bellman equations is computed using numerical derivatives; all variables in the approximation are either in percentage deviations or percentage point deviations (for rates) from the steady state. Before computing the equilibrium, the model is made stationary by expressing all nominal variables in real terms and replacing ratios of price levels with the inflation rate, as is done in Appendix A.

Because the laws of motion for the mortgage variables are nonlinear, and cannot be substituted out into the per-period utility function as required by the standard LQ approximation method, the method is modified along the lines of Benigno and Woodford (2006). This involves forming a Lagrangian, consisting of the per-period utility function and the laws of motion for the mortgage variables. The Lagrangian is then used as the return function in the Bellman equation being approximated. This adjustment is necessary to ensure that second-order cross-derivatives of the utility function and the constraints are taken into account in the LQ approximation. This modification, as applied to the homeowner, is described in detail by Kydland et al. (2012). The specification for the capital owner is analogous. We therefore refer the reader to that paper.

An alternative procedure—implemented, for instance, by Dynare—would be to log-linearize the model's equilibrium conditions in Appendix A and use a version of the Blanchard-Kahn method to arrive at the equilibrium decision rules and pricing functions. As is well known, the two procedures yield the same linear equilibrium decision rules and pricing functions, approximations to the set of functions $W(z, S)$. We have a slight preference for the LQ method as, in the future, it can be easily adopted to a specification of the model with recursive preferences, which price in long-term risk, and are thus a natural choice for studying the implications of the risk characteristics of long-term mortgage loans.

In computing the partial equilibrium results, we treat X_{Kt}, i_t^M, and π_t in the homeowner case, and X_{Ht}, B_{t+1}, and N_t in the capital owner case, in the same way as the exogenous state variables in the vector z_t. Specifically, the variables are assumed to follow a diagonal VAR(1) process, with the parameter values specified in the text, and are included in the vector z_t of exogenous state variables in the respective Bellman equations. The Bellman equation of each block is then LQ approximated. The homeowner block gives aggregate decision rules for X_{Ht}, B_{t+1}, and N_t, while the capital owner block gives aggregate decision rules and pricing functions for X_{Kt}, i_t^M, and π_t. These are linear functions of the variables in each block's (modified) vector z and in each block's vector of endogenous state variables: $[K_t, D_t^*, \Gamma_t^*, \Re_t^*]$ in the capital owner's case and $[H_t, B_t, D_t, \Gamma_t, \Re_t]$ in the homeowner's case.

Appendix C: Data counterparts to variables

This appendix explains the construction of the data used to calculate the aggregate ratios employed in calibrating the model. Adjustments to official data are made to ensure that the data correspond conceptually more closely to the variables in the model. To start, for reasons discussed by Gomme and Rupert (2007), the following expenditure categories are taken out of GDP: gross housing value added, compensation of general government employees, and net exports. In addition, we also exclude expenditures on consumer durable goods, as our 'home capital' includes only housing, and multifamily structures, which since the mid-1980s rely much less on mortgage finance than single-family structures (Kydland et al., 2012). With these adjustments, the data counterparts to the expenditure components of output in the model are constructed from BEA's NIPA tables as follows: consumption (C) = the sum of expenditures on nondurable goods and services less gross housing value added; capital investment (X_K) = the sum of nonresidential structures, equipment & software, and the change in private inventories; housing structures (X_S) = residential gross fixed private investment less multifamily structures; and government expenditures (G) = the sum of government consumption expenditures and gross investment less compensation of general government employees. Our measure of output ($Y = C + X_K + X_S + G$) accounts, on average (1958-2006), for 74% of GDP.

BEA's Fixed Assets Tables and Census Bureau's M3 data provide stock counterparts to capital and housing investment: capital stock (K) = the sum of private nonresidential fixed assets and business inventories; housing stock (H) = residential assets less 5+ unit properties.[35] Federal Reserve's Flow of Funds Accounts provide data on mortgages and we equalize mortgage debt in the model (D) with the stock of home mortgages for 1-4 family properties. The Flow of Funds data, however, include mortgage debt issued for purchases of existing homes, second mortgages, and home equity loans. In contrast, the model speaks only to mortgage debt on new housing. The data thus provide an upper bound for D in the model.

Appendix D: Estimation of mortgage debt servicing costs

As discussed in the main text, a key measurement for calibrating the model concerns the mortgage debt servicing costs of homeowners. Unfortunately, such information for the United States is not readily available. Four different procedures are therefore used to arrive at its estimate. To a smaller or larger extent, the four procedures exploit the notion that the homeowners in the model correspond to the 3rd and 4th quintiles of the U.S. wealth distribution. Some of these estimates arguably overestimate the debt servicing costs, while other underestimate it. Nevertheless, all four procedures yield estimates in the ballpark of 18.5% of pre-tax income, the value used to calibrate the model. This ballpark is similar to the estimates for the United Kingdom reported in the literature, noted in the Introduction.

The first procedure, for FRM (1972-2006) and ARM (1984-2006), combines data on income from the Survey of Consumer Finances (SCF) and the model's expression for debt servicing costs. Suppose that all mortgage debt is FRM. The model's expression for steady-state debt-servicing costs, $(R+\gamma)[D/(pwN-p\tau\Psi)]$, can then be used to compute the average debt-servicing costs of homeowners. The various elements of this expression are mapped into data in the following way: $D/(pwN-p\tau\Psi)$ corresponds to the average ratio of mortgage debt (for 1-4 unit structures) to the combined personal income (annual, pre-tax) of the 3rd and 4th quintiles, which is equal to 1.56; R corresponds to the average FRM annual interest rate for a conventional 30-year mortgage, equal to 9.31%; and γ corresponds to the average amortization rate over the life of the mortgage, equal to 4.7% per annum. This yields debt-servicing costs of 22%. This estimate is likely an upper bound as some of the outstanding mortgage debt in the data is owed by the 5th quintile (the 1st and 2nd quintiles are essentially renters) and the effective interest rate on the stock in the data is likely lower than the average FRM rate due to refinancing. When all mortgage debt is assumed to be ARM, this procedure yields 17.5% (based on the average Treasury-indexed 1-year ARM rate for a conventional 30-year mortgage).

The second estimate is based on Federal Reserve's Financial Obligation Ratios (FOR) for mortgages (1980-2006). FOR report all payments on mortgage debt (mortgage payments, homeowner's insurance, and property taxes) as a fraction of NIPA's share of disposable income attributed to homeowners. For our purposes, the problem with these data is that members of the 5th quintile of the wealth distribution are also counted as homeowners in the data (as long as they own a home), even though they do not represent the typical homeowner

[35] Separate stock data on 2-4 unit properties are not available, but based on completions data from the Census Bureau's Construction Survey, 2-4 unit properties make up only a tiny fraction of the multifamily housing stock.

in the sense of Campbell and Cocco (2003). To correct for this, we apply the share of the aggregate SCF personal income attributed to the 3rd, 4th, and 5th quintiles of the wealth distribution to disposable income from NIPA. This gives us an estimate of NIPA disposable income attributed to these three quintiles. This aggregate is then multiplied by the financial obligation ratio to arrive at a time series for total mortgage payments. Assuming again that all mortgage payments are made by the 3rd and 4th quintiles, the total mortgage payments are divided by NIPA personal (pre-tax) income attributed to just these two quintiles (using the SCF shares). This procedure yields average debt-servicing costs of 20%.

Third, we use the ratio of all debt payments to pre-tax family income for the 50-74.9 percentile of the wealth distribution, reported in SCF for 1989-2007. The average ratio is 19%. About 80% of the payments are classified as residential by the purpose of debt, yielding an average ratio of 15.2%. A key limitation of this procedure is that the data exclude the 1970s and most of the 1980s—periods that experienced almost twice as high mortgage interest rates, on average, than the period covered by the survey. Another issue is that the information reported in the survey is not exactly for the 3rd and 4th quintiles.

The fourth procedure is based on the Consumer Expenditure Survey (CEX), 1984-2006. This survey reports the average income and mortgage payments (interest and amortization) of homeowners with a mortgage. To the extent that homeowners without a mortgage are likely to belong to the 5th quintile of the wealth distribution—they have 100% of equity in their home and thus have higher net worth than homeowners with a mortgage—the survey's homeowners with a mortgage should closely correspond to the notion of homeowners used in this paper (CEX does not contain data on wealth). The resulting average, for the available data period, for mortgage debt servicing costs of this group (pre-tax income) is 15%. Given that the data do not cover the period of high mortgage rates of the late 1970s and early 1980s, like the third estimate, this estimate probably also underestimates the debt servicing costs for the period used in calibrating the model.